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# A SENSITIVITY EVALUATION OF TRAFFIC ASSIGNMENT

by

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and

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Research Report 17-2

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### ABSTRACT

The purpose of this study was to investigate the sensitivity of traffic assignment to input from the preceding modeling phases (i.e., the trip generation and trip distribution phases). The analyses focused not only on sensitivity of assignment results to inaccuracies from the modeling phases, but the sensitivity of various commonly used measures of assignment accuracy in discerning such inaccuracies.

The results indicate that the percent RMS error is the measure most sensitive to trip matrix inaccuracies, while the total vehicle miles of travel (VMT) was the least discriminating. The analyses further demonstrate that, due to the aggregative nature of the assignment procedure, many differences that may be observed at the zonal level and zonal interchange level tend to disappear in the assignment results.

Based on the results of these analyses, a "short-cut" (sketch planning) approach is proposed, which would be expected to produce assignment results of sufficient accuracy for preliminary system evaluation and comparison with other alternatives similarly modeled.

Key Words: Transportation Planning, Urban Transportation Studies, Traffic Assignment, Sketch Planning Techniques.

#### SUMMARY

The purpose of this study was to investigate the effects of different trip matrices on various measures of assignment accuracy commonly employed to evaluate traffic assignment results. Using the Tyler, Texas network, three different trip matrices were developed to generate three different traffic assignments. The first matrix (Assignment 1 Matrix) was prepared by generating a matrix of random numbers and then scaled so that the sum of interchange volumes would equal the total trips in the urban area. The second matrix (Assignment 2 Matrix) was developed in the same manner, but further constrained by the trip length frequency (TLF) of the urban area. The third matrix (Assignment 3 Matrix) was the same as the second, but further constrained by using the desired trip ends at each external station in the urban area.

Each of the three stochastic matrices were assigned, and the results were compared to an assignment produced by a fully modeled trip matrix (Existing Trip Assignment) which was developed and used in the Tyler urban transportation study. The assignment results of all four assignments were evaluated by using macro-level measures of assignment accuracy (VMT, screenlines, cutlines, travel routes) and micro-level measures of assignment accuracy (including statistical measures of link differences such as mean, standard deviation, and percent RMS error).

The results of the analyses showed that Assignment 3 (matrix constrained to total trips, TLF, and external stations) produced acceptable traffic assignment results; although, the Existing Trip Assignment proved to have the best assignment results. The micro-level measures of assignment accuracy generally showed the least sensitivity to TLF as a matrix constraint. It was also concluded that percent RMS error was the "best" (most sensitive to matrix inaccuracies) measure and the total VMT was the "poorest" (least discriminating) measure.

These analyses indicate that the assignment procedure is a powerful tool in the modeling process for the evaluation of land use and transportation system alternatives. Due to the aggregative nature of the

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assignment procedure, many differences that may be observed at the zonal level and zonal interchange level tend to disappear in the assignment results. This implies that much of the "precision" in the preceding modeling phases (i.e., trip generation and trip distribution phases) may be sacrificed and still produce reasonably accurate assignment results. This is an extremely important observation when considering the proposed short-cut approaches for first-cut system evaluation. Based on the findings using the stochastic matrices, a short-cut (or sketch planning) approach is proposed, which would be expected to produce assignment results of sufficient accuracy for preliminary system evaluation and comparison with other alternatives similarly modeled.

### IMPLEMENTATION STATEMENT

The results of this study demonstrate that many inaccuracies that may be observed at the zonal level and the zonal interchange level tend to disappear in the traffic assingment results. The analyzes further indicated that, of the various commonly used measures of assignment accuracy, the percent RMS error was the measure most sensitive to trip matrix inaccuracies, while the total vehicle miles of travel (VMT) was the least discriminating. The results of these analyses are useful to the transportation analyst in evaluating traffic assignment results.

Based on the results of these analyses, a "short-cut" (sketch planning) procedure is proposed, which would be expected to produce assignment results of sufficient accuracy for preliminary system evaluation and comparison with other alternatives similarly modeled.

# CHAPTER I - INTRODUCTION

Of the five basic phases of the modeling process used in urban transportation planning, traffic assignment is the most visible and most widely used (and misused) portion of the total process. The highway designer uses the assignment results for design capacity for an interchange or route; and, the planner uses the same results to evaluate transportation systems using something more than just intuition. The nature of the application by the two types of users (designers and planners) differs and is probably at the root of most misapplications and misinterpretations of assignment results.

The nature of the input (essentially, the preceding modeling phases generate the input for the assignment phase) and the nature of the output (computer printout that gives an impression of very precise and accurate traffic volumes for each link) lend a very deterministic appearance to the traffic assignment process. This leads many individuals into a feeling of having arrived at *the solution* upon completion of the assignment phase (rather than having arrived at a point where additional information is available for use in evaluation of the alternatives).

The traffic assignment process allocates trips (how many trips travel between what zone pairs is determined in the trip distribution phase) to a specific transportation network. Three basic assignments are generally made on any network; they are:

- existing trips to the existing network
- future trips to the existing network or existing plus committed
- future trips to a proposed future network

The first is essentially a calibrating assignment and is the only type of assignment that will be considered in this report. The other two types of traffic assignment are performed under the assumption that if the model can replicate existing traffic, it can reasonably forecast future traffic.

### Problem Statement

The accuracy of a traffic assignment is dependent upon the reliability of the trip matrix employed. How valid is this statement? The

purpose of this study was to investigate the effects of different trip matrices on various measures of assignment accuracy that are used to evaluate assignment results. Such comparisons should provide an evaluation of the power of the assignment process to mask differences in the input data. Additionally, analyses of the assignment results produced by different trip matrices should provide a means of evaluating the sensitivity of various commonly used measures of assignment accuracy.

## Method of Study

A "better-worse" approach was used in developing data for analyzing sensitivity of the measures of accuracy of traffic assignment results. Four different trip matrices were used to generate four different traffic assignments on one network. The existing network for the Tyler Urban Transportation Study was selected for test and evaluation. This coded network corsisted of 221 zones (including external stations) and 732 links (including the links to external stations but excluding centroid connector). The Tyler network offered the following advantages for the purposes of this research:

- Because of its relatively small size, modest computer expense was involved in each assignment run.
- There are no geographical or other barriers which might compound the interpretation of results.
- The network was previously used for test and evaluation.

In the Tyler Study area, there were 262,497 total vehicular trips 191,161 internal trips, 63,193 external-local trips, and 8,143 externalthrough trips. The "better-worse" gradient hypothesized that the least desirable assignment (i.e., the "worse" case) would result from a stochastic trip matrix constrained only to total trips. The fully modeled trip matrix developed in the urban transportation study was used as the standard for comparison in the analyses. The four matrices used in the analyses are defined as follows:

 Assignment 1 Matrix - a stochastic trip matrix constrained only to the total trips for the urban area. The matrix was prepared by generating a matrix of random numbers using a uniform random number generator. The random number matrix was subsequently scaled so that the sum of the interchange volumes would equal the total trips for the urban area. Residual

rounding was applied to obtain integer interchange volumes.

Assignment 2 Matrix - a stochastic trip matrix constrained to the total trips as well as the desired trip length frequency for the urban area. Again, the matrix was prepared by generating a matrix of random numbers using a uniform random number generator. To apply the trip length frequency constraint, the random numbers within a given separation interval were scaled so that the sum of the interchange volumes at that separation would equal the total desired trips at that separation for the urban area. The external-local zone pairs were assigned a common separation value equal to the maximum internal separation plus one. Similarly, the external-through zone pairs were assigned a common separation value equal to the maximum internal separation plus two. This, in effect, controlled the total number of external-through trips for the urban area. The imposition of a trip length frequency constraint was hypothesized to produce a matrix which would yield better assignment results than the Assignment 1 Matrix.

Assignment 3 Matrix - a stochastic trip matrix constrained to the total trips, the desired trip length frequency, and the desired trip ends at each external station for the urban area. Thus, this matrix is a variation of the Assignment 2 matrix. The internal travel portions of both matrices were identical. They differ only in the handling of the external-local and external-through traffic. This additional constraint was applied because of the large variance of desired trip ends associated with the external stations. It was hypothesized that this additional constraint would provide a slight improvement relative to the somewhat isolated links connected to or in the vicinity of the external stations.

resulting matrices were merged to attain the

• Existing Trip Matrix - the fully modeled trip matrix as developed and used in the urban transportation study. Trip generation and distribution were performed for each of the following trip purposes: home-based work (HBW), home-based nonwork (HBNW), nonhome-based (NHB), truck and taxi, external-local, and external-through. The

existing trip matrix.

Figure I-1 indicates the differences in the distribution of trip ends resulting with the four trip matrices. Whereas Figure I-1A shows most of the zones in the range of 2,000-2,500 trip ends, Figure I-1B shows that constraining to the trip length-frequency results in a considerable increase in the distribution of zonal trip ends. The graphs of the matrices for Assignment 2 and 3 are very similar except that Assignment 3 shows the effect on zonal trip ends of the additional constraint of external station productions in the form of a small "tail" to the right. Figure I-1D, which is the fully modeled trip matrix, shows a much more dispersed and varied distribution than the previous three figures.

Figure I-2 shows the distribution of zonal interchanges. Notice that, like Figure I-1, the Assignment 1 Matrix has a distribution very different from the three other matrices. The Assignment 2 and 3 Matrices are similar to the Existing Trip Matrix except for the significant difference in the number of interchanges with zero volume. Again, the effect of the trip length frequency is obvious.

In order to test the effect of the four different matrices on the assignment process, several measures of accuracy were used to evaluate the assignment results. The Federal Highway Administration (1) identifies the following five basic measures which were employed to compare traffic assignment results with ground counts:

- 1. A comparison of total counted volume to total assigned volume across some aggregation such as total study area, sub-areas, and/or facility types, screenlines, gridlines, and cutlines.
- 2. A comparison of total vehicle miles of travel (VMT) from ground counts to vehicle miles of travel from the assigned results.
- 3. The developing of the total weighted error between ground counts to assigned volumes at some level of aggregation.
- 4. The calculation of root-mean-square (RMS) errors comparing ground counts and assigned volumes by link within the stratification chosen for comparison.

5. A graphic comparison of ground counts versus assigned volumes. Assignment accuracy was evaluated using both macro-level and microlevel measures. Macro-level measurements of assignment accuracy are those measures that analyze the entire network or specific portions of the network; such measures include:

 Vehicle miles of travel (VMT) - this measure was calculated by multiplying the length of a link by its respective assigned or counted volume. The degree to which assigned VMT matches



Number of Zones



FIGURE I-2: DISTRIBUTION OF ZONAL INTERCHANGES

counted VMT is measured by the ratio (in percent) of assigned VMT to counted VMT.

- Screenlines compare total assigned volumes to total counted volumes of all links intersecting an imaginary line dividing the study area into two parts.
- Cutlines are similar to screenlines but intersect links of a travel corridor rather than the entire study area. This measure is somewhat more precise than screenlines in that it evaluates the assignment's ability to replicate travel on a more narrowly defined travel corridor.
- Travel routes compare counted and assigned link volumes; the volumes are accumulated along selected travel routes as opposed to volumes accumulated from intersected links as for screenlines and cutlines.

Micro-level measurements of assignment accuracy analyze the differences between counted and assigned volumes on a link by link basis; these measures are:

- Distribution of link differences by error ranges the differences between assigned and counted link volumes were tabulated for each link for absolute error ranges (±500 vpd, ±1000 vpd, ±2000 vpd, and ± 3000 vpd) and percent error ranges (±10, ±25, ±50, and ±100 percent). The number of links in each range was converted to a percentage of the total links. The distribution of differences by error ranges gives a perspective of the dispersion of error, the variability, and the extremes of the errors.
- Statistical measures of link differences three common statistical measures (mean, standard deviation, and percent RMS error) were employed in the evaluation of link differences. The following relationships were used for calculation:
  - mean difference =  $\frac{\Sigma Ai Ci}{N}$  Eq. 1
  - standard deviation of the differences =

$$\frac{\left[\Sigma(A1 - C1)^2\right] - \left(\Sigma(A1 - C1)\right)^2}{N-1}$$

Eq. 2

7

RMS error = 
$$\sum \frac{\Sigma(A_1 - C_1)^2}{N-1}$$
 Eq.3

Eq. 4

• percent RMS error = 
$$\frac{(100) \text{ RMS}}{\frac{\Sigma Ci}{N}}$$

assigned volume for link i Where: A, = counted volume for link i = total number of links N

The mean difference is a measure of the central tendency of the distribution, and it indicates if the assignment tends toward overassignment (positive mean) or underassignment (negative mean). While a mean difference tending toward zero would indicate that the over and underassignments were evenly divided, it does not necessarily follow that it is a "good" assignment.

The standard deviation is a measure of the dispersion of data about the mean, and it gives some indication of the "goodness" of the assignment. The smaller the value of the standard deviation, the closer the grouping of data about the mean. Comparisons of the standard deviations of the assignments give a relative measure of the "goodness" of the assignments.

Root-mean-square (RMS) error is very similar to the standard deviation, in that it is also a measure of dispersion of the data. However, it is a measure of dispersion of the differences relative to a zero difference, whereas, the standard deviation is relative to the mean difference. Calculation of the standard deviation involves a bias which is the mean; as the mean approaches zero, the standard deviation approaches the RMS error.

Percent RMS error measures the relationship between RMS error and the average counted volume. Since the counted volume remains the same for a given network, the average counted volume is a constant and the percent RMS error is simply RMS error divided by a constant. Thus, percent RMS error provides no new insight into analysis of a given network. However, it is valuable in comparing assignments of different networks, and it is a relative measure among networks.

# All-or-Nothing and Multiple Path Assignments

Two distinct assignments, the traditional "all-or-nothing" assignment and a "multiple path" assignment, were made by using each of the four matrices. An all-or-nothing assignment assigns trips only to the links that make up the minimum path between two zones. The weighted multiple path assighment procedure used produces an assignment in which the assigned volumes are in relative balance with the traffic counts. This is accomplished through an iterative technique, whereby the link impedances are adjusted between iterations. An all-or-nothing assignment is performed for each iteration (of which there were five), based on the "current" link impedances. The link impedances are adjusted, based on the traffic counts specified in the link data and the assigned link volumes; the impedance of each link is adjusted if its assigned volume does not equal its count. After the five iterations, the assignments are combined to obtain the final weighted multi-path assignment. This weighted assignment is calculated by applying iteration weights (i.e., percentages determined from a multiple regression analysis technique) to the respective assigned link volumes from each iteration and summing (2).

The measures of assignment accuracy were used to evaluate the first iteration all-or-nothing assignment and also the weighted multiple path assignment. The weighted assignments were analyzed to attempt to quantify the "power" of the assignment process, and to determine if the relative relationships (or rankings) of the four assignments changed from the first iteration all-or-nothing assignment to the weighted assignment.

In the following chapters, a "measure of goodness" is first established, against which the stochastic matrix assignments can be evaluated. By using the various measures of assignment accuracy, the assignments will be evaluated for the all-or-nothing assignment results and then for the weighted assignment results. Finally, the general and specific implications of stochastic matrix assignments on the transportation planning process are discussed.

# CHAPTER II - ESTABLISHING A MEASURE OF GOODNESS

Some measure of the degree of "goodness" of the results must be established for comparing the stochastic assignments with the existing trip assignment. Such a standard might be established by relating the stochastic matrix assignments to several modeled assignments. The percent RMS error and the percent assigned VMT values were selected because they are easily comparable among different networks and are measures of the network as a whole.

# Percent RMS Error

Percent RMS error appears to be a suitable statistic for comparing assignments of different networks because it accounts for varying network size and volume. Table II-1 ranks the selected networks by percent RMS and also lists the respective average counted link volumes. The average counted volumes ranged from 3,636 vpd to 10,356 vpd. For the all-ornothing assignment, the Existing Trip Assignment had the lowest value, while Assignment 3 ranked fifth at 71.7 percent. However, Assignment 3 had a lower percent RMS value than seven of the eleven conventional matrix assignments and Assignment 2 was better than five of the eleven. Even Assignment 1 (matrix constrained only to total trips) yields a percent RMS error that is within the range obtained in the assignment of observed trips to existing networks for various urban transportation studies. On a total network basis, no discernible relationship appeared to exist between either the number of links and percent RMS or the average counted volume and percent RMS for the networks listed in Table II-1.

In terms of percent RMS, the stochastic matrix assignments compare very favorably with the other study networks. But, when viewed only with the existing trip assignment, the three stochastic matrix assignments appear to have significantly larger values of percent RMS.

	NETWORK	Percent RMS Error	Average Counted Volume (vpd)
1.	Existing Trip Assignment	49.0	5020
2.	San Angelo	58.1	5091
3.	Houston-Galveston	65.5	10356
4.	Texarkana	67.1	4382
5.	Assignment 3	71.1	5020
6.	Wichita Falls	72.9	5978
7.	Abilene	77.0	3871
8.	Assignment 2	77.4	5020
9.	Lubbock	80.9	7843
10.	McAllen-Pharr	83.0	3636
11.	Amarillo	85.4	7200
12.	Assignment 1	87.8	5020
13.	Corpus Christi	90.2	7628
14.	Laredo	93.0	4280

# TABLE II-1: PERCENT RMS ERROR (ALL-OR-NOTHING)

# Percent RMS Error By Counted Volume Groups

The percent RMS data for the eleven networks were further analyzed by the counted volume groups shown in Table II-2. It should be noted that six of the eleven networks had counted link volumes greater than 25,000 vpd (thus, not all rows of percentages in Table II-2 add to 100 percent). However, these data were not included for evaluation because the Tyler network did not have links exceeding the 15,000-24,999 vpd volume group.

NETWORK	Total Links	0-999 vpd	5000 - 4999 vpd	5000 - 9999 vpd	10000 - 14999 vpd	15000 - 24999 vpd
Abilene	1337	31%	38%	20%	8%	2%
Amarillo	775	11	35	27	17	7
Corpus Christi	1069	10	33	29	17	8
Houston- Galveston	6054	15	29	23	13	12
Laredo	801	10	59	24	6	1
Lubbock	874	3	43	28	10	14
McAllen- Pharr	1739	29	47	14	8	2
San Angelo	655	9	47	35	6	3
Texarkana	905	25	42	21	7	5
Tyler	712	21	40	23	11	5
Wichita Falls	640	8	42	31	13	5

TABLE II-2: PERCENTAGE OF TOTAL LINKS IN EACH VOLUME GROUP

Table II-3 shows the various values of percent RMS for the five volume groups. The values ranged from 824 percent in the 0-999 vpd volume group to 26 percent in the 15,000-24,999 vpd volume group. Generally, the values of percent RMS decreased as the volume groups increased; however, the magnitude of change was very slight for the two largest volume groups.

Inspection of Table II-3 reveals an interesting trend exhibited by the four Tyler network assignments. The four assignments progressively improved their relative rankings from the four poorest in terms of percent RMS error in the 0-999 vpd volume group to the four best values in both the 10,000-14,999 vpd and 15,000-24,999 vpd volume groups. The behavior of the RMS error by volume group suggests that the shifting of rankings is due to some unknown characteristic of the Tyler network that favors high volume links over low volume links.

### Multiple Path (Weighted) Assignment

Values of percent RMS were also analyzed for the same networks for the weighted multiple path assignments. The values of percent RMS for the weighted assignment (Table II-4) ranged from 21.7 percent to 76.1 percent with the existing trip assignment again having the lowest value. Interestingly, there was considerable change in the relative rankings of the assignments from the all-or-nothing assignment to the weighted assignment. For instance, Lubbock went from the ninth best value in the all-or-nothing assignment (see Table II-1) to the second best value in the multiple path assignment. The improvements in values of percent RMS range between 17 and 56 percentage points (the three stochastic matrix assignments all improved about 30 percentage points). Whereas Assignments 3 and 2 ranked fifth and eighth, respectively, in Table II-1, they ranked only ninth and eleventh in Table II-4. Assignment 1, however, was still better than at least one existing study assignment (Laredo).

Table II-5 lists the percent RMS error of the various networks by volume group for the weighted assignment. The stochastic matrix assignments do not compare as favorably in the weighted assignment as they did in the all-or-nothing assignment. A ranking of eighth is the best by any of the stochastic assignments for any of the five volume groups.

# TABLE II-3: PERCENT RMS ERROR BY VOLUME GROUP (ALL-OR-NOTHING ASSIGNMENT)

0-999 VPD		1000-4999 VP	D	5000-9999 VP	D	10000-14999 V	PD	15000-24999	VPD
Network	% RMS	Network	% RMS	Network	% RMS	Network	% RMS	Network	% RMS
Houston-Galveston	154.0	Abilene	59.9	Existing Trip Assignment	27.9	Existing Trip Assignment	25.7	Existing Trip Assignment	25.9
McAllen-Pharr- Edinburg	223.5	Texarkana	61.1	San Angelo	39.4	Assignment 3	32.1	Assignment 1	26.8
Wichita Falls	223.7	Lubbock	75.2	Abilene	49.5	Assignment 2	37.0	Assignment 2	32.7
San Angelo	240.6	Wichita Falls	78.0	Assignment 3	50.1	Assignment 1	38.0	Assignment 3	32.9
Abilene	244.5	San Angelo	86.4	Lubbock	55.0	Texarkana	38.5	Texarkana	34.4
Amarillo	285.4	Houston-Galveston	87.3	Assignment 2	55.3	San Angelo	40.7	San Angelo	42.2
Texarkana	285.4	Existing Trip Assignment	87.8	McAllen-Pharr- Edinburg	58.6	McAllen-Pharr- Edinburg	41.9	Amarillo	46.0
Laredo	328.5	Amarillo	89.0	Wichita Falls	60.2	Abilene	49.6	Houston-Galveston	47.3
Lubbock	387.7	Laredo	94.9	Texarkana	61.6	Lubbock	52.3	McAllen-Pharr- Edinburg	49.1
Corpus Christi	446.3	Edinburg	96.4	Corpus Christi	63.9	Corpus Christi	52.7	Corpus Christi	55.5
Existing Trip Assignment	464.6	Assignment 3	<b>1</b> 30.1	Amarillo	65.1	Wichita Falls	56.3	Wichita Falls	55.6
Assignment 3	651.7	Assignment 2	137.9	Assignment 1	67.0	Houston-Galveston	56.9	Laredo	63.0
Assignment 2	715.1	Corpus Christi	148.6	Houston-Galveston	68.7	Amarillo	60.4	Abilene	65.1
Assignment 1	823.8	Assignment 1	163.2	Laredo	71.4	Laredo	69.4	Lubbock	65.8

Volume Groups

Assignments 2 and 3, however, are again better than at least one of the conventional matrix assignments in any volume group.

As in Table II-4, the conventional matrix assignments generally showed greater improvement between the all-or-nothing and the weighted multiple path assignments than the stochastic matrix assignments, especially for the larger volume groups.

	NETWORK	Percent RMS Error
1.	Existing Trip Assignment	21.7
2.	Lubbock	25.4
3.	San Angelo	26.3
4.	Wichita Falls	28.5
5.	Houston-Galveston	32.3
6.	Amarillo	32.3
7.	Corpus Christi	35.0
8.	Abilene	36.7
9.	Assignment 3	41.9
10.	Texarkana	42.4
11.	Assignment 2	43.9
12.	McAllen-Pharr	49.5
13.	Assignment 1	62.5
14.	Laredo	76.1

TABLE II-4: PERCENT RMS ERROR (WEIGHTED ASSIGNMENT)

# TABLE II-5: PERCENT RMS ERROR BY VOLUME GROUP (WEIGHTED ASSIGNMENT)

0-000 VPD		1000-4999 VP	n	5000-9999 VP	n	10000-14999 VE	סי	15000-24999	VPD
()=333 ¥PU		1000-4355 11		3000-3333 11		10000 11355 1			+
Network	% RMS								
Lubbock	58.0	Existing Trip Assignment	30.7	Existing Trip Assignment	18.6	Existing Trip Assignment	12.9	Existing Trip Assignment	11.7
Texarkana	86.0	Abilene	31.2	San Angelo	21.0	Texarkana	18.2	Lubbock	12.0
Existing Trip Assignment	91.6	San Angelo	33.3	Lubbock	25.3	Wichita Falls	21.4	Texarkana	12.5
San Angelo	95.4	Houston-Galveston	39.6	Abilene	25.8	San Angelo	21.8	San Angelo	14.2
Wichita Falls	97.8	Wichita Falls	41.2	Wichita Falls	25.8	Abilene	22.7	Wichita Falls	16.0
Houston-Galveston	109.5	McAllen-Pharr- Edinburg	43.5	Corpus Christi	28.3	Lubbock	22.8	Abilene	22.9
Corpus-Christi	113.4	Amarillo	44.5	Amarillo	29.8	Amarillo	24.0	Houston-Galveston	23.5
McAllen-Pharr- Edinburg	115.4	Lubbock	50.8	Houston-Galveston	30.2	Corpus Christi	26.2	Assignment 3	24.1
Assignment 3	149.8	Texarkana	54.9	Assignment 3	32.1	Houston-Galveston	27.0	Corpus Christi	24.3
Amarillo	242.0	Assignment 2	57.8	McAllen-Pharr- Edinburg	32.8	McAllen-Pharr- Edinburg	27.8	Amarillo	24.5
Abilene	249.6	Assignment 3	58.7	Assignment 2	33.6	Assignment 3	28.1	Assignment 2	25.4
Assignment 1	262.3	Corpus Christi	68.4	Texarkana	38.5	Assignment 2	28.6	Assignment 1	35.0
Assignment 2	273.1	Laredo	79.1	Laredo	53.0	Assignment 1	36.4	McAllen-Pharr- Edinburg	36.8
Laredo	324.2	Assignment 1	86.0	Assignment 1	54.4	Laredo	59.2	Laredo	57.8

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To give additional support to the rankings of the stochastic matrix assignments in Table II-4, data from traffic assignments conducted outside of Texas are shown in Table II-6. The values presented are percent standard deviation as opposed to percent RMS; however, Humphrey (3) states that a comparison of the two measures reveals the numbers to be in reasonable agreement. The values of percent standard deviation range from 30.9 percent to 55.3 percent for the ten selected cities, compared to 21.7 percent to 76.1 percent from the weighted assignment of percent RMS error. For comparison, the percent standard deviation for the four Tyler network assignments were calculated; these values are:

Assignment 1	49.4%
Assignment 2	42.8%
Assignment 3	39.3%
Existing Trip Assignment	21.5%

The Existing Trip Assignment has a better value of percent standard deviation than all of the ten selected cities. The values for the three stochastic matrix assignment, however, are all well within the bounds of values of the ten cities reported by Humphrey.

Assignments 2 and 3 and the Existing Trip Assignment have values of percent standard deviation very similar to their values of percent RMS. Assignment 1, however, has a percent RMS value of 62.5 percent and a percent standard deviation value of 49.4 percent; this large difference is explained by a mean difference of 1976 vpd. The calculation of standard deviation is relative to the mean, whereas, RMS error is not; thus, as the mean gets small the standard deviation approaches RMS error. This suggests that, if a "poor" assignment has a rather large mean difference, the percent standard deviation tends to favor poorer assignments. That is, a poor assignment will have a better value of percent standard deviation than percent RMS. Percent RMS error would, therefore, appear to be a better measure of assignment accuracy than percent standard deviation.

CITY	Percent Standard Deviation
Madison, Wis.	30.9
Salt Lake City, Utah	38.0
Atlanta, Ga.	39.0
Salem, Ore.	41.8
Denver, Colo.	44.4
Tucson, Ariz.	47.7
Sioux Falls, S.D.	49.1
Green Bay, Wis.	49.4
Honolulu, Hawaii	53.5
Portland, Ore.	55.3

# TABLE II-6:PERCENT STANDARD DEVIATION OF ASSIGNMENT<br/>RESULTS FROM SELECTED NON-TEXAS CITIES

Note: Above results are averages of 3-5 loadings (Iterations).

Source:

e: (3) Humphrey, T. F. : A Report on the Accuracy of Traffic Assignment When Using Capacity Restraint; HRR #191.

# Vehicle-Miles of Travel

Vehicle-miles of travel (VMT) were analyzed for the same 11 networks plus two additional networks (Brownsville and Bryan-College Station) that did not have data available for analysis of percent RMS. The counted VMT ranged from 303,093 VMT to 33,835,703 VMT. Table 11-7 ranks the assignments by their absolute difference from a perfect percent assigned VMT (100.0 percent). Assignments 2 and 3 (both matrices constrained to total trips and trip length frequency) rank seventh and eighth and compare very favorably with the other assignments. Assignment 1, however, is over 30 percent overassigned and considerably worse than the poorest of the conventional matrix assignments.

	NETWORK	Counted VMT	Absolute Difference from 100.0% VMT	Percent Assigned VMT (%)
1.	Houston-Galveston	33,835,703	1.0	101.0
2.	Abilene	1,118,049	1.2	98.8
3.	Existing Trip Assignment	706,309	1.4	98.6
4.	Texarkana	778,622	1.7	101,7
5.	San Angelo	551,794	2.0	98.0
6.	McAllen-Pharr	1,194,265	2.4	97.6
7.	Assignment 2	706,309	2.8	102.8
8.	Assignment 3	706,309	3.0	103.0
9.	Lubbock	1,609,041	3.6	103.6
10.	Wichita Falls	1,029,880	5.4	94.6
11.	Corpus Christi	2,547,828	5.7	105.7
12.	Amarillo	1,612,388	6.5	106.5
13.	Laredo	362,341	8.5	91.5
14.	Brownsville	303.093	17.4	117.4
15.	Bryan-College Station	404,683	20.3	79.7
16.	Assignment 1	706,309	30.7	130.7

# TABLE II-7: RANKING OF PERCENT ASSIGNED VEHICLE-MILES OF TRAVEL (ALL-OR-NOTHING ASSIGNMENT)

Analytically, the differences in the values of the first 12 assignments are not very significant; all are less than 7 percent over- or underassigned, which is rather insignificant when spread over an entire network. Although there are no written standards for acceptable values of percent assigned VMT, it is generally recognized that, if assigned VMT are within  $\pm 10$  to  $\pm 15$  percent of counted VMT, the assignment is sufficiently calibrated. These "limits" would be relaxed for analysis of VMT by facility type or some form of subarea analysis.

#### Summary

When compared to the Existing Trip Assignment, the stochastic matrix assignments appear inferior, especially as measured by percent RMS error. However, in comparison to values of percent RMS error and VMT from other networks, the stochastic assignments appear to be within acceptable limits.

Although the establishment of rigid, quantifiable limits of goodness does not appear feasible, the bounds of reasonable values of percent RMS (or percent standard deviation) can be inferred from the foregoing analyses. Figure II-1, which is a graphical representation of the data presented in Tables II-1, II-4, and II-6, implies that a reasonable value for an upper limit on percent RMS error is about 93 percent for an all-or-nothing assignment and about 76 percent for a weighted assignment. In the sense that all three stochastic matrix assignments have values of percent RMS under those limits, they could be considered to be "acceptably good."





# CHAPTER III - ANALYSES OF ALL-OR-NOTHING ASSIGNMENTS

## Macro-Level Measurements of Assignment Accuracy

Four measures of assignment accuracy at the macro-level (vehicle-miles traveled (VMT), screenlines, cutlines, and travel routes) were utilized in evaluating the results of the various assignments in the comparison of the three stochastic matrix assignments to the Existing Trip Assignment. The following summarizes the findings of these macro-level analyses.

# Vehicle-Miles of Travel

The vehicle-miles of travel were compiled for each assignment based on the assigned volume and the lengths of the links in the network. The assigned VMT were compared to the counted VMT and expressed as a percent of counted VMT (percent assigned).

Separate tabulations of VMT were made for links classified as "developed urban," "fringe," and the total network VMT (Figure III-1). Using counted traffic volumes, existing observed vehicle-miles of travel were calculated for these same classifications; 587,120 vehicle-miles were estimated for the developed urban area (83 percent) and 119,189 vehiclemiles (17 percent) in the fringe area. Table III-1 shows the percent assigned VMT for the four assignments.

When the constraint of trip length frequency is incorporated (Assignments2 and 3 Matrices), the resulting assigned total VMT is only slightly higher than that from the Existing Trip Assignment. Both assignments (2 and 3) yield excellent results for links in the "developed urban" area and only a modest overassignment in the "fringe" area. Assignments 2 and 3 yield noticeably better results than Assignment 1 (constrained to total trips only).

This verifies that VMT is sensitive to the trip length frequency. It implies that acceptable results in terms of VMT will be obtained so long as the appropriate trip length frequency is utilized or achieved in the trip distribution process even though the number of trip ends in the several zones and/or travel pattern (zone-to-zone movements) is incorrect.



FIGURE III-1: TYLER NETWORK AND LINK CLASSIFICATIONS

Assignment	Developed Urban	Fringe	Tota]
1- Total Trips	135.2%	108.3%	130.7%
2- Total Trips	100.4%	114.9%	102.8%
3- Total Trips, Trip Length, & Externals	100.9%	113.3%	103.0%
Existing Trip	97.9%	102.1%	98.6%

# TABLE III-1: ASSIGNED VEHICLE-MILES OF TRAVEL AS A PERCENT OF ESTIMATED FROM GROUND COUNTS

## Screenlines

Six screenlines were established on the Tyler network. Each of the two major screenlines, one N-S and one E-W, essentially bisected the city. The N-S screenline passed through the middle of the CBD while the E-W screenline passed immediately south of the CBD. In addition to the two major screenlines, there were four auxiliary screenlines which paralleled the major screenline and the external cordon. Since the auxiliary screenlines were north and south of the E-W screenline and east and west of the N-S screenline, they were designated E-W/N, E-W/S, N-S/E, and N-S/W (Figure III-2). It was suspected that the intersected CBD links as a group might exhibit different characteristics from the non-CBD links; therefore, the N-S screenline was analyzed in three ways: non-CBD links, CBD links, and the complete screenline.

The counted volumes crossing the six screenlines ranged in magnitude from 39,200 to 87,050 vpd. The assigned volumes of the six screenlines were compared to the counted volumes and converted to a percent such that a value greater than 100 percent indicated an overassignment. Like VMT, assigned screenline volumes are generally considered acceptable if they are within ±15 percent of the counted volumes.


FIGURE III-2: SCREENLINES

Screenline		Counted Volume	Assignment 1	Assignment 2	Assignment 3	Existing Trip Assignment
	Non-CBD Links	49,800	134.8%	94.2%	96.1%	105.0%
N-S	CBD Links	37,250	161.6%	160.9%	160.7%	101.9%
	Screenline Total	87,050	146.3%	122.6%	123.8%	103.7%
E-W		81,250	150.6%	119.1%	117.0%	99.9%
E-W/	'S	47,700	120.1%	84.3%	74.4%	97.4%
E-W/	'N	39,200	164.4%	112.4%	121.2%	114.2%
N-S/	W	53,250	81.1%	65.9%	87.9%	92.1%
N-S/	Έ	57,550	133.4%	101.1%	94.3%	88.4%

## TABLE III-2: SCREENLINE SUMMARY — ASSIGNED VOLUME AS A PERCENT OF GROUND COUNTS

Assignment 1 (matrix constrained only to total trips) generally was considerably overassigned in the CBD. Assigned screenline volumes for Assignment 1 ranged from 164 percent to 81 percent of the counted volumes none were within ±15 percent of 100 percent. Screenlines for Assignments 2 and 3 (both constrained to total trips and trip length frequency) were approximately balanced between over- and underassignments.

Although the range was about the same for Assignments 2 and 3 as it was for Assignment 1, Assignments 2 and 3 each had three screenline volumes within  $\pm 15$  percent of the counted volume. The screenlines for the Existing Trip Assignment also were evenly divided between over- and underassignment; however, the range was small and all eight of its volumes were within  $\pm 15$ percent of the counted volume (114 to 88 percent).

The CBD links of the N-S screenline accounted for 43 percent of the volume crossing the screenline and significantly affected the results. All the stochastic matrix assignments had assigned values of approximately 160 percent of counted volume for the CBD links; while the Existing Trip

Assignment was 102 percent of counted volume. For the non-CBD links, Assignment 1 was overassigned (134 percent), while the remaining three assignments were all well within ±15 percent of counted volumes. Thus, the CBD links had a strong tendency to be overassigned for the stochastic matrix assignments, but not with the Existing Trip Matrix. This results from the overestimate of CBD trip ends in the stochastic matrices. Total trip ends for the 45 CBD zones numbered approximately 109,000 for Assignment 1 and approximately 145,000 for Assignments 2 and 3; while the number of "observed" trip ends for the Existing Trip Assignment was some 80,000.

Based on screenline analysis, the Existing Trip Assignment is significantly better than all three of the stochastic matrix assignments. The Existing Trip Assignment produced screenline crossing volumes which had about one-third the error of the stochastic matrix assignments.

#### Cutlines

Seventeen cutlines (corridor intercepts) were established on the Tyler network; the positions of the cutlines on the network are shown in Figure III-3. Counted volumes for these cutlines ranged from 9,220 to 77,670 vpd. Inspection of Table III-3 shows that Assignment 1 was generally overassigned, with 12 of the 17 cutlines having assigned values exceeding 100 percent of the counted volume; only seven cutlines had assigned volumes within the acceptable  $\pm$ 15 percent of the counted volumes. In contrast, Assignments 2 and 3 were generally underassigned; both of these stochastic matrices resulted in 11 of the 17 cutlines being underassigned. Assignment 2 had six cutlines within  $\pm$ 15 percent of the ground count, while Assignment 3 had eight.

Only one of the 17 cutlines was overassigned from the Existing Trip Assignment and 12 of the cutlines were within  $\pm$ 15 percent of the ground count.

Cutline F was the only cutline to intercept some CBD links and was generally overassigned (211 percent for Assignment 1 and 111 percent for Assignments 2 and 3). The Existing Trip Assignment, however, resulted in underassignment of this cutline (78 percent of the counted volume).

Several of the cutlines were grouped to examine corridor trends with respect to assignment results. However, results varied and no strong tendencies were observed in terms of corridors. Fourteen of the seventeen cutlines showed a significant change in the cutline assigned volumes



FIGURE III-3: CUTLINES

# TABLE III-3: ASSIGNED CUTLINE VOLUMES AS A PERCENT OF COUNTED VOLUMES

# Assigned As A Percent of Counted Volumes

Cutline	Ground Count	Assignment 1	Assignment 2	Assignment 3	Existing Trip Assignment
A	23,050	113.3%	65.1%	92.3%	96.6%
В	19,100	104.8%	98.7%	99.1%	96.3%
C	28,210	112.0%	90.2%	100.0%	93.4%
D	77,670	126.8%	97.5%	94.6%	89.3%
E	22,190	136.2%	137.3%	125.3%	94.4%
F	34,050	211.1%	111.3%	111.4%	78.1%
G	62,180	·98.8%	129.5%	67.6%	78.1%
н	12,490	101.3%	63.2%	58.2%	88.0%
I	47,330	116.2%	74.7%	73.0%	88.0%
J	17,985	61.0%	56.7%	76.4%	79.5%
к	35,100	94.3%	69.8%	74.5%	88.2%
L	18,650	88.1%	61.7%	73.0%	95.8%
М	9,220	129.4%	97.3%	105.4%	91.0%
Ŋ	19,880	158.4%	107.1%	114.0%	98.7%
0	9,220	214.2%	170.8%	146.4%	117.5%
Р	36,180	154.6%	115.4%	97.6%	95.2%
Q	11,850	139.0%	70.6%	80.0%	78,3%

between Assignment 1 and Assignment 2; however, only six of those fourteen were definitely improvements. For example, cutline Q had a value of 139 percent for Assignment 1; the introduction of TLF to the matrix reduced the value to 71 percent for Assignment 2. The Existing Trip Assignment generally produced cutline volumes which were in close agreement to the counted volumes (i.e., narrower error bounds) than were produced by the stochastic matrices.

## Travel Routes

Four different travel routes were selected, and the accumulated assigned link volumes along each route were compared to the accumulated counted volumes and converted to a percent. Route A was a completely circular travel route that served as an outer loop on the Tyler traffic network. Route B was a short, generally E-W travel route that followed the top segment of the inner loop which is located approximately 1/4 mile north of the north boundary of the CBD. Route C was a generally N-S travel route running between two edges of the external cordon. Route D was an E-W travel route that followed the same path as the E-W screenline discussed previously. The counted volume for Route B was 31,300 vpd, while the volumes for the other three routes ranged between 450,000 and 467,900 vpd (Figure III-4).

From the data shown in Table III-4, it is obvious that Assignment 1 is again the most overassigned of the four and that there is little difference between Assignments 2 and 3. The Existing Trip Assignment has two routes (A and D) within  $\pm 15$  percent, while the stochastic matrix assignments have only one each.

Travel Route	Counted Volume	Assignment 1	Assignment 2	Assignment 3	Existing Trip Assignment
Route A	467,600	170.0%	125.2%	125.0%	108.5%
Route B	31,300	303.9	254.1	279.1	275.1
Route C	450,500	97.6	64.6	72.0	82.5
Route D	467,900	117.7	96.6	101.0	92.1

TABLE III-4: CUMULATIVE ROUTE VOLUME ASSIGNED AS A PERCENT OF COUNTED VOLUMES



# FIGURE III-4: TRAVEL ROUTES

#### Summary

An overall view of the results of the four macro-level measures leads to the following two observations:

- Assignment 1 (matrix constrained only to total trips) gives the poorest results. Assignments 2 and 3 (both matrices further constrained to trip length frequency) achieve about the same results and are better than Assignment 1 but not as good as the Existing Trip Assignment;
- All four measures (VMT, screenlines, cutlines, travel routes) are sensitive to trip length frequency. However, the results are variable in regards to improvement in the assignment results.

The first observation is readily apparent from the preceding discussions of the macro-level measures; the second requires some explanation.

All four macro measures showed the greatest numerical change in values between Assignments 1 and 2. For example, the numerical difference in VMT between Assignment 1 and Assignment 2 represented 87 percent of the range of values for all four assignments. Likewise, for screenlines and travel routes, the difference between the first two assignments amounted to about 60 to 80 percent of the range in values. A large numerical change was observed between Assignments 1 and 2 for 14 of the 17 cutlines. Although 6 of these 14 large numerical changes were distinct improvements, the others often changed from a large overassignment to a large underassignment. Nevertheless, it does indicate that macro measures are sensitive to TLF.

The VMT results were somewhat polarized, with Assignment 1 on the "poor end" of the scale (130 percent) and Assignments 2 and 3 and the Existing Trip Assignment on the "good end" (a change of both quality and quantity). In contrast, the screenline results of all three stochastic matrix assignments were clustered on the "poor end" of the scale (no more than three screenlines with ±15 percent of counted volumes); the Existing Trip Assignment was alone on the "good end" of the scale (all eight screenlines with ±15 percent). Thus, VMT was most sensitive to TLF in terms of improving the assignment, while screenlines were the least qualitatively sensitive to TLF.

#### Analysis of Micro-Level Measurements of Assignment Accuracy

The micro-level measures of assignment accuracy consisted of several tests that utilized the link-by-link differences between the counted volumes and the assigned volumes for analysis from various perspectives. The mean and standard deviations of the differences were computed on a total network basis, by counted volume groups, and for the network excluding CBD links. The distribution of differences by error ranges was also analyzed for the total network and by counted volume groups. Also, some basic statistical tests were conducted on the differences.

#### Mean and Standard Deviation

In determining the mean and the standard deviations of differences for the four assignments, the counted volume for any given link was subtracted from the corresponding assigned volume. For analysis, the differences were arranged in a frequency distribution table, theoretically centered about zero and spread over the range between the largest negative difference and the largest positive difference, with the differences incremented in intervals of 500 vpd.

The mean differences (an indication of the balance between over- and underassignment) for the four assignments ranged from 1,800 vpd to -93 vpd. This indicates a large general overassignment of Assignment 1 and a slight tendency toward underassignment for the Existing Trip Assignment.

Table III-5 lists the values of standard deviation of each assignment, and Figure III-5 gives a graphical representation of the distribution of differences (for clarity, only Assignment 3 and the Existing Trip Assignment are shown). Theoretically, a perfect assignment (i.e., one that did not differ from the counted volumes) would be represented by a vertical line at zero. Thus, the better the assignment, the greater the tendency of the curve to peak at zero and the lesser the tendency for the curve to spread.

Inspection of Table III-5 and Figure III-5 points up the relationship between the two data displays. In Figure III-5, the Existing Trip Assignment is peaked higher (21 links more) and is somewhat less spread than Assignment 3. The two assignments appear to be similar, with the Existing Trip Assignment judged to be slightly better. However, the



FIGURE III-5: DISTRIBUTION OF DIFFERENCES

standard deviation for the Existing Trip Assignment is almost 1,100 vpd less than that of Assignment 3. Graphically, the two assignments appear similar; while, numerically, there exists a significant difference.

Assignment	Mean	Standard Deviation
1	1800	4017
2	576	3847
3	518	3516
Existing Trip	-93	2427

TABLE III-5:	MEAN AND STANDARD DEVIATIONS	OF	DIFFERENCES
	IN LINK VOLUMES (vpd)		

Comparison of Figure III-5 and Table III-5 points out that the value of the standard deviation is sensitive to the behavior of data on the tails of the curves. The tendency to peak at zero is a necessary, but not sufficient, indicator of the goodness of the assignment. Standard deviation is a good indicator of the closeness of the fit between assigned and counted volumes, but, it can also be affected by a small proportion of very bad data points.

A comparison of the statistical values obtained for the three stochastic matrix assignments indicates that the TLF affected the mean difference (1,800 vpd for Assignment 1 versus 576 vpd for Assignment 2), but did not significantly affect the value of standard deviation. The introduction of trip length frequency as a constraint on the matrix of Assignment 2 reduced the tendency toward overassignment, but had a very minor effect on the range in assigned volume.

The mean and standard deviations of the differences between counted and assigned volumes were also computed, excluding the CBD links. This was done to determine the extent to which the CBD links as a group (representing 21 percent of the links in the network) tended to affect the assignment.

Table III-6 shows the changes in values of mean and standard deviations when the CBD links are excluded from analysis. Notice that all values were reduced, although by varying amounts. Assignments 2 and 3 decreased the most

FIGURE III-5: UISTRIBUTION OF DIFFERENCES

for both mean and standard deviations. The CBD links as a group tended to increase the degree of overassignment. For the stochastic matrix assignments, the CBD links were less affected by trip length frequency as a matrix constraint than the non-CBD links. This result is not surprising, in light of the differences in the distribution of trip ends between the stochastic matrices and the existing trip matrix.

## TABLE III-6: VALUES AND CHANGES IN MEAN AND STANDARD DEVIATION WHEN EXCLUDING CBD LINKS FROM ANALYSIS

Assignment	Mean (vpd)	Change	Standard Deviation (vpd)	Change
1	1,458	-342	3,978	-39
2	-115	-691	3,623	-224
3	-165	-683	3,184	-332
Existing Trip	-151	-58	2,336	-91 -

#### Distribution of Differences by Error Ranges

The differences between assigned volumes and counted volumes for each link were tabulated for absolute error  $(\pm 500, \pm 1,000, \pm 2,000, \text{ and } \pm 3,000 \text{ vpd})$  and percent error  $(\pm 10, \pm 25, \pm 50, \text{ and } \pm 100 \text{ percent})$  for all four assignments. The number of links, in each error range was converted to a percentage of the total number of links and the results are shown in Table III-7.

Absolute and percent errors give two slightly different views of the same data. For percent error, the magnitude of the error is relative to the volume of the given link. An over-(or under-) assignment of 500 vpd, on a link with a Counted volume of 500 vpd (100 percent error) is much more significant than an over- (or under-) assignment of 500 vpd, on a link with a counted volume of 10,000 vpd (5 percent error). Thus, while both examples would have an absolute error of 500 vpd, one would be very good and one very poor on a percent error basis.

Although Table III-7 shows the Existing Trip Assignment with values of 84.2 percent for both the  $\pm 3,000$  vpd error range and the  $\pm 100$  percent error range, this should certainly not be construed to mean that these are all the same links in both groups. Absolute error is somewhat analogous to a standard deviation, in that it is a gross measure more meaningful on a network basis. Percent error, on the other hand, is a more relative measure on a link-by-link basis.

Generally, the values in Table III-7 increase from Assignment 1 through the Existing Trip Assignment. The values for the three stochastic matrix assignments are very similar, with Assignments 1 and 2 very close for most of the error ranges and Assignment 3 somewhat better than Assignment 2. In essence, the addition of the trip length frequency constraint did not significantly affect the distribution of the percentage of links in different error ranges.

		Absolute	Error		Percent Error			
Assignment	±500 vpd	±1000 vpd	±2000 vpd	±3000 vpd	±10%	±25%	±50%	±100%
1	13.9	25.3	47.1	62.3	10.7	23.1	43.2	66.4
2	15.0	27.9	45.4	62.3	6.8	22.0	47.4	72.1
3	21.0	35.4	55.9	69.8	11.9	29.0	52.2	77.3
Existing Trip	27.7	45.5	69.1	84.2	18.3	39.2	63.7	84.2

TABLE III-7: DISTRIBUTION OF DIFFERENCES BY ERROR RANGES (All Values in Percent)

#### Mean and Standard Deviations of Differences by Counted Volume Groups

To further investigate the distribution of differences between assigned and counted link volumes, the network links were divided into four counted volume groups and analyzed to determine if tendencies of the assignments could be attributed to links of a particular volume group. The volume group. The volume groups were established as follows:

•	1 - 999 vpd	— 153 links	- 21% of network
•	1000 - 4999 vpd	— 290 links	— 40% of network
•	5000 - 9999 vpd	— 173 links	— 23% of network
•	10000 vpd and above	— 116 links	- 16% of network

Note: Differences in the number of links shown above, from the totals in Chapter II, are due to the inclusion of the links to the 20 external stations in the above analysis.

Inspection of Figure III-6 shows a very obvious trend toward a flattening of peaks and an increased spread of data as the volumes increased. The plot of the assignments for the 1-999 vpd volume group shows a large peak at zero but also a long positive tail; the positive tail being verified by the values of mean difference ranging from 1975 to 832 vpd (Table III-8). On the other hand, the plot of the 10,000 vpd and above volume group peaks about zero only for the Existing Trip Assignment and generally is very flat and widely dispersed. The mean differences generally tended to become less positive as volume increased (though the three stochastic matrix assignments all had larger means in the 1000-4999 vpd volume group than in the 1-999 vpd volume group).

The 1-999 and 1000-4999 vpd volume groups were overassigned for all assignments, with means ranging from 547 vpd to 2405 vpd. The 5000-9999 vpd volume group had both under- and overassignments; while the 10,000 vpd and above volume group was, with the exception of Assignment 1, very underassigned.

It is interesting to note (Figure III-6) that the negative tail (the dispersion of negative differences) increased with each successively larger volume group. This is due to the limits of the volume groups and the fact that differences are being analyzed. Since the differences are computed by subtracting the assigned volume from the counted volume, the largest negative difference that can exist is controlled by the upper limit of



FIGURE III-6: DISTRIBUTION OF DIFFERENCES BY VOLUME GROUPS

the volume group. As the bound of the volume group increases, the likelihood of larger negative differences also increases. With this in mind, the tendency of the assignments to have less positive means as the volume groups increase, is probably less a function of underassignments on high volume links than it is a natural function of analysis by bounded volume groups.

Examination of the values of standard deviation in Table III-8 verifies the trend toward greater dispersion of data with increasing volume groups as observed in Figure III-6. Generally, the standard deviation increased with increasing volume groups. For all assignments, the value of standard deviation was lowest for the links of the 1-999 vpd volume group and highest for the 10,000 vpd and above volume group.

In comparing the three stochastic matrix assignments, no large division of quality is found among them. Assignment 2 is noticeably better than Assignment 1; and. Assignment 3 is better still, although to a slightly lesser degree. The largest difference between values of standard deviation exists between Assignment 3 and the Existing Trip Assignment.

·	1-999 vpd		1000	1000-4999 vpd 5000-		-9999 vpd	10000	vpd and above
Assignment	Mean Vpd	Standard Standa an Deviation Mean Deviat pd vpd vpd vpd		Standard Deviation vpd	Mean vpd	Standard Deviation vpd	Mean vpd	Standard Deviation vpd
1	1975	3071	2405	3709	1259	4583	863	4674
2	1720	2723	1819	3271	-371	3893	-2625	4168
3	1311	2556	1491	3192	46	3516	-2258	3809
Existing Trip	832	1871	547	2285	-740	1811	-1949	2915

TABLE III-8: MEAN AND STANDARD DEVIATIONS BY COUNTED VOLUME GROUP

Because the values of standard deviation are largest for the largest volume groups, it would appear that the high volume links receive the poorest assignments. However, it may be recalled from Chapter II that the values of percent RMS decreased with increasing volume groups because the value of RMS error (similar to the value of standard deviation) was being divided by the average count of the volume group. Thus, relative to the average count of a counted volume group (actually in terms of percent standard deviation) the standard deviation of the differences is somewhat better for the large volume groups.

## Distribution of Differences by Error Ranges

The previously discussed error ranges (absolute and percent) were further analyzed by tabulating the data by counted volume groups (Table III-9). Generally, respective percentage values of absolute error decreased with increasing volume group. For example, 29.4 percent of the Assignment 1 links having counted volumes of 1-999 vpd were within  $\pm 500$ vpd; while, only 6.9 percent of the links having counted volumes of 10,000 vpd and above were within  $\pm 500$  vpd. The trend was exactly the opposite for the values of percent error; as the volume group increased, the respective values of percent error increased.

As with other measures, the Existing Trip Assignment again had the best results. Interestingly, though, Assignment 1 was not consistently the poorest assignment. Of the 32 values of absolute or percent error for each assignment in Table III-9, Assignment 1 had a better value than Assignment 2 for 15 values. Assignment 3 was consistently better than both Assignments 1 and 2. Differences by error ranges for counted volumegroups as measures of assignment accuracy are not very sensitive to TLF as a matrix constraint.

TABLE III-9: DISTRIBUTION OF DIFFERENCES BY ERROR RANGES FOR COUNTED VOLUME GROUPS

1-333 Vpd									
		Absolute	e Error	(vpd)	Percent Error				
Assignment	±500	±1000	±2000	+3000	±10%	<b>+</b> 25%	<b>+</b> 50%	+100%	
1 2 3 Existing Trip	29.4% 37.3 41.2 52.9	41.8% 54.9 63.4 74.5	65.4% 68.6 80.4 85.0	78.4% 75.8 86.3 89.5	3.9% 5.9 7.2 8.5	5.9% 7.8 10.5 11.1	9.2% 16.3 19.6 21.6	27.5% 38.6 45.1 54.3	

1-999 vpd

1000-4999 vpd

1 2 3 Existing	12.4% 13.5 21.7	24.1% 31.7 37.2	49.7% 53.1 59.7	65.5% 71.0 75.9	5.5% 4.5 7.9	13.5% 20.0 24.5	35.2% 40.0 45.2	62.1% 65.2 75.2
Trip	24.1	42.8	70.3	86.6	12.8	28.6	55.2	84.1

5000-9999 vpd

1 2 3 Existing	7.5% 3.5 10.4	16.8% 8.1 20.8	35.8% 29.5 43.9	50.9% 50.3 61.3	13.3% 5.2 16.2	31.2% 23.1 38.7	57.8% 63.0 68.8	87.3% 94.8 94.2
Trip	19.7	36.4	68.2	88.4	27.8	63.0	91.9	100.0

10,000 vpd and above

1	6.9%	19.0%	33.6%	50.0%	28.5%	57.8%	86.2%	97.4%
2	6.9	12.1	18.9	40.5	16.4	44.0	83.6	100.0
3	8.6	15.5	31.9	45.7	21.6	50.9	87.9	100.0
Trip	15.5	27.6	46.6	64.7	31.0	67.2	98.3	100.0

# Statistical Tests of Differences

Three different statistical tests were employed to determine if any of the differences between assigned and counted volumes were statistically significant. All tests were performed using Assignment 3 and/or the Existing Trip Assignment. Assignments 1 and 2 were not tested because Assignment 3 was judged to be the best of the three stochastic matrix assignments and, therefore, if any of them would prove statistically cignificant it would be Assignment 3. A Chi-Square test for normality was performed using the differences between Assignment 3 link volumes and the counted link volumes. The Chi-Square test yielded values of  $\chi^2_{calc} = 185.5$  and  $\chi^2_{.05} = 26.3$ . These values indicate that the distribution of differences by link between Assignment 3 and the counted volumes is significantly different from a normal distribution. Figure III-7 shows two different distributions having the same mean and standard deviation. This demonstrates that these two measures alone are not an accurate indication of how closely the data clusters about a zero difference.

A Wilcoxson Signed Rank test was performed to compare Assignment 3 and the Existing Trip Assignment with counted volumes. Actually, the comparison was of the differences by link between Assignment 3 and the counted volumes and the Existing Trip Assignment and the counted volumes. The hypothesis tested was that differences between Assignment 3 and the counted volumes are the same as those between the Existing Trip Assignment and the counted volumes. The results of the test showed that the hypothesis could not be accepted at the 0.05 level of significance. Thus, it was concluded that Assignment 3 differed from the counted volumes to a significantly greater degree than the Existing Trip Assignment, and the Existing Trip Assignment was statistically better.

The Chi-Square, Goodness of Fit test was performed on both Assignment 3 and the Existing Trip Assignment using volume group intervals of 500 vpd and comparing the number of links (assigned vs counted) in each volume group. The hypothesis tested was that the assigned link volumes would be distributed like the counted link volumes. The hypothesis was rejected at the 0.05 level of significance for both Assignment 3 and the Existing Trip Assignment.

It seems apparent that there is distinct statistical significance to the differences between assigned and counted volumes, for Assignment 3 as well as the Existing Trip Assignment. Also, the differences between Assignment 3 and the Existing Trip Assignment are statistically significant. This suggests that the statistical checks are much more precise than the assignment process is capable of achieving. Analytical significance, though somewhat subjective, is more relevant to the analysis of assignments than is statistical significance.



#### Summary

NORMAL DISTRIBUTION LINEINUS I UN

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ASSIGNMENT

With the exception of the mean difference for the network excluding CBD links, the introduction of trip length frequency as a matrix constraint generally had little effect on the micro-level measures. The largest numerical difference between any two adjacent assignments (1-2-3-Existing Trip) for a given measure was normally between Assignment 3 and the Existing Trip Assignment. The mean difference for the entire network showed Assignments 2 and 3 much better than Assignment 1, but they were also significantly poorer than the Existing Trip Assignment. In general, all micro-level measures indicated that Assignment 1 had the poorest results; Assignment 2 usually slightly better; Assignment 3 slightly better than Assignment 2; and the Existing Trip Assignment the best of all.

# Evaluation

Analysis of both the macro-level and micro-level measures has shown the similarities and the differences in their behavior with respect to the stochastic trip matrices.

For all measures of assignment accuracy, based on results of the all-or-nothing assignment, Assignment 1 gave the poorest results, Assignments 2 and 3 were better, and the Existing Trip Assignment gave the best results. In contrast, however, the micro-level measures generally showed the greatest improvement in assignment results between Assignment 3 and the Existing Trip Assignment, while the macro-level measures all showed the greatest improvement between Assignments 1 and 2. Thus, the microlevel measures appear to be relatively less sensitive to the trip length frequency than the macro measures. However, they are more sensitive than the macro measures to the distribution of zonal trip ends. The sensitivity of macro measures and micro measures to TLF and the distribution of zonal trip ends would appear to relate in the following manner:



This diagram suggests that, as the measures are listed from top to bottom, there is a decreasing tendency to hide matrix inaccuracies. Analysis of VMT results has shown that, as long as an accurate trip length frequency is used in the trip matrix, assigned VMT will very closely match counted VMT, even with a fairly uniform (and unrealistic) distribution of zonal trip ends. On the other hand, three of the micro-level measures tended to show their greatest improvement between a stochastic matrix and the existing trip matrix. This indicates a tendency to be most influenced by the distribution of trip ends.

As a measure of the accuracy of an assignment, VMT is the least discriminating of the eight measures analyzed, while percent RMS error would appear to be the "best" measure. Standard deviation probably is most sensitive to the distribution of trips; however, it is difficult to know a reasonable value of standard deviation for any assignment because it is so dependent on network size. For instance, the Tyler study contained 712 network links having an average counted volume of 5,020 vpd; and, the Houston study contained 6,054 network links with an average counted volume of 10,356 vpd. Houston will obviously have some assigned volumes much larger than those for Tyler. Due to the nature of the statistic, the standard deviation will be larger for Houston than for Tyler because it is dealing with larger numbers. Whether Houston has a better value of standard deviation is indeterminate, since the two values are not comparing like distributions of data.

Since percent RMS error is calculated in terms of network size, and ranges of "reasonable" values were established in Chapter II, percent RMS is the preferred measure of assignment accuracy. However, the single most important conclusion from these analyses is that several measures need to be used in combination, with full awareness of the strengths and weaknesses of each.

# CHAPTER IV - ANALYSES OF MULTIPLE PATH ASSIGNMENTS

The weighted multiple path assignment is a product of five all-ornothing assignments having different link impedances. Following each assignment iteration, assigned link volumes are compared to counted volumes; the link impedances are then adjusted and the process repeated. The weighted multiple path assignment results are then obtained by the weighted average of the all-or-nothing iterations. The same measures of assignment accuracy used in analyzing the all-or-nothing assignments were also calculated for the multiple path assignment results.

Weighted assignment results for VMT, standard deviation, mean difference, and percent RMS error are shown in Table IV-1. These are "single value" measures and, therefore, are tabulated together for ease of comparison. Inspection of Table IV-1 reveals that, as in all analyses from Chapter III, Assignment 1 gives the poorest results and the Existing Trip Assignment yield the best results in terms of any of the four measures of assignment accuracy. Furthermore, the amount of change in the value of any of the four measures between the all-or-nothing and the weighted assignments is fairly constant. The changes in value range from +1.4 to +3.1 percentage points for VMT and from -25.3 to -33.5 percentage points for percent RMS.

Figure IV-1 compares the distribution of link differences of Assignment 3 for the all-or-nothing assignment and for the weighted assignment. Note the greater tendency to peak near zero and the lesser dispersion of data for the multiple path assignment that is amplified by the 1,542 vpd improvement in the value of standard deviation. Figure IV-2 is a graphical presentation of the data in Table IV-1. The origin of the axes for each of the four measures in the figure represents the theoretically perfect value (where assigned volumes and counted volumes are matched exactly). Figure IV-2 gives a feel for how close the Existing Trip Assignment and Assignment 3 come to achieving the optimal value. Although the scales and units of measure for the four measures of assignment accuracy differ greatly, the percent RMS and standard deviation show significant change between the all-or-nothing and the weighted assignments, while VMT and the mean difference indicate much less change.

Measure	Assignment 1		Assignment 2		Assignment 3		Existing Trip Assignment	
of Accuracy	Weighted Assignment	Change*	Weighted Assignment	Change*	Weighted Assignment	Change*	Weighted Assignment	Change*
VMT (%)	132.9	+2.2	105.9	+3.1	104.7	+1.7	100.0	+1.4
Mean (vpd)	1976	+176	714	+138	649	+131	16	+109
Standard Deviation (vpd)	2476	-1538	2150	-1697	1974	-1542	1078	-1349
% RMS (%)	62.5	-25.3	43.9	-33.5	41.9	-29.2	21.7	-27.3

TABLE IV-1: MULTIPLE PATH ASSIGNMENT VALUES AND CHANGES FOR SELECTED MEASURES OF ASSIGNMENT ACCURACY

\*The numerical difference between the value for the all-or-nothing assignment and the weighted multiple path assignment. A positive change (+) indicates that the weighted value is greater than the the all-or-nothing value.



FIGURE IV-1: DISTRIBUTION OF DIFFERENCES FOR ASSIGNMENT 3, BOTH ASSIGNMENTS



Screenlines, cutlines, and travel routes were also calculated for the weighted multiple path assignment. Since Assignment 3 produced the best stochastic matrix assignment, only Assignment 3 and the Existing Trip Assignment will be examined for these three measures.

From Table IV-2, it may be observed that the Existing Trip Assignment is clearly superior to Assignment 3 in terms of screenlines. As would be expected, the values of percent assigned are relatively unchanged from those calculated from the all-or-nothing assignment.

Table IV-3 lists the values of percent assigned cutline volumes for the weighted assignment. Of the 17 cutlines, 11 have values within  $\pm 15$  percent of counted volumes for Assignment 3, and 15 within  $\pm 15$  percent for the Existing Trip Assignment. Both assignments have three more cutlines within  $\pm 15$  percent than they had for the all-or-nothing assignment, indicating a small improvement between weighted and all-or-nothing assignments.

	Percent Assigned		
Screenline		Assignment 3	Existing Trip Assignment
N-S	Non-CBD Links	114.4%	107.9%
	CBD Links	147.3%	96.6%
•	Screenline Total	128.5%	103.1%
E-W		119.2%	101.9%
E-W/S		74.3%	94.9%
E-W/N		123.4%	117.1%
N-S/W		87.5%	93.2%
N-S/E		93.5%	89.8%

TABLE IV-2: SCREENLINE SUMMARY: ASSIGNED VOLUME AS A PERCENT OF GROUND COUNTS

Cutline	Assignment 3	Existing Trip Assignment
Α	111.8%	117.7%
В	100.0%	96.0%
С	101.8%	99.8%
D	102.5%	94.2%
E	104.0%	90.1%
F	104.6%	95.6%
G	88.1%	92.5%
Н	67.9%	95.4%
I	82.3%	113.8%
J	90.2%	94.2%
к	110.9%	109.1%
L	92.2%	89.9%
M	129.0%	95.3%
N	132.7%	113.1%
0	140.5%	108.8%
Р	95.4%	92.1%
Q	79.6%	74.7%

TABLE IV-3: ASSIGNED CUTLINE VOLUMES AS A PERCENT OF COUNTED VOLUMES

Travel Route	Assignment 3	Existing Trip Assignment
Route A	100.2%	100.3%
Route B	156.0%	139.0%
Route C	97.2%	99.8%
Route D	118.9%	105.1%

TABLE IV-4: CUMULATIVE ROUTE VOLUMES ASSIGNED AS A PERCENT OF COUNTED

Finally, Table IV-4 shows the percent assigned values for the four travel routes. As with the cutline analysis, travel routes showed slightly improved assigned values for the weighted assignment. Assignment 3 and the Existing Trip Assignment have one more travel route within  $\pm 15$  percent of counted volumes than for the all-or-nothing assignment.

#### Summary

For the all-or-nothing assignment (Chapter III), some of the measures were very strongly affected by the introduction of trip length frequency as a matrix constraint and others were not. These general tendencies were also observed for the multiple path assignment results. The mean and all macro-level measures showed the greatest difference between Assignments 1 and 2; while the remaining micro-level measures showed the most difference between Assignment 3 and the Existing Trip Assignment.

It is also interesting to note that the two measures least affected by TLF (standard deviation and percent RMS error) showed the greatest improvement in values between the all-or-nothing assignment and the weighted assignment. The mean and all macro-level measures generally showed a minimal improvement between the all-or-nothing and the multiple path assignments.

The multiple path assignment has been shown to have no significant effect on the relative behavior of the three stochastic matrix assignments. Further, the weighted assignment showed significant improvement for all four Tyler Assignments; with the amount of improvement varying, depending on the measure of accuracy employed.

# CHAPTER V - CONCLUSIONS AND IMPLICATIONS

Previous chapters have sought to present data relative to the behavior of the various measures of assignment accuracy and to the differences in the three stochastic trip matrices and the existing trip matrix. Based on the analyses, many conclusions, interpretations, and recommendations are possible.

#### Summary of Findings

The analyses of the data from the preceding chapters lead to the following conclusions:

 Regardless of the measure of assignment accuracy used for comparison, the four assignments to the Tyler network consistently showed the following relationship:

Assignment L: Matrix constrained to total trips only Assignment 2: Matrix constrained to total trips + TLF Assignment 3: Matrix constrained to total trips + TLF Existing Trip Assignment: Fully modeled trip matrix

Assignments 2 and 3 generally gave very similar results, with Assignment 3 usually being slightly better. However, the external station constraint is ovbiously not nearly as powerful as total trips and TLF.

- All four macro-level measures showed a quantitative sensitivity to TLF (i.e., the most improvement in values occurred between Assignments 1 and 2), while also showing a variable qualitative sensitivity to TLF (a large numerical change between Assignments 1 and 2, but without necessarily improving the value).
- Three of the four micro-level measures of assignment accuracy (error ranges, percent RMS, and standard deviation) are relatively less sensitive to trip length frequency than the macro measures; they showed the most improvement between Assignment 3 and the Existing Trip Assignment.
- Of the eight measures of assignment accuracy examined, percent RMS is clearly the "best" measure and VMT is the "poorest." Percent RMS error tends to indicate the degree of matrix inaccuracy because it is relatively insensitive to trip length frequency but sensitive to the distribution of zonal trip ends. On the other hand, VMT shows an assignment to be quite acceptable as long as the trip matrix is constrained to total trips and trip length frequency.

- Based on the analyses herein, a reasonable upper limit on the value of percent RMS for a network would be approximately 100 percent for an all-or-nothing assignment and approximately 80 percent for a weighted multiple path assignment.
- Figure V-1 is a graphical summary of the data from Table II-3 showing percent RMS error as a function of counted volume for the four Tyler assignments and for the other selected networks. At the higher volumes (the more important volumes) the stochastic matrix assignments are as good or better than the selected networks in terms of percent RMS.
- No single value adequately indicates the accuracy of an assignment; several measures should be calculated and analyzed in conjunction.
- The multiple path assignment significantly improves assignment results for any measure of assignment accuracy. It affected the three stochastic matrix assignments and the existing trip assignment by an equal amount.

## Evaluation

The foregoing analyses have detailed the differences in traffic assignment results that were achieved with a stochastic trip matrix compared to an existing trip matrix. The stochastic assignments were found to yield surprisingly good results. This gives rise to the significance of the differences between the results of the best stochastic matrix (Assignment 3) and the Existing Trip Assignment.

One measure by which the acceptability of the stochastic matrix assignment might be judged would be whether a specific design would be affected by no more than one lane of traffic. A Federal Highway Administration publication, <u>TRAFFIC ASSIGNMENT</u>: <u>August 1973</u> (1), addresses this point by developing urban design criteria based on reasonable values of directinal factors, peak hour factors, etc. The resulting values indicate that to be no more than one lane off in a design, volume range and percent error relate in the following manner:

Volume Range (000's) (ADT)	Percent Error
5-10	35-45%
10-20	27-35%
20-30	24-27%
30-40	22-24%



FIGURE V-1: BERCENT RMS ERROR AS A FUNCTION OF COUNTED VOLUME

Percent RMS Error

From Table V-1, it can be seen that Assignment 3 and the Existing Trip Assignment are well within these limits. Although there is considerable difference in the values of percent RMS for Assignment 3 and the Existing Trip Assignment, it is not significant enough to necessitate a different facility design. The results of Assignment 3 are inferior to those of the Existing Trip Assignment; but, at the same time, they are good enough for system design purposes. While this does not suggest that stochastic matrices should be used for system planning, it does suggest that excessive "Fine-tuning" of the assignment process is probably unnecessary for preliminary system evaluation.

Volume Groups	Existing Trip Assignment	Assignment 3
5000- 9999 VPD	18.6%	32.1%
10000-14999 VPD	12.9%	28.1%
15000-24999 VPD	11.7%	24.1%

TABLE V-1: PERCENT RMS ERROR FOR SELECTED VOLUME GROUPS (WEIGHTED ASSIGNMENT)

In using stochastic trip matrices in this study, there was obviously no real relation between the number of trips produced by, and attracted to, a given zone and the actual land use of that zone. Superimposing elements in Figure I-1 to develop Figure V-2 shows how the distribution of zonal trip ends for the Assignment 3 Matrix compares to the Existing Trip Matrix. Of the 221 total zones, 54 percent have the same distribution of trip ends for the Assignment 3 Matrix as for the Existing Trip Matrix. This does not necessarily mean they were the same zones for each matrix. In fact, in comparing the two matrices, only 28 percent of the zones had trip ends within plus or minus 500 trips.



Zonal Trip Ends (thousands)

FIGURE V-2: COMPARISON OF ZONAL TRIP END DISTRIBUTIONS

The results achieved with the stochastic trip matrices suggest that, if the Assignment 3 Matrix could be improved slightly (e.g., produce a matrix that conforms closer to the Existing Trip Matrix), the resulting traffic assignment would likely be indistinguishable from a conventional fully-modeled assignment.

It is not readily apparent which type of trip end conformance (comparison of matrix distributions or zone-by-zone comparison of the number of zonal trip ends) is most necessary to produce an acceptable traffic assignment. However, the results do indicate the magnitude of matrix error that can exist while achieving assignment results comparable to Assignment 3.

#### Conceptual Evaluation

The preceding has sought to present data relative to the behavior of various measures of assignment accuracy and to generally describe the differences in the three stochastic trip matrices relative to the Existing Trip Matrix. It is worthwhile, at this point, to further evaluate the results from a conceptual point of view to better understand why some of the results were good and some were not. Such an evaluation will serve to clarify and highlight the conceptual implications of the results observed. These analyses will focus on the trip generation, the urban travel pattern estimates, the assignment process, and the implications relative to sketch planning techniques.

#### Trip Generation

Trip generation analyses provide an estimate of zonal trip ends (i.e., productions and attractions by purpose) for each of the zones in the urban area. These estimates of zonal trip ends are, in essence, a geographical distribution of trip ends for the urban area which reflect the geographical distribution and intensity of activities in the urban area. In a sense, this is an abstract description of the urban form.

As in all urban transportation studies, the Tyler zonal structure reflects, to some degree, the geographical distribution of activities in the urban area. This may be illustrated by subdividing the area into four concentric rings, as illustrated in Figure V-3. As may be seen from this figure, Ring 1 consists of the zone generally delineated as the CBD; Rings 2 and 3 comprise the remainder of the developed urban area; and Ring 4 consists of those zones in the fringe area of Tyler. For convenience, the external stations are ignored, and the evaluation will focus on the internal travel of the urban area (i.e., external-local and external-through trips will be ignored). Table V-2 summarizes the number of zones in each ring, a rough estimate of the size of each ring in square miles, the average number of zones per square mile, and the percent of trip ends per square mile as reflected in the Existing Trip Matrix (i.e., the fully modeled trip matrix) and the three stochastic matrices. The intensities of activities are reflected by both the trip ends per square mile within an area and the zonal structure (reflected by the number of zones per square mile). For example, as the intensity of activities (reflected in the trip ends per square mile) declined the number of zones per square mile declined in a similar manner.

In essence, the zonal structure itself provides a crude measure of the intensity of activities within the urban area. In the case of the Assignment 1 Matrix, the expected percent of trip ends per ring would equal the percent of the zones within that ring. The geographic distribution of internal trip ends is summarized in Table V-3. As can be seen, the percent of trip ends by sector for Assignment 1 is proportionate to the number of zones per ring.

The expected number of trip ends per ring for Assignment 2 and 3 is complicated by the application of trip length frequency constraint. Using a uniform random number generator to distribute trips between the zone pairs within a given separation strata, as was done for Assignment 2 and 3, the e.pected percent of trip ends within a given ring may be computed as follows:


FIGURE V-3: RING STRUCTURE FOR TYLER NETWORK

## TABLE V-2: GEOGRAPHICAL DISTRIBUTION OF INTERNAL ZONES AND INTERNAL TRIP ENDS

	RINGS*			
	l (CBD)	2 Developed Urban	3 Developed Urban	4 Fringe Area
Number of Zones	34	28	88	51
Approximate Area in Square Miles	0.25	2.5	22.7	32.6
Average Number of Zones per Square Mile	136	11.2	3.9	1.6
Average Number of Trip Ends Per Square Mile				
• Desired**	138,000	29,000	10,000	2,000
<ul> <li>Resulting from Assignment Matrix 1</li> </ul>	297,000	24,000	7,000	3,000
<ul> <li>Resulting from Assignment Matrices 2 &amp; 3</li> </ul>	392,000	32,000	7,000	2,000

\* Ring structure shown in Figure V-1

**\*\*** Desired trip ends were computed from the Existing Trip Matrix

# TABLE V-3: GEOGRAPHIC DISTRIBUTION OF INTERNAL TRIP ENDS

			Percentage D	istributions of Trip Ends	Internal
Riı	ngs*	Percent of Zones	Assignment 1	Assignments 2 & 3	Existing Trip
1	(CBD)	16.9	16.9	25.3	9.1
2	(Developed Urban Area)	13.9	14.1	20.7	19.1
3	(Developed Urban Area)	43.8	43.5	41.7	57.9
4	(Fringe Area)	25.4	25.5	12.3	13.9
	TOTAL	100.0	100.0	100.0	100.0

\* Rings are illustrated in Figure V-1



Where

Z <sub>Rt</sub>	=	Number of zone pairs at separation t with an "origin end" in Ring R plus the number of zone pairs at separ- ation t with a "destination end" in Ring R.
T_+	=	Percent of trips desired at separation t (from the

trip length frequency for the urban area)

= The maximum internal separation Μ

= The number of rings Κ

The preceding formula would provide a close estimate of the trip end results from Assignments 2 and 3 matrices, summarized in Table V-3. As may be observed, the application of trip length frequency constraint tended to increase the trip ends in Rings 1 and 2 (i.e., the CBD and the inner portion of a developed urban area). This simply reflects the disproportionate opportunities to travel at the shorter separations (i.e., 1 through 5 minutes) within Rings 1 and 2, which result from the smaller zone sizes in these rings.

All three stochastic matrices tended to overestimate the trip ends within Ring 1 (i.e., the CBD). Although the trip end estimates resulting from the stochastic matricies for areas outside the CBD (i.e., Rings 2, 3, and 4) were crude, they were substantially better than the trip end estimates for the CBD (i.e., Ring 1). Indeed, in view of the process by which they were generated, the trip end estimates from the stochastic matrices for Rings 2, 3, and 4 were surprisingly good.

As is apparent from the preceding discussion, the zonal structure imposed on the urban area was a major determinant of the trip end distribution resulting from the stochastic matrices. For example, if the zonal structures were redefined such that the CBD consisted of only two zones, it is apparent that the resulting trip ends would substantially

underestimate the desired trip ends within the CBD. It must be emphasized, however, that the zonal structure for Tyler is not unusual or peculiar. The strategy used in delineating zones for the Tyler Study is common practice in most urban transportation studies. It is important to note that the zonal structure, in essence, provided a crude tool for a distribution of activities in the urban area.

### Implications Relative to O.D. Trip Tables

In operational studies, it is common to compute an origin-destination trip matrix by expanding the data from the origin-destination surveys and to assign this trip table to the existing network. This procedure normally yields reasonable assignment results. Previous research reported in Research Report 167-7 has demonstrated that the estimates of zonal trip ends from the expansion of home interview data from that zone is subject to substantial error. For example, error ranges of from  $\pm 32\%$  to  $\pm 66\%$  were found to be associated with the estimation of trip productions for zones containing 424 occupied dwelling units using a 5% sampling rate; and error ranges of from 19% to 40% were observed to a 95% probability level for sampling rate of 12 1/2%. This research, which was based on 100% survey data, demonstrates the variance of estimates of zonal trip ends, based on the expansion of home interview survey data, is substantially greater than it was commonly believed to be.

Using the same 100% survey data, analyses of the accuracy of travel pattern estimates from the home interview (reported in Research Report 167-8) were performed. Results of these analyses demonstrated that an expanded O-D trip table will substantially underestimate the number of nonzero interchanges, For example, using the San Antonio-Bexar County Urban Transportation Study, it was found that a 5% sample from a zone containing 424 dwelling units would detect on the average only about 13% of the actual nonzero interchanges. Analysis of the accuracy of estimates of interchange volumes from expanded survey data demonstrated a tremendous variance of estimates. For example, it was found that for interchange volumes involving 100 to 200 trips (which are relatively large interchange volumes) a 5% sampling rate, or greater, would be required to be 95% confident that the resulting estimate would be within ±100% of the true interchange volume.

In essence, the analysis of the San Antonio 100% survey data has demonstrated that a trip matrix based upon expanded origin-destination survey data is subject to substantial error, in terms of the resulting zonal trip ends as well as the interzonal interchange volumes. Nevertheless, the assignment of these expanded O-D trip matrices has general yielded reasonable assignment results. This had lead practitioners to feel confident in the accuracy of their survey data. In reality, what has been observed over the years is the power of the assignment process to mask inaccuracies at both the zonal level (trip end estimates) and the zonal interchange level.

An expanded O-D trip table provides a coarse estimate of the geographical distribution of trip ends and a coarse estimate of the urban travel pattern reflecting a reliable estimate of the trip length frequency for the urban area. It is no wonder that the use of more sophisticated trip generation analyses (either aggregate or disaggregate cross classification or regression). combined with the mathematical modeling of the urban travel pattern via trip distribution models, such as the Texas Trip Distribution Model of the gravity model, have yielded even better assignment results. A key implication here is that an expanded O-D trip matrix did provide a good estimate of the total trips in the urban area and a reasonable estimate of the geographic distribution of these trip ends at a rather macroscopic level (i.e., rather large aggregations of zones). At the same time, these expanded O-D trip tables did reflect a rather crude estimate of the urban travel pattern which are probably reasonable at rather macroscopic levels (e.g., screenlines) combined with a good estimate of the trip length frequency for the urban area. From the perspective provided by the assignment results from the stochastic matrices, it is no surprise that these expanded 0-D trip matrices did vield reasonably good assignment results. Indeed, recognizing the variance of estimates associated with these expanded O-D trip matrices tends to accentuate and further substantiate the findings from the assignments of the stochastic matrices.

## Short-Cut Trip Generation Analysis

Interest in "short-cut" analysis tools, or sketch planning tools, to perform first-cut evaluations of various land-use transportation system alternatives has come to the forefront of attention during the past few years. The preceding analyses suggest that assignment results are not

overly sensitive to the results of the preceding modeling phases (i.e., the trip generation and trip distribution phases). It is proposed, therefore, that a simplified or "short-cut" trip generation analysis procedure might be utilized for such "first-cut" evaluations. The following generally describes a proposed simplified trip generation procedure for such analyses.

### Land Use

To implement such a procedure, the land-use patterns might be described via a simple land-use map reflecting the desired land-use categories. The desired zonal structure may then be superimposed upon the proposed land-use map. There are, of course, various techniques which might be utilized for superimposing a zonal structure. The following briefly describes two such techniques:

- Predetermined Zonal Structure: In some instances, the analyst may wish to use a predetermined zonal structure. Such a zonal structure (along with the proposed network) might be delineated on transparent material so that it may be simply overlayed on the land use map.
- Land Use Determined Zonal Structure: Another approach is to use the land use map in determining the zonal structure. In doing so, the analyst may describe the zones so as to minimize, to some degree, the number of different land uses within a zone.

Having superimposed a zonal structure on the desired land-use map, the analyst may proceed to measure or visually estimate the number of acres in each zone by land-use category. A technique which might be utilized for such estimation would utilize a transparent acreage grid, so that the analyst could simply overlay the zone and visually estimate the number of acres of each land use in the zone.

As alluded to previously, the land-use categories should be **reasonably** simple, but of sufficient detail to describe the urban area being studied. For example, small urban areas would generally require fewer categories than large urban areas. Table V-4 summarizes some typical land-use categories which might be utilized in large and small urban areas. In addition, the analyst may wish to utilize a number of special land-use categories to actually simplify his task. An analyst working with the San Antonio, Texas, urban area

might wish to define separate land-use categories for each of the military bases located in the urban area. Similarly, an analyst might wish to define a special land use for a major college or university in the area. A more common situation would be to describe categories for public schools (e.g., large high school, small high school, junior high school, elementary school), so that the land use would simply reflect the number of each type of unit within a zone (e.g., a zone might contain one junior high school and two elementary schools). Special categories might also be used to describe various parks and recreational areas and facilities (e.g., a special category might be used for the Astrodome in Houston). In essence, it is being suggested that the use of such special categories offers a more direct approach for handling these traffic generators.

#### Trip Generation Rates

Having a description of the land-use categories in each zone (i.e., the number of acres or number of units of each land-use category within a zone), a set of vehicle trip generation rates consistent with the land-use categories may be applied to determine the zonal productions and attractions. It is proposed that only three trip purposes need be used: internal trips, external-local trips, and external-through trips.

In most urban areas, the analyst will already have a basic set of rates available from previous studies and will need only to combine and/or adjust these rates to reflect the desired trip purposes and land-use categories. In the case of residential land uses, the existing rates would probably be in terms of trips per dwelling unit; thus, the analyst would simply identify an estimated number of occupied dwelling units per acre, etc., for a given residential land-use category in order to develop the desired rate in terms of trips per acre. In the case of external-local trips, it is proposed that the external station be considered the production end of the trips, so that only external-local attraction rates need be applied to the internal zones.

Having the estimated land uses in each zone and the necessary rates, it is a relatively simple matter to determine the estimated zonal productions and attractions in each zone.

## TABLE V-4: TYPICAL LAND USE CATEGORIES FOR SIMPLIFIED TRIP GENERATION ANAYLSIS

## LARGE URBAN AREAS

Residential (in acres):

- Low Density (i.e., single family dwelling units)
- Medium Density (e.g., garden apartments etc.)
- High Density (e.g., high rise apartments)
- Other (e.g., mobile home parks, etc.)

Commercial (in acres):

- Organized shopping (such
- as regional shopping centers)
- Unorganized shopping (such as strip development)
- Major Office Complexes
- Other

Industrial (in acres):

- Heavy Industry (such as automobile assembly plants, etc.)
- Light Industry (such as electronic components)

Other (as appropriate)

#### SMALL URBAN AREAS

Residential (in acres):

- Low Density (i.e., single family dwelling units
- Medium Density (e.g., garden apartments, etc.)
- Other (e.g., mobile home parks etc.)

Commercial (in acres):

- Organized Shopping (such as regional shopping centers)
- Unorganized shopping (such as strip development)

Industrial (in acres):

- Heavy Industry
- Light Industry

Other (as appropriate)

## Consistency Checks

The analyst should, throughout this process, be aware and concerned as to the consistency of these estimates with other forecasts. For example, in most studies, a population forecast will be available to the analyst. It is a rather simple matter to estimate the population reflected in the land use by simply summing the number of acres in each land use category, and by using the estimated dwelling units per acre for that category plus the estimated persons per dwelling unit. If the resulting population is substantially different from the forecasted population, it may be necessary to make some adjustments before proceeding. Keeping in mind that the estimates need to be reasonable and not necessarily precise, other checks might be applied to determine if the estimated commercial acreages, industrial acreages, etc., are reasonably consistent with forecasts for the urban area.

The primary objective of these consistency checks is to assure that trip generation results provide a good estimate of the total trips for the urban area and a reasonable estimate (not necessarily a precise estimate) of their geographical distribution.

#### Summary

An experienced transportation analyst, given a land-use map, can utilize a simplified trip generation procedure to estimate zonal productions and attractions. It is certainly reasonable to expect that the use of such an approach in Tyler would provide a significantly better geographical distribution of trip ends than was observed from the stochastic matrices. It follows that the use of such data would produce assignment results significantly closer to those of the Existing Trip Matrix than did the stochastic matrices. Such results would be of sufficient accuracy for "first-cut" system evaluation.

## Urban Travel Pattern

Given the zonal productions and attractions by purpose and the estimated trip length frequencies for the urban area, the next step in the traditional modeling process is to utilize a trip distribution model to determine the urban travel pattern. Trip length frequency is, of course, an important characteristic of urban travel pattern. The

application of the trip length frequency constraint in generating Assignment Matrices 2 and 3 was, in essence, a surrogate for trip distribution modeling in determining the urban travel pattern reflected in these matrices.

The poor assignment results observed from Assignment Matrix 1 suggest that the matrix did not reflect a reasonable urban travel pattern. The assignment results from Assignment Matrices 2 and 3, however, were much closer to the fully modeled assignment. It is worthwhile to compare the internal travel pattern described by these two stochastic matrices with the internal travel pattern described by the fully modeled matrix.

For convenience, the ring structure described in Figure V-1 will be used to briefly summarize these basic travel patterns. Table V-5 summarizes the ring-to-ring volumes for internal trips reflected in Assignment Matrices 2 and 3 and in the Existing Trip Matrix. As may be observed at this rather macroscopic level, the urban travel patterns reflected in these two matrices differ significantly. The percent errors in the ring-to-ring movements relative to the Existing Trip Matrix may be computed, and will range from negative 50 percent to 365 percent. The smallest error would be negative 4 percent associated with the intra-ring trips for Ring 2. While these two stochastic matrices reflect significantly different geographical distributions of trip ends and travel patterns, when compared with the fully modeled matrix, they yielded assignment results which were surprisingly close to those of the fully modeled trip matrix.

Using the results from the proposed "Short-Cut Trip Generation Analysis" (previously described) and an estimate of the trip length frequency, it would seem reasonable to expect that the analyst could use a trip distribution model to estimate the urban travel pattern. In the case of Tyler, the use of a traditional trip distribution model and zonal productions and attractions (determined from a "short-cut" trip generation analysis based on simple land-use data) would have produced an urban travel pattern estimate significantly closer to that reflected in the Existing Trip Matrix than was achieved by the stochastic matrices. The use of productions and attractions in the proposed short-cut approach would avoid the problems observed relative to the CBD.

(Ring-to-Ring Volumes as Percents of Internal Trips)				
	1	2	3	4
1	6.9%	6.2%	10.1%	1.8%
2	6.4%	4.9%	8.2%	1.4%
3	10.4%	8.0%	18.1%	5.3%
4	1.8%	1.5%	5.2%	3.8%

TABLE V-5: SUMMARY OF URBAN TRAVEL PATTERNS REFLECTED IN ASSIGNMENT MATRICES 2 & 3 AND THE EXISTING TRIP MATRIX

Assignment Matrices 2 & 3

Existing Trip Matrix

(Ring-to-Ring Volumes as Percents of Internal Trips)

	1	2	3	4
1	1.5%	2.5%	4.7%	0.5%
2	2.4%	5.2%	10.2%	1.4%
3	4.5%	10.1%	36.5%	6.7%
4	0.5%	1.4%	6.6%	5.3%

#### The Assignment Procedure

The trip generation-trip distribution phases produce a trip matrix describing the urban travel pattern which is subsequently input to the assignment process. As has already been discussed, the geographical distribution of trip ends represented by the Assignment 3 Matrix is, at best, extremely crude, since it is basically a function of the zonal structure and the trip length frequency constraint. The application of the trip length frequency constraint in the generation of this matrix is, in a sense, analogous to the use of an extremely coarse trip distribution model, which simply assures a reasonable number of vehicle hours on the network.

As for the assignment process itself, the identical traffic assignment procedures and identical networks were used for both the Assignment 3 Matrix and the Existing Trip Matrix (i.e., the fully modeled matrix). While the trip end comparisons and the ring-to-ring movement summaries indicated that there were significant differences between these matrices prior to entering the assignment process, the differences in the assignment results were not nearly as significant. This gives some indication as to the power of the assignment process, due to its aggregative nature, to overcome inadequacies from the preceding modeling phases. Presuming a reasonably coded network, it is reasonable to expect that the number of zone pairs whose minimum paths traverse a given link is largely a function of: the level of service speed associated with that link; location of that link in the network; and the network density. In essence, the more centrally located the link and the higher the level of service speed on the link, the larger the number of zone pairs which might be expected to utilize such a link. In the four all-ornothing assignments used in the analyses reported herein, each link in the network had the identical set of zone pairs whose minimum paths traverse the given link. Therefore, the differences in the assignments were entirely attributable to the differences in the distribution of trip ends within the urban area and the interchange volumes.

Relative to Assignment 2, it is interesting to note that almost identical assignment results could be obtained by developing a trip matrix as follows:

1) Using a separation matrix, the total number of zone pairs at each separation may be determined.

- From the trip length frequency distribution, the average interchange volume between zone pairs at a given separation may be determined.
- 3) Using the separation matrix and the average interchange volumes for each separation, a trip matrix could be built in which all of the interchange volumes entered in that matrix would simply be the average interchange volume for a zone pair at that separation.

The assignment of a matrix determined in such a manner would produce results which would be almost identical to those of Assignment 2, and would differ from Assignment 3, primarily, due to the manner in which external-local and external-through traffic were handled.

In summary, the assignment procedure is a powerful tool in the modeling process for the evaluation of land-use and transportation system alternatives. Due to the aggregative nature of the assignment procedure. many differences that may be observed at the zonal level and zonal interchange level tend to disappear in the assignment results. This implies that much of the "precision" in the preceding modeling phases (i.e., trip generation and trip distribution phases) may be sacrificed and still produce reasonably accurate assignment results. This is an extremely important observation when considering the proposed short-cut approaches for first-cut system evaluation. While at the microscopic level, travel pattern estimates derived by using short-cut approaches may differ somewhat from those which may be obtained by using more precise techniques, it is unlikely that such differences would be of sufficient magnitude to have a major impact on the assignment results. In short, these analyses suggest that less precise techniques may be utilized in the trip generation than trip distribution phases of the modeling process while still maintaining a reasonable level of accuracy.

Based on the finding using the stochastic matrices, it would seem reasonable that the proposed short-cut (or sketch planning) approach should produce assignment results of sufficient accuracy for preliminary system evaluation and comparison with other alternatives similarly modeled.

## REFERENCES

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