TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A PROCEDURE FOR ESTIMATION OF TRIP LENGTH FREQUENCY DISTRIBUTIONS		5. Report Date April 1974 6. Performing Organization Code
7. Author(s) David F. Pearson, Vergil G. Stover, and J. D. Benson		8. Performing Organization Report No. Research Report No. 17-1
9. Performing Organization Name and Address Texas Transportation Institute Texas A&M University College Station, Texas 77843		10. Work Unit No. 11. Contract or Grant No. Study 2-10-74-17
12. Sponsoring Agency Name and Address Texas Highway Department 11th & Brazos Austin, Texas 78701		13. Type of Report and Period Covered Interim – September 1973 April 1974 14. Sponsoring Agency Code

15. Supplementary Notes

Research performed in cooperation with DOT, FHWA.
Research Study Title: "Urban Transportation Study Procedures".

16. Abstract

A study was undertaken to determine the feasibility and means of theoretically estimating the trip length frequency distribution for "synthetic" urban transportation studies. The result of this study was the development of a procedure by which the trip length frequency distribution may be theoretically estimated. The procedure requires two inputs; the observed or estimated mean trip length and the maximum separation as defined by the network for the urban area. The procedure was tested and compared with the observed trip length frequency distributions from 18 transportation studies conducted in Texas for home-based and nonwork trip purposes, non-home based, and truck-taxi trip purposes. As a whole, the procedure was felt to give results ranging from adequate to excellent.

17. Key Words	18. Distribution Statement
Urban Transportation, Trip Length Frequency Distribution, Procedure	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	66	

Form DOT F 1700.7 (8-69)

A PROCEDURE FOR ESTIMATION OF TRIP LENGTH FREQUENCY DISTRIBUTIONS

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Research Report 17-1

Urban Transportation Study Procedures
Research Study Number 2-10-74-17

Sponsored by the
Texas Highway Department
In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration

TEXAS TRANSPORTATION INSTITUTE
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April 1974

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ABSTRACT

A study was undertaken to determine the feasibility and means of theoretically estimating the trip length frequency distribution for "synthetic" urban transportation studies. The result of this study was the development of a procedure by which the trip length frequency distribution may be theoretically estimated. The procedure requires two inputs; the observed or estimated mean trip length and the maximum separation as defined by the network for the urban area. The procedure was tested and compared with the observed trip length frequency distributions from 18 transportation studies conducted in Texas for home-based and nonwork trip purposes, non-home based, and truck-taxi trip purposes. As a whole, the procedure was felt to give results ranging from adequate to excellent.

SUMMARY

The purpose of the research reported herein was to develop a procedure for the theoretical estimation of the trip length frequency distribution (TLFD) for urban areas. This would prove useful in both the "synthetic" transportation study and in the future projection of TLFD's; thereby further reducing the need for expensive origin-destination (O-D) surveys.

The data used in this research were furnished by the Texas Highway Department and consisted of the observed TLFD's from 20 transportation studies conducted in Texas. Using these data, a procedure was developed and analyzed for the estimation of home based work and nonwork TLFD's. The procedure was also found to be applicable in estimating TLFD's for nonhome based and truck-taxi trips.

The procedure developed requires two data inputs; an estimate of the mean trip length and the maximum possible separation as defined by the network of the area being considered. Using those inputs, a TLFD is computed which has approximately the same mean trip length as the value input to the procedure. Statistical tests were conducted to determine the accuracy achieved with regard to the theoretical representation of the observed TLFD's. The results indicate that the theoretical TLFD's closely matched those observed from the transportation studies conducted in Texas.

The reliability of the procedure developed is demonstrated by the tests concerning the sensitivity of the procedure with regard to the calibration process, variations in the parameter values, and variations in the input values to the procedure. The results of those tests indicated the procedure was relatively insensitive and provided consistently good estimates of the TLFD's.

In summarizing, the procedure as reported herein may be used in lieu of an O-D survey to provide a reasonable and reliable estimate of the TLFD for an urban area. It may be used in either a "synthetic" transportation study and/or for the future projection of the TLFD.

IMPLEMENTATION STATEMENT

The procedure as developed in this research provides a means by which the trip length frequency distribution may be estimated theoretically. Thus, the distribution may be easily estimated for both "synthetic" urban transportation studies and future forecasts. The inputs required are the observed or estimated mean trip length and the maximum separation as defined by the network for the urban area. The use of sample data is not required for the estimation of the trip length frequency distribution. However, a small sample may be needed for the estimation of the mean trip length.

The use of the procedure presented in this report can provide a significant savings in both the man-hours and data requirements presently needed in the estimation of trip length frequency distributions. It is easily applicable and lends itself quite readily to the current "synthetic" transportation study approach being pioneered by the Texas Highway Department.

INTRODUCTION

Evaluation of the travel patterns that might be expected, given different transportation systems and urban development, is the primary purpose of an urban transportation study. The evaluation of present and future travel patterns as employed in the State of Texas involved the use of a trip length frequency distribution directly in the trip distribution model. This distribution has in the past been obtained from origin-destination data collected in home-interview surveys. With the use of the "synthetic" transportation study, the trip length frequency distribution must be estimated from previously collected data or from limited survey data. For future years, the trip length frequency distribution requires a substantial number of observations; definition of a distribution for a future period of time is a complex and difficult task.

Previous research indicated that observed income distributions could be accurately duplicated by modeling techniques using the mean income (4)*. A similar technique might be developed into a procedure for the estimation of trip length frequency distributions. Consequently, research was undertaken to develop a procedure that would be capable of estimating existing and future trip length frequency distributions for both conventional and "synthetic" transportation studies. Such a procedure might further reduce the need for extensive origin-destination surveys.

Previous Research

Of the limited research previously conducted, most of it has been directed toward estimating the average trip length (measured in terms of time and/or distance) for an urban area. A lesser amount of research has been concerned with estimation of the trip length frequency.

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* Numbers enclosed in parentheses refer to references listed in back of report.

Using data from various origin-destination studies, Voorhees (1) employed regression analysis to relate average trip length (time and distance) to urban population, average city density, and average network speed. It was reported that population and network speed explain much of the variation in the average trip length, an appendix did describe a means of using the Gamma distribution to estimate the trip length-frequency distribution (TLFD) using the maximum likelihood method described by Greenwood and Durand (5) to estimate the parameters for the Gamma distribution.

Although the Gamma distribution is also employed in the research reported herein, the application and calibration used are entirely different than in the Voorhees report. Further, the form of the Gamma distribution used is also different. For comparative purposes the maximum likelihood method as used in the Voorhees study is shown in Appendix A of this report.

Using data from various origin-destination studies, Voorhees (2) developed guidelines for strategies to reduce the mean trip length on a sub-area basis and check the reasonableness of aggregate mean trip length forecasts. That research is not directly related to the research which is the subject of this report.

Using the 100% data collected from 3 zones in the San Antonio Transportation Study, Stover, Benson and Pearson (3) tested the accuracy of estimating the mean trip length and the overall TLFD from sample data. No conclusive decisions were made since the overall conclusion was that the data base was too small. However, a relationship between the sample size needed to estimate the mean trip length within a given error range was developed. Data obtained from the origin-destination survey in the San Antonio Transportation Study were also used to test the possibility of using smaller sample sizes to obtain reasonable estimates of the TLFD and the mean trip length. It was concluded that much smaller samples would yield as reliable results as the larger samples taken.

Study Approach

The purpose of the research reported herein was basically to develop a procedure which would replicate the trip length frequency distributions

(TLFD) and the mean trip length (MTL) found in previous transporation studies accurately. Data from previous transportation studies conducted in Texas were used to calibrate the models for estimation of the trip length frequency. A separate set of data, also obtained from previous O-D surveys in Texas was used for comparison with the TLFD produced by the calibrated model; various statistical measures were used to determine the accuracy of the theoretical TLFD's.

Description of Data

The data used in this research consisted of the observed trip length-frequency distributions (TLFD) for 20 transportation studies conducted by the Texas Highway Department over the past 11 years. Table 1 indicates the location, sampling rate used, observed mean trip length, the population of the study area, the overall size of the urban area, and the year the study was conducted for each transportation study. Trip length-frequencies for two auto driver trips from each transportation study were used; these are:

Trip Purpose	Abbreviation
Home-based Work Auto-Driver	HBW
Home-based Nonwork Auto-Driver	HBNW

These distributions were used primarily in the development of the procedure. However, the subsequent procedure was applied to the nonhome based and truck-taxi distributions observed in those studies and the results are presented in this report as well. All separations are in terms of "time"*. The following characteristics were exhibited by the distributions:

- the distributions were highly skewed toward the longer separations
- the distributions have a single mode
- trips were not exchanged at every possible separation
- none of the observed distributions were perfectly smooth

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* The separations are computed from the link distance and relative level of service speed; therefore, the computed time separation in minutes is not synonymous with clock time in minutes.

TABLE 1: TRANSPORTATION STUDIES USED

Transportation Study	Sampling Rate (Percent)	Mean Trip Length (min.)		Population	Area Size (mi^2)	Year
		HBW	HBNW			
Abilene	12.50	6.213	4.634	100,865	222	1965
Amarillo	10.00	10.080	7.157	156,356	201	1964
Austin	10.00	9.457	6.798	209,608	177	1962
Brownsville	12.50	6.530	5.630	65,018	81	1970
Bryan-College Station	12.50	7.104	5.668	57,008	107	1970
Dallas-Fort Worth	4.00	14.142	7.741	1,821,468	2,600	1964
El Paso	6.67	12.937	9.294	362,794	422	1970
Harlingen-San Benito	12.50	5.723	4.693	67,653	200	1965
JORTS*	10.00	12.508	7.324	314,714	648	1963
Laredo	12.50	4.849	4.163	64,311	28	1964
Lubbock	10.00	8.707	6.429	152,780	157	1964
McAllen-Pharr	12.50	5.144	4.432	79,413	90	1967
San Angelo	12.50	6.051	4.638	63,438	80	1964
San Antonio	5.00	13.518	8.715	825,843	1,247	1969
Sherman-Denison	12.50	7.387	4.828	62,121	199	1968
Texarkana	12.50	6.025	4.776	64,278	90	1965
Tyler	12.50	6.536	4.921	64,512	58	1964
Victoria	12.50	5.751	4.801	45,863	131	1970
Waco	12.50	9.705	6.901	132,350	248	1964
Wichita Falls	12.50	9.140	6.290	107,704	125	1964

*JORTS - abbreviation for Jefferson-Orange Regional Transportation Study

Statistical Evaluation

The following statistical evaluations used in this research were primarily aimed at determining how well a theoretical TLFD matched the observed TLFD:

- Value of the correlation coefficient (R)
- Value of the coefficient of determination (R^2)
- Value of the root-mean-square (RMS) error
- Kolmogorov-Smirnov (K-S) goodness of fit test

Although all of the tests are relatively good indicators of how well the theoretical distributions matched the observed distributions, the value of the coefficient of determinations (R^2) and the K-S test were used as the primary test statistics. If the R^2 value was ≥ 0.9 and/or the K-S test indicated a rejection level of less than 80%, the theoretical TLFD was felt to match the observed TLFD extremely well. Visual comparison of the TLFD's when plotted together was also used as an important criteria as to how well the theoretical distributions matched the observed.

PRELIMINARY ANALYSIS

In developing a procedure for estimating the trip length frequency distributions (TLFD) for an urban area, the following desirable characteristics were established:

- The procedure should require as few inputs as possible
- The procedure should be applicable to all urban areas
- The procedure should provide an acceptable TLFD while maintaining the same mean trip length as observed from sample data or estimated
- The procedure should be capable of forecasting a TLFD for a synthetic transportation study and for future forecasts

Analyses were undertaken to study and develop feasible means and methods for predicting observed trip length frequency distributions.

Probability Distributions

Since a TLFD is generally presented as a percentage of trips at each separation, the distribution can be thought of as being a probability distribution. Thus, other established probability distributions might be considered as potential approaches in developing a procedure to estimate TLFD's. Those probability distributions which exhibit similar characteristics as trip length frequency distributions include:

1. Poisson Distribution
2. Chi-Square Distribution
3. Pearson Type III Distribution
4. Gamma Distribution
5. Wiebull Distribution

Previous research (1) indicated that of the first four distributions listed above, the Gamma Distribution gave the best results for fitting TLFD's. As far as could be determined, the Wiebull Distribution has never been used with regard to TLFD's. Therefore, the research reported herein concentrated on the Gamma and Wiebull Distributions. If acceptable results were not obtained using these two distributions, it was planned to give further consideration to the other distributions.

Gamma Distribution

The Gamma Distribution is generally represented as a two parameter continuous distribution with its origin being zero. Functionally it is represented as follows:

$$f(t) = \frac{B^{\alpha}}{\Gamma\alpha} \; t^{\alpha-1} \; e^{-Bt} \qquad (1)$$

Where,

α = the shape parameter

B = the scale parameter

e = the base of natural logarithms (2.71828)

$\Gamma(\alpha) = (\alpha - 1)!$

$f(t)$ = the relative density of occurrence of trips taking time t

Figure 1 shows variations of the Gamma Distribution holding one parameter (i.e. scale parameter $\beta = 1.0$) constant. Two methods (moments and maximum likelihood) have generally been used to fit a set of known data points. The method of moments is not described in this report since research has indicated that the maximum likelihood method is considered the better of the two methods with regard to fitting distributions similar to a trip length frequency distribution (1). A brief description of the maximum likelihood method is given in Appendix A.

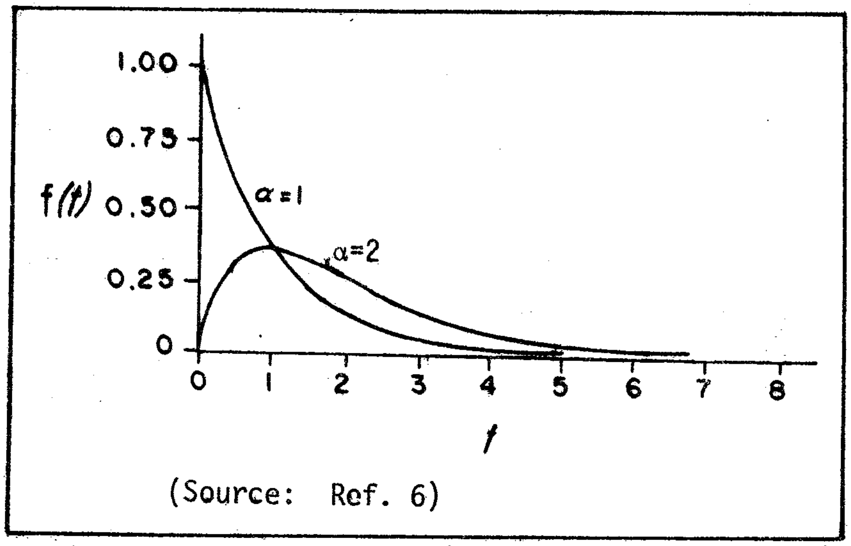


FIGURE 1: GAMMA DISTRIBUTION HOLDING SALE PROMOTION CONSTANT

Wiebull Distribution

The Wiebull Distribution is generally represented as a three parameter distribution, but, like the Gamma Distribution, it becomes a two paramenter continuous distribution when the origin is zero. Functionally it is represented as follows:

$$f(t) = \alpha B t^{\alpha-1} e^{-Bt^{\alpha}} \quad (6)$$

Where,

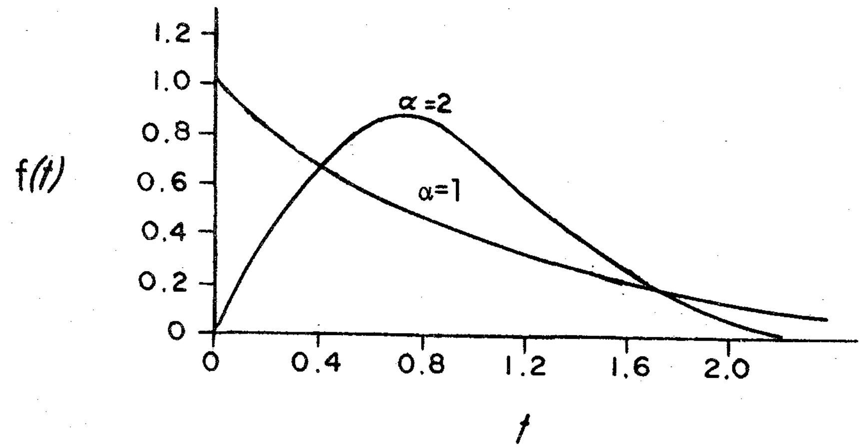
α = the shape parameter

B = the scale parameter

e = the base of natural logarithms (2.71828)

f(t) = the relative density of occurrence of trips making time t

Figure 2 shows various plots of the Wiebull distribution holding one parameter (i.e. scale parameter $\beta = 1.0$) constant. It will be noted in comparing Figures 1 and 2 that there are specific parameter values for each of the two distributions which would produce similar curves.



(Source: Ref. 6)

FIGURE 2: WIEBULL DISTRIBUTION WITH SCALE PARAMETER CONSTANT

The procedure used by Foster (4) to fit the Wiebull Distribution to income distribution data for Texas counties appeared to be applicable to trip length-frequency distributions since the distributions exhibit similar characteristics. Foster used a procedure to fit the cumulative form of the Wiebull Distribution to the cumulative percentage of persons in each income range where the income ranges had been nondimensionalized by dividing by the median income of the county being fitted. By dividing each income range by the median income of the county, each curve was put on scale where each had a median of 1.0. A nonlinear least squares fit was used to fit the data from each county and determine the parameter values. Once the parameters were computed for the counties for which data were available, the average parameter values and median incomes were used to compute income distributions for the other counties. The mean trip length would be used rather than the median trip length to nondimensionalize the separations in applications of this procedure to fitting a TLFD. This decision was made because other research to date has dealt with the mean trip length.

Preliminary Applications

Both the Gamma and Wiebull distributions were fitted to various TLFD's to determine if either gave reasonable results. An example is presented in Figure 3 which shows the observed and theoretical HBW trip length frequency distributions for the Dallas-Fort Worth transporation study. Although the appearance of the curves seems to indicate that the Gamma distribution provided a better fit, the statistical measures indicate that the Wiebull Distribution is slightly better. This should be the case since the fitting of the Wiebull Distribution involves an iterative technique (4). The basic conclusion drawn from this initial application of the procedures discussed was that both the Gamma and Wiebull Distributions provided reasonable fits but more testing would be required to determine which provided the better fit.

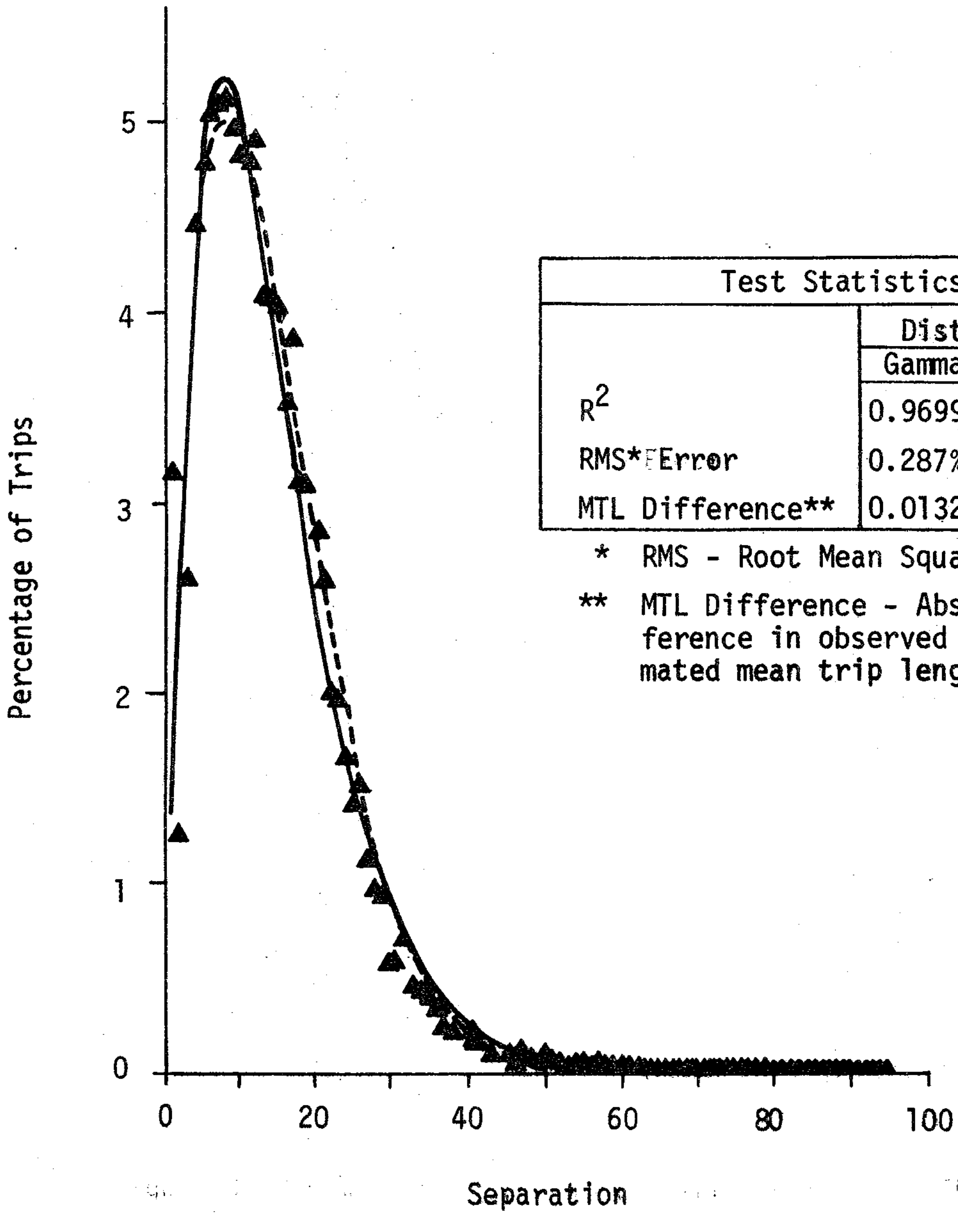
In examining the iterative curve fit technique used by Foster (4) to fit the Wiebull cumulative distribution, it was found that this technique could be used to fit any two parameter distributions. This can be accomplished by simply substituting the partial derivatives with respect to each other parameter of the distribution desired for the partial derivative of the cumulative Wiebull distribution. Thus, both the Gamma and Wiebull Distributions could be fitted to the data points directly; this provided a more convenient comparison as to which provided a better fit. Derivation of the partial derivatives for each distribution is shown in Appendix C. In fitting the Gamma and Wiebull distributions directly, several interesting observations were made. First, the sum of the values obtained from the Gamma distribution for the nondimensionalized values (obtained by dividing each separation by the mean trip length) did not equal to 1.0 as was originally expected. Further investigation revealed that this resulted from the fact that the values being input to the Gamma distribution were not integers as a result of the nondimensionalization. This problem was eliminated by simply converting the values computed by the Gamma distribution to percents (i.e., dividing each value by the sum of all the values returned). While it might seem that the most obvious solution would have

LEGEND

▲ Observed Trip Length Frequency Distribution

----- Cumulative Weibull Distribution Curve Fit

——— Maximum Likelihood Gamma Distribution Curve Fit



Test Statistics	Distribution	
	Gamma	Wiebull
R^2	0.9699	0.9720
RMS* Error	0.287%	0.276%
MTL Difference**	0.0132	0.1619

* RMS - Root Mean Square

** MTL Difference - Absolute difference in observed and estimated mean trip length.

FIGURE 3: EXAMPLE FITS ON DALLAS-FORT WORTH HOME-BASED WORK TRIP LENGTH FREQUENCY DISTRIBUTION

been to use the separations instead of the nondimensional values, such was not the case. If the separations were used, a curve fit would be necessary for each individual TLFD; this, of course, would destroy the commonality achieved through using nondimensionalized values. By using the nondimensionalized values, it was hoped that one curve could be eventually developed to use on all the different TLFD's.

It was further observed that the curve fit did not converge rapidly and several iterations were necessary to achieve acceptable results. This did not impose any real difficulty since the curve fitting process was performed exclusively with a computer program and the increased number of iterations did not add a significant amount of time or cost to the overall process.

The final and perhaps the most significant observation was that the more accurate the curve fit became (i.e., in terms of the sum of the errors squared), the poorer the theoretical estimate of the mean trip length. Closer examination indicated that the theoretical estimate of the mean trip length did converge, thereby providing a good estimate of the actual mean trip length, but the minimum sum of the errors squared did not occur at that point. It was discovered, however, that the decrease in the sum of the errors squared became relatively insignificant after convergence to the point where the theoretical and actual trip length closely matched. This is shown in Table 2 which gives the values for the total error sum squares and the total absolute difference in the estimated and observed mean trip length for several iterations in a calibration run on 10 different TLFD's.

It will also be noted from Table 2 that some cycling was evident. This was observed to have little effect with respect to convergence. Therefore, the curve fit program was modified to use the parameter values which gave the best estimate of the actual mean trip length. It will be recalled that this was one of the basic requirements desired. The result of fitting those distributions directly for the Dallas-Fort Worth transportation study is in Figure 4. A modest improvement resulted with respect to fitting the actual data points. The primary improvement with the Gamma distribution was in the estimate of the mean trip length.

LEGEND

Observed Trip Length Frequency Distribution

---- Least Squares Standard Wiebull Distribution Curve Fit

—— Least Squares Gamma Distribution Curve Fit

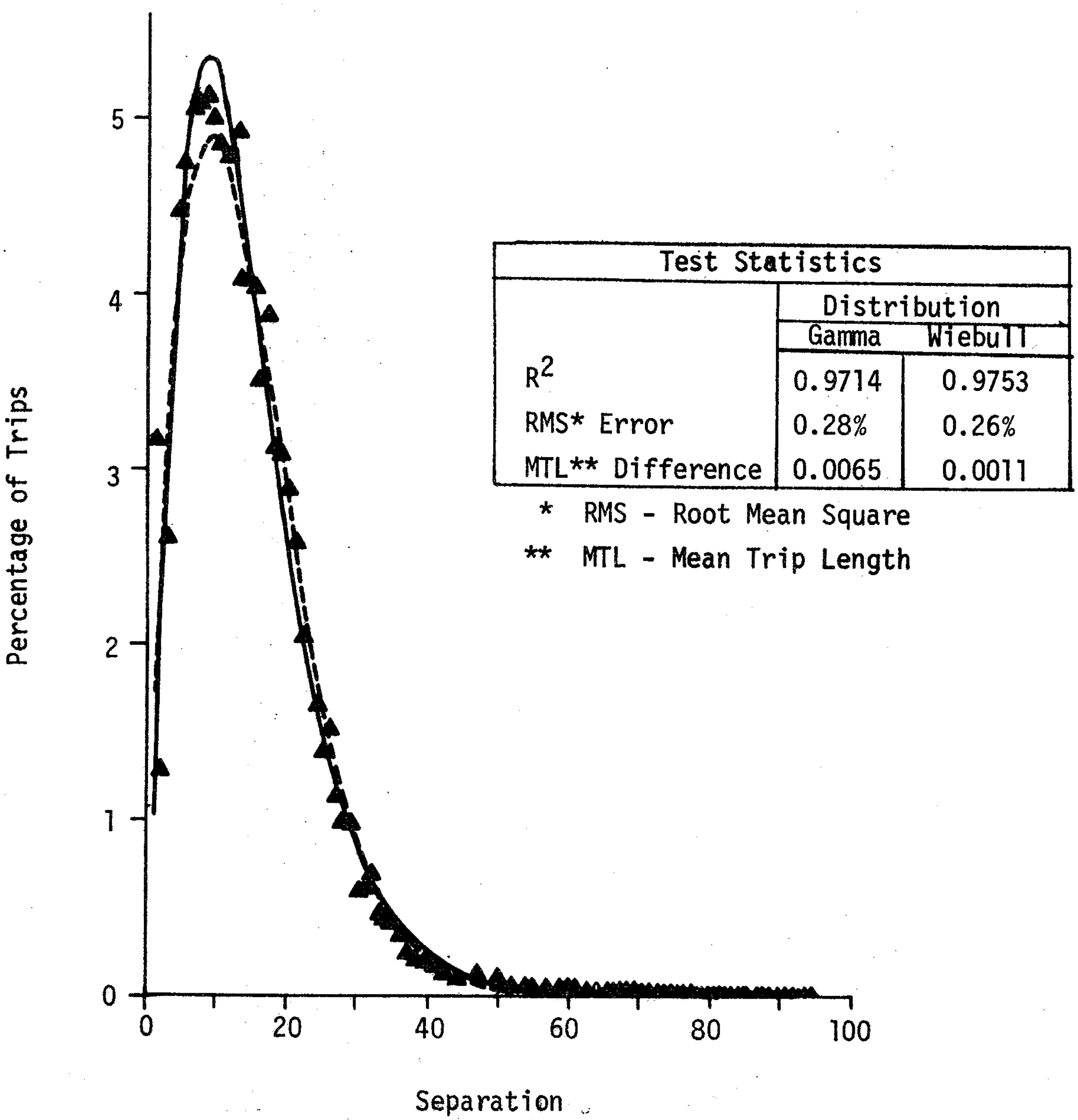


FIGURE 4: EXAMPLE FITS ON DALLAS-FORT WORTH HOME-BASED WORK TRIP LENGTH FREQUENCY DISTRIBUTION

TABLE 2: CONVERGENCE OF ITERATIVE CURVE FIT

Iteration	Sum of the Errors Squared	Sum of the Absolute MTL[a] Differences
1	0.2941	1.1776
5	0.2926	1.5904
10[b]	0.2864	0.5534
15	0.2741	5.7367
20	0.2667	12.4252
25	0.2708	12.9386
29	0.2652	17.4257

[a] MTL - Mean Trip Length

[b] That iteration produced the best results for that run

Curve Fit Methods

The following methods were developed to fit the observed trip length-frequency distributions:

1. Maximum likelihood method using standard two parameter Gamma distributions;
2. Least squares curve fit using standard two parameter Gamma distribution;
3. Least squares curve fit using cumulative two parameter Wiebull distribution; and
4. Least square curve fit using standard two parameter Wiebull distribution.

Each of the four methods was employed in fitting known TLFD's and the results were compared to evaluate which one would be the best for further development of a procedure satisfying the criteria already established. Several disadvantages were found in using the maximum likelihood to fit the data points to the Gamma distribution. Application of the method as shown in Appendix A requires an estimate of the mean trip length and the median trip length. Although methods are known to estimate the mean trip length, none were found with regard to estimating the median trip length. This does not mean to imply that it is not possible but was beyond the scope of this research. This method also has the disadvantage of not considering an increase in the maximum possible separation, except, of course, as reflected through a change in the mean trip length. Therefore, there is some doubt as the the applicability of the maximum likelihood method using the Gamma distribution for synthetic studies and future forecasts.

The direct fit of the Gamma distribution provided consistently better fits and more accurate estimates of the mean trip length than the Wiebull distribution. Comparative statistics are shown in Table 3 for the HBW trip length-frequency distributions for three test data sets. The coefficient of determination (R^2) and the accuracy of the estimate of the mean trip length were the major considerations in evaluating the curve fits.

TABLE 3: COMPARISON OF DISTRIBUTIONS AND CURVE FIT METHODS

		Dallas-Fort Worth		Amarillo		Bryan-College Station	
Distribution	Curve Fit Method	R^2	MTL Difference	R^2	MTL Difference	R^2	MTL Difference
Standard Wiebull	Least Squares	0.9753	0.0011	0.9463	0.7800	0.9309	0.0386
Cumulative Wiebull	Least Squares	0.9720	0.1619	0.9327	0.3618	0.9502	0.2297
Standard Gamma	Maximum Likelihood	0.9699	0.0132	0.9349	0.0166	0.8427	0.0467
Standard Gamma	Least Squares	0.9714	0.0065	0.9377	0.0127	0.9553	0.0008

* MTL Difference = Absolute difference between the theoretical and observed mean trip lengths.

It will be noted from Table 3 that there was not a great deal of difference in the R^2's except in the case of Bryan-College Station. However, there are sizable differences between the estimated and observed mean trip lengths. It might appear from those results that there was relatively little or no difference between the Wiebull and Gamma Distributions, thereby, making a choice between the two rather arbitrary. However, closer examination revealed the following:

- With regard to the Dallas-Fort Worth TLFD, the least-squares standard Wiebull curve fit provided an improvement, both in the R^2 and the estimate of the MTL, over the cumulative Wiebull fit. The least squares Gamma fit also provided an improvement, both in the R^2 and the estimate of the MTL, over the maximum likelihood fit.
- With regard to the Amarillo TLFD, the least squares standard Wiebull curve fit provided an improvement in the R^2 and an increase in the MTL difference with respect to the cumulative Wiebull fit. However, least squares Gamma fit provided improvements in both the R^2 and the MTL estimate, with respect to the maximum likelihood fit.
- With regard to the Bryan-College Station TLFD, the least squares Wiebull fit provided a poorer R^2 with an improvement in the MTL estimate over the cumulative Wiebull fit. The least squares Gamma fit provided improvements in both the R^2 and the estimate of the MTL with respect to the maximum likelihood fit.

The implication from these observations is that the Gamma distribution estimates are more stable between different urban areas. In the preliminary testing, it was also noted that the Gamma distribution seemed to have a greater peaking ability (i.e. reach a high mode at the smaller separations) than the Wiebull distribution. This was felt to be desirable since this is a relative characteristic of most trip length frequency distributions. The decision was made to proceed with the development of a procedure using the Gamma distribution.

PROCEDURE DEVELOPMENT

To develop a procedure for the theoretical estimation of the trip length frequency distribution (TLFD) for an urban area, the following two basic approaches may be used with regard to the data available:

1. Determine the parameter values for each individual TLFD, compute the average of all the parameter values found, and use the average values to compute theoretical TLFD's for comparison with the observed distributions; or

2. Select a number of the observed distributions, use them combined (i.e. nondimensionalized) to calibrate the parameter values and use the calibrated parameter values to compute theoretical TLFD's for comparison with all the observed distributions.

Although the first approach would be valid in this research, the decision was made to use the second approach for the following reasons:

- Use of average parameter values (computed on the basis of all distributions) would tend to limit the application of the procedure only to those urban areas used in its development;
- The use of average parameter values allows each distribution to have an equal effect on the final procedure; and
- By using a number of the distributions to calibrate a set of parameter values, the calibrated parameter values could then be applied to the distributions not used in the calibration procedure and the results evaluated. It was felt that this approach provides a more rigorous test of the applicability of the procedure to areas outside the data base.

Calibration of Procedure

The first step in the calibration of the procedure was the selection of the initial ten TLFD's to be used to obtain the parameter values for the Gamma distribution. This was done by simply arraying the data sets according to the population of the study area and selecting every other one. The study areas chosen were Bryan-College Station, San Angelo, Laredo, Brownsville, McAllen-Pharr, Wichita Falls, Lubbock, Austin, JORTS*, and Dallas-Fort Worth; these study area populations ranged from approximately 50,000 to nearly 2,000,000.

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* JORTS was used instead of El Paso because preliminary analyses resulted in relatively poor comparisons for JORTS; use of the "poor case" should provide a more rigorous test of the calibration procedure.

With the study areas selected, the actual calibration was performed using the least squares Gamma curve fit applied to all ten TLFD's at the same time. This was done by applying the curve fit to each TLFD and a cumulative sum of the errors squared used to adjust the parameter values to converge on those values which minimized the cumulative sum of the errors squared. It should be noted that both the scale and shape parameter values were adjusted and were not necessarily the same value. It will be recalled that each distribution was nondimensionalized (using its mean trip length) before the curve fit was performed. Thus, each has the common characteristic of a nondimensionalized mean trip length of 1.0. The results of the initial calibration are shown in Table 4.

TABLE 4: INITIAL CALIBRATION RESULTS FOR HOME-BASED WORK

Study	MTL Dif	R	R^2	RMS Error (%)
Austin	0.0544	.9866	.9734	.2799
Brownsville	0.0155	.9690	.9389	.7552
Bryan-College Station	0.0118	.9834	.9672	.7278
Dallas-Fort Worth	0.0267	.9861	.9724	.1635
JORTS	0.0240	.9269	.8592	.4637
Laredo	0.0120	.9863	.9729	.6341
Lubbock	0.1949	.9609	.9234	.5342
McAllen-Pharr	0.0042	.9954	.9908	.2886
San Angelo	0.0990	.9729	.9465	.9258
Wichita Falls	0.1109	.9864	.9730	.3260
Averages	0.0553	.9754	.9518	.5099

The test statistics in Table 4 indicate that the curve fit provided reasonable estimates for all of the HBW TLFD's, with the poorest fit being for the JORTS TLFD. In performing the curve fits, it was noted that the best results were always obtained when the two parameter values were approximately the same. Therefore, only one parameter value was subsequently varied in the curve fit procedure. Varying the shape parameter and setting

The test statistics in Table 4 indicate that the curve fit provided reasonable estimates for all of the HBW TLFD's, with the poorest fit being for the JORTS TLFD. In performing the curve fits, it was noted that the best results were always obtained when the two parameter values were approximately the same. Therefore, only one parameter value was subsequently varied in the curve fit procedure. Varying the shape parameter and setting the scale parameter equal to it produced much better results in terms of estimating the mean trip lengths. Table 5 shows the results of these curve fits. In comparing the results summarized in Table 5 with Table 4, it may be noted that the estimates of the mean trip length improved, while there was relatively little change in overall accuracy. Consequently, this procedure, which will be referred to as the one-parameter Gamma distribution, was selected for use in further evaluation.

Applying the calibrated parameter value (i.e. $\alpha = \beta = 3.57$), theoretical TLFD's were calculated for the other ten studies not used in the calibration process. Table 6 shows the comparison of the theoretical with the observed TLFD's. With the exception of Sherman-Denison, the comparisons are considered acceptable to excellent. The relatively poor comparison for JORTS and Sherman-Denison appears to result from the fact that each has more than one concentrated urban area with considerable distance separating them. Therefore, both were considered as special cases and were eliminated from the calibration process; a more detailed discussion is contained in the "special considerations section" of this report.

Eliminating JORTS from the calibration data sets used, the parameter value was recalibrated. Using the previously calibrated parameter value as a first estimate, the iterative nonlinear least squares curve fit did not converge but tended to cycle between two different values. The cause of the cycling was due to the curve fit method which is highly prone to over- or under-correct the parameter value as it approaches the best fit. This can sometimes be corrected by adjusting the parameter value by a fraction of the adjustment given by the curve fit. However, since that method of correction was highly arbitrary, another method was sought.

The varying the initial estimate of the parameter, it was discovered that the more accurate the estimate of the mean trip length, the poorer the overall curve fit and vice versa. Thus, it was decided to use a combination

TABLE 5: ONE PARAMETER* CALIBRATION RESULTS FOR HOME-BASED WORK

Study	MTL Dif	R	R^2	RMS Error (%)
Austin	0.0209	.9939	.9879	0.1595
Brownsville	0.0093	.9745	.9496	0.3701
Bryan-College Station	0.0020	.9788	.9581	0.3047
Dallas-Fort Worth	0.0002	.9640	.9293	0.3946
JORTS	0.0003	.8334	.6945	1.2058
Laredo	0.0107	.9856	.9714	0.4109
Lubbock	0.0859	.9769	.9544	0.3111
McAllen-Pharr	0.0075	.9878	.9758	0.3317
San Angelo	0.0448	.9917	.9835	0.2874
Wichita Falls	0.0441	.9910	.9822	0.2191
Averages	0.0226	.9678	.9387	0.3995

* $\alpha = \beta = 3.57$

TABLE 6: RESULTS FROM CALIBRATED PARAMETER VALUE* FOR HOME-BASED WORK

Study	MTL Dif	R	R^2	RMS Error (%)
Abilene	0.0043	.9915	.9831	0.6165
Amarillo	0.0026	.9741	.9489	0.6543
El Paso	0.0004	.9585	.9186	0.6246
Harlingen-San Benito	0.0021	.9512	.9048	1.5565
San Antonio	0.0148	.9553	.9127	0.6723
Sherman-Denison	0.0058	.8596	.7389	2.0687
Texarkana	0.0092	.9751	.9509	1.1140
Tyler	0.0400	.9887	.9776	0.6954
Victoria	0.0019	.9714	.9436	1.1865
Waco	0.0012	.9669	.9350	0.7612
Averages	0.0082	.9592	.9214	0.9950

* $\alpha = \beta = 3.57$

of the measure of overall accuracy of the curve fits (i.e., the sum of the errors squared) and the total of the absolute differences between the estimated and actual mean trip lengths as a relative measure of each fit. This would choose the parameter value which gave relatively good overall fits while giving accurate estimates of the mean trip lengths. The method of combining those measures was chosen such that the desired results (i.e. good overall fits and accurate estimates of the mean trip lengths) would be achieved. After implementing the new method for selecting the parameter value which gave the desired results, the calibration procedure was repeated. The procedure produced essentially the same parameter value for HBW than had been obtained before the elimination of JORTS from the calibration. Consequently, the results remain the same.* The parameter values chosen were 3.57 for HBW and 2.929 for HBNW.

Methodology

The procedure up to this stage has used two inputs; the observed mean trip length and the maximum separation at which an interchange of trips was observed (M_s). It has been assumed that an estimate of the mean trip length was available but no mention has been made with regard to M_s. The value of M_s simply defines the number of separation intervals over which the distribution is to be computed. Thus a method was needed to predict that value, since (as previously mentioned) none of these observed distributions showed any trips occuring at all possible separations. A method, therefore, was developed to estimate the maximum trip length (M_s) given the maximum separation (M) defined by the network for the urban areas. This method for estimating maximum trip length is discussed in Appendix D.

The two inputs required for the theoretical estimation of a TLFD are the mean trip length and either the maximum trip length or the maximum possible separation. The procedure used in the estimation of the theoretical trip length frequency distribution is as follows:

---

* The results for HBNW do not change since all initial testing was done on the HBW trip length frequency distributions and the procedures established for HBW were applied only once for HBNW.

- If maximum possible separation is input, then an estimate of the maximum trip length (M_s) is computed using the maximum possible separation (see Appendix D). If, however, the user elects to input the maximum trip length (M_s), then the user's estimate of M_s will be used in estimating the trip length frequency distribution.

- Each separation (from 1 through M_s) is then divided by the mean trip length (i.e. then separations are nondimensionalized).

- The nondimensionalized separation values are then used in one of the following equations (both equations are the general Gamma distribution with the calibrated parameter values substituted and the equations reduced).

$$\text{HBW} \quad f(x) = 26.15\ x^{2.57}\ e^{-3.57} \qquad \text{Eq. 1}$$

$$\text{HBNW} \quad f(x) = 12.42\ x^{1.929}\ e^{-2.929} \qquad \text{Eq. 2}$$

- The values returned from the equation are converted to percentages and represent the predicted percentage of trips at the separation interval with which each corresponds.

Figures 5 and 6 show the master curves plotted from Equations 1 and 2.

Testing the Procedure

To test the procedure, it was assumed that the mean trip length was known (i.e., in this case the observed values). The maximum trip length was obtained using the procedure outlined in Appendix D and the maximum possible separation which was obtainable from the network for each study area. Using those inputs and previously calibrated parameter values for the one parameter Gamma distribution, the theoretical TLFD's were computed and compared with the observed TLFD's for both HBW and HBNW trip purposes. The statistical results of these comparisons are shown in Table 7 (the theoretical and observed TLFD's plots are shown in Appendix B). As indicated, 9 of the TLFD's comprised the data set used in the calibration process; the other 9 were used to provide an independent evaluation of the procedure.

Analysis of the data presented in Table 7 indicates that the theoretical TLFD's do not match the observed as well as might be desired for the larger studies. An explantion for this is given in the section concerning Special Considerations. Overall, the results are judged to be good and that the procedure is applicable to any urban area.

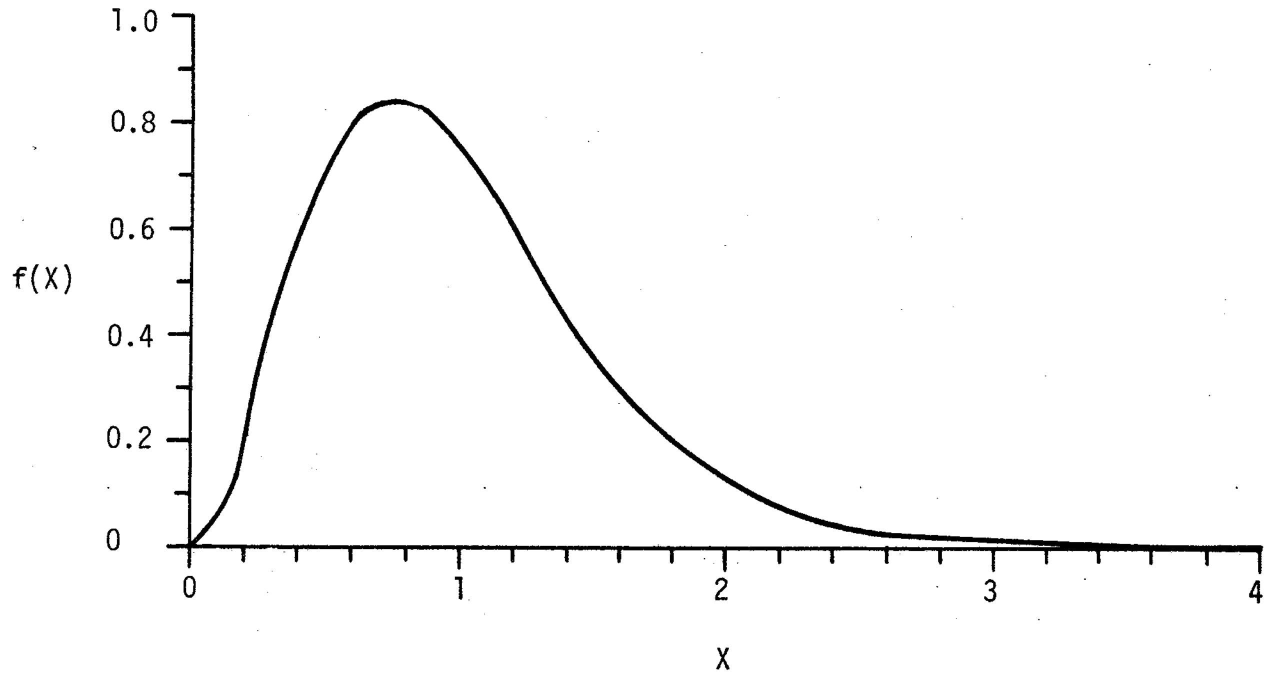


FIGURE 5: HOME-BASED WORK MASTER CURVE

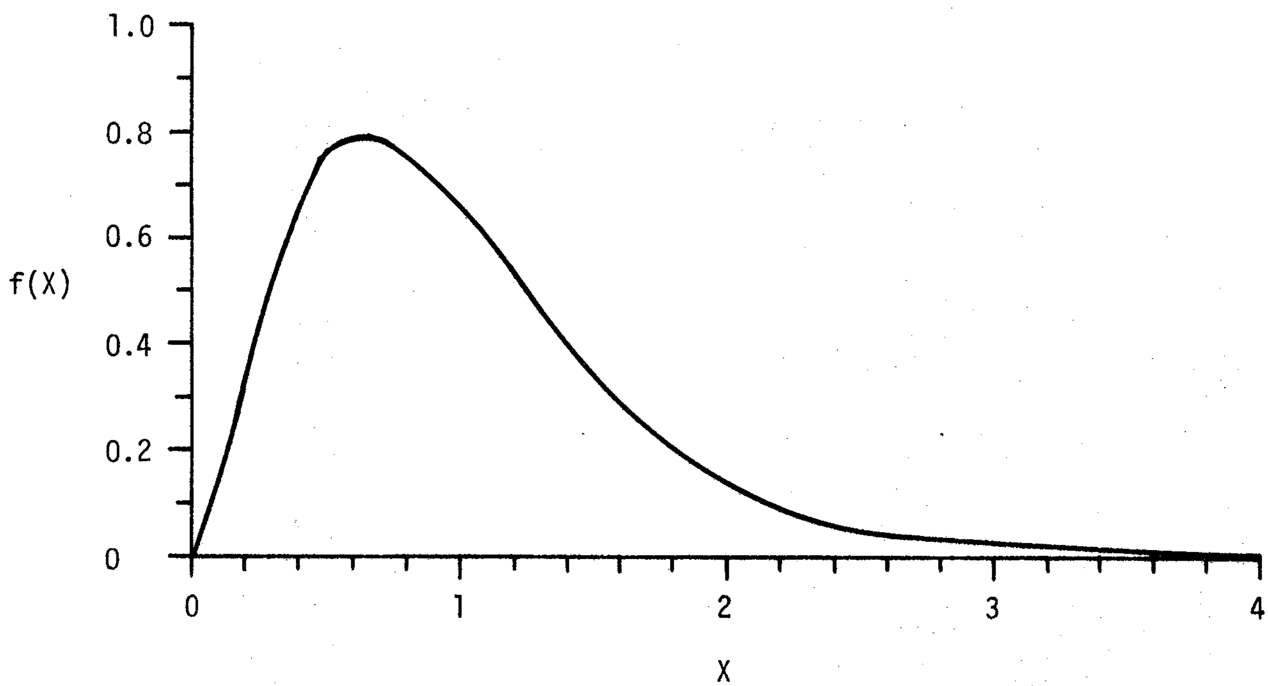


FIGURE 6: HOME-BASED NON-WORK MASTER CURVE

TABLE 7: COMPARISON OF COMPLETELY THEORETICAL TRIP LENGTH FREQUENCY DISTRIBUTIONS WITH OBSERVED TRIP LENGTH FREQUENCY DISTRIBUTIONS

Study	HBW TRIP PURPOSE						HBNW TRIP PURPOSE					
	Max Sep						Max Sep					
	Obs	Est	MTL Dif	R	R^2	RMS Error	Obs	Est	MTL Dif	R	R^2	RMS Error
Abilene	25	27	0.0025	0.9929	0.9858	0.5286%	24	26	0.0058	0.9343	0.9688	0.9242%
Amarillo	44	42	0.0041	0.9768	0.9541	0.5906%	51	41	0.0017	0.9802	0.9609	0.6566%
Austin*	33	33	0.0209	0.9908	0.9817	0.4170%	36	32	0.0001	0.9780	0.9566	0.7770%
Brownsville*	24	26	0.0048	0.9635	0.9283	1.1798%	31	25	0.0001	0.9951	0.9901	0.4683%
Bryan-College Station *	32	38	0.0011	0.9727	0.9461	0.8438%	36	37	0.0035	0.9888	0.9777	0.6027%
Dallas-Fort Worth*	95	98	0.0002	0.9618	0.9250	0.4616%	94	96	0.0016	0.8579	0.7359	1.2252%
El Paso	73	68	0.0005	0.9609	0.9233	0.5722%	72	67	0.0010	0.8891	0.7906	1.0919%
Harlingen-San Benito	26	24	0.0032	0.9555	0.9129	1.4257%	25	24	0.0052	0.9700	0.9410	1.2953%
Laredo*	17	15	0.0266	0.9763	0.9531	1.3571%	14	15	0.0073	0.9921	0.9843	0.8351%
Lubbock*	25	25	0.0859	0.9596	0.9209	0.9711%	27	25	0.0113	0.9389	0.8816	1.4368%
McAllen-Pharr *	19	20	0.0052	0.9823	0.9649	1.0573%	18	19	0.0030	0.9979	0.9958	0.3838%
San Angelo*	18	17	0.0657	0.9855	0.9713	0.8435%	17	17	0.0092	0.9822	0.9647	1.1215%
San Antonio	51	60	0.0026	0.9643	0.9299	0.5470%	54	59	0.0012	0.9022	0.8140	1.1211%
Texarkana	22	21	0.0134	0.9784	0.9573	1.0061%	21	21	0.0024	0.9728	0.9463	1.2770%
Tyler	20	18	0.0816	0.9903	0.9807	0.6536%	21	18	0.0115	0.9929	0.9859	0.6783%
Victoria	27	28	0.0017	0.9749	0.9505	1.0274%	27	28	0.0053	0.9883	0.9766	0.7628%
Waco	46	41	0.0035	0.9689	0.9387	0.7161%	45	40	0.0019	0.9153	0.8377	1.3905%
Wichita Falls*	29	27	0.0746	0.9852	0.9706	0.5785%	29	26	0.0052	0.9829	0.9661	0.7875%

Max Sep — the maximum separation at which an interchange of trips may be expected to occur.

MTL Difference - absolute difference between observed and estimated mean trip lengths.

* - those studies were used in the calibration of the parameters.

Further Comment on Nonhome Based Truck-Taxi Trips

Although this report primarily has dealt with HBW and HBNW trip purposes, the trip purposes nonhome based and truck-taxi were also considered with regard to the applicability of this procedure. Parameter values of 2.50 and 1.75 were selected for NHB and TRTX trip purposes, respectively; the theoretical distributions were computed and compared with the observed distributions. The results are presented in Table 8 and Table 9. These trip purposes are subject to more variation than HBW or HBNW trips; consequently, somewhat poorer results are to be expected for nonhome based and truck-taxi trips. Representative plots were made showing the theoretical and observed NHB and truck-taxi distributions for three studies and are presented in Figure 15, (page 42). It should be noted that the fits are rather good.

One common characteristic with the truck-taxi distribution was that the percentage of trips in the first separation interval was greater than that in the second separation interval. This is believed to be due to intraxonal trips and probably accounts for a large part of the error encountered with the theoretical distribution.

SENSITIVITY OF PROCEDURE

Sensitivity of the procedure using the calibrated parameter values, of course, is of significant interest in evaluating the potential application of the procedure. If the parameter values used produce results which are not highly sensitive to moderate differences in input data, the procedure may be applied in "synthetic" studies more easily and with a greater degree of confidence. Tests were, therefore, performed using HBW trip length frequency distributions to determine the sensitivity of the procedure with regard to errors in the input values to the procedure.

Sensitivity to Input Data

Sensitivity tests on the effect of errors in terms of the input values were accomplished by varying the maximum possible separation (M), the maximum trip length (M_s), and the predicted or observed mean trip length (MTL). To test the effect of changes in M, the HBW TLFD from the San Antonio Transportation Study was used; this particular TLFD was chosen because the procedure shown in Appendix D gave the poorest estimate of the M_s. It was felt that this would provide a more rigorous test.

The input value of M was varied ±10% and the MTL held constant along with the parameter value. The results are shown in Table 10. As can be seen, no significant changes resulted. It is concluded that the procedure presented in Appendix D will provide good estimates of the value of M_s even though the input value of M may vary; and, that the procedure for estimating the TLFD's is not overly sensitive to errors in M.

TABLE 10: SENSITIVITY WITH REGARD TO MAXIMUM POSSIBLE SEPARATION+

Variation	Resulting Value of M	Predicted M_s	Actual M_s	Abs. MTL Difference	RMS Error	R^2	Rejection By* K-S Test
-10%	69	54	51	0.0083	0.58%	0.9266	No
0%	77	60	51	0.0026	0.55%	0.9299	No
+10%	85	67	51	0.0008	0.52%	0.9323	No

\+ table shows results for the San Antonio HBW TLFD

* rejection level of 80% confidence used

Sensitivity of the procedure with regard to errors in the maximum trip length M_s was tested by varying the observed M_s by ±10% and comparing the resulting theoretical distributions with the observed distributions. The results are shown in Table 11; as may be noted, the variation of M_s has relatively little effect on the resulting theoretical trip length frequency distributions.

TABLE 11: SENSITIVITY WITH REGARD TO MAXIMUM TRIP LENGTH

Variation	Average MTL Difference	Average R^2	Average RMS Error	Percent Rejected by K-S Test*
-10%	0.0467	0.9499	0.8228%	0.0%
0%	0.0167	0.9497	0.8203%	0.0%
+10%	0.0084	0.9496	0.8199%	0.0%

* rejection level of 80%

Sensitivity to the estimate of the mean trip length (MTL) was tested by increasing and decreasing the MTL by 10% and comparing the resulting theoretical trip length frequency distributions (TLFD) with the observed distributions. Table 12 shows the results of varying the input values for the HBW trips. These results indicate that the variation in the input values for the MTL's had little effect on the overall fits of the TLFD's. Only one of the theoretical TLFD;s rejected by the K-S goodness of fit test.

TABLE 12: SENSITIVITY WITH REGARD TO MEAN TRIP LENGTH

Variation	Avg. R^2	Percent Change*	Avg. RMS Error	Percent Change*	Percent Rejected by K-S Test
-10%	0.9390	-1.55%	1.05%	+28.0%	0.0%
+10%	0.9235	-3.17%	0.98%	+19.5%	5.56%

* With respect to values obtained from inputting observed MTL values.

Conclusions

It was concluded that the procedure is not overly sensitive to variations in the input values. This is to be expected, since the TLFD's used in the calibration were nondimensionalized (such that each distribution had a mean of 1.0), and the master curves used in the procedure were calibrated based on these nondimensionalized distributions. Thus, the distinguishing factors which determine the differences between individual TLFD's would be the scaling factor (i.e., the mean trip length) and the number of points needed (i.e., the maximum trip length); both are inputs to the calculation of each theoretical trip length frequency.

PARAMETER VALUE SELECTION

The procedure, as developed in the previous section, uses two inputs: an estimate of the mean trip length and the maximum trip length (or maximum separation from which the maximum trip length is estimated). Using these inputs and the parameter value selected for the given trip purpose, a theoretical trip length frequency distribution may be obtained. The purpose of this section is to describe the manner in which an appropriate parameter value for the Gamma distribution may be selected for a given trip purpose.

Although the preceding sections have focused primarily on the home-based work and home-based nonwork trip purposes, the procedure is also applicable to other trip purposes. The scope of this section, therefore, has been expanded to include the nonhome-based and the truck-taxi trip purposes normally used by the Texas Highway Department.

To demonstrate the effect of the parameter value of the Gamma distribution and thereby provide guidance in the selection of the appropriate value, theoretical estimates were computed for and compared to each of the 18 observed distributions for each of the 4 trip purposes using parameter values of from 1.25 to 6.0 (in increments of 0.25). The 2 measures used to describe the effects of the parameter values are the absolute difference between the theoretical (i.e., the resulting) and observed (i.e., the desired) mean trip lengths and the R^2 values which provide a measure of the overall fit.

Effect on the Mean Trip Length Estimate

At each parameter value, the average absolute difference of the theoretical versus the observed mean trip length and the 95% confidence interval were computed for each trip purpose. This information, along with the maximum and minimum observed absolute differences, was plotted for each trip purpose (see Figures 7, 9, 11, and 13). Two observations relative to the relationship between the parameter value and the expected absolute error in estimating the mean trip length over the observed range of parameter values are apparent from these plots:

- As the parameter value increases, the magnitude of the expected absolute error in estimating the mean trip length decreases at a decreasing rate.

- For a given parameter value, the magnitude of the expected absolute error increases as the desired mean trip length increases.

Effect on the R^2 Values

At each parameter value, the average R^2 from the 18 observations was computed. The average R^2 values, along with the maximum and minimum observed R^2, values are summarized for each trip purpose in Figures 8, 10, 12, and 14. Care should be exercised in reviewing these graphs since the distribution of R^2 values about the mean for a given parameter value tends to be skewed. This is especially true for the truck-taxi trip purpose. For example, at the parameter value of 1.75 for the truck-taxi trips (Figure 19) the average R^2 value was 0.9083 with maximum and minimum observed values of 0.9825 and 0.6945 respectively. The implication of a wide distribution of points is somewhat misleading in that only 5 of the 18 observed R^2 values were below 0.9.

The relationship between the average R^2 values and the parameter values (for the range of parameter values considered) tends to be unimodal for each trip purpose, with a maximum average R^2 occurring at a parameter value such that higher parameter values for that trip purpose would be expected to yield a lower average R^2 value. In other words, the relationship suggests that for each trip purpose there exists a parameter value which yields a maximum average R^2 value for the 18 observations.

Methodology For Selection

An ideal parameter value for a given trip purpose would be one which both minimizes the average absolute error in approximating the desired mean trip length and maximizes the average R^2 value. Unfortunately, as may be observed from Figures 7-14, these are conflicting objectives. The analyst, therefore, is faced with a "trade-off" situation in the selection of a parameter value for a given trip purpose. By increasing the parameter beyond the value which yields the maximum average R^2, the expected average absolute error relative to the desired mean trip length would be reduced.

It is proposed that for the 4 trip purposes considered, the analyst might use Figures 7-14 in selecting a parameter value for each trip purpose. By comparing the R^2 plots and the absolute mean trip length difference plots

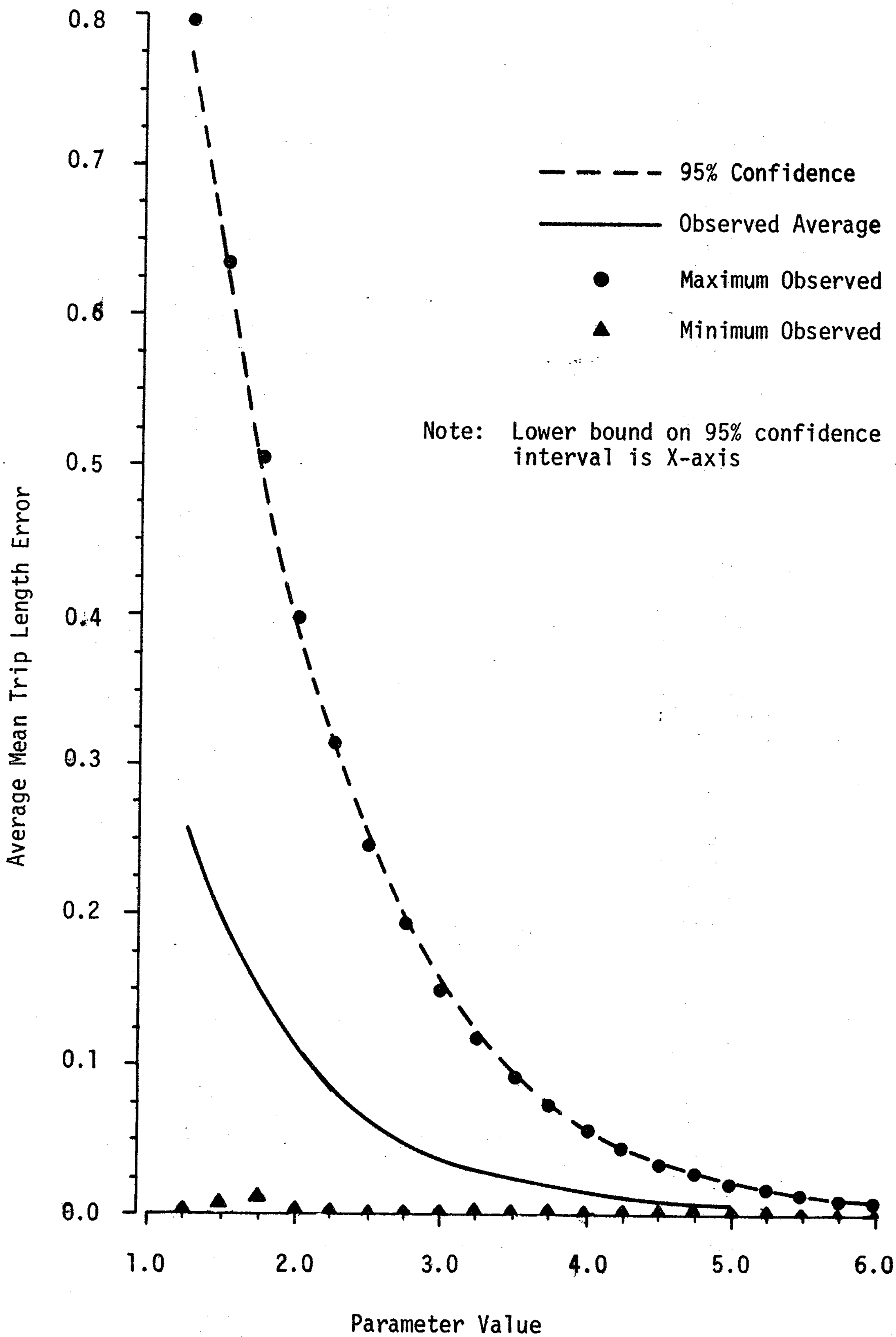


FIGURE 7: VARIATION OF HOME-BASED WORK MEAN TRIP LENGTH ERROR

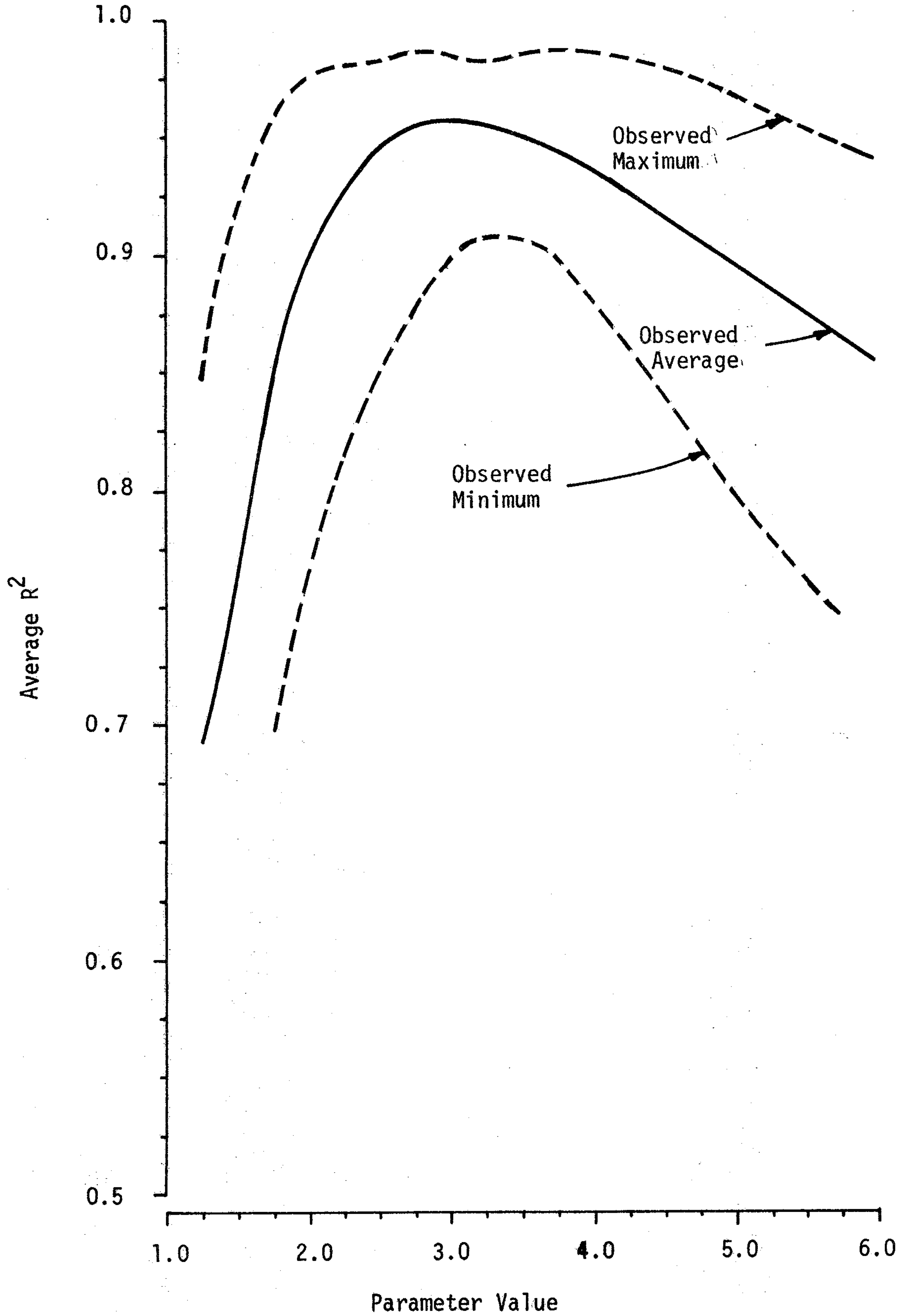


FIGURE 8: VARIATION OF HOME-BASED WORK R^2

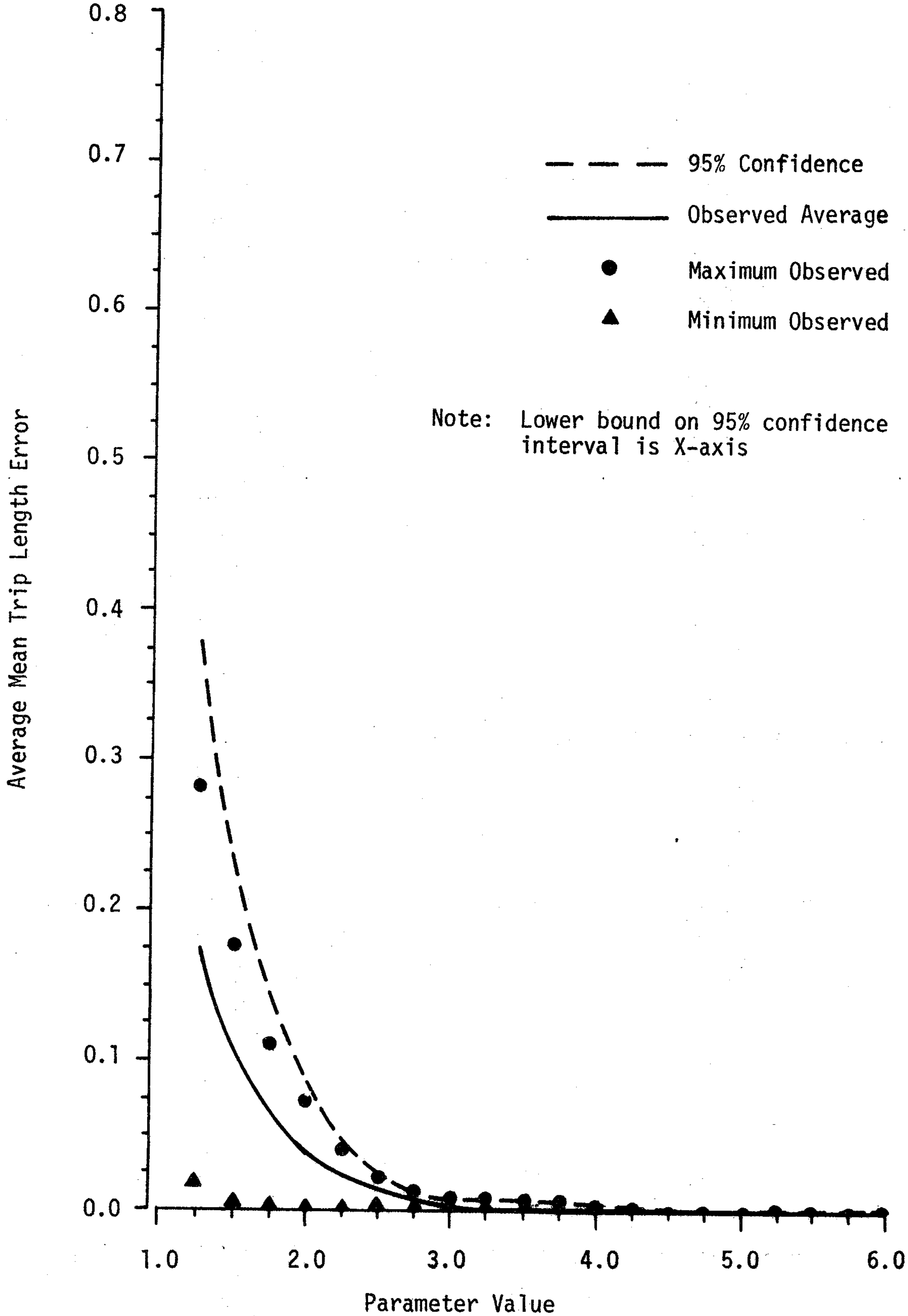


FIGURE 9: VARIATION OF HOME BASE NON-WORK MEAN TRIP LENGTH ERROR

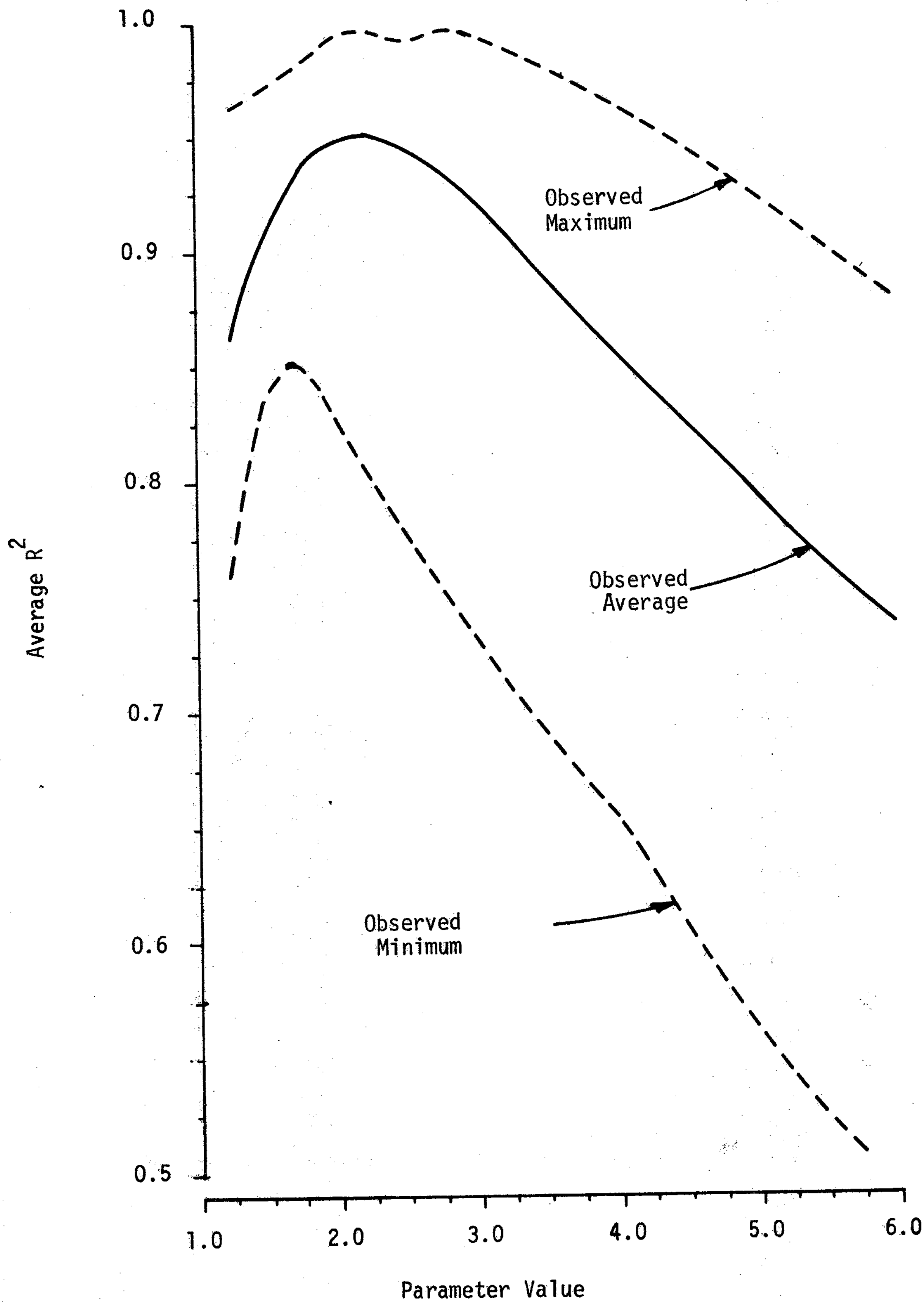


FIGURE 10: VARIATION OF HOME-BASED NON-WORK R^2

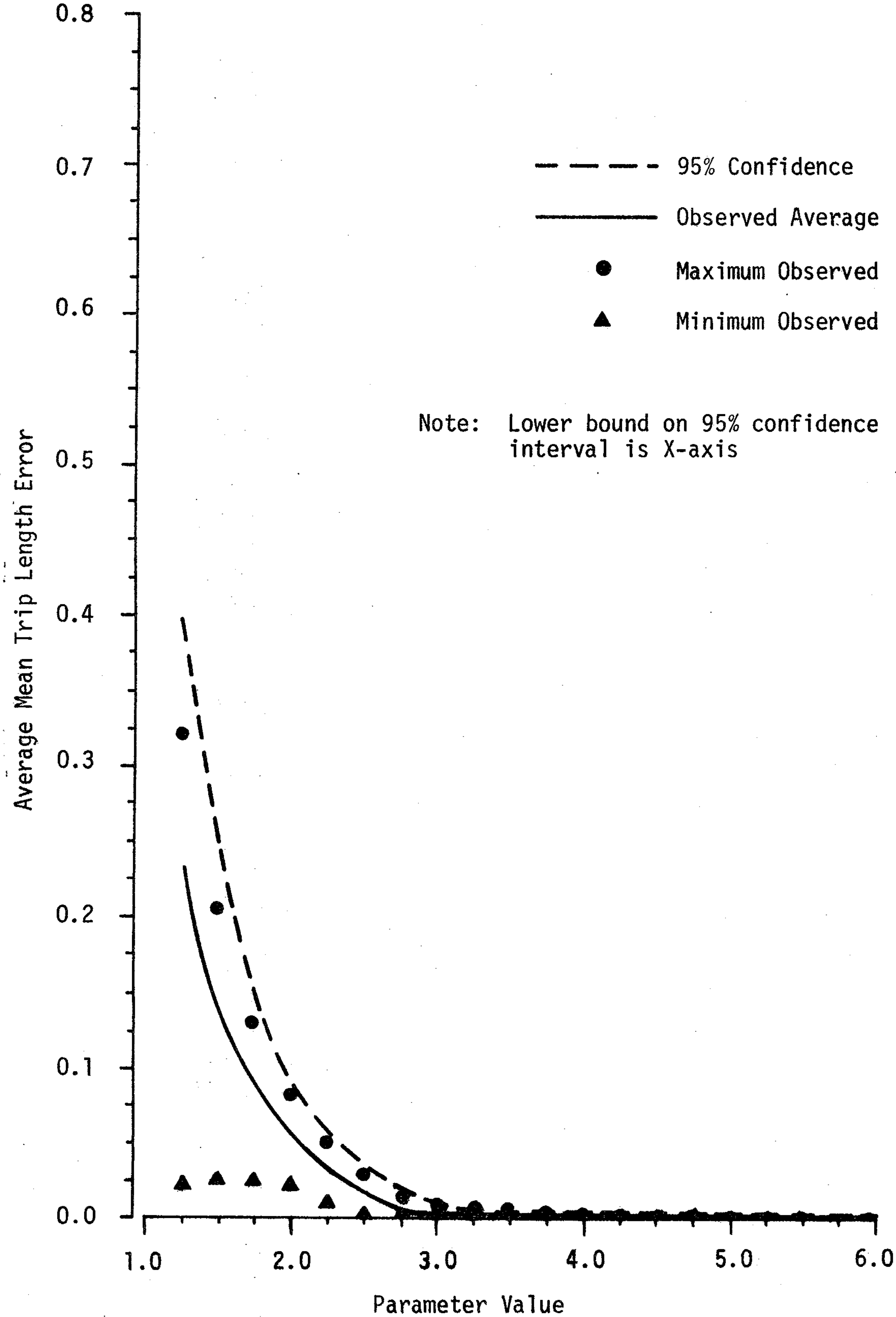


FIGURE 11: VARIATION OF NON-HOME BASE MEAN TRIP LENGTH ERROR

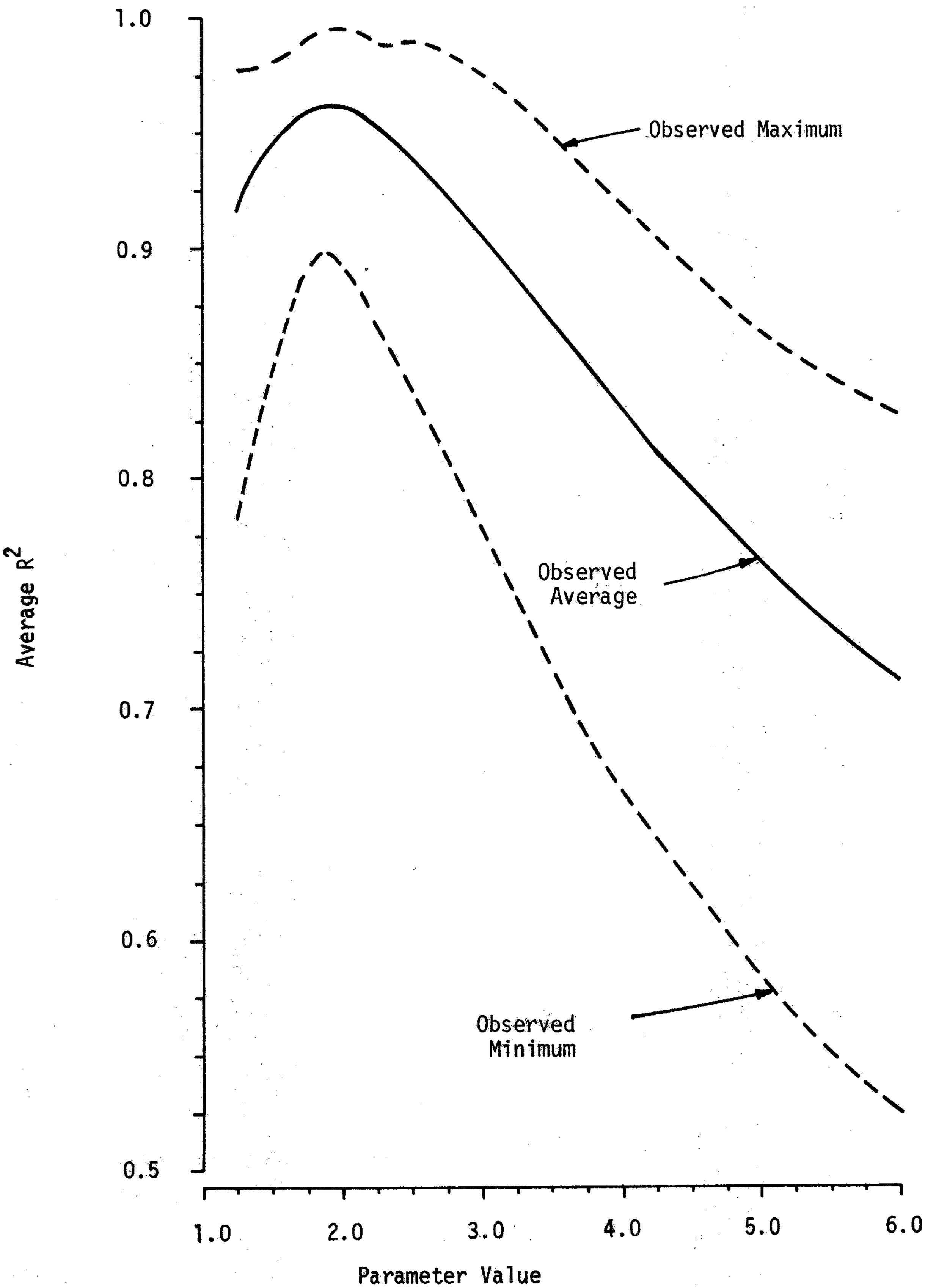


FIGURE 12: VARIATION OF NON-HOME BASED R^2

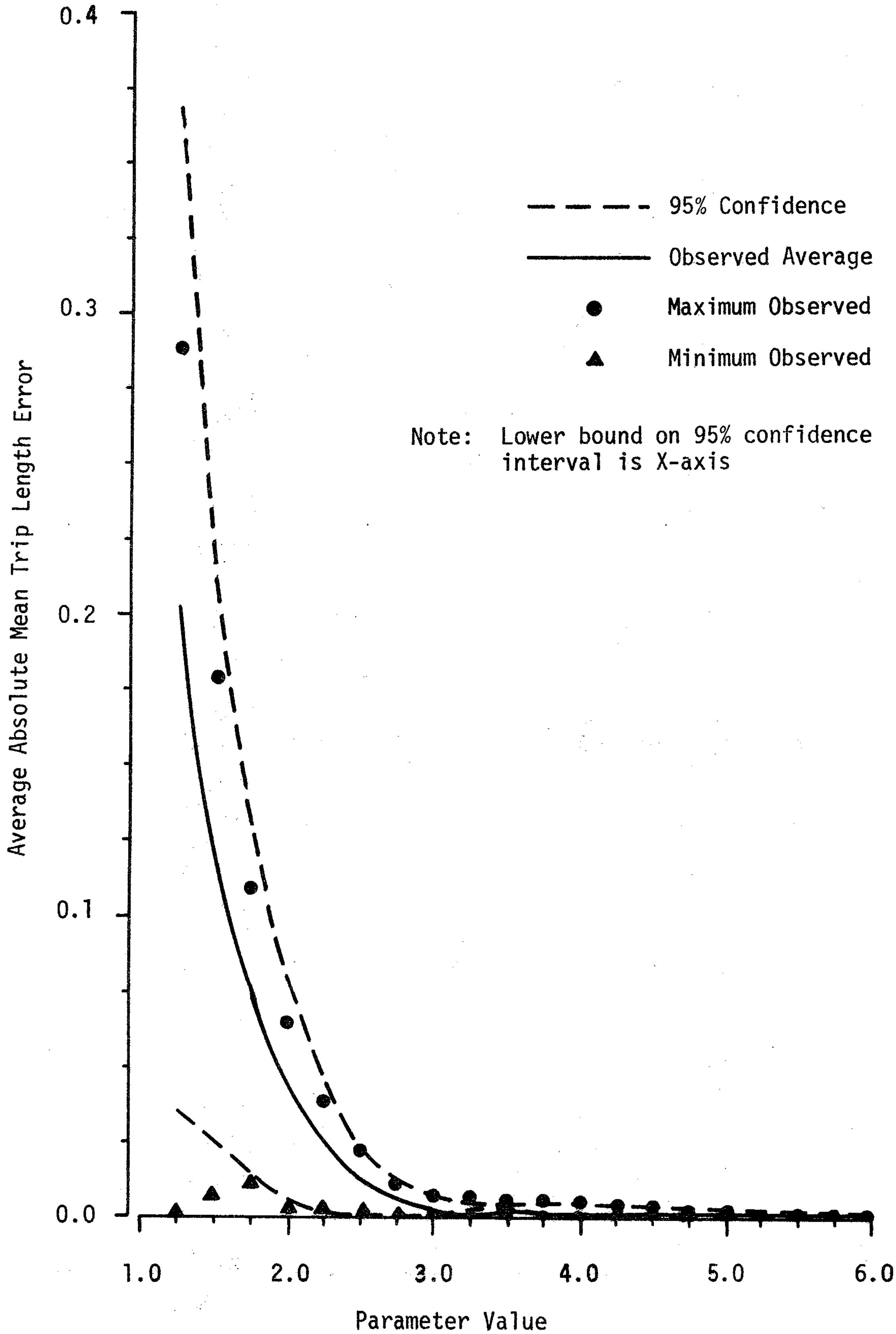


FIGURE 13: VARIATION OF TRUCK-TAXI MEAN TRIP LENGTH ERROR

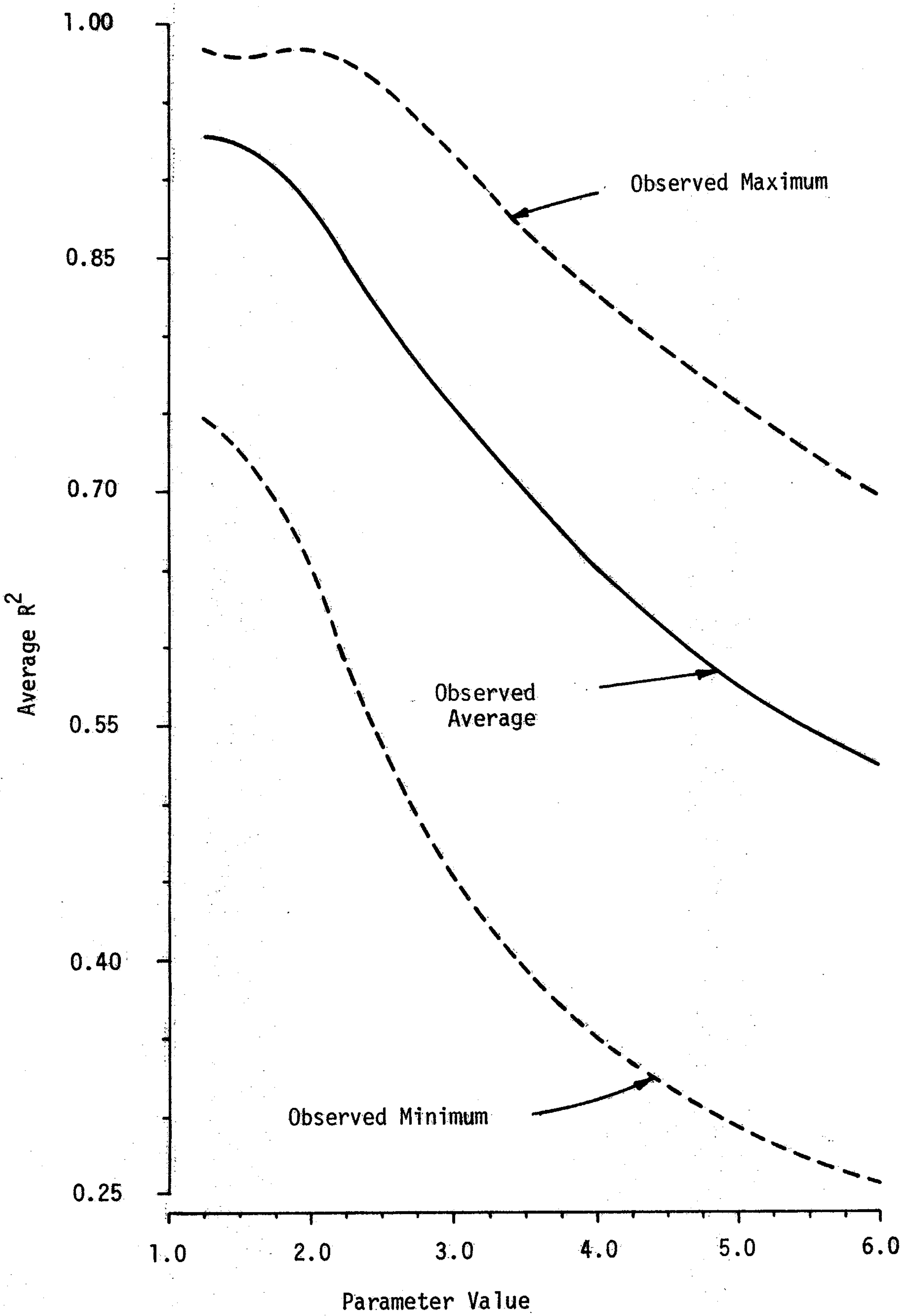


FIGURE 14: VARIATION OF TRUCK-TAXI R^2

for a given trip purpose, the analyst may select the parameter value which might be expected to yield the result deemed most desirable for a specific urban area.

For example, consider the home based work trip purpose. The maximum average R^2 occurs at a parameter value of about 3.0 (Figure 8). At this parameter value, the expected average absolute error in the resulting mean trip length would be about 0.04 "minutes," while the maximum expected error (at the 95% confidence level) would be about 1.75 "minutes". If it were desired to limit the expected maximum absolute error in mean trip length to, say, 0.1 "minute" (the corresponding average error would be 0.025), a parameter value of 3.5 would be selected. This would result in a modest reduction in the goodness of the estimate of trip length frequency as measured by the R^2 from 0.955 to 0.95.

Nonhome-based and Truck-Taxi Trip Purposes

As previously noted, the scope of this section has been expanded to include the nonhome-based and truck-taxi trip purposes. As a matter of completeness, it is appropriate to focus briefly on some of the detailed results from applying the procedure in the 18 urban areas. Parameter values were selected for both trip purposes from the plots shown in Figures 11-14. Theoretical distributions were subsequently computed and compared with the observed distributions. The results are presented in Table 8 and Table 9. As expected, the results are, in some cases, rather poor. However, those trip purposes are felt to be subject to more error than HBW or HBNW. The effect of intrazonal trips is also felt to be large (especially with the truck-taxi trip purpose) and detrimental to the overall results. It is still felt the procedure is applicable and the results are overall good. To illustrate the accuracy of the theoretical distributions, the theoretical and observed distributions for 3 of the urban areas are presented in Figure 15.

TABLE 8: COMPARISON OF NON-HOME BASE THEORETICAL TRIP LENGTH FREQUENCY DISTRIBUTIONS WITH OBSERVED TRIP LENGTH FREQUENCY DISTRIBUTIONS ($\alpha = \beta = 2.50$)

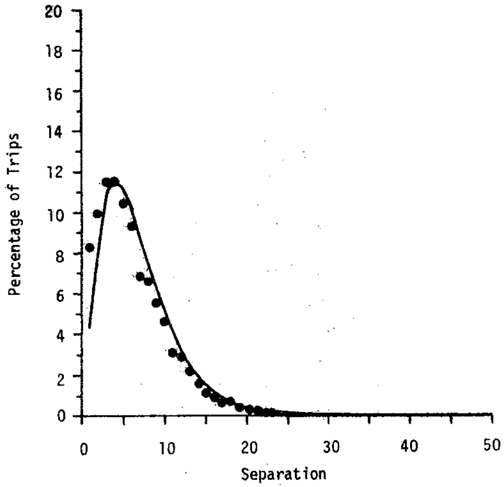
Study	Max Sep Obs	Max Sep Est	Obs MTL	MTL Dif	R	R^2	RMS Error
Abilene	26	37	4.4890	0.0240	0.9861	0.9723	0.86%
Amarillo	44	48	6.7290	0.0124	0.9838	0.9678	0.60%
Austin	31	37	6.3290	0.0127	0.9862	0.9726	0.63%
Brownsville	30	29	4.8190	0.0208	0.9808	0.9620	0.98%
Bryan-College Station	33	42	5.1530	0.0193	0.9906	0.9813	0.57%
Dallas-Ft. Worth	108	110	8.9790	0.0079	0.9162	0.8394	0.83%
El Paso	60	77	8.8140	0.0081	0.9195	0.8455	0.94%
Harlingen-San Benito	24	27	3.9910	0.0292	0.9874	0.9749	0.91%
Laredo	17	17	3.9080	0.0192	0.9759	0.9525	1.47%
Lubbock	25	28	6.6410	0.0164	0.9694	0.9397	0.96%
McAllen-Pharr	16	22	3.8980	0.0296	0.9946	0.9893	0.65%
San Angelo	17	19	4.6100	0.0037	0.9810	0.9623	1.11%
San Antonio	52	68	9.5760	0.0069	0.9466	0.8961	0.76%
Texarkana	21	24	4.3430	0.0236	0.9839	0.9680	1.02%
Tyler	22	20	4.5430	0.0124	0.9826	0.9655	1.06%
Victoria	27	32	4.0370	0.0287	0.9911	0.9823	0.73%
Waco	45	46	6.9050	0.0117	0.9403	0.8842	1.14%
Wichita Falls	28	30	5.9460	0.0097	0.9432	0.8895	1.40%

TLFD - Trip Length Frequency Distribution
NHB - Non-Home Base
MTL - Mean Trip Length
Max Sep - Maximum Trip Length
MTL Dif - Absolute Difference Between Observed and Estimated Mean Trip Lengths

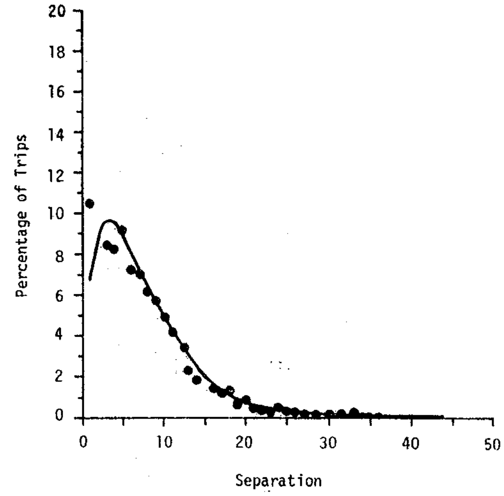
TABLE 9: COMPARISON OF TRUCK-TAXI THEORETICAL TRIP LENGTH FREQUENCY DISTRIBUTIONS WITH OBSERVED TRIP LENGTH FREQUENCY DISTRIBUTIONS ($\alpha = \beta = 1.75$)

Study	Max Sep		Obs MTL	MTL Dif	R	R^2	RMS Error
	Obs	Est					
Abilene	25	28	5.0070	0.1078	0.9862	0.9725	0.76%
Amarillo	43	44	7.5640	0.0788	0.9801	0.9606	0.59%
Austin	35	35	7.1940	0.0627	0.9661	0.9334	0.86%
Brownsville	27	27	5.8290	0.0769	0.9783	0.9571	0.91%
Bryan-College Station	32	40	6.2590	0.0939	0.9420	0.8874	1.27%
Dallas-Fort Worth	89	103	9.5030	0.0703	0.8734	0.7629	1.01%
El Paso	54	72	8.4030	0.0769	0.8476	0.7184	1.38%
Harlingen-San Benito	25	26	5.5030	0.0804	0.9828	0.9660	0.83%
Laredo	16	16	3.9450	0.0952	0.9912	0.9825	0.86%
Lubbock	28	26	6.9040	0.0118	0.8334	0.6945	2.07%
McAllen-Pharr	21	21	4.8130	0.0821	0.9871	0.9744	0.84%
San Angelo	18	18	5.0020	0.0131	0.9686	0.9382	1.27%
San Antonio	51	63	9.8990	0.0656	0.9627	0.9268	0.60%
Texarkana	20	22	4.8530	0.0949	0.9829	0.9660	0.92%
Tyler	21	19	4.9890	0.0376	0.9813	0.9630	0.97%
Victoria	25	30	5.0330	0.1093	0.9912	0.9824	0.64%
Waco	45	43	7.9480	0.0698	0.9270	0.8594	1.12%
Wichita Falls	29	28	6.0630	0.0721	0.9508	0.9040	1.24%

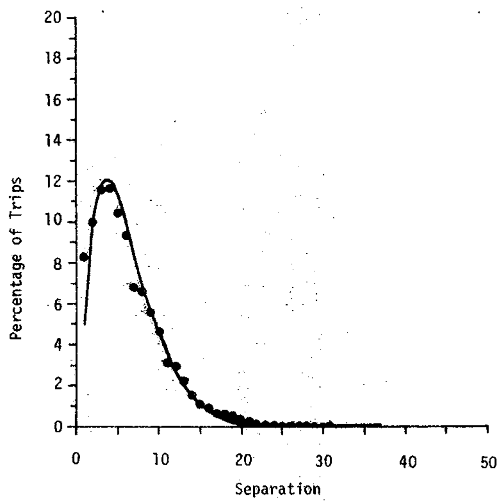
TLFD - Trip Length Frequency Distribution
TRTX - Truck-Taxi
MTL - Mean Trip Length
Max Sep - Maximum Trip Length
MTL Dif - Absolute Difference Between Observed and Estimated Mean Trip Lengths



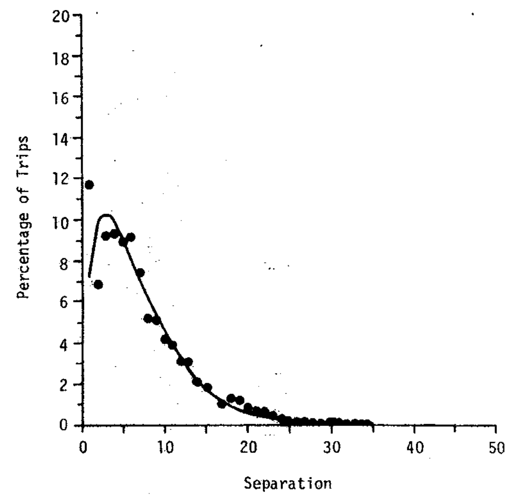
AMARILLO TRANSPORTATION STUDY (NHB)



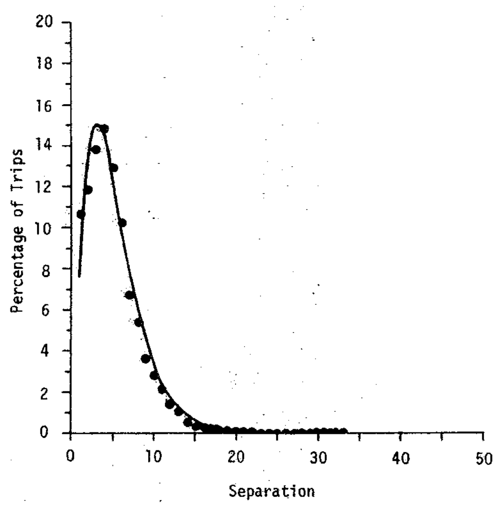
AMARILLO TRANSPORTATION STUDY (TRTX)



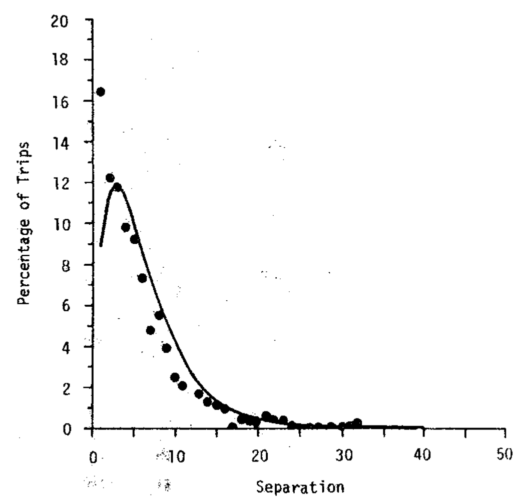
AUSTIN TRANSPORTATION STUDY (NHB)



AUSTIN TRANSPORTATION STUDY (TRTX)



BRYAN-COLLEGE STATION TRANSPORTATION STUDY (NHB)



BRYAN-COLLEGE STATION TRANSPORTATION STUDY (TRTX)

Figure 15: Theoretical and Observed Distributions for NHB and TRTX

SPECIAL CONSIDERATIONS

In the development of any procedure designed to theoretically predict real-world conditions, there will be, in most cases, some area in which the procedure must be modified to account for externalities not found in most applications. The purpose of this section is to present two such cases with regard to the procedure developed and illustrate the modifications necessary in order for the procedure to be applicable.

Urban Areas

It may be recalled that in the development of this procedure, the TLFD's from two studies (JORTS and Sherman-Denison) were dropped from the development and considered as special cases, since the procedure did not produce good results for those particular TLFD's. Both studies consist of more than one concentrated urban area with substantial undeveloped land between the developed areas. Thus, these two areas each have a TLFD which is a composite of two or more separate TLFD's. For instance, the TLFD from the Sherman-Denison study is a composite of the following TLFD's:

- A TLFD for Sherman
- A TLFD for Denison
- A TLFD for the trips interacting between Sherman and Denison

Thus, a method was sought to adapt the procedure to work for such studies as Sherman-Denison. This involves the use of the procedure to predict the individual TLFD's and combine them to compare with the observed. Since the data (i.e., inputs needed) were not readily available for such a test, they were estimated from the network as used in the transportation study. Sherman and Denison appear similar in size and the mean trip length for each was estimated at 4.0. The mean trip length for trips interacting between the two areas was taken as the approximate separation between the center of each area (this was originally estimated as 16.0). The maximum possible separation for Sherman and for Denison was assumed to be half the maximum possible for the entire area. For the trips interacting between the two, the maximum separation was taken as the maximum possible as observed in the actual study. Using those inputs, theoretical

TLFD's were calculated, combined, and compared with the observed from the Sherman-Denison Transportation Study. Although the overall fit was good, the estimate of the observed mean trip length was quite poor.

By varying the mean trip length used to compute the TLFD for the trips interacting between the Sherman and Denison areas, it was found that the final TFLD could be varied to predict the observed mean trip length closely while still giving a good overall fit. The results shown in Figure 16 indicate that the theoretical fit is very good. By adapting the procedure in the manner, the improvement over the results from applying the procedure straightforward (as was done with the other studies) was significant. The R^2 increased from .7053 to .95 and the difference in the mean trip length improved from .73 to .029. Consequently these results lead to the conclusion that the procedure may be adapted to special areas such as Sherman-Denison and JORTS to achieve acceptable results.

Intrazonal Trips

Comparisons of the theoretical percentage of trips at different separations with the observed percentage indicate that the theoretical percentage is always less than the observed at the separation interval of one. In most cases, this difference was insignificant; however, in some instances with the larger study areas, the difference is rather large. The large study areas also tended to have a higher percentage of trips in the separation interval of one than in the interval of two. For example, Dallas-Fort Worth had 3.2% of the HBW trips at a separation of one and only 1.3% at a separation of two; for the HBNW trips, 14% occurred at a separation of one while only 3.7% occurred at a separation of two. A possible explanation could be the occurrence of intrazonal trips. With large transportation study areas, larger zone sizes are used and, for this reason, more trips will be intrazonal. Since most people do not live in the same zone in which they are employed, this would explain the marked difference between the HBW and NBNW TLFD's at the smaller separation intervals. The small study areas would not be affected as much, since the zone sizes are smaller and fewer intrazonal trips are found.

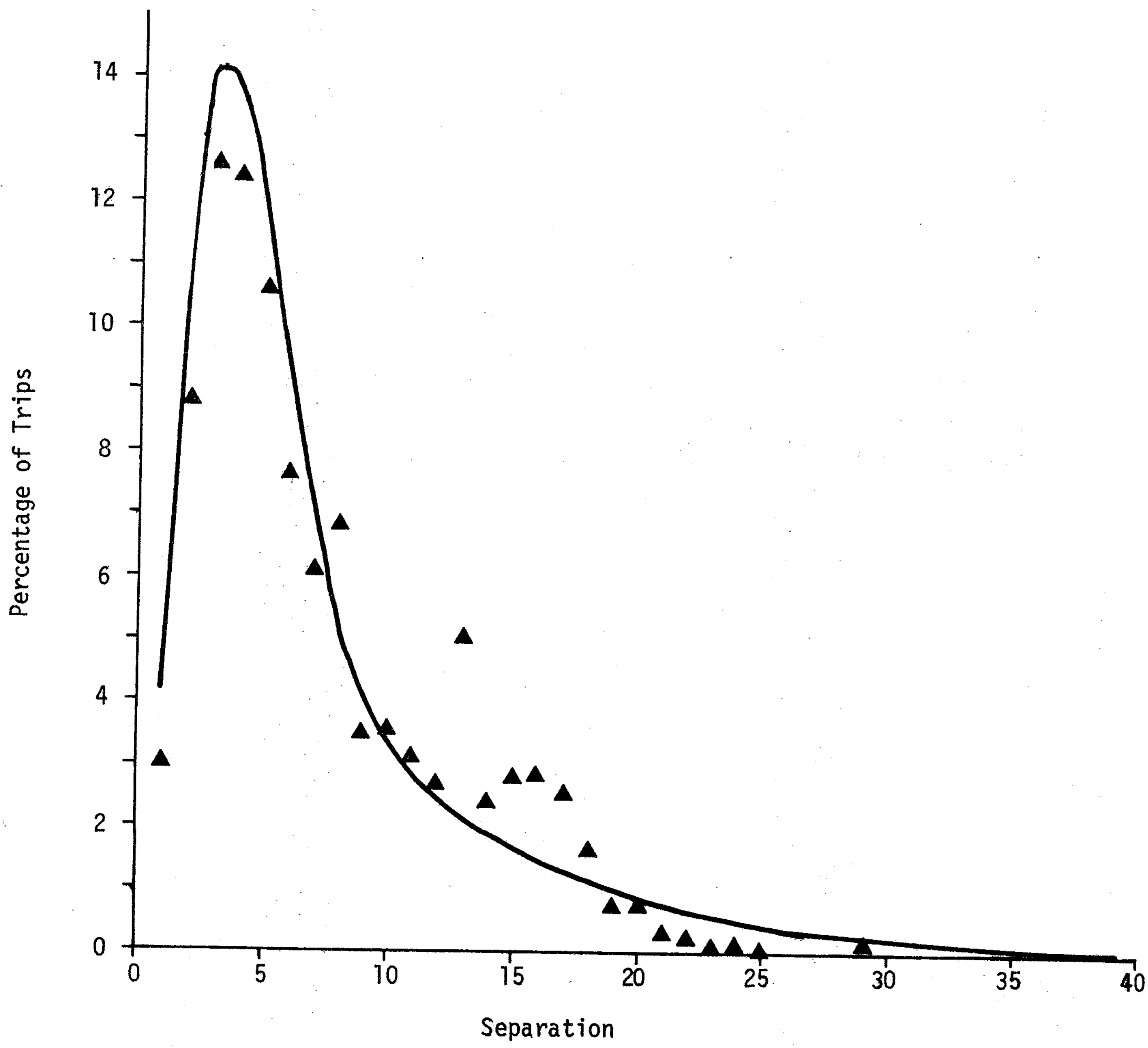


FIGURE 16: RESULTS OF SPECIAL APPLICATION

A better fit, therefore, might be obtained if the intrazonal trips were dropped from the TLFD's and the calibration procedure repeated. However, data are not readily available with regard to the number of intrazonal trips and the separations at which they occurred. Therefore, a simple test was performed to determine if better results could indeed be obtained. It was assumed that 3% of the total number of trips were intrazonal trips and that these all occurred in the first interval of each TLFD. The calibration process was performed on the modified HBNW TLFD's to determine the effect. The results are shown in Table 10. It was found that an improvement occurred in the overall fit for the larger studies where most of the smaller studies were either slightly affected or a slight negative effect was noted. This suggests that an improvement could be made in the procedure if the intrazonal trips were eliminated. For application in an urban transportation study it would be a simple matter to compute a TLFD and then add in the intrazonal trips. This, in all probability, would give much better results but further testing and analyses were not possible at this time.

TABLE 10: RESULTS OF TESTS FROM ELIMINATING INTRAZONAL TRIPS*

Study	R^2 (Intrazonal Trips Included)+	R^2 (Intrazonal Trips Not Included)++
Austin	0.9660	0.9677
Brownsville	0.9927	0.9845
Bryan-College Station	0.9817	0.9641
Dallas-Fort Worth	0.7258	0.7656
Laredo	0.9900	0.9888
Lubbock	0.9181	0.9474
McAllen-Pharr	0.9970	0.9856
San Angelo	0.9769	0.9738
Wichita Falls	0.9755	0.9643

* Intrazonal trips were estimated at 3% of total and all were at first separation internal

\+ Calibrated parameter value of 2.929

++ Calibrated parameter value of 3.60

RESULTS AND APPLICATIONS

The results of the research reported herein indicate that the trip length frequency distribution (TLFD) for an urban area can be estimated using only the observed or predicted mean trip length (MTL) and the maximum possible separation as defined by the network for that area.

The procedure developed may be directly applied to any urban area consisting of a single concentrated urban development. For those urban areas which contain more than one concentrated urban development (e.g., two cities) with considerable undeveloped land separating those developments, the procedure may be applied directly to each concentrated development and once for each combination of two of those developments to obtain a number of theoretical TLFD's. Those TLFD's may be combined to represent the TLFD for the entire urban area. The procedure may be used to predict the TLFD for both existing conditions and future conditions. Thus, a reasonable estimate of the TLFD for an urban area may be obtained without the use of an expensive origin-destination survey. This procedure can prove to be a valuable tool as part of the "synthetic" transportation study.

APPENDIX A

MAXIMUM LIKELIHOOD METHOD OF ESTIMATING PARAMETER VALUES FOR THE GAMMA DISTRIBUTION

The general Gamma function may be written as follows (1):

$$f(t) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} (t - m)^{\alpha - 1} e^{-\beta(t - m)}$$

Where

α = shape parameter

β = scale parameter

m - origin parameter

e = the base of natural logarithms (2.71828)

t - time

f(t) - relative density of occurrence of trips taking time t

$\Gamma(\alpha)$ = $(\alpha - 1)!$

The maximum likelihood method as described by Greenwood and Durand (5) and the application of it by Voorhees (1) were used in this research. No attempt is made here to demonstrate the derivation of the maximum likelihood method, since References 1, 5, and 6 provide good discussions with regard to the method. Therefore, this appendix outlines the computational procedure involved in using the maximum likelihood method to estimate the parameters in the Gamma Distribution to fit an observed TLFD. The likelihood function, L, is designated as follows:

$$L = \ln f(t)$$

Differentiating the likelihood function and simplifying by setting the origin parameter m to zero, the following result is obtained:

$$\ln \alpha - \frac{d}{d\alpha} (\ln \Gamma(\alpha)) = \ln \mu - \ln G$$

where

μ = arithmetic mean of the distribution

G = geometric mean of the distribution

Using the values shown in Table A-1 as developed by Green and Durand (5), the following steps may be followed to obtain estimates for the parameters α and β (1):

1. Select the origin parameter, m, and transform the variate to make $f(o) = 0$;
2. Compute the arithmetic, μ, and geometric, G, means of the distribution;
3. Compute $\gamma = \ln \mu - \ln G$
4. Using Table A-1, find $\gamma\alpha$ and solve for α using the relationship $\alpha = {}^{\gamma\alpha}/\gamma$
5. Solve for β using the relationship $\beta = \alpha/\mu$.

TABLE A-1

Table For Estimating Parameters of Gamma Distributions

Value of Y	Value of Yα	Value of Y	Value of Yα	Value of Y	Value of Yα	Value of Y	Value of Yα
0.10	0.5161	0.23	0.5352	0.36	0.5523	0.49	0.5677
0.11	0.5176	0.24	0.5366	0.37	0.5536	0.50	0.5689
0.12	0.5192	0.25	0.5380	0.38	0.5548	0.51	0.5700
0.13	0.5207	0.26	0.5393	0.39	0.5560	0.52	0.5711
0.14	0.5222	0.27	0.5407	0.40	0.5573	0.53	0.5722
0.15	0.5237	0.28	0.5420	0.41	0.5585	0.54	0.5733
0.16	0.5252	0.29	0.5433	0.42	0.5597	0.55	0.5743
0.17	0.5266	0.30	0.5447	0.43	0.5608	0.56	0.5754
0.18	0.5281	0.31	0.5460	0.44	0.5620	0.57	0.5765
0.19	0.5295	0.32	0.5473	0.45	0.5632	0.58	0.5775
0.20	0.5310	0.33	0.5486	0.46	0.5643	0.59	0.5786
0.21	0.5324	0.34	0.5498	0.47	0.5655	0.60	0.5796
0.22	0.5338	0.35	0.5511	0.48	0.5666	0.61	0.5806

Source: Ref. 5

APPENDIX B

COMPARISON OF THEORETICAL AND OBSERVED TRIP LENGTH FREQUENCY DISTRIBUTIONS

This appendix provides a comparison of the theoretical trip length frequency distribution (TLFD) produced by the procedure developed in this research with the observed distribution found from the origin-destination survey for 18 transportation studies conducted in Texas. The comparisons are presented graphically with plots showing the percentage of trips at each separation, both theoretical and observed. It should be recalled that all separations are in units of minutes computed from link distance and relative level of service speed and are not synonymous with clock time. On the following pages, Figure B-1 through B-18 are the HBW TLFD's and Figures B-19 through B-36 are the HBNW TLFD's. It should be noted that the scales shown differ between figures and although the distributions may appear the same, such is not the case.

Reviewing Figures B-1 through B-18, the theoretical estimates appear to reasonably predict the observed values and are generally those obtained if a best fit was hand drawn through the observed points. The only exceptions were the larger studies where the theoretical estimates were generally too high around the mode (e.g., see Figures B-1, and B-11). Overall, the figures shown indicate that the procedure is consistently accurate over a wide range of urban areas for HBW trips.

Reviewing Figures B-19 through B-36, the theoretical estimates appear to reasonably predict the observed distributions. Once again, the exceptions are the larger studies of Dallas-Fort Worth, San Antonio, and El Paso (see Figures B-19, B-28, and B-29.) It appears in those three cases that the procedure provided poor estimates of the observed values. However, it should be noted that those are the largest studies and it is felt that the probable high occurrence of intrazonal trips at the smaller separations is the direct cause of the extreme errors found in those estimates. Overall, it was concluded the procedure produced reasonable results for HBNW TLFD's.

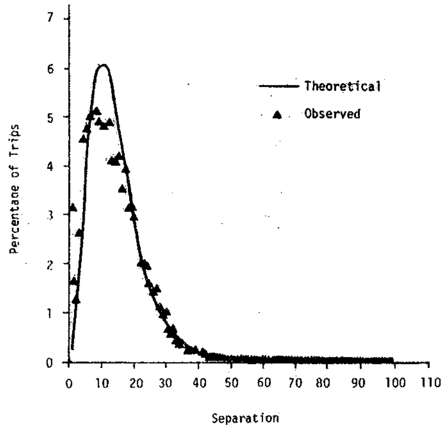


FIGURE B-1: DALLAS - FT. WORTH TRANSPORTATION STUDY (HBW)

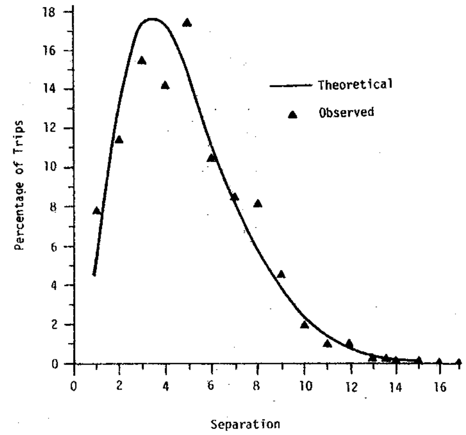


FIGURE B-2: LAREDO TRANSPORTATION STUDY (HBW)

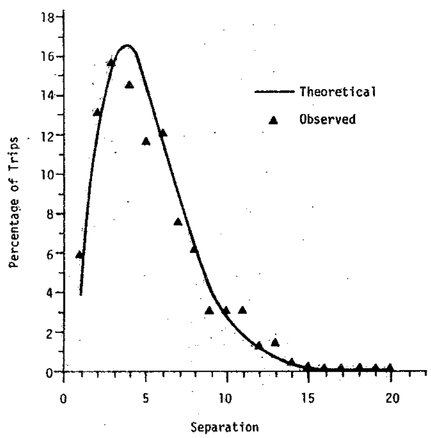


FIGURE B-3: McALLEN - PHARR TRANSPORTATION STUDY (HBW)

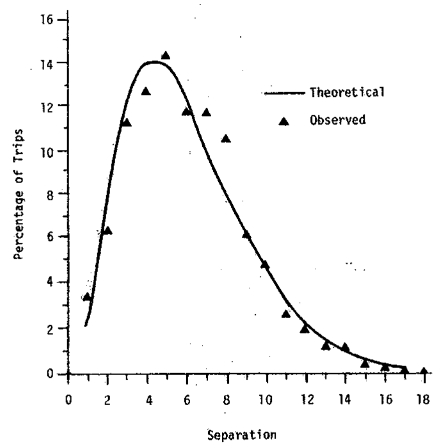


FIGURE B-4: SAN ANGELO TRANSPORTATION STUDY (HBW)

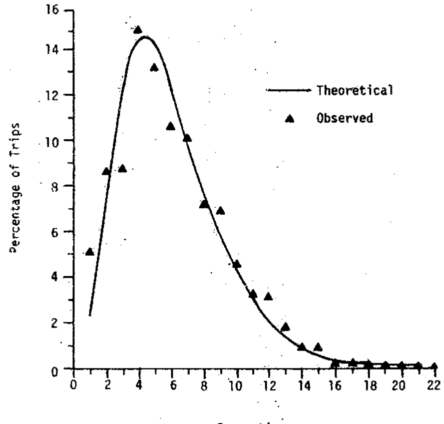


FIGURE B-5: TEXARKANA TRANSPORTATION STUDY (HBW)

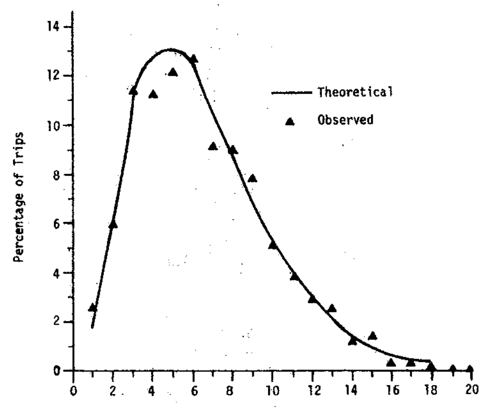


FIGURE B-6: TYLER TRANSPORTATION STUDY (HBW)

NOTE: Scales are different between individual figures.

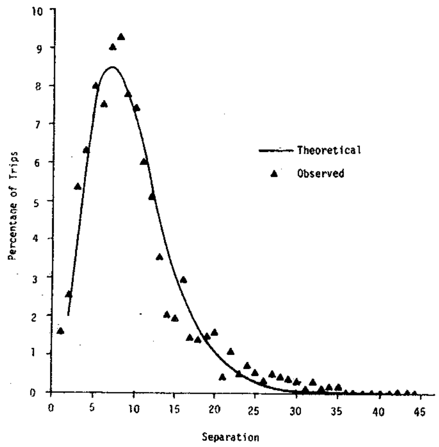


FIGURE B-7: AMARILLO TRANSPORTATION STUDY (HBW)

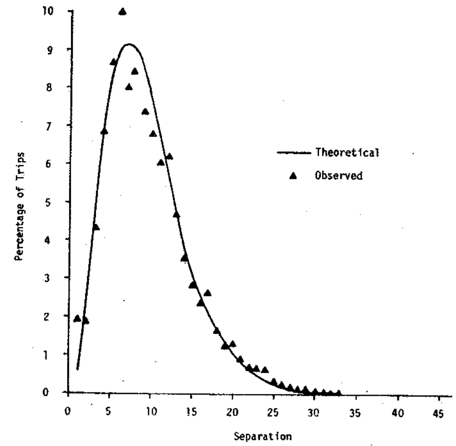


FIGURE B-8: AUSTIN TRANSPORTATION STUDY (HBW)

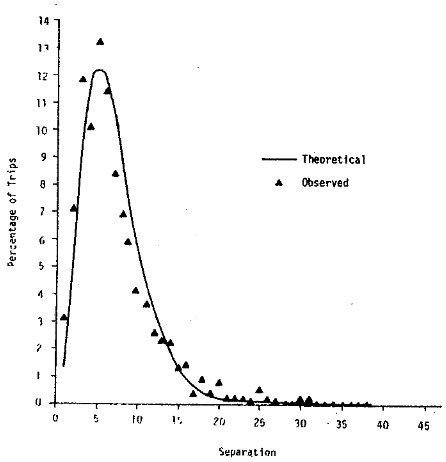


FIGURE B-9: BRYAN - COLLEGE STATION TRANSPORTATION STUDY (HBW)

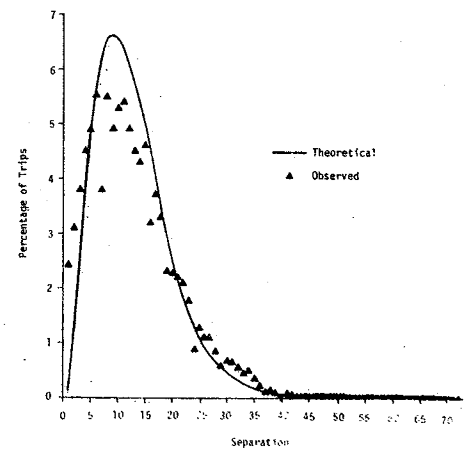


FIGURE B-10: EL PASO TRANSPORTATION STUDY (HBW)

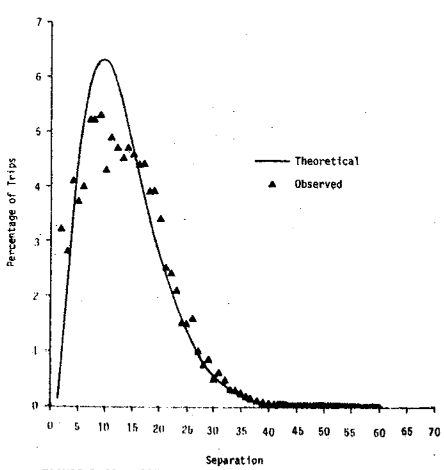


FIGURE B-11: SAN ANTONIO TRANSPORTATION STUDY (HBW)

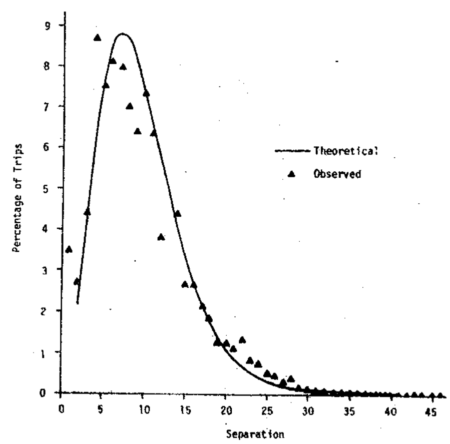


FIGURE B-12: WACO TRANSPORTATION STUDY (HBW)

NOTE: Scales are different between individual figures.

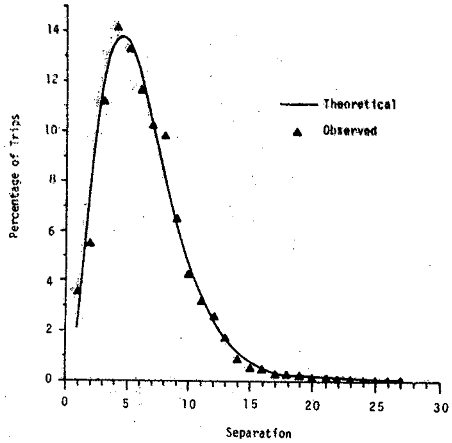


FIGURE B-13: ABILENE TRANSPORTATION STUDY (HBW)

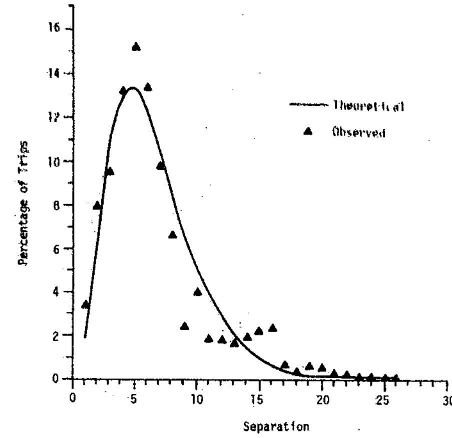


FIGURE B-14: BROWNSVILLE TRANSPORTATION STUDY (HBW)

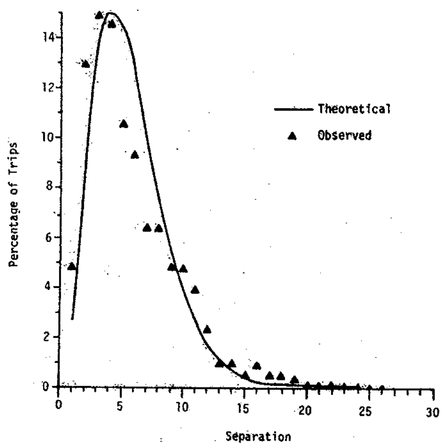


FIGURE B-15: HARLINGEN - SAN BENITO TRANSPORTATION STUDY (HBW)

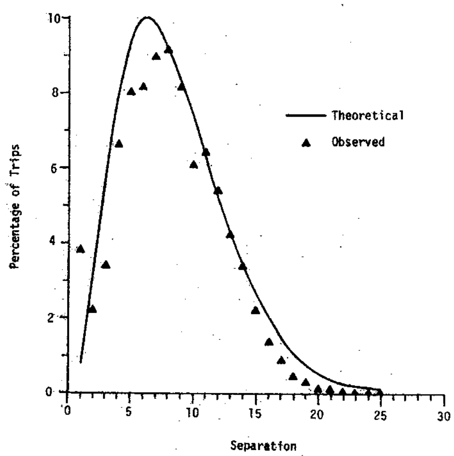


FIGURE B-16: LUBBOCK TRANSPORTATION STUDY (HBW)

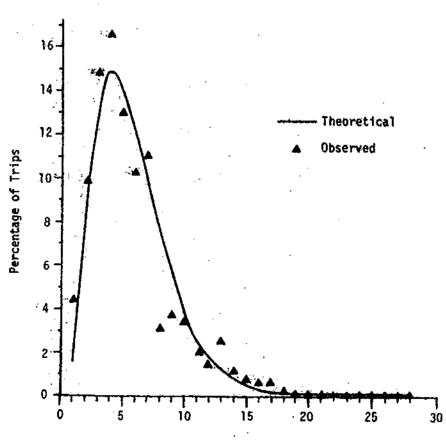


FIGURE B-17: VICTORIA TRANSPORTATION STUDY (HBW)

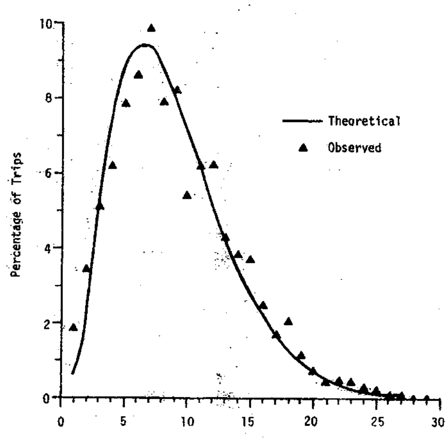


FIGURE B-18: WICHITA FALLS TRANSPORTATION STUDY (HBW)

NOTE: Scales are different between individual figures.

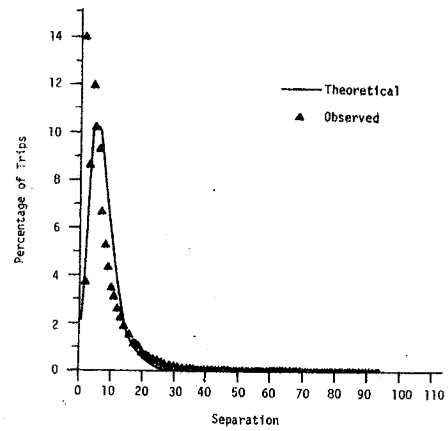


FIGURE B-19: DALLAS - FT. WORTH TRANSPORTATION STUDY (HBNW)

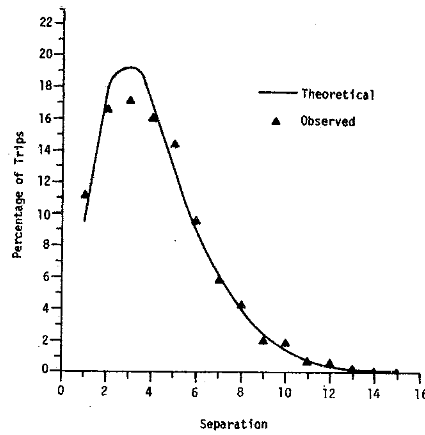


FIGURE B-20: LAREDO TRANSPORTATION STUDY (HBNW)

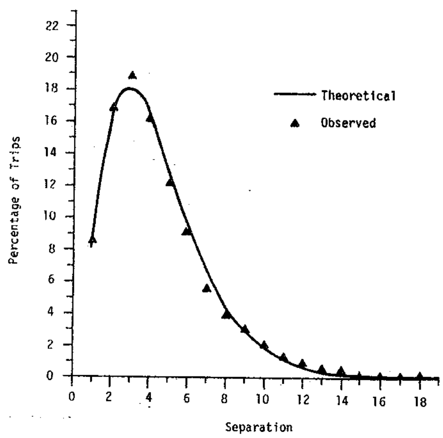


FIGURE B-21: McALLEN PHARR TRANSPORTATION STUDY (HBNW)

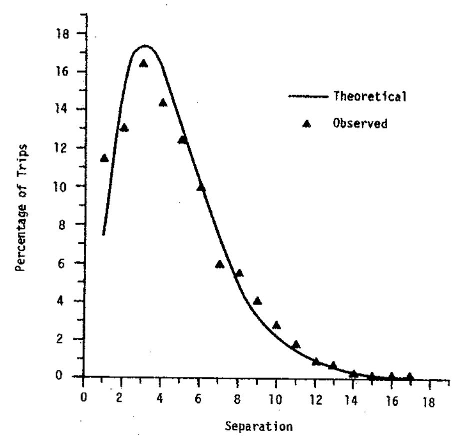


FIGURE B-22: SAN ANGELO TRANSPORTATION STUDY (HBNW)

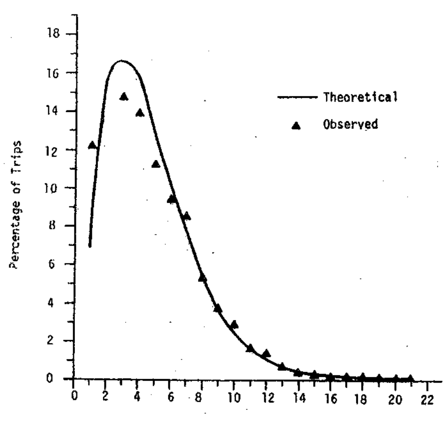


FIGURE B-23: TEXARKANA TRANSPORTATION STUDY (HBNW)

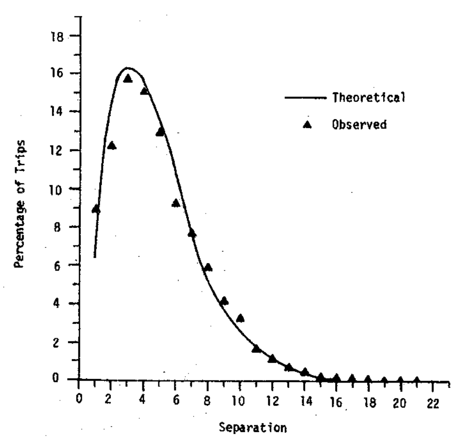


FIGURE B-24: TYLER TRANSPORTATION STUDY (HBNW)

NOTE: Scales are different between individual figures.

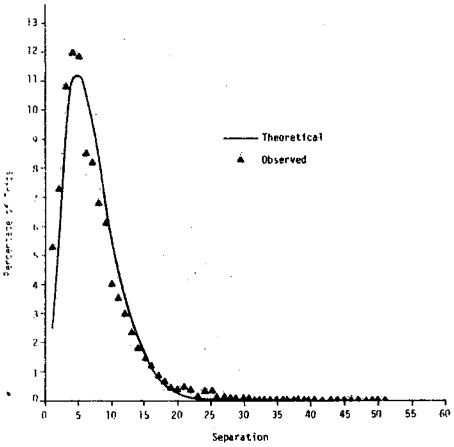


FIGURE B-25: AMARILLO TRANSPORTATION STUDY (HBNW)

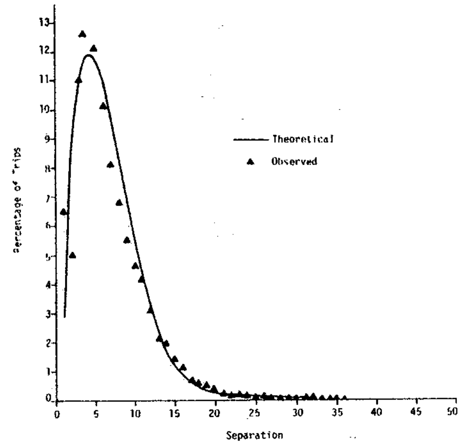


FIGURE B-26: AUSTIN TRANSPORTATION STUDY (HBNW)

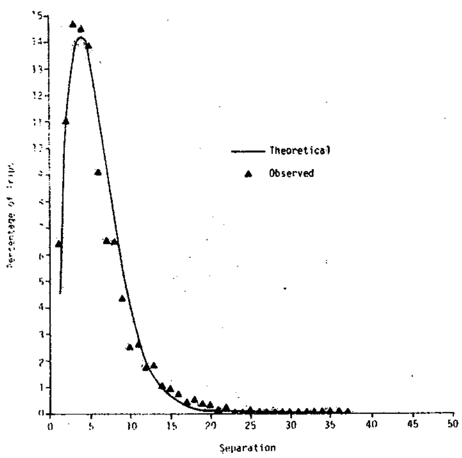


FIGURE B-27: BRYAN - COLLEGE STATION TRANSPORTATION STUDY (HBNW)

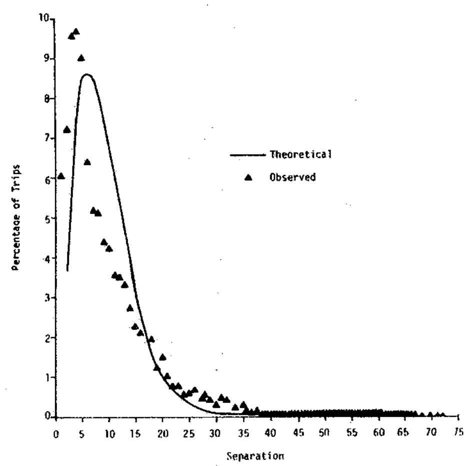


FIGURE B-28: EL PASO TRANSPORTATION STUDY (HBNW)

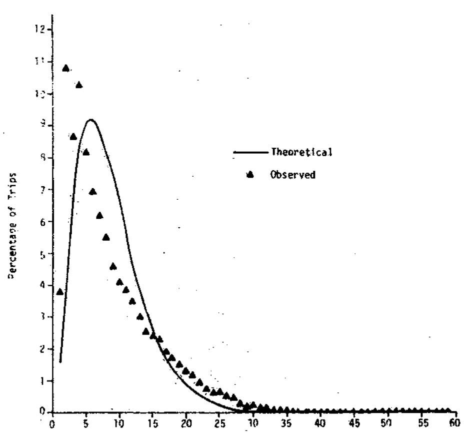


FIGURE B-29: SAN ANTONIO TRANSPORTATION STUDY (HBNW)

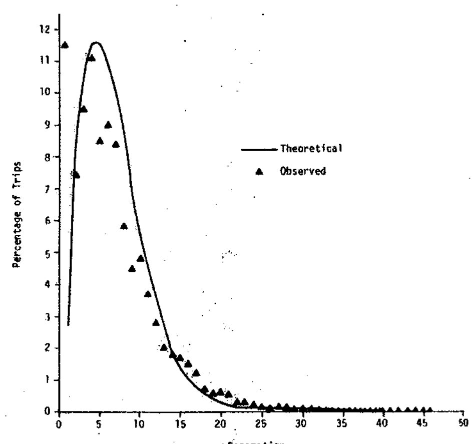


FIGURE B-30: WACO TRANSPORTATION STUDY (HBNW)

NOTE: Scales are different between individual figures.

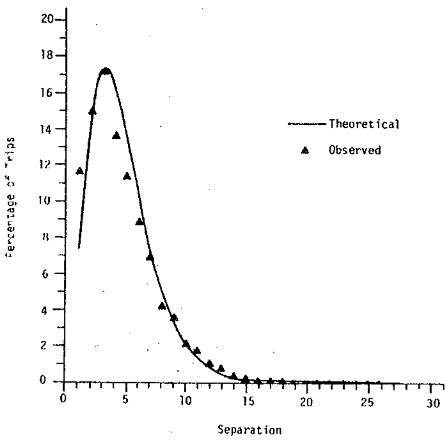


FIGURE B-31: ABILENE TRANSPORTATION STUDY (HBNW)

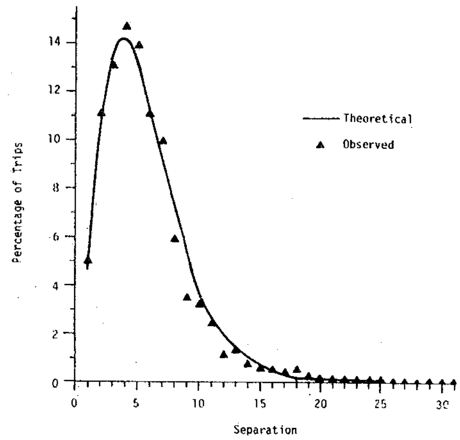


FIGURE B-32: BROWNSVILLE TRANSPORTATION STUDY (HBNW)

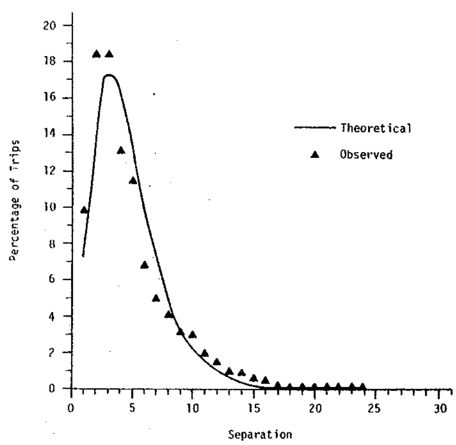


FIGURE B-33: HARLINGEN - SAN BENITO TRANSPORTATION STUDY (HBNW)

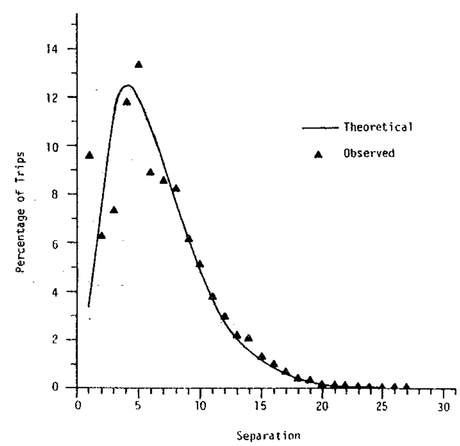


FIGURE B-34: LUBBOCK TRANSPORTATION STUDY (HBNW)

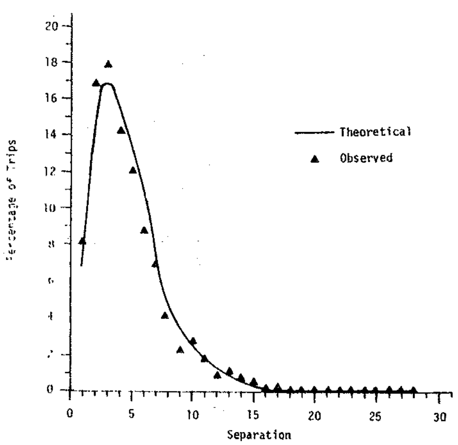


FIGURE B-35: VICTORIA TRANSPORTATION STUDY (HBNW)

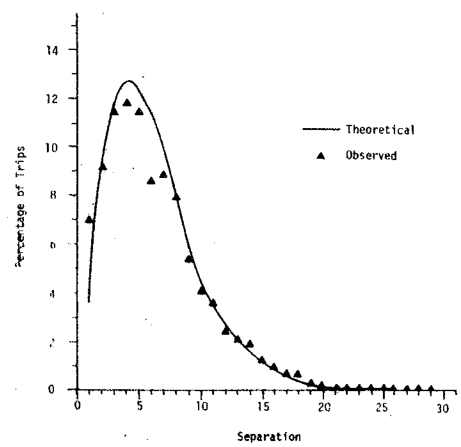


FIGURE B-36: WICHITA FALLS TRANSPORTATION STUDY (HBNW)

NOTE: Scales are different between individual figures.

APPENDIX C

DERIVATION OF PARTIALS FOR THE GAMMA AND WIEBULL DISTRIBUTIONS

Since the Gamma and Wiebull functions are different, this section will be broken into two parts, each describing the steps taken to derive the partials of the Gamma and Wiebull distributions with respect to their shape, α, and scale, β, parameters. The following theoretical differential formulas were used (7):

Equation No.	Equation
C-1	$\frac{d}{dx}(uvw) = uv\frac{d}{dx}(w) + un\frac{d}{dx}(v) + vw\frac{d}{dx}(u)$
C-2	$\frac{d}{dx}(a^u) = a^u \ell n\ a \frac{du}{dx},\ (a > o)$
C-3	$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

The two-parameter Wiebull distribution is expressed in the following relationship:

$$f(t) = \alpha\ \beta\ t^{\alpha-1}\ e^{-\beta t^{\alpha}} \qquad \text{C-4}$$

Where

α = shape parameter

β = scale parameter

e = based of natural logarithms (2.71 ...)

t = time

f(t) = relative frequency of occurrence of trips at separation time t

The following equalities are established to simplify the Wiebull expression:

$$\mu = \alpha\beta$$

$$v = t^{\alpha-1}$$

$$w = e^{-\beta t^{\alpha}}$$

Using the above relations, the following were derived using equations C-2 and C-3.

$$\frac{du}{d\alpha} = \beta \qquad \frac{du}{d\beta} = \alpha$$

$$\frac{dv}{d\alpha} = t^{\alpha-1}\ell Nt \qquad \frac{dv}{d\beta} = 0.0$$

$$\frac{dw}{d\alpha} = e^{-\beta t^{\alpha}}(-\beta t^{\alpha}\ell nt) \quad \frac{dw}{d\beta} = e^{-\beta t^{\alpha}}(-t^{\alpha})$$

Substituting those relationships into Equation C-1 and simplifying yields the following expressions:

$$\frac{d}{d\alpha}\{f(t)\} = \beta t^{\alpha-1}\ e^{-\beta t^{\alpha}}\{1 + \alpha \ln(t) - \alpha\beta t^{\alpha}\ln(t)\} \qquad \text{C-5}$$

$$\frac{d}{d\beta}\{f(t)\} \quad \alpha t^{\alpha-1}\ e^{-\beta t^{\alpha}}\{1 - \beta t^{\alpha}\} \qquad \text{C-6}$$

The Equations C-5 and C-6 are then the partials of the Wiebull distribution with respect to its shape parameter, α, and its scale parameter, β.

The two-parameter Gamma distribution is expressed in the following relationship:

$$f(t) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\ t^{\alpha-1}\ e^{-\beta t} \qquad \text{C-7}$$

Where

α = shape parameter

β = scale parameter

e = base of natural logarithms (2.71 ...)

t = time

f(t) = relative frequency of occurrence of trips taking time t

$\Gamma(\alpha) = (\alpha - 1)!$ and is generally called the Gamma function

By rearranging, Equation C-7 may be written in the following manner:

$$f(t) = \beta^{\alpha} t^{\alpha-1} e^{-\beta t} \{\Gamma(\alpha)\}^{-1}$$

The following equalities are established to simplify the Gamma distribution:

$$u = \beta^{\alpha}$$

$$v = t^{\alpha-1}$$

$$w = [\Gamma(\alpha)]^{-1} e^{-\beta t}$$

Using Equations C-2 and C-3, the differentials were found and are as follows:

$$\frac{du}{d\alpha} = \beta^{\alpha} \ell n\ \beta \qquad \frac{du}{d\beta} = \alpha\beta^{\alpha-1}$$

$$\frac{dv}{d\alpha} = t^{\alpha-1} \ell n\ t \qquad \frac{dv}{d\beta} = 0.0$$

$$\frac{dw}{d\alpha} = e^{-\beta t} \frac{d}{d\alpha}\left(\{\Gamma(\alpha)\}^{-1}\right) \qquad \frac{dw}{d\beta} = \{\Gamma(\alpha)\}^{-1} e^{-\beta t}(-t)$$

Substituting the above expressions into Equation C-1, the following expressions were found:

$$\frac{d}{d\alpha}\{f(t)\} = \beta^{\alpha}t^{\alpha-1}e^{-\beta t}\left[\frac{d}{d\alpha}\left(\{\Gamma(\alpha)\}^{-1}\right)\right] + \beta^{\alpha}\{\Gamma(\alpha)\}^{-1}e^{-\beta t}t^{\alpha-1}\ell n\ t$$
$$+ t^{\alpha}\{\Gamma(\alpha)\}^{-1}e^{-\beta t}\beta^{\alpha}\ln\beta \qquad \text{C-8}$$

$$\frac{d}{d\beta}\{f(t)\} = \beta^{\alpha}t^{\alpha-1}\{\Gamma(\alpha)\}^{-1}e^{-\beta t}(-t) + t^{\alpha-1}\{\Gamma(\alpha)\}^{-1}e^{-\beta t}\alpha\beta^{\alpha-1} \qquad \text{C-9}$$

The above expressions are almost complete as far as performing computations except for the Gamma function, $\Gamma(\alpha)$. Research revealed that for large positive values of x, $\Gamma(x)$ approaches the following asymptotic series (8).

$$x^x e^{-x} \sqrt{\frac{2\pi}{x}} \left[1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4} + \cdots \right]$$

No delineation point was found as to what was considered large and small so the decision was made to use the series shown to the first 3 terms. Since the end result is involved in an iterative technique, any error introduced here should have little effect. Thus the following relationship was established:

$$\Gamma(\alpha) = \alpha^{\alpha} e^{-\alpha} \sqrt{2\pi/\alpha} \;\{1 + \frac{1}{12\alpha} + \frac{1}{288\alpha^2}\}$$

Multiplying through and inverting the results gives the following:

$$\{\Gamma(\alpha)\}^{-1} = \alpha^{-\alpha} e^{\alpha} \left[2\pi/\alpha\right]^{-\frac{1}{2}} + \left[\alpha^{1-\alpha} e^{\alpha} (2\pi/\alpha)^{-\frac{1}{2}}\right]_{/12} + \left[\alpha^{2-\alpha} e^{\alpha} (2\pi/\alpha)^{-\frac{1}{2}}\right]_{/288} \qquad \text{C-10}$$

The above expression was then broken into three parts and the following equalities set up.

$$A = \alpha^{-\alpha} e^{\alpha} \left[2\pi/\alpha\right]^{-\frac{1}{2}}$$

$$B = \left[\alpha^{1-\alpha} e^{\alpha} (2\pi/\alpha)^{-\frac{1}{2}}\right]_{/12}$$

$$C = \left[\alpha^{2-\alpha} e^{\alpha} (2\pi/\alpha)^{-\frac{1}{2}}\right]_{/288}$$

Thus the following expression holds:

$$\frac{d}{d\alpha}\{\Gamma(\alpha)\}^{-1} = \frac{dA}{d\alpha} + \frac{dB}{d\alpha} + \frac{dC}{d\alpha}$$

For the sake of time and simplicity, the step by step differentiation is not shown here for A, B, and C but let it suffice to say that Equations C-1, C-2, and C-3 were used and the same procedure followed as before. The final result is as follows:

$$\frac{d}{d\alpha}\{\Gamma(\alpha)\}^{-1} = \alpha^{-\alpha} e^{\alpha} \left[2\pi/\alpha\right]^{-\frac{1}{2}} \left\{ \frac{2\pi^2}{\alpha^3} - \ln\alpha + \left(\frac{2\pi^2}{\alpha^2} + 1 - \alpha\ln\alpha\right)_{/12} + \left(\frac{2\pi^2}{\alpha} + 2\alpha - \alpha^2 \ln\alpha\right)_{/288} \right\} \qquad \text{C-11}$$

Substituting Equations C-10 and C-11 into Equations C-8 and C-9 gives the computational equations for the partials of the Gamma distribution with respect to its shape, α, and scale, β, parameters.

APPENDIX D

A METHOD FOR ESTIMATING THE MAXIMUM SEPARATION AT WHICH AN INTERCHANGE OF TRIPS MAY BE EXPECTED TO OCCUR

A method of predicting the maximum separation at which an interchange of trips might be expected to occur was developed, which was simple as well as accurate. In studying the trip length frequency distributions (TLFD) shown in Appendix B, it was noted that trips were never exchanged at the maximum possible separation. Since this characteristic was common to every TLFD, some method was needed to predict the maximum separation at which trips would be interchanges. This was necessary since the procedure used a Gamma distribution to predict the TLFD and, if the procedure used the maximum possible separation, the resulting TLFD would show trips occuring at every separation. Also, the resulting distribution would be spread out so far that it would not represent a realistic TLFD.

In an attempt to find a method, plots were made for trip purposes HBW and HBNW of the maximum separation at which an interchange occurred versus the maximum separation possible. Since the plots were very linear, least squares curve fits were performed forcing the fit to pass through the origin. The results are shown in Figure D-1 and D-2. The coefficient of determination for the trip purpose HBW was 0.9653 and for HBNW was 0.9732. Both indicate relatively good fits. A similar analysis was performed for the trip purposes nonhome based (NHB) and truck-taxi (TRTX). The results were similar to those for HBW and HBNW. Although the plots for NHB and TRTX are not presented, the linear relationship determined is presented with those for HBW and HBNW. A question still remained, however, as to the effect an error in that estimate would have on the overall procedure.. Subsequent testing revealed that the effect was not noticeable since the absolute values (i.e., number of trips) at those separations were small. The main consideration was that the distribution did not attempt to show trips occurring at every possible separation. To prevent this, all that was needed was a fairly reasonable estimate of the maximum separation at which an interchange of trips could be expected to occur. The following linear relations (plotted in Figures D-1 and D-2) satisfy that requirement:

Trip Purpose	Equation
HBW	Y = 0.7825X
HBNW	Y = 0.767X
NHB	Y = 0.880X
TRTX	Y = 0.824X

where

X = Maximum Separation Possible

Y = Estimate of the maximum separation at which an interchange of trips will occur.

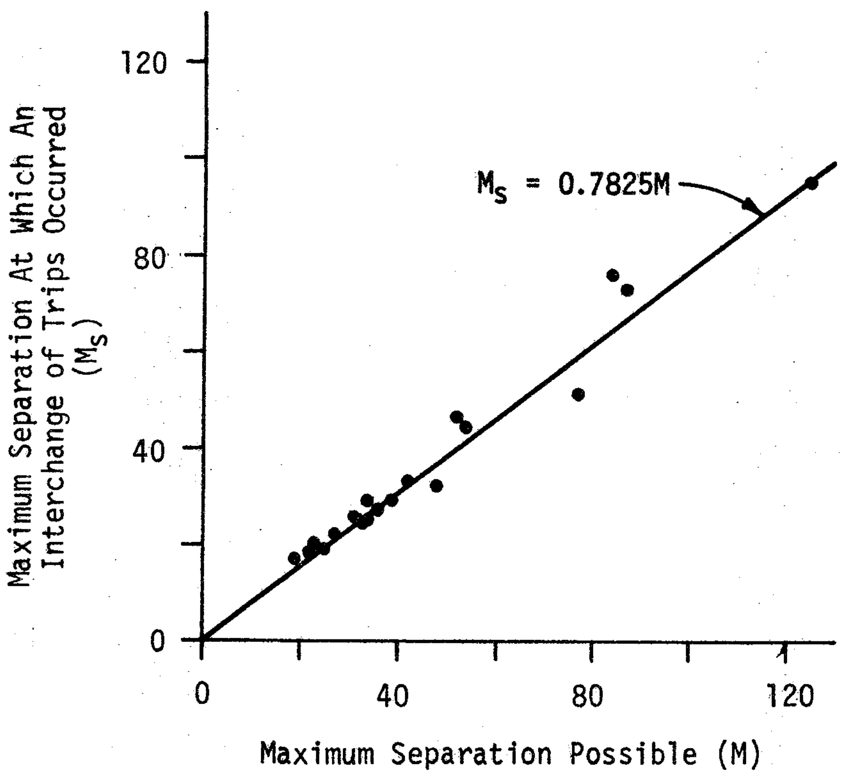


Figure D-1: Variation of M_s with M For HBW Trips

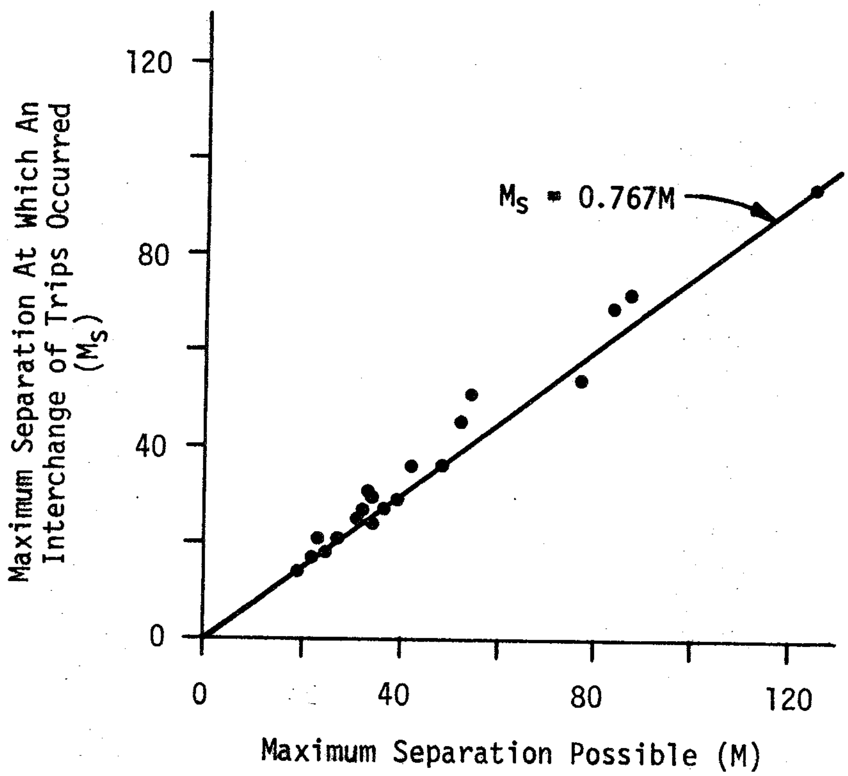


Figure D-2: Variation of M_S with M For HBNW Trips

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