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16. Abstract

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The new method makes use of layered elastic theory developed by Vlasov and Leont'ev and a generalized form of Odemark's assumption. The non-linearity of pavement materials response to load is accounted for by letting the coefficients of vertical displacement distribution with depth and radius depend upon the geometry of the pavement. These coefficients were determined by non-linear regression analysis upon displacements that were measured at the Texas Transportation Institute's Pavement Test Facility, in which 27 different pavement sections were constructed according to a partial factorial experimental design.

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FLEXIBLE PAVEMENT DEFLECTION EQUATION USING ELASTIC MODULI AND FIELD MEASUREMENTS

by

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Research Report No. 207-7F

Flexible Pavement Evaluation and Rehabilitation

Research Study 2-8-75-207

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This report summarizes the development of a new method of predicting the vertical deflections in a multi-layered flexible pavement. The new method was developed primarily because the current method that is in use in Texas employs an empirically-derived deflection equation in which the material properties of the layers are "stiffness coefficients" which can be determined from Dynaflect deflections. The new method has the following four characteristics: (1) it is based upon elastic layered theory; (2) it makes use of material properties that can be determined by non-destructive testing in the field; (3) it is simple enough that deflection calculations can be made very rapidly and inexpensively on a computer; and (4) it uses the elastic modulus of materials since that property can also be measured in the laboratory.

The new method makes use of layered elastic theory developed by Vlasov and Leont'ev and a generalized form of Odemark's assumption. The non-linearity of pavement materials response to load is accounted for by letting the coefficients of vertical displacement distribution with depth and radius depend upon the geometry of the pavement. These coefficients were determined by non-linear regression analysis upon displacements that were measured at the Texas Transportation Institute's Pavement Test Facility, in which 27 different pavement sections were constructed according to a partial factorial experimental design.

The squared error between the observed deflection basin and the basin predicted by the new method is compared section by section with what can be achieved by Boussinesq theory and by the stiffness coefficient method that is currently in use in Texas. The new method is shown to be 4 to 200 times more accurate than the current method.

SUMMARY

This report gives details of the development of a new deflection equation which is able to predict with reasonable accuracy the vertical deflections of flexible pavements as they occur in the field. There are several reasons for developing this new deflection equation, some of which are as follows: (1) there is a need to use elastic moduli as the material property of each layer instead of the stiffness coefficients as are used in the current deflection equation because elastic moduli can be measured in the lab as well as inferred from field deflection data; (2) there is a need to develop a simplified method of inferring elastic moduli from Dynaflect deflections; (3) there is a need for a more accurate method of calculating surface deflections of pavements for use in the Texas Flexible Pavement Design System. Making use of elastic moduli will allow the Texas S.D.H.P.T. to develop correlations between laboratory and field measurements and allow these properties to be used directly in the design and evaluation of new and rehabilitated pavements.

The new method of predicting pavement deflections satisfies the following four criteria: (1) it must be based upon layered elastic theory; (2) it must make use of material properties that can be determined by nondestructive testing in the field; (3) it should be simple so that deflection computations can be made very rapidly and inexpensively on a computer; and (4) it should use the elastic modulus of materials, since that property can be measured in the laboratory as well as in the field.

The new method makes use of the layered elastic theory in a book by two Russians, Vlasov and Leont'ev and also uses a generalized form of Odemark's assumption. The non-linearity of pavement materials response to load is accounted for by letting the coefficients of vertical displacement distribution with depth and radius depend upon the thickness of the stiff surface layers of the pavement.

The coefficients were determined by non-linear regression analysis of the displacements that were measured at the Texas Transportation Institute's Pavement Test Facility, in which 27 different pavement sections were constructed according to a partial factorial experimental design. Six different types of pavements are represented at the Pavement Test Facility: (1) asphalt concrete on cement-stabilized limestone base on unbound limestone subbase; (2) sandwich construction: unbound limestone base course between asphalt concrete surface and cement stabilized limestone subbase; (3) asphalt concrete on limestone base and gravel subbase; (4) asphalt concrete on cement stabilized limestone base on gravel subbase; (5) asphalt concrete on lime-stabilized limestone base on sandy clay subbase; (6) mixed designs.

The accuracy of the prediction of the surface deflections was compared with actual deflection measurements that were made on the 27 pavement sections. The new deflection equation predicts the deflections of the entire Dynaflect basin with a mean square error that is 4 to 200 times smaller than that produced by the method that is currently in use in the Texas FPS design system and the Dynaflect analysis computer programs.

The report is divided into five parts: (1) introduction; (2) theoretical development, (3) statistical development, (4) results, (5) conclusions and recommendations. The report recommends that the new "Russian deflection equation" method should be used in the following future developments: (1) a pattern search computer program that converts Dynaflect measurements into elastic moduli of the layers; (2) a new Texas FPS design system which uses elastic moduli and the new "Russian deflection equation".

IMPLEMENTATION STATEMENT

This report gives details of the development of a new flexible pavement deflection equation which is intended to replace the deflection equation that is used in the Texas Flexible Pavement Design System (FPS) and is also used to convert Dynaflect measurements into layer stiffness coefficients. The new deflection equation uses elastic moduli instead of stiffness coefficients and as a result, these material properties can be measured in the lab as well as in the field. In order for the work that is summarized in this report to be implemented, it must be incorporated into the Texas FPS design system and also into a pattern search computer program that will convert Dynaflect deflection measurements into the elastic moduli of the layers. In addition, there needs to be a correlation between the results using a pressuremeter and the dynaflect, and the correlation should be found on a wide range of pavements in the existing highway system including a variety of soil types, climates, and traffic levels.

DISCLAIMER

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CHAPTER I

INTRODUCTION

Since the construction of the Texas Transportation Institute (TTI) Pavement Test Facility in the early 1960's it has been the objective of the pavement design program at TTI to be able to predict with reasonable accuracy the surface deflections of Texas pavements as they occur in the field. It has been a further objective that the prediction method should have the following characteristics:

1. It must be based upon or similar to elastic layered theory.
2. It must make use of material properties that can be determined by nondestructive testing in the field.

The deflection equation that was devised by F. H. Scrivner in study 2-8-62-32, "Extension of AASHO Road Test Results", and documented in a series of reports (cf. Reports 21-11, 32-12, and 32-13) met these two objectives very well. The material properties used in Scrivner's deflection equation were "stiffness coefficients" which could be derived from Dynaflect deflections by using an automated trial-and-error procedure. (1,2).

The Scrivner deflection equation was used in the Flexible Pavement System (FPS) Series of pavement design computer programs that were originally developed in Study 32 and later modified and improved in Study 1-8-68-123" "A Systems Analysis of Pavement Design and Research Implementation." The principal usefulness of the deflection equation in FPS was in being able to calculate the surface curvature index of a pavement very simply, making it possible to predict the performance of many different trial pavements and to select the best of them based upon the least total cost over the life of the pavement.

The use of stiffness coefficients was somewhat troublesome in practice, however, since they could only be inferred from Dynaflect data They appeared to depend upon the thickness and location of the pavement layer, rather than being a property of the material alone, and they could not be measured in the laboratory for purposes of comparison and control.

The usefulness of the deflection equation in FPS and the difficulty interpreting stiffness coefficient data led to two more criteria which should be satisfied by a prediction method, as follows:

3. It should be simple so that deflection computations can be made very rapidly and inexpensively on a computer.
4. It should use the elastic modulus of materials, since that property can also be measured in the laboratory.

Layered elastic theory could not be used for this purpose because of the comparatively large amounts of computer time that are required to make one set of deflection calculations on one pavement section. One attempt to avoid the computational difficulty was incorporated into the FPS-BISTRO computer program which was documented in report 123-17, entitled, "Optimization of a Flexible Pavement System Using Linear Elasticity." Spline interpolation was used in that program to short-cut the computational time ordinarily required by layered elastic computations (3).

This report summarizes the development of a new deflection equation that meets all four criteria and, in addition, appears to predict pavement surface deflections more accurately than does Scrivner's deflection equation. The new deflection equation is an approximation of elastic layered theory, as is Scrivner's, but it uses elastic moduli as the material property of each layer. The new deflection equation starts with the theoretical development in a book by two Russians, Vlasov and Leont'ev (5), and makes use of a generalized form of Odemark's assumption (6) in arriving at the final result.

The accuracy of the prediction of surface deflections was compared with actual deflection measurements that were made on the 27 pavement sections in the TTI Pavement Test Facility (4). The new deflection equation predicts the deflections of the entire basin with a mean square error that is 4 to 200 times smaller than that produced by Scrivner's deflection equation.

This report is divided into four parts: (1) theoretical development, (2) statistical development, (3) results, and (4) conclusions and recommendations.

CHAPTER II

THEORETICAL DEVELOPMENT

The new deflection equation has been nicknamed "the Russian equation "because it is based upon work that was published by Vlasov and Leont'ev (5). Their approximate elastic theory was motivated by the fact that they were primarily interested in designing beams, plates, and shells to rest upon a subgrade. A knowledge of the material properties of the subgrade was essential to producing a good design, but it was not necessary to know the material properties with a high degree of accuracy. This knowledge led to a search for approximate elastic layered theory that could produce acceptably accurate representations of the deflections, moments, and shear in the surface flexural element. This approach was adopted in this study under the assumption that if the design of pavements is to be based upon surface deflections, there is no need to use a form of elastic theory that is more exact than that proposed by Vlasov and Leont'ev. The reader who wishes to study the derivations in detail is referred to their book.

The equation of the deflection of an elastic layer of depth H above a rigid layer due to a load P applied to a rigid circular plate of radius, r_o is:

$$w(r,z) = \frac{3P\ (1 + \nu_o)\ K_o(\alpha r)\ \Psi_1(z)}{\pi E_o\ H\ \Psi_t} \tag{1}$$

for all radii greater than r_o, In this equation

E_o	=	the elastic modulus of the layer
ν_o	=	the Poisson's ratio of the layer
r	=	the radius
z	=	the depth below the surface

$$K_o(\alpha r) \quad = \quad K_o \text{ the modified Bessel function with argument, } \alpha r. \tag{1}$$

$$\alpha \quad = \quad \sqrt{\frac{k}{2t}} \tag{2}$$

$$k \quad = \quad \frac{E_o \, s_{11}}{1-\nu_o^{\,2}} \tag{3}$$

$$t \quad = \quad \frac{E_o \, r_{11}}{4(1+\nu_o)} \tag{4}$$

$$s_{11} \quad = \quad \frac{\Psi_k}{H} \quad = \int_o^H (\Psi_1')^2 \, dz \tag{5}$$

$$r_{11} \quad = \quad \frac{H \, \Psi_t}{3} \quad = \int_o^H \psi_1^2 dz \tag{6}$$

$\Psi_1(z)$ = an assumed form of distribution of vertical displacement with depth.

In this study, $\Psi_1(z)$ was assumed to be of the form:

$$\Psi_1(z) \quad = \quad \left[\frac{H-z}{H}\right]^m \tag{7}$$

where m is an exponent that is to be found from field measurements.

Substituting Eq. (7) into Eqs. (5) and (6) and eventually substituting everything into Eq. (1) produces the equations given below which are the basis of this study:

$$w(r,z) = \frac{P}{\pi} \frac{(1+\nu_o)}{E_o} \cdot \frac{2m+1}{H} \cdot Ko(\alpha r) \cdot \left[\frac{H-z}{H}\right]^m \qquad (8)$$

$$\alpha = \frac{1}{H} \left[\frac{2m^2 \quad (2m+1)}{(2m-1)(1-\nu_o)}\right]^{\frac{1}{2}} \qquad (9)$$

The equations give above are for a single elastic layer where as all pavements have at least two layers. This fact required a modification in both equations to account for multiple layers. This was done by using a generalized form of Odemark's assumption (6). That assumption transforms the thicknesses of all layers to an equivalent thickness of a material with a single modulus. The transformed total thickness of all layers is:

$$H' = \sum_{i=1}^{k} h_i \left(\frac{E_i}{E_o}\right)^n \qquad (10)$$

k = the number of layers

n = $\frac{1}{3}$ in Odemark's assumption, but is found by analysis of field measurements in this study

H' = the transformed depth of all layers

h_i = the thickness of layer i

E_i = the elastic modulus of layer i

E_o = the modulus of the datum layer which, in this study, was chosen to be the subgrade

The depth to any point below the surface is given by $\bar{z}$ as follows

$$\bar{z} = \sum_{i=1}^{l-1} h_i \left(\frac{E_i}{E_o} \right)^n + \left(z - \sum_{i=1}^{l-1} h_i \right) \left(\frac{E_l}{E_o} \right)^n \qquad (11)$$

where l = the number of the layer in which z falls

z = the depth to a given point

$\bar{z}$ = the transformed depth to that point.

The depth of the subgrade layer, h_k, is given by

$$h_k = H - \sum_{i=1}^{k-1} h_i \qquad (12)$$

where H is an effective depth of a rigid layer that must be determined by analysis from field measurement data.

The new equation for α becomes

$$\alpha = \frac{mB}{H'} \left[\frac{2(2mB+1)}{(2mB-1)\ (1-\nu_o)} \right]^{\frac{1}{2}} \qquad (13)$$

where B is a number to be derived from an analysis of field measurement data, and the remaining terms have been defined previously.

Equation 8 is revised for multilayer pavements to read

$$w(r,z) = \frac{C}{\pi} \cdot p \cdot \frac{1+\nu_o}{E_o} \cdot \frac{2m+1}{H'} \cdot K_o(\alpha r) \cdot \left[\frac{H'-\bar{z}}{H'}\right]^m \quad (14)$$

where C is a constant to be determined from an analysis of field measurement data and the remaining terms have been defined previously.

There are five constants to be determined by an analysis of field measurement data. Expected values of these constants were used as initial values in a non-linear regression analysis procedure that has been developed at TTI. These expected values are as follows.

B = 1.0

C = 1.0

m = to be determined by a separate study of vertical displacements with depth

n = 0.33 as in Odemark's assumption

H = the depth of the "rigid" layer, assumed initially to be 70 inches.

Study of Vertical Displacements with Depth

The variation of vertical displacement with depth was assumed to be of the general form:

$$w(\bar{z}) = w(o)\left[\frac{H'-\bar{z}}{H'}\right]^m \quad (15)$$

where $w(o)$ = the deflection at the surface

$w(\bar{z})$ = the deflection at the transformed depth, $\bar{z}$

This analysis requires the determination of three constants, the initial values of which are given below:

m = 1.0

n = 0.33

H' = 70 inches

The study to determine these three constants preceded the study which determined the five constants. It was found, not surprisingly, that m depends upon the structure of the pavement. Equations relating m to the total depth of stiff layers were developed from the results of this study and were used in the second study to aid in determining the other four constants, B, C, n, and H. These equations and the other results of the non-linear regression analysis made on the deflection data measured at the TTI Pavement Test Facility are given in Chapter IV.

CHAPTER III

STATISTICAL DEVELOPMENT

Horizontal and vertical deflections were measured on each of the 27 sections of pavement represented in the TTI Pavement Test Facility. The Dynaflect was used to load the pavement and accelerometers placed in a vertical hole were used to measure horizontal and vertical displacements with depth. The measurements were made in Study 136 and are recorded in Report 136-2 and its appendixes (4). These are the data that were anlalyzed to determine the constants in the equations presented in the previous chapter.

The elastic moduli of each of the materials were inferred from acoustic pulse wave velocities measured in those materials and recorded in Research Reports 32-8(7) and 32-15F(8).

It was assumed that the following elastic equation for compressional wave speed applies:

$$\rho V^2 = \frac{E\,(1-\nu)}{(1-\nu-2\nu^2)} \qquad (16)$$

where ρ = the mass density of the material

ν = the Poisson's ratio

E = the elastic modulus

V = the compressional wave velocity in the material

Table 1 lists all of the materials, the wave velocities measured in them (7), their assumed unit weights and Poisson's ratios and their calculated values of elastic modulus, all of which were used in the analysis reported here. These moduli should be verified by an independent method of measurement such as the Briaud pressuremeter (9). If there are substantial differences, then the analyses

Table 1. Calculated Elastic Moduli for Materials in the TTI Pavement Test Facility.

Material	Measured Field Pulse Velocity, ft/sec	Assumed Unit Weight, lb/ft^3	Assumed Poisson's Ratio	Calculated Elastic Modulus, lb/in^2
Crushed Limestone + 4% Cement	7309	140	0.45	425,300
Crushed Limestone + 2% Lime	5448	140	0.45	236,300
Crushed Limestone	5222	135	0.45	209,300
Gravel	3721	135	0.47	64,600
Sand Clay	2576	125	0.47	29,800
Embankment-Compacted Plastic Clay	2412	120	0.48	17,100
Subgrade	-(Assumed)-		E_o =	15,000
Asphalt Concrete	-(Assumed)-	--------------------	--------------------	500,000

reported here should be repeated and the results either altered or verified. At the present time, it is not expected that significant changes would occur in the constants to be derived.

The 27 pavement test sections were divided into typical construction types as follows:

1. Stiff thick top layers on crushed limestone base course (4 sections)
2. Sandwich construction; an unbound crushed limestone base course between two stiffer layers (4 sections)
3. Normal hot mix asphaltic concrete construction on a crushed limestone base course on a gravel subbase (4 sections)
4. Stiff, thick top layers on a gravel base course (4 sections)
5. Normal hot mix asphaltic concrete construction on lime stabilzed base course (5 sections)
6. Mixed designs (6 sections)

The materials, layer thicknesses, and corresponding section numbers are shown in Table 2.

Regression Method

None of the regression analyses which were made in this study followed the standard linear regression procedure which assumes a linear relation between the observed dependent variable y and a set of independent variables, x. If there are n unknowns $x_1, x_2, \ldots, x_n$, then n + 1 simultaneous equations are formulated and solved to determine the constants $a_o, a_1, a_2 \ldots, a_n$ in the equation.

$$\hat{y} = a_o + a_1 x_1 + a_2 x_2 + \ldots\ldots + a_n x_n \tag{17}$$

where $\hat{y}$ = the predicted value of the dependent variable.

In order for the constants a_i (i=0,n) to be the "best" values, a "least squares" criterion is imposed upon the analysis which minimizes the sum of the squares of differences between the observed y and the predicted $\hat{y}$.

The n + 1 simultaneous equations mentioned above result from imposing this minimum least squares criterion upon the analysis which determines the a-constants by taking n + 1 partial derivatives. In general, it is neither necessary nor physically correct to assume a linear dependence of y upon x. The only really necessary condition for general regression analysis to meet is some criterion for determining the "best" values of the constants that are to be found in the assumed equation. In this report, the least-squares criterion has been adopted for determining the "best" values of the constants just as is done in ordinary linear regression analysis. In general, this process used is as follows:

Step 1. Assume a functional relation between $\hat{y}$ and x. This may be symbolized as

$$\hat{y} = f(x) \tag{18}$$

Step 2. Subtract the predicted value of dependent variable $\hat{y}$ from the observed value y. This gives an error, ε . Then square the error and add it to the errors of all of the other observations. This gives an equation of the form:

$$\varepsilon_j^2 = \sum_j \left[y_j - f(x) \right]^2 \tag{19}$$

Table 2. Pavement Sections in TTI Pavement Test Facility

Section No.	Layer No.	Material	Layer Thickness, in
1.	1.	Asphalt Concrete	4.6
	2.	Limestone + Cement	3.4
	3.	Crushed Limestone	4.0
	4.	Compacted Plastic Clay	41.0
2.	1.	Asphalt Concrete	1.5
	2.	Limestone + Cement	12.0
	3.	Crushed Limestone	4.0
	4.	Compacted Plastic Clay	36.0
3.	1.	Asphalt Concrete	1.0
	2.	Limestone + Cement	4.0
	3.	Crushed Limestone	12.0
	4.	Compacted Plastic Clay	36.0
4.	1.	Asphalt Concrete	4.7
	2.	Limestone + Cement	12.3
	3.	Crushed Limestone	12.7
	4.	Compacted Plastic Clay	24.8
5.	1.	Asphaltic Concrete	5.5
	2.	Crushed Limestone	3.0
	3.	Limestone + Cement	3.5
	4.	Compacted Plastic Clay	43.0
6.	1.	Asphalt Concrete	1.0
	2.	Crushed Limestone	10.0
	3.	Limestone + Cement	6.0
	4.	Compacted Plastic Clay	36.0
7.	1.	Asphalt Concrete	1.0
	2.	Crushed Limestone	4.0
	3.	Limestone + Cement	13.0
	4.	Compacted Plastic Clay	37.0
8.	1.	Asphalt Concrete	5.5
	2.	Crushed Limestone	12.0
	3.	Limestone + Cement	12.5
	4.	Compacted Plastic Clay	26.0

Table 2. .... (cont'd)

Section No.	Layer No.	Material	Layer Thickness in
9.	1.	Asphalt Concrete	5.2
	2.	Crushed Limestone	10.8
	3.	Gravel	39.0
10.	1.	Asphalt Concrete	0.9
	2.	Crushed Limestone	16.1
	3.	Gravel	36.0
11.	1.	Asphalt Concrete	0.8
(a replicate	2.	Crushed Limestone	16.2
of Section 10)	3.	Gravel	36.0
12.	1.	Asphalt Concrete	5.5
	2.	Crushed Limestone	22.5
	3.	Gravel	25.0
13.	1.	Asphalt Concrete	5.0
	2.	Limestone + Cement	8.0
	3.	Gravel	41.5
14.	1.	Asphalt Concrete	1.1
	2.	Limestone + Cement	15.9
	3.	Gravel	36.0
15.	1.	Asphalt Concrete	1.0
	2.	Limestone + Cement	16.0
	3.	Gravel	36.5
16.	1.	Asphalt Concrete	5.0
	2.	Limestone + Cement	23.0
	3.	Gravel	26.5
17.	1.	Asphalt Concrete	2.8
	2.	Limestone + Lime	15.7
	3.	Sandy Clay	34.5
18.	1.	Asphalt Concrete	1.0
	2.	Limestone + Lime	16.0
	3.	Sandy Clay	38.0

Table 2. ....(cont'd)

Section No.	Layer No.	Material	Layer Thickness in
19.	1.	Asphalt Concrete	5.5
	2.	Limestone + Lime	14.7
	3.	Sandy Clay	34.5
20.	1.	Asphalt Concrete	3.5
	2.	Limestone + Lime	12.5
	3.	Sandy Clay	37.0
21.	1.	Asphalt Concrete	3.1
	2.	Limestone + Lime	19.9
	3.	Sandy Clay	32.0
24.	1.	Asphalt Concrete	2.8
	2.	Crushed Limestone	8.2
	3.	Limestone + Lime	7.5
	4.	Sandy Clay	35.5
25.	1.	Asphalt Concrete	3.5
	2.	Limestone + Cement	7.5
	3.	Limestone + Lime	8.0
	4.	Sandy Clay	38.0
26.	1.	Asphalt Concrete	3.0
	2.	Limestone + Lime	8.0
	3.	Crushed Limestone	7.5
	4.	Sandy Clay	35.5
27.	1.	Asphalt Concrete	3.2
	2.	Limestone + Lime	7.8
	3.	Limestone + Cement	7.0
	4.	Sandy Clay	37.0
28.	1.	Asphalt Concrete	3.0
	2.	Limestone + Lime	16.0
	3.	Compacted Plastic Clay	37.0
28.	1.	Asphalt Concrete	3.0
	2.	Limestone + Lime	16.0
	3.	Compacted Plastic Clay	37.0

Table 2. ....(cont'd)

Section No.	Layer No.	Material	Layer Thickness in
29.	1.	Asphalt Concrete	3.0
	2.	Limestone + Lime	16.0
	3.	Gravel	37.0

Step 3. Using a computerized pattern search technique, find the set of constants in f(x) which minimizes the sum of squared errors in Eq. 19.

Thus, the regression analysis used in this study met the same criteria as do ordinary linear regression analyses, but because of the way they are formulated they permit the use of more realistic equations that relate the observed values y to the independent variables.

Regression Analyses Performed

Five separate regression studies were made, the first one to determine the variation of vertical displacement with depth and the remaining four to determine the constants in different surface deflection equations, as follows:

1. Russian equation with an assumed $J_0(\alpha r)$ variation of deflection with radius.
2. Russian equation with an assumed $K_0(\alpha r)$ variation of deflection with radius.
3. Boussinesq single layer theory in which deflection varies inversely with radius.
4. Scrivner's original deflection equation.

Without going in to detail, the error terms used in each of the five regression analysis are recorded below.

Regression Analysis No. 1. The squared error equation for variation of vertical displacement with depth is as follows:

$$\varepsilon_j^2 = \left[\frac{w(10,z_j)}{w(10,0)} - \left[\frac{H' - \bar{z}_j}{H'} \right]^m \right]^2 \tag{20}$$

where ε_j = the error for the j^{th} observation

H' = the transformed depth of the section

$$H' = \sum_{i=1}^{k} h_i \left(\frac{E_i}{E_o} \right)^n \tag{21}$$

k = the total number of layers above the assumed rigid base

$w(10,0)$ = the vertical deflection on the surface of the pavement where the radius is 10 inches, and the depth is zero inches.

$w(10,\bar{z}j)$ = the vertical deflection of the pavement where the radius is 10 inches and the transformed depth is $\bar{z}_j$.

$\bar{z}_j$ = the transformed depth to a point below the surface

$$\bar{z}_j = \sum_{i=1}^{l-1} h_i \left(\frac{E_i}{E_o} \right)^n + \left(z_j - \sum_{i=1}^{l-1} h_i \right) \left(\frac{E_l}{E_o} \right)^n \tag{22}$$

l = the number of the layer in which z_j is found

h_i = the depth of the layers, $i = 1,2,...k$.

h_k = the depth of the subgrade

$$h_k = H - \sum_{i=1}^{k-1} h_i \tag{23}$$

H = the assumed depth of the pavement section down to the rigid layer.

The constants to be found by non-linear regression analysis are m, n, and H. Their starting values on each pavement section were:

m = 1.0
n = 0.33
H = 100.00 inches

Constraints were placed on the values these constants could take on in the analysis. The depth H had to be greater than 60 inches and n had to be greater than 0.30.

A separate analysis was made for each of the 27 pavement sections and new values were found for m, n, and H for each of them. The m-values appeared to be controlled by the thickness of the stiff surface courses. Equations relating m to these thicknesses were found for each of the 6 basic types of pavements at the TTI Pavement Test Facility and were used in the subsequent regression analysis.

Regression Analysis No. 2 The error equation for surface deflections with an assumed J_o (αr) deflection basin is as follows

$$\varepsilon_j = w(r_j,o) - C \frac{1 + \nu_o}{\pi E_o} \cdot \frac{2m + 1}{H'} \cdot P \cdot J_o(\alpha r_j) \qquad (24)$$

where ε_j = the error in the observation at the radius, r_j

$$\alpha = \frac{\Psi\alpha}{H'} \qquad (25)$$

$$\psi_\alpha \quad 2m\ B \left[\frac{2mB + 1}{2mB - 1}\right]^{\frac{1}{2}} \qquad (26)$$

$$H' = \sum_{i=1}^{k} h_i \left(\frac{E_i}{E_o}\right)^n \qquad (27)$$

H'_i = the depth of layer i, (1=1,2,...,k)

k = the total number of layers above the rigid base

$$h_k = H - \sum_{i=1}^{k} h_i \qquad (28)$$

$w(r_j, o)$ = the surface vertical deflection measured at radius, r_j

m = the number found in the previous regression analysis to be related to the thicknesses of the upper layer.

$J_o(\alpha r_j)$ = the zero-th order Bessel function of the first kind with argument αr_j

P = the Dynaflect

E_o = the subgrade modulus

ν_o = the Poisson's ratio of the subgrade.

The constants to be found in this regression analysis are C, B, n, and H. Their initial values on every section of pavement were as follows:

C = 1.0
B = 1.0
n = 0.30
H = 70 inches

Several constraints were placed on the values that these constants could take on in the analysis, as follows:

$$H' \geq 1.32\, r_{max} > 66 \text{ inches}$$

$$\frac{2}{3m} \leq B \leq \frac{0.6\, H'}{m\, r_{max}}$$

$$m \geq 0.10$$

The minimum constraint on B was made neccessary by the condition in Eq. 26 in which Ψ_α becomes infinite as B approaches 1/2m. The maximum constraint was based upon those ranges of J_o (αr) which are positive. The minimum constraint on H' was also based upon the range of positive values of J_o (αr). The minimum constraint on m was based upon the results of the regression analyses performed in Analysis No. 1.

Assumed values of P. ε_o, and ν_o were used in all of the surface deflection analyses. The assumed values are as follows:

$P = 1000$ lb

$\varepsilon_o = 15{,}000$ lb/in?

$\nu_o = 0.5$

The remaining regression analyses have several of the above variables in common. In every case, the sum of squared errors was computed section by section to permit comparison with the results of the other assumed basin deflection equations.

Regression Analysis No. 3. The error equation for the surface deflections with an assumed K_o (αr) deflection basin is as follows:

$$\varepsilon_j = \left[w(r_j,o) - C \frac{1 + \nu_o}{\pi E_o} \cdot \frac{2m + 1}{H'} \cdot P \cdot K_o (\alpha r) \right] \quad (28)$$

This equation is identical with the one used in Analysis No. 2, with the exception that K_o (αr) is used here instead of J_o (αr). K_o (αr_j) is the zero-th order modified Bessel function argument αr_j.

The constants to be found, their initial values and constraints, and the assumed values of P, E_o, and ν_o are all the same as in Analysis No. 2.

Regression Analysis No. 4. The analysis was done to see if Bousinesq theory for the deflection of the surface an infinite half space under a point load could be used to predict the surface deflections of a layered mass whose thicknesses had been transformed according to Odemark's assumption. The error equation for this deflection equation is as follows.

$$\varepsilon_j = \left[w(r_j,o) - C \frac{P}{E_o r_j} \right] \qquad (30)$$

where all terms have been defined previously. The initial value of C was set at 0.25 to account for the factor $(1-\nu_o^2)/\pi$ which was left out of the equation. As in the previous two analyses, the sum of squared errors as well as the best value of C was found for each section in the TTI Pavement Test Facility.

Regression Analysis No. 5. As a final check on the accuracy to the three previous analyses, Scrivner's deflection equation was used to predict surface deflection. The original values of layer stiffness coefficient that were determined for each of the materials were inserted into the deflection equation and the sum of square errors was computed for comparison with the results of the previous three analyses. The results of these computations are shown in the next chapter of this report. The Scrivner deflection equation was originally developed from Dynaflect data on these same test sections. The interested reader is referred to the series of reports from Study 32 in the list of references to this report to trace the development of Scrivner's equation in detail.

Table 3 gives values of stiffness coefficients of the different materials in the TTI Pavement Test Facility as they appear in the Study 32 reports and as they were used in the analysis reported here.

Table 3. Stiffness Coefficients of Materials at the TTI Pavement Test Facility

Material	Stiffness Coefficient
Asphalt Concrete	0.5222
Limestone + Cement	0.7902
Limestone + Lime	0.5159
Crushed Limestone	0.4716
Gravel	0.3988
Sandy Clay	0.3293
Compacted Plastic Clay	0.2709
Undisturbed Plastic Clay	0.1980

CHAPTER IV

RESULTS

The results of the five regression analysis that were explained in the previous chapter will be discussed in order in this chapter. The five analysis were as follows:

1. Variation of vertical displacement with depth.
2. Russian deflection equation with a J_o (αr) surface deflection basin.
3. Russian deflection equation with a K_o (αr) surface deflection basin.
4. Boussinesq theory deflection equation with an inverse radius surface deflection basin.
5. Scrivner's deflection equation.

Regression Analysis No. 1. The principal result of this regression analysis was to determine the power law by which vertical deflections vary with transformed depth below the surface. The power m in the equation.

$$w(r,\bar{z}) = w(r,o) \left[\frac{H' - \bar{z}}{H'}\right]^m \qquad (31)$$

was found to vary with the thickness of the stiff surface layers. The equation relating m to this thickness changed from one pavement type to another. The resulting equations are given below, for each pavement type.

1. 5-layers; stiff, thick top layers on crushed limestone base course; $(h_1 + h_2) > 5.0$ inches; (Sections 1,2,3,4)

$$m = 0.861 - 0.0421\,(h_1 + h_2) \qquad (32)$$

2. 5-layers; sandwich construction; an unbound crushed limestone base course between two stiffer layers; $h_1 < 5.5$ inches; (Sections 5,6,7, 8)

$$m = 0.539 + 0.0609 h_1 \quad (33)$$

3. 4-layers; normal hot mix asphalt concrete construction on a crushed limestone base course on a gravel subbase; $h_1 < 5.2$ inches; (Sections 9,10,11,12)

$$m = 0.704 - 0.0260 h_1 \quad (34)$$

4. 4-layers; stiff, thick top layers; asphalt concrete on cement stabilized crushed limestone on gravel subbase course; $(h_1 + h_2) > 12.0$ inches; (Sections 13,14,15,16)

$$m = 1.148 - 0.0365 (h_1 + h_2) \quad (35)$$

5. 4-layers; normal hot mix asphalt concrete construction on lime stabilized base course on sandy subbase course; $h_1 < 5.5$ inches; (Sections 17,18,19,20,21)

$$m = 0.449 - 0.0293 h_1 \quad (36)$$

6. Mixed designs: $3.0 \leq (h_1 + h_2) \leq 11.0$ inches; (Sections 24,25,26, 27,28,29)

$$m = 0.399 - 0.0181 h_1 \text{ or } (h_1 + h_2) \quad (37)$$

The thickness to be used in this equation is the total thickness of all bound surface and base materials.

These equations for m were used in Regression Analyses Nos. 2 and 3 to determine a value of m.

Regression Analyses Nos. 2. and 3. Because the deflection equations used in Analyses Nos. 2 and 3 were so similar, the constants found in each of the regression analyses will be presented together so that they can be compared. Table 4 compares the B and n values, section by section, and Table 5 compares the C and H values section by section. A graph of the B, C, and n values plotted against surface course thickness for three of the types of pavements are shown in Figures 1, 2, and 3. Figure 1 shows the results of using the J_0 - deflection basin on Pavement Type No. 3, i,e,. the normal hot mix asphalt concrete construction on crushed limestone base course. This figure is significant because it shows that the J_0 - basin with the normal hot mix construction obeys Odemark's assumption of n (=0.33) very well. All of the other pavements depart somewhat from this assumption. This finding is significant since it shows the range of validity of Odemark's assumption which is used widely in a number of pavement analysis and design systems (e.g. 10,11,). Figure 2 shows the effect on the J_0 - basin of lime-stabilizing the base course. The B - value is nearly twice as great as with the unbound base course and the n - value is always 1-1.5 times higher than Odemark's assumed value of 0.33. The effect of cement stabilized base course on the J_0 - basin deflection equation is shown in Figure 3. In this figure, the B, C, and n values are plotted against the combined thickness of asphalt concrete and cement - stabilized crushed limestone. In general, C is somewhat smaller than 1.0, B rises sharply from 0.8 to 5.2, and n remains nearly 0.5, increasing somewhat as the depth of stabilized layers increases.

The analysis that used the K_0 - basin gave values of n and H that appear questionable. All n values larger than 1.0 marked with an asterisk in Table 4 and all H values larger than 110 inches are marked similarly in Table 5. The asterisk denote values of n and H that are questionable because they exceed the

Table 4 Comparison of B and n Values From J_o and K_o Deflection Basins

Pavement Type	Section	B - Values J_o - Basin	B - Values K_o - Basin	n - Values J_o - Basin	n - Values K_o - Basin
1.	1.	1.51	1.54	0.084	0.63
	2.	2.39	2.76	0.33	1.19*
	3.	1.08	2.02	0.19	0.45
	4.	5.19	5.13	0.37	1.24*
2.	5.	1.17	1.31	0.15	0.66
	6.	1.82	1.61	0.21	0.67
	7.	1.55	1.93	0.34	1.08
	8.	0.78	2.67	0.44	0.55
3.	9.	1.06	2.48	0.35	0.66
	10.	0.88	2.38	0.30	0.67
	11.	0.87	2.46	0.32	0.67
	12.	1.05	2.61	0.38	0.62
4.	13.	0.82	2.36	0.48	1.20*
	14.	1.13	1.71	0.51	1.86*
	15.	1.21	1.53	0.51	2.17*
	16.	5.24	5.89	0.57	2.09*
5.	17.	1.68	2.20	0.45	1.51*
	18.	1.62	1.56	0.34	1.08
	19.	2.44	2.81	0.47	1.92*
	20.	1.74	2.34	0.48	1.77
	21.	1.71	2.26	0.51	1.85

Table 4 Comparison of B and n Values From J_o and K_o Deflection Basins

cont'd.....

Pavement Type	Section	B - Values J_o - Basin	B - Values K_o - Basin	n - Values J_o - Basin	n - Values K_o - Basin
6.	24.	3.63	3.16	0.17	0.54
	25.	3.33	3.95	0.35	1.21*
	26.	3.62	3.86	0.25	0.89
	27.	3.65	4.05	0.29	1.13*
	28.	11.85	12.54	0.51	1.47*
	29.	11.63	12.45	0.53	1.90*

* Questionable values

Table 5. Comparison of C and H Values From J_o and K_o Deflection Basins

Pavement Type	Section	C - Values J_o - Basin	C - Values K_o - Basin	H - Values J_o - Basin	H - Values K_o - Basin
1.	1.	1.49	2.25	73	79
	2.	1.14	3.09	74	95
	3.	1.15	1.79	61	85
	4.	1.07	3.06	75	97
2.	5.	1.54	2.48	69	85
	6.	1.40	2.26	64	82
	7.	1.06	2.40	70	93
	8.	0.75	1.12	81	163
3.	9.	0.91	1.28	73	108
	10.	0.89	1.33	73	102
	11.	0.85	1.29	73	106
	12.	0.86	1.17	72	100
4.	13.	0.82	1.43	88	147*
	14.	0.74	2.02	91	142*
	15.	0.92	3.18	89	127*
	16.	0.91	2.89	86	127*
5.	17.	1.12	2.22	82	120*
	18.	1.11	1.97	73	118*
	19.	1.00	2.96	84	127*
	20.	1.15	2.59	88	120*
	21.	1.02	2.53	86	123*

* Questionable Values

Table 5. Comparison of C and H Values From

J_o and K_o Deflection Basins cont'd...

Pavement Type	Section	C - Values		H - Values (inches)	
		J_o - Basin	K_o - Basin	J_o - Basin	K_o - Basin
6.	24	1.27	2.03	66	90
	25	0.95	2.58	76	98
	26	1.24	2.64	73	92
	27	1.27	3.21	77	96
	28	1.46	2.99	73	117*
	29	1.06	2.94	83	126*

* Questionable Values

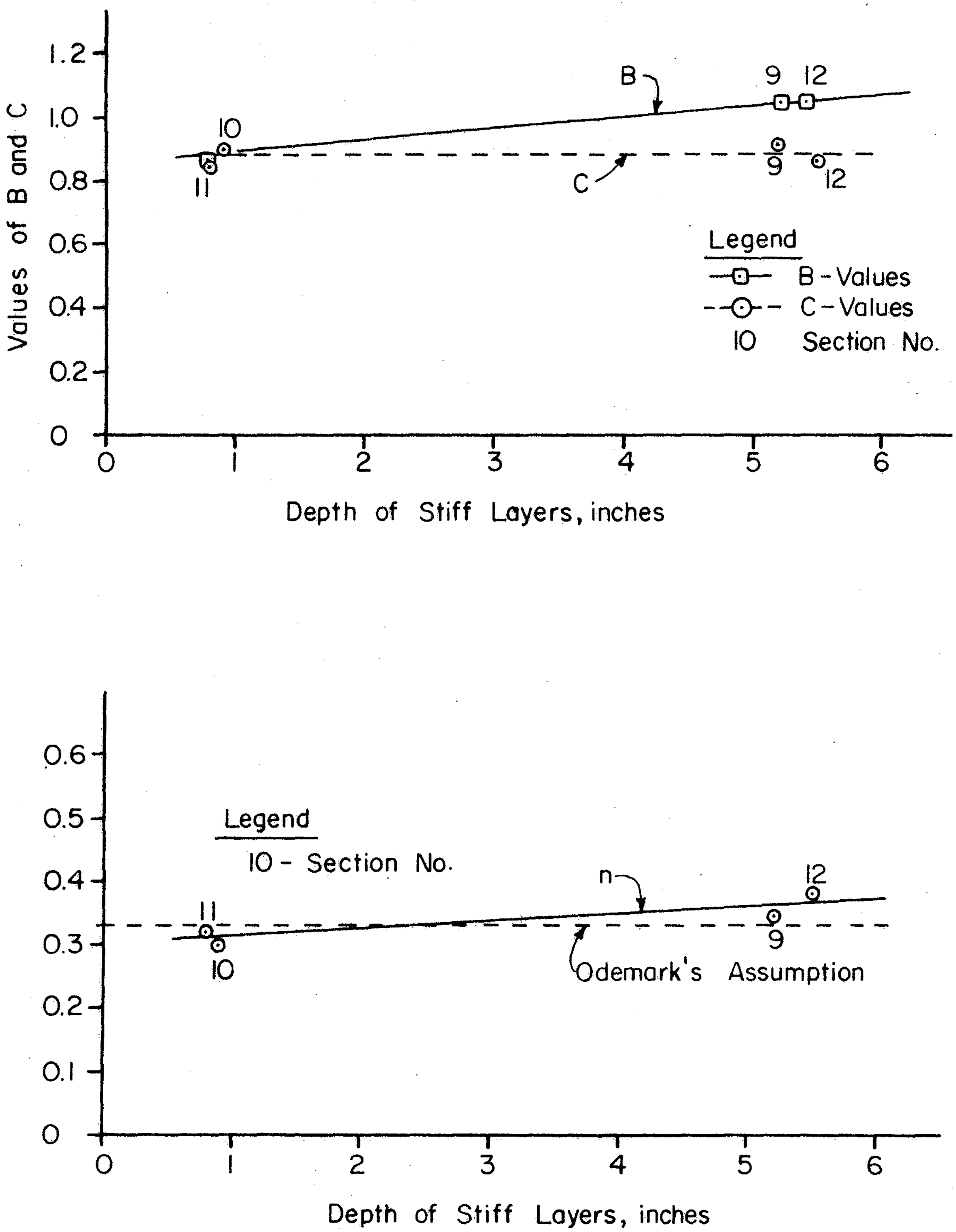


Figure 1. Russian Equations Constants for Normal Hot Mix Asphaltic Concrete Construction on Crushed Limestone Base Course on Gravel Subbase (Jo-Basin).

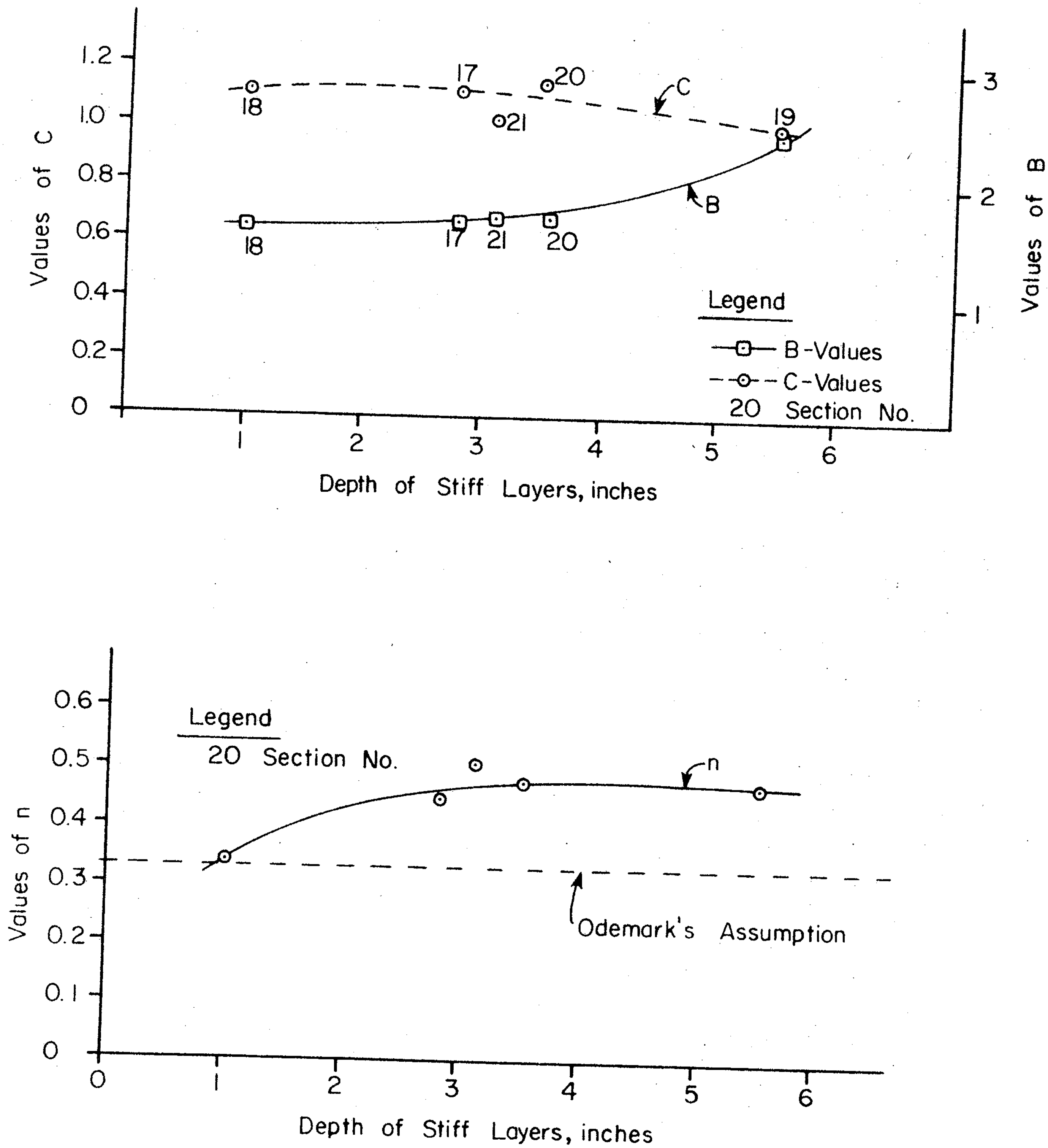
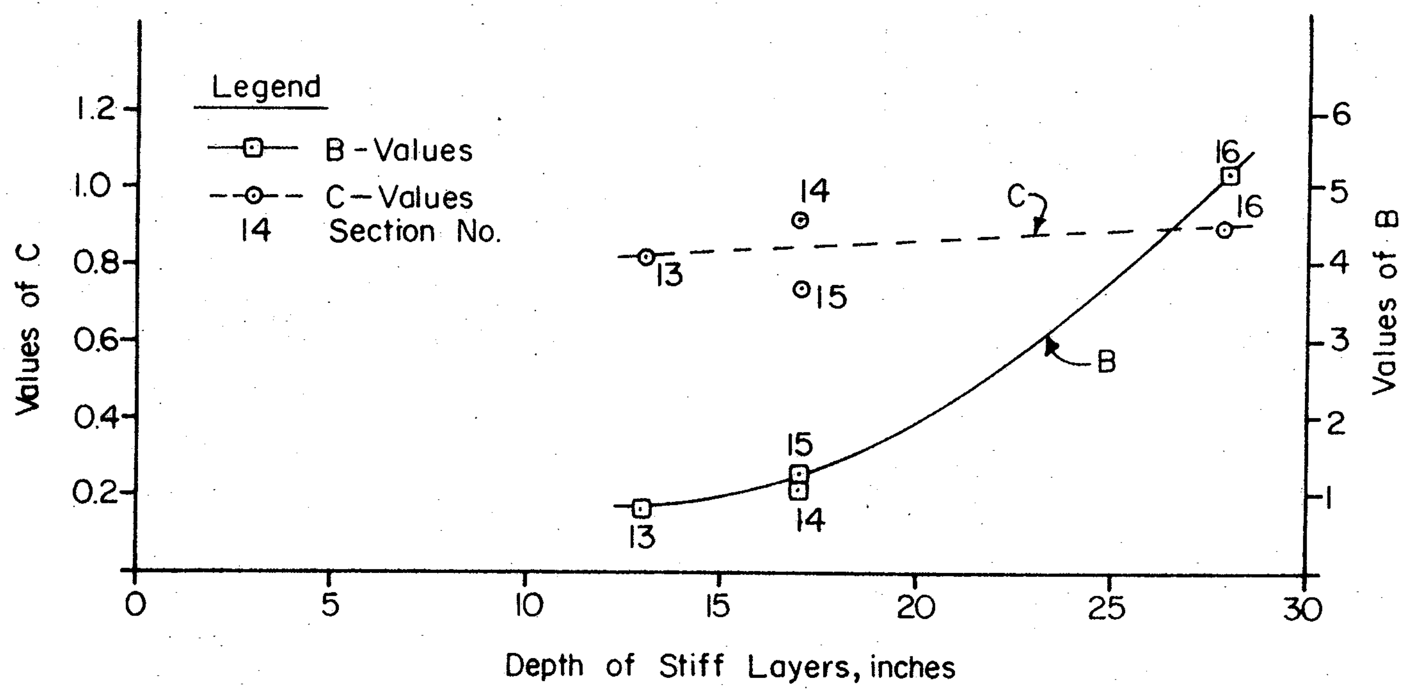


Figure 2. Russian Equations Constants for Normal Hot Mix Asphaltic Concrete Construction on Lime Stabilized Base Course (Jo-Basin).



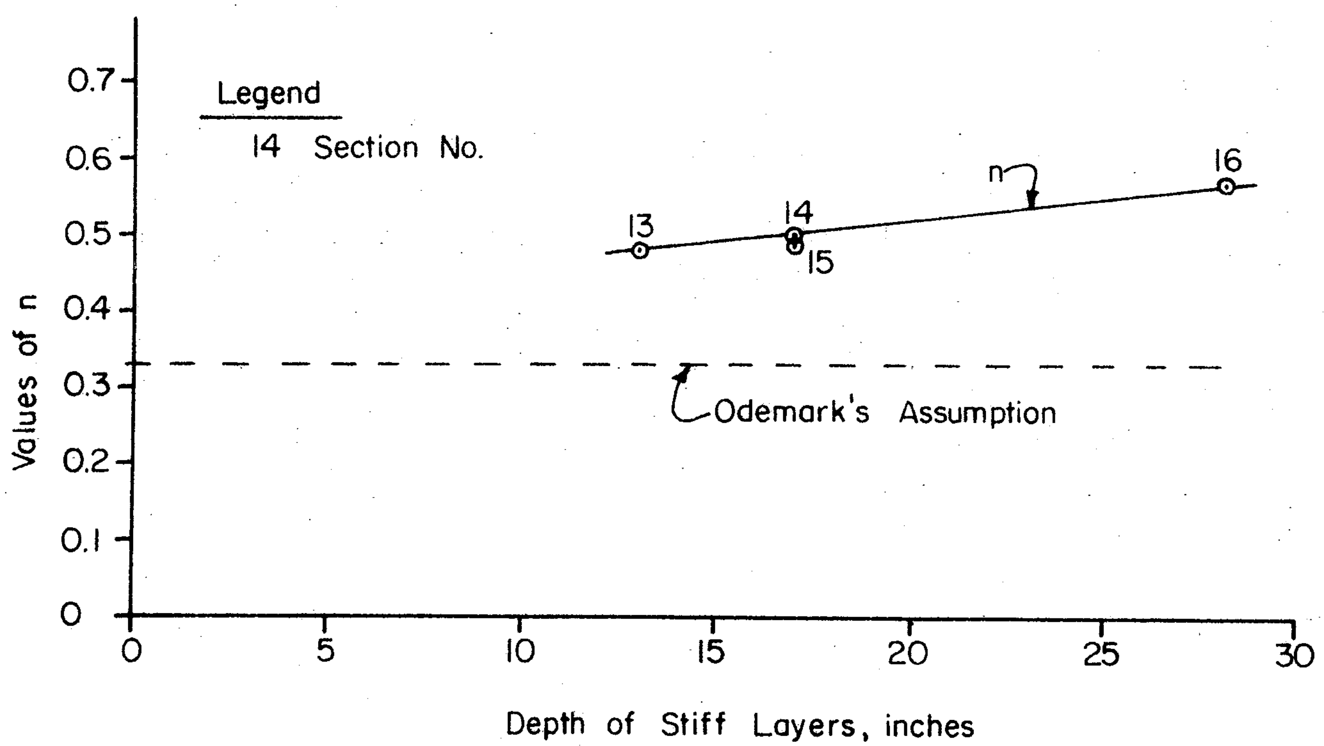


Figure 3. Russian Equations Constants for Stiff, Stabilized Top Layers on Gravel Subbase (Jo-Basin).

limits stated above.

Regression Analysis No. 4. The analysis using Boussinesq theory determined only one constant, the initial value of which was set at 0.25. Table 6 shows the final values of C as determined by the pattern search method. The "normal" pavement type (No. 3, hot mix on crushed limestone) has a fairly constant C - value of around 0.12 whereas the other types of pavement have C - values that vary above and below this value depending upon the combined thickness of stiff surface and base courses in the section.

Regression Analysis No. 5. No new constants were determined with Scrivner's deflection equation. The only calculation that was made was to determine the squared errors between the observed and predicted values of deflections.

Comparison of Surface Deflection Basins. The only valid comparison between the four deflection basins used in this study is a comparison of the sum of squared errors between observed and predicted deflections. There were 5 deflections used to compute the sum of squared errors in each case and they were located so as to simulate a Dynaflect deflection basin. Table 7 gives the computed sum of squared errors for each of the four surface deflection equations. A visual comparison can be made between these equations by noting the relative sizes of squared error terms on each horizontal line. In general, the J_0 and K_0 - Russian equations are between 4 times and 200 times more accurate in predicting a Dynaflect basin that is the Boussinesq or the Scrivner deflection equation. An asterisk has been placed beside the squared error term that is the smallest of all of the candidates. There were three results which were judged to be a tie between the J_0 and K_0 - basins, and they were on Sections 5, and 13, and 21. The J_0 - basin had the smallest error 12 times and the K_0 - basin had the smallest

Table 6. C - Value From Bousinesq Deflection Basins

Type of Pavement	Section	C - Value
1.	1.	0.22
	2.	0.092
	3.	0.19
	4.	0.068
2.	5.	0.25
	6.	0.18
	7.	0.10
	8.	0.07
3.	9.	0.12
	10.	0.13
	11.	0.13
	12.	0.11
4.	13.	0.085
	14.	0.066
	15.	0.087
	16.	0.053
5.	17.	0.10
	18.	0.13
	19.	0.081
	20.	0.091
	21.	0.084

Table 6. C - Value From Boussinesg Deflection

Basins Cont'd...

Type of Pavement	Section	C - Value
	24.	0.13
	25.	0.065
	26.	0.10
6.	27.	0.093
	28.	0.092
	29.	0.058

Table 7. Sum of Squared Errors for the Four Deflection Equations (in.^{2})

Pavement Type	Section	Russian J_o - Basin x10-8	Russian K_o - Basin x10-8	Boussinesq x10-8	Scrivner x10-8
	1.	2.62	0.81*	14.5	31.7
1.	2.	0.040*	0.53	11.5	31.6
	3.	6.39	0.081*	3.92	10.5
	4.	0.0092*	0.31	6.86	14.5
	5.	2.42*	1.72*	26.5	9.69
2.	6.	2.13	0.39*	10.7	10.2
	7.	0.12*	0.24	10.3	18.2
	8.	1.21	0.24*	1.76	22.1
	9.	3.46	0.18*	1.23	4.33
3.	10.	5.86	0.37*	0.67	4.36
	11.	4.03	0.10*	0.82	6.89
	12.	3.99	0.14*	0.44	5.78
	13.	0.30*	0.24*	4.31	3.90
4.	14.	0.035*	0.064	5.27	2.08
	15.	0.033*	0.13	9.94	0.48
	16.	0.071	0.032*	3.52	1.48
	17.	0.35	0.073*	7.97	32.6
	18.	0.68	0.35*	8.70	11.7
5.	19.	0.063*	0.17	7.99	41.2
	20.	0.21	0.041*	8.05	58.3
	21.	0.095*	0.093*	7.39	27.7

Table 7. Sum of Squared Errors for the Four Deflection Equations (in.^{2})

Pavement Type	Section	J_o - Basin	K_o - Basin	Boussinesq	Scrivner
	24.	1.94	0.025*	3.13	19.8
	25.	0.011*	0.21	5.75	22.0
6.	26.	0.26	0.092*	8.43	35.9
	27.	0.079*	0.22	10.1	13.4
	28.	0.26	0.049*	6.24	126.6
	29.	0.039*	0.050	3.76	30.5

* Smallest sum of squared errors.

error 18 times. The deflection basin for "normal" hot mix asphalt pavement (Sections 9-12) is predicted best by the K_o - equation, as are all but one of the deflection basins in the hot mix asphalt pavement sections on lime stabilized base course (Sections 17-21). The J_o - basin is equally accurate on all of the remaining types of pavement in the TTI Pavement Test Facility.

Conclusions

The new Russian deflection equations make much more accurate predictions of the surface deflections of multi-layered pavements than either the Boussinesq theory or the Scrivner deflection equation.

The Odemark assumption is shown to be accurate only for the J_o - basin Russian equation for "normal" hot mix asphalt concrete pavements with crushed limestone base courses, but in no case is the exponent n more than $\pm$ 0.25 removed from Odemark's value of 0.33.

The K_o - basin Russian equation is more accurate at predicting the deflections of "normal" hot mix asphalt concrete pavements on unbound or lime stabilized crushed limestone base courses than is the J_o - basin Russian equation. In all other types of pavements, including those with cement stabilized base courses and sandwich construction, the J_o - basin and K_o - basin equations are equally accurate, However, the constants n and H that were derived in the K_o - basin analysis have questionable physical significance in Sections 13 through 29 which includes the "normal" hot mix asphalt pavement on lime stabilized base course, and the mixed designs.

One of the more significant results of these analyses is to show that linear elastic layered theory may not be able to predict the displacements in a pavement as accurately as the empirically - modified layered elastic Russian equations reported here.

CHAPTER V

CONCLUSIONS AND RECOMMENDATION

This concluding chapter of this report summarizes the results of the development of a new multi-layered pavement deflection equation which is based upon the work of two Russians, Vlasov and Leont'ev (5) and a transformed layer thickness equation which is a more general version of the classic assumption made by Odemark (6). The new deflection equation satisfies the following criteria:

1. It is based upon elastic layered theory.
2. It uses material properties that can be determined by non-destructive testing in the field.
3. It is simple enough to permit rapid an inexpensive computations on a computer.
4. It uses the elastic modulus as the material property of each layer, a property which can also be measured in the laboratory.

Constants for the Russian deflection equation were determined by analyzing the vertical deflections of 27 different sections of pavement at the TTI Pavement Test Facility and the accuracy of single layer Boussinesq theory and the Scrivner deflection equation which is currently used in the Texas Flexible Pavement Design (FPS). The new Russian equation was found to be between 4 and 200 times more accurate than these other two methods of predicting deflections.

Two forms of deflection basin shape were tried, one using the K_0 Bessel function that was derived by Vlasov and Leont'ev and the other using a J_0 Bessel function. While the K_0 - basin was more accurate on more pavement sections than was the J_0 - basin it achieved this accuracy by using constants that are of questionable physical significance. On the other hand, the J_0 - basin required constants that were reasonably close to Odemark's original assumption and were physically reasonable. As a result, it is concluded that the Russian deflection equation with the J_o - basin should be adopted as a new basis for the Texas Flexible Pavement Design System.

Recommendations

Because of the significantly greater accuracy that can be achieved with the Russian deflection equations reported herein, as well as their ability to use elastic moduli which can be measured both in the lab and in the field, the following recommendations are made:

1. Write a pattern search computer program that use the Russian equations to convert Dynaflect measurements into elastic moduli
2. Rewrite the deflection equation that is presently used in the Texas Flexible Pavement Design System so that it uses elastic moduli and the Russian equations.
3. Make a series of confirmatory measurements of the elastic moduli of the layers in a series of pavements both at the TTI Pavement Test Facility and on selected pavements in the state highway system. Compare these latter measurements with what can be inferred from Dynaflect measurements and the Russian equations.
4. Make a further investigation of the Russian equations to determine the accuracy with which various critical stresses and strains in the pavement layers can be predicted.

In short, a broad-scale implementation of the Russian equations approach to pavement material properties determination and pavement performance prediction is recommended.

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APPENDIX A

CALCULATED AND OBSERVED DYNAFLECT DEFLECTION BASINS

Table A.1. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 1 (inches)

	Russian Jo-Basin $\times 10^{-3}$	Russian Ko-Basin $\times 10^{-3}$	Boussinesq $\times 10^{-3}$	Scrivner $\times 10^{-3}$	Observed $\times 10^{-3}$
Section No. 1					
Geophone No. 1	1.118	1.254	1.437	1.688	1.219
2	1.053	1.004	0.920	1.314	1.031
3	0.871	0.728	0.553	0.877	0.775
4	0.603	0.545	0.385	0.601	0.563
5	0.292	0.419	0.293	0.430	0.359
Section No. 2					
Geophone No. 1	0.445	0.488	0.612	0.841	0.447
2	0.433	0.429	0.392	0.748	0.425
3	0.399	0.362	0.236	0.597	0.397
4	0.346	0.314	0.164	0.460	0.362
5	0.277	0.279	0.125	0.353	0.269
Section No. 3					
Geophone No. 1	1.010	1.175	1.284	1.292	1.162
2	0.942	0.895	0.822	1.066	0.916
3	0.752	0.598	0.494	0.768	0.594
4	0.476	0.411	0.344	0.552	0.416
5	0.169	0.290	0.262	0.405	0.277
Section No. 4					
Geophone No. 1	0.325	0.359	0.452	0.595	0.325
2	0.317	0.317	0.289	0.512	0.313
3	0.295	0.269	0.174	0.434	0.297
4	0.261	0.235	0.121	0.359	0.269
5	0.215	0.210	0.092	0.291	0.214

Table A.2. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 2 (inches)

	Russian Jo-Basin $x10^{-3}$	Russian Ko-Basin $x10^{-3}$	Boussinesq $x10^{-3}$	Scrivner $x10^{-3}$	Observed $x10^{-3}$
Section No. 5					
Geophone No. 1	1.307	1.453	1.688	1.666	1.391
2	1.237	1.180	1.081	1.275	1.234
3	1.035	0.877	0.649	0.850	0.953
4	0.736	0.673	0.452	0.585	0.669
5	0.385	0.530	0.344	0.420	0.462
Section No. 6					
Geophone No. 1	0.916	1.029	1.188	1.263	1.000
2	0.865	0.829	0.760	0.989	0.853
3	0.723	0.609	0.457	0.722	0.650
4	0.512	0.462	0.318	0.528	0.450
5	0.264	0.360	0.242	0.392	0.334
Section No. 7					
Geophone No. 1	0.505	0.552	0.689	0.846	0.531
2	0.489	0.479	0.441	0.708	0.472
3	0.441	0.396	0.265	0.564	0.431
4	0.366	0.338	0.184	0.439	0.359
5	0.271	0.295	0.141	0.340	0.280
Section No. 8					
Geophone No. 1	0.343	0.393	0.472	0.716	0.419
2	0.328	0.325	0.302	0.543	0.300
3	0.286	0.250	0.181	0.428	0.225
4	0.222	0.198	0.126	0.350	0.202
5	0.144	0.161	0.096	0.285	0.181

Table A.3. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 3 (inches)

	Russian Jo-Basin $x10^{-3}$	Russian Ko-Basin $x10^{-3}$	Boussinesq $x10^{-3}$	Scrivner $x10^{-3}$	Observed $x10^{-3}$
Section No. 9					
Geophone No. 1	0.619	0.726	0.802	0.820	0.748
2	0.580	0.558	0.514	0.619	0.523
3	0.470	0.379	0.309	0.469	0.380
4	0.309	0.265	0.215	0.373	0.269
5	0.127	0.190	0.164	0.298	0.198
Section No. 10					
Geophone No. 1	0.683	0.815	0.886	0.890	0.844
2	0.638	0.615	0.567	0.659	0.578
3	0.512	0.403	0.341	0.497	0.381
4	0.329	0.272	0.237	0.392	0.278
5	0.124	0.188	0.181	0.312	0.220
Section No. 11					
Geophone No. 1	0.648	0.765	0.832	0.891	0.781
2	0.604	0.578	0.533	0.659	0.556
3	0.483	0.381	0.320	0.497	0.372
4	0.307	0.259	0.223	0.392	0.267
5	0.111	0.180	0.170	0.312	0.190
Section No. 12					
Geophone No. 1	0.585	0.699	0.753	0.766	0.714
2	0.545	0.523	0.482	0.583	0.507
3	0.434	0.340	0.290	0.452	0.321
4	0.274	0.226	0.202	0.365	0.228
5	0.095	0.155	0.154	0.294	0.178

Table A.4. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 4 (inches)

	Russian Jo-Basin	Russian Ko-Basin	Boussinesq	Scrivner	Observed
	$x10^{-3}$	$x10^{-3}$	$x10^{-3}$	$x10^{-3}$	$x10^{-3}$
Section No. 13					
Geophone No. 1	0.426	0.466	0.565	0.577	0.462
2	0.407	0.392	0.361	0.481	0.378
3	0.353	0.310	0.217	0.397	0.350
4	0.270	0.252	0.151	0.328	0.248
5	0.170	0.211	0.115	0.269	0.188
Section No. 14					
Geophone No. 1	0.319	0.346	0.442	0.420	0.328
2	0.310	0.307	0.283	0.382	0.311
3	0.286	0.262	0.170	0.335	0.278
4	0.247	0.230	0.118	0.287	0.236
5	0.197	0.206	0.090	0.242	0.206
Section No. 15					
Geophone No. 1	0.412	0.448	0.577	0.414	0.425
2	0.403	0.400	0.369	0.377	0.397
3	0.375	0.345	0.222	0.331	0.366
4	0.330	0.307	0.154	0.284	0.325
5	0.272	0.278	0.118	0.240	0.278
Section No. 16					
Geophone No. 1	0.252	0.277	0.356	0.362	0.269
2	0.246	0.246	0.228	0.294	0.241
3	0.229	0.211	0.137	0.258	0.213
4	0.203	0.187	0.095	0.228	0.198
5	0.168	0.168	0.073	0.199	0.178

Table A.5. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 5 (inches)

	Russian Jo-Basin $x10^{-3}$	Russian Ko-Basin $x10^{-3}$	Boussinesq $x10^{-3}$	Scrivner $x10^{-3}$	Observed $x10^{-3}$
Section No. 17					
Geophone No. 1	0.494	0.541	0.674	0.958	0.537
2	0.476	0.466	0.432	0.765	0.456
3	0.426	0.381	0.259	0.583	0.397
4	0.348	0.321	0.180	0.448	0.337
5	0.250	0.277	0.137	0.346	0.267
Section No. 18					
Geophone No. 1	0.658	0.727	0.863	0.975	0.697
2	0.626	0.602	0.553	0.772	0.628
3	0.535	0.463	0.332	0.580	0.500
4	0.399	0.367	0.231	0.443	0.353
5	0.235	0.299	0.176	0.342	0.279
Section No. 19					
Geophone No. 1	0.387	0.423	0.538	0.884	0.400
2	0.377	0.374	0.345	0.706	0.375
3	0.348	0.317	0.207	0.545	0.328
4	0.301	0.277	0.144	0.424	0.309
5	0.241	0.248	0.110	0.332	0.244
Section No. 20					
Geophone No. 1	0.438	0.480	0.607	1.039	0.469
2	0.425	0.420	0.389	0.824	0.416
3	0.387	0.351	0.234	0.614	0.366
4	0.328	0.303	0.163	0.465	0.311
5	0.252	0.267	0.124	0.355	0.269
Section No. 21					
Geophone No. 1	0.406	0.441	0.560	0.824	0.428
2	0.394	0.387	0.358	0.656	0.381
3	0.359	0.326	0.215	0.514	0.350
4	0.304	0.283	0.150	0.406	0.294
5	0.235	0.250	0.114	0.320	0.245

Table A.6. Dynaflect Deflection Basins - Observed and Calculated - Pavement Type 6 (inches)

	Russian Jo-Basin $x10^{-3}$	Russian Ko-Basin $x10^{-3}$	Boussinesq $x10^{-3}$	Scrivner $x10^{-3}$	Observed $x10^{-3}$
Section No. 24					
Geophone No. 1	0.646	0.739	0.837	1.031	0.737
2	0.608	0.584	0.536	0.793	0.581
3	0.501	0.415	0.322	0.591	0.422
4	0.343	0.304	0.224	0.451	0.309
5	0.161	0.229	0.171	0.348	0.216
Section No. 25					
Geophone No. 1	0.313	0.343	0.432	0.638	0.309
2	0.305	0.302	0.277	0.552	0.313
3	0.282	0.256	0.166	0.454	0.278
4	0.245	0.223	0.116	0.366	0.242
5	0.197	0.198	0.088	0.294	0.200
Section No. 26					
Geophone No. 1	0.500	0.550	0.680	0.990	0.531
2	0.482	0.472	0.435	0.785	0.478
3	0.431	0.383	0.261	0.590	0.403
4	0.351	0.321	0.182	0.451	0.331
5	0.251	0.275	0.139	0.347	0.273
Section No. 27					
Geophone No. 1	0.446	0.493	0.620	0.762	0.462
2	0.434	0.432	0.397	0.610	0.434
3	0.399	0.361	0.238	0.484	0.381
4	0.344	0.312	0.166	0.386	0.337
5	0.273	0.275	0.126	0.307	0.286
Section No. 28					
Geophone No. 1	0.451	0.494	0.611	1.287	0.484
2	0.434	0.423	0.391	1.036	0.425
3	0.385	0.343	0.235	0.754	0.356
4	0.310	0.287	0.163	0.547	0.297
5	0.217	0.245	0.125	0.403	0.237
Section No. 29					
Geophone No. 1	0.277	0.303	0.384	0.695	0.291
2	0.269	0.267	0.246	0.540	0.262
3	0.247	0.225	0.148	0.425	0.241
4	0.212	0.196	0.103	0.345	0.206
5	0.167	0.175	0.078	0.281	0.175