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AUTOMATED DESIGN OF CONTINUOUS BRIDGES WITH PRECAST PRESTRESSED CONCRETE BEAMS VOLUME I: DESIGN CONSIDERATIONS

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Texas Transportation Institute
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A computer program has been developed in this study to perform the calculations for the design of continuous prestressed concrete bridge girders. The continuous girder is constructed from simple span precast concrete I-shaped beams made continuous by supplementary reinforcing in the deck and the ends of the precast beams. Specification for the designs produced are those currently accepted by the Texas Highway Department.

This volume of the report describes the analysis techniques used in the determination of design moments and shears and the design criteria and methods of computation used in completing a design.

SUMMARY

This report describes the analysis techniques and design calculations used in a computer program developed for the design of continuous prestressed concrete bridge girders. The continuous girder is constructed from simple span precast prestressed concrete I-shaped beams made continuous by supplemental reinforcing in the deck and the ends of the beam. The program is limited to continuous girders in which precast beams in all spans are of identical shape. The program considers live loads produced by standard AASHTO trucks and lane loadings, by an "axle train" of up to 15 wheels of arbitrary weight and spacing and by a uniform distributed load on the continuous beam. Dead load due to beam weight, diaphrams and slab weight are also included. Provisions are made to treat cases where a portion of the deck over interior supports is cast first to establish continuity and the remaining deck weight is carried by the continuous beam.

The program computes for each span of the girder the number of prestressing strands and their placement, the area of conventional reinforcing required in the deck to resist negative moment, the area of reinforcing required at interior supports to resist positive moment and the spacing of stirrups.

RECOMMENDATION FOR IMPLEMENTATION

A computer program was developed in this study to carry out the necessary calculations for the design of continuous bridge girders constructed from simple span precast prestressed concrete I-shaped beams.

This program is being used by the Texas Highway Department, and its continued use is recommended.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

This report provides a general description of the design considerations incorporated in a computer program for the automated design of continuous bridges, constructed with precast, prestressed concrete beams. This type of bridge differs from conventional simple span prestressed concrete beam and cast-in-place deck construction in that the deck slab is continuous over interior supports and is reinforced to withstand negative moments arising from continuity. The construction sequence for such a structure consists of placement of the beams on bent caps and pouring of a portion of the deck over each interior support to establish continuity. After the initial continuity pours have gained sufficient strength, the remaining segments of deck are cast. A common variation on this construction sequence is the casting of the entire deck at one time, without the initial continuity pours. The first construction sequence subjects the simple span beams to the dead load force of the initial deck segment placed for continuity, while the remainder of the dead load of the slab and all subsequent live loads are carried by the continuous beam. This mode of construction is said to be partially continuous for dead load. In the latter construction sequence, all dead load produced by the deck is carried by the simple beams, while subsequent live loads are carried by the continuous beam. This construction sequence is said to be continuous for live load only.

With either method, some economy in design is obtained through the reduction of the maximum positive moments that a beam must sustain. The design of this type of bridge beam requires certain additional considerations which do not arise in the design of simple span prestressed concrete highway beams. The computation of live load moments and shears involves the analysis of a statically indeterminate structure. In addition, if the bridge is partially continuous for dead load, the forces produced by the casting of the remainder of the deck must be obtained from an indeterminate structural analysis. Concrete bridge structures are subject to time dependent deformations produced by creep of the deck and beam concrete and by differential shrinkage between the deck and beams. In continuous structures, these deformations cannot occur freely due to the restraint of continuity. As a result, additional moments and shears are produced in the beam which must be considered during design. Negative moments are produced in the continuous beam by live loads and by a portion of the dead load of the deck in the case of beams partially continuous for dead load. The negative moments are resisted by conventional reinforcing bars, whose required area must be determined at those sections which resist negative moment.

This mode of construction depends on adequate continuity connections at interior supports. References 4 and 8 devote considerable attention to the details of such a connection and the reader is referred to these studies for additional information.

The remainder of this report describes the design calculations and analysis methodology incorporated in the computer program. Sections II and III are devoted to the development of analysis techniques to determine the forces which the bridge beam must sustain, while Section IV describes the design criteria on which the program is based. The details of program operation, and specific instructions for its use are contained in Vol. II of this report.

II. LIVE LOAD AND DEAD LOAD SHEAR AND MOMENT COMPUTATIONS

The most expedient means of determining the maximum and minimum moments and largest (in absolute value) shear force produced at a point in a continuous beam by moving or variable loads is through the use of influence lines. The method found to be the most efficient computationally for the requirements of this work is construction of influence lines for the reactions of the beam. The influence lines for both shear and moment at a specified point can then be constructed directly through the application of statics and these in turn are utilized for the calculation of live load and dead load design moments and shears.

2.1 Influence Lines for Continuous Beams

Consider the continuous prismatic beam of N spans with (N+1) reaction forces, shown in Fig. 1. The computation of reaction forces $R_1, \ldots, R_{N+1}$ produced by the unit load at position Z can be accomplished through the application of Castigliano's Second Theorem. That is,

$$\frac{\partial U}{\partial R_i} = 0 \; ; \quad i=1, \ldots, (N-1) \tag{1}$$

In Equation (1), R_i is the ith support reaction, and the expression applies to only (N-1) of the reactions which are chosen as redundant. Let

$$l_i = \sum_{j=1}^{i} L_i \; ; \quad l_0=0 \tag{2}$$

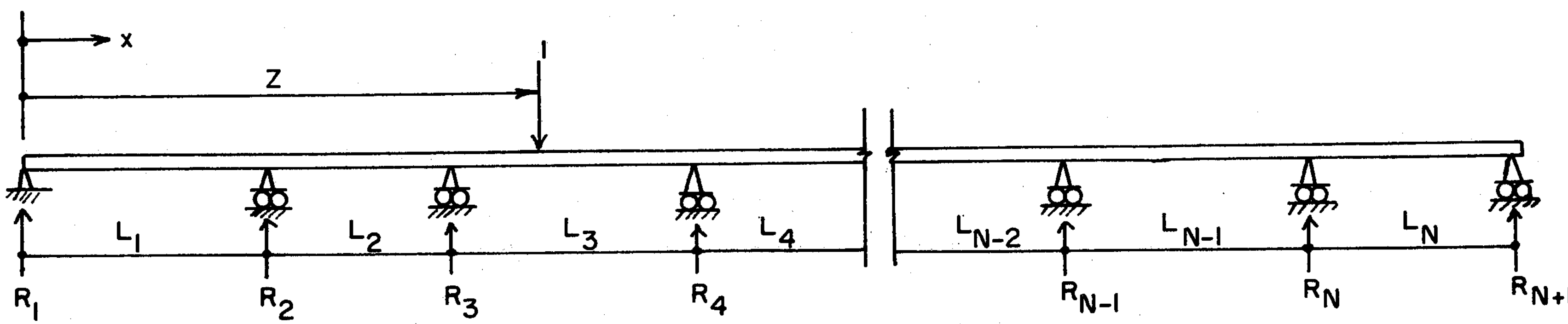


FIGURE 1. CONTINUOUS BEAM WITH N SPANS

$$\langle x-a\rangle^{n} = \begin{cases} (x-a)^{n} & ; \ x \geq a \\ 0 & ; \ x<a \end{cases} \tag{3}$$

The strain energy of the beam U can be written as

$$U = \frac{1}{2EI} \sum_{i=1}^{N} \int_{l_{i-1}}^{l_i} M_i^2(x)dx \tag{4}$$

where $M_i(x)$ is the expression for bending moment in the interval $l_{i-1} \leq x \leq l_i$. In writing Equation (4), it is assumed that EI is the same for all spans. The moment expression for each interval can be written as

$$M_i(x) = \sum_{j=1}^{i} R_j(x-l_{j-1}) - \langle x-Z\rangle^{1} ; \quad l_{i-1} \leq x \leq l_i \tag{5}$$

which when substituted into Equation (4) gives

$$U = \frac{1}{2EI} \sum_{i=1}^{N} \int_{l_{i-1}}^{l_i} \left\{ \sum_{j=1}^{i} R_j(x-l_{j-1}) - \langle x-Z\rangle^{1} \right\}^2 dx$$

or

$$U = \frac{1}{2EI} \sum_{i=1}^{N-1} \int_{l_{i-1}}^{l_i} \left\{ \sum_{j=1}^{i} R_j(x-l_{j-1}) - \langle x-Z\rangle^{1} \right\}^2 dx$$

$$+ \frac{1}{2EI} \int_{l_{N-1}}^{l_N} \left\{ \sum_{j=1}^{N} R_j(x-l_{j-1}) - \langle x-Z\rangle^{1} \right\}^2 dx \tag{6}$$

Equation (6) contains reactions R_1 through R_N. Since the structure is indeterminate to the (N-1) degree, only (N-1) reaction forces may be treated as independent variables in Equation (6). Applying statics through the summation of moments about support (N+1) yields the following expression for R_N in terms of the remaining (N-1) independent reactions.

$$R_N = \frac{l_N - Z}{L_N} - \frac{1}{L_N} \sum_{j=1}^{N-1} R_j (l_N - l_{j-1}) \tag{7}$$

Hence, Equation (6) becomes

$$U = \frac{1}{2EI} \sum_{i=1}^{N-1} \int_{l_{i-1}}^{l_i} \left\{ R_j (x - l_{j-1}) - \langle x - Z \rangle^1 \right\}^2 dx$$

$$+ \frac{1}{2EI} \int_{l_{N-1}}^{l_N} \left\{ \sum_{j=1}^{N-1} R_j (x - l_{j-1}) - \langle x - Z \rangle^1 + \frac{l_N - Z}{L_N} (x - l_{N-1}) \right.$$

$$\left. - \frac{1}{L_N} \sum_{j=1}^{N-1} R_j (l_N - l_{j-1})(x - l_{N-1}) \right\}^2 dx \tag{8}$$

The application of Equation (1) to Equation (8) yields (N-1) linear equations in the (N-1) redundant reactions $R_1, \ldots, R_{N-1}$. This system of equations can be conveniently written in matrix notation as

$$[A][\bar{R}] = [\bar{b}] \tag{9}$$

where [A] is an (N-1) by (N-1) coefficient matrix with elements a_{ij}, $[\bar{R}]$ is the vector of unknown reactions $R_1, \ldots, R_{N-1}$ and $[\bar{b}]$ is a vector of constants $b_1, \ldots, b_{N-1}$. The terms in Equation (9) are

$$a_{ij} = (l_N - l_{i-1})(l_N - l_{j-1})\alpha_{NN}/L_N^2 - (l_N - l_{i-1})\alpha_{Nj}/L_N$$
$$-(l_N - l_{j-1})\alpha_{Ni}/L_N + \alpha_{ij} \tag{10}$$

where

$$\alpha_{ij} = 1/3(l_N^3 - l_{i-1}^3) - 1/2(l_{i-1} + l_{j-1})(l_N^2 - l_{i-1}^2)$$
$$+ l_{i-1} l_{j-1}(l_N - l_{k-1}) \tag{11}$$

and

$$b_i = \int_{l_{i-1}}^{l_N} \langle x-Z \rangle^1 (x - l_{i-1})dx - \frac{(l_N - l_{i-1})}{L_N} \int_{l_{N-1}}^{l_N} \langle x-Z \rangle^1 (x - l_{N-1})dx$$
$$- (l_N - Z)(\alpha_{Ni} - \frac{(l_N - l_{i-1})}{L_N}\alpha_{NN})/L_N \tag{12}$$

Equations (10) through (12) define the elements of the linear system of equations appearing in Equation (9), and for a specified position Z of the unit load, this system may be solved through standard matrix algebra operations to obtain $R_1, \ldots, R_{N-1}$. The elements a_{ij} of the coefficient matrix [A] are independent of the position Z of the unit load. Thus, one may construct a matrix whose elements define the reaction forces for a number of different unit load positions (e.g., $Z=1$ ft, $Z=2$ ft, ..., $Z=l_N$ ft) by writing

$$[A][\bar{R}] = [B] \tag{13}$$

where

$$[B] = [\bar{b}_1 \ldots \bar{b}_j \ldots \bar{b}_{l_N}] \tag{14}$$

and

$$\bar{b}_j = \begin{matrix} \cdot \\ \cdot \\ \cdot \\ b_i \\ \cdot \\ \cdot \\ \cdot \end{matrix} \tag{15}$$

The term b_i in Equation (15) is obtained from Equation (12), with Z being the jth value of Z (unit load j ft from the left support). Equation (13) can be solved for the matrix of support reactions [B] quite effeciently by a standard computer routine. If the solution to Equation (13) is stored in the matrix [B], the element in the ith row and jth column of [B] is the value of R_i when the unit load is j ft from the left support.

The values of shear and moment at a point y ft from the left end of the beam and with the unit load Z ft from the left end of the beam follow from statics to give

$$V = \sum_{i=1}^{N+1} R_i \langle y-l_{i-1} \rangle^0 - 1.0 \tag{16}$$

$$M = \sum_{i=1}^{N} R_i \langle y - l_{i-1} \rangle^1 - 1.0 \langle y - Z \rangle^1 \tag{17}$$

Repeated application of Equations (16) and (17) for Z values from zero to l_N yields the ordinates of the influence lines for shear and moment at point y. It should be noted that the R_i term in Equations (16) and (17) is taken from the ith row and zth column of [B].

2.2 Maximum and Minimum Values for Moment and Shear - Moving Loads

Once the influence lines for moment and shear at a point on the continuous beam have been obtained by the method just presented, maximum and minimum values of moment and shear at the point for moving loads can be determined. Of interest here are extremes for standard AASHTO trucks (1) and a train of arbitrarily spaced wheels (axle train). Fig. 2 shows influence lines for moment and shear for a design point located in the first span. Within each span, both the influence line for moment and for shear will contain an extreme point (either a relative maximum or relative minimum). In seeking the position of series of moving wheel loads which produces either an absolute maximum or minimum value of moment or shear, one need consider only those positions where an extreme point is contained between the first and last wheels in the series. These limiting positions are shown in Fig. 3 for the first extreme point on the moment influence line for an axle train moving from left to right across the bridge. The magnitude of the moment produced at the design point by a series of wheels is obtained by multiplying the weight of each wheel times the ordinate of the moment influence line beneath that wheel, and summing the results over all wheels in the series. A relative maximum value of moment at the

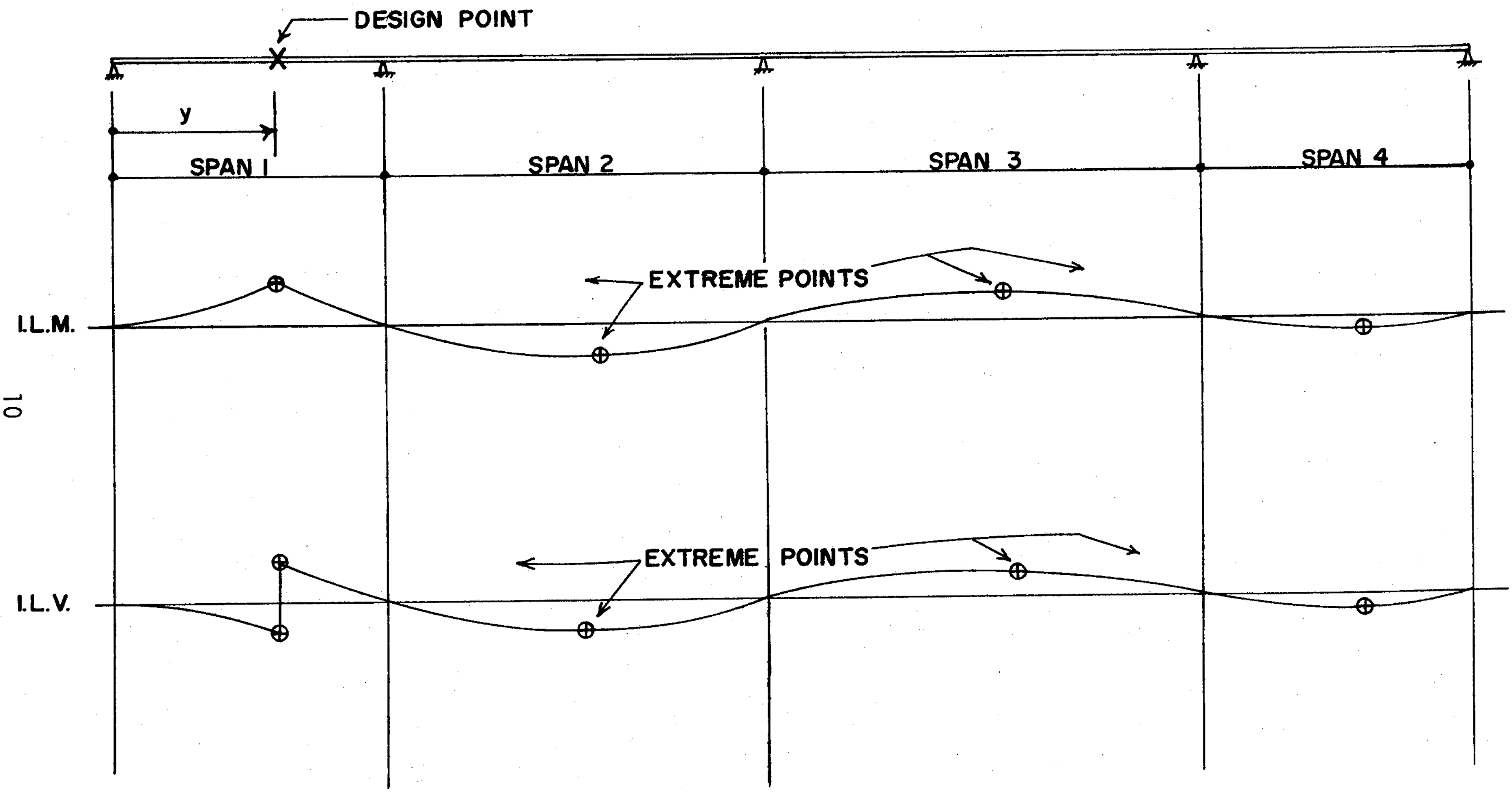


FIGURE 2. INFLUENCE LINES FOR MOMENT AND SHEAR

design point will occur for some intermediate position of the wheel series lying between the starting and terminal positions shown in Fig. 3. The position for maximum moment can be found in an efficient manner by sequentially moving the leading wheel (W_1 in Fig. 3) from its starting to terminal position and computing the moment for each move. Once the trailing wheel (W_4 in Fig. 3) has reached its terminal position, the wheel series "skips" to the starting position for the next extreme point on the influence line. This procedure is repeated until the last extreme point is traversed and then the process is repeated, with the wheel series moving from right to left, beginning at the right - most extreme point. An inspection of the relative maximum and minimum values of moment obtained for each extreme point produces the absolutely largest and smallest (most negative) values of moment at the design point. The largest and smallest values of shear force at the design point are obtained by the same procedure.

A slight variation on this method is necessary to accommodate the AASHTO HS-truck, which has a variable wheel spacing. For each position of the leading wheel W_1, the spacing of the rear wheels W_2 and W_3 must be varied between the limits of 14 and 30 ft to find the relative maximum moment for that leading wheel position. The terminal position of the HS-truck is reached when the rear wheel W_3 is over the influence line extreme point and the rear wheel spacing is 30 ft. These limiting positions are shown in Fig. 4 for an HS-truck moving from left to right.

2.3 Maximum and Minimum Values for Moment and Shear - Lane and Uniformly Distributed Loads

Influence lines for shear and moment may be integrated to compute the shear or moment at a design point produced by a uniform load. For uniformly

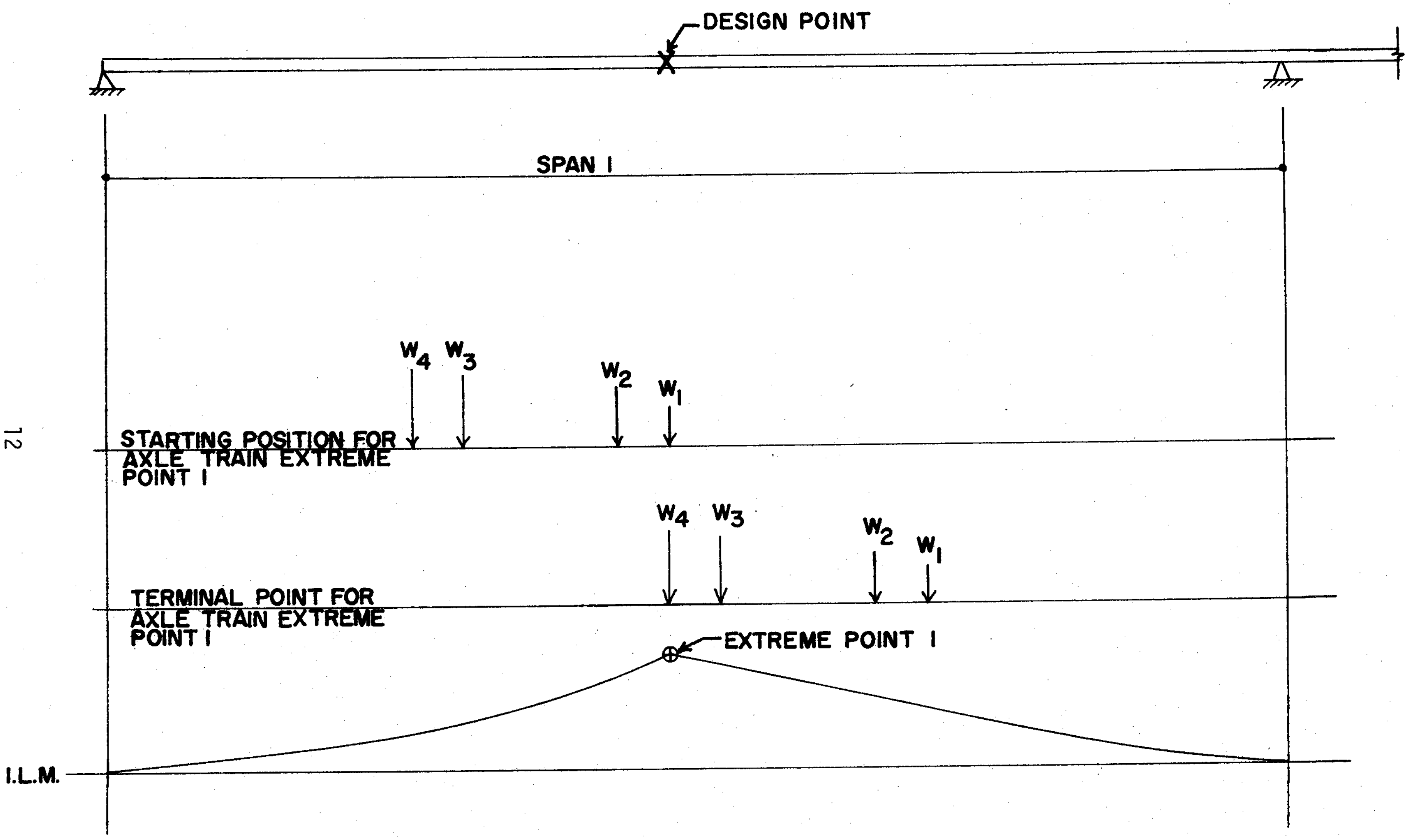


FIGURE 3. START AND TERMINAL POSITIONS FOR AXLE TRAIN AT EXTREME POINT I

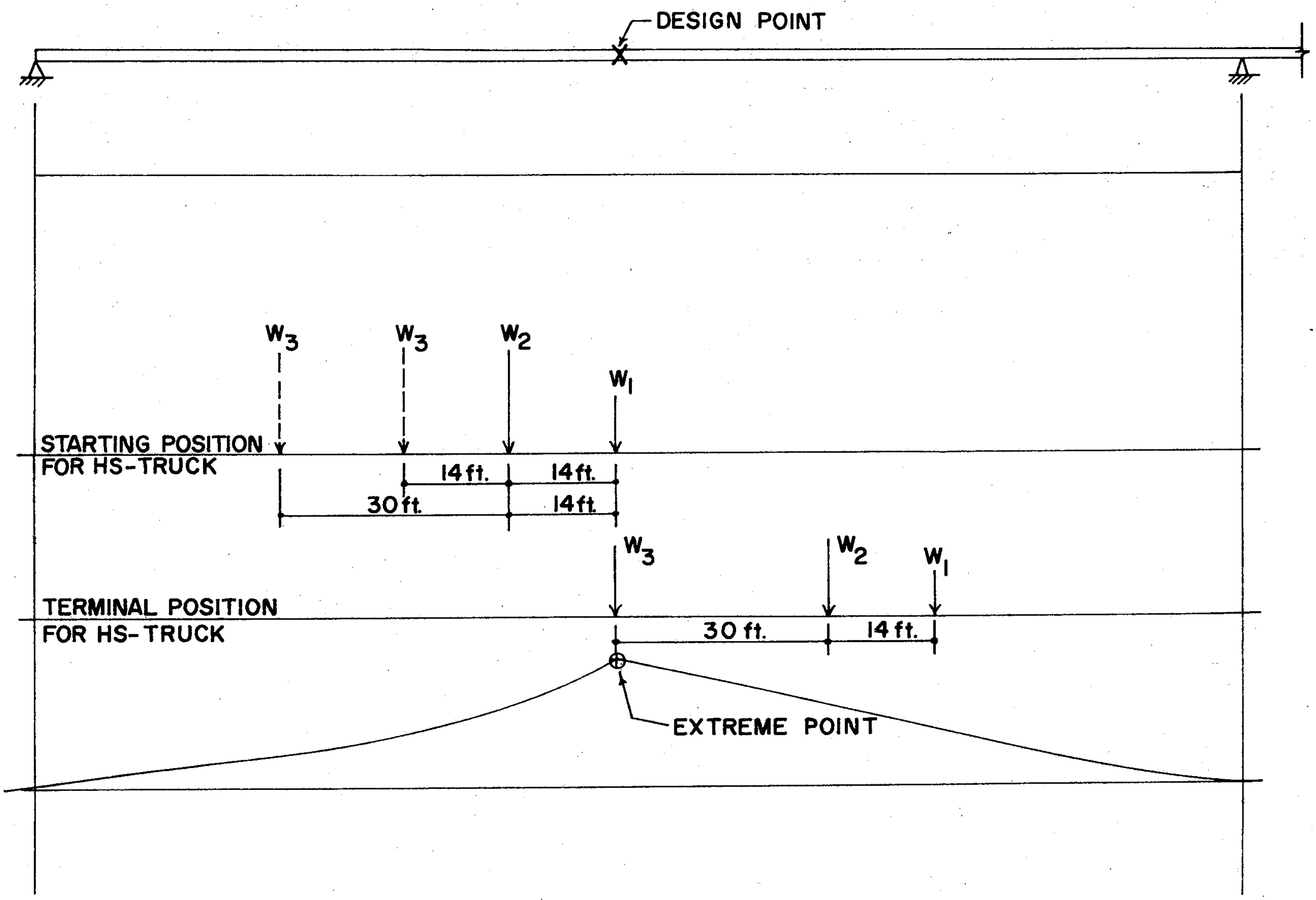


FIGURE 4. STARTING AND TERMINAL CONFIGURATIONS FOR HS-TRUCK

distributed loads extending over the entire beam, the total area under the curves in Fig. 2, scaled by the magnitude of the uniform load, will give the moment and shear at the design point. With AASHTO lane loading, partial spans may be loaded to produce maximum effects. In Fig. 2, integration should be carried over spans 1 and 3 to produce maximum at the design point. In general, the maximum and minimum values of moment and shear are obtained by summing all positive or negative areas beneath the curves, respectively. Lane loadings also include an additional concentrated force (two in the case of negative moment) positioned to produce maximum effect. The position of the concentrated force can be determined from inspection of the ordinates of the influence lines. The requisite integrations can be executed numerically since ordinates of the influence lines are known at discrete points along the span.

2.4 Moments and Shears Produced by Dead Load

Dead load moments and shears are produced by the weight of the beams themselves, diaphrams and deck slab. The moments and shears due to beam weight are computed in the usual manner. The forces resulting from diaphrams is computed from simple beam theory, based on their number and spacing input to the program.

Forces resulting from the weight of the slab depend on deck placement sequence. The program computes moments and shears due to the placement of slab segments over supports for continuity, using simple beam theory. Forces resulting from the placement of the remainder of the deck utilizes numerical integration of the influence lines as described above.

III. CREEP AND SHRINKAGE RESTRAINT FORCE COMPUTATIONS

The manner in which restraint forces are created in a continuous beam can be visualized as those forces necessary to re-establish continuity when each span of the beam is allowed to deform without restraint. Figure 5(a) and (b) shows a prestressed beam immediately after placement on its supports and at some later period after creep has occurred. With the passage of time, the simple beam will continue to deform from its initial position. Two opposing effects are at work; creep under dead weight stresses which tend to sag the beam downward, and creep under prestress forces which tend to camber the beam upward. The latter effect dominates for the situation shown in Fig. 5. The two simple beams are not free to deform independently as shown, because continuity is established at the outset. The moment that would exist at some time t, assuming that no stress redistribution was produced in the beam by creep, is that moment necessary to establish continuity of slope (the angles θ in Fig. 5) at the supports. Obviously, stress redistribution does occur as the result of creep, and the true final restraint moments at time t are obtained from scaling the moments which re-establish continuity by (2)

$$SF_{creep} = \frac{\phi}{1+\phi} \quad (18)$$

where

$$\phi = \frac{\varepsilon_c}{\varepsilon_e}(1-\alpha) \quad (19)$$

ε_c = creep strain at time t due to unit stress,

ε_e = initial strain due to unit stress, and

α = fraction of ε_c which has occurred when continuity connection is established.

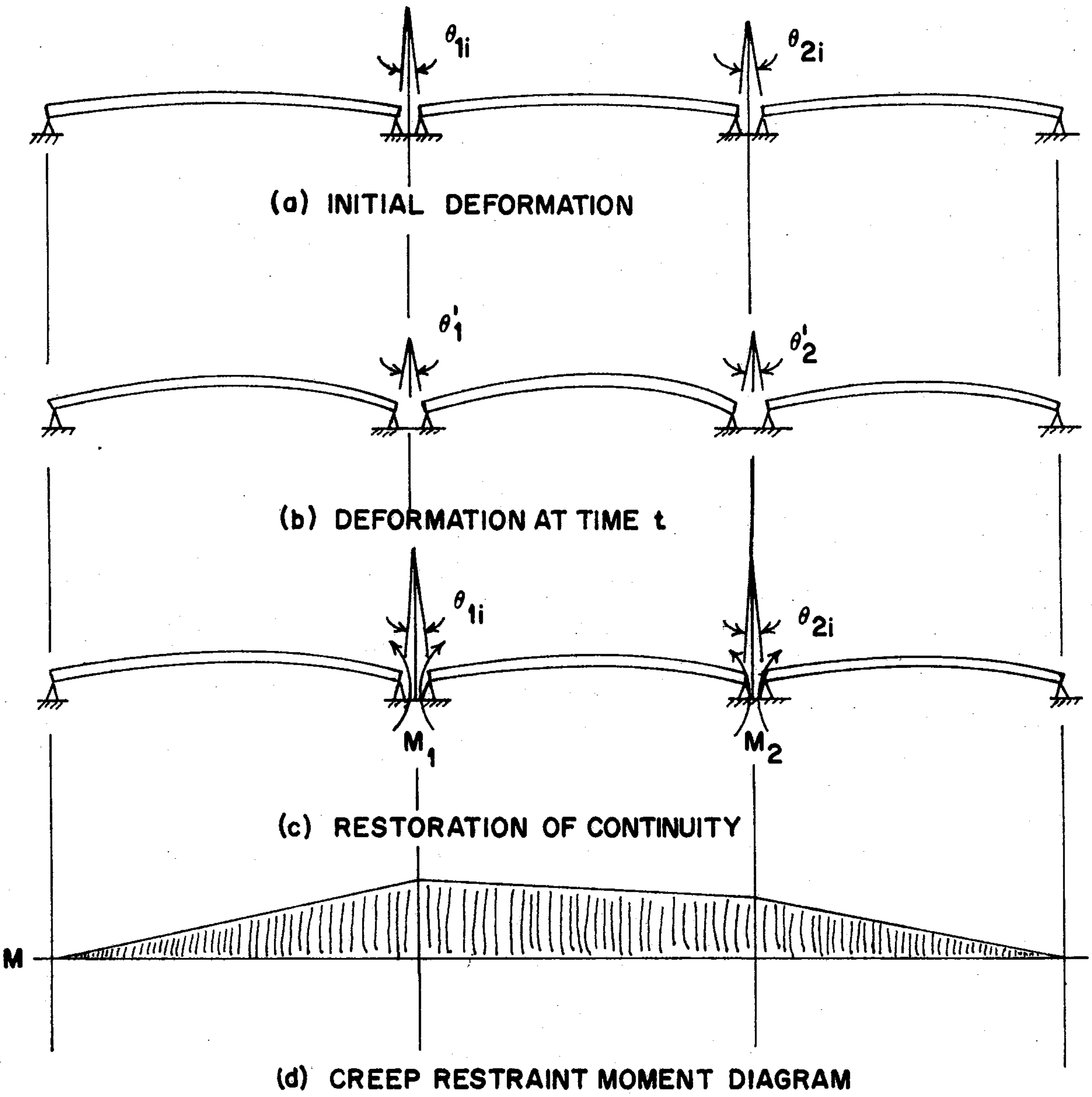


FIGURE 5. CREEP RESTRAINT MOMENT

The end result is a linearly varying moment superimposed on those described in the previous section. The creep restraint moments may be either positive or negative, depending on the amount and location of prestressing.

Figure 6 indicates the deformation produced by differential shrinkage between slab and beam concrete. When the deck is cast, a substantial portion of the shrinkage in the beams has already occurred. Thus, the shrinkage rate in the slab is more rapid, and overruns the rate of shrinkage of the beams. The result is an overall compression of the top of the beams, where they interface with the deck, which produces downward deflection. The final moments which occur in the beam as a result of its continuity are the moments necessary to re-establish continuity, scaled by (2)

$$SF_{shrinkage} = \frac{1}{1+\phi} \tag{20}$$

where ϕ is defined by Eq. (19).

3.1 Computation of Unscaled Restraint Moments

The unscaled restraint moments produced by shrinkage and dead load and prestress creep are computed by the slope-deflection method of elastic analysis. The fixed end moments for span (i,j) of the continuous beam are denoted by FEM_{ij} and FEM_{ji}. Positive fixed end moments are those which tend to rotate the support in a counterclockwise direction. Expressions for the fixed end moments from each of the three restraint moment sources are given in Figs. 7 through 9.

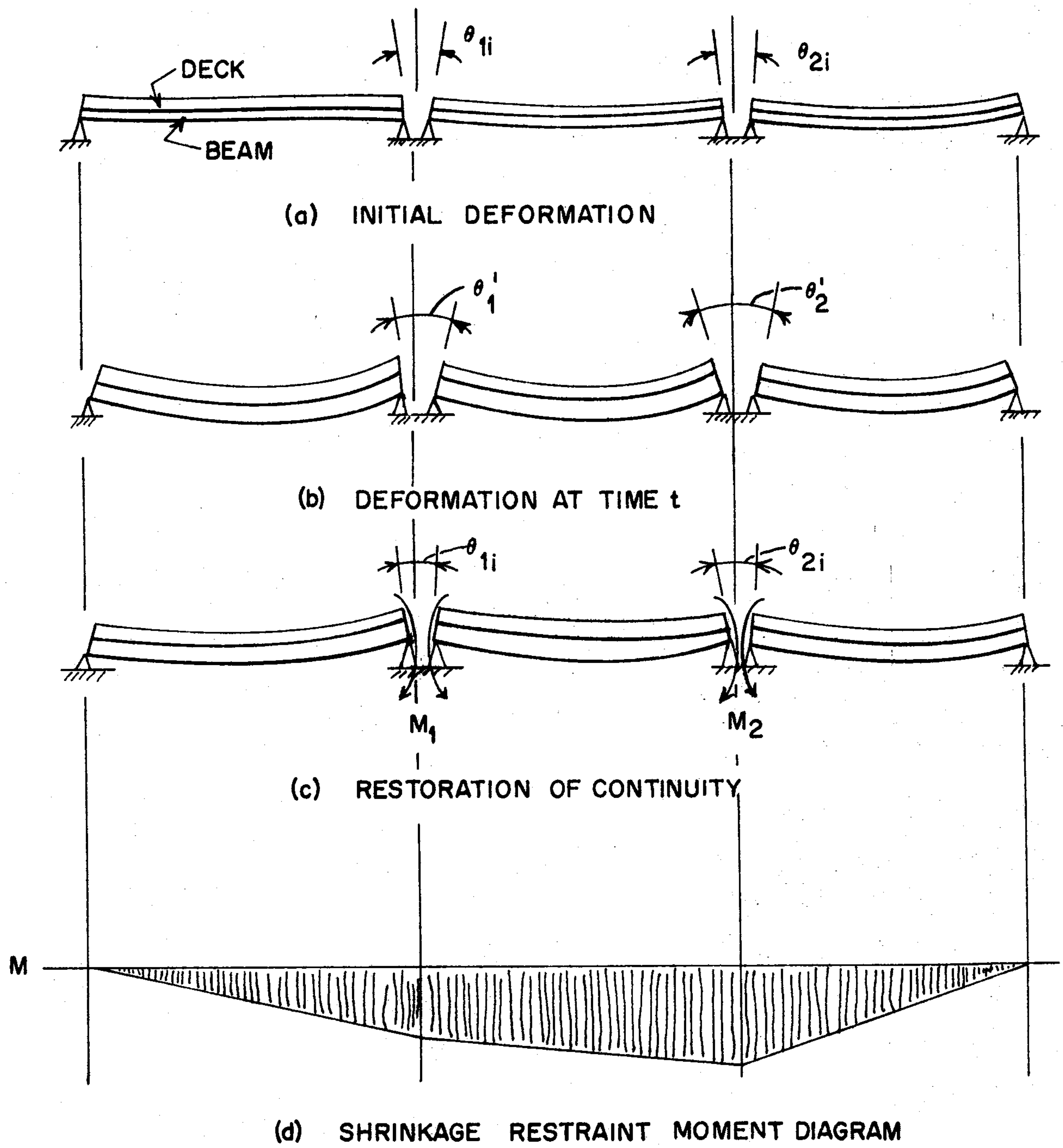
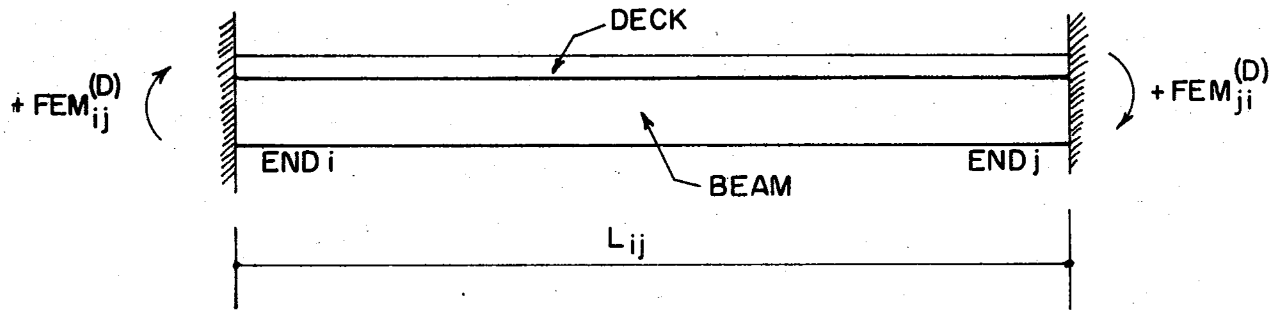


FIGURE 6. SHRINKAGE RESTRAINT MOMENTS

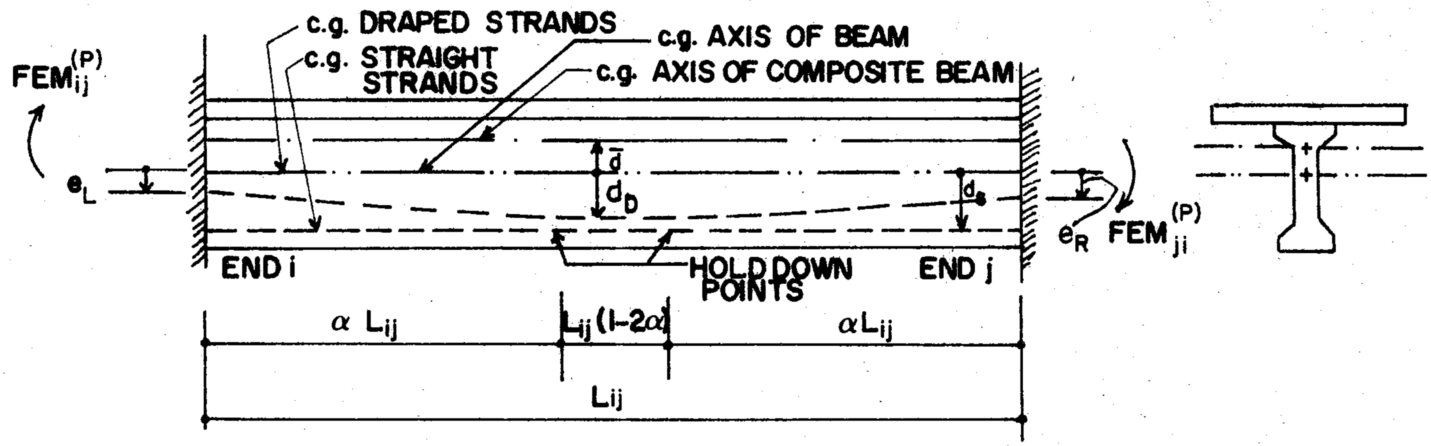


$$FEM_{ij}^{(D)} = -\frac{1}{12} wL_{ij}^2$$

$$FEM_{ji}^{(D)} = +\frac{1}{12} wL_{ij}^2$$

w = WEIGHT PER UNIT LENGTH OF BEAM PLUS DECK

FIGURE 7. FIXED END MOMENTS FOR DEAD LOAD CREEP



$\bar{d}$, d_D, d_S, e_L, & e_R ALL POSITIVE AS SHOWN

P_S = TOTAL PRESTRESS FORCE IN STRAIGHT STRANDS

P_D = TOTAL PRESTRESS FORCE IN DRAPED STRANDS

$M_0 = P_S(\bar{d} + d_S)$

$M_1 = P_D(\bar{d} + e_L)$

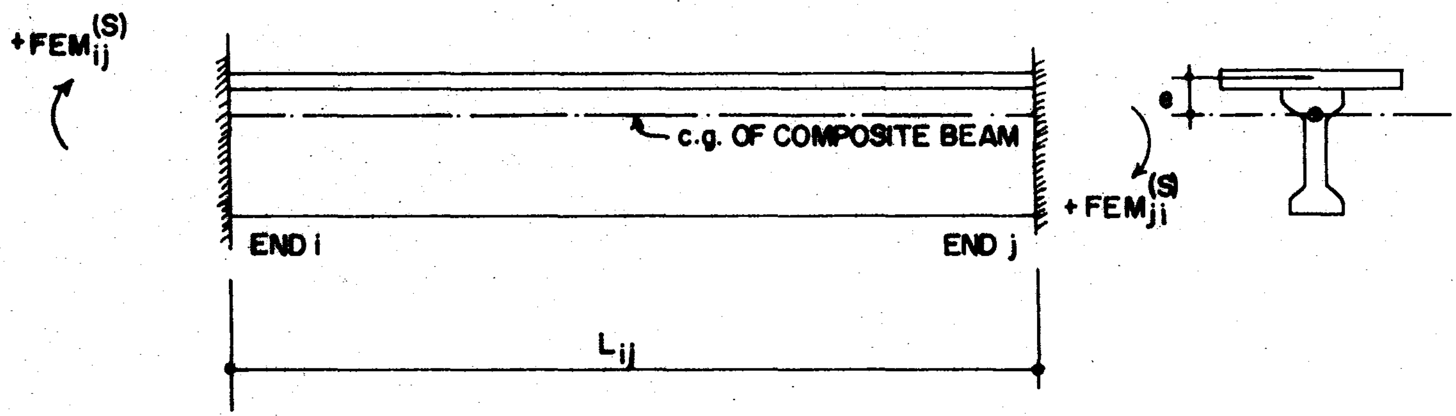
$M_2 = P_D(\bar{d} + d_D)$

$M_3 = P_D(\bar{d} + e_R)$

$$FEM_{ij}^{(P)} = M_0 + \alpha(2-\alpha)M_1 + (1-\alpha)M_2 - \alpha(1-\alpha)M_3$$

$$FEM_{ji}^{(P)} = -M_0 + \alpha(1-\alpha)M_1 - (1-\alpha)M_2 - \alpha(2-\alpha)M_3$$

FIGURE 8 FIXED END MOMENTS FOR PRESTRESS CREEP



$$FEM_{ij}^{(S)} = -\epsilon_{SD} E A e$$

$$FEM_{ji}^{(S)} = \epsilon_{SD} E A e$$

ϵ_{SD} = DIFFERENTIAL SHRINKAGE STRAIN BETWEEN DECK AND BEAM CONCRETE (see Section 3.2)

E = MODULUS OF ELASTICITY OF DECK CONCRETE

A = CROSS SECTIONAL AREA OF DECK

e = DISTANCE BETWEEN MID DEPTH OF DECK AND CENTROID OF COMPOSITE SECTION

FIGURE 9. FIXED END MOMENTS FOR SHRINKAGE

The slope deflection method relates the rotations at each support of the continuous beam to the stiffnesses of the spans and the fixed end moments. For a beam of N spans, this can be written as

$$EI \cdot \begin{bmatrix} \frac{4}{L_1} & \frac{2}{L_1} & 0 & 0\cdots0 \\ \frac{2}{L_1} & \frac{4}{L_1}+\frac{4}{L_2} & \frac{2}{L_2} & 0\cdots0 \\ 0 & \frac{2}{L_2} & \frac{4}{L_2}+\frac{4}{L_3} & \frac{2}{L_3}\cdots0 \\ 0 & 0 & \frac{2}{L_3} & \cdots\cdot0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0\cdots\frac{4}{L_N} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \vdots \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} -FEM_{12} \\ -FEM_{21}-FEM_{23} \\ -FEM_{32}-FEM_{34} \\ -FEM_{43}-FEM_{45} \\ \vdots \\ -FEM_{N,\ N+1} \end{bmatrix} \quad (21)$$

where $\theta_1, \ldots, \theta_{N+1}$ are the rotations at the supports. Solution of Eq. (21) yields support rotations, which can be used to compute final moments at each of each span from

$$M_{ij} = \frac{2EI}{L_{ij}} \cdot (2\theta_i + \theta_j) + FEM_{ij} \quad (22)$$

$$M_{ji} = \frac{2EI}{L_{ij}} (\theta_i + 2\theta_j) + FEM_{ji}$$

3.2 Computation of Scale Factors for Restraint Moments

The final unscaled moments which are produced by shrinkage, dead load creep and prestress creep may be computed separately by substituting the appropriate fixed end moments into Eqs. (21) and (22). The resulting moments are scaled by the shrinkage and creep factors given in Eqs. (18), (19) and (20) and summed to give the final restraint moments.

The scale factors in Eqs. (18) and (20) depend on the creep factor ϕ, which is defined by Eq. (19). To determine ϕ, one must have a unit creep curve for the beam and deck concrete (the deck and beam concretes are assumed to have the same creep and shrinkage behavior) to establish ε_c. Previous research has been conducted on the creep and shrinkage of concretes typically used by the Texas Highway Department for prestressed beams (3). Creep and shrinkage data were taken on concretes from four localities in Texas, and the properties of these concretes are listed in Table 1. Studies on creep and shrinkage were conducted on 3 in. x 3 in. x 16 in. prisms stored under laboratory conditions of 50% humidity and 73°F. Expressions which were found to fit the shrinkage and unit creep strain data are listed in Table 2. A reasonable estimate of the overall average unit creep function $\varepsilon_c(t)$ and shrinkage strain function $\varepsilon_s(t)$, applicable for all locations, is given by

$$\varepsilon_c(t) = \frac{425t}{34+t} \quad (\text{in./in./ksix}10^{-6}) \tag{23}$$

$$\varepsilon_s(t) = \frac{525t}{20+t} \quad (\text{in./in.x}10^{-6}) \tag{24}$$

where the constant terms in each expression are the average of those values appearing in Table 2. The value for ε_c appearing in Eq. (19) should correspond to the maximum unit strain expected for any time t.

Location	Release Strength (psi)	Measured Release Modulus (ksi)	Computed* Release Modulus (ksi)
Dallas	7,080	5,200	4,800
Odessa	5,050	3,260	4,050
San Antonio	5,250	4,220	4,130
Lufkin	5,760	4,440	4,330

* Using ACI 318-71 Equation, $E = 57{,}000 \sqrt{f'_c}$

TABLE 1. STRENGTH AND MODULI VALUES FROM REFERENCE (3)

Location	Shrinkage (in./in. x 10^{-6})	Creep (in./in./ksi x 10^{-6})
Dallas	$\frac{500T}{15+T}$*	$\frac{365T}{40+T}$
Odessa	$\frac{650T}{20+T}$	$\frac{525T}{25+T}$
San Antonio	$\frac{500T}{20+T}$	$\frac{385T}{25+T}$
Lufkin	$\frac{450T}{25+T}$	$\frac{430T}{45+T}$

* T-time in days

TABLE 2. EXPRESSIONS FOR SHRINKAGE STRAIN AND UNIT CREEP STRAIN FOR 3 in. x 3 in. x 16 in. PRISMS STORED UNDER LABORATORY CONDITIONS (From Reference 3)

This will occur, according to Eqs. (23), when $t = \infty$ and yields $\varepsilon_c = 425$ (in./in./ksix10^{-6}). This basic unit creep strain is for concrete specimens with a volume/surface ratio of approximately 1.0, loaded at approximately 1 day after casting, and cured under a constant relative humidity of 50%. Corrections to the basic unit creep strain are required for volume/surface ratios significantly different from 1.0. The age of the test specimens at loading is representative of the production sequence used by most manufacturers, so no correction is applied for age at loading.

The modified unit creep strain $\tilde{\varepsilon}_c$ is written as

$$\tilde{\varepsilon}_c = 425\alpha_{v/s} \times 10^{-6}(\text{in./in./ksi}) \tag{25}$$

where $\alpha_{v/s}$ is the correction to the basic unit creep strain. A plot of this correction factor versus volume/surface ratio is given in Fig. 10. It was obtained by downward shift of a similar curve appearing in reference (4), so that $\alpha_{v/s} = 1.0$ for the volume/surface ratio of 1.0 at which the creep data were taken. Noting that α in Eg. (19) is given by

$$\alpha = \frac{1}{425} \cdot \frac{425t}{34+t} = \frac{t}{34+t} \tag{26}$$

that

$$\varepsilon_e = 1/E_i \tag{27}$$

and that Eq. (25) gives the corrected unit creep strain, we have

$$\phi = 425\alpha_{v/s}E_i(1-\frac{t}{34+t}) \tag{28}$$

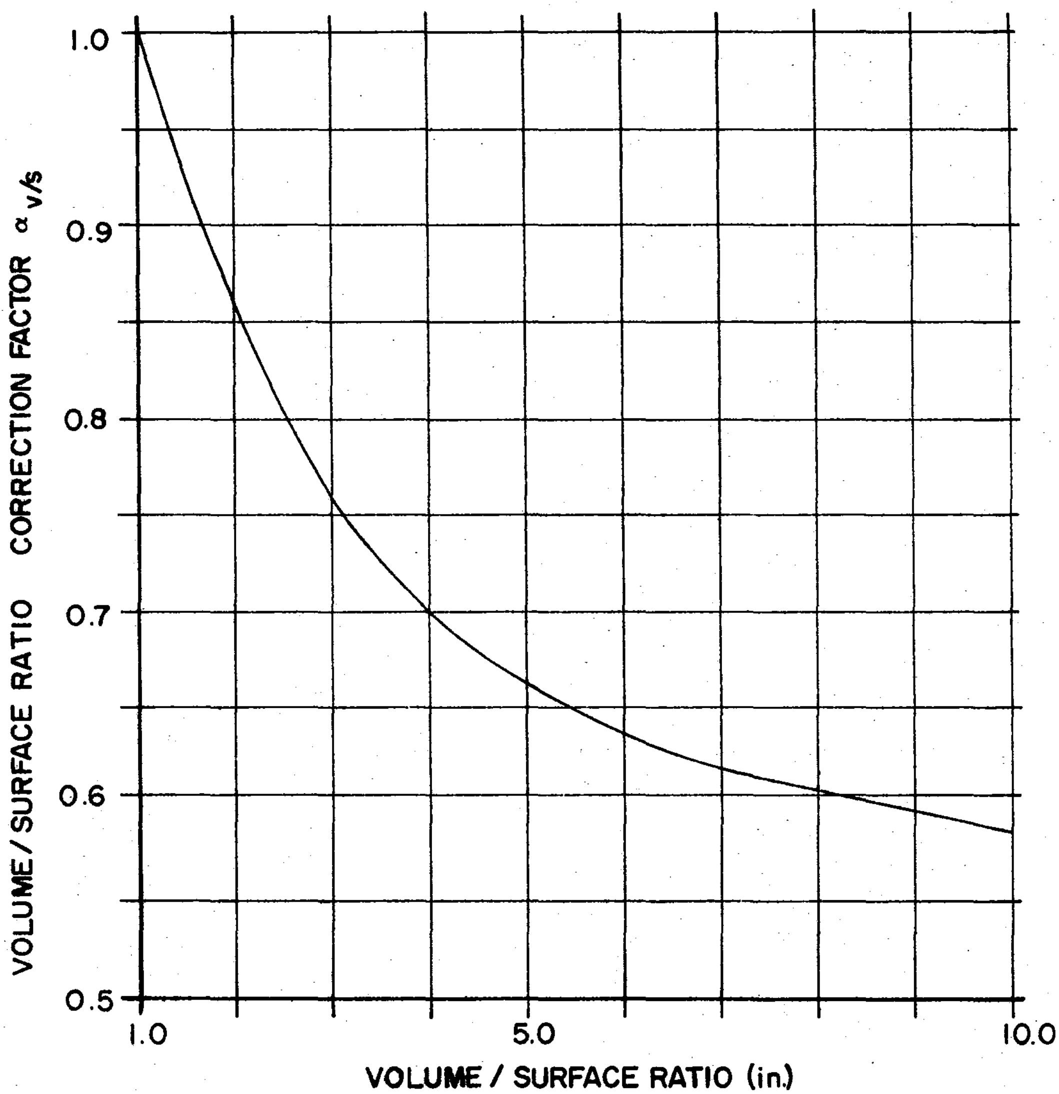


FIGURE 10. SHRINKAGE STRAIN CORRECTION FACTOR

(Modified from Reference 4)

where

E_i = modulus of elasticity of beam concrete at release of strands (ksi),

t = age of the beam in days, when continuity connection made, and

$\alpha_{v/s}$ = correction factor from Fig. 10.

Corrections to the ultimate shrinkage strain of 525 (in./in.x10^{-6}) (obtained from Eq. (24) with $t = \infty$) must be made for relative humidities greatly different from 50%. Letting ε_{su} be the corrected ultimate shrinkage strain, we have

$$\varepsilon_{su} = 525\alpha_H \times 10^{-6} (\text{in./in.}) \quad (29)$$

where α_H is a humidity correction factor taken from reference (4), and shown in Fig. 11. The differential shrinkage strain ε_{SD} used in Fig. 9 for the computation of fixed end moments due to shrinkage can be written as

$$\varepsilon_{SD} = \varepsilon_{su} \cdot \beta \quad (30)$$

The factor β is the fraction of shrinkage which has occurred in the beam concrete before the deck is cast. From Eq. (24),

$$\beta = \frac{1}{525} \cdot \frac{525t}{20+t} = \frac{t}{20+t} \quad (31)$$

Substituting Eqs. (29) and (31) into (30) gives

$$\varepsilon_{SD} = 525\alpha_H \cdot \frac{t}{20+t} \quad (32)$$

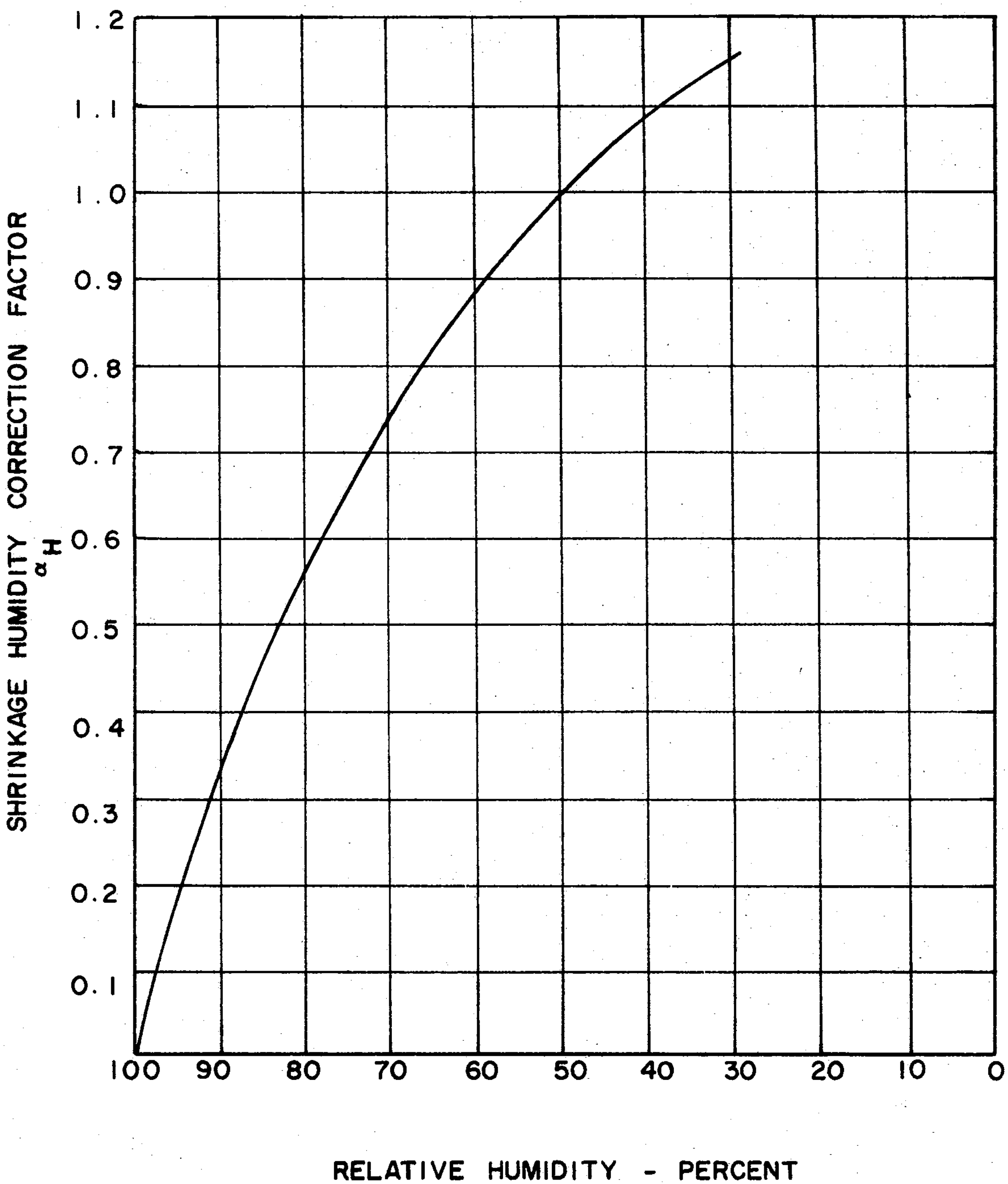


FIGURE 11. SHRINKAGE HUMIDITY CORRECTION FACTOR

(From Reference 4)

where

α_H = correction factor from Fig. 11, and

t = age of beam concrete when deck cast.

Thus, the final scaled restraint moments may be obtained by performing the elastic analysis described previously (using Eq. (32) to obtain shrinkage fixed end moments), computing the creep factor from Eq. (28) and scaling the moments resulting from prestress and dead load creep by Eq. (18) and from shrinkage by Eq. (20).

IV. DESIGN CRITERIA

The computer program developed in this study carries out the design of a continuous beam constructed with precast, prestressed concrete I-beams. The applicable design code is Standard Specifications for Highway Bridges, 11th edition, published by The American Association of State Highway and Transportation Officials (1). This section reviews the provisions of this Specification which govern the design and outlines the computations made in arriving at a satisfactory design. In the following discussion, reference to a "Section" denotes a provision from this document.

4.1 Predesign Decisions

The structural engineer, charged with the responsibility for the complete design of a bridge, can utilize this program to carry out routine computations involved in selection of strand patterns, longitudinal deck reinforcing, stirrups, and continuity connection reinforcement. To interface with the program, he must first specify:

(i) the length of each span in the continuous beam,

(ii) the geometrical properties of the precast beam (which must be the same for all spans),

(iii) the properties of reinforcement, beam concrete and slab concrete,

(iv) the beam spacing,

(v) the slab thickness,

(vi) the design live load, and

(vii) the type of continuity construction (partially continuous for dead load or continuous for live load only).

The program contains a greatly simplified input format to accommodate designs which incorporate standard THD and AASHTO beams, reinforcement and loadings. The details of program input are explained in Vol. II of this report.

4.2 Beam Design Loads

The live load moments and shears used in design can be computed from

(i) standard AASHTO trucks and lane loadings,

(ii) a series of up to 15 arbitrarily spaced, moving wheels "axle train",

(iii) a uniformly distributed live load applied to the continuous beam and superimposed on the load produced by (i) or (ii).

The portion of an AASHTO truck and lane loading applied to the beam is determined from Section 1.3.1 and taken as S/5.5, unless specified otherwise on the input form. The fraction of wheel loads from an axle train which is applied to the beam is specified on input. No lateral distribution of uniform load is included in the program.

Dead load moments and shears from beam weight, diaphrams and portions of the deck poured for continuity are computed from input information.

4.3 Criteria for Strand Pattern Selection

Strand pattern selection for a span of the continuous beam is based primarily on service load stress considerations, although ultimate strength requirements may in some cases govern strand placement. In each span, stresses produced by loads are checked at top and bottom of the beam at tenth points (each end and 9 interior points). Stresses produced by prestress at strand release are checked at top and bottom of beam, at each end and hold down points. Hold down points vary with span length and are listed in Table 3.

4.3.1 Calculation of Load Induced Stresses - Four stress checks are made at each tenth point during strand placement, as indicated in Table 4. Stress produced by external loads at the top and bottom of a beam at the ith tenth point are given by

$$\sigma_{ti} = \frac{1}{Z_t}\left\{M_{DLi}^{(B)} + M_{DLi}^{(DNC)}\right\} + \frac{1}{ZC_t}\left\{M_{DLi}^{(DC)} + (M_{LLi} + I) + M_{Ri}\right\} \quad (33)$$

$$\sigma_{bi} = \frac{1}{Z_b}\left\{M_{DLi}^{(B)} + M_{DLi}^{(DNC)}\right\} + \frac{1}{ZC_b}\left\{M_{DLi}^{(DC)} + (M_{LLi} + I) + M_{Ri}\right\} \quad (34)$$

where

Z_t = top section modulus of beam,

Z_b = bottom section modulus of beam,

ZC_t = top section modulus of composite beam,

ZC_b = bottom section modulus of composite beam,

$M_{DLi}^{(B)}$ = moment at ith tenth point due to beam weight,

Span Length (ft)	Distance Each Side of Midspan (ft)
0 to 119	5.0
120 to 140	6.0
141 to 159	7.0
160 to 180	8.0

TABLE 3. LOCATION OF HOLD DOWN POINTS

Location	Stress	Load
top of beam	comp.	DL. + Max (+) LL + I + restraint moment if positive
bottom of beam	ten.	DL. + Max (+) LL + I + restraint moment if positive
top of beam	ten.	DL. + Max (-) LL + I + restraint moment if negative
bottom of beam	comp.	DL. + Max (-) LL + I + restraint moment if negative

TABLE 4. STRESS CHECKS FOR DESIGN

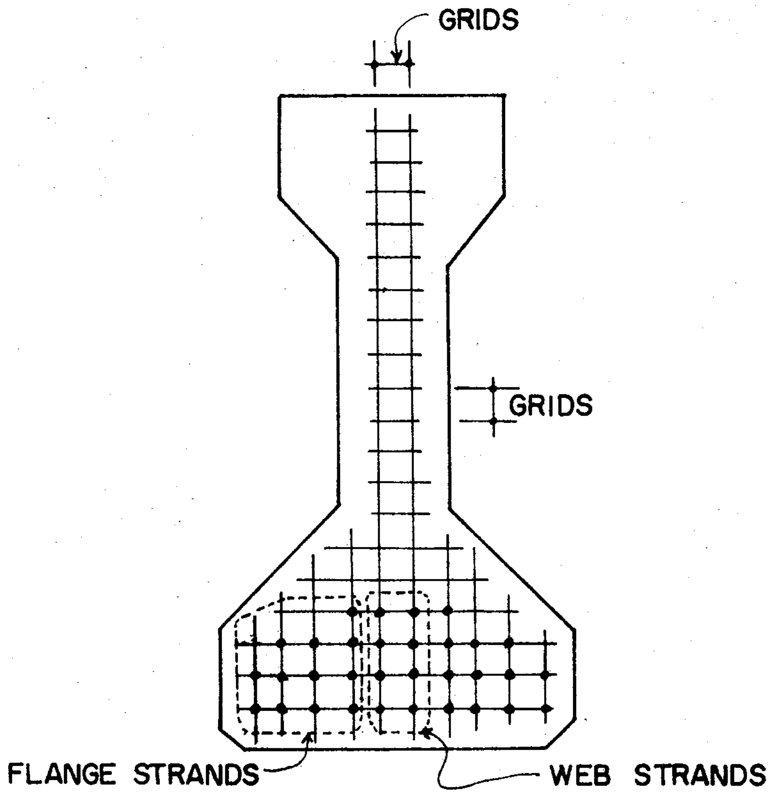
$M_{DLi}^{(DNC)}$ = moment at ith tenth point due to segment of deck slab poured to establish continuity,

$M_{DLi}^{(DC)}$ = moment at ith tenth point due to segment of deck slab cast after initial continuity pour,

$(M_{LLi} + I)$ = moment due to live load, plus impact (for AASHTO loadings only),

M_{Ri} = restraint moment at ith tenth point.

4.3.2 Calculation of Prestress Induced Stresses - The computation of stresses at the top and bottom of the beam resulting from prestress force are set up in terms of the number of strands in each strand row of the beam. The total number of strand rows available in the beam is denoted by the variable NRAV. For I shaped cross sections, the lower rows can contain both "flange strands" (strands which are outside the confines of the web and must remain straight) and "web strands" (which may be draped). The variable NRFLG denotes the number of lower rows which may contain both flange and web strands. The remaining upper rows have only web rows. The variable x(i) denotes one-half of the number of flange strands in flange row i. The variable I(i) is a binary variable (has a value of either 0 or 1) and indicates the presence of NSWEB strands in row i if it has the value 1.0. These variables are defined in Fig. 12 for a standard THD beam. The number of strand rows and the maximum number of strands permitted in a row are determined from the cross sectional dimensions and the strand grid spacing (GRIDS). The stresses at the top and bottom of the beam at design point i are given by



NRAV = 19 X(1) = X(2) = 4 I(1) = I(2) = I(3) = I(4) = 1

NRFLG = 6 X(3) = 3 I(5) = . . . = I(19) = 0

NSWEB = 2 X(4) = 1

X(5) = X(6) = 0

FIGURE 12. VARIABLES DEFINING STRAND PATTERN

$$\sigma_{ti}^{(P)} = \sum_{j=1}^{NRFLG} -(1-\xi)\, F_o\, [1/A + d_j/Z_t]\; 2x_j$$

$$+ \sum_{j=1}^{NRAV} -(1-\xi)\, F_o \cdot NSWEB\, [1/A + d_j/Z_t]\; I_j$$

$$-\;(1-\xi)\, F_o \cdot NSWEB \cdot \frac{\tau_i}{Z_t} \cdot GRIDS \cdot J \cdot ECC \qquad (35)$$

$$\sigma_{bi}^{(P)} = \sum_{j=1}^{NRFLG} -(1-\xi)\, F_o\, [1/A - d_j/Z_b]\; 2x_j$$

$$+ \sum_{j=1}^{NRAV} -(1-\xi)\, F_o \cdot NSWEB\, [-1/A + d_j/Z_b]\; I_j$$

$$+\;(1-\xi)\, F_o \cdot NSWEB \cdot \frac{\tau_i}{Z_b} \cdot GRIDS \cdot J \cdot ECC \qquad (36)$$

where

F_o = initial force in prestressing strand before release of strands,

ξ = fraction of initial strand force lost after release (assumed to be 0.20),

A = cross sectional area of beam,

Z_t = top section modulus of beam,

Z_b = bottom section modulus of beam,

J = number of strand rows which contain strands,

ECC = end eccentricity of web strands (number of rows by which the web strands are raised). If i=1 thru 6, ECC is the left end eccentricity. If i=7 thru 12, ECC is the right end eccentricity.

d_j = distance from c.g. axis of beam to row j. d_j is positive if row j is above the c.g. axis,

$$\tau_i = \begin{cases} \frac{1}{\alpha}(\alpha - i/10); & 0 \le iL/10 \le \alpha L \\ 0; & \alpha L \le iL/10 \le L(1-\alpha) \\ \frac{1}{\alpha}(i/10 - \alpha); & L(1-\alpha) < iL/10 \le L, \text{ and} \end{cases}$$

α, L = defined in Fig. 8.

4.3.3 Selection of Number and Positions of Strands - A trial strand pattern is selected based on tension stress at the bottom of the beam at the tenth point where maximum positive moment occurs. The end eccentricities are assumed to be zero during strand pattern selection. The allowable tensile stress is taken as

$$\sigma_{ten} = 6.0\sqrt{f'_c} \tag{37}$$

where f'_c is the minimum 28 day strength of the beam concrete, specified on input. The total stress at the bottom of the beam is the sum of those stresses given by Eqs. (34) and (36).

The strand pattern selection proceeds with sequential placement of strands in each row. NSWEB strands are first placed in row 1, (I_1 = 1), followed by placement in pairs of flange strands (x_1). If additional strands are required, the sequence of events is repeated for higher rows, until the final tension stress in the bottom of the beam is less than that given by Eq. (37).

After an initial pattern of strands has been selected, cracking and ultimate moment capacities of the section are computed. If the ultimate moment is less than 1.2 times the cracking moment, additional strands are added, by the same process described above, until this code provision (Section 1.6.10 (B)) is met.

At this point, no provisions have yet been made to insure that service load stresses at the top of the beam under positive moment, stresses top and bottom under negative moment and release stresses do not exceed their allowables. The problem of determining the end eccentricities, release strength and 28 day strength of the beam is cast as a mathematical programming problem. The problem may be stated as:

$$\text{Minimize } f'_{ci} + f'_c \tag{38}$$

Subject to:

$$0 \cdot f'_{ci} - 0.6\, f'_c - a_1 e_L - b_1 e_R \leq \sigma_{Pt1} + \sigma_{Lt1}^{(+)} \tag{39}$$

$$0 \cdot f'_{ci} - 0.4\, f'_c - a_j e_L - b_j e_R \leq \sigma_{Ptj} + \sigma_{Ltj}^{(+)};\ j = 2, 10 \tag{40}$$

$$0 \cdot f'_{ci} - 0.6\, f'_c - a_{11} e_L - b_{11} e_R \leq \sigma_{Pt_{11}} + \sigma_{Lt_{11}}^{(+)} \tag{41}$$

$$0 \cdot f'_{ci} - 6.0 \sqrt{f'_c} + c_j e_L + d_j e_R \leq -\sigma_{Pbj} - \sigma_{Lbj}^{(+)};\ j=1, 11 \tag{42}$$

$$0 \cdot f'_{ci} - 6.0 \sqrt{f'_c} + a_j e_L - b_j e_R \leq -\sigma_{Ptj} - \sigma_{Ltj}^{(+)};\ j=1, 11 \tag{43}$$

$$0 \cdot f'_{ci} - 0.6\, f'_c - c_1 e_L - d_1 e_R \leq \sigma_{Pb1} + \sigma_{Lb1}^{(-)} \tag{44}$$

$$0 \cdot f'_{ci} - 0.4\, f'_c - c_j e_L - d_j e_R \leq \sigma_{Pbj} + \sigma_{Lbj}^{(-)};\ j=2, 10 \tag{45}$$

$$0 \cdot f'_{ci} - 0.6\, f'_c - c_{11} e_L - d_{11} e_R \leq \sigma_{Pb_{11}} + \sigma_{Lb_{11}}^{(-)} \tag{46}$$

$$-7.5 \sqrt{f'_c} + 0 \cdot f'_c + a_1 e_L + b_1 e_R \leq \sigma_{Pt_1}^{(+)} \tag{47}$$

$$-0.6\, f'_{ci} + 0 \cdot f'_c - a_1 e_L - b_1 e_R \leq \sigma_{Pt_1}^{(r)} \tag{48}$$

$$-7.5 \sqrt{f'_c} + 0 \cdot f'_c + c_1 e_L + d_1 e_R \leq \sigma_{Pb_1}^{(r)} \tag{49}$$

$$-0.6\, f'_{ci} + 0 \cdot f'_c - c_1 e_L - d_1 e_R \leq \sigma_{Pb_1}^{(r)} \tag{50}$$

$$-7.5\sqrt{f'_{ci}} + 0 \cdot f'_c + a_{11}e_L + b_{11}e_R \leq \sigma_{Pt_{11}}^{(r)} \qquad (51)$$

$$-0.6\ f'_{ci} + 0 \cdot f'_c - a_{11}e_L - b_{11}e_R \leq \sigma_{Pt_{11}}^{(r)} \qquad (52)$$

$$-7.5\sqrt{f'_{ci}} + 0 \cdot f'_c + c_{11}e_L + d_{11}e_R \leq \sigma_{Pb_{11}}^{(r)} \qquad (53)$$

$$-0.6\ f'_{ci} + 0 \cdot f'_c - c_{11}e_L - d_{11}e_R \leq \sigma_{Pb_{11}}^{(r)} \qquad (54)$$

$$-7.5\sqrt{f'_{ci}} + 0 \cdot f'_c - 0 \cdot e_L - 0 \cdot e_R \leq \sigma_{Pth}^{(r)} - \sigma_{wth} \qquad (55)$$

$$-0.6\ f'_{ci} + 0 \cdot f'_c - 0 \cdot e_L - 0 \cdot e_R \leq \bar{\sigma}_{Pth}^{(r)} + \sigma_{wth} \qquad (56)$$

$$-7.5\sqrt{f'_{ci}} + 0 \cdot f'_c + 0 \cdot e_L + 0 \cdot e_R \leq \sigma_{Pbh}^{(r)} - \sigma_{wbh} \qquad (57)$$

$$-0.6\ f'_{ci} + 0 \cdot f'_c + 0 \cdot e_L + 0 \cdot e_R \leq \sigma_{Pbh} + \sigma_{wbh} \qquad (58)$$

$$e_L \leq e_{max} \qquad (59)$$

$$e_R \leq e_{max} \qquad (60)$$

$$-f'_{ci} \leq -4.0 \qquad (61)$$

$$-f'_c \leq -5.0 \qquad (62)$$

$$f'_{ci} - f'_c \leq 0.0 \qquad (63)$$

where

f'_{ci} = release strength,

f'_c = 28 day strength,

e_L = strand eccentricity at left end of beam,

e_R = strand eccentricity at right end of beam,

$a_j = -\ (1-\xi) \cdot F_o \cdot \text{NSWEB} \cdot \frac{\tau_j}{Z_t} \cdot \text{GRIDS} \cdot J$

$b_j = -\ (1-\xi) \cdot F_o \cdot \text{NSWEB} \cdot \frac{\tau_j}{Z_t} \cdot \text{GRIDS} \cdot J$

$$c_j = (1-\xi) \cdot F_o \cdot NSWEB \cdot \frac{\tau_j}{Z_b} \cdot GRIDS \cdot J$$

$$d_j = (1-\xi) \cdot F_o \cdot NSWEB \cdot \frac{\bar{\tau}_j}{Z_b} \cdot GRIDS \cdot J$$

$$\tau_j = \frac{1}{\alpha}(\alpha - j/10); \; 0 \le \frac{jL}{10} \le \alpha L$$

$$\qquad 0 \; ; \; \alpha L \le \frac{jL}{10} \le L$$

$$\bar{\tau}_j = \qquad 0 \; ; \; 0 \le \frac{jL}{10} \le L(1-\alpha)$$

$$\frac{1}{\alpha}(j/10-1+\alpha); \; L(1-\alpha) \le \frac{jL}{10} \le L$$

$\sigma_{Ptj} + \sigma_{Pbj}$ = stress top and bottom of beam at jth tenth point, respectively, due to prestress and computed from Eqs. (35) and (36) with the omission of the last term in each of these equations,

$\sigma_{Ltj}^{(+)} + \sigma_{Lbj}^{(+)}$ = stress top and bottom of beam at jth tenth point produced by dead load moment, positive live load moment and creep restraint moment, if positive,

$\sigma_{Ltj}^{(-)} + \sigma_{Lbj}^{(-)}$ = stress top and bottom of beam at jth tenth point produced by dead load moment, negative live load moment and creep restraint moment, if negative,

$\sigma_{Ptj}^{(r)} + \sigma_{Pbj}^{(r)}$ = stress top and bottom of beam at jth tenth point (j=1 is left end, j=11 is right end and j=h is hold down point) due to prestress at release, computed from Eqs. (35) and (36) with the omission of the last term in each of these equations,

$\sigma_{wth} + \sigma_{wbh}$ = stress top and bottom of beam at hold down point produced by beam weight, and

e_{max} = maximum number of rows that web strands may be raised at end of beam (depends on J, the number of rows with strands).

This problem contains four variables (f'_{ci}, f'_c, e_L and e_R) and 61 inequality constraints (Eqs. 39 thru 63). The constraints are linear with the exception of those containing $\sqrt{f'_{ci}}$ or $\sqrt{f'_c}$. These are linearized by using the first order Taylor series expansion

$$\sqrt{f'_c} = 7.4535\ f'_c + 33.552 \qquad (64)$$

where $\sqrt{f'_c}$ is in psi and f'_c is in ksi. This expansion is taken about the point f'_c = 4.5 ksi. The error involved in the use of Eq. (64) is small, amounting to approximately 7% for f'_c = 9.5 ksi and being smaller for smaller values of f'_c or f'_{ci}. Substitution of Eq. (64) into all constraints containing f'_{ci} or f'_c produces a constraint equation linear in the variables. Equation (42), for example, becomes

$$0 \cdot f'_{ci} - 6.0\ (7.4535)\ f'_c + c_j e_L + d_j e_R \leq -\sigma_{Pbj} - \sigma^{(+)}_{Lbj} - 33.552 \qquad (42')$$

The completely linear programming problem is solved by applying the standard Simplex (5) algorithm to the dual of this program. The resulting solution of this program is the end eccentricities, for the strand pattern selected on the basis of maximum positive moment, which minimizes the sum of the release and 28-day beam strengths. The restrictions imposed by the inequality constraints are summarized in Table 5.

4.4 Computation of Positive Ultimate Moment Capacity

Section 1.6.3 stipulates that beams be designed to supply a specified ultimate moment capacity. Equations are presented in Section 1.6.9 for the computation of ultimate moment for flanged sections in which the neutral axis at ultimate lies within the deck slab or within the constant width section of the top flange of the beam. For cases where the neutral axis lies within the slab, ultimate moment capacity is computed from the

Constraint Eq.(s) No.	Design Restriction
39	Limits compression stress at the top of the left end of the beam due to prestress, dead load, positive live load and positive restraint moments to less than 0.6 f'_c.
40	Limits compression stress at top of beam at interior tenth points due to prestress, lead load, positive live load and positive creep restraint moments to less than 0.6 f'_c.
41	Same as Eq. (39), except for right end of beam.
42	Limits tension stresses at bottom of beam at all tenth points due to prestress, dead load, positive live load and positive restraint moment to less than 6.0 $\sqrt{f'_c}$.
43	Limits tension stresses at top of beam at all tenth points due to prestress, dead load, negative live load and negative restraint moment to less than 6.0 $\sqrt{f'_c}$.
44	Limits compression stress at bottom of left end of beam due to prestress, dead load negative live load and negative restraint moment to less than 0.6 f'_c.
45	Same as Eq. (44), for all interior tenth points and limits stress to 0.4 f'_c.
46	Same as Eq. (44), for right end.
47-50	Limits release stresses top and bottom left end, to 7.5 f'_{ci} if tension and 0.6 f'_{ci} if compression.
51-54	Same as Eqs. (47) - (50), except for right end.
55-58	Same as Eqs. (47) - (50), except for hold down point.
59&60	Limits left and right end eccentricities.
61&62	f'_{ci} and f'_c must be greater than 4.0 ksi and 5.0 ksi, respectively.
63	f'_c must be greater than or equal to f'_{ci}.

TABLE 5. VERBAL DESCRIPTION OF CONSTRAINTS

AASHTO equations

$$M_u = A_s^* f_{su}^* d \left(1-0.6 \frac{p^* f_{su}^*}{f_c'}\right) \quad (65)$$

$$f_{su}^* = f_s' \left(1-0.5 \frac{p^* f_s'}{f_c'}\right) \quad (66)$$

where

A_s^* = total area of prestressing strands,

f_{su}^* = average stress in strands at ultimate,

f_s' = ultimate strength of strands,

f_c' = 28-day strength of beam concrete,

$p^* = A_s^*/bd$

d = distance from c.g. of strands to top of slab,

b = effective width of slab used in moment calculation,

$$= b_{eff} \cdot \frac{f_c' \text{ slab}}{f_c' \text{ beam}}$$

b_{eff} = effective slab width from Section 1.6.23.

Equation (65) does not include the contribution to ultimate moment capacity of longitudinal reinforcing in the deck for temperature, lateral load distribution and negative bending moment. However, little additional moment capacity would be gained by including this steel area in the calculations because its proximity to the neutral axis results in small strains, and thus small compressive forces.

For situations where the neutral axis lies in the beam, Eq. (65) is not applicable. In addition, the stress in conventional reinforcing in the deck slab will generally be at or near yield because of its distance from the neutral axis. The calculation of ultimate moment capacity when the neutral axis lies in the beam is therefore based on the following assumptions:

(i) at failure, the compression strain at the top of the deck is .003 in./in.,

(ii) the strain profile is linear at ultimate,

(iii) the distribution of compression stress in the concrete can be replaced with the equivalent stress block shown in Figure 13, and the resultant compressive force in the concrete (C_c) acts through the centroid of the area of the concrete under compression (shaded area in Figure 13),

(iv) the conventional reinforcing in the deck is assumed to be concentrated at mid-depth of the slab for calculation purposes, and its stress at ultimate is proportional to its strain up to the yield stress of 60 ksi and is constant thereafter,

(v) the average stress in the strands at ultimate is obtained from the stress-strain curve for the strands developed below.

The amount of reinforcing in the deck must be adequate to resist negative moments produced by live loads and creep and shrinkage restraint moments. The calculation of this required reinforcing area is described in the next section. In addition, the AASHTO Specification requires certain deck reinforcing for temperature and lateral load distribution. These requirements are met by THD through the use of standard reinforcement details for various beam spacings (6). The total area of reinforcing for temperature and load distribution contained in the flange of the deck for various beam spacings is shown in Table 6, and were computed from reference (6). The area of deck reinforcing used in the computation of ultimate moment capacity when the neutral axis lies in the beam is the greater of those areas required for negative moment at midspan and for temperature and load distribution.

The stress-strain properties of the strands (assumption v above) are assumed as:

$$\varepsilon_s = f_s/28{,}000; \quad f_s \leq f_{pl} \tag{67}$$

$$\varepsilon_s = \frac{f_{pl}}{28{,}000}\left\{1 + \frac{(f'_s - f_{pl})}{(f'_s - 2f_{pl})} - \frac{f_{pl}\,(f'_s - f_{pl})^2}{(f'_s - 2f_{pl})} \cdot f_s \frac{1}{(f'_s - f_s)}\right\}; \; f_s > f_{pl} \tag{68}$$

where

ε_s = strain in the strand (in./in.)

f_s = stress in the strand (ksi)

f'_s = ultimate strength of the strand (ksi)

f_{pl} = proportional limit stress, assumed as .63 f'_s

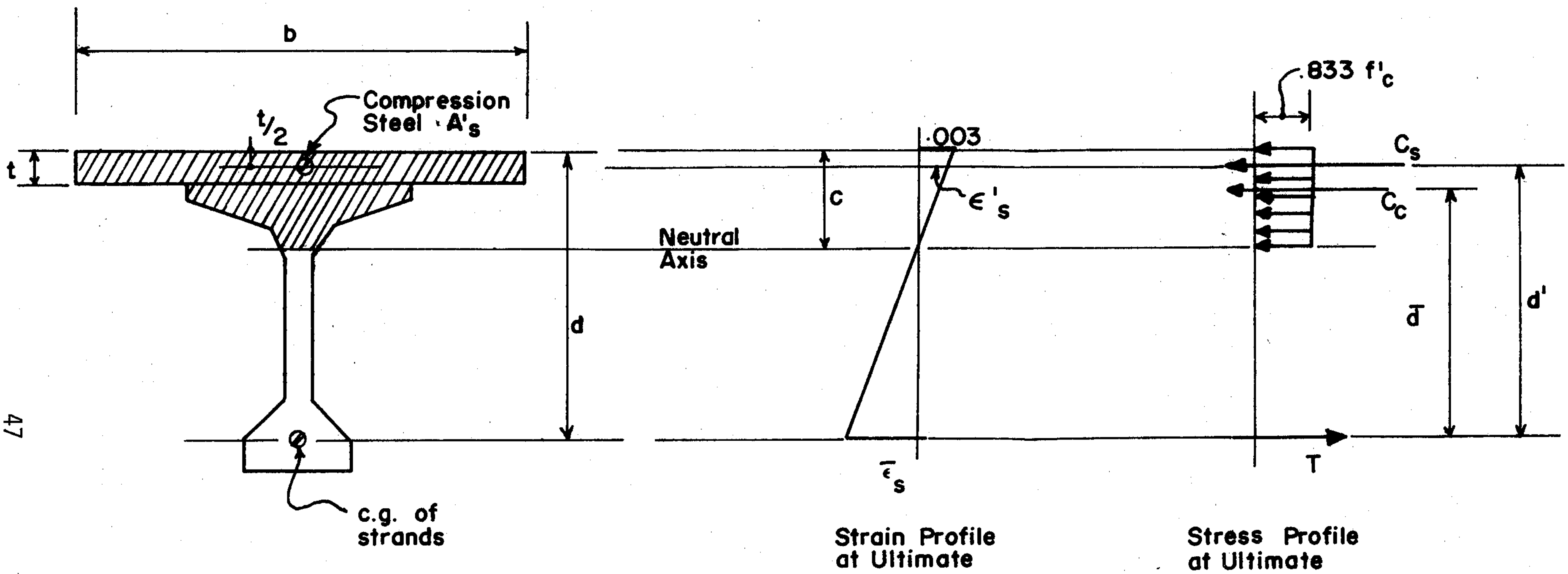
Equations (67) and (68) are plotted for f'_s = 250 and 270 ksi in Fig. 14. The ultimate moment capacity of sections where the neutral axis falls below the slab is given by

$$M_u = C_c \cdot \bar{d} + C_s \cdot d' \tag{69}$$

where C_c, $\bar{d}$, C_s and d' are defined in Fig. 13. The resultant compressive forces C_c and C_s can be computed once the location of the neutral axis c

Lateral Beam Spacing S (ft.)	Area of Longitudinal Reinforcing per Foot Width of Slab (sq. in.)
0.00 to 4.99	2.86/S
5.00 to 6.83	3.57/S
6.84 to 8.00	4.08/S
8.01 to 9.00	4.39/S

TABLE 6. STANDARD LONGITUDINAL TEMPERATURE AND DISTRIBUTION REINFORCING



$$b = b_{eff} \cdot \frac{f'_c \text{ slab}}{f'_c \text{ beam}}$$

b_{eff} = effective slab width

FIGURE 13. STRESS AND STRAIN PROFILES AT ULTIMATE FOR POSITIVE MOMENT

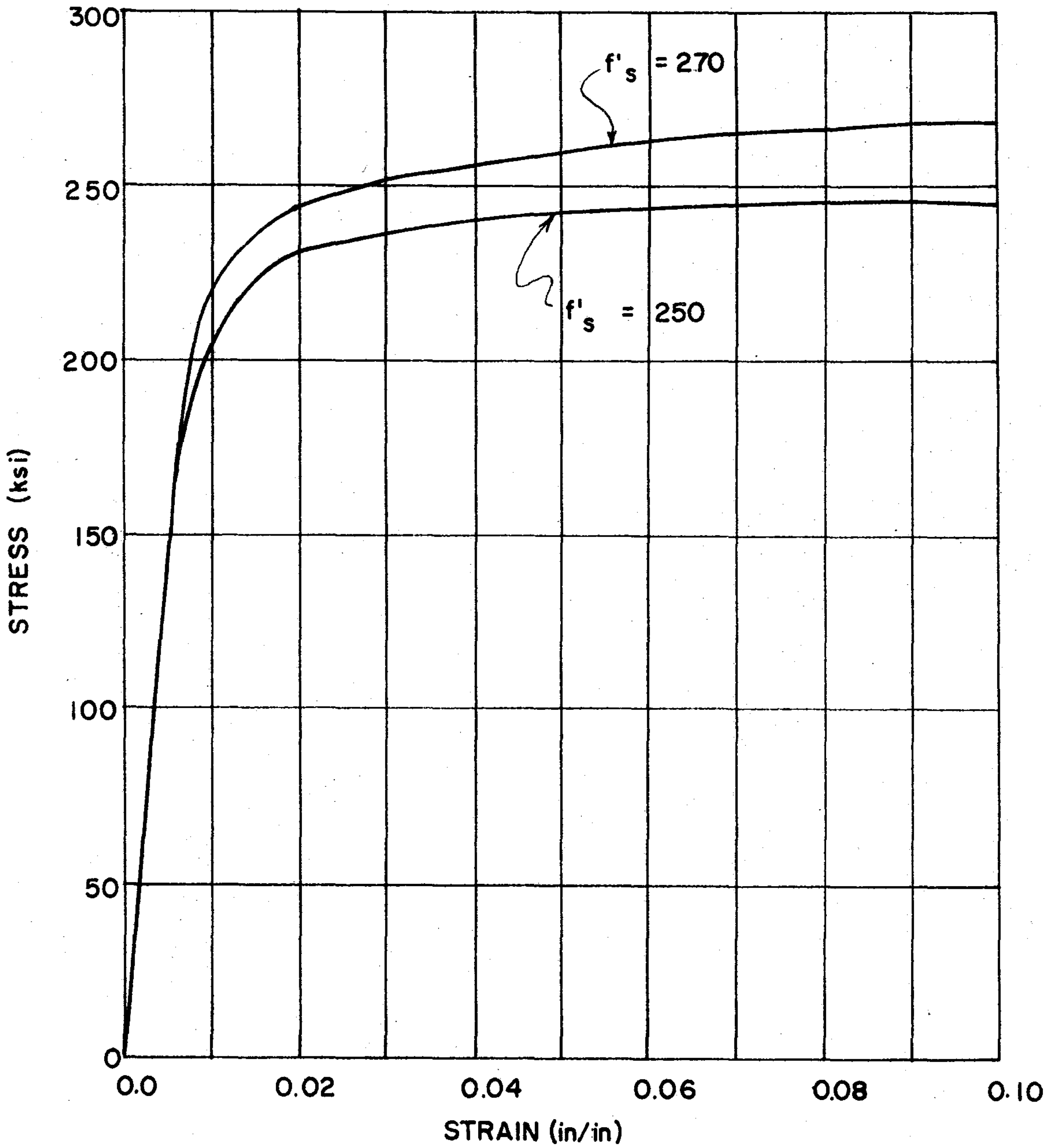


FIGURE 14. STRESS - STRAIN CURVES FOR PRESTRESSING STRANDS

(A plot of Eqs. 67 & 68)

is determined by trial and error solution to the condition that

$$T = C_c + C_s \tag{70}$$

Equation (69) reduces to Eq. (65) if the area of the deck reinforcing is zero ($C_s = 0$) and the neutral axis is in the slab. For this situation,

$$C_c = .833 f'_c bc$$

$$\bar{d} = d - 0.5 \frac{A_s^* f_{su}}{.833 f'_c b}$$

and

$$M_u = A_s^* f_{su} d \left[1 - .6 \frac{p^* f_{su}}{f'_c}\right] \tag{71}$$

Equation (71) differs from Eq. (69) only in the stress in the strands at ultimate, f_{su}. The strain in the strands at ultimate is given by

$$\varepsilon_{su} = .003 \left[\frac{.833}{f_{su}} \cdot \frac{f'_c}{p^*} - 1\right] - \varepsilon_{si} \tag{72}$$

where

ε_{si} = strain in the strands after release of strands and all prestress losses have occurred.

The ultimate stress in the strands, f_{su} is found from the simultaneous solution of Eqs. (72) and (67) or (68). A comparison between the ultimate moment computed by Eq. (65)(the AASHTO equation for neutral axis in the flange) and by Eq. (71) is shown in Fig. 15. The initial strand strain ε_{si}

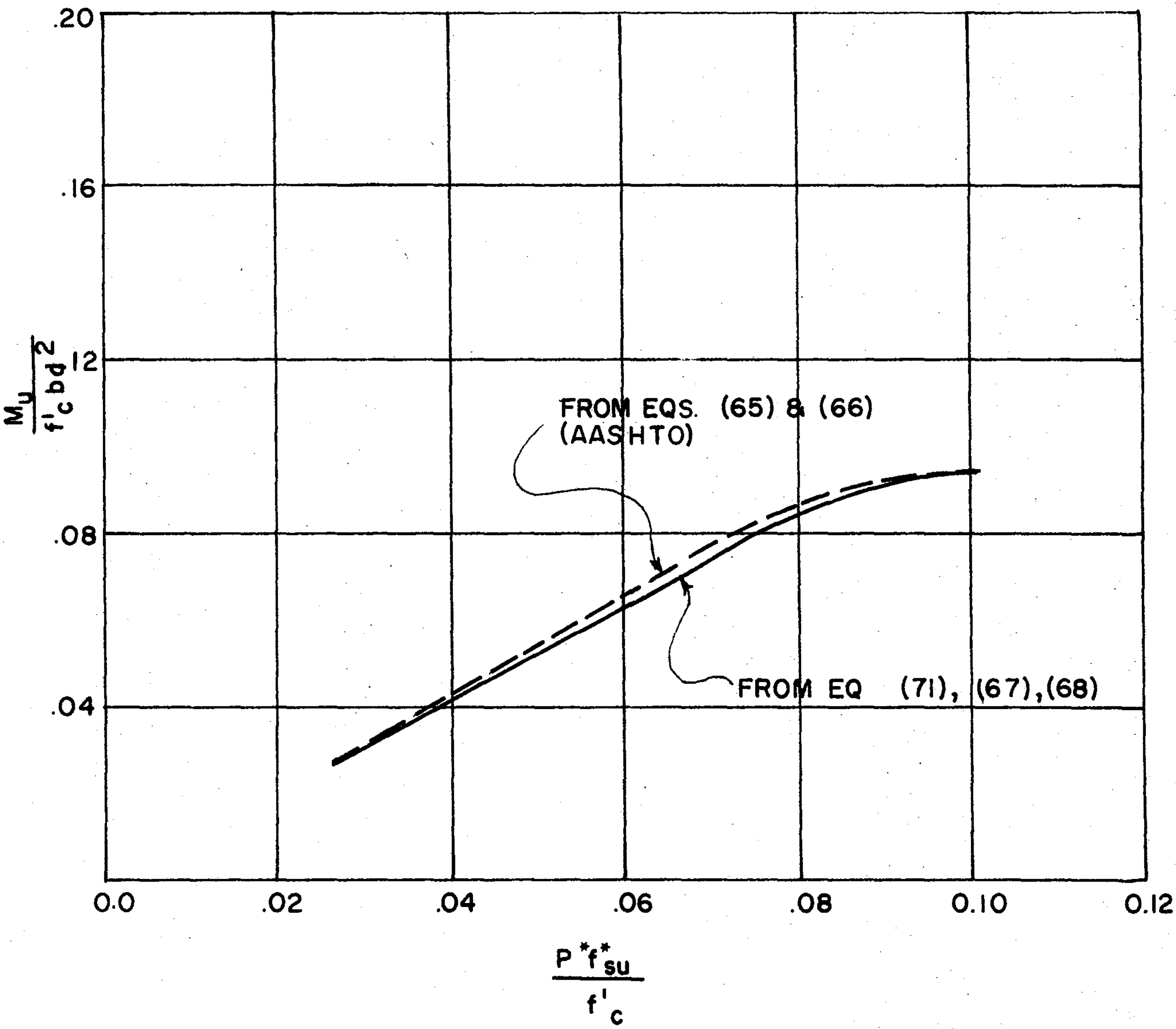


FIGURE 15. COMPARISON OF MOMENT CAPACITIES FOR NEUTRAL AXIS IN SLAB

was taken as .00002 f'_s. For practical ranges of reinforcement index, the two approaches give nearly identical results for cases where the neutral axis lies in the slab.

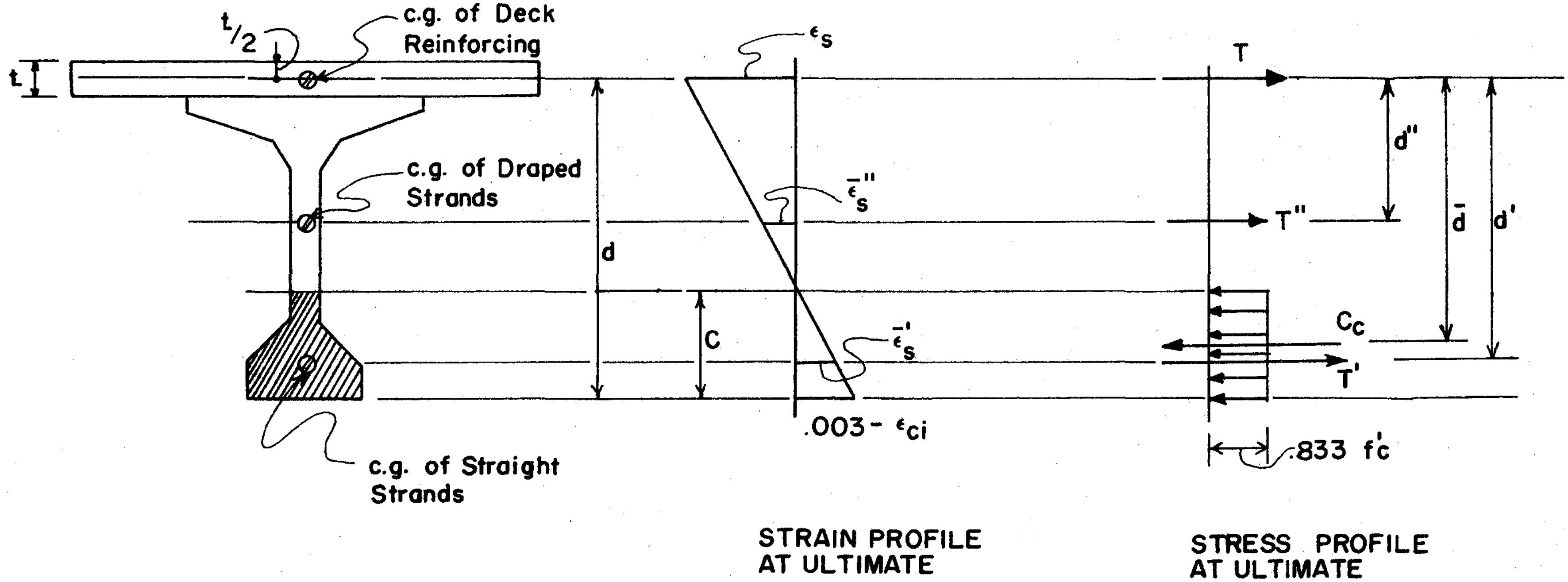
4.5 Computation of Negative Moment Deck Reinforcement

Negative moments are produced in the continuous beam by live loads, and in some cases, by creep restraint effects. Section 1.6.12(c) (3) stipulates that reinforcing be proportioned by ultimate strength design. The stress in the reinforcing under live load service conditions should not exceed 21 ksi to reduce the possibility of fatigue failure (Section 1.5.25(b)).

The computation of negative moment reinforcing for interior tenth points is based on the assumptions listed in the previous section. The computations for negative moment involve several additional considerations. As shown in Fig. 16, the compressive strain in the concrete at failure is the failure strain .003 in./in. minus the strain ε_{ci} produced by precompression due to prestress. The strands are separated into straight and drapped categories for computation of strand force. The ultimate moment capacity of the section can be computed from

$$M_u = C_c\bar{d} - T'd' - T''d'' \tag{73}$$

The values of T', d', T" and d" can be determined from the strain profile in Fig. 17 and strand stress-strain characteristics, if the location of the neutral axis c is known. The selection of the required area of deck steel to resist a specified ultimate moment proceeds by establishing a trial value of neutral axis location c which produces a net compressive force C_c equal to the tensile forces in the strands (T' + T"). The



ϵ_{ci} = INITIAL COMPRESSIVE STRAIN IN CONCRETE DUE TO PRESTRESS-ASSUMED = 0.4 f_c'

FIGURE 16. STRESS AND STRAIN PROFILES AT ULTIMATE FOR NEGATIVE MOMENT AT INTERIOR TENTH POINTS

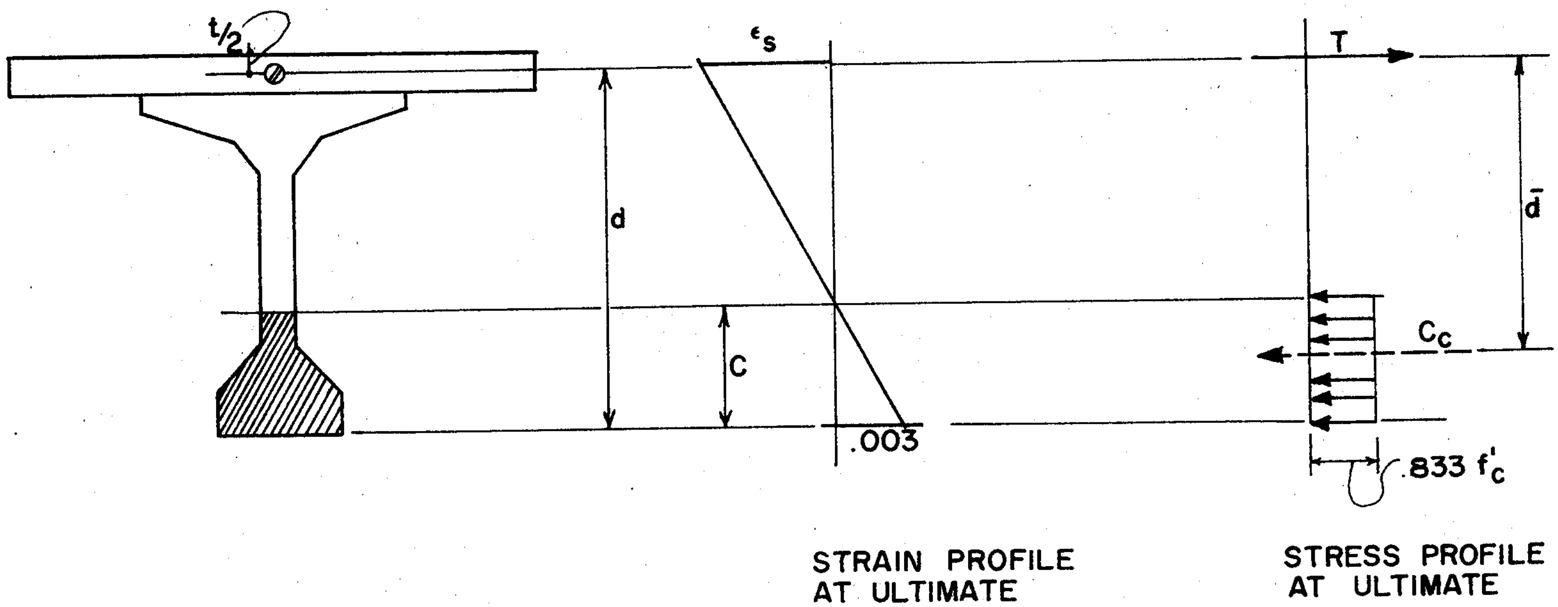


FIGURE 17. STRESS AND STRAIN PROFILE AT ULTIMATE FOR NEGATIVE MOMENT AT END OF BEAM

moment capacity for this condition is computed and compared with that required. If it exceeds the required moment (which can occur when the c.g. of the draped strands is high), then no supplemental deck steel is required to obtain the requisite capacity. If the strands alone do not provide adequate tensile force, the location of the neutral axis c is incremented, the stress in the deck reinforcing is computed from the known strain ε_s, and the area of steel necessary to satisfy equilibrium, i.e.

$$T + T' + T'' = C_c \tag{74}$$

is computed. Equation (73) is then used to compute capacity of the section. If the capacity exceeds that required, the computations are complete. If not, then c is incremented again and the process repeated.

The computation of negative moment reinforcing required at an end of a beam differs from that just described in that the prestressing strands are ineffective at this point. The pretension stress in the strands is essentially zero at the end of the beam because of the development length they require. Thus, they are ignored in the calculations. The strain profile must produce a compressive strain of .003 in./in. in the concrete at failure since there is no precompression from prestress at this point. From Fig. 17, the ultimate moment capacity of the section can be written as

$$M_u = C_c\bar{d} \tag{75}$$

The area of reinforcing required is found by trial and error. A value of the c is assumed, and the area of steel necessary to satisfy $T = C_c$

is computed from the known steel strain ε_s. The capacity is then computed from Eq. (75). If it is insufficient, c is increased and the process repeated.

4.6 Computation of Positive Moment Reinforcement at Supports

Positive moments at interior supports will generally occur from live loads in remote spans, and under certain conditions, from creep and shrinkage restraint. Nonprestressed reinforcement is determined from ultimate strength computations analogous to those for negative moment reinforcing at supports described in the previous section. The c.g. of the reinforcing is assumed to be 2.5 in. from the bottom face of the beam. To preclude fatigue failure of the bars, the stress produced by service live loads is limited to 21 ksi.

4.7 Computation of Shear Reinforcing

The required area for stirrups are computed for three segments of each span; left end to left quarter point, left quarter point to right quarter point, and right quarter point to right end. The required areas are computed from the worst condition within each segment. Stirrup spacings are computed according to AASHTO (1) and ACI (7) provisions.

The AASHTO provisions stipulate that

$$A_v = \frac{(V_{u/\phi} - V_c - V_{pr})}{2f_{sy}jd} \qquad (76)$$

where

s = stirrup spacing,

A_v = area of stirrups required,

V_u = largest (in absolute value) ultimate shear force existing in segment under consideration,

V_c = .06 $f'_c b'jd$, but not more than 180 $b'jd$,

f_{sy} = yield strength of reinforcing,

jd = 0.9 times the beam depth if the moment at the point under consideration is positive, and .875 times (beam depth - 1/2 slab thickness) if the moment is negative.

ϕ = 0.9

V_{pr} = vertical component of strand force (kips)

The required stirrup spacing s can be obtained by solving Eq. (76). The maximum spacing permitted is 12 inches (Section 1.6.14(D)).

Sections 11.1.2 and 11.6.1 of ACI gives the following expressions for minimum required stirrup area (modified here to deduct the vertical component of strand force from the shear carried by the section)

$$A_v = \frac{b_w s}{f_y} \tag{77}$$

$$A_v = \frac{(V_u - V_c - V_{ps})\, b_w s}{f_y} \tag{78}$$

where

s = stirrup spacing,

b_w = beam web width,

f_y = yield strength of reinforcing,

v_u = ultimate shear stress to be resisted, = $V_u/\phi\, b_w d$

v_c = shear stress carried by the concrete section, given by

$$v_c = 1.6 \sqrt{f'_c} + 700 \frac{V_u d}{M_u} \; ; \; 2 \sqrt{f'_c} \leq v_c \leq 5 \sqrt{f'_c} \qquad (79)$$

V_u, M_u = ultimate shear force and bending moment at the section,

d = effective depth. For positive moment it is the distance from the c.g. of the strands to the top of the beam or 0.8 times the beam depth, whichever is greater. For negative moment, it is the distance from the bottom of the beam to mid-depth of slab.

ϕ = 0.85

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