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A GRAPHICAL TECHNIQUE FOR DETERMINING THE ELASTIC MODULI OF A TWO-LAYERED STRUCTURE FROM MEASURED SURFACE DEFLECTIONS

in cooperation with the
Department of Transportation
Federal Highway Administration

RESEARCH REPORT 136-3
STUDY 2-8-69-136
FLEXIBLE PAVEMENTS

A GRAPHICAL TECHNIQUE for DETERMINING the ELASTIC MODULI of a TWO-LAYERED STRUCTURE from MEASURED SURFACE DEFLECTIONS

by

Gilbert Swift

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Design and Evaluation of Flexible Pavements
Research Study 2-8-69-136

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PREFACE

This is the third report issued under Research Study 2-8169-136, "Design and Evaluation of Flexible Pavements", being conducted as part of the cooperative research program with the Texas Highway Department and the Department of Transportation, Federal Highway Administration.

The first report is:

> "Seasonal Variations of Pavement Deflections in Texas" by Rudell Poehl and Frank H. Scrivner, Research Report 136-1 Texas Transportation Institute, January 1971.

The second report is:

> "A Technique for Measuring the Displacement Vector throughout the Body of a Pavement Structure Subjected to Cyclic Loading" by W. M. Moore and Gilbert Swift, Research Report 136-2, Texas Transportation Institute, August 1971.

The author wishes to thank all members of the Institute who assisted in the work leading to the present report, especially Mr. F. H. Scrivner, Dr. W. M. Moore and Mr. C. H. Michalak who provided valuable assistance and advice in connection with the mathematical phases, and Mr. Charles E. Schlieker who drew the final version of the chart.

The author is grateful to the Texas Highway Department for their interest and cooperation, especially to Mr. James L. Brown and Mr. L. J. Buttler for their support of this research.

ABSTRACT

A theoretically derived chart has been developed by means of which the elastic modulus of each layer of a two-layer elastic structure can be evaluated. The required input data comprises two or more surface deflections observed at different known distances from a known vertical load, together with knowledge or an estimate of the thickness of the upper layer. The lower layer is assumed to be of infinite thickness.

One version of the chart and directions for its use are presented.

Key words: Deflections, Elastic Moduli, Graphical Computation, Pavement Structure

SUMMARY

The chart presented in this report permits pavement surface deflection measurements, such as Dynaflect readings, to be interpreted in terms of the properties (elastic moduli) of a two-layer elastic system. The moduli obtained from deflections observed at various locations can be utilized to characterize the materials at these locations and to determine whether the structures differ with respect to the deeper, or the shallower materials, or both.

IMPLEMENTATION STATEMENT

It is expected that the chart and graphical technique presented in this report will simplify and facilitate the interpretation of pavement deflection measurements by allowing the influence of the upper materials to be separated from that of the lower (subgrade) material. Separate evaluations of the properties of the materials in these two zones can lead to a better understanding of pavement structure behavior.

Extensions of this method to more than two layers and computerized versions of the technique may find broad applications in pavement condition evaluation.

The graphical form of the present method permits its use in the field, for immediate interpretation of the deflection data at the time the deflection measurements are being made.

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1. INTRODUCTION

Measurements of the deflections of pavement structures have been noted to bear a strong resemblance to the deflections computed for layered elastic systems. Since it thus appears that pavement deflection behavior is at least approximately elastic, it becomes desirable to ascertain what theoretical elastic structure most closely resembles a given pavement structure with respect to its deflections. The values of the moduli of the layers in this theoretical structure may then be attributed to the materials in the pavement structure. These modulus values can be expected to be useful for characterizing the materials and for comparing one pavement structure, or one location on a structure, with another.

While the technique detailed in this report is presently limited to interpretation of the observed deflections in terms of a two-layer elastic system, it is believed that a majority of pavement structures strongly resemble two-layer systems. For such structures this technique provides a means for characterizing the materials above and below a selected depth in terms of elastic theory. The two derived modulus values provide a basis for recognizing whether it is the deeper or the shallower materials in a given structure which cause its deflections to differ from those of some other structure. Accordingly, use of the derived modulus values can be expected to lead to improved understanding of pavement deflection behavior and the causes for its variability.

It is well known that the deflections on the surface of a layered elastic half-space can be computed, given the elastic properties and the thickness of each layer. Several equations and computer programs which can be used for this purpose are available (1)(2)(3)(4). However, the reverse problem, that of determining the elastic properties of a layered structure from knowledge of its deflections, is not amenable to direct analytic solution. Indirect methods are therefore required to solve the reverse problem.

A computer program which permits finding the elastic moduli of a two-layer structure from the deflections observed at two locations on its surface has been described by Scrivner, Michalak and Moore (4). In contrast to that method, the technique presented here utilizes the deflections observed at any plurality of locations. Using a set of five deflection measurements, such as is normally obtained with the Dynaflect(5)*, a wide range of structures can be analyzed without encountering non-unique solutions. Also, the graphical form of this technique permits its use when access to a computer is inconvenient. Within about 60 seconds after a set of deflections has been measured one can know whether these measurements correspond to an elastic system having one layer (homogeneous), or two layers, or more than two layers, and (when two or less) learn the modulus values required by the theory of elasticity to account for the observed deflections.

Extension of this method to include three layers merely requires computation and drafting of the requisite set of charts.

*Registered trademark, Radiation Engineering and Manufacturing Company (REMCO) 7450 Winscott Road, Fort Worth, Texas.

2. BASIS FOR THE TECHNIQUE

The deflection behavior on the surface of a layered elastic half-space, at various distances from a vertical load applied at a point (or on a sufficiently small area) can be expressed conveniently in terms of several dimensionless ratios. When Poisson's ratio is taken equal to 0.5 for both layers the relationship(1) can be written as:

$$\frac{wrE_2}{P} = \frac{3}{4\pi} f\left(\frac{E_1}{E_2}, \frac{r}{h}\right) \qquad \text{Eq. 1.}$$

where w is the vertical displacement at a distance, r, from the load,

E_1 is the elastic modulus of the upper layer,

E_2 is the elastic modulus of the lower layer,

P is the magnitude of the load,

and h is the thickness of the upper layer. (The lower layer is assumed to extend infinitely downward.)

For each particular value of the ratio $\frac{E_1}{E_2}$, the quantity $\frac{wrE_2}{P}$ is a different function of $\frac{r}{h}$. This fact permits evaluation of the ratio $\frac{E_1}{E_2}$, through finding the best match between a given set of measured deflection data and one of several sets of computed deflection data in which $\frac{E_1}{E_2}$ has a different constant value for each set.

After evaluating the modular ratio, $\frac{E_1}{E_2}$, the value of E_2 can then be determined from the relationship:

$$E_2 = \frac{wrE_2}{P} \text{ (computed)} \div \frac{wr}{P} \text{ (observed)}, \qquad \text{Eq. 2.}$$

Finally, E_1 can be obtained from the relationship:

$$E_1 = E_2 \times \frac{E_1}{E_2} \qquad \text{Eq. 3.}$$

The technique can be applied in a graphical manner by plotting the computed data sets in the form of a family of curves on bi-logarithmic graph paper. This family of curves, in conjunction with a related plot of the observed deflection data on a separate graph sheet, permits direct determination of the modular ratio $\frac{E_1}{E_2}$ and of the modulus of the lower layer, E_2, upon matching the set of plotted data-points to one (or between a pair) of the curves. The detailed procedure is given in Section 4 below.

3. DESCRIPTION OF THE CHART

While it is possible to construct the chart in several forms, the form presented in Figure 1 is believed to offer maximum convenience.

On this chart the quantity $10^5 \frac{wrE_2}{P}$ has been plotted as a function of the quantity $\frac{r}{h}$ with the ratio $\frac{E_1}{E_2}$ as a parameter. Each separate curve is identified with its particular value of $\frac{E_1}{E_2}$, ranging from 1/5 to 1000. Each of the curves extends from $\frac{r}{h} = 0.1$ to $\frac{r}{h} = 10.0$. The central vertical line, $\frac{r}{h} = 1.0$, is emphasized. A vertical numerical scale of the quantity $10^5 \frac{wrE_2}{P}$ is also provided, from which E_2 may be read directly.

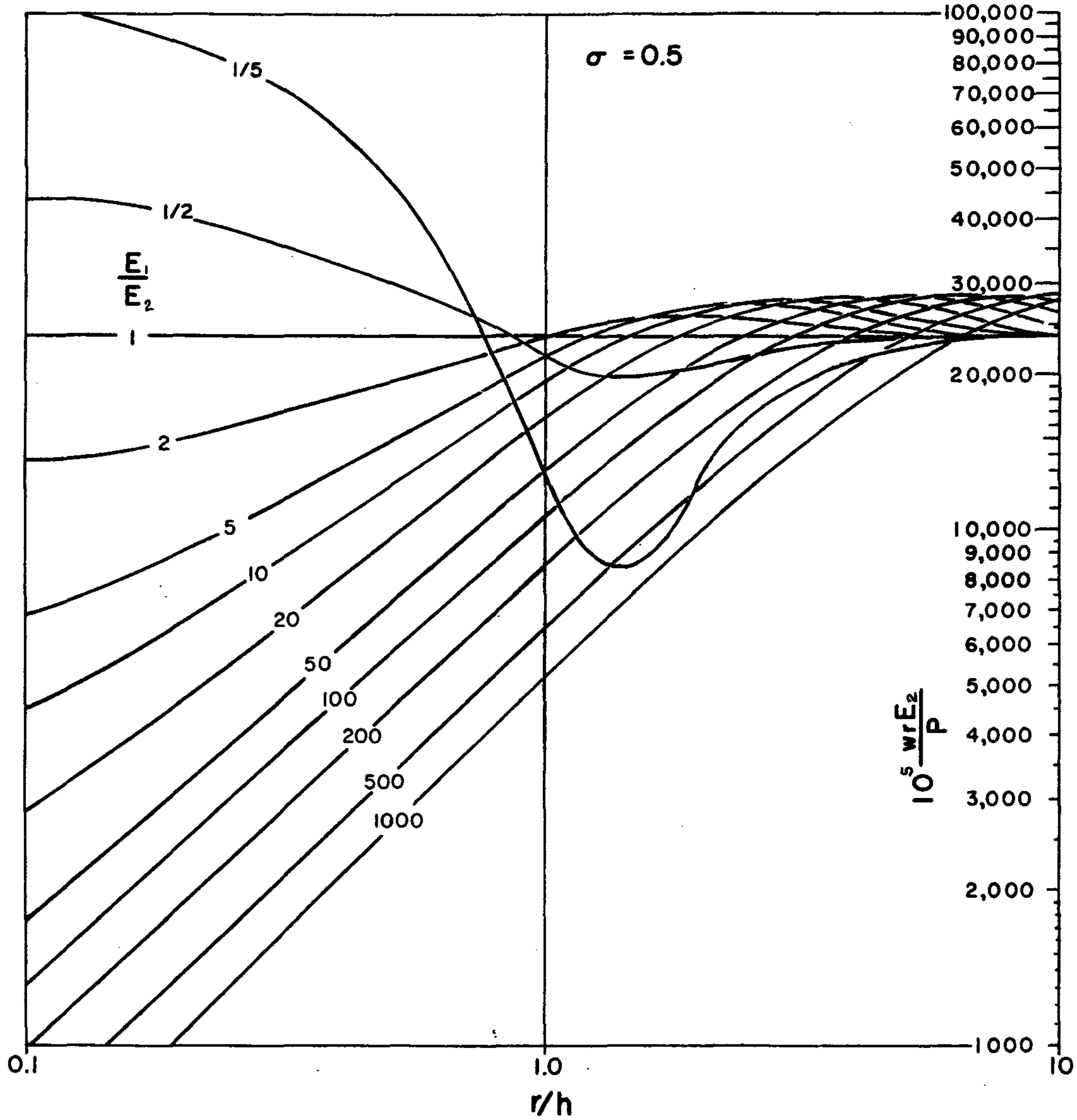

Figure 1: Two-Layer Elastic Deflection Chart. (Poisson's Ratio, σ = 0.5)

Note: This is a reduction of the original chart. An equally reduced bilogarithmic grid, which may be used to plot measured deflection data on tracing paper for overlay on this chart, is included in Appendix A.

4. DIRECTIONS FOR USING THE CHART

In order to use the chart, values of the quantity $\frac{wr}{P}$, derived from the measured deflections, are plotted versus r on a separate sheet using logarithmic scales dimensionally equal to those of the chart. For Dynaflect measurements the first step merely requires multiplying the deflections measured at the five standard locations by 10, 15.6, 26, 37.4 and 49 (inches), respectively. The resulting $\frac{wr}{P}$ values (wr per 1000 pounds) are then used to plot a set of points on the separate sheet. The $\frac{wr}{P}$ values are plotted vertically, versus the respective radial distances, r, horizontally. Figure 2 shows a typical plot of this type. The plot is then superimposed with the chart and aligned such that the vertical line $\frac{r}{h} = 1.0$ of the chart coincides with the vertical line on the plot for which r is equal to the known or assumed layer thickness, h.

Next, the plot is shifted vertically, while maintaining the alignment, until the set of plotted points coincides with one, or fits best between two, of the curves on the chart. In most cases the points will not precisely fit a single curve but can be placed in a "best fit" relationship between a pair of adjacent curves. Figure 3 illustrates one such case.

The ratio $\frac{E_1}{E_2}$ is now obtained, either directly from the identification on the one curve, or by interpolation between those of the two adjacent curves.

The value of E_2 in psi is next obtained by observing where, on the chart scale of the quantity $10^5 \frac{wrE_2}{P}$, the horizontal line of the plot corresponding to $\frac{wr}{P} = 0.01$ square inches per 1000 pounds intersects that scale.

Finally, the value of E_1 is obtained from equation 3, thus:

$$E_2 \times \frac{E_1}{E_2} = E_1$$

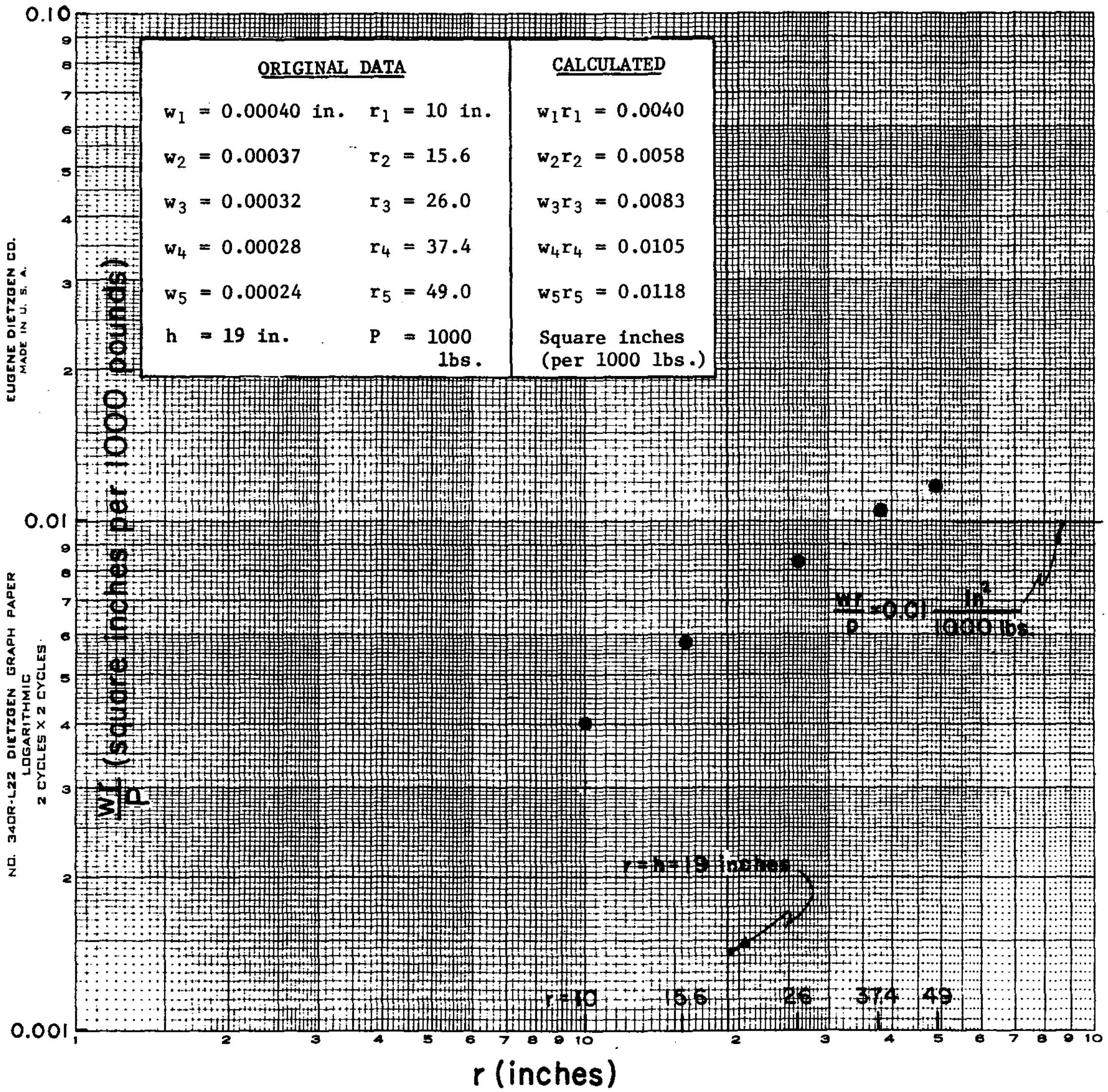

Figure 2: Typical plot of $\frac{wr}{P}$ versus r, derived from measured deflections.

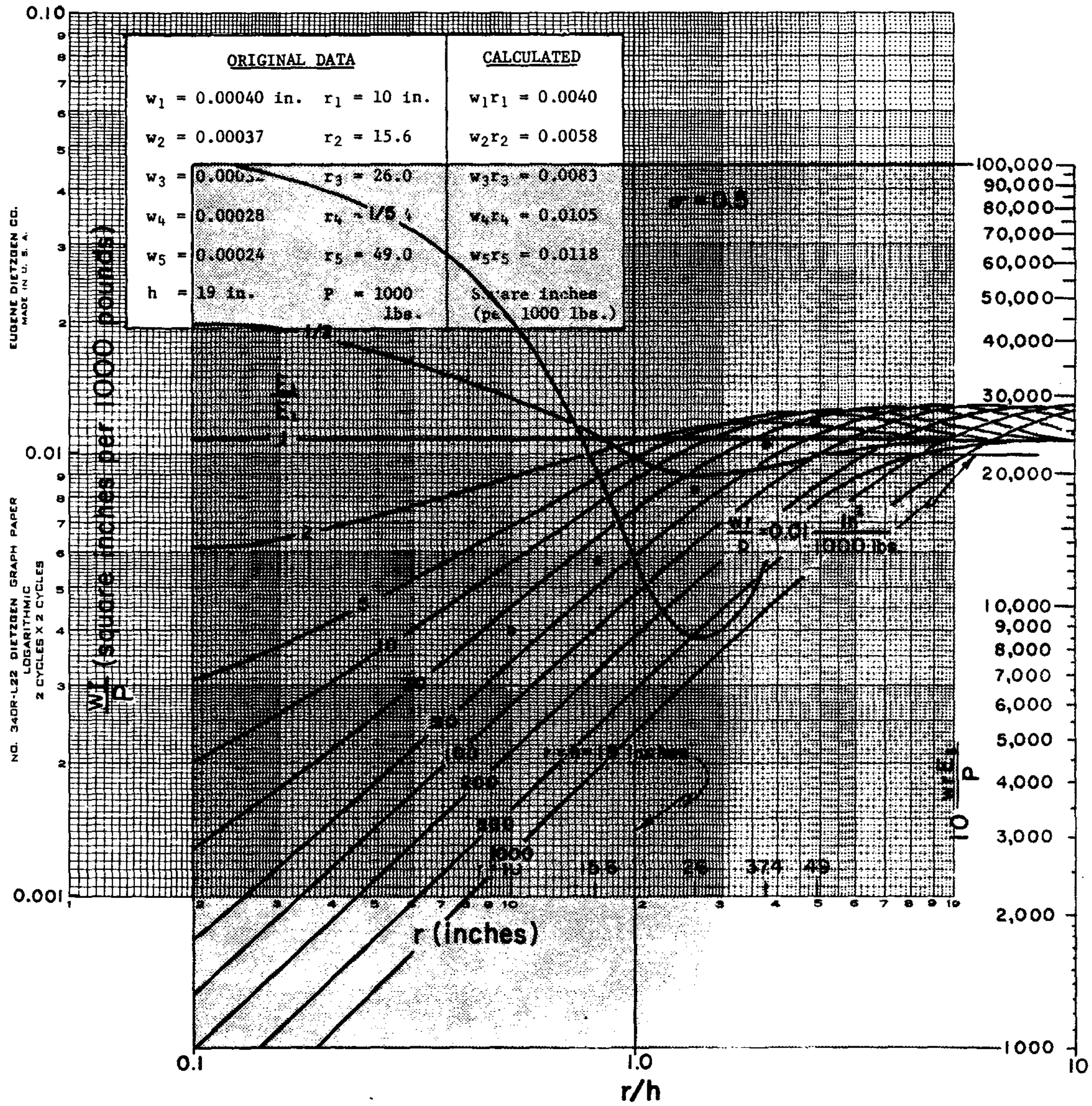

Figure 3: Example showing the chart superimposed in "best fit" position on the deflection plot. In this case, $\frac{E_1}{E_2}$ is found to be equal to 30 and the value of E_2 is found to be 22,000 psi. Accordingly, E_1 is 22, 000 x 30 or 660,000 psi.

5. EXAMPLES

The following examples are presented in the form of plots made on tracing paper which was superimposed over a bilogarithmic grid. The plots are shown as they would appear when aligned for best fit with the chart, but for clarity only a portion of the numerical scale and the line $\frac{r}{h} = 1.0$ of the chart are depicted.

Figure 4 exemplifies the use of the chart to interpret deflections measured at the same location at three different stages of construction. The deflections from which Figure 4a was plotted were observed prior to construction, on an excavated natural clay foundation. A close fit to the theoretical curve for $\frac{E_1}{E_2} = \frac{1}{5}$, and $E_2 = 21,000$ psi is obtained at $h = 24$ inches. Thus it appears that the upper two feet of this foundation has a substantially lower modulus value than the deeper material. From the nature of the departure of the plotted points from an exact fit with the theoretical curve for $\frac{E_1}{E_2} = \frac{1}{5}$ it can be estimated that the modular ratio is slightly less, say $\frac{1}{5.3}$, thus making $E_1 \approx 4000$ psi. After compaction of 2 feet of sandy clay fill at this site the measured deflections produced the plot shown in Figure 4b. This plot fits the chart in the vicinity of $h = 48$ inches, $E_2 \approx 28,000$ psi and $\frac{E_1}{E_2} \approx \frac{1}{2.2}$ ($E_1 = 12,800$ psi). Finally, upon completion of approximately 3 feet of the sandy clay fill, an 18-inch-thick stabilized limestone base course was added. The deflections then observed plot as shown in Figure 4c. This plot has been fitted to the chart at $h = 18$ inches, $E_2 \approx 26,000$ psi and $\frac{E_1}{E_2} = 20$ ($E_1 = 520,000$ psi).

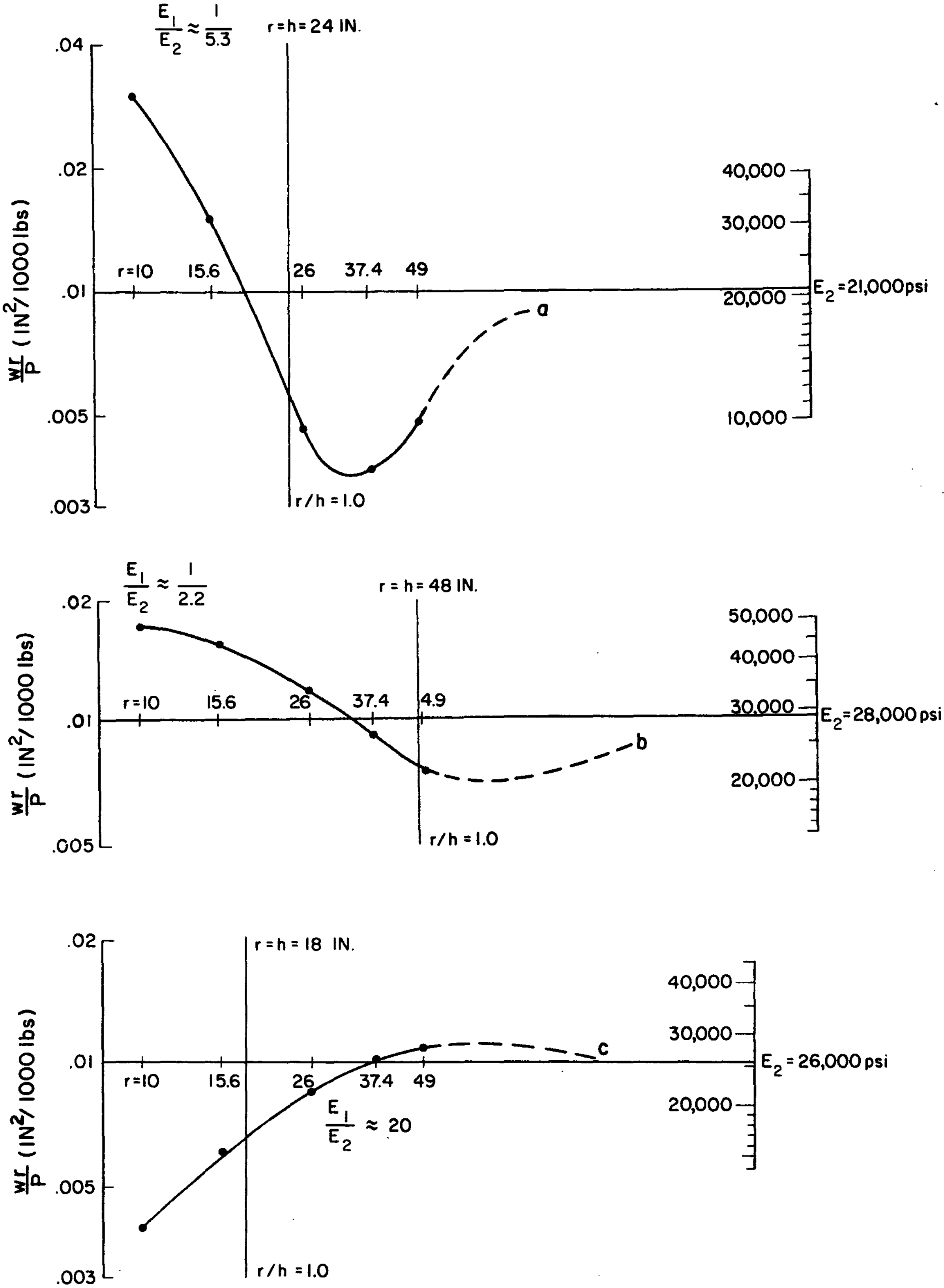

Figure 4: Deflection data obtained on the same site at three different stages of construction:

a, Prior to construction, on excavated natural clay foundation.
b, With 2 feet of compacted sandy clay fill.
c, Upon completion of an 18 inch thick stabilized limestone base course on approximately 3 feet of compacted sandy clay fill.

Thick highway or airport portland cement concrete pavements produce deflections similar to those from which Figures 5a and 5b have been plotted. The appropriate thicknesses and the derived modulus values are indicated in the respective captions to these figures. It is evident that the interpretation of the plots for such thick, stiff pavements is aided by having observations of the deflections at distances greater than the customary 49-inch maximum; in fact the 21-inch-thick concrete pavement warrants at least one measurement at r greater than 120 inches, in order to permit determining E_2 more precisely.

Figures 6a, 6b and 6c are derived from deflections measured at three locations along a city street, and serve to indicate the wide range of variation which can be detected in the support beneath an 8-inch-thick portland cement concrete pavement.

Another similarly paved street produced the three sets of deflection measurements plotted together in Figure 7. These particular locations were chosen to show that sets of deflections are sometimes obtained which cannot be matched to any two-layer elastic case.

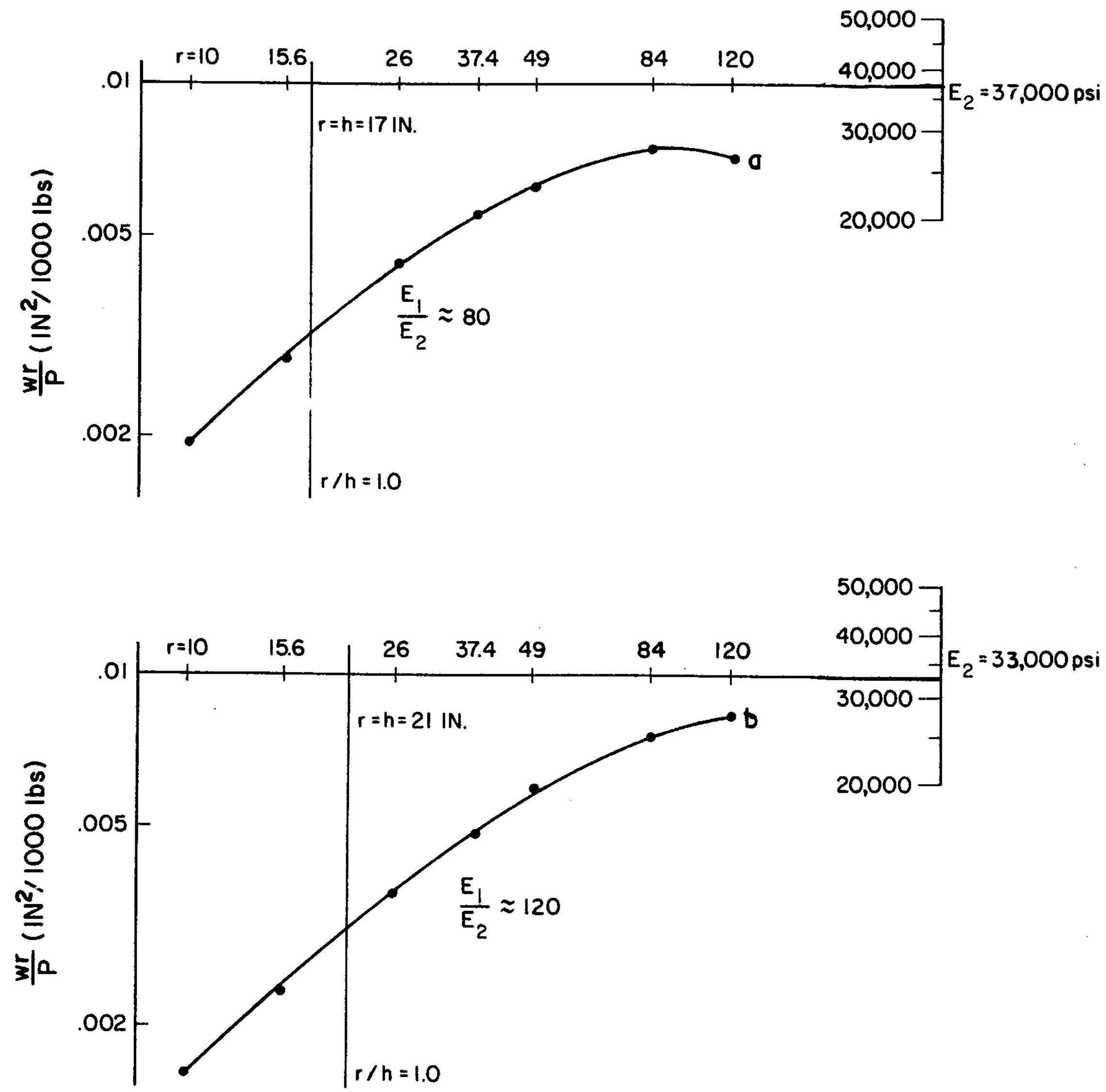

Figure 5: Deflection plots typical of thick portland cement concrete pavement:

a, A 17 inch thick airport pavement, sandy clay subgrade(CL), from the chart: $E_2 \approx 37{,}000$ psi, $\frac{E_1}{E_2} \approx 80$ $E_1 \approx 3{,}000{,}000$

b, A 21 inch thick airport pavement, sand subgrade(SP), from the chart: $E_2 \approx 33{,}000$ psi, $\frac{E_1}{E_2} \approx 120$ $E_1 \approx 4{,}000{,}000$

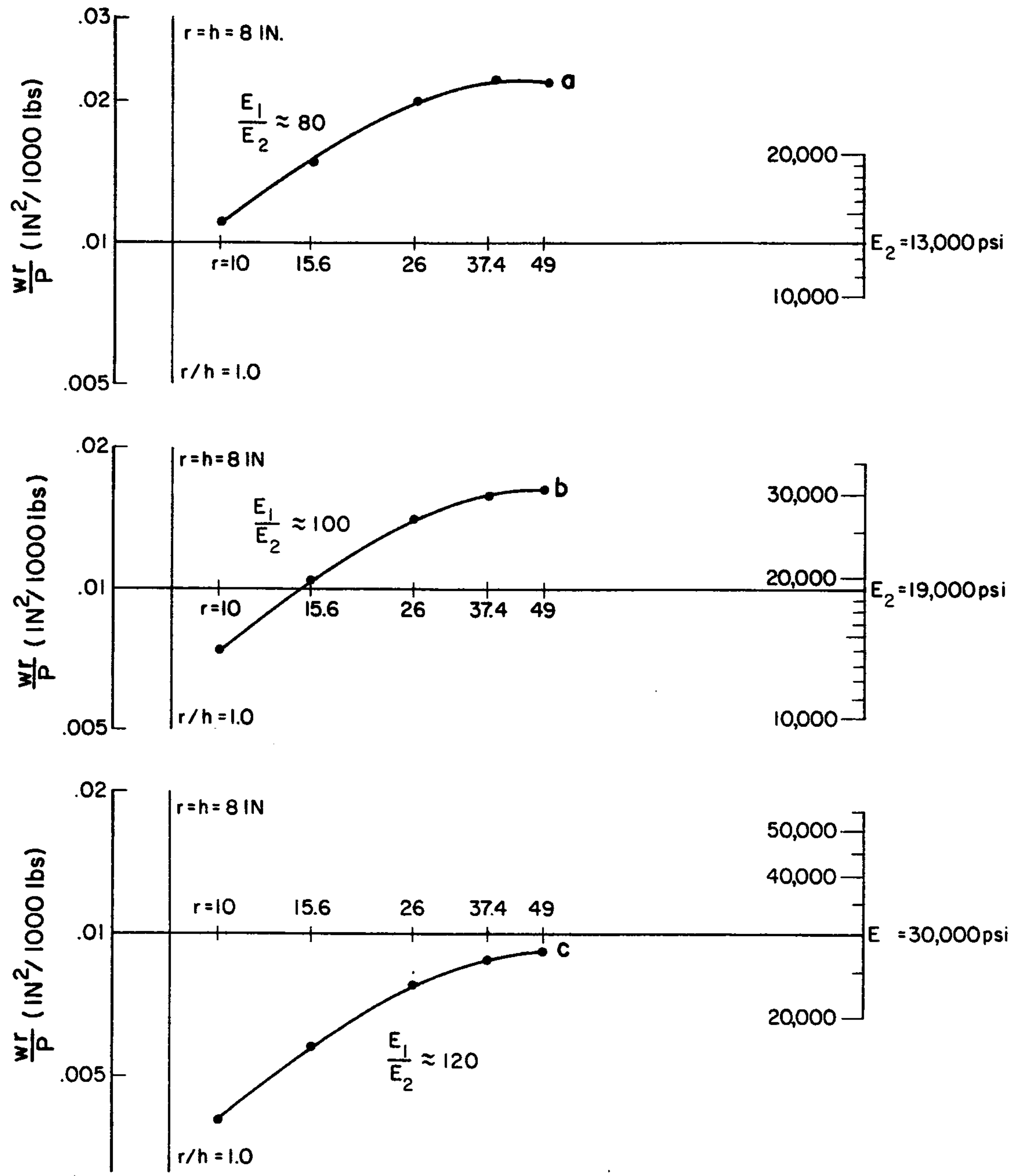

Figure 6: Deflection Data from Three Locations along a City Street with Portland Cement Concrete Pavement, 8 inches thick. Values derived from the chart are as follows:

a, $E_2 \approx$ 13,000 psi, $E_1/E_2 \approx 80$, $E_1 \approx$ 1,040,000 psi.
b, $E_2 \approx$ 19,000 psi, $E_1/E_2 \approx 100$, $E_1 \approx$ 1,900,000 psi.
c, $E_2 \approx$ 30,000 psi, $E_1/E_2 \approx 120$, $E_1 \approx$ 3,600,000 psi.

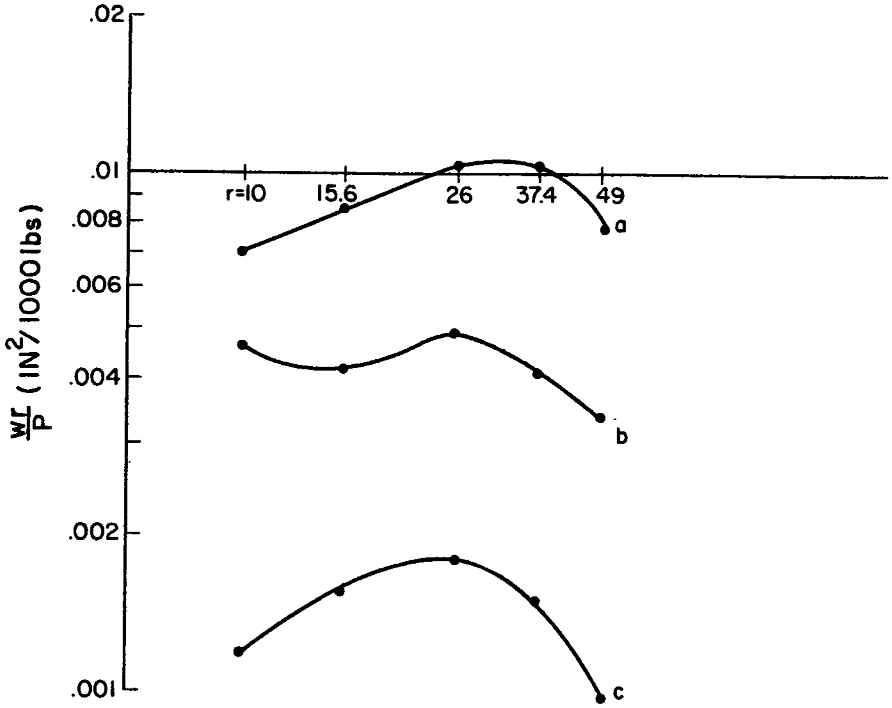

Figure 7: Deflection data from three locations on a city street with 8 inch thick portland cement concrete pavement. These deflections fail to conform with any two-layer elastic structure. Curve C can be fitted to a three-layer situation having a low-modulus middle layer.

6. SPECIAL CASES AND LIMITATIONS

The best available information concerning the thickness of the upper layer may sometimes be rather uncertain. In such cases, if the modulus of the upper layer is substantially less than that of the lower layer, the "best fit" of the plotted points to one of the curves can often be seen to require a horizontal shift of the plot with respect to the chart. In these cases a better value for the thickness, h, may be obtained by shifting the plot to secure the best fit and noting the value of r on the plot which then coincides with the line, $\frac{r}{h} = 1.0$, of the chart. The value of h is obtained at once from the relationship: $h = r$.

In other cases, particularly those in which the upper layer modulus is greatest, it will be found that equally good fits between the plotted points and one or another of the curves of the chart are obtainable with the plot shifted horizontally to any position within a relatively wide range. This results from the fact that very nearly the same deflections can occur on a number of different structures of this type. For example; the surface deflections, at any distance away from the load greater than about one half the layer thickness, are indistinguishably different among the following cases, provided E_2 is a constant.

Modular Ratio E_1/E_2	Upper layer thickness Inches
20	18.4
50	13.6
100	10.8
200	8.55
500	6.3
1000	5

This observation may be generalized by saying that, beyond a radius equal to half the layer thickness, the surface deflections are determined by E_2, the subgrade modulus, and by the product of $\sqrt[3]{E_1}$ and h. In this region ($E_1/E_2 > 20$, $r/h > 0.5$), doubling the thickness of the upper layer is equivalent to increasing its modulus eightfold. Accordingly, in such cases it is necessary to specify the layer thickness precisely in order to obtain a precise value for E_1. However, the value found for E_2 will be affected only slightly by variations of the thickness specified.

In certain instances the measured deflections may give rise to a set of plotted points which fails to conform with any of the curves of the chart. It is likely that these instances represent data from structures having more than two layers. While charts similar to the one presented here can be constructed for multi-layered structures this has yet to be done extensively. Since there are two additional variables in a three-layer structure, a substantial number of charts, on the order of one hundred would be required to cover a range of all the variables comparable with the range included on this two-layer chart. As an example, one set of three-layer curves, is shown in Figure 8. This particular set is of the type which applies when the central layer has the smallest modulus.

While the present chart has been constructed for the specific case of Poisson's ratio equal to 0.5 in both layers, this is believed to represent only a minor limitation. Similar charts could be constructed for other values of Poisson's ratio, but the moduli determined from them, using a given set of deflection data, would not differ greatly. For example, a chart based on Poisson's ratio equal to 0.25 instead of 0.5 would generally provide values for the moduli approximately 20% smaller.

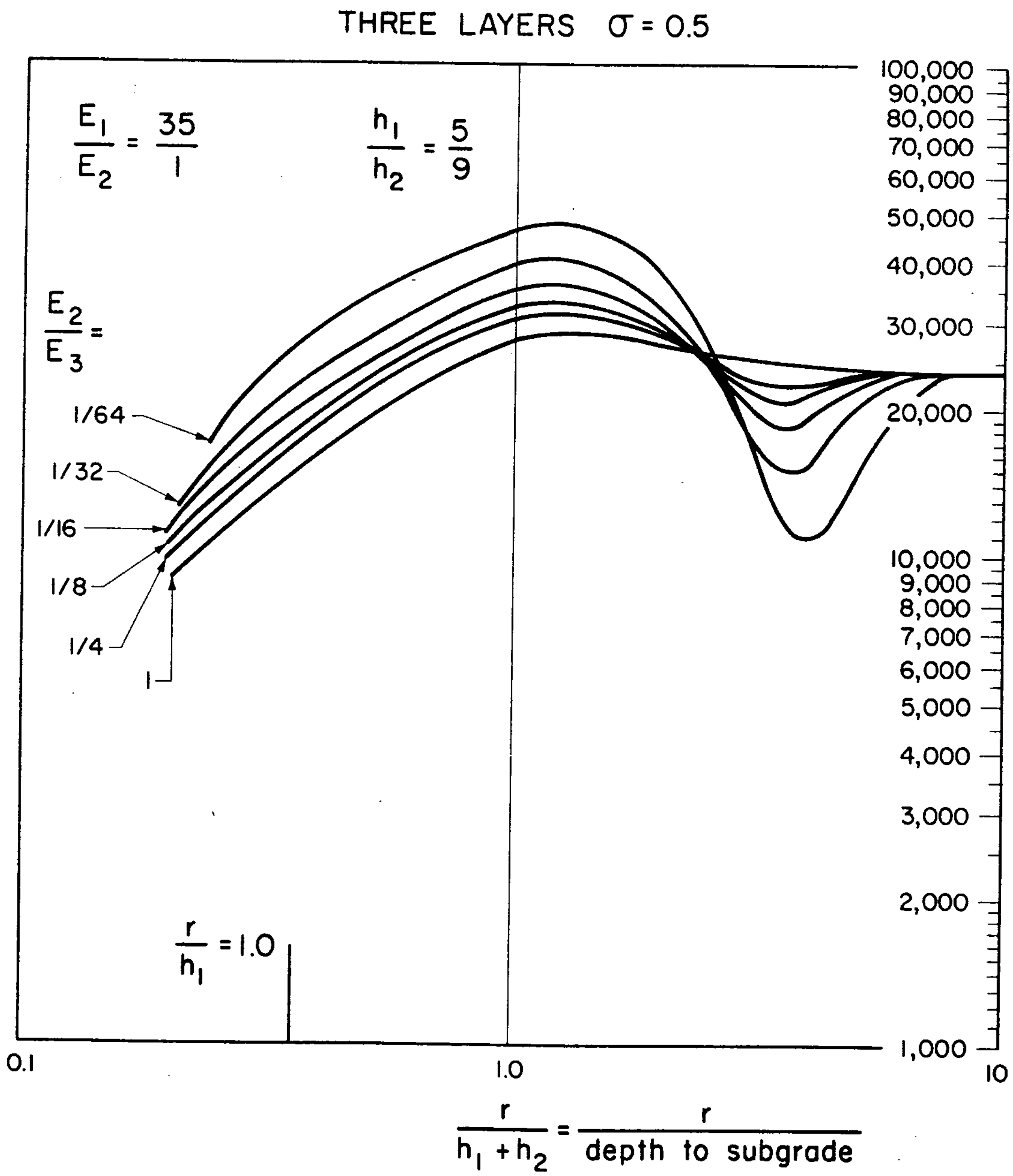

Figure 8: An example of a Three-Layer Elastic Deflection Chart.

Note: This particular chart applies to the specific conditions indicated, i.e. E_1/E_2 = 35, h_1/h_2 = 5/9, and is limited to cases in which E_2, the modulus of the middle layer, is less than either E_1 or E_3.

Reference (4) contains a recommendation that moduli derived from Dynaflect data be halved when used to characterize materials in a pavement design system. This recommendation is based on extensive field correlation studies between deflections produced by the 1000-pound load of the Dynaflect and those produced by heavily loaded vehicles (Benkelman Beam deflections). These studies show that heavy wheel loads, on the order of 10,000 pounds, tend to produce approximately twice as much deflection per thousand pounds. Hence elastic moduli derived from such deflections would be approximately half as great as those derived from the Dynaflect data. It should be pointed out, however, that the correlation studies on which this recommendation is based were conducted entirely on flexible pavements. Inherent limitations of the Benkelman Beam technique make it difficult to obtain comparable data for rigid pavements.

7. CONCLUSIONS

1. A chart has been designed by means of which a set of observed surface deflection data can be compared rapidly against deflections computed in accordance with two-layer elastic theory.
2. The chart can be used to indicate whether a given set of deflections corresponds to an elastic system having one layer (homogeneous), or two layers, or more than two layers. In the homogeneous and two-layer cases it provides a means for determining the modulus values required by the theory of elasticity to account for the observed deflections.
3. In certain specific cases the chart can be used to determine the layer thickness in addition to the moduli, but more generally the layer thickness must be specified in order to obtain unique values for the two moduli.
4. Extension of the technique to more than two layers is feasible.

8. REFERENCES

1. Burmister, D. M. "The Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways", Proceedings, Highway Research Board, Vol. 23, page 130, Equation (n), 1943.

2. Michelow, J. "Analysis of Stresses and Displacements in an N-Layered Elastic System Under a Load Uniformly Distributed over a Circular Area" California Research Corporation, Richmond, California, September, 1963.

3. Computer program "BISTRO", Koninklijke/Shell-Laboratorium, Amsterdam.

4. Scrivner, F. H.; Michalak, C. H.; and Moore, W. M. "Calculation of the Elastic Moduli of a Two Layer Pavement System from Measured Surface Deflections", Research Report No. 123-6, Texas Transportation Institute, Texas A&M University, College Station, Texas, March 1971.

5. Scrivner, F. H.; Swift, Gilbert; and Moore, W. M. "A New Research Tool for Measuring Pavement Deflection", Highway Research Record No. 129, Washington, D. C., pp. 1-11, 1966.

APPENDIX A

This appendix contains:

Table A-1; Values of the quantity $\frac{wrE_2}{P}$ used in plotting the chart given in Figure 1.

Table A-2; Measured deflections, from which the plots shown in Figures 3 through 7 were derived.

Figure A-1; Bi-logarithmic grid for plotting deflection data.

APPENDIX A

Values of the quantity $\frac{wrE_2}{P}$ used in plotting the chart are listed in the following table:

Table A-1

$\frac{E_1}{E_2}$	Values of $\frac{r}{h}$: 0.1	0.2	0.25	0.5	1.0	1.25	1.50	2.0	2.5	4.0	5.0	8.0	10.0
0.2	1.032	.876	.801	.476	.133	.088	.086	.126	.163	.210	.223	.236	.238
0.5	.439	.402	.384	.306	.218	.203	.200	.207	.217	.231	.235	.238	.238
1.0	.239	.239	.239	.239	.239	.239	.239	.239	.239	.239	.239	.239	.239
2 .0	.135	.151	.159	.194	.239	.250	.256	.258	.255	.245	.242	.239	.239
5.0	.069	.090	.100	.148	.218	.240	.255	.269	.272	.257	.248	.240	.239
10	.044	.064	.074	.120	.194	.220	.240	.265	.275	.269	.258	.241	.239
20	.029	.047	.055	.096	.166	.193	.216	.248	.268	.279	.271	.246	.240
50	.018	.032	.039	.072	.130	.156	.178	.215	.242	.280	.283	.261	.249
100	.013	.025	.030	.057	.107	.129	.150	.186	.215	.267	.281	.276	.262
200	.010	.019	.023	.045	.086	.106	.124	.156	.185	.245	.267	.285	.277
500	.007	.014	.017	.033	.065	.080	.094	.122	.146	.207	.235	.280	.286
1000	.0055	.0107	.013	.026	.052	.064	.076	.099	.121	.176	.206	.262	.279

The values given in table A-1 were obtained by means of a computer program based on equation (5) of reference (4).

Deflections, w, in mils (inches X10^{-3}), from which the plots shown in Figures 3 thru 7 were derived, are listed in the following table.

Table A-2

r inches	Fig. 3	Fig. 4a	Fig. 4b	Fig. 4c	Fig. 5a	Fig. 5b	Fig. 6a	Fig. 6b	Fig. 6c	Fig. 7a	Fig. 7b	Fig. 7c
10	0.40	3.0	1.7	0.39	0.19	0.16	1.11	0.74	0.39	0.70	0.47	0.12
15.6	0.37	0.98	0.98	0.38	0.18	0.15	0.96	0.67	0.36	0.55	0.27	0.099
26	0.32	0.18	0.45	0.33	0.17	0.14	0.78	0.54	0.30	0.40	0.19	0.069
37.4	0.27	0.10	0.24	0.27	0.15	0.13	0.60	0.42	0.26	0.28	0.11	0.040
49	0.24	0.10	0.15	0.22	0.13	0.12	0.45	0.33	0.18	0.16	0.069	0.020
84					0.09	0.09						
120					0.06	0.07						

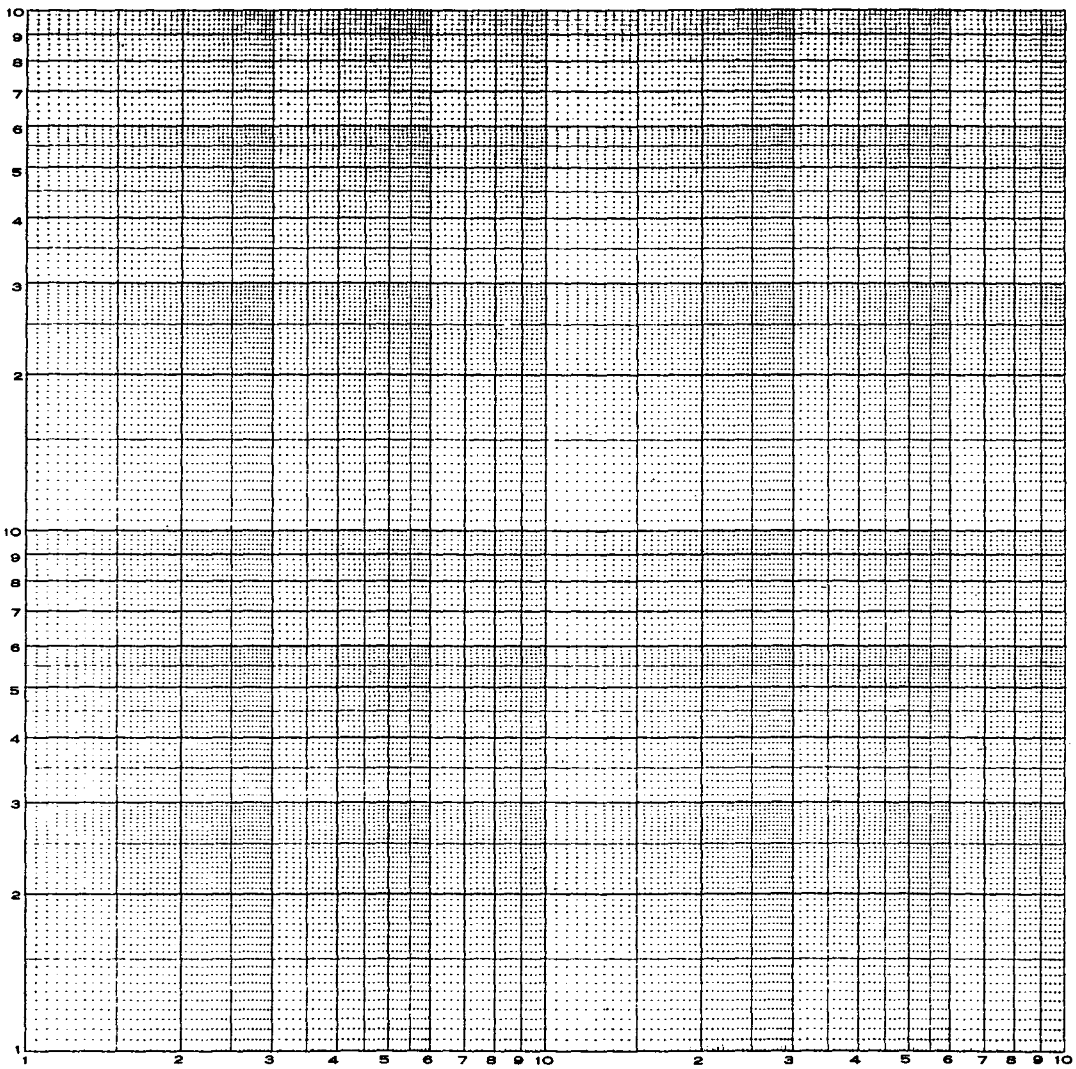

Figure A-1: Bilogarithmic grid, dimensionally equal to the chart of Figure 1.

Note: This grid may be used to plot deflection data, in the form wr versus r, on tracing paper, for overlay with the chart of Figure 1. The lines r = h and wr = 0.01 in^2/1000 lbs. may be ruled on the tracing paper to facilitate alignment with the chart.