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16. Abstract

The Texas Department of Transportation (TxDOT) is currently designing highway bridge structures using the American Association of State Highway and Transportation Officials (AASHTO) *Standard Specifications for Highway Bridges*, and it is expected that the agency will transition to the use of the *AASHTO LRFD Bridge Design Specifications* before 2007. This is a two-volume report that documents the findings of a TxDOT-sponsored research project to evaluate the impact of the Load and Resistance Factor (LRFD) Specifications on the design of typical Texas bridges as compared to the Standard Specifications. The objectives of this portion of the project are to evaluate the current LRFD Specifications to assess the calibration of the code with respect to typical Texas prestressed bridge girders, to perform a critical review of the major changes when transitioning to LRFD design, and to recommend guidelines to assist TxDOT in implementing the LRFD Specifications.

A parametric study for AASHTO Type IV, Type C, and Texas U54 girders was conducted using span length, girder spacing, and strand diameter as the major parameters that are varied. Based on the results obtained from the parametric study, two critical areas were identified where significant changes in design results were observed when comparing Standard and LRFD designs. The critical areas are the transverse shear requirements and interface shear requirements, and these are further investigated. In addition, limitations in the LRFD Specifications, such as those for the percentage of debonded strands and use of the LRFD live load distribution factor formulas, were identified as restrictions that would impact TxDOT bridge girder designs, and these issues are further assessed. The results of the parametric study, along with critical design issues that were identified and related recommendations, are summarized in Volume 1 of this report. Detailed design examples for an AASHTO Type IV girder and a Texas U54 girder using both the AASHTO Standard Specifications and AASHTO LRFD Specifications were also developed and compared. Volume 2 of this report contains these examples.

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IMPACT OF LRFD SPECIFICATIONS ON DESIGN OF TEXAS BRIDGES VOLUME 2: PRESTRESSED CONCRETE BRIDGE GIRDER DESIGN EXAMPLES

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The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official view or policies of the Federal Highway Administration (FHWA) or the Texas Department of Transportation (TxDOT). While every effort has been made to ensure the accuracy of the information provided in this report, this material is not intended to be a substitute for the actual codes and specifications for the design of prestressed bridge girders. This report does not constitute a standard, specification, or regulation; and is not intended for constructing, bidding, or permit purposes. The engineer in charge was Mary Beth D. Hueste, P.E. (TX 89660).

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TABLE OF CONTENTS

Page

Appendix A.1

Design Example for Interior AASHTO Type IV Girder using AASHTO Standard Specifications

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

TABLE Page

A.1 Design Example for Interior AASHTO Type IV Girder using AASHTO Standard Specifications

A.1.1 INTRODUCTION

The following detailed example shows sample calculations for the design of a typical interior AASHTO Type IV prestressed concrete girder supporting a single span bridge. The design is based on the *AASHTO Standard Specifications for Highway Bridges, 17th Edition* (AASHTO 2002). The guidelines provided by the TxDOT Bridge Design Manual (TxDOT 2001) are considered in the design. The number of strands and concrete strength at release and at service are optimized using the TxDOT methodology.

A.1.2 DESIGN PARAMETERS

The bridge considered for this design example has a span length of 110 ft. (center-to-center (c/c) pier distance), a total width of 46 ft., and total roadway width of 44 ft. The bridge superstructure consists of six AASHTO Type IV girders spaced 8 ft. center-to-center, designed to act compositely with an 8 in. thick cast-in-place (CIP) concrete deck. The wearing surface thickness is 1.5 in., which includes the thickness of any future wearing surface. T501 type rails are considered in the design. The design live load is taken as either HS 20-44 truck or HS 20-44 lane load, whichever produces larger effects. A relative humidity (RH) of 60 percent is considered in the design. The bridge cross section is shown in Figure A.1.2.1.

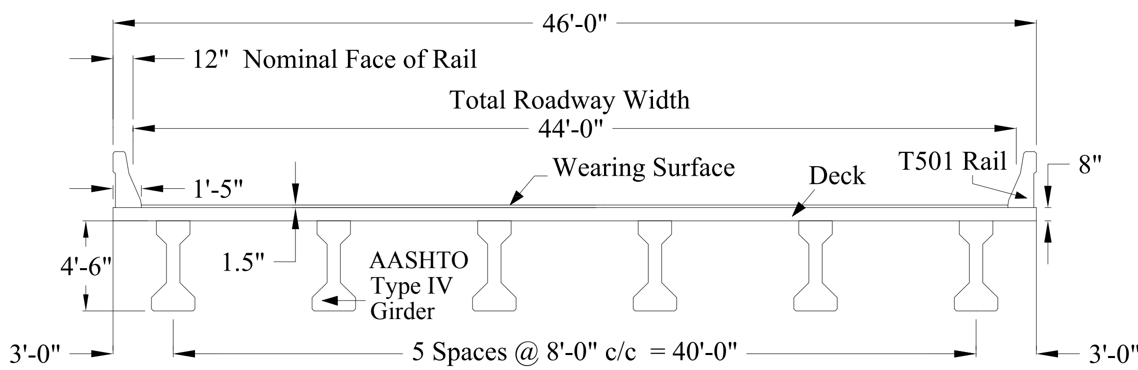

Figure A.1.2.1. Bridge Cross-Section Details.

The following calculations for design span length and the overall girder length are based on Figure A.1.2.2.

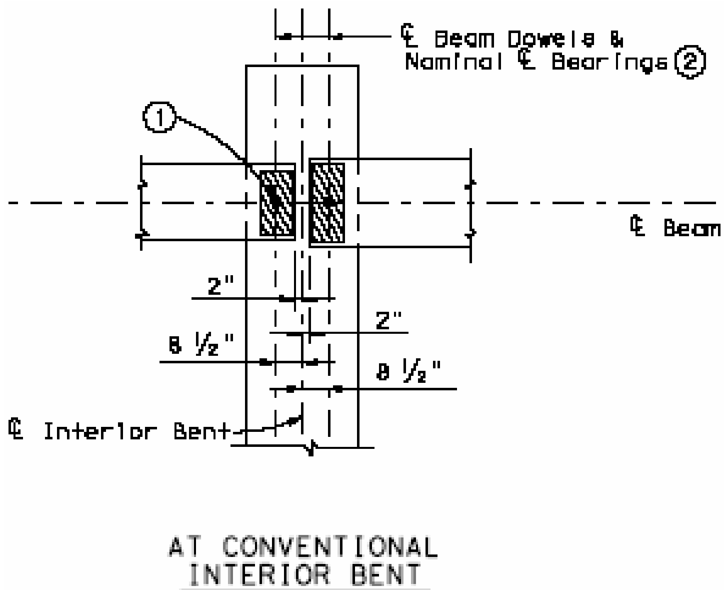

Figure A.1.2.2. Girder End Details (TxDOT Standard Drawing 2001).

Span Length (c/c piers) = 110 ft.-0 in.

From Figure A.1.2.2

Overall girder length = 110'-0" – 2(2") = 109'-8" = 109.67 ft.

Design Span = 110'-0" – 2(8.5") = 108'-7" = 108.583 ft. (c/c of bearing)

A.1.3 MATERIAL PROPERTIES

CIP slab:

Thickness, t_s = 8.0 in.

Concrete strength at 28 days, f'_c = 4000 psi

Thickness of asphalt-wearing surface (including any future wearing surface), t_w = 1.5 in.

Unit weight of concrete, w_c = 150 pcf

Precast girders: AASHTO Type IV

Concrete strength at release, f'_{ci} = 4000 psi (This value is taken as an initial estimate and will be finalized based on optimum design.)

Concrete strength at 28 days, f'_c = 5000 psi (This value is taken as initial estimate and will be finalized based on optimum design.)

Concrete unit weight, w_c = 150 pounds per cubic foot (pcf)

Pretensioning Strands: 0.5 in. diameter, seven wire low-relaxation
Area of one strand = 0.153 in.^{2}

Ultimate stress, f'_s = 270,000 psi

Yield strength, f_y^* = 0.9 f'_s = 243,000 psi [STD Art. 9.1.2]

Initial pretensioning, f_{si} = 0.75 f'_s [STD Art. 9.15.1]
= 202,500 psi

Modulus of Elasticity, E_s = 28,000 ksi [STD Art. 9.16.2.1.2]

Nonprestressed reinforcement: Yield strength, f_y = 60,000 psi

Unit weight of asphalt-wearing surface = 140 pcf [TxDOT recommendation]

T501 type barrier weight = 326 pounds per linear foot (plf) /side

A.1.4 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR GIRDER

A.1.4.1 Non-Composite Section

The section properties of an AASHTO Type IV girder as described in the TxDOT Bridge Design Manual (TxDOT 2001) are provided in Table A.1.4.1. Figure A.1.4.1 shows the section geometry and strand pattern.

Table A.1.4.1. Section Properties of AASHTO Type IV Girder (Adapted from TxDOT Bridge Design Manual [TxDOT 2001]).

y_t	y_b	Area	I	Wt./lf
(in.)	(in.)	(in.^{2})	(in.^{4})	(lbs)
29.25	24.75	788.4	260,403	821

where:

I = Moment of inertia about the centroid of the non-composite precast girder, in.^{4}

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder, in.

y_t = Distance from centroid to the extreme top fiber of the non-composite precast girder, in.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder, in.^{3}
$= I/\,y_b = 260{,}403/24.75 = 10{,}521.33$ in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder, in.^{3}
$= I/\,y_t = 260{,}403/29.25 = 8902.67$ in.^{3}

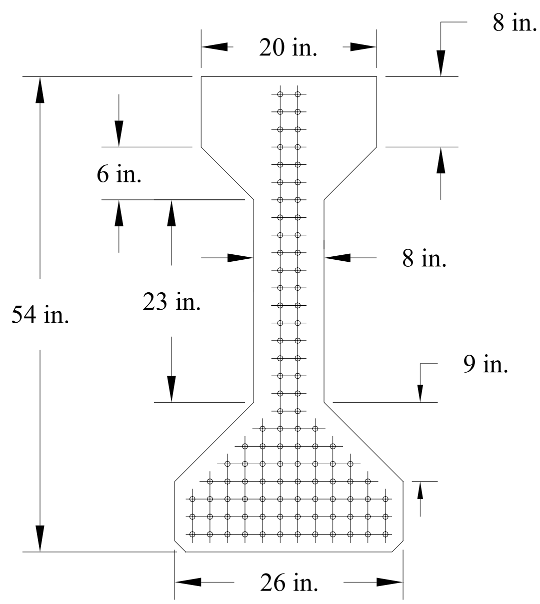

Figure A.1.4.1. Section Geometry and Strand Pattern for AASHTO Type IV Girder (Adapted from TxDOT Bridge Design Manual [TxDOT 2001]).

A.1.4.2 Composite Section

A.1.4.2.1 Effective Web Width

[STD Art. 9.8.3]

Effective web width of the precast girder is lesser of:

[STD Art. 9.8.3.1]

$b_e = 6 \times$ (flange thickness on either side of the web) + web + fillets
$= 6(8 + 8) + 8 + 2(6) = 116$ in.

or

$b_e =$ Total top flange width = 20 in. (controls)

Effective web width, $b_e = 20$ in.

A.1.4.2.2 Effective Flange Width

The effective flange width is lesser of: [STD Art. 9.8.3.2]

0.25×span length of girder: $\frac{108.583(12 \text{ in./ft.})}{4} = 325.75$ in.

6×(effective slab thickness on each side of the effective web width) + effective web width: 6(8 + 8) + 20 = 116 in.

One-half the clear distance on each side of the effective web width + effective web width: For interior girders, this is equivalent to the center-to-center distance between the adjacent girders.
8(12 in./ft.) + 20 in. = 96 in. (controls)

Effective flange width = 96 in.

A.1.4.2.3 Modular Ratio between Slab and Girder Concrete

Following the TxDOT Bridge Design Manual (TxDOT 2001) recommendation (pg. 7-85), the modular ratio between the slab and the girder concrete is taken as 1. This assumption is used for service load design calculations. For flexural strength limit design, shear design, and deflection calculations, the actual modular ratio based on optimized concrete strengths is used. The composite section is shown in Figure A.1.4.2, and the composite section properties are presented in Table A.1.4.2.

$$n = \left(\frac{E_c \text{ for slab}}{E_c \text{ for girder}}\right) = 1$$

where n is the modular ratio between slab and girder concrete, and E_c is the elastic modulus of concrete.

A.1.4.2.4 Transformed Section Properties

Transformed flange width = n × (effective flange width)
= (1)(96) = 96 in.

Transformed Flange Area = n × (effective flange width)(t_s)
= (1)(96) (8) = 768 in.^{2}

Table A.1.4.2. Properties of Composite Section.

	Transformed Area A (in.^{2})	y_b (in.)	Ay_b (in.^{3})	$A(y_{bc} - y_b)^2$	I (in.^{4})	$I + A(y_{bc} - y_b)^2$ (in.^{4})
Girder	788.4	24.75	19,512.9	212,231.53	260,403.0	472,634.5
Slab	768.0	58.00	44,544.0	217,868.93	4096.0	221,964.9
∑	1556.4		64,056.9			694,599.5

A_c = Total area of composite section = 1556.4 in.^{2}

h_c = Total height of composite section = 54 in. + 8 in. = 62 in.

I_c = Moment of inertia about the centroid of the composite section = 694,599.5 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder, in.
= 64,056.9/1556.4 = 41.157 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder, in.
= 54 – 41.157 = 12.843 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab, in.
= 62 – 41.157 = 20.843 in.

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder, in.^{3}
= I_c/y_{bc} = 694,599.5/41.157 = 16,876.83 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder, in.^{3}
= I_c/y_{tg} = 694,599.5/12.843 = 54,083.9 in.^{3}

S_{tc} = Section modulus of composite section referenced to the top fiber of the slab, in.^{3}
= I_c/y_{tc} = 694,599.5/20.843 = 33,325.31 in.^{3}

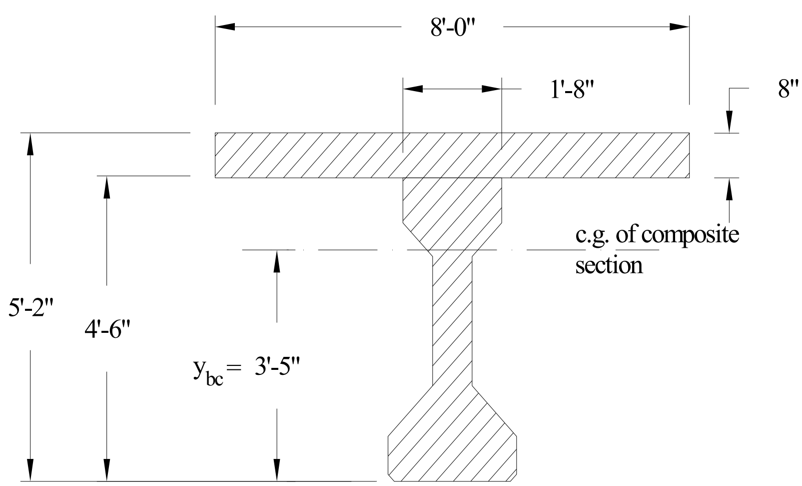

Figure A.1.4.2. Composite Section.

A.1.5 SHEAR FORCES AND BENDING MOMENTS

The self-weight of the girder and the weight of the slab act on the non-composite simple span structure, while the weight of the barriers, future wearing surface, and live load including impact load act on the composite simple span structure.

A.1.5.1 Shear Forces and Bending Moments due to Dead Loads

A.1.5.1.1 Dead Loads

Dead loads acting on the non-composite structure:

Self-weight of the girder = 0.821 kips/ft.
[TxDOT Bridge Design Manual (TxDOT 2001)]

Weight of cast-in-place deck on each interior girder

$$= (0.150\text{ kcf})\left(\frac{8\text{ in.}}{12\text{ in./ft.}}\right)(8\text{ ft.}) = 0.800\text{ kips/ft.}$$

Total dead load on non-composite section
= 0.821 + 0.800 = 1.621 kips/ft.

A.1.5.1.2 Superimposed Dead Loads

The dead loads placed on the composite structure are distributed equally among all the girders.
[STD Art. 3.23.2.3.1.1 & TxDOT Bridge Design Manual pg. 6-13]

Weight of T501 rails or barriers on each girder

$$= 2\left(\frac{326\text{ plf }/1000}{6\text{ girders}}\right) = 0.109\text{ kips/ft./girder}$$

Weight of 1.5 in. wearing surface

$$= (0.140\text{ kcf})\left(\frac{1.5\text{ in.}}{12\text{ in./ft.}}\right) = 0.0175\text{ ksf.}$$ This load is applied over the entire clear roadway width of 44 ft.-0 in.

Weight of wearing surface on each girder $= \dfrac{(0.0175\text{ ksf})(44.0\text{ ft.})}{6\text{ girders}}$

= 0.128 kips/ft./girder

Total superimposed dead load = 0.109 + 0.128 = 0.237 kips/ft.

A.1.5.1.3 Shear Forces and Bending Moments

Shear forces and bending moments for the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (hold-down point or harp point and critical section for shear) are provided in this section. The bending moment (M) and shear force (V) due to uniform dead loads and uniform superimposed dead loads at any section at a distance x from the centerline of bearing are calculated using the following formulas, where the uniform dead load is denoted as w.

$$M = 0.5wx(L - x)$$

$$V = w(0.5L - x)$$

The critical section for shear is located at a distance $h_c/2$ from the face of the support. However, as the support dimensions are not specified in this project, the critical section is measured from the centerline of bearing. This yields a conservative estimate of the design shear force.

Distance of critical section for shear from centerline of bearing
= 62/2 = 31 in. = 2.583 ft.

As per the recommendations of the TxDOT Bridge Design Manual (Chap. 7, Sec. 21), the distance of the hold-down (HD) point from the centerline of bearing is taken as the lesser of:

[0.5 × (span length) – (span length/20)] or [0.5 × (span length) – 5 ft.]

$$\frac{108.583}{2} - \frac{108.583}{20} = 48.862 \text{ ft. or } \frac{108.583}{2} - 5 = 49.29 \text{ ft.}$$

HD = 48.862 ft.

The shear forces and bending moments due to dead loads and superimposed dead loads are shown in Table A.1.5.1.

Table A.1.5.1. Shear Forces and Bending Moments due to Dead and Superimposed Dead Loads.

Distance from Bearing Centerline x	Section x/L	Dead Load				Superimposed Dead Loads		Total Dead Load	
		Girder Weight		Slab Weight					
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		kips	k-ft.	kips	k-ft.	kips	k-ft.	kips	k-ft.
0.000	0.000	44.57	0.00	43.43	0.00	12.87	0.00	100.87	0.00
2.583	0.024 ($h_c/2$)	42.45	112.39	41.37	109.52	12.25	32.45	96.07	254.36
10.858	0.100	35.66	435.59	34.75	424.45	10.29	125.74	80.70	985.78
21.717	0.200	26.74	774.38	26.06	754.58	7.72	223.54	60.52	1752.51
32.575	0.300	17.83	1016.38	17.37	990.38	5.15	293.40	40.35	2300.16
43.433	0.400	8.91	1161.58	8.69	1131.87	2.57	335.32	20.17	2628.76
48.862	0.450 (HD)	4.46	1197.87	4.34	1167.24	1.29	345.79	10.09	2710.90
54.292	0.500	0.00	1209.98	0.00	1179.03	0.00	349.29	0.00	2738.29

A.1.5.2 Shear Forces and Bending Moments due to Live Load

A.1.5.2.1 Live Load

The AASHTO Standard Specifications require the live load to be taken as either HS 20-44 standard truck loading, lane loading, or tandem loading, whichever yields the largest moments and shears. For spans longer than 40 ft., tandem loading does not govern; thus, only HS 20-44 truck loading and lane loading are investigated here.

[STD Art. 3.7.1.1]

The unfactored bending moments (M) and shear forces (V) due to HS 20-44 truck loading on a per-lane-basis are calculated using the following formulas given in the *PCI Design Manual* (PCI 2003).

Maximum bending moment due to HS 20-44 truck load

For $x/L = 0 - 0.333$

$$M = \frac{72(x)[(L-x)-9.33]}{L}$$

For $x/L = 0.333 - 0.5$

$$M = \frac{72(x)[(L-x)-4.67]}{L} - 112$$

Maximum shear force due to HS 20-44 truck load

For $x/L = 0 - 0.5$

$$V = \frac{72[(L-x)-9.33]}{L}$$

The bending moments and shear forces due to HS 20-44 lane load are calculated using the following formulas given in the *PCI Design Manual* (PCI 2003).

Maximum bending moment due to HS 20-44 lane load

$$M = \frac{P(x)(L-x)}{L} + 0.5(w)(x)(L-x)$$

Maximum shear force due to HS 20-44 lane load

$$V = \frac{Q(L-x)}{L} + (w)(\frac{L}{2} - x)$$

where:

x = Distance from the centerline of bearing to the section at which bending moment or shear force is calculated, ft.

L = Design span length = 108.583 ft.

P = Concentrated load for moment = 18 kips

Q = Concentrated load for shear = 26 kips

w = Uniform load per linear foot of lane = 0.64 klf

Shear force and bending moment due to live load including impact loading is distributed to individual girders by multiplying the distribution factor and the impact factor as follows.

Bending moment due to live load including impact load
M_{LL+I} = (live load bending moment per lane) (DF) (1+I)

Shear force due to live load including impact load
V_{LL+I} = (live load shear force per lane) (DF) (1+I)

where DF is the live load distribution factor, and I is the live load impact factor.

A.1.5.2.2 Live Load Distribution Factor for a Typical Interior Girder

The live load distribution factor for moment, for a precast prestressed concrete interior girder, is given by the following expression:

$$DF_{mom} = \frac{S}{5.5} = \frac{8.0}{5.5} = 1.4545 \text{ wheels/girder}$$ [STD Table 3.23.1]

where:

S = Average spacing between girders in feet = 8 ft.

The live load distribution factor for an individual girder is obtained as $DF = DF_{mom}/2$ = 0.727 lanes/girder.

For simplicity of calculation and because there is no significant difference, the distribution factor for moment is used also for shear as recommended by the TxDOT Bridge Design Manual (Chap. 6, Sec. 3, TxDOT 2001).

A.1.5.2.3 Live Load Impact

[STD Art. 3.8]

The live load impact factor is given by the following expression:

$$I = \frac{50}{L+125}$$ [STD Eq. 3-1]

where:

I = Impact fraction to a maximum of 30 percent

L = Design span length in feet = 108.583 ft. [STD Art. 3.8.2.2]

$$I = \frac{50}{108.583+125} = 0.214$$

The impact factor for shear varies along the span according to the location of the truck, but the impact factor computed above is also used for shear for simplicity as recommended by the TxDOT Bridge Design Manual (TxDOT 2001).

The distributed shear forces and bending moments due to live load are provided in Table A.1.5.2.

Table A.1.5.2. Distributed Shear Forces and Bending Moments due to Live Load.

Distance from Bearing Centerline x	Section x/L	HS 20-44 Truck Loading (controls)				HS 20-44 Lane Loading			
		Live Load		Live Load + Impact		Live Load		Live Load + Impact	
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		kips	k-ft.	kips	k-ft.	kips	k-ft.	kips	k-ft.
0.000	0.000	65.81	0.00	58.11	0.00	60.75	0.00	53.64	0.00
2.583	0.024 ($h_c/2$)	64.10	165.57	56.60	146.19	58.47	133.00	51.63	117.44
10.858	0.100	58.61	636.44	51.75	561.95	51.20	515.46	45.20	455.13
21.717	0.200	51.41	1116.52	45.40	985.84	41.65	916.38	36.77	809.12
32.575	0.300	44.21	1440.25	39.04	1271.67	32.10	1202.75	28.34	1061.97
43.433	0.400	37.01	1629.82	32.68	1439.05	22.55	1374.57	19.91	1213.68
48.862	0.450 (HD)	33.41	1671.64	29.50	1475.97	17.77	1417.52	15.69	1251.60
54.292	0.500	29.81	1674.37	26.32	1478.39	13.00	1431.84	11.48	1264.25

A.1.5.3 Load Combination

[STD Art. 3.22]

This design example considers only the dead and vehicular live loads. The wind load and the earthquake load are not included in the design, which is typical for the design of bridges in Texas. The general expression for group loading combinations for service load design (SLD) and load factor design (LFD) considering dead and live loads is given as:

$$\text{Group }(N) = \gamma[\beta_D \times D + \beta_L \times (L + I)]$$

where:

N = Group number

γ = Load factor given by STD Table 3.22.1.A.

β = Coefficient given by STD Table 3.22.1.A.

D = Dead load

L = Live load

I = Live load impact

Various group combinations provided by STD Table. 3.22.1.A are investigated, and the following group combinations are found to be applicable in the present case.

For service load design
Group I: This group combination is used for design of members for 100 percent basic unit stress. [STD Table 3.22.1A]

$\gamma = 1.0$

$\beta_D = 1.0$

$\beta_L = 1.0$

Group (I) = 1.0 × (D) + 1.0 × (L+I)

For load factor design
Group I: This load combination is the general load combination for load factor design relating to the normal vehicular use of the bridge. [STD Table 3.22.1A]

$\gamma = 1.3$

$\beta_D = 1.0$ for flexural and tension members

$\beta_L = 1.67$

Group (I) = 1.3[1.0 × (D) + 1.67 × (L+I)]

A.1.6 ESTIMATION OF REQUIRED PRESTRESS

A.1.6.1 Service Load Stresses at Midspan

The required number of strands is usually governed by concrete tensile stress at the bottom fiber of the girder at midspan section. The service load combination, Group I, is used to evaluate the bottom fiber stresses at the midspan section. The calculation for compressive stress in the top fiber of the girder at midspan section under Group I service load combination is shown in the following section.

Tensile stress at bottom fiber of the girder at midspan due to applied loads

$$f_b = \frac{M_g + M_S}{S_b} + \frac{M_{SDL} + M_{LL+I}}{S_{bc}}$$

Compressive stress at top fiber of the girder at midspan due to applied loads

$$f_t = \frac{M_g + M_S}{S_t} + \frac{M_{SDL} + M_{LL+I}}{S_{tg}}$$

where:

f_b = Concrete stress at the bottom fiber of the girder at the midspan section, ksi

f_t = Concrete stress at the top fiber of the girder at the midspan section, ksi

M_g = Moment due to girder self-weight at the midspan section of the girder = 1209.98 k-ft.

M_S = Moment due to slab weight at the midspan section of the girder = 1179.03 k-ft.

M_{SDL} = Moment due to superimposed dead loads at the midspan section of the girder = 349.29 k-ft.

M_{LL+I} = Moment due to live load including impact load at the midspan section of the girder = 1478.39 k-ft.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder = 8902.67 in.^{3}

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder = 16,876.83 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder = 54,083.9 in.^{3}

Substituting the bending moments and section modulus values, the stresses at bottom fiber (f_b) and top fiber (f_t) of the girder at the midspan section are:

$$f_b = \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{10{,}521.33} + \frac{(349.29 + 1478.39)(12 \text{ in./ft.})}{16{,}876.83}$$

$= 4.024$ ksi

$$f_t = \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{8902.67} + \frac{(349.29 + 1478.39)(12 \text{ in./ft.})}{54{,}083.9}$$

$= 3.626$ ksi

The stresses at the top and bottom fibers of the girder at the hold-down point, midspan, and top fiber of the slab are calculated in a similar fashion as shown above and summarized in Table A.1.6.1.

Table A.1.6.1. Summary of Stresses due to Applied Loads.

Load	Stresses in Girder				Stresses in Slab at Midspan
	Stress at Hold-Down (*HD*)		Stress at Midspan		
	Top Fiber (psi)	Bottom Fiber (psi)	Top Fiber (psi)	Bottom Fiber (psi)	Top Fiber (psi)
Girder Self-Weight	1614.63	-1366.22	1630.94	-1380.03	-
Slab Weight	1573.33	-1331.28	1589.22	-1344.73	-
Superimposed Dead Load	76.72	-245.87	77.50	248.35	125.77
Total Dead Load	3264.68	-2943.37	3297.66	-2973.10	125.77
Live Load	327.49	-1049.47	328.02	-1051.19	532.35
Total Load	3592.17	-3992.84	3625.68	-4024.29	658.12

(Negative values indicate tensile stresses)

A.1.6.2 Allowable Stress Limit

At service load conditions, the allowable tensile stress for members with bonded prestressed reinforcement is:

$$F_b = 6\sqrt{f'_c} = 6\sqrt{5000}\left(\frac{1}{1000}\right) = 0.4242 \text{ ksi} \qquad \text{[STD Art. 9.15.2.2]}$$

A.1.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – allowable tensile stress at final = $f_b - F_b$

$f_{breqd} = 4.024 - 0.4242 = 3.60$ ksi

Assuming the eccentricity of the prestressing strands at midspan (e_c) as the distance from the centroid of the girder to the bottom fiber of the girder (PSTRS14 methodology, TxDOT 2001)

$e_c = y_b = 24.75$ in.

Bottom fiber stress due to prestress after losses:

$$f_b = \frac{P_{se}}{A} + \frac{P_{se}\, e_c}{S_b}$$

where:

P_{se} = Effective pretension force after all losses, kips

A = Area of girder cross section = 788.4 in.^{2}

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

Required pretension is calculated by substituting the corresponding values in the above equation as follows:

$$3.60 = \frac{P_{se}}{788.4} + \frac{P_{se}(24.75)}{10,521.33}$$

Solving for P_{se},
P_{se} = 994.27 kips

Assuming final losses = 20 percent of initial prestress, f_{si} (TxDOT 2001)

Assumed final losses = 0.2(202.5) = 40.5 ksi

The prestress force per strand after losses
= (cross-sectional area of one strand) [f_{si} – losses]
= 0.153(202.5 – 40.5] = 24.78 kips

Number of prestressing strands required = 994.27/24.78 = 40.12

Try 42 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial estimate.

Strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 6(8)}{42} = 20.18 \text{ in.}$$

Available prestressing force
P_{se} = 42(24.78) = 1040.76 kips

Stress at bottom fiber of the girder at midspan due to prestressing, after losses

$$f_b = \frac{1040.76}{788.4} + \frac{1040.76(20.18)}{10,521.33}$$

$= 1.320 + 1.996 = 3.316 \text{ ksi} < f_{breqd} = 3.60 \text{ ksi}$

Try 44 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial estimate.

Strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 8(8)}{44} = 20.02 \text{ in.}$$

Available prestressing force
P_{se} = 44(24.78) = 1090.32 kips

Stress at bottom fiber of the girder at midspan due to prestressing, after losses

$$f_b = \frac{1090.32}{788.4} + \frac{1090.32(20.02)}{10{,}521.33}$$

$$= 1.383 + 2.074 = 3.457 \text{ ksi} < f_{breqd} = 3.60 \text{ ksi}$$

Try 46 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial estimate.

Effective strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8)}{46} = 19.88 \text{ in.}$$

Available prestressing force is:
$P_{se} = 46(24.78) = 1139.88$ kips

Stress at bottom fiber of the girder at midspan due to prestressing, after losses

$$f_b = \frac{1139.88}{788.4} + \frac{1139.88(19.88)}{10{,}521.33}$$

$$= 1.446 + 2.153 = 3.599 \text{ ksi} \sim f_{breqd} = 3.601 \text{ ksi}$$

Therefore, 46 strands are used as a preliminary estimate for the number of strands. Figure A.1.6.1 shows the strand arrangement.

Number of Strands	Distance from bottom (in.)
10	8
12	6
12	4
12	2

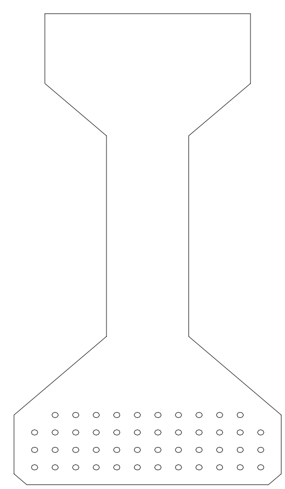

Figure A.1.6.1.Initial Strand Arrangement.

The distance from the centroid of the strands to the bottom fiber of the girder (y_{bs}) is calculated as:

$y_{bs} = y_b - e_c = 24.75 - 19.88 = 4.87$ in.

A.1.7 PRESTRESS LOSSES

[STD Art. 9.16.2]

Total prestress losses = $SH + ES + CR_C + CR_S$ [STD Eq. 9-3]

where:

SH = Loss of prestress due to concrete shrinkage, ksi

ES = Loss of prestress due to elastic shortening, ksi

CR_C = Loss of prestress due to creep of concrete, ksi

CR_S = Loss of prestress due to relaxation of pretensioning steel, ksi

Number of strands = 46

A number of iterations based on TxDOT methodology (TxDOT 2001) will be performed to arrive at the optimum number of strands, required concrete strength at release (f'_{ci}), and required concrete strength at service (f'_c).

A.1.7.1 Iteration 1

A.1.7.1.1 Concrete Shrinkage

[STD Art. 9.16.2.1.1]

For pretensioned members, the loss in prestress due to concrete shrinkage is given as:

$$SH = 17{,}000 - 150\ RH \quad \text{[STD Eq. 9-4]}$$

where:

RH is the relative humidity = 60 percent

$$SH = [17{,}000 - 150(60)]\frac{1}{1000} = 8.0 \text{ ksi}$$

A.1.7.1.2 Elastic Shortening

[STD Art. 9.16.2.1.2]

For pretensioned members, the loss in prestress due to elastic shortening is given as:

$$ES = \frac{E_s}{E_{ci}} f_{cir} \quad \text{[STD Eq. 9-6]}$$

where:

f_{cir} = Average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and the dead load of girder immediately after transfer, ksi

$$= \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, kips

As the initial losses are unknown at this point, 8 percent initial loss in prestress is assumed as a first estimate.

P_{si} = (number of strands)(area of each strand)[0.92(0.75 f'_s)]
= 46(0.153)(0.92)(0.75)(270) = 1311.18 kips

M_g = Moment due to girder self-weight at midspan, k-ft.
= 1209.98 k-ft.

e_c = Eccentricity of the prestressing strands at the midspan
= 19.88 in.

$$f_{cir} = \frac{1311.18}{788.4} + \frac{1311.18(19.88)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.88)}{260{,}403}$$

= 1.663 + 1.990 – 1.108 = 2.545 ksi

Initial estimate for concrete strength at release, f'_{ci} = 4000 psi

Modulus of elasticity of girder concrete at release is given as:

$E_{ci} = 33(w_c)^{3/2}\sqrt{f'_{ci}}$ [STD Eq. 9-8]

$$= [33(150)^{3/2}\sqrt{4000}\,]\left(\frac{1}{1000}\right) = 3834.25\text{ ksi}$$

Modulus of elasticity of prestressing steel, E_s = 28,000 ksi

Prestress loss due to elastic shortening is:

$$ES = \left[\frac{28{,}000}{3834.25}\right](2.545) = 18.59\text{ ksi}$$

A.1.7.1.3 Creep of Concrete

[STD Art. 9.16.2.1.3]

The loss in prestress due to the creep of concrete is specified to be calculated using the following formula:

$$CR_C = 12f_{cir} - 7f_{cds}$$ [STD Eq. 9-9]

where:

f_{cds} = Concrete stress at the center of gravity of the prestressing steel due to all dead loads except the dead load present at the time the prestressing force is applied, ksi

$$= \frac{M_S e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_{SDL} = Moment due to superimposed dead load at midspan section = 349.29 k-ft.

M_S = Moment due to slab weight at midspan section = 1179.03 k-ft.

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from center of gravity of the prestressing strands at midspan to the bottom fiber of the girder = 24.75 – 19.88 = 4.87 in.

I = Moment of inertia of the non-composite section = 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$f_{cds} = \frac{1179.03(12 \text{ in./ft.})(19.88)}{260{,}403} + \frac{349.29(12 \text{ in./ft.})(41.157 - 4.87)}{694{,}599.5}$$

$$= 1.080 + 0.219 = 1.299 \text{ ksi}$$

Prestress loss due to creep of concrete is:
CR_C = 12(2.545) – 7(1.299) = 21.45 ksi

A.1.7.1.4 Relaxation of Prestressing Steel

[STD Art. 9.16.2.1.4]

For pretensioned members with 270 ksi low-relaxation strands, the prestress loss due to relaxation of prestressing steel is calculated using the following formula.

$$CR_S = 5000 - 0.10ES - 0.05(SH + CR_C) \qquad \text{[STD Eq. 9-10A]}$$

where the variables are the same as defined in Section A.1.7 expressed in psi units.

$$CR_S = [5000 - 0.10(18{,}590) - 0.05(8000 + 21{,}450)]\left(\frac{1}{1000}\right)$$

$$= 1.669 \text{ ksi}$$

The *PCI Design Manual* (PCI 2003) considers only the elastic shortening loss in the calculation of total initial prestress loss, whereas, the TxDOT Bridge Design Manual (pg. 7-85, TxDOT 2001) recommends that 50 percent of the final steel relaxation loss shall also be considered for calculation of total initial prestress loss given as: [elastic shortening loss + 0.50(total steel relaxation loss)]

Using the TxDOT Bridge Design Manual (TxDOT 2001) recommendations, the initial prestress loss is calculated as follows.

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$$

$$= \frac{[18.59 + 0.5(1.669)]100}{0.75(270)} = 9.59\% > 8\% \text{ (assumed initial loss)}$$

Therefore, another trial is required assuming 9.59 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress loss due to concrete shrinkage. Therefore, the next trials will involve updating the losses due to elastic shortening, steel relaxation, and creep of concrete.

Based on the initial prestress loss value of 9.59 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_{si} = (number of strands)(area of each strand)[0.904(0.75 f_s')]
= 46(0.153)(0.904)(0.75)(270) = 1288.38 kips

Loss in prestress due to elastic shortening is:

$$ES = \frac{E_s}{E_{ci}} f_{cir}$$

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$f_{cir} = \frac{1288.38}{788.4} + \frac{1288.38(19.88)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.88)}{260{,}403}$$

$= 1.634 + 1.955 – 1.108 = 2.481$ ksi

E_s = 28,000 ksi

E_{ci} = 3834.25 ksi

$$ES = \left[\frac{28{,}000}{3834.25}\right](2.481) = 18.12 \text{ ksi}$$

Loss in prestress due to creep of concrete

$CR_C = 12f_{cir} – 7f_{cds}$

The value of f_{cds} is independent of the initial prestressing force value and will be the same as calculated in Section A.1.7.1.3.
f_{cds} = 1.299 ksi

CR_C = 12(2.481) – 7(1.299) = 20.68 ksi

Loss in prestress due to relaxation of steel

$$CR_S = 5000 - 0.10\,ES - 0.05(SH + CR_C)$$

$$= [5000 - 0.10(18{,}120) - 0.05(8000 + 20{,}680)]\left(\frac{1}{1000}\right)$$

$$= 1.754 \text{ ksi}$$

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$$

$$= \frac{[18.12 + 0.5(1.754)]100}{0.75(270)}$$ = 9.38% < 9.59% (assumed value for initial prestress loss)

Therefore, another trial is required assuming 9.38 percent initial prestress loss.

Based on the initial prestress loss value of 9.38 percent, the pretension force after allowing for the initial losses is calculated as follows.

$$P_{si} = \text{(number of strands)(area of each strand)}[\,0.906\,(0.75\,f_s'\,)]$$

$$= 46(0.153)(0.906)(0.75)(270) = 1291.23 \text{ kips}$$

Loss in prestress due to elastic shortening

$$ES = \frac{E_s}{E_{ci}}f_{cir}$$

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$f_{cir} = \frac{1291.23}{788.4} + \frac{1291.23(19.88)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.88)}{260{,}403}$$

$$= 1.638 + 1.960 - 1.108 = 2.490 \text{ ksi}$$

E_s = 28,000 ksi

E_{ci} = 3834.25 ksi

$$ES = \left[\frac{28{,}000}{3834.25}\right](2.490) = 18.18 \text{ ksi}$$

Loss in prestress due to creep of concrete

$CR_C = 12f_{cir} - 7f_{cds}$

f_{cds} = 1.299 ksi

CR_C = 12(2.490) – 7(1.299) = 20.79 ksi

Loss in prestress due to relaxation of steel

$CR_S = 5000 - 0.10\ ES - 0.05(SH + CR_C)$

$= [5000 - 0.10(18{,}180) - 0.05(8000 + 20{,}790)]\left(\frac{1}{1000}\right)$

= 1.743 ksi

Initial prestress loss = $\dfrac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$

$= \dfrac{[18.18 + 0.5(1.743)]100}{0.75(270)}$ = 9.41% ≈ 9.38% (assumed value of initial prestress loss)

A.1.7.1.5
Total Losses at Transfer

Total prestress loss at transfer = $(ES + \frac{1}{2}CR_s)$

= [18.18 + 0.5(1.743)] = 19.05 ksi

Effective initial prestress, f_{si} = 202.5 – 19.05 = 183.45 ksi

P_{si} = Effective pretension after allowing for the initial prestress loss

= (number of strands)(area of strand)(f_{si})

= 46(0.153)(183.45) = 1291.12 kips

A.1.7.1.6
Total Losses at Service

Loss in prestress due to concrete shrinkage, SH = 8.0 ksi

Loss in prestress due to elastic shortening, ES = 18.18 ksi

Loss in prestress due to creep of concrete, CR_C = 20.79 ksi

Loss in prestress due to steel relaxation, CR_S = 1.743 ksi

Total final loss in prestress = $SH + ES + CR_C + CR_S$

= 8.0 + 18.18 + 20.79 + 1.743 = 48.71 ksi

or, $\dfrac{48.71(100)}{0.75(270)}$ = 24.06%

Effective final prestress, f_{se} = 0.75(270) – 48.71 = 153.79 ksi

P_{se} = Effective pretension after allowing for the final prestress loss

= (number of strands)(area of strand)(effective final prestress)

= 46(0.153)(153.79) = 1082.37 kips

A.1.7.1.7 Final Stresses at Midspan

The number of strands is updated based on the final stress at the bottom fiber of the girder at the midspan section.

Final stress at the bottom fiber of the girder at the midspan section due to effective prestress, f_{bf}, is calculated as follows.

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\,e_c}{S_b} = \frac{1082.37}{788.4} + \frac{1082.37\,(19.88)}{10{,}521.33}$$

= 1.373 + 2.045 = 3.418 ksi < $f_{b\,reqd}$ = 3.600 ksi (No Good)

($f_{b\,reqd}$ calculations are presented in Section A.1.6.3)

Try 48 – 0.5 in. diameter, 270 ksi low-relaxation strands.

Eccentricity of prestressing strands at midspan

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 2(10)}{48} = 19.67 \text{ in.}$$

Effective pretension after allowing for the final prestress loss
P_{se} = 48(0.153)(153.79) = 1129.43 kips

Final stress at the bottom fiber of the girder at midspan section due to effective prestress

$$f_{bf} = \frac{1129.43}{788.4} + \frac{1129.43\,(19.67)}{10{,}521.33}$$

= 1.432 + 2.11 = 3.542 ksi < $f_{b\,reqd}$ = 3.600 ksi (No Good)

Try 50 – 0.5 in. diameter, 270 ksi low-relaxation strands.

Eccentricity of prestressing strands at midspan

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8) + 4(10)}{50} = 19.47 \text{ in.}$$

Effective pretension after allowing for the final prestress loss
P_{se} = 50(0.153)(153.79) = 1176.49 kips

Final stress at the bottom fiber of the girder at midspan section due to effective prestress

$$f_{bf} = \frac{1176.49}{788.4} + \frac{1176.49\,(19.47)}{10{,}521.33}$$

$$= 1.492 + 2.177 = 3.669 \text{ ksi} > f_{b\,reqd} = 3.600 \text{ ksi} \qquad \text{(O.K.)}$$

Therefore, use 50 – 0.5 in. diameter, 270 ksi low-relaxation strands.

Concrete stress at the top fiber of the girder due to effective prestress and applied loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + f_t = \frac{1176.49}{788.4} - \frac{1176.49\,(19.47)}{8902.67} + 3.626$$

$$= 1.492 - 2.573 + 3.626 = 2.545 \text{ ksi}$$

(f_t calculations are presented in Section A.1.6.1)

A.1.7.1.8 Initial Stresses at Hold-Down Point

The concrete strength at release, f'_{ci}, is updated based on the initial stress at the bottom fiber of the girder at the hold-down point.

Prestressing force after allowing for initial prestress loss

P_{si} = (number of strands)(area per strand)(effective initial prestress)

= 50(0.153)(183.45) = 1403.39 kips

Effective initial prestress calculations are presented in Section A.1.7.1.5.

Initial concrete stress at top fiber of the girder at the hold-down point due to self-weight of the girder and effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in.
= $0.5wx(L - x)$

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Distance of hold-down point from the end of the girder
= HD + (distance from centerline of bearing to the girder end)

HD = Hold-down point distance from centerline of the bearing = 48.862 ft. (see Sec. A.1.5.1.3)

x = 48.862 + 0.542 = 49.404 ft.

M_g = 0.5(0.821)(49.404)(109.67 – 49.404) = 1222.22 k-ft.

$$f_{ti} = \frac{1403.39}{788.4} - \frac{1403.39\ (19.47)}{8902.67} + \frac{1222.22(12\text{ in./ft.})}{8902.67}$$

= 1.78 – 3.069 + 1.647 = 0.358 ksi

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of the girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1403.39}{788.4} + \frac{1403.39\ (19.47)}{10{,}521.33} - \frac{1222.22(12\text{ in./ft.})}{10{,}521.33}$$

= 1.78 + 2.597 – 1.394 = 2.983 ksi

Compression stress limit for pretensioned members at transfer stage is 0.6 f'_{ci} [STD Art. 9.15.2.1]

Therefore, $f'_{ci\ reqd} = \dfrac{2983}{0.6}$ = 4971.67 psi

A.1.7.2 Iteration 2

A second iteration is carried out to determine the prestress losses and subsequently estimate the required concrete strength at release and at service using the following parameters determined in the previous iteration.

Number of strands = 50
Concrete strength at release, f'_{ci} = 4971.67 psi

A.1.7.2.1 Concrete Shrinkage

[STD Art. 9.16.2.1.1]

For pretensioned members, the loss in prestress due to concrete shrinkage is given as:

$$SH = 17{,}000 - 150\ RH \qquad \text{[STD Eq. 9-4]}$$

where RH is the relative humidity = 60 percent

$$SH = [17{,}000 - 150(60)]\frac{1}{1000} = 8.0\text{ ksi}$$

A.1.7.2.2 Elastic Shortening

[STD Art. 9.16.2.1.2]

For pretensioned members, the loss in prestress due to elastic shortening is given as:

$$ES = \frac{E_s}{E_{ci}} f_{cir} \qquad \text{[STD Eq. 9-6]}$$

where:

f_{cir} = Average concrete stress at the center of gravity of the pretensioning steel due to the pretensioning force and the dead load of girder immediately after transfer, ksi

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, kips

As the initial losses are dependent on the elastic shortening and steel relaxation loss, which are yet to be determined, the initial loss value of 9.41 percent obtained in the last trial of iteration 1 is taken as an initial estimate for initial loss in prestress.

P_{si} = (number of strands)(area of strand)[0.9059(0.75 f'_s)]
= 50(0.153)(0.9059)(0.75)(270) = 1403.35 kips

M_g = Moment due to girder self-weight at midspan, k-ft.
= 1209.98 k-ft.

e_c = Eccentricity of the prestressing strands at the midspan
= 19.47 in.

$$f_{cir} = \frac{1403.35}{788.4} + \frac{1403.35(19.47)^2}{260,403} - \frac{1209.98(12 \text{ in./ft.})(19.47)}{260,403}$$

= 1.78 + 2.043 – 1.086 = 2.737 ksi

Concrete strength at release, f'_{ci} = 4971.67 psi

Modulus of elasticity of girder concrete at release is given as:

$$E_{ci} = 33(w_c)^{3/2}\sqrt{f'_{ci}} \qquad \text{[STD Eq. 9-8]}$$

$$= [33(150)^{3/2}\sqrt{4971.67}\,]\left(\frac{1}{1000}\right) = 4274.66 \text{ ksi}$$

Modulus of elasticity of prestressing steel, E_s = 28,000 ksi

Prestress loss due to elastic shortening is:

$$ES = \left[\frac{28,000}{4274.66}\right](2.737) = 17.93 \text{ ksi}$$

A.1.7.2.3 Creep of Concrete

[STD Art. 9.16.2.1.3]

The loss in prestress due to creep of concrete is specified to be calculated using the following formula.

$$CR_C = 12f_{cir} - 7f_{cds} \qquad \text{[STD Eq. 9-9]}$$

where:

$$f_{cds} = \frac{M_S e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_{SDL} = Moment due to superimposed dead load at midspan section = 349.29 k-ft.

M_S = Moment due to slab weight at midspan section = 1179.03 k-ft.

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from center of gravity of the prestressing strands at midspan to the bottom fiber of the girder = 24.75 – 19.47 = 5.28 in.

I = Moment of inertia of the non-composite section = 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$f_{cds} = \frac{1179.03(12 \text{ in./ft.})(19.47)}{260,403} + \frac{(349.29)(12 \text{ in./ft.})(41.157 - 5.28)}{694,599.5}$$

$$= 1.058 + 0.216 = 1.274 \text{ ksi}$$

Prestress loss due to creep of concrete is

CR_C = 12(2.737) – 7(1.274) = 23.93 ksi

A.1.7.2.4 Relaxation of Pretensioning Steel

[STD Art. 9.16.2.1.4]

For pretensioned members with 270 ksi low-relaxation strands, prestress loss due to relaxation of prestressing steel is calculated using the following formula.

$CR_S = 5000 - 0.10\ ES - 0.05(SH + CR_C)$ [STD Eq. 9-10A]

$$CR_S = [5000 - 0.10(17{,}930) - 0.05(8000{+}23{,}930)]\left(\frac{1}{1000}\right)$$

$$= 1.61 \text{ ksi}$$

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$$

$$= \frac{[17.93 + 0.5(1.61)]100}{0.75(270)} = 9.25\% < 9.41\% \text{ (assumed initial loss)}$$

Therefore, another trial is required assuming 9.25 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress loss due to concrete shrinkage. Therefore, the next trial will involve updating the losses due to elastic shortening, steel relaxation, and creep of concrete.

Based on the initial prestress loss value of 9.25 percent, the pretension force after allowing for the initial losses is calculated as follows:

P_{si} = (number of strands)(area of each strand)[$0.9075(0.75\ f_s')$]

$= 50(0.153)(0.9075)(0.75)(270) = 1405.83$ kips

Loss in prestress due to elastic shortening

$$ES = \frac{E_s}{E_{ci}} f_{cir}$$

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1405.83}{788.4} + \frac{1405.83(19.47)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.47)}{260{,}403}$$

$$= 1.783 + 2.046 - 1.086 = 2.743 \text{ ksi}$$

E_s = 28,000 ksi

E_{ci} = 4274.66 ksi

Prestress loss due to elastic shortening is:

$$ES = \left[\frac{28{,}000}{4274.66}\right](2.743) = 17.97 \text{ ksi}$$

Loss in prestress due to creep of concrete

$CR_C = 12f_{cir} - 7f_{cds}$

The value of f_{cds} is independent of the initial prestressing force value and will be the same as calculated in Section A.1.7.2.3.

$f_{cds} = 1.274$ ksi

$CR_C = 12(2.743) - 7(1.274) = 24.0$ ksi

Loss in prestress due to relaxation of steel

$CR_S = 5000 - 0.10\ ES - 0.05(SH + CR_C)$

$$= [5000 - 0.10(17{,}970) - 0.05(8000 + 24{,}000)]\left(\frac{1}{1000}\right)$$

$= 1.603$ ksi

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$$

$$= \frac{[17.97 + 0.5(1.603)]100}{0.75(270)} = 9.27\% \approx 9.25\%$$ (assumed value for initial prestress loss)

A.1.7.2.5
Total Losses at Transfer

Total prestress loss at transfer = $(ES + \frac{1}{2}CR_s)$

$= [17.97 + 0.5(1.603)] = 18.77$ ksi

Effective initial prestress, $f_{si} = 202.5 - 18.77 = 183.73$ ksi

P_{si} = Effective pretension after allowing for the initial prestress loss

= (number of strands)(area of strand)(f_{si})

= 50(0.153)(183.73) = 1405.53 kips

A.1.7.2.6
Total Losses at Service

Loss in prestress due to concrete shrinkage, $SH = 8.0$ ksi

Loss in prestress due to elastic shortening, $ES = 17.97$ ksi

Loss in prestress due to creep of concrete, $CR_C = 24.0$ ksi

Loss in prestress due to steel relaxation, $CR_S = 1.603$ ksi

Total final loss in prestress = $SH + ES + CR_C + CR_S$

= 8.0 + 17.97 + 24.0 + 1.603 = 51.57 ksi

or $\dfrac{51.57(100)}{0.75(270)} = 25.47\%$

Effective final prestress, f_{se} = 0.75(270) – 51.57 = 150.93 ksi

P_{se} = Effective pretension after allowing for the final prestress loss

= (number of strands)(area of strand)(effective final prestress)

= 50(0.153)(150.93) = 1154.61 kips

A.1.7.2.7 Final Stresses at Midspan

Concrete stress at top fiber of the girder at the midspan section due to applied loads and effective prestress

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + f_t = \frac{1154.61}{788.4} - \frac{1154.61\ (19.47)}{8902.67} + 3.626$$

$$= 1.464 - 2.525 + 3.626 = 2.565 \text{ ksi}$$

(f_t calculations are presented in Section A.1.6.1.)

Compressive stress limit under service load combination is 0.6 f'_c

[STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{2565}{0.60} = 4275 \text{ psi}$$

Concrete stress at top fiber of the girder at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1154.61}{788.4} - \frac{1154.61\ (19.47)}{8902.67} + \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{8902.67} + \frac{349.29(12 \text{ in./ft.})}{54{,}083.9}$$

$$= 1.464 - 2.525 + 3.22 + 0.077 = 2.236 \text{ ksi}$$

Compressive stress limit for effective prestress + permanent dead loads = 0.4 f'_c [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{2236}{0.40} = 5590 \text{ psi} \qquad \text{(controls)}$$

Concrete stress at top fiber of the girder at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{1478.39(12\text{ in./ft.})}{54083.9} + 0.5\left\{\frac{1154.61}{788.4} - \frac{1154.61(19.47)}{8902.67} + \frac{(1209.98\ +\ 1179.03)(12\text{ in./ft.})}{8902.67} + \frac{349.29(12\text{ in./ft.})}{54{,}083.9}\right\}$$

$$= 0.328 + 0.5(1.464 - 2.525 + 3.22 + 0.077) = 1.446 \text{ ksi}$$

Allowable limit for compressive stress due to live load + 0.5(effective prestress + dead loads) = 0.4 f'_c [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{1446}{0.40} = 3615 \text{ psi}$$

Tensile stress at the bottom fiber of the girder at midspan due to service loads

$f_{bf} = \dfrac{P_{se}}{A} + \dfrac{P_{se}\, e_c}{S_b} - f_b$ (f_b calculations are presented in Sec. A.1.6.1.)

$$= \frac{1154.61}{788.4} + \frac{1154.61(19.47)}{10{,}521.33} - 4.024$$

= 1.464 + 2.14 – 4.024 = – 0.420 ksi (negative sign indicates tensile stress)

For members with bonded reinforcement, allowable tension in the precompressed tensile zone = $6\sqrt{f'_c}$ [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \left(\frac{420}{6}\right)^2 = 4900 \text{ psi}$$

The concrete strength at service is updated based on the final stresses at the midspan section under different loading combinations. The required concrete strength at service is determined to be 5590 psi.

A.1.7.2.8
Initial Stresses at Hold-Down Point

Prestressing force after allowing for initial prestress loss
P_{si} = (number of strands)(area of strand)(effective initial prestress)

= 50(0.153)(183.73) = 1405.53 kips

(Effective initial prestress calculations are presented in Section A.1.7.2.5.)

Initial concrete stress at top fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at the hold-down point based on overall girder length of 109 ft.-8 in.
= 1222.22 k-ft. (see Section A.1.7.1.8)

$$f_{ti} = \frac{1405.53}{788.4} - \frac{1405.53\,(19.47)}{8902.67} + \frac{1222.22(12\text{ in./ft.})}{8902.67}$$

= 1.783 – 3.074 + 1.647 = 0.356 ksi

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1405.53}{788.4} + \frac{1405.53\,(19.47)}{10{,}521.33} - \frac{1222.22(12\text{ in./ft.})}{10{,}521.33}$$

= 1.783 + 2.601 – 1.394 = 2.99 ksi

Compressive stress limit for pretensioned members at transfer stage is 0.6 f'_{ci}. [STD Art.9.15.2.1]

$$f'_{ci\ reqd} = \frac{2990}{0.6} = 4983.33 \text{ psi}$$

A.1.7.2.9
Initial Stresses at Girder End

The initial tensile stress at the top fiber and compressive stress at the bottom fiber of the girder at the girder end section are minimized by harping the web strands at the girder end. Following the TxDOT methodology (TxDOT 2001), the web strands are incrementally raised as a unit by 2 inches in each trial. The iterations are repeated until the top and bottom fiber stresses satisfy the allowable stress limits, or the centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder; in which case, the concrete strength at release is updated based on the governing stress.

The position of the harped web strands, eccentricity of strands at the girder end, top and bottom fiber stresses at the girder end, and the corresponding required concrete strengths are summarized in Table A.1.7.1. The required concrete strengths are based on allowable stress limits at transfer stage specified in STD Art. 9.15.2.1 presented as follows.

Allowable compressive stress limit = 0.6 f'_{ci}

For members with bonded reinforcement, allowable tension at transfer = $7.5\sqrt{f'_{ci}}$

Table A.1.7.1. Summary of Top and Bottom Stresses at Girder End for Different Harped Strand Positions and Corresponding Required Concrete Strengths.

Distance of the Centroid of Topmost Row of Harped Web Strands from		Eccentricity of Prestressing Strands at Girder End (in.)	Top Fiber Stress (psi)	Required Concrete Strength (psi)	Bottom Fiber Stress (psi)	Required Concrete Strength (psi)
Bottom Fiber (in.)	Top Fiber (in.)					
10 (no harping)	44	19.47	-1291.11	29,634.91	4383.73	7306.22
12	42	19.07	-1227.96	26,806.80	4330.30	7217.16
14	40	18.67	-1164.81	24,120.48	4276.86	7128.10
16	38	18.27	-1101.66	21,575.96	4223.43	7039.04
18	36	17.87	-1038.51	19,173.23	4169.99	6949.99
20	34	17.47	-975.35	16,912.30	4116.56	6860.93
22	32	17.07	-912.20	14,793.17	4063.12	6771.87
24	30	16.67	-849.05	12,815.84	4009.68	6682.81
26	28	16.27	-785.90	10,980.30	3956.25	6593.75
28	26	15.87	-722.75	9286.56	3902.81	6504.69
30	24	15.47	-659.60	7734.62	3849.38	6415.63
32	22	15.07	-596.45	6324.47	3795.94	6326.57
34	20	14.67	-533.30	5056.12	3742.51	6237.51
36	18	14.27	-470.15	3929.57	3689.07	6148.45
38	16	13.87	-407.00	2944.82	3635.64	6059.39
40	14	13.47	-343.85	2101.86	3582.20	5970.34
42	12	13.07	-280.69	1400.70	3528.77	5881.28
44	10	12.67	-217.54	841.34	3475.33	5792.22
46	8	12.27	-154.39	423.77	3421.89	5703.16
48	6	11.87	-91.24	148.00	3368.46	5614.10
50	4	11.47	-28.09	14.03	3315.02	5525.04
52	2	11.07	35.06	58.43	3261.59	5435.98

From Table A.1.7.1, it is evident that the web strands need to be harped to the topmost position possible to control the bottom fiber stress at the girder end.

Detailed calculations for the case when 10 web strands (5 rows) are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder) is presented as follows.

Eccentricity of prestressing strands at the girder end (see Figure A.1.7.2)

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 2(10) + 2(52+50+48+46+44)}{50}$$
$$= 11.07 \text{ in.}$$

Concrete stress at the top fiber of the girder at the girder end at transfer stage:

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\, e_e}{S_t}$$

$$= \frac{1405.53}{788.4} - \frac{1405.53\ (11.07)}{8902.67} = 1.783 - 1.748 = 0.035 \text{ ksi}$$

Concrete stress at the bottom fiber of the girder at the girder end at transfer stage:

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\, e_e}{S_b}$$

$$f_{bi} = \frac{1405.53}{788.4} + \frac{1405.53\ (11.07)}{10{,}521.33} = 1.783 + 1.479 = 3.262 \text{ ksi}$$

Compressive stress limit for pretensioned members at transfer stage is $0.6\, f'_{ci}$. [STD Art.9.15.2.1]

$$f'_{ci\ reqd} = \frac{3262}{0.60} = 5436.67 \text{ psi} \qquad \text{(controls)}$$

The required concrete strengths are updated based on the above results as follows.

Concrete strength at release, f'_{ci} = 5436.67 psi

Concrete strength at service, f'_c = 5590 psi

A.1.7.3 Iteration 3

A third iteration is carried out to refine the prestress losses based on the updated concrete strengths. Based on the new prestress losses, the concrete strength at release and service will be further refined.

A.1.7.3.1 Concrete Shrinkage

[STD Art. 9.16.2.1.1]

For pretensioned members, the loss in prestress due to concrete shrinkage is given as:

$$SH = 17{,}000 - 150\,RH \qquad \text{[STD Eq. 9-4]}$$

where:

RH is the relative humidity = 60 percent

$$SH = [17{,}000 - 150(60)]\frac{1}{1000} = 8.0 \text{ ksi}$$

A.1.7.3.2 Elastic Shortening

[STD Art. 9.16.2.1.2]

For pretensioned members, the loss in prestress due to elastic shortening is given as:

$$ES = \frac{E_s}{E_{ci}} f_{cir} \qquad \text{[STD Eq. 9-6]}$$

where:

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, kips

As the initial losses are dependent on the elastic shortening and steel relaxation loss, which are yet to be determined, the initial loss value of 9.27 percent obtained in the last trial (iteration 2) is taken as the first estimate for the initial loss in prestress.

P_{si} = (number of strands)(area of strand)[0.9073(0.75 f'_s)]
= 50(0.153)(0.9073)(0.75)(270) = 1405.52 kips

M_g = Moment due to girder self-weight at midspan, k-ft.
= 1209.98 k-ft.

e_c = Eccentricity of the prestressing strands at the midspan
= 19.47 in.

$$f_{cir} = \frac{1405.52}{788.4} + \frac{1405.52(19.47)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.47)}{260{,}403}$$

$$= 1.783 + 2.046 - 1.086 = 2.743 \text{ ksi}$$

Concrete strength at release, f'_{ci} = 5436.67 psi

Modulus of elasticity of girder concrete at release is given as:

$$E_{ci} = 33(w_c)^{3/2}\sqrt{f'_{ci}} \quad \text{[STD Eq. 9-8]}$$

$$= [33(150)^{3/2}\sqrt{5436.67}\,]\left(\frac{1}{1000}\right) = 4470.10 \text{ ksi}$$

Modulus of elasticity of prestressing steel, E_s = 28,000 ksi

Prestress loss due to elastic shortening is:

$$ES = \left[\frac{28{,}000}{4470.10}\right](2.743) = 17.18 \text{ ksi}$$

A.1.7.3.3
Creep of Concrete

[STD Art. 9.16.2.1.3]

The loss in prestress due to creep of concrete is specified to be calculated using the following formula:

$$CR_C = 12f_{cir} - 7f_{cds} \quad \text{[STD Eq. 9-9]}$$

where:

$$f_{cds} = \frac{M_S e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_{SDL} = Moment due to superimposed dead load at midspan section = 349.29 k-ft.

M_S = Moment due to slab weight at midspan section = 1179.03 k-ft.

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from center of gravity of the prestressing strands at midspan to the bottom fiber of the girder = 24.75 – 19.47 = 5.28 in.

I = Moment of inertia of the non-composite section = 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$f_{cds} = \frac{1179.03(12 \text{ in./ft.})(19.47)}{260{,}403} + \frac{(349.29)(12 \text{ in./ft.})(41.157 - 5.28)}{694{,}599.5}$$

$$= 1.058 + 0.216 = 1.274 \text{ ksi}$$

Prestress loss due to creep of concrete is

CR_C = 12(2.743) – 7(1.274) = 24.0 ksi

A.1.7.3.4 Relaxation of Pretensioning Steel

[STD Art. 9.16.2.1.4]

For pretensioned members with 270 ksi low-relaxation strands, the prestress loss due to relaxation of the prestressing steel is calculated using the following formula:

$$CR_S = 5000 - 0.10\ ES - 0.05(SH + CR_C) \qquad \text{[STD Eq. 9-10A]}$$

$$CR_S = [5000 - 0.10(17{,}180) - 0.05(8000+24{,}000)]\left(\frac{1}{1000}\right)$$

$$= 1.682 \text{ ksi}$$

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75 f_s'}$$

$$= \frac{[17.18 + 0.5(1.682)]100}{0.75(270)} = 8.90\% < 9.27\% \text{ (assumed)}$$

Therefore, another trial is required assuming 8.90 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress loss due to concrete shrinkage. Therefore, the next trial will involve updating the losses due to elastic shortening, steel relaxation, and creep of concrete.

Based on an initial prestress loss value of 8.90 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_{si} = (number of strands)(area of each strand)[0.911(0.75 f_s')]

= 50(0.153)(0.911)(0.75)(270) = 1411.25 kips

Loss in prestress due to elastic shortening

$$ES = \frac{E_s}{E_{ci}} f_{cir}$$

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1411.25}{788.4} + \frac{1411.25(19.47)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.47)}{260{,}403}$$

$$= 1.790 + 2.054 - 1.086 = 2.758 \text{ ksi}$$

E_s = 28,000 ksi

E_{ci} = 4470.10 ksi

$$ES = \left[\frac{28{,}000}{4470.10}\right](2.758) = 17.28 \text{ ksi}$$

Loss in prestress due to creep of concrete

$CR_C = 12f_{cir} - 7f_{cds}$

The value of f_{cds} is independent of the initial prestressing force value and will be the same as calculated in Section A.1.7.3.3.

f_{cds} = 1.274 ksi

CR_C = 12(2.758) – 7(1.274) = 24.18 ksi

Loss in prestress due to relaxation of steel

$CR_S = 5000 - 0.10\ ES - 0.05(SH + CR_C)$

$$= [5000 - 0.10(17{,}280) - 0.05(8000 + 24{,}180)]\left(\frac{1}{1000}\right)$$

= 1.663 ksi

$$\text{Initial prestress loss} = \frac{(ES + \frac{1}{2}CR_S)100}{0.75f_s'}$$

$$= \frac{[17.28 + 0.5(1.663)]100}{0.75(270)} = 8.94\% \approx 8.90\% \text{ (assumed value)}$$

A.1.7.3.5
Total Losses at Transfer

Total prestress loss at transfer = $(ES + \frac{1}{2}CR_s)$

= [17.28 + 0.5(1.663)] = 18.11 ksi

Effective initial prestress, f_{si} = 202.5 – 18.11 = 184.39 ksi

P_{si} = Effective pretension after allowing for the initial prestress loss

= (number of strands)(area of strand)(f_{si})

= 50(0.153)(184.39) = 1410.58 kips

A.1.7.3.6
Total Losses at Service Loads

Loss in prestress due to concrete shrinkage, SH = 8.0 ksi

Loss in prestress due to elastic shortening, ES = 17.28 ksi

Loss in prestress due to creep of concrete, CR_C = 24.18 ksi

Loss in prestress due to steel relaxation, CR_S = 1.663 ksi

Total final loss in prestress = $SH + ES + CR_C + CR_S$

= 8.0 + 17.28 + 24.18 + 1.663 = 51.12 ksi

or $\frac{51.12(100)}{0.75(270)} = 25.24\%$

Effective final prestress, f_{se} = 0.75(270) – 51.12 = 151.38 ksi

P_{se} = Effective pretension after allowing for the final prestress loss

= (number of strands)(area of strand)(effective final prestress)

= 50(0.153)(151.38) = 1158.06 kips

A.1.7.3.7 Final Stresses at Midspan

Concrete stress at top fiber of the girder at midspan section due to applied loads and effective prestress

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + f_t = \frac{1158.06}{788.4} - \frac{1158.06\,(19.47)}{8902.67} + 3.626$$

$$= 1.469 - 2.533 + 3.626 = 2.562 \text{ ksi}$$

(f_t calculations are presented in Section A.1.6.1.)

Compressive stress limit under service load combination is 0.6 f'_c. [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{2562}{0.6} = 4270 \text{ psi}$$

Concrete stress at top fiber of the girder at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1158.06}{788.4} - \frac{1158.06\,(19.47)}{8902.67} + \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{8902.67} + \frac{349.29(12 \text{ in./ft.})}{54{,}083.9}$$

$$= 1.469 - 2.533 + 3.22 + 0.077 = 2.233 \text{ ksi}$$

Compressive stress limit for effective prestress + permanent dead loads = 0.4 f'_c [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{2233}{0.40} = 5582.5 \text{ psi} \qquad \text{(controls)}$$

Concrete stress at top fiber of the girder at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{1478.39(12\text{ in./ft.})}{54,083.9} + 0.5\left\{\frac{1158.06}{788.4} - \frac{1158.06\,(19.47)}{8902.67} + \frac{(1209.98 + 1179.03)(12\text{ in./ft.})}{8902.67} + \frac{349.29(12\text{ in./ft.})}{54,083.9}\right\}$$

= 0.328 + 0.5(1.469 – 2.533 + 3.22 + 0.077) = 1.445 ksi

Allowable limit for compressive stress due to live load + 0.5(effective prestress + dead loads) = 0.4 f'_c [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \frac{1445}{0.40} = 3612.5 \text{ psi}$$

Tensile stress at the bottom fiber of the girder at midspan due to service loads

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\,e_c}{S_b} - f_b$$ (f_b calculations are presented in Sec. A.1.6.1.)

$$= \frac{1158.06}{788.4} + \frac{1158.06\,(19.47)}{10,521.33} - 4.024$$

= 1.469 + 2.143 – 4.024 = – 0.412 ksi (negative sign indicates tensile stress)

For members with bonded reinforcement, allowable tension in the precompressed tensile zone = 6 $\sqrt{f'_c}$. [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \left(\frac{412}{6}\right)^2 = 4715.1 \text{ psi}$$

The concrete strength at service is updated based on the final stresses at the midspan section under different loading combinations. The required concrete strength at service is determined to be 5582.5 psi.

A.1.7.3.8
Initial Stresses at Hold-Down Point

Prestressing force after allowing for initial prestress loss
P_{si} = (number of strands)(area of strand)(effective initial prestress)
= 50(0.153)(184.39) = 1410.58 kips

(Effective initial prestress calculations are presented in Section A.1.7.3.5.)

Initial concrete stress at the top fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in.
= 1222.22 k-ft. (see Section A.1.7.1.8.)

$$f_{ti} = \frac{1410.58}{788.4} - \frac{1410.58\,(19.47)}{8902.67} + \frac{1222.22(12\text{ in./ft.})}{8902.67}$$

= 1.789 – 3.085 + 1.647 = 0.351 ksi

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1410.58}{788.4} + \frac{1410.58\,(19.47)}{10{,}521.33} - \frac{1222.22(12\text{ in./ft.})}{10{,}521.33}$$

= 1.789 + 2.610 – 1.394 = 3.005 ksi

Compressive stress limit for pretensioned members at transfer stage is 0.6 f'_{ci}. [STD Art. 9.15.2.1]

$$f'_{ci\ reqd} = \frac{3005}{0.6} = 5008.3 \text{ psi}$$

A.1.7.3.9
Initial Stresses at Girder End

The eccentricity of the prestressing strands at the girder end when 10 web strands are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder) is calculated as follows (see Fig. A.1.7.2.):

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 2(10) + 2(52+50+48+46+44)}{50}$$

= 11.07 in.

Concrete stress at the top fiber of the girder at the girder end at transfer stage:

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_e}{S_t}$$

$$= \frac{1410.58}{788.4} - \frac{1410.58\,(11.07)}{8902.67} = 1.789 - 1.754 = 0.035 \text{ ksi}$$

Concrete stress at the bottom fiber of the girder at the girder end at transfer stage:

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_e}{S_b}$$

$$f_{bi} = \frac{1410.58}{788.4} + \frac{1410.58\,(11.07)}{10{,}521.33} = 1.789 + 1.484 = 3.273 \text{ ksi}$$

Compressive stress limit for pretensioned members at transfer stage is $0.6\,f'_{ci}$. [STD Art.9.15.2.1]

$$f'_{ci\,reqd} = \frac{3273}{0.60} = 5455 \text{ psi} \quad \text{(controls)}$$

The required concrete strengths are updated based on the above results as follows.

Concrete strength at release, f'_{ci} = 5455 psi

Concrete strength at service, f'_c = 5582.5 psi

The difference in the required concrete strengths at release and at service obtained from iterations 2 and 3 is less than 20 psi. Hence, the concrete strengths are sufficiently converged, and an additional iteration is not required.

Therefore, provide:

f'_{ci} = 5455 psi

f'_c = 5582.5 psi

50 – 0.5 in. diameter, 10 draped at the end, Grade 270 low-relaxation strands

Figures A.1.7.1 and A.1.7.2 show the final strand patterns at the midspan section and at the girder ends. Figure A.1.7.3 shows the longitudinal strand profile.

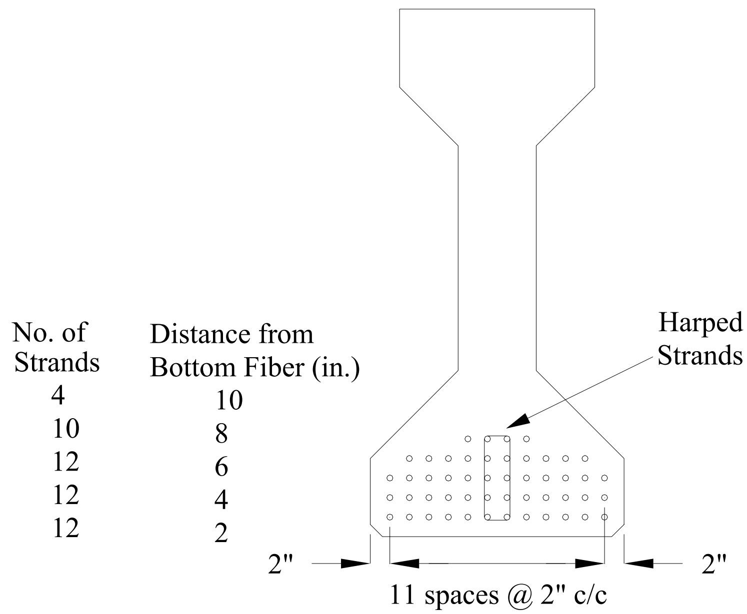

Figure A.1.7.1. Final Strand Pattern at Midspan Section.

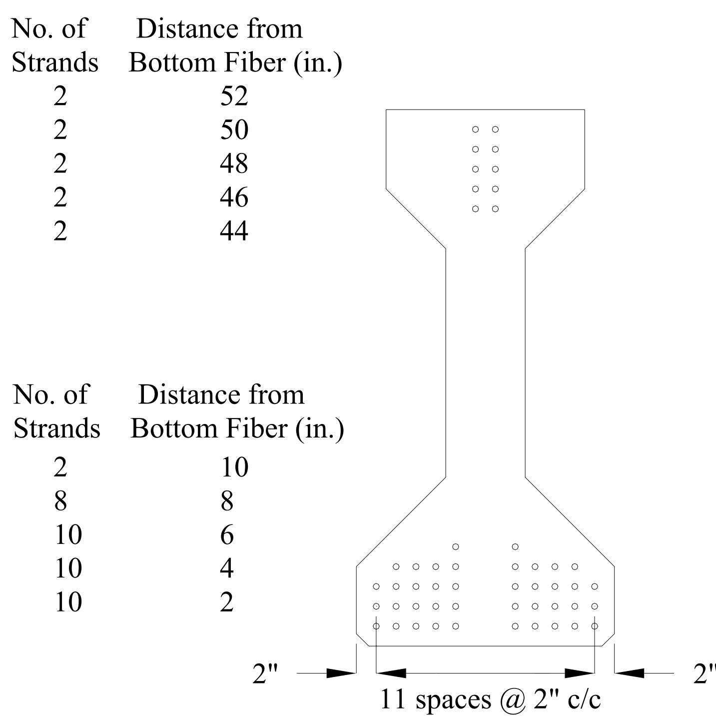

Figure A.1.7.2. Final Strand Pattern at Girder End.

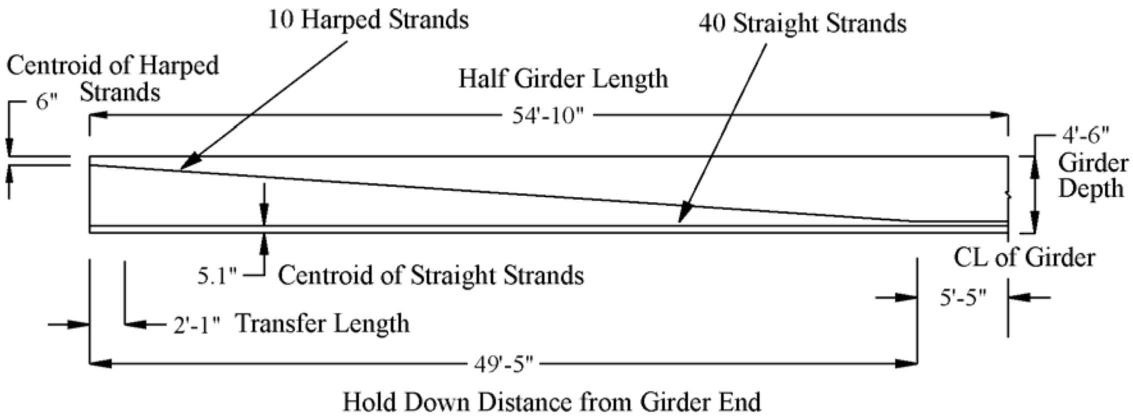

Figure A.1.7.3. Longitudinal Strand Profile (half of the girder length is shown).

The distance between the centroid of the 10 harped strands and the top fiber of the girder at the girder end

$$= \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \text{ in.}$$

The distance between the centroid of the 10 harped strands and the bottom fiber of the girder at the harp points

$$= \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \text{ in.}$$

Transfer length distance from girder end = 50 strand diameters

[STD Art. 9.20.2.4]

Transfer length = 50(0.50) = 25 in. = 2.083 ft.

The distance between the centroid of the 10 harped strands and the top of the girder at the transfer length section

$$= 6 \text{ in.} + \frac{(54 \text{ in} - 6 \text{ in} - 6 \text{ in})}{49.4 \text{ ft.}}(2.083 \text{ ft.}) = 7.77 \text{ in.}$$

The distance between the centroid of the 40 straight strands and the bottom fiber of the girder at all locations

$$= \frac{10(2) + 10(4) + 10(6) + 8(8) + 2(10)}{40} = 5.1 \text{ in.}$$

A.1.8 STRESS SUMMARY

A.1.8.1 Concrete Stresses at Transfer

A.1.8.1.1 Allowable Stress Limits

[STD Art. 9.15.2.1]

The allowable stress limits at transfer specified by the Standard Specifications are as follows.

Compression: $0.6\, f'_{ci}$ = 0.6(5455) = +3273 psi

Tension: The maximum allowable tensile stress is
$7.5\sqrt{f'_{ci}} = 7.5\sqrt{5455}$ = 553.93 psi

If the calculated tensile stress exceeds 200 psi or $3\sqrt{f'_{ci}} = 3\sqrt{5455}$ = 221.57 psi, whichever is smaller, bonded reinforcement should be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section.

A.1.8.1.2 Stresses at Girder End

Stresses at the girder end are checked only at release, because it almost always governs.

Eccentricity of prestressing strands at the girder end when 10 web strands are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder)

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 2(10) + 2(52+50+48+46+44)}{50}$$
$$= 11.07 \text{ in.}$$

Prestressing force after allowing for initial prestress loss

P_{si} = (number of strands)(area of strand)(effective initial prestress)
= 50(0.153)(184.39) = 1410.58 kips

(Effective initial prestress calculations are presented in Section A.1.7.3.5.)

Concrete stress at the top fiber of the girder at the girder end at transfer:

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\, e_e}{S_t}$$
$$= \frac{1410.58}{788.4} - \frac{1410.58\,(11.07)}{8902.67} = 1.789 - 1.754 = +0.035 \text{ ksi}$$

Allowable Compression: +3.273 ksi >> +0.035 ksi (reqd.) (O.K.)

Because the top fiber stress is compressive, there is no need for additional bonded reinforcement.

Concrete stress at the bottom fiber of the girder at the girder end at transfer stage:

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\, e_e}{S_b}$$

$$= \frac{1410.58}{788.4} + \frac{1410.58\ (11.07)}{10{,}521.33} = 1.789 + 1.484 = +3.273 \text{ ksi}$$

Allowable compression: +3.273 ksi = +3.273 ksi (reqd.) (O.K.)

A.1.8.1.3 Stresses at Transfer Length Section

Stresses at transfer length are checked only at release, because it almost always governs.

Transfer length = 50(strand diameter) [STD Art. 9.20.2.4]
= 50(0.50) = 25 in. = 2.083 ft.

The transfer length section is located at a distance of 2 ft.-1in. from the end of the girder or at a point 1 ft.-6.5 in. from the centerline of the bearing as the girder extends 6.5 in. beyond the bearing centerline. Overall girder length of 109 ft.-8 in. is considered for the calculation of bending moment at transfer length.

Moment due to girder self-weight, $M_g = 0.5wx(L - x)$

where:

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Transfer length distance from girder end = 2.083 ft.

$M_g = 0.5(0.821)(2.083)(109.67 - 2.083) = 92$ k-ft.

Eccentricity of prestressing strands at transfer length section

$$e_t = e_c - (e_c - e_e)\ \frac{(49.404\ -\ x)}{49.404}$$

where:

e_c = Eccentricity of prestressing strands at midspan = 19.47 in.

e_e = Eccentricity of prestressing strands at girder end
= 11.07 in.

x = Distance of transfer length section from girder end, ft.

$$e_t = 19.47 - (19.47 - 11.07)\frac{(49.404 - 2.083)}{49.404} = 11.42 \text{ in.}$$

Initial concrete stress at top fiber of the girder at transfer length section due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_t}{S_t} + \frac{M_g}{S_t}$$

$$f_{ti} = \frac{1410.58}{788.4} - \frac{1410.58\,(11.42)}{8902.67} + \frac{92\,(12 \text{ in./ft.})}{8902.67}$$

$= 1.789 - 1.809 + 0.124 = +0.104$ ksi

Allowable compression: +3.273 ksi >> 0.104 ksi (reqd.) (O.K.)

Because the top fiber stress is compressive, there is no need for additional bonded reinforcement.

Initial concrete stress at bottom fiber of the girder at the transfer length section due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_t}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1410.58}{788.4} + \frac{1410.58\,(11.42)}{10{,}521.33} - \frac{92\,(12 \text{ in./ft.})}{10{,}521.33}$$

$= 1.789 + 1.531 - 0.105 = 3.215$ ksi

Allowable compression: +3.273 ksi > 3.215 ksi (reqd.) (O.K.)

A.1.8.1.4 Stresses at Hold-Down Points

The eccentricity of the prestressing strands at the harp points is the same as at midspan.

$e_{harp} = e_c = 19.47$ in.

Initial concrete stress at top fiber of the girder at the hold-down point due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_{harp}}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in.
= 1222.22 k-ft. (see Section A.1.7.1.8.)

$$f_{ti} = \frac{1410.58}{788.4} - \frac{1410.58\,(19.47)}{8902.67} + \frac{1222.22(12\text{ in./ft.})}{8902.67}$$

$= 1.789 - 3.085 + 1.647 = 0.351$ ksi

Allowable compression: +3.273 ksi >> 0.351 ksi (reqd.) (O.K.)

Initial concrete stress at bottom fiber of the girder at the hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_{harp}}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1410.58}{788.4} + \frac{1410.58\,(19.47)}{10{,}521.33} - \frac{1222.22(12\text{ in./ft.})}{10{,}521.33}$$

$= 1.789 + 2.610 - 1.394 = 3.005$ ksi

Allowable compression: +3.273 ksi > 3.005 ksi (reqd.) (O.K.)

A.1.8.1.5 Stresses at Midspan

Bending moment due to girder self-weight at midspan section based on overall girder length of 109 ft.-8 in.

$$M_g = 0.5wx(L - x)$$

where:

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Half the girder length = 54.84 ft.

$M_g = 0.5(0.821)(54.84)(109.67 - 54.84) = 1234.32$ k-ft.

Initial concrete stress at top fiber of the girder at midspan section due to self-weight of the girder and the effective initial prestress

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\,e_c}{S_t} + \frac{M_g}{S_t}$$

$$f_{ti} = \frac{1410.58}{788.4} - \frac{1410.58\,(19.47)}{8902.67} + \frac{1234.32(12\text{ in./ft.})}{8902.67}$$

$= 1.789 - 3.085 + 1.664 = 0.368$ ksi

Allowable compression: +3.273 ksi >> 0.368 ksi (reqd.) (O.K.)

Initial concrete stress at bottom fiber of the girder at midspan section due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\,e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1410.58}{788.4} + \frac{1410.58\,(19.47)}{10{,}521.33} - \frac{1234.32(12\text{ in./ft.})}{10{,}521.33}$$

= 1.789 + 2.610 – 1.408 = 2.991 ksi

Allowable compression: +3.273 ksi > 2.991 ksi (reqd.) (O.K.)

A.1.8.1.6 Stress Summary at Transfer

Allowable Stress Limits:

Compression: +3.273 ksi

Tension: – 0.20 ksi without additional bonded reinforcement
– 0.554 ksi with additional bonded reinforcement

Location	Top of girder f_t (ksi)	Bottom of girder f_b (ksi)
Girder end	+0.035	+3.273
Transfer length section	+0.104	+3.215
Hold-down points	+0.351	+3.005
Midspan	+0.368	+2.991

A.1.8.2 Concrete Stresses at Service Loads

A.1.8.2.1 Allowable Stress Limits

[STD Art. 9.15.2.2]

The allowable stress limits at service load after losses have occurred specified by the Standard Specifications are presented as follows.

Compression:

Case (I): For all load combinations

$0.60\,f_c'$ = 0.60(5582.5)/1000 = +3.349 ksi (for precast girder)

$0.60\,f_c'$ = 0.60(4000)/1000 = +2.400 ksi (for slab)

Case (II): For effective prestress + permanent dead loads

$0.40\,f_c'$ = 0.40(5582.5)/1000 = +2.233 ksi (for precast girder)

$0.40\,f_c'$ = 0.40(4000)/1000 = +1.600 ksi (for slab)

Case (III): For live loads + 0.5(effective prestress + dead loads)

$0.40\ f_c' = 0.40(5582.5)/1{,}000 = +2.233$ ksi (for precast girder)

$0.40\ f_c' = 0.40(4000)/1{,}000 = +1.600$ ksi (for slab)

Tension: For members with bonded reinforcement

$$6\sqrt{f_c'} = 6\sqrt{5582.5}\left(\frac{1}{1000}\right) = -\,0.448 \text{ ksi}$$

A.1.8.2.2 Final Stresses at Midspan

Effective pretension after allowing for the final prestress loss

P_{se} = (number of strands)(area of strand)(effective final prestress)

= 50(0.153)(151.38) = 1158.06 kips

Case (I): Service load conditions

Concrete stress at the top fiber of the girder at the midspan section due to service loads and effective prestress

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL} + M_{LL+I}}{S_{tg}}$$

$$= \frac{1158.06}{788.4} - \frac{1158.06\ (19.47)}{8902.67} + \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{8902.67} + \frac{(349.29 + 1478.39)(12 \text{ in./ft.})}{54{,}083.9}$$

= 1.469 – 2.533 + 3.220 + 0.406 = 2.562 ksi

Allowable compression: +3.349 ksi > +2.562 ksi (reqd.) (O.K.)

Case (II): Effective prestress + permanent dead loads

Concrete stress at top fiber of the girder at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1158.06}{788.4} - \frac{1158.06\ (19.47)}{8902.67} + \frac{(1209.98 + 1179.03)(12 \text{ in./ft.})}{8902.67} + \frac{349.29(12 \text{ in./ft.})}{54{,}083.9}$$

= 1.469 – 2.533 + 3.22 + 0.077 = 2.233 ksi

Allowable compression: +2.233 ksi = +2.233 ksi (reqd.) (O.K.)

Case (III): Live loads + 0.5(prestress + dead loads)

Concrete stress at top fiber of the girder at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{1478.39(12\text{ in./ft.})}{54,083.9} + 0.5\left\{\frac{1158.06}{788.4} - \frac{1158.06\,(19.47)}{8902.67} + \frac{(1209.98\ +\ 1179.03)(12\text{ in./ft.})}{8902.67} + \frac{349.29(12\text{ in./ft.})}{54,083.9}\right\}$$

$$= 0.328 + 0.5(1.469 - 2.533 + 3.22 + 0.077) = 1.445\text{ ksi}$$

Allowable compression: +2.233 ksi > +1.445 ksi (reqd.) (O.K.)

Tensile stress at the bottom fiber of the girder at midspan due to service loads

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\,e_c}{S_b} - \frac{M_g + M_S}{S_b} - \frac{M_{SDL} + M_{LL+I}}{S_{bc}}$$

$$= \frac{1158.06}{788.4} + \frac{1158.06\,(19.47)}{10,521.33} - \frac{(1209.98 + 1179.03)(12\text{ in./ft.})}{10,521.33} - \frac{(349.29 + 1478.39)(12\text{ in./ft.})}{16,876.83}$$

$$= 1.469 + 2.143 - 2.725 - 1.299 = -0.412\text{ ksi (tensile stress)}$$

Allowable Tension: –0.448 ksi < –412 ksi (reqd.) (O.K.)

Superimposed dead and live loads contribute to the stresses at the top of the slab calculated as follows.

Case (I): Superimposed dead load and live load effect

Concrete stress at top fiber of the slab at midspan due to live load + superimposed dead loads

$$f_t = \frac{M_{SDL} + M_{LL+I}}{S_{tc}} = \frac{(349.29 + 1478.39)(12\text{ in./ft.})}{33,325.31} = +0.658\text{ ksi}$$

Allowable compression: +2.400 ksi > +0.658 ksi (reqd.) (O.K.)

Case (II): Superimposed dead load effect

Concrete stress at top fiber of the slab at midspan due to superimposed dead loads

$$f_t = \frac{M_{SDL}}{S_{tc}} = \frac{(349.29)(12 \text{ in./ft.})}{33,325.31} = 0.126 \text{ ksi}$$

Allowable compression: +1.600 ksi > +0.126 ksi (reqd.) (O.K.)

Case (III): Live load + 0.5(superimposed dead loads)

Concrete stress at top fiber of the slab at midspan due to live loads + 0.5(superimposed dead loads)

$$f_t = \frac{M_{LL+I} + 0.5(M_{SDL})}{S_{tc}}$$

$$= \frac{(1478.39)(12 \text{ in./ft.}) + 0.5(349.29)(12 \text{ in./ft.})}{33,325.31} = 0.595 \text{ ksi}$$

Allowable compression: +1.600 ksi > +0.595 ksi (reqd.) (O.K.)

A.1.8.2.3 Summary of Stresses at Service Loads

At Midspan	Top of slab f_t (ksi)	Top of Girder f_t (ksi)	Bottom of girder f_b (ksi)
Case I	+0.658	+2.562	-0.412
Case II	+0.126	+2.233	–
Case III	+0.595	+1.455	–

A.1.8.2.4 Composite Section Properties

The composite section properties calculated in Section A.1.4.2.4 were based on the modular ratio value of 1. Because the actual concrete strength is now selected, the actual modular ratio can be determined, and the corresponding composite section properties can be computed. Table A.1.8.1 shows the section properties obtained.

Modular ratio between slab and girder concrete

$$n = \left(\frac{E_{cs}}{E_{cp}}\right)$$

where:

n = Modular ratio between slab and girder concrete

E_{cs} = Modulus of elasticity of slab concrete, ksi

$= 33(w_c)^{3/2}\sqrt{f'_{cs}}$ [STD Eq. 9-8]

w_c = Unit weight of concrete = 150 pcf

f'_{cs} = Compressive strength of slab concrete at service = 4000 psi

$$E_{cs} = [33(150)^{3/2}\sqrt{4000}\,]\left(\frac{1}{1000}\right) = 3834.25 \text{ ksi}$$

E_{cp} = Modulus of elasticity of precast girder concrete, ksi $= 33(w_c)^{3/2}\sqrt{f'_c}$

f'_c = Compressive strength of precast girder concrete at service = 5582.5 psi

$$E_{cp} = [33(150)^{3/2}\sqrt{5582.5}\,]\left(\frac{1}{1000}\right) = 4529.65 \text{ ksi}$$

$$n = \frac{3834.25}{4529.65} = 0.846$$

Transformed flange width, $b_{tf} = n \times$ (effective flange width)

Effective flange width = 96 in. (see Section A.1.4.2.)

$b_{tf} = 0.846(96) = 81.22$ in.

Transformed flange area, $A_{tf} = n \times$ (effective flange width)(t_s)

t_s = Slab thickness = 8 in.

$A_{tf} = 0.846(96)(8) = 649.73$ in.^{2}

Table A.1.8.1. Properties of Composite Section.

	Transformed Area A (in.^{2})	y_b in.	Ay_b in.^{3}	$A(y_{bc} - y_b)^2$	I in.^{4}	$I + A(y_{bc} - y_b)^2$ in.^{4}
Girder	788.40	24.75	19,512.9	177,909.63	260,403.0	438,312.6
Slab	649.73	58.00	37,684.3	215,880.37	3465.4	219,345.8
∑	1438.13		57,197.2			657,658.4

A_c = Total area of composite section = 1438.13 in.^{2}

h_c = Total height of composite section = 54 in. + 8 in. = 62 in.

I_c = Moment of inertia of composite section = 657,658.4 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder, in.
= 57,197.2/1438.13 = 39.77 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder, in.
= 54 – 39.772 = 14.23 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 39.77 = 22.23 in.

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder, in.^{3}
= I_c/y_{bc} = 657,658.4/39.77 = 16,535.71 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder, in.^{3}
= I_c/y_{tg} = 657,658.4/14.23 = 46,222.83 in.^{3}

S_{tc} = Section modulus of composite section referenced to the top fiber of the slab, in.^{3}
= I_c/y_{tc} = 657,658.4/22.23 = 29,586.93 in.^{3}

A.1.9 FLEXURAL STRENGTH

[STD Art. 9.17]

The flexural strength limit for Group I loading is investigated as follows. The Group I load factor design combination specified by the Standard Specifications is:

$M_u = 1.3[M_g + M_S + M_{SDL} + 1.67(M_{LL+I})]$ [STD Table 3.22.1.A]

where:

M_u = Design flexural moment at midspan of the girder, k-ft.

M_g = Moment due to self-weight of the girder at midspan
= 1209.98 k-ft.

M_S = Moment due to slab weight at midspan = 1179.03 k-ft.

M_{SDL} = Moment due to superimposed dead loads at midspan
= 349.29 k-ft.

M_{LL+I} = Moment due to live loads including impact loads at midspan = 1478.39 k-ft.

Substituting the moment values from Table A.1.5.1 and A.1.5.2

M_u = 1.3[1209.98 + 1179.03 + 349.29 + 1.67(1478.39)]

= 6769.37 k-ft.

For bonded members, the average stress in the pretensioning steel at ultimate load conditions is given as:

$$f_{su}^* = f_s' \left(1 - \frac{\gamma^*}{\beta_1}\rho^* \frac{f_s'}{f_c'}\right) \quad \text{[STD Eq. 9-17]}$$

The above equation is applicable when the effective prestress after losses, $f_{se} > 0.5\, f_s'$

where:

f_{su}^* = Average stress in the pretensioning steel at ultimate load, ksi

f_s' = Ultimate stress in prestressing strands = 270 ksi

f_{se} = Effective final prestress (see Section A.1.7.3.6)
= 151.38 ksi > 0.5(270) = 135 ksi (O.K.)
The equation for f_{su}^* shown above is applicable.

f_c' = Compressive strength of slab concrete at service
= 4000 psi

γ^* = Factor for type of prestressing steel
= 0.28 for low-relaxation steel strands [STD Art. 9.1.2]

$$\beta_1 = 0.85 - 0.05\frac{(f_c' - 4000)}{1000} \geq 0.65 \quad \text{[STD Art. 8.16.2.7]}$$

It is assumed that the neutral axis lies in the slab, and hence, the f_c' of slab concrete is used for the calculation of the factor β_1. If the neutral axis is found to be lying below the slab, β_1 will be updated.

$$\beta_1 = 0.85 - 0.05\frac{(4000 - 4000)}{1000} = 0.85$$

ρ^* = Ratio of prestressing steel = $\dfrac{A_s^*}{b\,d}$

A_s^* = Area of pretensioned reinforcement, in.^{2}
= (number of strands)(area of strand) = 50(0.153) = 7.65 in.^{2}

b = Effective flange (composite slab) width = 96 in.

y_{bs} = Distance from centroid of the strands to the bottom fiber of the girder at midspan = 5.28 in. (see Section A.1.7.3.3)

d = Distance from top of the slab to the centroid of prestressing strands, in.
= girder depth (h) + slab thickness (t_s) – y_{bs}
= 54 + 8 – 5.28 = 56.72 in.

$$\rho^* = \frac{7.65}{96(56.72)} = 0.001405$$

$$f_{su}^* = 270\left[1-\left(\frac{0.28}{0.85}\right)(0.001405)\left(\frac{270.0}{4.0}\right)\right] = 261.565 \text{ ksi}$$

Depth of equivalent rectangular compression block

$$a = \frac{A_s^* f_{su}^*}{0.85 f_c' b} = \frac{7.65\,(261.565)}{0.85(4)(96)}$$

a = 6.13 in. < t_s = 8.0 in. [STD Art. 9.17.2]

The depth of compression block is less than the flange (slab) thickness. Hence, the section is designed as a rectangular section, and f_c' of the slab concrete is used for calculations.

For rectangular section behavior, the design flexural strength is given as:

$$\phi M_n = \phi\left[A_s^* f_{su}^* d\left(1-0.6\frac{\rho^* f_{su}^*}{f_c'}\right)\right]$$ [STD Eq. 9-13]

where:

ϕ = Strength reduction factor = 1.0 for prestressed concrete members [STD Art. 9.14]

M_n = Nominal moment strength of the section

$$\phi M_n = 1.0\left[(7.65)(261.565)\frac{(56.72)}{(12 \text{ in./ft.})}\left(1-0.6\frac{0.001405(261.565)}{4.0}\right)\right]$$

= 8936.56 k-ft. > M_u = 6769.37 k-ft. (O.K.)

A.1.10 DUCTILITY LIMITS

[STD Art. 9.18]

A.1.10.1 Maximum Reinforcement

[STD Art. 9.18.1]

To ensure that steel is yielding as the ultimate capacity is approached, the reinforcement index for a rectangular section shall be such that:

$$\frac{\rho^* f_{su}^*}{f_c'} < 0.36\beta_1 \qquad \text{[STD Eq. 9-20]}$$

$$0.001405\left(\frac{261.565}{4.0}\right) = 0.092 < 0.36(0.85) = 0.306 \qquad \text{(O.K.)}$$

A.1.10.2 Minimum Reinforcement

[STD Art. 9.18.2]

The nominal moment strength developed by the prestressed and nonprestressed reinforcement at the critical section shall be at least 1.2 times the cracking moment, M_{cr}^*

$$\phi M_n \geq 1.2\ M_{cr}^*$$

$$M_{cr}^* = (f_r + f_{pe})\, S_{bc} - M_{d\text{-}nc}\left(\frac{S_{bc}}{S_b} - 1\right) \qquad \text{[STD Art. 9.18.2.1]}$$

where:

f_r = Modulus of rupture of concrete = $7.5\sqrt{f_c'}$ for normal weight concrete, ksi [STD Art. 9.15.2.3]

$$= 7.5\sqrt{5582.5}\left(\frac{1}{1000}\right) = 0.5604 \text{ ksi}$$

f_{pe} = Compressive stress in concrete due to effective prestress forces only at extreme fiber of section where tensile stress is caused by externally applied loads, ksi

The tensile stresses are caused at the bottom fiber of the girder under service loads. Therefore, f_{pe} is calculated for the bottom fiber of the girder as follows.

$$f_{pe} = \frac{P_{se}}{A} + \frac{P_{se}e_c}{S_b}$$

P_{se} = Effective prestress force after losses = 1158.06 kips

e_c = Eccentricity of prestressing strands at midspan = 19.47 in.

$$f_{pe} = \frac{1158.06}{788.4} + \frac{1158.06(19.47)}{10{,}521.33} = 1.469 + 2.143 = 3.612 \text{ ksi}$$

$M_{d\text{-}nc}$ = Non-composite dead load moment at midspan due to self-weight of girder and weight of slab
= 1209.98 + 1179.03 = 2389.01 k-ft. = 28,668.12 k-in.

S_b = Section modulus of the precast section referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

S_{bc} = Section modulus of the composite section referenced to the extreme bottom fiber of the precast girder
= 16,535.71 in.^{3}

$$M_{cr}^{*} = (0.5604 + 3.612)(16{,}535.71) - (28{,}668.12)\left(\frac{16{,}535.71}{10{,}521.33} - 1\right)$$

$$= 68{,}993.6 - 16{,}387.8 = 52{,}605.8 \text{ k-in.} = 4383.8 \text{ k-ft.}$$

$$1.2\,M_{cr}^{*} = 1.2(4383.8) = 5260.56 \text{ k-ft.} < \phi M_n = 8936.56 \text{ k-ft.}$$

(O.K.)

[STD Art. 9.20]

A.1.11 SHEAR DESIGN

The shear design for the AASHTO Type IV girder based on the Standard Specifications is presented in the following section.

Prestressed concrete members subject to shear shall be designed so that:

$$V_u \leq \phi(V_c + V_s) \qquad \text{[STD Eq. 9-26]}$$

where:

V_u = Factored shear force at the section considered (calculated using load combination causing maximum shear force), kips

V_c = Nominal shear strength provided by concrete, kips

V_s = Nominal shear strength provided by web reinforcement, kips

ϕ = Strength reduction factor for shear = 0.90 for prestressed concrete members [STD Art. 9.14]

The critical section for shear is located at a distance $h/2$ (h is the depth of composite section) from the face of the support. However, because the support dimensions are unknown, the critical section for shear is conservatively calculated from the centerline of the bearing support. [STD Art. 9.20.1.4]

Distance of critical section for shear from bearing centerline

$= h/2 = \dfrac{62}{2(12\text{ in./ft.})} = 2.583$ ft.

From Tables A.1.5.1 and A.1.5.2, the shear forces at the critical section are as follows:

V_d = Shear force due to total dead load at the critical section
= 96.07 kips

V_{LL+I} = Shear force due to live load including impact at critical section = 56.60 kips

The shear design is based on Group I loading, presented as follows.

Group I load factor design combination specified by the Standard Specifications is:

$V_u = 1.3(V_d + 1.67\ V_{LL+I})$
$= 1.3[96.07 + 1.67(56.6)] = 247.8$ kips

Shear strength provided by normal weight concrete, V_c, shall be taken as the lesser of the values V_{ci} or V_{cw}. [STD Art. 9.20.2]

Computation of V_{ci} [STD Art. 9.20.2.2]

$$V_{ci} = 0.6\sqrt{f'_c}\,b'd + V_d + \frac{V_i\,M_{cr}}{M_{max}} \geq 1.7\sqrt{f'_c}\,b'd \qquad \text{[STD Eq. 9-27]}$$

where:

V_{ci} = Nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment, kips

f'_c = Compressive strength of girder concrete at service
= 5582.5 psi

b' = Width of the web of a flanged member = 8 in.

d = Distance from the extreme compressive fiber to centroid of pretensioned reinforcement, but not less than $0.8h_c$
$= h_c - (y_b - e_x)$ [STD Art. 9.20.2.2]

h_c = Depth of composite section = 62 in.

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder = 24.75 in.

e_x = Eccentricity of prestressing strands at the critical section for shear

$$= e_c - (e_c - e_e)\ \frac{(49.404\ -\ x)}{49.404}$$

e_c = Eccentricity of prestressing strands at midspan
= 19.12 in.

e_e = Eccentricity of prestressing strands at the girder end
= 11.07 in.

x = Distance of critical section from girder end = 2.583 ft.

$$e_x = 19.47 - (19.47 - 11.07)\frac{(49.404\ -\ 2.583)}{49.404} = 11.51 \text{ in.}$$

d = 62 – (24.75 – 11.51) = 48.76 in.
= $0.8h_c$ = 0.8(62) = 49.6 in. > 48.76 in.
Therefore, d = 49.6 in. is used in further calculations.

V_d = Shear force due to total dead load at the critical section
= 96.07 kips

V_i = Factored shear force at the section due to externally applied loads occurring simultaneously with maximum moment, M_{max}
= $V_{mu} - V_d$

V_{mu} = Factored shear force occurring simultaneously with factored moment M_u, conservatively taken as design shear force at the section, V_u = 247.8 kips

V_i = 247.8 – 96.07 = 151.73 kips

M_{max} = Maximum factored moment at the critical section due to externally applied loads
= $M_u - M_d$

M_d = Bending moment at the critical section due to unfactored dead load = 254.36 k-ft. (see Table A.1.5.1)

M_{LL+I} = Bending moment at the critical section due to live load including impact = 146.19 k-ft. (see Table A.1.5.2)

M_u = Factored bending moment at the section
= $1.3(M_d + 1.67M_{LL+I})$
= 1.3[254.36 + 1.67(146.19)] = 648.05 k-ft.

M_{max} = 648.05 – 254.36 = 393.69 k-ft.

M_{cr} = Moment causing flexural cracking at the section due to externally applied loads

$$= \frac{I}{Y_t}(6\sqrt{f'_c} + f_{pe} - f_d)$$ [STD Eq. 9-28]

f_{pe} = Compressive stress in concrete due to effective prestress at the extreme fiber of the section where tensile stress is caused by externally applied loads, which is the bottom fiber of the girder in the present case

$$= \frac{P_{se}}{A} + \frac{P_{se}e_x}{S_b}$$

P_{se} = Effective final prestress = 1158.06 kips

$$f_{pe} = \frac{1158.06}{788.4} + \frac{1158.06(11.51)}{10{,}521.33} = 1.469 + 1.267 = 2.736 \text{ ksi}$$

f_d = Stress due to unfactored dead load at extreme fiber of the section where tensile stress is caused by externally applied loads, which is the bottom fiber of the girder in the present case

$$= \left[\frac{M_g + M_S}{S_b} + \frac{M_{SDL}}{S_{bc}}\right]$$

M_g = Moment due to self-weight of the girder at the critical section = 112.39 k-ft. (see Table A.1.5.1)

M_S = Moment due to slab weight at the critical section = 109.52 k-ft. (see Table A.1.5.1)

M_{SDL} = Moment due to superimposed dead loads at the critical section = 32.45 k-ft.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

S_{bc} = Section modulus of the composite section referenced to the extreme bottom fiber of the precast girder = 16,535.71 in.^{3}

f_d $= \left[\frac{(112.39+109.52)(12\text{ in./ft.})}{10{,}521.33} + \frac{32.45(12\text{ in./ft.})}{16{,}535.71}\right]$

= 0.253 + 0.024 = 0.277 ksi

I = Moment of inertia about the centroid of the cross section = 657,658.4 in.^{4}

Y_t = Distance from centroidal axis of composite section to the extreme fiber in tension, which is the bottom fiber of the girder in the present case = 39.77 in.

M_{cr} $= \frac{657{,}658.4}{39.772}\left(\frac{6\sqrt{5582.5}}{1000} + 2.736 - 0.277\right)$

= 48,074.23 k-in. = 4006.19 k-ft.

$$V_{ci} = \frac{0.6\sqrt{5582.5}}{1000}(8)(49.6) + 96.07 + \frac{151.73(4006.19)}{393.69}$$

= 17.79 + 96.07 + 1544.00 = 1657.86 kips

Minimum $V_{ci} = 1.7\sqrt{f'_c}\, b'd$ [STD Art. 9.20.2.2]

$= \frac{1.7\sqrt{5582.5}}{1000}(8)(49.6)$

= 50.40 kips << V_{ci} = 1657.86 kips (O.K.)

Computation of V_{cw} [STD Art. 9.20.2.3]

$V_{cw} = (3.5\sqrt{f'_c} + 0.3 f_{pc})\, b'\, d + V_p$ [STD Eq. 9-29]

where:

V_{cw} = Nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web, kips

f_{pc} = Compressive stress in concrete at centroid of cross-section resisting externally applied loads, ksi

$$= \frac{P_{se}}{A} - \frac{P_{se}e_x(y_{bcomp} - y_b)}{I} + \frac{M_D(y_{bcomp} - y_b)}{I}$$

P_{se} = Effective final prestress = 1158.06 kips

e_x = Eccentricity of prestressing strands at the critical section for shear = 11.51 in.

y_{bcomp} = Lesser of y_{bc} and y_w, in.

y_{bc} = Distance from centroid of the composite section to the extreme bottom fiber of the precast girder = 39.77 in.

y_w = Distance from bottom fiber of the girder to the junction of the web and top flange
$= h - t_f - t_{fil}$

h = Depth of precast girder = 54 in.

t_f = Thickness of girder flange = 8 in.

t_{fil} = Thickness of girder fillets = 6 in.

y_w = 54 – 8 – 6 = 40 in. > y_{bc} = 39.77 in.

Therefore, y_{bcomp} = 39.77 in.

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder = 24.75 in.

M_D = Moment due to unfactored non-composite dead loads at the critical section
= 112.39 + 109.52 = 221.91 k-ft. (see Table A.1.5.1)

$$f_{pc} = \frac{1158.06}{788.4} - \frac{1158.06\,(11.51)\,(39.772 - 24.75)}{260,403} + \frac{221.91(12\text{ in./ft.})(39.772 - 24.75)}{260,403}$$
= 1.469 – 0.769 + 0.154 = 0.854 ksi

b' = Width of the web of a flanged member = 8 in.

d = Distance from the extreme compressive fiber to centroid of pretensioned reinforcement = 49.6 in.

V_p = Vertical component of prestress force for harped strands, kips
$= P_{se} \sin\Psi$

P_{se} = Effective prestress force for the harped strands, kips
= (number of harped strands)(area of strand)(effective final prestress)
= 10(0.153)(151.38) = 231.61 kips

Ψ = Angle of harped tendons to the horizontal, radians

$$= \tan^{-1}\left(\frac{h - y_{ht} - y_{hb}}{0.5(HD_e)}\right)$$

y_{ht} = Distance of the centroid of the harped strands from top fiber of the girder at girder end = 6 in. (see Fig. A.1.7.3)

y_{hb} = Distance of the centroid of the web strands from bottom fiber of the girder at hold-down point = 6 in. (see Figure A.1.7.3)

HD_e = Distance of hold-down point from the girder end
= 49.404 ft. (see Figure A.1.7.3)

$$\Psi = \tan^{-1}\left(\frac{54 - 6 - 6}{49.404\,(12\text{ in./ft.})}\right) = 0.071 \text{ radians}$$

V_p = 231.61 sin (0.071) = 16.43 kips

$$V_{cw} = \left(\frac{3.5\sqrt{5582.5}}{1000} + 0.3(0.854)\right)(8)(49.6) + 16.43 = 221.86 \text{ kips}$$

The allowable nominal shear strength provided by concrete, V_c, is the lesser of V_{ci} = 1657.86 kips and V_{cw} = 221.86 kips

Therefore, V_c = 221.86 kips

Shear reinforcement is not required if $2V_u \leq \phi V_c$.

[STD Art. 9.20]

where:

V_u = Factored shear force at the section considered (calculated using load combination causing maximum shear force)
= 247.8 kips

ϕ = Strength reduction factor for shear = 0.90 for prestressed concrete members [STD Art. 9.14]

V_c = Nominal shear strength provided by concrete = 221.86 kips

$2\ V_u = 2\times(247.8) = 495.6$ kips $> \phi V_c = 0.9\times(221.86) = 199.67$ kips

Therefore, shear reinforcement is required. The required shear reinforcement is calculated using the following criterion.

$$V_u < \phi(V_c + V_s) \qquad \text{[STD Eq. 9-26]}$$

where V_s is the nominal shear strength provided by web reinforcement, kips

$$\text{Required } V_s = \frac{V_u}{\phi} - V_c = \frac{247.8}{0.9} - 221.86 = 53.47 \text{ kips}$$

Maximum shear force that can be carried by reinforcement

$$V_{s\,max} = 8\sqrt{f'_c}\ b'd \qquad \text{[STD Art. 9.20.3.1]}$$

where:

f'_c = Compressive strength of girder concrete at service
= 5582.5 psi

$$V_{s\,max} = \frac{8\sqrt{5582.5}}{1000}(8)(49.6)$$

$= 237.18$ kips $>$ Required $V_s = 53.47$ kips (O.K.)

The section depth is adequate for shear.

The required area of shear reinforcement is calculated using the following formula: [STD Art. 9.20.3.1]

$$V_s = \frac{A_v f_y d}{s} \text{ or } \frac{A_v}{s} = \frac{V_s}{f_y d} \qquad \text{[STD Eq. 9-30]}$$

where:

A_v = Area of web reinforcement, in.^{2}

s = Center-to-center spacing of the web reinforcement, in.

f_y = Yield strength of web reinforcement = 60 ksi

$$\text{Required } \frac{A_v}{s} = \frac{(53.47)}{(60)(49.6)} = 0.018 \text{ in}^2\text{./in.}$$

Minimum shear reinforcement [STD Art. 9.20.3.3]

$$A_{v-min} = \frac{50\,b's}{f_y} \text{ or } \frac{A_{v\text{-}min}}{s} = \frac{50\,b'}{f_y}$$ [STD Eq. 9-31]

$$\frac{A_{v\text{-}min}}{s} = \frac{(50)(8)}{60{,}000} = 0.0067 \text{ in.}^2\text{/in.} < \text{Required } \frac{A_v}{s} = 0.018 \text{ in}^2\text{./in.}$$

Therefore, provide $\frac{A_v}{s} = 0.018$ in.^{2}/in.

Typically, TxDOT uses double-legged #4 Grade 60 stirrups for shear reinforcement. The same is used in this design.

A_v = Area of web reinforcement, in.^{2} = (number of legs)(area of bar) = 2(0.20) = 0.40 in.^{2}

Center-to-center spacing of web reinforcement

$$s = \frac{A_v}{\text{Required } \frac{A_v}{s}} = \frac{0.40}{0.018} = 22.22 \text{ in. (use 22 in.)}$$

$$V_s \text{ provided} = \frac{A_v f_y d}{s} = \frac{(0.40)(60)(49.6)}{22} = 54.1 \text{ kips}$$

Maximum spacing of web reinforcement is specified to be the lesser of 0.75 h_c or 24 in., unless V_s exceeds $4\sqrt{f'_c}\ b'\,d$.

[STD Art. 9.20.3.2]

$$4\sqrt{f'_c}\ b'\,d = \frac{4\sqrt{5582.5}}{1000}(8)(49.6)$$

$$= 118.59 \text{ kips} < V_s = 54.1 \text{ kips} \quad \text{(O.K.)}$$

Because V_s is less than the limit, the maximum spacing of the web reinforcement is given as:

s_{max} = Lesser of 0.75 h_c or 24 in.

where:

h_c = Overall depth of the section = 62 in. (Note that the wearing surface thickness can also be included in the overall section depth calculations for shear. In the present case, the wearing surface thickness of 1.5 in. includes the future wearing surface thickness, and the actual wearing surface thickness is not specified. Therefore, the wearing surface thickness is not included. This will not have any effect on the design.)

s_{max} = 0.75(62) = 46.5 in. > 24 in.

Therefore, maximum spacing of web reinforcement is s_{max} = 24 in.

Spacing provided, s = 22 in. < s_{max} = 24 in. (O.K.)

Therefore, use #4 double-legged stirrups at 22 in. center-to-center spacing at the critical section.

The calculations presented above provide the shear design at the critical section. Additional sections along the span can be designed for shear using the same approach.

A.1.12 HORIZONTAL SHEAR DESIGN

[STD Art. 9.20.4]

Composite flexural members are required to be designed to fully transfer the horizontal shear forces at the contact surfaces of interconnected elements.

The critical section for horizontal shear is at a distance of $h_c/2$ (where h_c is the depth of composite section = 62 in.) from the face of the support. However, as the dimensions of the support are unknown in the present case, the critical section for shear is conservatively calculated from the centerline of the bearing support.

Distance of critical section for horizontal shear from bearing centerline:

$$h_c/2 = \frac{62\text{ in.}}{2(12\text{ in./ft.})} = 2.583\text{ ft.}$$

The cross sections subject to horizontal shear shall be designed such that:

$$V_u \leq \phi V_{nh} \quad \text{[STD Eq. 9-31a]}$$

where:

V_u = Factored shear force at the section considered (calculated using load combination causing maximum shear force)
= 247.8 kips

V_{nh} = Nominal horizontal shear strength of the section, kips

ϕ = Strength reduction factor for shear = 0.90 for prestressed concrete members [STD Art. 9.14]

$$\text{Required } V_{nh} \geq \frac{V_u}{\phi} = \frac{247.8}{0.9} = 275.33\text{ kips}$$

The nominal horizontal shear strength of the section, V_{nh}, is determined based on one of the following applicable cases.

Case (a): When the contact surface is clean, free of laitance, and intentionally roughened, the allowable shear force in pounds is given as:

$$V_{nh} = 80\ b_v\ d$$ [STD Art. 9.20.4.3]

where:

b_v = Width of cross section at the contact surface being investigated for horizontal shear = 20 in. (top flange width of the precast girder)

d = Distance from the extreme compressive fiber to centroid of pretensioned reinforcement
$= h_c - (y_b - e_x)$ [STD Art. 9.20.2.2]

h_c = Depth of the composite section = 62 in.

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder = 24.75 in.

e_x = Eccentricity of prestressing strands at the critical section = 11.51 in.

d = 62 – (24.75 – 11.51) = 48.76 in.

$$V_{nh} = \frac{80(20)(48.76)}{1000}$$
= 78.02 kips < Required V_{nh} = 275.33 kips (N.G.)

Case (b): When minimum ties are provided and contact surface is clean, free of laitance, but not intentionally roughened, the allowable shear force in pounds is given as:

$$V_{nh} = 80\ b_v\ d$$ [STD Art. 9.20.4.3]

$$V_{nh} = \frac{80(20)(48.76)}{1000}$$
= 78.02 kips < Required V_{nh} = 275.33 kips (N.G.)

Case (c): When minimum ties are provided and contact surface is clean, free of laitance, and intentionally roughened to a full amplitude of approximately 0.25 in., the allowable shear force in pounds is given as:

$$V_{nh} = 350\, b_v\, d \qquad \text{[STD Art. 9.20.4.3]}$$

$$V_{nh} = \frac{350(20)(48.76)}{1000}$$

$$= 341.32 \text{ kips} > \text{Required } V_{nh} = 275.33 \text{ kips} \quad \text{(O.K.)}$$

Design of ties for horizontal shear [STD Art. 9.20.4.5]

Minimum area of ties between the interconnected elements

$$A_{vh} = \frac{50\, b_v s}{f_y}$$

where:

A_{vh} = Area of horizontal shear reinforcement, in.^{2}

s = Center-to-center spacing of the web reinforcement taken as 22 in. This is the center-to-center spacing of web reinforcement, which can be extended into the slab.

f_y = Yield strength of web reinforcement = 60 ksi

$$A_{vh} = \frac{50(20)(22)}{60{,}000} = 0.37 \text{ in.}^2 \approx 0.40 \text{ in.}^2 \text{ (provided web reinf. area)}$$

Maximum spacing of ties shall be:

s = Lesser of 4(least web width) and 24 in. [STD Art. 9.20.4.5.a]

Least web width = 8 in.

s = 4(8 in.) = 32 in. > 24 in. Therefore, use maximum s = 24 in.

Maximum spacing of ties = 24 in., which is greater than the provided spacing of ties = 22 in. (O.K.)

Therefore, the provided web reinforcement shall be extended into the CIP slab to satisfy the horizontal shear requirements.

A.1.13 PRETENSIONED ANCHORAGE ZONE

A.1.13.1 Minimum Vertical Reinforcement

[STD Art. 9.22]

In a pretensioned girder, vertical stirrups acting at a unit stress of 20,000 psi to resist at least 4 percent of the total pretensioning force must be placed within the distance of $d/4$ of the girder end.

[STD Art. 9.22.1]

Minimum vertical stirrups at each end of the girder:

P_s = Prestressing force before initial losses have occurred, kips
= (number of strands)(area of strand)(initial prestress)

Initial prestress, $f_{si} = 0.75\ f'_s$ [STD Art. 9.15.1]

where f'_s = Ultimate strength of prestressing strands = 270 ksi

$f_{si} = 0.75(270) = 202.5$ ksi

$P_s = 50(0.153)(202.5) = 1549.13$ kips

Force to be resisted, F_s = 4 percent of $P_s = 0.04(1549.13)$
$= 61.97$ kips

Required area of stirrups to resist F_s

$$A_v = \frac{F_s}{\text{Unit stress in stirrups}}$$

Unit stress in stirrups = 20 ksi

$$A_v = \frac{61.97}{20} = 3.1 \text{ in.}^2$$

Distance available for placing the required area of stirrups = $d/4$

where d is the distance from the extreme compressive fiber to centroid of pretensioned reinforcement = 48.76 in.

$$\frac{d}{4} = \frac{48.76}{4} = 12.19 \text{ in.}$$

Using six pairs of #5 bars at 2 in. center-to-center spacing (within 12 in. from girder end) at each end of the girder:

A_v = 2(area of each bar)(number of bars)
$= 2(0.31)(6) = 3.72 \text{ in.}^2 > 3.1 \text{ in.}^2$ (O.K.)

Therefore, provide six pairs of #5 bars at 2 in. center-to-center spacing at each girder end.

A.1.13.2 Confinement Reinforcement

STD Art. 9.22.2 specifies that nominal reinforcement must be placed to enclose the prestressing steel in the bottom flange for a distance d from the end of the girder. [STD Art. 9.22.2]

where

d = Distance from the extreme compressive fiber to centroid of pretensioned reinforcement
$= h_c - (y_b - e_x) = 62 - (24.75 - 11.51) = 48.76$ in.

A.1.14 CAMBER AND DEFLECTIONS

A.1.14.1 Maximum Camber

The Standard Specifications do not provide guidelines for the determination camber of prestressed concrete members. The Hyperbolic Functions Method (Furr et al. 1968, Sinno 1968, Furr and Sinno 1970) for the calculation of maximum camber is used by TxDOT's prestressed concrete bridge design software, PSTRS14 (TxDOT 2004). The following steps illustrate the Hyperbolic Functions Method for the estimation of maximum camber.

Step 1: The total prestressing force after initial prestress loss due to elastic shortening has occurred.

$$P = \frac{P_i}{\left(1 + pn + \frac{e_c^2 A_s n}{I}\right)} + \frac{M_D\, e_c\, A_s\, n}{I\left(1 + pn + \frac{e_c^2 A_s n}{I}\right)}$$

where:

P_i = Anchor force in prestressing steel
= (number of strands)(area of strand)(f_{si})

f_{si} = Initial prestress before release = 0.75 f_s'
[STD Art. 9.15.1]

f_s' = Ultimate strength of prestressing strands = 270 ksi

f_{si} = 0.75(270) = 202.5 ksi

P_i = 50(0.153)(202.5) = 1549.13 kips

I = Moment of inertia of the non-composite precast girder
= 260,403 in.^{4}

e_c = Eccentricity of prestressing strands at the midspan
= 19.47 in.

M_D = Moment due to self-weight of the girder at midspan
= 1209.98 k-ft.

A_s = Area of prestressing steel
= (number of strands)(area of strand)
= 50(0.153) = 7.65 in.^{2}

p = A_s/A

A = Area of girder cross section = 788.4 in.^{2}

$$p = \frac{7.65}{788.4} = 0.0097$$

n = Modular ratio between prestressing steel and the girder concrete at release = E_s/E_{ci}

E_{ci} = Modulus of elasticity of the girder concrete at release
= $33(w_c)^{3/2}\sqrt{f'_{ci}}$ [STD Eq. 9-8]

w_c = Unit weight of concrete = 150 pcf

f'_{ci} = Compressive strength of precast girder concrete at release = 5455 psi

$$E_{ci} = [33(150)^{3/2}\sqrt{5455}\,]\left(\frac{1}{1000}\right) = 4477.63 \text{ ksi}$$

E_s = Modulus of elasticity of prestressing strands
= 28,000 ksi

n = 28,000/4477.63 = 6.25

$$\left(1 + pn + \frac{e_c^2 A_s n}{I}\right) = 1 + (0.0097)(6.25) + \frac{(19.47^2)(7.65)(6.25)}{260{,}403}$$
$$= 1.130$$

$$P = \frac{1549.13}{1.130} + \frac{(1209.98)(12 \text{ in./ft.})(19.47)(7.65)(6.25)}{260{,}403(1.130)}$$

$$= 1370.91 + 45.93 = 1416.84 \text{ kips}$$

Initial prestress loss is defined as:

$$PL_i = \frac{P_i - P}{P} = \frac{1549.13 - 1416.84}{1549.13} = 0.0854 = 8.54\%$$

Note that the values obtained for initial prestress loss and effective initial prestress force using this methodology are comparable with the values obtained in Section A.1.7.3.5. The effective prestressing force after initial losses was found to be 1410.58 kips (comparable to 1416.84 kips), and the initial prestress loss was determined as 8.94 percent (comparable to 8.54 percent).

The stress in the concrete at the level of the centroid of the prestressing steel immediately after transfer is determined as follows.

$$f_{ci}^s = P\left(\frac{1}{A} + \frac{e_c^2}{I}\right) - f_c^s$$

where:

f_c^s = Concrete stress at the level of centroid of prestressing steel due to dead loads, ksi

$$= \frac{M_D\, e_c}{I} = \frac{(1209.98)(12 \text{ in./ft.})(19.47)}{260{,}403} = 1.0856 \text{ ksi}$$

$$f_{ci}^s = 1416.84\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403}\right) - 1.0856 = 2.774 \text{ ksi}$$

The ultimate time dependent prestress loss is a function of the ultimate creep and shrinkage strains. As the creep strains vary with the concrete stress, the following steps are used to evaluate the concrete stresses and adjust the strains to arrive at the ultimate prestress loss. It is assumed that the creep strain is proportional to the concrete stress, and the shrinkage stress is independent of concrete stress.

Step 2: Initial estimate of total strain at steel level assuming constant sustained stress immediately after transfer

$$\varepsilon_{c1}^s = \varepsilon_{cr}^{\infty} f_{ci}^s + \varepsilon_{sh}^{\infty}$$

where:

$\varepsilon_{cr}^{\infty}$ = Ultimate unit creep strain = 0.00034 in./in. [This value is prescribed by Furr and Sinno (1970).]

$\varepsilon_{sh}^{\infty}$ = Ultimate unit shrinkage strain = 0.000175 in./in. [This value is prescribed by Furr and Sinno (1970).]

ε_{c1}^{s} = 0.00034(2.774) + 0.000175 = 0.001118 in./in.

Step 3: The total strain obtained in Step 2 is adjusted by subtracting the elastic strain rebound as follows:

$$\varepsilon_{c2}^{s} = \varepsilon_{c1}^{s} - \varepsilon_{c1}^{s} E_s \frac{A_s}{E_{ci}}\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\varepsilon_{c2}^{s} = 0.001118 - (0.001118)(28{,}000)\frac{7.65}{4477.63}\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403}\right)$$

= 0.000972 in./in.

Step 4: The change in concrete stress at the level of centroid of prestressing steel is computed as follows:

$$\Delta f_c^{s} = \varepsilon_{c2}^{s} E_s A_s\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\Delta f_c^{s} = (0.000972)(28{,}000)(7.65)\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403}\right) = 0.567 \text{ ksi}$$

Step 5: The total strain computed in Step 2 needs to be corrected for the change in the concrete stress due to creep and shrinkage strains.

$$\varepsilon_{c4}^{s} = \varepsilon_{cr}^{\infty}\left(f_{ci}^{s} - \frac{\Delta f_c^{s}}{2}\right) + \varepsilon_{sh}^{\infty}$$

$$\varepsilon_{c4}^{s} = 0.00034\left(2.774 - \frac{0.567}{2}\right) + 0.000175 = 0.00102 \text{ in./in.}$$

Step 6: The total strain obtained in Step 5 is adjusted by subtracting the elastic strain rebound as follows:

$$\varepsilon_{c5}^{s} = \varepsilon_{c4}^{s} - \varepsilon_{c4}^{s} E_s \frac{A_s}{E_{ci}}\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\varepsilon_{c5}^{s} = 0.00102 - (0.00102)(28{,}000)\ \frac{7.65}{4477.63}\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403}\right)$$

= 0.000887 in./in.

Furr and Sinno (1970) recommend stopping the updating of stresses and the adjustment process after Step 6. However, as the difference between the strains obtained in Steps 3 and 6 is not negligible, this process is carried on until the total strain value converges.

Step 7: The change in concrete stress at the level of centroid of prestressing steel is computed as follows:

$$\Delta f_{c1}^{s} = \varepsilon_{c5}^{s} E_s A_s \left(\frac{1}{A} + \frac{e_c^2}{I} \right)$$

$$\Delta f_{c1}^{s} = (0.000887)(28{,}000)(7.65)\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403} \right) = 0.5176 \text{ ksi}$$

Step 8: The total strain computed in Step 5 needs to be corrected for the change in the concrete stress due to creep and shrinkage strains.

$$\varepsilon_{c6}^{s} = \varepsilon_{cr}^{\infty} \left(f_{ci}^{s} - \frac{\Delta f_{c1}^{s}}{2} \right) + \varepsilon_{sh}^{\infty}$$

$$\varepsilon_{c6}^{s} = 0.00034\left(2.774 - \frac{0.5176}{2} \right) + 0.000175 = 0.00103 \text{ in./in.}$$

Step 9: The total strain obtained in Step 8 is adjusted by subtracting the elastic strain rebound as follows

$$\varepsilon_{c7}^{s} = \varepsilon_{c6}^{s} - \varepsilon_{c6}^{s} E_s \frac{A_s}{E_{ci}} \left(\frac{1}{A} + \frac{e_c^2}{I} \right)$$

$$\varepsilon_{c7}^{s} = 0.00103 - (0.00103)(28{,}000)\ \frac{7.65}{4477.63}\left(\frac{1}{788.4} + \frac{19.47^2}{260{,}403} \right)$$

$$= 0.000896 \text{ in./in}$$

The strains have sufficiently converged, and no more adjustments are needed.

Step 10: Computation of final prestress loss

Time dependent loss in prestress due to creep and shrinkage strains is given as:

$$PL^{\infty} = \frac{\varepsilon_{c7}^{s} E_s A_s}{P_i} = \frac{0.000896(28{,}000)(7.65)}{1549.13} = 0.124 = 12.4\%$$

Total final prestress loss is the sum of initial prestress loss and the time dependent prestress loss expressed as follows:

$$PL = PL_i + PL^{\infty}$$

where:

PL = Total final prestress loss percent

PL_i = Initial prestress loss percent = 8.54 percent

PL^{∞}= Time dependent prestress loss percent = 12.4 percent

PL = 8.54 + 12.4 = 20.94 percent

(This value of final prestress loss is less than the one estimated in Section A.1.7.3.6. where the final prestress loss was estimated to be 25.24 percent.)

Step 11: The initial deflection of the girder under self-weight is calculated using the elastic analysis as follows:

$$C_{DL} = \frac{5\, w\, L^4}{384 E_{ci} I}$$

where:

C_{DL} = Initial deflection of the girder under self-weight, ft.

w = Self-weight of the girder = 0.821 kips/ft.

L = Total girder length = 109.67 ft.

E_{ci} = Modulus of elasticity of the girder concrete at release = 4477.63 ksi = 644,778.72 k/ft.^{2}

I = Moment of inertia of the non-composite precast girder = 260,403 in.^{4} = 12.558 ft.^{4}

$$C_{DL} = \frac{5(0.821)(109.67^4)}{384(644{,}778.72)(12.558)} = 0.191 \text{ ft.} = 2.29 \text{ in.}$$

Step 12: Initial camber due to prestress is calculated using the moment area method. The following expression is obtained from the *M/EI* diagram to compute the camber resulting from the initial prestress.

$$C_{pi} = \frac{M_{pi}}{E_{ci} I}$$

where:

M_{pi} = $[0.5(P)\,(e_e)\,(0.5L)^2 + 0.5(P)\,(e_c - e_e)\,(0.67)\,(HD)^2 + 0.5P\,(e_c - e_e)\,(HD_{dis})\,(0.5L + HD)]/(E_{ci})(I)$

P = Total prestressing force after initial prestress loss due to elastic shortening has occurred = 1416.84 kips

HD = Hold-down distance from girder end = 49.404 ft. = 592.85 in. (see Figure A.1.7.3)

HD_{dis} = Hold-down distance from the center of the girder span = 0.5(109.67) – 49.404 = 5.431 ft. = 65.17 in.

e_e = Eccentricity of prestressing strands at girder end = 11.07 in.

e_c = Eccentricity of prestressing strands at midspan = 19.47 in.

L = Overall girder length = 109.67 ft. = 1316.04 in.

$$M_{pi} = \{0.5(1416.84)(11.07)\,[0.5(1316.04)]^2 + 0.5(1416.84)(19.47 - 11.07)(0.67)(592.85)^2 + 0.5(1416.84)(19.47 - 11.07)(65.17)[0.5(1316.04) + 592.85]\}$$

$$M_{pi} = 3.396 \times 10^9 + 1.401 \times 10^9 + 0.485 \times 10^9 = 5.282 \times 10^9$$

$$C_{pi} = \frac{5.282 \times 10^9}{(4477.63)(260{,}403)} = 4.53 \text{ in.} = 0.378 \text{ ft.}$$

Step 13: The initial camber, C_I, is the difference between the upward camber due to initial prestressing and the downward deflection due to self-weight of the girder.

$$C_i = C_{pi} - C_{DL} = 4.53 - 2.29 = 2.24 \text{ in.} = 0.187 \text{ ft.}$$

Step 14: The ultimate time-dependent camber is evaluated using the following expression.

$$\text{Ultimate camber } C_t = C_i\,(1 - PL^{\infty})\,\frac{\varepsilon_{cr}^{\infty}\left(f_{ci}^{s} - \dfrac{\Delta f_{c1}^{s}}{2}\right) + \varepsilon_e^{s}}{\varepsilon_e^{s}}$$

where:

$$\varepsilon_e^{s} = \frac{f_{ci}^{s}}{E_{ci}} = \frac{2.774}{4477.63} = 0.000619 \text{ in./in.}$$

$$C_t = 2.24(1 - 0.124)\,\frac{0.00034\left(2.774 - \dfrac{0.5176}{2}\right) + 0.000619}{0.000619}$$

$C_t = 4.673$ in. = 0.389 ft. ↑

A.1.14.2 Deflection Due to Slab Weight

The deflection due to the slab weight is calculated using an elastic analysis as follows.

Deflection of the girder at midspan

$$\Delta_{slab1} = \frac{5\,w_s\,L^4}{384\,E_c\,I}$$

where:

w_s = Weight of the slab = 0.80 kips/ft.

E_c = Modulus of elasticity of girder concrete at service
$= 33(w_c)^{3/2}\sqrt{f_c'}$
$= 33(150)^{1.5}\sqrt{5582.5}\left(\dfrac{1}{1000}\right) = 4529.66$ ksi

I = Moment of inertia of the non-composite girder section
= 260,403 in.^{4}

L = Design span length of girder (center-to-center bearing)
= 108.583 ft.

$$\Delta_{slab1} = \frac{5\left(\dfrac{0.80}{12\text{ in./ft.}}\right)\left[(108.583)(12\text{ in./ft.})\right]^4}{384(4529.66)(260{,}403)}$$

= 2.12 in. = 0.177 ft. ↓

Deflection at quarter span due to slab weight

$$\Delta_{slab2} = \frac{57\, w_s\, L^4}{6144\, E_c\, I}$$

$$\Delta_{slab2} = \frac{57\left(\dfrac{0.80}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{6144(4529.66)(260,403)}$$

= 1.511 in. = 0.126 ft. ↓

A.1.14.3 Deflections due to Superimposed Dead Loads

Deflection due to barrier weight at midspan

$$\Delta_{barr1} = \frac{5\, w_{barr}\, L^4}{384\, E_c\, I_c}$$

where:

w_{barr} = Weight of the barrier = 0.109 kips/ft.

I_c = Moment of inertia of composite section = 657,658.4 in^4

$$\Delta_{barr1} = \frac{5\left(\dfrac{0.109}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{384(4529.66)(657{,}658.4)}$$

= 0.114 in. = 0.0095 ft. ↓

Deflection at quarter span due to barrier weight

$$\Delta_{barr2} = \frac{57\, w_{barr}\, L^4}{6144\, E_c\, I}$$

$$\Delta_{barr2} = \frac{57\left(\dfrac{0.109}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{6144(4529.66)(657{,}658.4)}$$

= 0.0815 in. = 0.0068 ft. ↓

Deflection due to wearing surface weight at midspan

$$\Delta_{ws1} = \frac{5\, w_{ws}\, L^4}{384\, E_c\, I_c}$$

where:

w_{ws} = Weight of wearing surface = 0.128 kips/ft.

$$\Delta_{ws1} = \frac{5\left(\dfrac{0.128}{12 \text{ in./ft.}}\right)[(108.583)(12 \text{ in./ft.})]^4}{384(4529.66)(657{,}658.4)}$$

$= 0.134$ in. $= 0.011$ ft. ↓

Deflection at quarter span due to wearing surface

$$\Delta_{ws2} = \frac{57\, w_{ws}\, L^4}{6144\, E_c\, I}$$

$$\Delta_{ws2} = \frac{57\left(\dfrac{0.128}{12 \text{ in./ft.}}\right)[(108.583)(12 \text{ in./ft.})]^4}{6144(4529.66)(657{,}658.4)}$$

$= 0.096$ in. $= 0.008$ ft. ↓

A.1.14.4 Total Deflection due to Dead Loads

The total deflection at midspan due to slab weight and superimposed loads is:

$\Delta_{T1} = \Delta_{slab1} + \Delta_{barr1} + \Delta_{ws1}$

$= 0.177 + 0.0095 + 0.011 = 0.1975$ ft. ↓

The total deflection at quarter span due to slab weight and superimposed loads is:

$\Delta_{T2} = \Delta_{slab2} + \Delta_{barr2} + \Delta_{ws2}$

$= 0.126 + 0.0068 + 0.008 = 0.1408$ ft. ↓

The deflections due to live loads are not calculated in this example as they are not a design factor for TxDOT bridges.

A.1.15 COMPARISON OF RESULTS FROM DETAILED DESIGN AND PSTRS14

The prestressed concrete bridge girder design program, PSTRS14 (TxDOT 2004), is used by TxDOT for bridge design. The PSTRS14 program was run with same parameters as used in this detailed design, and the results of the detailed example and PSTRS14 program are compared in Table A.1.15.1.

Table A.1.15.1. Comparison of the Results from PSTRS14 Program with Detailed Design Example.

Parameter		PSTRS14 Result	Detailed Design Result	Percent Difference
Live Load Distribution Factor		0.727	0.727	0.00
Initial Prestress Loss		8.93%	8.94%	-0.11
Final Prestress Loss		25.23%	25.24%	-0.04
Girder Stresses at Transfer				
At Girder End	Top Fiber	35 psi	35 psi	0.00
	Bottom Fiber	3274 psi	3273 psi	0.03
At Transfer Length Section	Top Fiber	Not Calculated	104 psi	-
	Bottom Fiber	Not calculated	3215 psi	-
At Hold-Down	Top Fiber	319 psi	351 psi	-10.03
	Bottom Fiber	3034 psi	3005 psi	1.00
At Midspan	Top Fiber	335 psi	368 psi	-9.85
	Bottom Fiber	3020 psi	2991 psi	0.96
Girder Stresses at Service				
At Girder End	Top Fiber	29 psi	Not calculated	-
	Bottom Fiber	2688 psi	Not calculated	-
At Midspan	Top Fiber	2563 psi	2562 psi	0.04
	Bottom Fiber	-414 psi	-412 psi	0.48
Slab Top Fiber Stress		Not calculated	658 psi	-
Required Concrete Strength at Transfer		5457 psi	5455 psi	0.04
Required Concrete Strength at Service		5585 psi	5582.5 psi	0.04
Total Number of Strands		50	50	0.00
Number of Harped Strands		10	10	0.00
Ultimate Flexural Moment Required		6771 k-ft.	6769.37 k-ft.	0.02
Ultimate Moment Provided		8805 k-ft	8936.56 k-ft.	-1.50
Shear Stirrup Spacing at the Critical Section: Double-Legged #4 Stirrups		21.4 in.	22 in.	-2.80
Maximum Camber		0.306 ft.	0.389 ft.	-27.12
Deflections				
Slab Weight	Midspan	-0.1601 ft.	0.1770 ft.	-11.00
	Quarter Span	-0.1141 ft.	0.1260 ft.	-10.00
Barrier Weight	Midspan	-0.0096 ft.	0.0095 ft.	1.04
	Quarter Span	-0.0069 ft.	0.0068 ft.	1.45
Wearing Surface Weight	Midspan	-0.0082 ft.	0.0110 ft.	-34.10
	Quarter Span	-0.0058 ft.	0.0080 ft.	-37.60

Except for a few differences, the results from the detailed design are in good agreement with the PSTRS14 (TxDOT 2004) results. The causes for the differences in the results are discussed as follows.

1. *Girder Stresses at Transfer*: The detailed design example uses the overall girder length of 109 ft.-8 in. for evaluating the stresses at transfer at the midspan section and hold-down point locations. The PSTRS14 uses the design span length of 108 ft.-7 in. for this calculation. This causes a difference in the stresses at transfer at hold-down point locations and midspan. The use of the full girder length for stress calculations at transfer may better reflect the end conditions for this load stage.

2. *Maximum Camber*: The difference in the maximum camber results from detailed design and PSTRS14 (TxDOT 2001) is occurring due to two reasons.

    a. The detailed design example uses the overall girder length for the calculation of initial camber; whereas, the PSTRS14 program uses the design span length.

    b. The updated composite section properties, based on the modular ratio between slab and actual girder concrete strengths are used for the camber calculations in the detailed design. However, the PSTRS14 program does not update the composite section properties.

3. *Deflections*: The difference in the deflections is due to the use of updated section properties and elastic modulus of concrete in the detailed design, based on the optimized concrete strength. The PSTRS14 program does not update the composite section properties and uses the elastic modulus of concrete based on the initial input.

A.1.16 REFERENCES

AASHTO (2002), *Standard Specifications for Highway Bridges*, 17th Ed., American Association of Highway and Transportation Officials (AASHTO), Inc., Washington, D.C.

Furr, H.L., R. Sinno and L.L. Ingram (1968). "Prestress Loss and Creep Camber in a Highway Bridge with Reinforced Concrete Slab on Prestressed Concrete Beams," *Texas Transportation Institute Report*, Texas A&M University, College Station.

Furr, H.L. and R. Sinno (1970) "Hyperbolic Functions for Prestress Loss and Camber," *Journal of the Structural Division*, Vol. 96, No. 4, pp. 803-821.

PCI (2003). "Precast Prestressed Concrete Bridge Design Manual," 2nd Ed., Precast/Prestressed Concrete Institute, Chicago, Illinois.

Sinno, R. (1968). "The Time-Dependent Deflections of Prestressed Concrete Bridge Beams," *Ph.D. Dissertation*, Texas A&M University, College Station.

TxDOT (2001). "TxDOT Bridge Design Manual," Bridge Division, Texas Department of Transportation.

TxDOT (2004). "Prestressed Concrete Beam Design/Analysis Program," User Guide, Version 4.00, Bridge Division, Texas Department of Transportation.

Appendix A.2

Design Example for Interior AASHTO Type IV Girder using AASHTO LRFD Specifications

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A.2 Design Example for Interior AASHTO Type IV Girder using AASHTO LRFD Specifications

A.2.1 INTRODUCTION

The following detailed example shows sample calculations for the design of a typical interior AASHTO Type IV prestressed concrete girder supporting a single span bridge. The design is based on the *AASHTO LRFD Bridge Design Specifications*, 3rd Edition (AASHTO 2004). The recommendations provided by the TxDOT Bridge Design Manual (TxDOT 2001) are considered in the design. The number of strands and concrete strength at release and at service are optimized using the TxDOT methodology.

A.2.2 DESIGN PARAMETERS

The bridge considered for this design example has a span length of 110 ft. (center-to-center (c/c) pier distance), a total width of 46 ft. and total roadway width of 44 ft. The bridge superstructure consists of six AASHTO Type IV girders spaced 8 ft. center-to-center, designed to act compositely with an 8 in. thick cast-in-place (CIP) concrete deck. The wearing surface thickness is 1.5 in., which includes the thickness of any future wearing surface. T501 type rails are considered in the design. HL-93 is the design live load. A relative humidity (RH) of 60 percent is considered in the design, and the skew angle is 0 degrees. The bridge cross section is shown in Figure A.2.2.1.

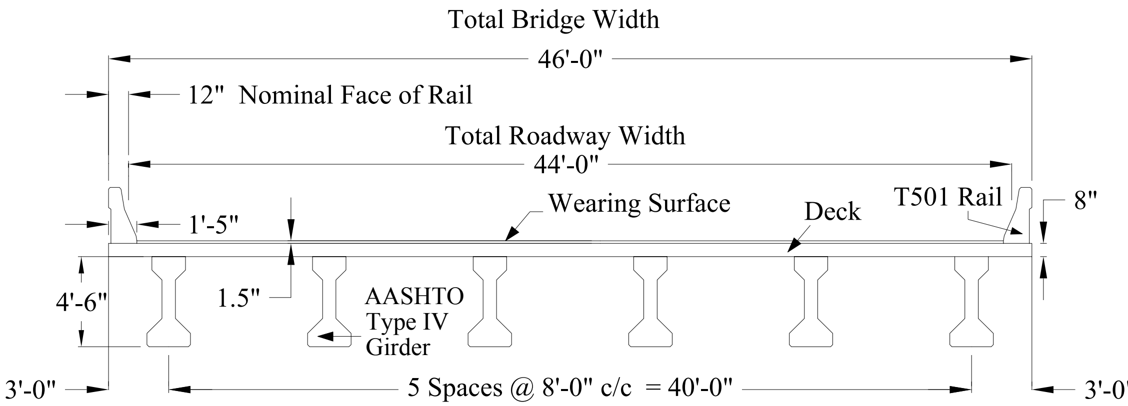

Figure A.2.2.1. Bridge Cross Section Details.

The following calculations for design span length and the overall girder length are based on Figure A.2.2.2.

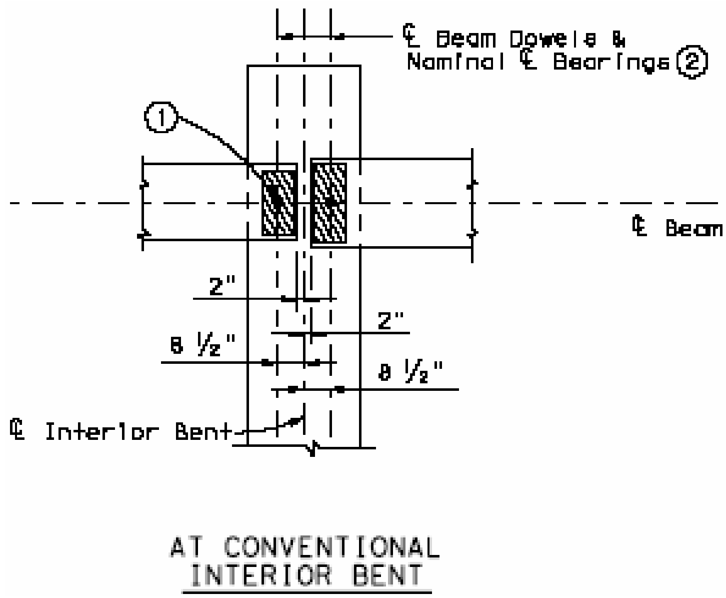

Figure A.2.2.2. Girder End Details (TxDOT Standard Drawing 2001).

Span Length (c/c piers) = 110 ft.-0 in.

From Figure A.2.2.2

Overall girder length = 110'-0" – 2(2") = 109'-8" = 109.67 ft.

Design Span = 110'-0" – 2(8.5") = 108'-7" = 108.583 ft. (c/c of bearing)

A.2.3 MATERIAL PROPERTIES

Cast-in-place slab:

Thickness, t_s = 8.0 in.

Concrete strength at 28 days, f'_c = 4000 psi

Thickness of asphalt wearing surface (including any future wearing surface), t_w = 1.5 in.

Unit weight of concrete, w_c = 150 pcf

Precast girders: AASHTO Type IV

Concrete strength at release, f'_{ci} = 4000 psi (This value is taken as an initial estimate and will be finalized based on optimum design.)

Concrete strength at 28 days, f_c' = 5000 psi (This value is taken as initial estimate and will be finalized based on optimum design.)

Concrete unit weight, w_c = 150 pcf

Pretensioning strands: 0.5 in. diameter, seven wire low relaxation

Area of one strand = 0.153 in.^{2}

Ultimate stress, f_{pu} = 270,000 psi

Yield strength, $f_{py} = 0.9 f_{pu}$ = 243,000 psi
[LRFD Table 5.4.4.1-1]

Stress limits for prestressing strands: [LRFD Table 5.9.3-1]

Before transfer, $f_{pi} \leq 0.75 f_{pu}$ = 202,500 psi

At service limit state (after all losses)
$f_{pe} \leq 0.80 f_{py}$ = 194,400 psi

Modulus of Elasticity, E_p = 28,500 ksi [LRFD Art. 5.4.4.2]

Nonprestressed reinforcement:

Yield strength, f_y = 60,000 psi

Modulus of Elasticity, E_s = 29,000 ksi [LRFD Art. 5.4.3.2]

Unit weight of asphalt wearing surface = 140 pcf
[TxDOT recommendation]

T501 type barrier weight = 326 plf /side

A.2.4 CROSS SECTION PROPERTIES FOR A TYPICAL INTERIOR GIRDER

A.2.4.1 Non-Composite Section

The section properties of an AASHTO Type IV girder as described in the TxDOT Bridge Design Manual (TxDOT 2001) are provided in Table A.2.4.1. The section geometry and strand pattern are shown in Figure A.2.4.1.

Table A.2.4.1. Section Properties of AASHTO Type IV Girder [Adapted from TxDOT Bridge Design Manual (TxDOT 2001)].

y_t	y_b	Area	I	Wt./lf
in.	in.	in.^{2}	in.^{4}	lbs
29.25	24.75	788.4	260,403	821

where:

I = Moment of inertia about the centroid of the non-composite precast girder = 260,403 in.^{4}

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder = 24.75 in.

y_t = Distance from centroid to the extreme top fiber of the non-composite precast girder = 29.25 in.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder, in.^{3}
= I/y_b = 260,403/24.75 = 10,521.33 in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder, in.^{3}
= I/y_t = 260,403/29.25 = 8902.67 in.^{3}

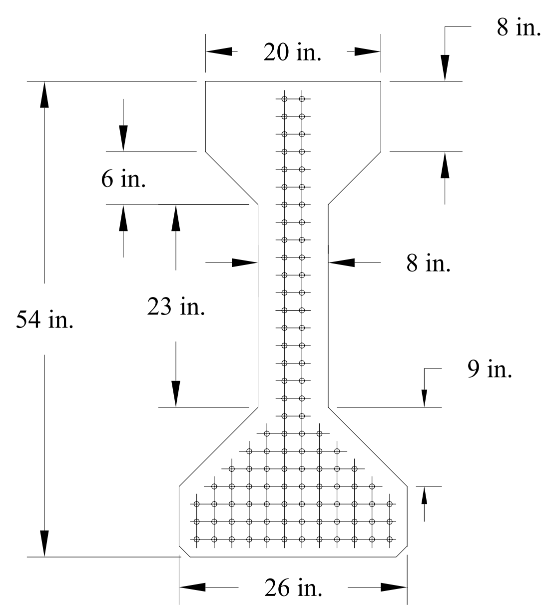

Figure A.2.4.1. Section Geometry and Strand Pattern for AASHTO Type IV Girder (Adapted from TxDOT Bridge Design Manual [TxDOT 2001]).

A.2.4.2
Composite Section

A.2.4.2.1
Effective Flange Width

[LRFD Art. 4.6.2.6.1]

The effective flange width is lesser of:

0.25 span length of girder: $\frac{108.583(12 \text{ in./ft.})}{4} = 325.75$ in.

12 × (effective slab thickness) + (greater of web thickness or one-half girder top flange width): 12(8) + 0.5(20) = 106 in.
(0.5 × (girder top flange width) = 10 in. > web thickness = 8 in.)

Average spacing of adjacent girders: (8 ft.)(12 in./ft.) = 96 in. (controls)

Effective flange width = 96 in.

A.2.4.2.2
Modular Ratio between Slab and Girder Concrete

Following the TxDOT Bridge Design Manual (TxDOT 2001) recommendation (pg. 7-85), the modular ratio between the slab and girder concrete is taken as 1. This assumption is used for service load design calculations. For the flexural strength limit design, shear design, and deflection calculations, the actual modular ratio based on optimized concrete strengths is used. The composite section is shown in Figure A.2.4.2 and the composite section properties are presented in Table A.2.4.2.

$$n = \left(\frac{E_c \text{ for slab}}{E_c \text{ for girder}}\right) = 1$$

where n is the modular ratio between slab and girder concrete, and E_c is the elastic modulus of concrete.

A.2.4.2.3
Transformed Section Properties

Transformed flange width = n × (effective flange width)
= (1)(96) = 96 in.

Transformed Flange Area = n × (effective flange width)(t_s)
= (1)(96)(8) = 768 in.^{2}

Table A.2.4.2. Properties of Composite Section.

	Transformed Area A (in.^{2})	y_b in.	$A\,y_b$	$A(y_{bc} - y_b)^2$	I in.^{4}	$I + A(y_{bc} - y_b)^2$ in.^{4}
Girder	788.4	24.75	19,512.9	212,231.53	260,403.0	472,634.5
Slab	768.0	58.00	44,544.0	217,868.93	4096.0	221,964.9
$\sum$	1556.4		64,056.9			694,599.5

A_c = Total area of composite section = 1556.4 in.^{2}

h_c = Total height of composite section = 54 + 8 = 62 in.

I_c = Moment of inertia about the centroid of the composite section = 694,599.5 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder, in.
= 64,056.9/1556.4 = 41.157 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder, in.
= 54 - 41.157 = 12.843 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 - 41.157 = 20.843 in.

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder, in.^{3}
= I_c/y_{bc} = 694,599.5/41.157 = 16,876.83 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder, in.^{3}
= I_c/y_{tg} = 694,599.5/12.843 = 54,083.9 in.^{3}

S_{tc} = Section modulus of composite section referenced to the top fiber of the slab, in.^{3}
= I_c/y_{tc} = 694,599.5/20.843 = 33,325.31 in.^{3}

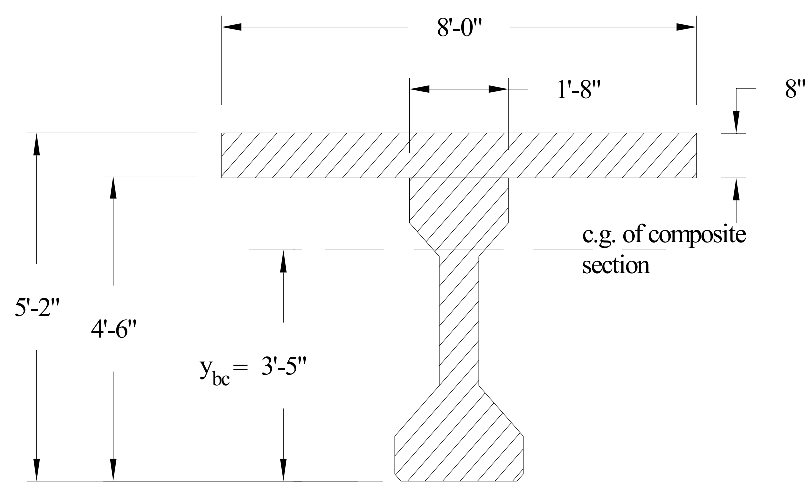

Figure A.2.4.2. Composite Section.

A.2.5 SHEAR FORCES AND BENDING MOMENTS

The self-weight of the girder and the weight of the slab act on the non-composite simple span structure, while the weight of the barriers, future wearing surface, live load, and dynamic load act on the composite simple span structure.

A.2.5.1 Shear Forces and Bending Moments due to Dead Loads

A.2.5.1.1 Dead Loads

[LRFD Art. 3.3.2]

Dead loads acting on the non-composite structure:

Self-weight of the girder = 0.821 kip/ft.
[TxDOT Bridge Design Manual (TxDOT 2001)]

Weight of cast-in-place deck on each interior girder

$$= (0.150 \text{ kcf})\left(\frac{8 \text{ in.}}{12 \text{ in./ft.}}\right)(8 \text{ ft.}) = 0.800 \text{ kips/ft.}$$

Total dead load on non-composite section
= 0.821 + 0.800 = 1.621 kips/ft.

A.2.5.1.2 Superimposed Dead Loads

The superimposed dead loads placed on the bridge, including loads from railing and wearing surface, can be distributed uniformly among all girders given the following conditions are met.
[LRFD Art. 4.6.2.2.1]

1. Width of deck is constant (O.K.)

2. Number of girders, N_b, is not less than four
   Number of girders in present case, $N_b = 6$ (O.K.)

3. Girders are parallel and have approximately the same stiffness (O.K.)

4. The roadway part of the overhang, $d_e \leq 3.0$ ft.
   where d_e is the distance from the exterior web of the exterior girder to the interior edge of the curb or traffic barrier, ft. (see Figure A.2.5.1)

   d_e = (overhang distance from the center of the exterior girder to the bridge end) – 0.5×(web width) – (width of barrier)
   = 3.0 – 0.33 - 1.0 = 1.67 ft. < 3.0 ft. (O.K.)

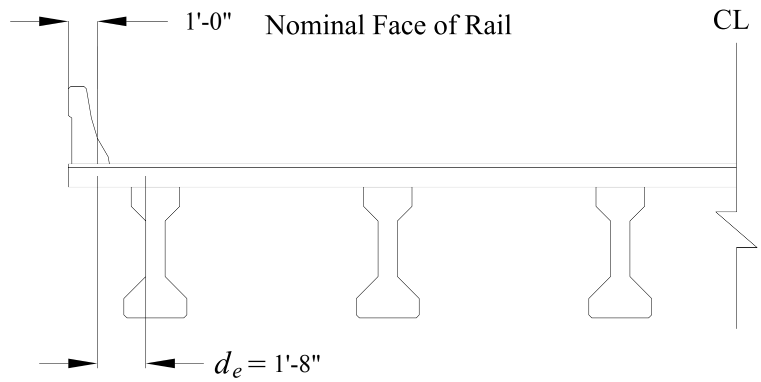

Figure A.2.5.1. Illustration of d_e Calculation.

5. Curvature in plan is less than 4° (curvature = 0°) (O.K.)

6. Cross section of the bridge is consistent with one of the cross sections given in LRFD Table 4.6.2.2.1-1
   Precast concrete I sections are specified as Type k (O.K.)

Because all of the above criteria are satisfied, the barrier and wearing surface loads are equally distributed among the six girders.

Weight of T501 rails or barriers on each girder

$$= 2\left(\frac{326 \text{ plf } /1000}{6 \text{ girders}}\right) = 0.109 \text{ kips/ft./girder}$$

Weight of 1.5 in. wearing surface

$= (0.140 \text{ kcf})\left(\frac{1.5 \text{ in.}}{12 \text{ in/ft.}}\right) = 0.0175$ kips/ft. This load is applied over the entire clear roadway width of 44 ft.-0 in.

Weight of wearing surface on each girder

$$= \frac{(0.0175 \text{ ksf})(44.0 \text{ ft.})}{6 \text{ girders}} = 0.128 \text{ kips/ft./girder}$$

Total superimposed dead load = 0.109 + 0.128 = 0.237 kips/ft.

A.2.5.1.3 Shear Forces and Bending Moments

Shear forces and bending moments for the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (hold-down point or harp point and critical section for shear) are provided in this section. The bending moment (*M*) and shear force (*V*) due to uniform dead loads and uniform superimposed dead loads at any section at a distance *x* from the centerline of bearing are calculated using the following formulas, where the uniform load is denoted as *w*.

$$M = 0.5w\,x\,(L - x)$$

$$V = w(0.5L - x)$$

The distance of the critical section for shear from the support is calculated using an iterative process illustrated in the shear design section. As an initial estimate, the distance of the critical section for shear from the centerline of bearing is taken as:
$(h_c/2) + 0.5(\text{bearing width}) = (62/2) + 0.5(7) = 34.5$ in. $= 2.875$ ft.

As per the recommendations of the TxDOT Bridge Design Manual (Chap. 7, Sec. 21), the distance of the hold-down (*HD*) point from the centerline of bearing is taken as the lesser of:

[0.5×(span length) – (span length/20)] or [0.5×(span length) – 5 ft.]

$$\frac{108.583}{2} - \frac{108.583}{20} = 48.862 \text{ ft. or } \frac{108.583}{2} - 5 = 49.29 \text{ ft.}$$

HD = 48.862 ft.

The shear forces and bending moments due to dead loads and superimposed loads are shown in Tables A.2.5.1 and A.2.5.2, respectively.

Table A.2.5.1. Shear Forces due to Dead and Superimposed Dead Loads.

Distance from Bearing Centerline *x*	Section *x*/*L*	Dead Loads		Superimposed Dead Loads			Total Dead Load
		Girder Weight	Slab Weight	Barrier Weight	Wearing Surface Weight	Total	
ft.		kips	kips	kips	kips	kips	kips
0.000	0.000	44.57	43.43	5.92	6.95	12.87	100.87
2.875	0.026	42.21	41.13	5.60	6.58	12.19	95.53
10.858	0.100	35.66	34.75	4.73	5.56	10.29	80.70
21.717	0.200	26.74	26.06	3.55	4.17	7.72	60.52
32.575	0.300	17.83	17.37	2.37	2.78	5.15	40.35
43.433	0.400	8.91	8.69	1.18	1.39	2.57	20.17
48.862	0.450 (*HD*)	4.46	4.34	0.59	0.69	1.29	10.09
54.292	0.500	0.00	0.00	0.00	0.00	0.00	0.00

Table A.2.5.2. Bending Moments due to Dead and Superimposed Dead Loads.

Distance from Bearing Centerline *x*	Section *x*/*L*	Dead Loads		Superimposed Dead Loads			Total Dead Load
		Girder Weight	Slab Weight	Barrier Weight	Wearing Surface Weight	Total	
ft.		k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
0.000	0.000	0.00	0.00	0.00	0.00	0.00	0.00
2.875	0.026	124.76	121.56	16.56	19.45	36.01	282.33
10.858	0.100	435.59	424.45	57.83	67.91	125.74	985.78
21.717	0.200	774.38	754.58	102.81	120.73	223.54	1752.51
32.575	0.300	1016.38	990.38	134.94	158.46	293.40	2300.16
43.433	0.400	1161.58	1131.87	154.22	181.10	335.32	2628.76
48.862	0.450 (*HD*)	1197.87	1167.24	159.04	186.76	345.79	2710.90
54.292	0.500	1209.98	1179.03	160.64	188.64	349.29	2738.29

A.2.5.2 Shear Forces and Bending Moments due to Live Load

A.2.5.2.1 Live Load

[LRFD Art. 3.6.1.2]

The LRFD Specifications specify a significantly different live load as compared to the Standard Specifications. The LRFD design live load is designated as HL-93, which consists of a combination of:

- Design truck with dynamic allowance or design tandem with dynamic allowance, whichever produces greater moments and shears, and

- Design lane load without dynamic allowance.

[LRFD Art. 3.6.1.2.2]

The design truck is designated as HS 20-44 consisting of an 8 kip front axle and two 32 kip rear axles.

[LRFD Art. 3.6.1.2.3]

The design tandem consists of a pair of 25-kip axles spaced 4 ft. apart. However, for spans longer than 40 ft. the tandem loading does not govern, thus only the truck load is investigated in this example.

[LRFD Art. 3.6.1.2.4]

The lane load consists of a load of 0.64 klf uniformly distributed in the longitudinal direction.

A.2.5.2.2 Live Load Distribution Factors for a Typical Interior Girder

The distribution factors specified by the LRFD Specifications have changed significantly as compared to the Standard Specifications, which specify $S/11$ (S is the girder spacing) to be used as the distribution factor.

[LRFD Art. 4.6.2.2]

The bending moments and shear forces due to live load can be distributed to individual girders using simplified approximate distribution factors specified by the LRFD Specifications. However, the simplified live load distribution factors can be used only if the following conditions are met:

[LRFD Art. 4.6.2.2.1]

1. Width of deck is constant (O.K.)

2. Number of girders, N_b, is not less than four
   Number of girders in present case, $N_b = 6$ (O.K.)

3. Girders are parallel and have approximately the same stiffness (O.K.)

4. The roadway part of the overhang, $d_e \leq 3.0$ ft.
   where d_e is the distance from exterior web of the exterior girder to the interior edge of curb or traffic barrier, ft.

   d_e = (overhang distance from the center of the exterior girder to the bridge end) – 0.5×(web width) – (width of barrier)
   = 3.0 – 0.33 - 1.0 = 1.67 ft. < 3.0 ft. (O.K.)

5. Curvature in plan is less than 4° (curvature = 0°) (O.K.)

6. Cross section of the bridge is consistent with one of the cross sections given in LRFD Table 4.6.2.2.1-1
7. Precast concrete I sections are specified as Type k (O.K.)

The number of design lanes is computed as follows:

Number of design lanes = Integer part of the ratio $w/12$

where w is the clear roadway width between the curbs = 44 ft.

[LRFD Art. 3.6.1.1.1]

Number of design lanes = Integer part of (44/12) = 3 lanes.

A.2.5.2.2.1 Distribution Factor for Bending Moment

The approximate live load moment distribution factors for interior girders are specified by LRFD Table 4.6.2.2.2b-1. The distribution factors for Type k (prestressed concrete I section) bridges can be used if the following additional requirements are satisfied:

$3.5 \leq S \leq 16$, where S is the spacing between adjacent girders, ft.
$S = 8.0$ ft (O.K.)

$4.5 \leq t_s \leq 12$, where t_s is the slab thickness, in.
$t_s = 8.0$ in (O.K.)

$20 \leq L \leq 240$, where L is the design span length, ft.
$L = 108.583$ ft. (O.K.)

$N_b \geq 4$, where N_b is the number of girders in the cross section.
$N_b = 6$ (O.K.)

$10{,}000 \leq K_g \leq 7{,}000{,}000$, where K_g is the longitudinal stiffness parameter, in.^{4}

$K_g = n(I + A\,e_g^2)$ [LRFD Art. 3.6.1.1.1]

where:

n = Modular ratio between girder and slab concrete.

$$= \frac{E_c \text{ for girder concrete}}{E_c \text{ for deck concrete}} = 1$$

Note that this ratio is the inverse of the one defined for composite section properties in Section A.2.4.2.2.

A = Area of girder cross section (non-composite section)
= 788.4 in.^{2}

I = Moment of inertia about the centroid of the non-composite precast girder = 260,403 in.^{4}

e_g = Distance between centers of gravity of the girder and slab, in.
= $(t_s/2 + y_t) = (8/2 + 29.25) = 33.25$ in.

$K_g = 1[260{,}403 + 788.4\,(33.25)^2] = 1{,}132{,}028.5$ in.^{4} (O.K.)

The approximate live load moment distribution factors for interior girders specified by the LRFD Specifications are applicable in this case as all the requirements are satisfied. LRFD Table 4.6.2.2.2b-1 specifies the distribution factor for all limit states except fatigue limit state for interior Type k girders as follows:

For one design lane loaded:

$$DFM = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0\,L\,t_s^3}\right)^{0.1}$$

where:

DFM = Live load moment distribution factor for interior girders.

S = Spacing of adjacent girders = 8 ft.

L = Design span length = 108.583 ft.

t_s = Thickness of slab = 8 in.

$$DFM = 0.06 + \left(\frac{8}{14}\right)^{0.4}\left(\frac{8}{108.583}\right)^{0.3}\left(\frac{1{,}132{,}028.5}{12.0(108.583)(8)^3}\right)^{0.1}$$

DFM = 0.06 + (0.8)(0.457)(1.054) = 0.445 lanes/girder

For two or more lanes loaded:

$$DFM = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0\,L\,t_s^3}\right)^{0.1}$$

$$DFM = 0.075 + \left(\frac{8}{9.5}\right)^{0.6}\left(\frac{8}{108.583}\right)^{0.2}\left(\frac{1{,}132{,}028.5}{12.0(108.583)(8)^3}\right)^{0.1}$$

= 0.075 + (0.902)(0.593)(1.054) = 0.639 lanes/girder

The greater of the above two distribution factors governs. Thus, the case of two or more lanes loaded controls.

DFM = 0.639 lanes/girder

A.2.5.2.2.2
Skew Reduction for DFM

LRFD Article 4.6.2.2.2e specifies a skew reduction for load distribution factors for moment in longitudinal beams on skewed supports. LRFD Table 4.6.2.2.2e-1 presents the skew reduction formulas for skewed Type k bridges where the skew angle θ is such that 30° ≤ θ ≤ 60°.

For Type k bridges having a skew angle such that θ < 30°, the skew reduction factor is specified as 1.0. For Type k bridges having a skew angle θ > 60°, the skew reduction is the same as for θ = 60°.

For the present design, the skew angle is 0°; thus a skew reduction for the live load moment distribution factor is not required.

A.2.5.2.2.3
Distribution Factor for Shear Force

The approximate live load shear distribution factors for interior girders are specified by LRFD Table 4.6.2.2.3a-1. The distribution factors for Type k (prestressed concrete I section) bridges can be used if the following requirements are satisfied:

$3.5 \leq S \leq 16$, where S is the spacing between adjacent girders, ft.
$S = 8.0$ ft. (O.K.)

$4.5 \leq t_s \leq 12$, where t_s is the slab thickness, in.
$t_s = 8.0$ in (O.K.)

$20 \leq L \leq 240$, where L is the design span length, ft.
$L = 108.583$ ft. (O.K.)

$N_b \geq 4$, where N_b is the number of girders in the cross section.
$N_b = 6$ (O.K.)

The approximate live load shear distribution factors for interior girders specified by the LRFD Specifications are applicable in this case as all the requirements are satisfied. LRFD Table 4.6.2.2.3a-1 specifies the distribution factor for all limit states for interior Type k girders as follows.

For one design lane loaded:

$$DFV = 0.36 + \left(\frac{S}{25.0}\right)$$

where:

DFV = Live load shear distribution factor for interior girders

S = Spacing of adjacent girders = 8 ft.

$$DFV = 0.36 + \left(\frac{8}{25.0}\right) = 0.680 \text{ lanes/girder}$$

For two or more lanes loaded:

$$DFV = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2$$

$$DFV = 0.2 + \frac{8}{12} - \left(\frac{8}{35}\right)^2 = 0.814 \text{ lanes/girder}$$

The greater of the above two distribution factors governs. Thus, the case of two or more lanes loaded controls.

DFV = 0.814 lanes/girder

The distribution factor for live load moments and shears for the same case using the Standard Specifications is 0.727 lanes/girder.

A.2.5.2.2.4
Skew Correction for DFV

LRFD Article 4.6.2.2.3c specifies that the skew correction factor shall be applied to the approximate load distribution factors for shear in the interior girders on skewed supports. LRFD Table 4.6.2.2.3c-1 provides the correction factor for load distribution factors for support shear of the obtuse corner of skewed Type k bridges where the following conditions are satisfied:

$0° \le \theta \le 60°$, where θ is the skew angle
$\theta = 0°$ (O.K.)

$3.5 \le S \le 16$, where S is the spacing between adjacent girders, ft.
S = 8.0 ft. (O.K.)

$20 \le L \le 240$, where L is the design span length, ft.
L = 108.583 ft. (O.K.)

$N_b \ge 4$, where N_b is the number of girders in the crosssection
$N_b = 6$ (O.K.)

The correction factor for load distribution factors for support shear of the obtuse corner of skewed Type k bridges is given as:

$$1.0 + 0.20\left(\frac{12.0\, L\, t_s^3}{K_g}\right)^{0.3} \tan\theta = 1.0 \text{ when } \theta = 0°$$

For the present design, the skew angle is 0 degrees; thus, the skew correction for the live load shear distribution factor is not required.

A.2.5.2.3 Dynamic Allowance

The LRFD Specifications specify the dynamic load effects as a percentage of the static live load effects. LRFD Table 3.6.2.1-1 specifies the dynamic allowance to be taken as 33 percent of the static load effects for all limit states, except the fatigue limit state, and 15 percent for the fatigue limit state. The factor to be applied to the static load shall be taken as:

$$(1 + IM/100)$$

where:

IM = Dynamic load allowance, applied to truck load or tandem load only
= 33 for all limit states except the fatigue limit state
= 15 for fatigue limit state

The Standard Specifications specify the impact factor to be:

$$I = \frac{50}{L+125} < 30\%$$

The impact factor was 21.4 percent for the Standard design.

A.2.5.2.4 Shear Forces and Bending Moments

A.2.5.2.4.1 Due to Truck Load

The maximum shear forces and bending moments due to HS 20-44 truck loading for all limit states, except for the fatigue limit state, on a per-lane-basis are calculated using the following formulas given in the *PCI Design Manual* (PCI 2003).

Maximum bending moment due to HS 20-44 truck load

For $x/L = 0 - 0.333$

$$M = \frac{72(x)[(L-x)-9.33]}{L}$$

For $x/L = 0.333 - 0.5$

$$M = \frac{72(x)[(L-x)-4.67]}{L} - 112$$

Maximum shear force due to HS 20-44 truck load

For $x/L = 0 - 0.5$

$$V = \frac{72[(L-x)-9.33]}{L}$$

where:

x = Distance from the centerline of bearing to the section at which bending moment or shear force is calculated, ft.

L = Design span length = 108.583 ft.

Distributed bending moment due to truck load including dynamic load allowance (M_{LT}) is calculated as follows:

M_{LT} = (Moment per lane due to truck load)(DFM)(1+IM/100)
= (M)(0.639)(1 + 33/100)
= (M)(0.85)

Distributed shear force due to truck load including dynamic load allowance (V_{LT}) is calculated as follows:

V_{LT} = (Shear force per lane due to truck load)(DFV)(1+IM/100)
= (V)(0.814)(1 + 33/100)
= (V)(1.083)

where:

M = Max. bending moment due to HS 20-44 truck load, k-ft.

DFM = Live load moment distribution factor for interior girders

IM = Dynamic load allowance, applied to truck load or tandem load only

DFV = Live load shear distribution factor for interior girders

V = Maximum shear force due to HS 20-44 truck load, kips

The maximum bending moments and shear forces due to an HS 20-44 truck load are calculated at every tenth of the span length and at the critical section for shear and the hold-down point location. The values are presented in Table A.2.5.2.

A.2.5.2.4.2
Due to Design Lane Load

The maximum bending moments (M_L) and shear forces (V_L) due to a uniformly distributed lane load of 0.64 klf are calculated using the following formulas given by the *PCI Design Manual* (PCI 2003).

Maximum bending moment, $M_L = 0.5(0.64)(x)(L - x)$

where:

x = Distance from centerline of bearing to section at which the bending moment or shear force is calculated, ft.

L = Design span length = 108.583 ft.

Maximum shear force, $V_L = \dfrac{0.32(L-x)^2}{L}$ for $x \leq 0.5L$

(Note that maximum shear force at a section is calculated at a section by placing the uniform load on the right of the section considered as shown in Figure A.2.5.2, given by the *PCI Design Manual* (PCI 2003). This method yields a slightly conservative estimate of the shear force as compared to the shear force at a section under uniform load placed on the entire span length.)

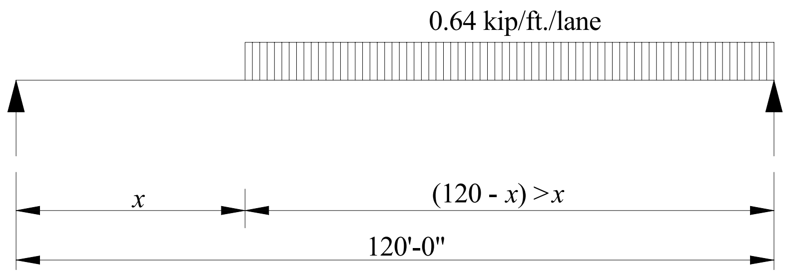

Figure A.2.5.2. Maximum Shear Force due to Lane Load.

Distributed bending moment due to lane load (M_{LL}) is calculated as follows:

M_{LL} = (Moment per lane due to lane load)(DFM)
= M_L (0.639)

Distributed shear force due to lane load (V_{LL}) is calculated as follows:

V_{LL} = (shear force per lane due to lane load)(DFV)
= V_L (0.814)

where:

M_L = Maximum bending moment due to lane load, k-ft.

DFM = Live load moment distribution factor for interior girders

DFV = Live load shear distribution factor for interior girders

V_L = Maximum shear force due to lane load, kips

The maximum bending moments and shear forces due to the lane load are calculated at every tenth of the span length and at the critical section for shear and the hold-down point location. The values are presented in Table A.2.5.3.

Table A.2.5.3. Shear Forces and Bending Moments due to Live Load.

Distance from Bearing Centerline x	Section x/L	HS 20-44 Truck Loading				Lane Loading			
		Undistributed Truck Load		Distributed Truck + Dynamic Load		Undistributed Lane Load		Distributed Lane Load	
		Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
		V	M	V_{LT}	M_{LT}	V_L	M_L	V_{LL}	M_{LL}
ft.		kips	k-ft.	kips	k-ft.	kips	k-ft.	kips	k-ft.
0.000	0.000	65.81	0.00	71.25	0.00	34.75	0.00	28.28	0.00
2.875	0.026	63.91	183.73	69.19	156.15	32.93	97.25	26.81	62.14
10.858	0.100	58.61	636.43	63.45	540.88	28.14	339.55	22.91	216.97
21.717	0.200	51.41	1116.54	55.66	948.91	22.24	603.67	18.10	385.75
32.575	0.300	44.21	1440.25	47.86	1224.03	17.03	792.31	13.86	506.28
43.433	0.400	37.01	1629.82	40.07	1385.14	12.51	905.49	10.18	578.61
48.862	0.450 (*HD*)	33.41	1671.64	36.17	1420.68	10.51	933.79	8.56	596.69
54.292	0.500	29.81	1674.37	32.27	1423.00	8.69	943.22	7.07	602.72

A.2.5.3 Load Combinations

LRFD Art. 3.4.1 specifies load factors and load combinations. The total factored load effect is specified to be taken as:

$$Q = \sum \eta_i \ \gamma_i \ Q_i \qquad \text{[LRFD Eq. 3.4.1-1]}$$

where:

Q = Factored force effects

γ_i = Load factor, a statistically based multiplier applied to force effects specified by LRFD Table 3.4.1-1

Q_i = Unfactored force effects

η_i = Load modifier, a factor relating to ductility, redundancy, and operational importance
= $\eta_D \ \eta_R \ \eta_I \geq 0.95$, for loads for which a maximum value of γ_i is appropriate [LRFD Eq. 1.3.2.1-2]
= $\dfrac{1}{\eta_D \ \eta_R \ \eta_I} \leq 1.0$, for loads for which a minimum value of γ_i is appropriate [LRFD Eq. 1.3.2.1-3]

η_D = A factor relating to ductility
= 1.00 for all limit states except strength limit state

For the strength limit state:

η_D ≥ 1.05 for nonductile components and connections
= 1.00 for conventional design and details complying with the LRFD Specifications
≥ 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by the LRFD Specifications

η_D = 1.00 is used in this example for strength and service limit states as this design is considered to be conventional and complying with the LRFD Specifications.

η_R = A factor relating to redundancy
= 1.00 for all limit states except strength limit state

For strength limit state:

η_R ≥ 1.05 for nonredundant members
= 1.00 for conventional levels of redundancy
≥ 0.95 for exceptional levels of redundancy

η_R = 1.00 is used in this example for strength and service limit states as this design is considered to provide a conventional level of redundancy to the structure.

η_I = A factor relating to operational importance
= 1.00 for all limit states except strength limit state

For strength limit state:

η_I ≥ 1.05 for important bridges
= 1.00 for typical bridges
≥ 0.95 for relatively less important bridges

η_I = 1.00 is used in this example for strength and service limit states, as this example illustrates the design of a typical bridge.

$\eta_i = \eta_D\ \eta_R\ \eta_I$ = 1.00 in present case [LRFD Art. 1.3.2]

The notations used in the following section are defined as follows:

DC = Dead load of structural components and non-structural attachments

DW = Dead load of wearing surface and utilities

LL = Vehicular live load

IM = Vehicular dynamic load allowance

This design example considers only the dead and vehicular live loads. The wind load and the extreme event loads, including earthquake and vehicle collision loads, are not included in the design, which is typical to the design of bridges in Texas. Various limit states and load combinations provided by LRFD Art. 3.4.1 are investigated, and the following limit states are found to be applicable in present case:

Service I: This limit state is used for normal operational use of a bridge. This limit state provides the general load combination for service limit state stress checks and applies to all conditions except Service III limit state. For prestressed concrete components, this load combination is used to check for compressive stresses. The load combination is presented as follows:

$Q = 1.00\ (DC + DW) + 1.00(LL + IM)$ [LRFD Table 3.4.1-1]

Service III: This limit state is a special load combination for service limit state stress checks that applies only to tension in prestressed concrete structures to control cracks. The load combination for this limit state is presented as follows:

$Q = 1.00(DC + DW) + 0.80(LL + IM)$ [LRFD Table 3.4.1-1]

Strength I: This limit state is the general load combination for strength limit state design relating to the normal vehicular use of the bridge without wind. The load combination is presented as follows:

[LRFD Table 3.4.1-1 and 2]

$Q = \gamma_P(DC) + \gamma_P(DW) + 1.75(LL + IM)$

γ_P = Load factor for permanent loads provided in Table A.2.5.4

Table A.2.5.4. Load Factors for Permanent Loads.

Type of Load	Load Factor, γ_P	
	Maximum	Minimum
DC: Structural components and non-structural attachments	1.25	0.90
DW: Wearing surface and utilities	1.50	0.65

The maximum and minimum load combinations for the Strength I limit state are presented as follows:

Maximum $Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$
Minimum $Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)$

For simple span bridges, the maximum load factors produce maximum effects. However, minimum load factors are used for component dead loads (*DC*) and wearing surface load (*DW*) when dead load and wearing surface stresses are opposite to those of live load. In the present example, the maximum load factors are used to investigate the ultimate strength limit state.

A.2.6 ESTIMATION OF REQUIRED PRESTRESS

The required number of strands is usually governed by concrete tensile stress at the bottom fiber of the girder at the midspan section. The load combination for the Service III limit state is used to evaluate the bottom fiber stresses at the midspan section. The calculation for compressive stress in the top fiber of the girder at midspan section under service loads is also shown in the following section. The compressive stress is evaluated using the load combination for the Service I limit state.

A.2.6.1 Service Load Stresses at Midspan

Tensile stress at the bottom fiber of the girder at midspan due to applied dead and live loads using load combination Service III

$$f_b = \frac{M_{DCN}}{S_b} + \frac{M_{DCC} + M_{DW} + 0.8(M_{LT} + M_{LL})}{S_{bc}}$$

Compressive stress at the top fiber of the girder at midspan due to applied dead and live loads using load combination Service I

$$f_t = \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW} + M_{LT} + M_{LL}}{S_{tg}}$$

where:

f_b = Concrete stress at the bottom fiber of the girder, ksi

f_t = Concrete stress at the top fiber of the girder, ksi

M_{DCN} = Moment due to non-composite dead loads, k-ft.
= $M_g + M_S$

M_g = Moment due to girder self-weight = 1209.98 k-ft.

M_S = Moment due to slab weight = 1179.03 k-ft.

M_{DCN} = 1209.98 + 1179.03 = 2389.01 k-ft.

M_{DCC} = Moment due to composite dead loads except wearing surface load, k-ft.
= M_{barr}

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DCC} = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

M_{LT} = Distributed moment due to HS 20-44 truck load including dynamic load allowance = 1423.00 k-ft.

M_{LL} = Distributed moment due to lane load = 602.72 k-ft.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder = 8902.67 in.^{3}

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder
= 16,876.83 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder = 54,083.9 in.^{3}

Substituting the bending moments and section modulus values, stresses at bottom fiber (f_b) and top fiber (f_t) of the girder at midspan section are:

$$f_b = \frac{(2389.01)(12\text{ in./ft.})}{10,521.33} + \frac{[160.64 + 188.64 + 0.8(1423.00 + 602.72)](12\text{ in./ft.})}{16,876.83}$$

= 2.725 + 1.400 = 4.125 ksi (as compared to 4.024 ksi for design using Standard Specifications)

$$f_t = \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{[160.64 + 188.64 + 1423.00 + 602.72](12\text{ in./ft.})}{54,083.9}$$

= 3.220 + 0.527 = 3.747 ksi (as compared to 3.626 ksi for design using Standard Specifications)

The stresses in the top and bottom fibers of the girder at the hold-down point, midspan, and top fiber of the slab are calculated in a similar way as shown above, and the results are summarized in Table A.2.6.1.

Table A.2.6.1. Summary of Stresses due to Applied Loads.

Load	Stresses in Girder				Stresses in Slab
	Stress at Hold-Down (*HD*)		Stress at Midspan		Stress at Midspan
	Top Fiber (psi)	Bottom Fiber (psi)	Top Fiber (psi)	Bottom Fiber (psi)	Top Fiber (psi)
Girder Self-Weight	1614.63	-1366.22	1630.94	-1380.03	-
Slab Weight	1573.33	-1331.28	1589.22	-1344.73	-
Barrier Weight	35.29	-113.08	35.64	-114.22	57.84
Wearing Surface Weight	41.44	-132.79	41.85	-134.13	67.93
Total Dead Load	3264.68	-2943.38	3297.66	-2973.10	125.77
HS 20-44 Truck Load (multiplied by 0.8 for bottom fiber stress calculation)	315.22	-808.12	315.73	-809.44	512.40
Lane Load (multiplied by 0.8 for bottom fiber stress calculation)	132.39	-339.41	133.73	-342.84	217.03
Total Live Load	447.61	-1147.54	449.46	-1152.28	729.43
Total Load	3712.29	-4090.91	3747.12	-4125.39	855.21

(Negative values indicate tensile stress)

A.2.6.2 Allowable Stress Limit

LRFD Table 5.9.4.2.2-1 specifies the allowable tensile stress in fully prestressed concrete members. For members with bonded prestressing tendons that are subjected to not worse than moderate corrosion conditions (these corrosion conditions are assumed in this design), the allowable tensile stress at service limit state after losses is given as:

$$F_b = 0.19\sqrt{f'_c}$$

where:

f'_c = Compressive strength of girder concrete at service = 5.0 ksi

F_b = $0.19\sqrt{5.0}$ = 0.4248 ksi (as compared to allowable tensile stress of 0.4242 ksi for the Standard design)

A.2.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – Allowable tensile stress at service = $f_b - F_b$

$f_{pb\text{-}reqd.}$ = 4.125 – 0.4248 = 3.700 ksi

Assuming the eccentricity of the prestressing strands at midspan (e_c) as the distance from the centroid of the girder to the bottom fiber of the girder (PSTRS 14 methodology, TxDOT 2004)

$e_c = y_b$ = 24.75 in.

Stress at the bottom fiber of the girder due to prestress after losses:

$$f_b = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b}$$

where:

P_{pe} = Effective prestressing force after all losses, kips

A = Area of girder cross section = 788.4 in.^{2}

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

Required prestressing force is calculated by substituting the corresponding values in the above equation as follows.

$$3.700 = \frac{P_{pe}}{788.4} + \frac{24.75\,P_{pe}}{10{,}521.33}$$

Solving for P_{pe},

P_{pe} = 1021.89 kips

Assuming final losses = 20 percent of initial prestress f_{pi} (TxDOT 2001)

Assumed final losses = 0.2(202.5) = 40.5 ksi

The prestress force required per strand after losses
= (cross sectional area of one strand) [f_{pi} – losses]
= 0.153(202.5 – 40.5) = 24.78 kips

Number of prestressing strands required = 1021.89/24.78 = 41.24

Try 42 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial trial.

Strand eccentricity at midspan after strand arrangement

$e_c = 24.75 - \dfrac{12(2+4+6)+6(8)}{42} = 20.18$ in.

Available prestressing force

P_{pe} = 42(24.78) = 1040.76 kips

Stress at bottom fiber of the girder due to prestress after losses:

$f_b = \dfrac{1040.76}{788.4} + \dfrac{1040.76(20.18)}{10{,}521.33}$

$= 1.320 + 1.996 = 3.316$ ksi $< f_{pb\text{- }reqd.} = 3.700$ ksi (N.G.)

Try 44 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial trial.

Strand eccentricity at midspan after strand arrangement

$e_c = 24.75 - \dfrac{12(2+4+6)+8(8)}{44} = 20.02$ in.

Available prestressing force

P_{pe} = 44(24.78) = 1090.32 kips

Stress at bottom fiber of the girder due to prestress after losses:

$f_b = \dfrac{1090.32}{788.4} + \dfrac{1090.32(20.02)}{10{,}521.33}$

$= 1.383 + 2.075 = 3.458$ ksi $< f_{pb\text{- }reqd.} = 3.700$ ksi (N.G.)

Try 46 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial trial.

Strand eccentricity at midspan after strand arrangement

$e_c = 24.75 - \dfrac{12(2+4+6)+10(8)}{46} = 19.88$ in.

Available prestressing force

P_{pe} = 46(24.78) = 1139.88 kips

Stress at bottom fiber of the girder due to prestress after losses:

$f_b = \dfrac{1139.88}{788.4} + \dfrac{1139.88(19.88)}{10{,}521.33}$

$= 1.446 + 2.154 = 3.600$ ksi $< f_{pb\text{- }reqd.} = 3.700$ ksi (N.G.)

Try 48 – 0.5 in. diameter, 270 ksi low-relaxation strands as an initial trial.

Strand eccentricity at midspan after strand arrangement

$$e_c = 24.75 - \frac{12(2+4+6) + 10(8)+2(10)}{48} = 19.67 \text{ in.}$$

Available prestressing force
P_{pe} = 48(24.78) = 1189.44 kips

Stress at bottom fiber of the girder due to prestress after losses:

$$f_b = \frac{1189.44}{788.4} + \frac{1189.44(19.67)}{10,521.33}$$

$$= 1.509 + 2.223 = 3.732 \text{ ksi} > f_{pb\text{- reqd.}} = 3.700 \text{ ksi} \quad \text{(O.K.)}$$

Therefore, use 48 strands as a preliminary estimate for the number of strands. The strand arrangement is shown in Figure A.2.6.1.

Number of Strands	Distance from bottom fiber (in.)
2	10
10	8
12	6
12	4
12	2

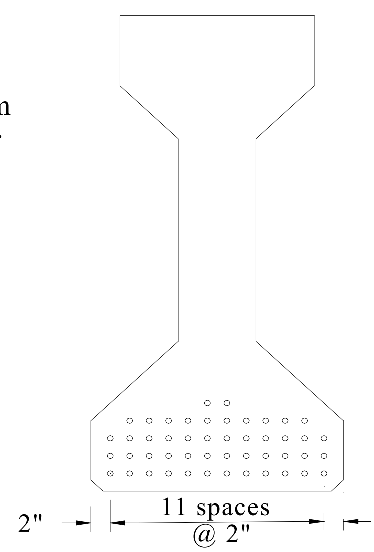

Figure A.2.6.1. Initial Strand Arrangement.

The distance from the center of gravity of the strands to the bottom fiber of the girder (y_{bs}) is calculated as:

$y_{bs} = y_b - e_c$ = 24.75 – 19.67 = 5.08 in.

A.2.7 PRESTRESS LOSSES

[LRFD Art. 5.9.5]

The LRFD Specifications specify formulas to determine the instantaneous losses. For time-dependent losses, two different options are provided. The first option is to use a lump-sum estimate of time-dependent losses given by LRFD Art. 5.9.5.3. The second option is to use refined estimates for time-dependent losses given by LRFD Art. 5.9.5.4. The refined estimates are used in this design as they yield more accuracy as compared to the lump-sum method.

The instantaneous loss of prestress is estimated using the following expression:

$$\Delta f_{pi} = (\Delta f_{pES} + \Delta f_{pR1})$$

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

TxDOT methodology was used for the evaluation of instantaneous prestress loss in the Standard design example given by the following expression.

$$\Delta f_{pi} = (ES + \frac{1}{2} CR_S)$$

where:

Δf_{pi} = Instantaneous prestress loss, ksi

Δf_{pES} = Prestress loss due to elastic shortening, ksi

Δf_{pR1} = Prestress loss due to steel relaxation before transfer, ksi

f_{pj} = Jacking stress in prestressing strands = 202.5 ksi

ES = Prestress loss due to elastic shortening, ksi

CR_S = Prestress loss due to steel relaxation at service, ksi

The time-dependent loss of prestress is estimated using the following expression:

$$\text{Time dependent loss} = \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$$

where:

Δf_{pSR} = Prestress loss due to concrete shrinkage, ksi

Δf_{pCR} = Prestress loss due to concrete creep, ksi

Δf_{pR2} = Prestress loss due to steel relaxation after transfer, ksi

The total prestress loss in prestressed concrete members prestressed in a single stage, relative to stress immediately before transfer is given as:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$$ [LRFD Eq. 5.9.5.1-1]

However, considering the steel relaxation loss before transfer Δf_{pR1}, the total prestress loss is calculated using the following expression:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} + \Delta f_{pR2}$$

The calculation of prestress loss due to elastic shortening, steel relaxation before and after transfer, creep of concrete, and shrinkage of concrete are shown in the following sections.

Trial number of strands = 48

A number of iterations based on TxDOT methodology (TxDOT 2001) will be performed to arrive at the optimum number of strands, required concrete strength at release (f'_{ci}), and required concrete strength at service (f'_c).

A.2.7.1 Iteration 1

A.2.7.1.1 Elastic Shortening

[LRFD Art. 5.9.5.2.3]

The loss in prestress due to elastic shortening in prestressed members is given as:

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Eq. 5.9.5.2.3a-1]

where:

E_p = Modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = Modulus of elasticity of girder concrete at transfer, ksi
= $33{,}000(w_c)^{1.5}\sqrt{f'_{ci}}$ [LRFD Eq. 5.4.2.4-1]

w_c = Unit weight of concrete (must be between 0.09 and 0.155 kcf for LRFD Eq. 5.4.2.4-1 to be applicable)
= 0.150 kcf

f'_{ci} = Initial estimate of compressive strength of girder concrete at release = 4 ksi

E_{ci} = $[33{,}000(0.150)^{1.5}\sqrt{4}\,]$ = 3834.25 ksi

f_{cgp} = Sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and the self-weight of the member at sections of maximum moment, ksi

$$= \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_i = Pretension force after allowing for the initial losses, kips

A = Area of girder cross section = 788.4 in.^{2}

I = Moment of inertia of the non-composite section = 260,403 in.^{4}

e_c = Eccentricity of the prestressing strands at the midspan = 19.67 in.

M_g = Moment due to girder self-weight at midspan, k-ft. = 1209.98 k-ft.

LRFD Art. 5.9.5.2.3a states that for pretensioned components of usual design, f_{cgp}, can be calculated on the basis of prestressing steel stress assumed to be $0.7f_{pu}$ for low-relaxation strands. However, TxDOT methodology is to assume the initial losses as a percentage of the initial prestressing stress before release, f_{pj}. In both procedures, initial losses assumed has to be checked, and if different from the assumed value, a second iteration should be carried out.

TxDOT methodology is used in this example, and initial loss is assumed to be 8 percent of initial prestress, f_{pj}.

P_i = Pretension force after allowing for 8 percent initial loss, kips

= (number of strands)(area of each strand)[0.92(f_{pj})]

= 48(0.153)(0.92)(202.5) = 1368.19 kips

$$f_{cgp} = \frac{1368.19}{788.4} + \frac{1368.19(19.67)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.67)}{260{,}403}$$

= 1.735 + 2.033 – 1.097 = 2.671 ksi

Prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{3834.25}\right](2.671) = 19.854 \text{ ksi}$$

A.2.7.1.2
Concrete Shrinkage

[LRFD Art. 5.9.5.4.2]

The loss in prestress due to concrete shrinkage for pretensioned members is given as:

$$\Delta f_{pSR} = 17 - 0.15\,H \qquad \text{[LRFD Eq. 5.9.5.4.2-1]}$$

where:

H = Average annual ambient relative humidity = 60 percent

Δf_{pSR} = [17 – 0.15(60)] = 8.0 ksi

A.2.7.1.3
Creep of Concrete

[LRFD Art. 5.9.5.4.3]

The loss in prestress due to creep of concrete is given as:

$$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp} \geq 0 \qquad \text{[LRFD Eq. 5.9.5.4.3-1]}$$

where:

Δf_{cdp} = Change in concrete stress at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied, calculated at the same section as f_{cgp}

$$= \frac{M_S\, e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_S = Moment due to slab weight at the midspan section = 1179.03 k-ft.

M_{SDL} = Moment due to superimposed dead load = $M_{barr} + M_{DW}$

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

M_{SDL} = 160.64 + 188.64 = 349.28 k-ft.

y_{bc} = Distance from the centroid of the composite section to the extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from center of gravity of the prestressing strands at midspan to the bottom fiber of the girder
= 24.75 – 19.67 = 5.08 in.

I = Moment of inertia of the non-composite section
= 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12\text{ in./ft.})(19.67)}{260,403} + \frac{(349.28)(12\text{ in./ft.})(41.157-5.08)}{694,599.5}$$

= 1.069 + 0.218 = 1.287 ksi

Prestress loss due to creep of concrete is:
Δf_{pCR} = 12(2.671) – 7(1.287) = 23.05 ksi

A.2.7.1.4 Relaxation of Prestressing Strands

[LRFD Art. 5.9.5.4.4]

A.2.7.1.4.1 Relaxation at Transfer

[LRFD Art. 5.9.5.4.4b]

For pretensioned members with low-relaxation prestressing steel, initially stressed in excess of $0.5f_{pu}$, the relaxation loss is given as:

$$\Delta f_{pR1} = \frac{\log(24.0t)}{40}\left[\frac{f_{pj}}{f_{py}} - 0.55\right]f_{pj}$$ [LRFD Eq. 5.9.5.4.4b-2]

where:

Δf_{pR1} = Prestress loss due to relaxation of steel at transfer, ksi

f_{pu} = Ultimate stress in prestressing steel = 270 ksi

f_{pj} = Initial stress in tendon at the end of stressing
= $0.75f_{pu}$ = 0.75(270) = 202.5 ksi > $0.5f_{pu}$ = 135 ksi

t = Time estimated in days from stressing to transfer taken as 1 day (default value for PSTRS14 design program [TxDOT 2004])

f_{py} = Yield strength of prestressing steel = 243 ksi

Prestress loss due to initial steel relaxation is:

$$\Delta f_{pR1} = \frac{\log(24.0)(1)}{40}\left[\frac{202.5}{243} - 0.55\right]202.5 = 1.98\text{ ksi}$$

A.2.7.1.4.2
Relaxation after Transfer

[LRFD Art. 5.9.5.4.4c]

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is given as:

Δf_{pR2} = 30% of [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

[LRFD Art. 5.9.5.4.4c-1]

where the variables are the same as defined in Section A.2.7 expressed in ksi units

Δf_{pR2} = 0.3[20.0 – 0.4(19.854) – 0.2(8.0 + 23.05)] = 1.754 ksi

The instantaneous loss of prestress is estimated using the following expression:

$$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$$
$$= 19.854 + 1.980 = 21.834 \text{ ksi}$$

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$
$$= \frac{100(19.854 + 1.980)}{202.5} = 10.78\% > 8\%$$ (assumed value of initial prestress loss)

Therefore, another trial is required assuming 10.78 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress losses due to concrete shrinkage (Δf_{pSR}) and initial steel relaxation (Δf_{pR1}). Therefore, the next trial will involve updating the losses due to elastic shortening (Δf_{pES}), creep of concrete (Δf_{pCR}), and steel relaxation after transfer (Δf_{pR2}).

Based on the initial prestress loss value of 10.78 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_i = (number of strands)(area of each strand)[0.8922(f_{pj})]

= 48(0.153)(0.8922)(202.5) = 1326.84 kips

Loss in prestress due to elastic shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1326.84}{788.4} + \frac{1326.84(19.67)^2}{260,403} - \frac{1209.98(12 \text{ in./ft.})(19.67)}{260,403}$$

$= 1.683 + 1.971 – 1.097 = 2.557$ ksi

$E_{ci} = 3834.25$ ksi

$E_p = 28{,}500$ ksi

Prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{3834.25}\right](2.557) = 19.01 \text{ ksi}$$

The loss in prestress due to creep of concrete is given as:

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp} \geq 0$

The value of Δf_{cdp} depends on the dead load moments, superimposed dead load moments, and the section properties. Thus, this value will not change with the change in initial prestress value and will be the same as calculated in Section A.2.7.1.3.

$\Delta f_{cdp} = 1.287$ ksi

$\Delta f_{pCR} = 12(2.557) - 7(1.287) = 21.675$ ksi

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is:

Δf_{pR2} = 30% of $[20.0 - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

$= 0.3[20.0 - 0.4(19.01) - 0.2(8.0 + 21.675)] = 1.938$ ksi

The instantaneous loss of prestress is estimated using the following expression:

$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$

$= 19.01 + 1.980 = 20.99$ ksi

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(19.01 + 1.980)}{202.5} = 10.37\% < 10.78\% \text{ (assumed value of initial prestress loss)}$$

Therefore, another trial is required assuming 10.37 percent initial prestress loss.

Based on the initial prestress loss value of 10.37 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_i = (number of strands)(area of each strand)[0.8963(f_{pj})]

= 48(0.153)(0.8963)(202.5) = 1332.94 kips

Loss in prestress due to elastic shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1332.94}{788.4} + \frac{1332.94(19.67)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.67)}{260{,}403}$$

$$= 1.691 + 1.980 - 1.097 = 2.574 \text{ ksi}$$

E_{ci} = 3834.25 ksi

E_p = 28,500 ksi

Prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{3834.25}\right](2.574) = 19.13 \text{ ksi}$$

The loss in prestress due to creep of concrete is given as:

$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp} \geq 0$

Δf_{cdp} = 1.287 ksi

Δf_{pCR} = 12(2.574) – 7(1.287) = 21.879 ksi

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is:

Δf_{pR2} = 30% of [20.0 – 0.4 Δf_{pES} – 0.2($\Delta f_{pSR} + \Delta f_{pCR}$)

= 0.3[20.0 – 0.4(19.13) – 0.2(8.0 + 21.879)] = 1.912 ksi

The instantaneous loss of prestress is estimated using the following expression:

$$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$$

= 19.13 + 1.98 = 21.11 ksi

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(19.13 + 1.98)}{202.5}$$ = 10.42% ≈ 10.37% (assumed value of initial prestress loss)

A.2.7.1.5
Total Losses at Transfer

Total prestress loss at transfer

$$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$$

= 19.13 + 1.98 = 21.11 ksi

Effective initial prestress, f_{pi} = 202.5 – 21.11 = 181.39 ksi

P_i = Effective pretension after allowing for the initial prestress loss

= (number of strands)(area of each strand)(f_{pi})

= 48(0.153)(181.39) = 1332.13 kips

A.2.7.1.6
Total Losses at Service Loads

Total final loss in prestress:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} + \Delta f_{pR2}$$

Δf_{pES} = Prestress loss due to elastic shortening = 19.13 ksi

Δf_{pSR} = Prestress loss due to concrete shrinkage = 8.0 ksi

Δf_{pCR} = Prestress loss due to concrete creep = 21.879 ksi

Δf_{pR1} = Prestress loss due to steel relaxation before transfer = 1.98 ksi

Δf_{pR2} = Prestress loss due to steel relaxation after transfer = 1.912 ksi

Δf_{pT} = 19.13 + 8.0 + 21.879 + 1.98 + 1.912 = 52.901 ksi

The percent final loss is calculated using the following expression:

$$\%\Delta f_{pT} = \frac{100(\Delta f_{pT})}{f_{pj}}$$

$$= \frac{100(52.901)}{202.5} = 26.12\%$$

Effective final prestress
$f_{pe} = f_{pj} - \Delta f_{pT}$ = 202.5 – 52.901 = 149.60 ksi

Check prestressing stress limit at service limit state (defined in Section A.2.3): $f_{pe} \leq 0.8 f_{py}$

f_{py} = Yield strength of prestressing steel = 243 ksi

f_{pe} = 149.60 ksi < 0.8(243) = 194.4 ksi (O.K.)

Effective prestressing force after allowing for final prestress loss

P_{pe} = (number of strands)(area of each strand)(f_{pe})

= 48(0.153)(149.60) = 1098.66 kips

A.2.7.1.7 Final Stresses at Midspan

The number of strands is updated based on the final stress at the bottom fiber of the girder at the midspan section.

Final stress at the bottom fiber of the girder at the midspan section due to effective prestress (f_{bf}) is calculated as follows:

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b}$$

$$= \frac{1098.66}{788.4} + \frac{1098.66(19.67)}{10{,}521.33}$$

= 1.393 + 2.054 = 3.447 ksi < $f_{pb\text{-}reqd.}$ = 3.700 ksi (N.G)

($f_{pb\text{-}reqd.}$ calculations are presented in Section A.2.6.3.)

Try 50 – 0.5 in. diameter, low-relaxation strands.

Eccentricity of prestressing strands at midspan

$$e_c = 24.75 - \frac{12(2+4+6)+10(8)+4(10)}{50} = 19.47 \text{ in.}$$

Effective pretension after allowing for the final prestress loss
P_{pe} = 50(0.153)(149.60) = 1144.44 kips

Final stress at the bottom fiber of the girder at the midspan section due to effective prestress (f_{bf}) is:

$$f_{bf} = \frac{1144.44}{788.4} + \frac{1144.44(19.47)}{10,521.33}$$

$$= 1.452 + 2.118 = 3.57 \text{ ksi} < f_{pb\text{-}reqd.} = 3.700 \text{ ksi} \qquad \text{(N.G)}$$

Try 52 – 0.5 in. diameter, low-relaxation strands.

Eccentricity of prestressing strands at midspan

$$e_c = 24.75 - \frac{12(2+4+6)+10(8)+6(10)}{52} = 19.29 \text{ in.}$$

Effective pretension after allowing for the final prestress loss
P_{pe} = 52(0.153)(149.60) = 1190.22 kips

Final stress at the bottom fiber of the girder at the midspan section due to effective prestress (f_{bf}) is:

$$f_{bf} = \frac{1190.22}{788.4} + \frac{1190.22(19.29)}{10,521.33}$$

$$= 1.509 + 2.182 = 3.691 \text{ ksi} < f_{pb\text{-}reqd.} = 3.700 \text{ ksi} \qquad \text{(N.G)}$$

Try 54 – 0.5 in. diameter, low-relaxation strands.

Eccentricity of prestressing strands at midspan

$$e_c = 24.75 - \frac{12(2+4+6)+10(8)+8(10)}{54} = 19.12 \text{ in.}$$

Effective pretension after allowing for the final prestress loss
P_{pe} = 54(0.153)(149.60) = 1236.0 kips

Final stress at the bottom fiber of the girder at the midspan section due to effective prestress (f_{bf}) is:

$$f_{bf} = \frac{1236.0}{788.4} + \frac{1236.0(19.12)}{10,521.33}$$

$$= 1.567 + 2.246 = 3.813 \text{ ksi} > f_{pb\text{-}reqd.} = 3.700 \text{ ksi} \qquad \text{(O.K.)}$$

Therefore, use 54 – 0.5 in. diameter, 270 ksi low-relaxation strands.

Concrete stress at the top fiber of the girder due to effective prestress and applied permanent and transient loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + f_t = \frac{1236.0}{788.4} - \frac{1236.0(19.12)}{8902.67} + 3.747$$

$$= 1.567 - 2.654 + 3.747 = 2.66 \text{ ksi}$$

(f_t calculations are shown in Section A.2.6.1.)

A.2.7.1.8 Initial Stresses at Hold-Down Point

The concrete strength at release, f'_{ci}, is updated based on the initial stress at the bottom fiber of the girder at the hold-down point.

Prestressing force after allowing for initial prestress loss

P_i = (number of strands)(area of strand)(effective initial prestress)

= 54(0.153)(181.39) = 1498.64 kips

(Effective initial prestress calculations are presented in Section A.2.7.1.5.)

Initial concrete stress at top fiber of the girder at the hold-down point due to self-weight of the girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at the hold-down point based on overall girder length of 109 ft.-8 in.
= $0.5wx(L - x)$

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Distance of hold-down point from the end of the girder
= HD + (distance from centerline of bearing to the girder end)

HD = Hold-down point distance from centerline of the bearing
= 48.862 ft. (see Sec. A.2.5.1.3)

x = 48.862 + 0.542 = 49.404 ft.

M_g = 0.5(0.821)(49.404)(109.67 – 49.404) = 1222.22 k-ft.

$$f_{ti} = \frac{1498.64}{788.4} - \frac{1498.64(19.12)}{8902.67} + \frac{1222.22(12 \text{ in./ft.})}{8902.67}$$

$= 1.901 - 3.218 + 1.647 = 0.330$ ksi

Initial concrete stress at bottom fiber of the girder at the hold-down point due to self-weight of the girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$= \frac{1498.64}{788.4} + \frac{1498.64(19.12)}{10{,}521.33} - \frac{1222.22(12 \text{ in./ft.})}{10{,}521.33}$$

$= 1.901 + 2.723 - 1.394 = 3.230$ ksi

Compression stress limit for pretensioned members at transfer stage is $0.6\,f'_{ci}$ [LRFD Art. 5.9.4.1.1]

Therefore, $f'_{ci\text{-reqd.}} = \frac{3230}{0.6} = 5383.33$ psi

A.2.7.2 Iteration 2

A second iteration is carried out to determine the prestress losses and to subsequently estimate the required concrete strength at release and at service using the following parameters determined in the previous iteration.

Number of strands = 54

Concrete strength at release, $f'_{ci} = 5383.33$ psi

A.2.7.2.1 Elastic Shortening

[LRFD Art. 5.9.5.2.3]

The loss in prestress due to elastic shortening in prestressed members is given as:

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Eq. 5.9.5.2.3a-1]

where:

E_p = Modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = Modulus of elasticity of girder concrete at transfer, ksi
= $33{,}000(w_c)^{1.5}\sqrt{f'_{ci}}$ [LRFD Eq. 5.4.2.4-1]

w_c = Unit weight of concrete (must be between 0.09 and 0.155 kcf for LRFD Eq. 5.4.2.4-1 to be applicable)
= 0.150 kcf

f'_{ci} = Compressive strength of girder concrete at release
= 5.383 ksi

E_{ci} = $[33{,}000(0.150)^{1.5}\sqrt{5.383}\,] = 4447.98$ ksi

f_{cgp} = Sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and the self-weight of the member at sections of maximum moment, ksi

$$= \frac{P_i}{A} + \frac{P_i\,e_c^2}{I} - \frac{(M_g)e_c}{I}$$

A = Area of girder cross section = 788.4 in.^{2}

I = Moment of inertia of the non-composite section
= 260,403 in.^{4}

e_c = Eccentricity of the prestressing strands at the midspan
= 19.12 in.

M_g = Moment due to girder self-weight at midspan, k-ft.
= 1209.98 k-ft.

P_i = Pretension force after allowing for the initial losses, kips

As the initial losses are dependent on the elastic shortening and the initial steel relaxation loss, which are yet to be determined, the initial loss value of 10.42 percent obtained in the last trial (Iteration 1) is taken as an initial estimate for the initial loss in prestress for this iteration.

P_i = (number of strands)(area of strand)[0.8958(f_{pj})]
= 54(0.153)(0.8958)(202.5) = 1498.72 kips

$$f_{cgp} = \frac{1498.72}{788.4} + \frac{1498.72(19.12)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.12)}{260{,}403}$$

= 1.901 + 2.104 – 1.066 = 2.939 ksi

The prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{4447.98}\right](2.939) = 18.83 \text{ ksi}$$

A.2.7.2.2
Concrete Shrinkage

[LRFD Art. 5.9.5.4.2]

The loss in prestress due to concrete shrinkage (Δf_{pSR}) depends on the relative humidity only. The change in compressive strength of girder concrete at release (f'_{ci}) and number of strands does not effect the prestress loss due to concrete shrinkage. It will remain the same as calculated in Section A.2.7.1.2.

Δf_{pSR} = 8.0 ksi

A.2.7.2.3
Creep of Concrete

[LRFD Art. 5.9.5.4.3]

The loss in prestress due to creep of concrete is given as:

$$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp} \geq 0 \qquad \text{[LRFD Eq. 5.9.5.4.3-1]}$$

where:

Δf_{cdp} = Change in concrete stress at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied and calculated at the same section as f_{cgp}.

$$= \frac{M_S\, e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_S = Moment due to slab weight at midspan section
= 1179.03 k-ft.

M_{SDL} = Moment due to superimposed dead load
= $M_{barr} + M_{DW}$

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

M_{SDL} = 160.64 + 188.64 = 349.28 k-ft.

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from center of gravity of the prestressing strands at midspan to the bottom fiber of the girder
= 24.75 – 19.12 = 5.63 in.

I = Moment of inertia of the non-composite section
= 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12\text{ in./ft.})(19.12)}{260,403} + \frac{(349.28)(12\text{ in./ft.})(41.157-5.63)}{694,599.5}$$

$$= 1.039 + 0.214 = 1.253\text{ ksi}$$

Prestress loss due to creep of concrete is:
$\Delta f_{pCR} = 12(2.939) - 7(1.253) = 26.50$ ksi

A.2.7.2.4 Relaxation of Prestressing Strands

[LRFD Art. 5.9.5.4.4]

A.2.7.2.4.1 Relaxation at Transfer

[LRFD Art. 5.9.5.4.4b]

The loss in prestress due to relaxation of steel at transfer (Δf_{pR1}) depends on the time from stressing to transfer of prestress (t), the initial stress in tendon at the end of stressing (f_{pj}), and the yield strength of prestressing steel (f_{py}). The change in compressive strength of girder concrete at release (f'_{ci}) and number of strands does not affect the prestress loss due to relaxation of steel before transfer. It will remain the same as calculated in Section A.2.7.1.4.1.

$\Delta f_{pR1} = 1.98$ ksi

A.2.7.2.4.2 Relaxation after Transfer

[LRFD Art. 5.9.5.4.4c]

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is given as:

$\Delta f_{pR2} = 30\%$ of $[20.0 - 0.4\,\Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$
[LRFD Art. 5.9.5.4.4c-1]

where the variables are the same as defined in Section A.2.7 expressed in ksi units

$\Delta f_{pR2} = 0.3[20.0 - 0.4(18.83) - 0.2(8.0 + 26.50)] = 1.670$ ksi

The instantaneous loss of prestress is estimated using the following expression:

$$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$$
$$= 18.83 + 1.980 = 20.81\text{ ksi}$$

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(18.83 + 1.98)}{202.5} = 10.28\% < 10.42\% \text{ (assumed value of initial prestress loss)}$$

Therefore, another trial is required assuming 10.28 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress losses due to concrete shrinkage (Δf_{pSR}) and initial steel relaxation (Δf_{pR1}). Therefore, the new trials will involve updating the losses due to elastic shortening (Δf_{pES}), creep of concrete (Δf_{pCR}), and steel relaxation after transfer (Δf_{pR2}).

Based on the initial prestress loss value of 10.28 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_i = (number of strands)(area of each strand)[0.8972(f_{pj})]

= 54(0.153)(0.8972)(202.5) = 1501.06 kips

Loss in prestress due to elastic shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1501.06}{788.4} + \frac{1501.06(19.12)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.12)}{260{,}403}$$

$$= 1.904 + 2.107 - 1.066 = 2.945 \text{ ksi}$$

E_{ci} = 4447.98 ksi

E_p = 28,500 ksi

Prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{4447.98}\right](2.945) = 18.87 \text{ ksi}$$

The loss in prestress due to creep of concrete is given as:
$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp} \geq 0$

The value of Δf_{cdp} depends on the dead load moments, superimposed dead load moments, and section properties. Thus, this value will not change with the change in initial prestress value and will be the same as calculated in Section A.2.7.2.3.
$\Delta f_{cdp} = 1.253$ ksi

$\Delta f_{pCR} = 12(2.945) - 7(1.253) = 26.57$ ksi

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is:
Δf_{pR2} = 30% of $[20.0 - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

$= 0.3[20.0 - 0.4(18.87) - 0.2(8.0 + 26.57)] = 1.661$ ksi

The instantaneous loss of prestress is estimated using the following expression:
$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$
$= 18.87 + 1.98 = 20.85$ ksi

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(18.87 + 1.98)}{202.5} = 10.30\% \approx 10.28\%$$ (assumed value of initial prestress loss)

A.2.7.2.5
Total Losses at Transfer

Total prestress loss at transfer
$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$
$= 18.87 + 1.98 = 20.85$ ksi

Effective initial prestress, $f_{pi} = 202.5 - 20.85 = 181.65$ ksi

P_i = Effective pretension after allowing for the initial prestress loss
= (number of strands)(area of each strand)(f_{pj})
= 54(0.153)(181.65) = 1500.79 kips

A.2.7.2.6
Total Losses at Service Loads

Total final loss in prestress
$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} + \Delta f_{pR2}$

Δf_{pES} = Prestress loss due to elastic shortening = 18.87 ksi

Δf_{pSR} = Prestress loss due to concrete shrinkage = 8.0 ksi

Δf_{pCR} = Prestress loss due to concrete creep = 26.57 ksi

Δf_{pR1} = Prestress loss due to steel relaxation before transfer = 1.98 ksi

Δf_{pR2} = Prestress loss due to steel relaxation after transfer = 1.661 ksi

Δf_{pT} = 18.87 + 8.0 + 26.57 + 1.98 + 1.661 = 57.08 ksi

The percent final loss is calculated using the following expression:

$$\%\Delta f_{pT} = \frac{100(\Delta f_{pT})}{f_{pj}}$$

$$= \frac{100(57.08)}{202.5} = 28.19\%$$

Effective final prestress
$f_{pe} = f_{pj} - \Delta f_{pT}$ = 202.5 – 57.08 = 145.42 ksi

Check prestressing stress limit at service limit state (defined in Section A.2.3): $f_{pe} \leq 0.8 f_{py}$

f_{py} = Yield strength of prestressing steel = 243 ksi

f_{pe} = 145.42 ksi < 0.8(243) = 194.4 ksi (O.K.)

Effective prestressing force after allowing for final prestress loss

P_{pe} = (number of strands)(area of each strand)(f_{pe})
= 54(0.153)(145.42) = 1201.46 kips

A.2.7.2.7 Final Stresses at Midspan

The required concrete strength at service ($f'_{c\text{-reqd.}}$) is updated based on the final stresses at the top and bottom fibers of the girder at the midspan section shown as follows.

Concrete stresses at the top fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress will be investigated for the following three cases using the Service I limit state shown as follows.

1) Concrete stress at the top fiber of the girder at the midspan section due to effective final prestress + permanent loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}$$

where:

f_{tf} = Concrete stress at the top fiber of the girder, ksi

M_{DCN} = Moment due to non-composite dead loads, k-ft.
= $M_g + M_S$

M_g = Moment due to girder self-weight = 1209.98 k-ft.

M_S = Moment due to slab weight = 1179.03 k-ft.

M_{DCN} = 1209.98 + 1179.03 = 2389.01 k-ft.

M_{DCC} = Moment due to composite dead loads except wearing surface load, k-ft.
= M_{barr}

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DCC} = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder = 8902.67 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder = 54,083.9 in.^{3}

$$f_{tf} = \frac{1201.46}{788.4} - \frac{1201.46(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12\text{ in./ft.})}{54,083.9}$$

= 1.524 – 2.580 + 3.220 + 0.077 = 2.241 ksi

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is 0.45 f'_c .

$$f'_{c\text{-reqd.}} = \frac{2241}{0.45} = 4980.0 \text{ psi} \quad \text{(controls)}$$

2) Concrete stress at the top fiber of the girder at the midspan section due to live load + 0.5×(effective final prestress + permanent loads)

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}\right)$$

where:

M_{LT} = Distributed moment due to HS 20-44 truck load, including dynamic load allowance = 1423.00 k-ft.

M_{LL} = Distributed moment due to lane load = 602.72 k-ft.

$$f_{tf} = \frac{(1423 + 602.72)(12\text{ in./ft.})}{54,083.9} + 0.5\left\{\frac{1201.46}{788.4} - \frac{1201.46(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12\text{ in./ft.})}{54,083.9}\right\}$$

= 0.449 + 0.5(1.524 – 2.580 + 3.220 + 0.077) = 1.570 ksi

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is 0.40 f'_c .

$$f'_{c\text{-reqd.}} = \frac{1570}{0.40} = 3925 \text{ psi}$$

3) Concrete stress at the top fiber of the girder at the midspan section due to effective prestress + permanent loads + transient loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW} + M_{LT} + M_{LL}}{S_{tg}}$$

$$f_{tf} = \frac{1201.46}{788.4} - \frac{1201.46(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64+188.64)(12\text{ in./ft.})}{54,083.9} + \frac{(1423.00+602.72)(12\text{ in./ft.})}{54,083.9}$$

$$= 1.524 – 2.580 + 3.220 + 0.077 + 0.449 = 2.690 \text{ ksi}$$

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is $0.60\,\phi_w\,f'_c$.

where ϕ_w is the reduction factor, applicable to thin-walled hollow rectangular compression members where the web or flange slenderness ratios are greater than 15.

[LRFD Art. 5.9.4.2.1]

The reduction factor ϕ_w is not defined for I-shaped girder cross sections and is taken as 1.0 in this design.

$$f'_{c\text{ -reqd.}} = \frac{2690}{0.60(1.0)} = 4483.33 \text{ psi}$$

Concrete stresses at the bottom fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress is investigated using Service III limit state as follows.

$f_{bf} = \dfrac{P_{pe}}{A} + \dfrac{P_{pe}\,e_c}{S_b} - f_b$ (f_b calculations are presented in Sec. A.2.6.1)

$$= \frac{1201.46}{788.4} + \frac{1201.46(19.12)}{10{,}521.33} - 4.125$$

$$= 1.524 + 2.183 – 4.125 = – 0.418 \text{ ksi}$$

The tensile stress limit in fully prestressed concrete members with bonded prestressing tendons, subjected to not worse than moderate corrosion conditions (assumed in this design example) at the service limit state after losses, is given by LRFD Table 5.9.4.2.2-1 as $0.19\sqrt{f'_c}$.

$$f'_{c\text{ -reqd.}} = 1000\left(\frac{0.418}{0.19}\right)^2 = 4840.0 \text{ psi}$$

The concrete strength at service is updated based on the final stresses at the midspan section under different loading combinations, as shown above. The governing required concrete strength at service is 4980 psi.

A.2.7.2.8 Initial Stresses at Hold-Down Point

Prestressing force after allowing for initial prestress loss

P_i = (number of strands)(area of strand)(effective initial prestress)

= 54(0.153)(181.65) = 1500.79 kips

(Section A.2.7.2.5 presents effective initial prestress calculations.)

Initial concrete stress at top fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in.
= 1222.22 k-ft. (see Section A.2.7.1.8)

$$f_{ti} = \frac{1500.79}{788.4} - \frac{1500.79\,(19.12)}{8902.67} + \frac{1222.22\,(12\text{ in./ft.})}{8902.67}$$

= 1.904 – 3.223 + 1.647 = 0.328 ksi

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1500.79}{788.4} + \frac{1500.79\,(19.12)}{10{,}521.33} - \frac{1222.22\,(12\text{ in./ft.})}{10{,}521.33}$$

= 1.904 + 2.727 – 1.394 = 3.237 ksi

Compressive stress limit for pretensioned members at transfer stage is 0.60 f'_{ci}. [LRFD Art.5.9.4.1.1]

$$f'_{ci\text{-reqd.}} = \frac{3237}{0.60} = 5395 \text{ psi}$$

A.2.7.2.9
Initial Stresses at Girder End

The initial tensile stress at the top fiber and compressive stress at the bottom fiber of the girder at the girder end section are minimized by harping the web strands at the girder end. Following TxDOT methodology (TxDOT 2001), the web strands are incrementally raised as a unit by 2 inches in each trial. The iterations are repeated until the top and bottom fiber stresses satisfy the allowable stress limits, or the centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder, in which case, the concrete strength at release is updated based on the governing stress. The position of the harped web strands, eccentricity of strands at the girder end, top and bottom fiber stresses at the girder end, and the corresponding required concrete strengths are summarized in Table A.2.7.1.

Table A.2.7.1. Summary of Top and Bottom Stresses at Girder End for Different Harped Strand Positions and Corresponding Required Concrete Strengths.

Distance of the Centroid of Topmost Row of Harped Web Strands from		Eccentricity of Prestressing Strands at Girder End (in.)	Top Fiber Stress (ksi)	Required Concrete Strength (ksi)	Bottom Fiber Stress (ksi)	Required Concrete Strength (ksi)
Bottom Fiber (in.)	Top Fiber (in.)					
10 (no harping)	44	19.12	-1.320	30.232	4.631	7.718
12	42	18.75	-1.257	27.439	4.578	7.630
14	40	18.38	-1.195	24.781	4.525	7.542
16	38	18.01	-1.132	22.259	4.472	7.454
18	36	17.64	-1.070	19.872	4.420	7.366
20	34	17.27	-1.007	17.620	4.367	7.278
22	32	16.90	-0.945	15.504	4.314	7.190
24	30	16.53	-0.883	13.523	4.261	7.102
26	28	16.16	-0.820	11.677	4.208	7.014
28	26	15.79	-0.758	9.967	4.155	6.926
30	24	15.42	-0.695	8.392	4.103	6.838
32	22	15.05	-0.633	6.952	4.050	6.750
34	20	14.68	-0.570	5.648	3.997	6.662
36	18	14.31	-0.508	4.479	3.944	6.574
38	16	13.93	-0.446	3.446	3.891	6.485
40	14	13.56	-0.383	2.548	3.838	6.397
42	12	13.19	-0.321	1.785	3.786	6.309
44	10	12.82	-0.258	1.157	3.733	6.221
46	8	12.45	-0.196	0.665	3.680	6.133
48	6	12.08	-0.133	0.309	3.627	6.045
50	4	11.71	-0.071	0.087	3.574	5.957
52	2	11.34	-0.008	0.001	3.521	5.869

The required concrete strengths used in Table A.2.7.1 are based on the allowable stress limits at transfer stage specified in LRFD Art. 5.9.4.1, presented as follows.

Allowable compressive stress limit = 0.60 f'_{ci}

For fully prestressed members, in areas with bonded reinforcement sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of 0.5f_y (f_y is the yield strength of nonprestressed reinforcement), not to exceed 30 ksi, the allowable tension at transfer stage is given as $0.24\sqrt{f'_{ci}}$.

From Table A.2.7.1, it is evident that the web strands are needed to be harped to the topmost position possible to control the bottom fiber stress at the girder end.

Detailed calculations for the case when 10 web strands (5 rows) are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder) are presented as follows.

Eccentricity of prestressing strands at the girder end (see Figure A.2.7.2)

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 6(10) + 2(52+50+48+46+44)}{54}$$

$$= 11.34 \text{ in.}$$

Concrete stress at the top fiber of the girder at the girder end at transfer stage:

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_e}{S_t}$$

$$= \frac{1500.79}{788.4} - \frac{1500.79\,(11.34)}{8902.67} = 1.904 - 1.912 = -0.008 \text{ ksi}$$

Tensile stress limit for fully prestressed concrete members with bonded reinforcement is $0.24\sqrt{f'_{ci}}$. [LRFD Art. 5.9.4.1]

$$f'_{ci\text{ -reqd.}} = 1000\left(\frac{0.008}{0.24}\right)^2 = 1.11 \text{ psi}$$

Concrete stress at the bottom fiber of the girder at the girder end at transfer stage:

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_e}{S_b}$$

$$= \frac{1500.79}{788.4} + \frac{1500.79\,(11.34)}{10{,}521.33} = 1.904 + 1.618 = 3.522 \text{ ksi}$$

Compressive stress limit for pretensioned members at transfer stage is 0.60 f'_{ci}. [LRFD Art. 5.9.4.1]

$$f'_{ci\text{-reqd.}} = \frac{3522}{0.60} = 5870 \text{ psi} \qquad \text{(controls)}$$

The required concrete strengths are updated based on the above results as follows.

Concrete strength at release, f'_{ci} = 5870 psi

Concrete strength at service, f'_c is greater of 4980 psi and f'_{ci}

f'_c = 5870 psi

A.2.7.3 Iteration 3

A third iteration is carried out to refine the prestress losses based on the updated concrete strengths. Based on the updated prestress losses, the concrete strength at release and at service will be further refined.

Number of strands = 54

Concrete strength at release, f'_{ci} = 5870 psi

A.2.7.3.1 Elastic Shortening

[LRFD Art. 5.9.5.2.3]

The loss in prestress due to elastic shortening in prestressed concrete members is given as:

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} \qquad \text{[LRFD Eq. 5.9.5.2.3a-1]}$$

where:

E_p = Modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = Modulus of elasticity of girder concrete at transfer, ksi

= $33{,}000(w_c)^{1.5}\sqrt{f'_{ci}}$ [LRFD Eq. 5.4.2.4-1]

w_c = Unit weight of concrete (must be between 0.09 and 0.155 kcf for LRFD Eq. 5.4.2.4-1 to be applicable)

= 0.150 kcf

f'_{ci} = Compressive strength of girder concrete at release
= 5.870 ksi

E_{ci} = $[33{,}000(0.150)^{1.5}\sqrt{5.870}\,]$ = 4644.83 ksi

f_{cgp} = Sum of concrete stresses at the center of gravity of the prestressing steel due to prestressing force at transfer and the self-weight of the member at sections of maximum moment, ksi

$$= \frac{P_i}{A} + \frac{P_i\, e_c^2}{I} - \frac{(M_g)e_c}{I}$$

A = Area of girder cross section = 788.4 in.^{2}

I = Moment of inertia of the non-composite section
= 260,403 in.^{4}

e_c = Eccentricity of the prestressing strands at the midspan
= 19.12 in.

M_g = Moment due to girder self-weight at midspan, k-ft.
= 1209.98 k-ft.

P_i = Pretension force after allowing for the initial losses, kips

As the initial losses are dependent on the elastic shortening and the initial steel relaxation loss, which are yet to be determined, the initial loss value of 10.30 percent obtained in the last trial (Iteration 2) is taken as an initial estimate for initial loss in prestress for this iteration.

P_i = (number of strands)(area of strand)[0.897(f_{pj})]
= 54(0.153)(0.897)(202.5) = 1500.73 kips

$$f_{cgp} = \frac{1500.73}{788.4} + \frac{1500.73\ (19.12)^2}{260{,}403} - \frac{1209.98(12\text{ in./ft.})(19.12)}{260{,}403}$$

= 1.904 + 2.107 – 1.066 = 2.945 ksi

The prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{4644.83}\right](2.945) = 18.07\text{ ksi}$$

A.2.7.3.2 Concrete Shrinkage

[LRFD Art. 5.9.5.4.2]

The loss in prestress due to concrete shrinkage (Δf_{pSR}) depends on the relative humidity only. The change in compressive strength of girder concrete at release (f'_{ci}) does not affect the prestress loss due to concrete shrinkage. It will remain the same as calculated in Section A.2.7.1.2.

Δf_{pSR} = 8.0 ksi

A.2.7.3.3 Creep of Concrete

[LRFD Art. 5.9.5.4.3]

The loss in prestress due to creep of concrete is given as:

$$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp} \geq 0 \qquad \text{[LRFD Eq. 5.9.5.4.3-1]}$$

where:

Δf_{cdp} = Change in concrete stress at the center of gravity of the prestressing steel due to permanent loads except the dead load present at the time the prestress force is applied calculated at the same section as f_{cgp}.

$$= \frac{M_S\, e_c}{I} + \frac{M_{SDL}(y_{bc} - y_{bs})}{I_c}$$

M_S = Moment due to the slab weight at midspan section = 1179.03 k-ft.

M_{SDL} = Moment due to superimposed dead load = $M_{barr} + M_{DW}$

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

M_{SDL} = 160.64 + 188.64 = 349.28 k-ft.

y_{bc} = Distance from the centroid of the composite section to the extreme bottom fiber of the precast girder = 41.157 in.

y_{bs} = Distance from centroid of the prestressing strands at midspan to the bottom fiber of the girder = 24.75 – 19.12 = 5.63 in.

I = Moment of inertia of the non-composite section = 260,403 in.^{4}

I_c = Moment of inertia of composite section = 694,599.5 in.^{4}

$$\Delta f_{cdp} = \frac{1179.03(12\text{ in./ft.})(19.12)}{260,403} + \frac{(349.28)(12\text{ in./ft.})(41.157-5.63)}{694,599.5}$$

$$= 1.039 + 0.214 = 1.253 \text{ ksi}$$

Prestress loss due to creep of concrete is:
$\Delta f_{pCR} = 12(2.945) - 7(1.253) = 26.57$ ksi

A.2.7.3.4 Relaxation of Prestressing Strands

[LRFD Art. 5.9.5.4.4]

A.2.7.3.4.1 Relaxation at Transfer

[LRFD Art. 5.9.5.4.4b]

The loss in prestress due to relaxation of steel at transfer (Δf_{pR1}) depends on the time from stressing to transfer of prestress (t), the initial stress in tendon at the end of stressing (f_{pj}), and the yield strength of prestressing steel (f_{py}). The change in compressive strength of girder concrete at release (f'_{ci}) and number of strands does not affect the prestress loss due to relaxation of steel before transfer. It will remain the same as calculated in Section A.2.7.1.4.1.

$\Delta f_{pR1} = 1.98$ ksi

A.2.7.3.4.2 Relaxation after Transfer

[LRFD Art. 5.9.5.4.4c]

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is given as:

$\Delta f_{pR2} = 30\%$ of $[20.0 - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

[LRFD Art. 5.9.5.4.4c-1]

where the variables are the same as defined in Section A.2.7 expressed in ksi units

$\Delta f_{pR2} = 0.3[20.0 - 0.4(18.07) - 0.2(8.0 + 26.57)] = 1.757$ ksi

The instantaneous loss of prestress is estimated using the following expression:

$$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$$
$$= 18.07 + 1.980 = 20.05 \text{ ksi}$$

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(18.07 + 1.98)}{202.5} = 9.90\% < 10.30\% \text{ (assumed value of initial prestress loss)}$$

Therefore, another trial is required assuming 9.90 percent initial prestress loss.

The change in initial prestress loss will not affect the prestress losses due to concrete shrinkage (Δf_{pSR}) and initial steel relaxation (Δf_{pR1}). Therefore, the new trials will involve updating the losses due to elastic shortening (Δf_{pES}), creep of concrete (Δf_{pCR}), and steel relaxation after transfer (Δf_{pR2}).

Based on the initial prestress loss value of 9.90 percent, the pretension force after allowing for the initial losses is calculated as follows.

P_i = (number of strands)(area of each strand)[0.901(f_{pj})]

= 54(0.153)(0.901)(202.5) = 1507.42 kips

Loss in prestress due to elastic shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

$$f_{cgp} = \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{(M_g)e_c}{I}$$

$$= \frac{1507.42}{788.4} + \frac{1507.42(19.12)^2}{260{,}403} - \frac{1209.98(12 \text{ in./ft.})(19.12)}{260{,}403}$$

$$= 1.912 + 2.116 - 1.066 = 2.962 \text{ ksi}$$

E_{ci} = 4644.83 ksi

E_p = 28,500 ksi

Prestress loss due to elastic shortening is:

$$\Delta f_{pES} = \left[\frac{28{,}500}{4644.83}\right](2.962) = 18.17 \text{ ksi}$$

The loss in prestress due to creep of concrete is given as:
$\Delta f_{pCR} = 12f_{cgp} - 7\Delta f_{cdp} \geq 0$

The value of Δf_{cdp} depends on the dead load moments, superimposed dead load moments, and section properties. Thus, this value will not change with the change in initial prestress value and will be the same as calculated in Section A.2.7.2.3.
$\Delta f_{cdp} = 1.253$ ksi

$\Delta f_{pCR} = 12(2.962) - 7(1.253) = 26.773$ ksi

For pretensioned members with low-relaxation strands, the prestress loss due to relaxation of steel after transfer is:
Δf_{pR2} = 30% of [20.0 – 0.4 Δf_{pES} – 0.2($\Delta f_{pSR} + \Delta f_{pCR}$)

$= 0.3[20.0 - 0.4(18.17) - 0.2(8.0 + 26.773)] = 1.733$ ksi

The instantaneous loss of prestress is estimated using the following expression:
$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$
$= 18.17 + 1.98 = 20.15$ ksi

The percent instantaneous loss is calculated using the following expression:

$$\%\Delta f_{pi} = \frac{100(\Delta f_{pES} + \Delta f_{pR1})}{f_{pj}}$$

$$= \frac{100(18.17 + 1.98)}{202.5} = 9.95\% \approx 9.90\%$$ (assumed value of initial prestress loss)

_A.2.7.3.5_
Total Losses at Transfer

Total prestress loss at transfer
$\Delta f_{pi} = \Delta f_{pES} + \Delta f_{pR1}$
$= 18.17 + 1.98 = 20.15$ ksi

Effective initial prestress, $f_{pi} = 202.5 - 20.15 = 182.35$ ksi

P_i = Effective pretension after allowing for the initial prestress loss

= (number of strands)(area of each strand)(f_{pi})

= 54(0.153)(182.35) = 1506.58 kips

A.2.7.3.6 Total Losses at Service Loads

Total final loss in prestress

$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} + \Delta f_{pR2}$

Δf_{pES} = Prestress loss due to elastic shortening = 18.17 ksi

Δf_{pSR} = Prestress loss due to concrete shrinkage = 8.0 ksi

Δf_{pCR} = Prestress loss due to concrete creep = 26.773 ksi

Δf_{pR1} = Prestress loss due to steel relaxation before transfer = 1.98 ksi

Δf_{pR2} = Prestress loss due to steel relaxation after transfer = 1.733 ksi

Δf_{pT} = 18.17 + 8.0 + 26.773 + 1.98 + 1.773 = 56.70 ksi

The percent final loss is calculated using the following expression:

$$\%\Delta f_{pT} = \frac{100(\Delta f_{pT})}{f_{pj}}$$

$$= \frac{100(56.70)}{202.5} = 28.0\%$$

Effective final prestress

$f_{pe} = f_{pj} - \Delta f_{pT} = 202.5 - 56.70 = 145.80$ ksi

Check prestressing stress limit at service limit state (defined in Section A.2.3): $f_{pe} \leq 0.8 f_{py}$

f_{py} = Yield strength of prestressing steel = 243 ksi

f_{pe} = 145.80 ksi < 0.8(243) = 194.4 ksi (O.K.)

Effective prestressing force after allowing for final prestress loss

P_{pe} = (number of strands)(area of each strand)(f_{pe})

= 54(0.153)(145.80) = 1204.60 kips

A.2.7.3.7 Final Stresses at Midspan

The required concrete strength at service ($f'_{c\text{-reqd.}}$) is updated based on the final stresses at the top and bottom fibers of the girder at the midspan section shown as follows.

Concrete stresses at the top fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress will be investigated for the following three cases using the Service I limit state shown as follows.

1) Concrete stress at the top fiber of the girder at the midspan section due to effective final prestress + permanent loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}$$

where:

f_{tf} = Concrete stress at the top fiber of the girder, ksi

M_{DCN} = Moment due to non-composite dead loads, k-ft.
= $M_g + M_S$

M_g = Moment due to girder self-weight = 1209.98 k-ft.

M_S = Moment due to slab weight = 1179.03 k-ft.

M_{DCN} = 1209.98 + 1179.03 = 2389.01 k-ft.

M_{DCC} = Moment due to composite dead loads except wearing surface load, k-ft.
= M_{barr}

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DCC} = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder = 8902.67 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder = 54,083.9 in.^{3}

$$f_{tf} = \frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12\text{ in./ft.})}{54,083.9}$$

$$= 1.528 - 2.587 + 3.220 + 0.077 = 2.238 \text{ ksi}$$

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is 0.45 f'_c.

$$f'_{c\text{ -reqd.}} = \frac{2238}{0.45} = 4973.33 \text{ psi} \quad \text{(controls)}$$

2) Concrete stress at the top fiber of the girder at the midspan section due to live load + 0.5×(effective final prestress + permanent loads)

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}\right)$$

where:

M_{LT} = Distributed moment due to HS 20-44 truck load including dynamic load allowance = 1423.00 k-ft.

M_{LL} = Distributed moment due to lane load = 602.72 k-ft.

$$f_{tf} = \frac{(1423.00 + 602.72)(12\text{ in./ft.})}{54,083.9} + 0.5\left\{\frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12\text{ in./ft.})}{54,083.9}\right\}$$

$$= 0.449 + 0.5(1.528 - 2.587 + 3.220 + 0.077) = 1.568 \text{ ksi}$$

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is 0.40 f'_c.

$$f'_{c\text{ -reqd.}} = \frac{1568}{0.40} = 3920 \text{ psi}$$

3) Concrete stress at the top fiber of the girder at the midspan section due to effective prestress + permanent loads + transient loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\, e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW} + M_{LT} + M_{LL}}{S_{tg}}$$

$$f_{tf} = \frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12\text{ in./ft.})}{54,083.9} + \frac{(1423.00 + 602.72)(12\text{ in./ft.})}{54,083.9}$$

= 1.528 – 2.587 + 3.220 + 0.077 + 0.449 = 2.687 ksi

Compressive stress limit for this service load combination given in LRFD Table 5.9.4.2.1-1 is $0.60\,\phi_w\, f'_c$.

where ϕ_w is the reduction factor, applicable to thin-walled hollow rectangular compression members where the web or flange slenderness ratios are greater than 15.

[LRFD Art. 5.9.4.2.1]

The reduction factor ϕ_w is not defined for I-shaped girder cross sections and is taken as 1.0 in this design.

$$f'_{c\text{ -reqd.}} = \frac{2687}{0.60(1.0)} = 4478 \text{ psi}$$

Concrete stresses at the bottom fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress will be investigated using Service III limit state as follows.

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - f_b$$ (f_b calculations are presented in Sec. A.2.6.1)

$$= \frac{1204.60}{788.4} + \frac{1204.60(19.12)}{10,521.33} - 4.125$$

= 1.528 + 2.189 – 4.125 = – 0.408 ksi

Tensile stress limit in fully prestressed concrete members with bonded prestressing tendons, subjected to not worse than moderate corrosion conditions (assumed in this design example) at service limit state after losses, is given by LRFD Table 5.9.4.2.2-1 as $0.19\sqrt{f'_c}$.

$$f'_{c\text{ -reqd.}} = 1000\left(\frac{0.408}{0.19}\right)^2 = 4611 \text{ psi}$$

The concrete strength at service is updated based on the final stresses at the midspan section under different loading combinations as shown above. The governing required concrete strength at service is 4973 psi.

A.2.7.3.8 Initial Stresses at Hold-Down Point

Prestressing force after allowing for initial prestress loss

P_i = (number of strands)(area of strand)(effective initial prestress)

= 54(0.153)(182.35) = 1506.58 kips

(See Section A.2.7.3.5 for effective initial prestress calculations.)

Initial concrete stress at top fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_c}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in.

= 1222.22 k-ft. (see Section A.2.7.1.8)

$$f_{ti} = \frac{1506.58}{788.4} - \frac{1506.58\,(19.12)}{8902.67} + \frac{1222.22\,(12\text{ in./ft.})}{8902.67}$$

= 1.911 – 3.236 + 1.647 = 0.322 ksi

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1506.58}{788.4} + \frac{1506.58\,(19.12)}{10{,}521.33} - \frac{1222.22\,(12\text{ in./ft.})}{10{,}521.33}$$

= 1.911 + 2.738 – 1.394 = 3.255 ksi

Compressive stress limit for pretensioned members at transfer stage is 0.60 f'_{ci}. [LRFD Art.5.9.4.1.1]

$$f'_{ci\text{ -reqd.}} = \frac{3255}{0.60} = 5425 \text{ psi}$$

A.2.7.3.9
Initial Stresses at Girder End

The eccentricity of the prestressing strands at the girder end when 10 web strands are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder) is calculated as follows (see Fig. A.2.7.2).

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 6(10) + 2(52+50+48+46+44)}{54}$$
$$= 11.34 \text{ in.}$$

Concrete stress at the top fiber of the girder at the girder end at transfer stage:

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_e}{S_t}$$

$$= \frac{1506.58}{788.4} - \frac{1506.58\,(11.34)}{8902.67} = 1.911 - 1.919 = -0.008 \text{ ksi}$$

Tensile stress limit for fully prestressed concrete members with bonded reinforcement is $0.24\sqrt{f'_{ci}}$. [LRFD Art. 5.9.4.1]

$$f'_{ci\text{ -reqd.}} = 1000\left(\frac{0.008}{0.24}\right)^2 = 1.11 \text{ psi}$$

Concrete stress at the bottom fiber of the girder at the girder end at transfer:

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_e}{S_b}$$
$$= \frac{1506.58}{788.4} + \frac{1506.58\,(11.34)}{10{,}521.33} = 1.911 + 1.624 = 3.535 \text{ ksi}$$

Compressive stress limit for pretensioned members at transfer is $0.60\, f'_{ci}$. [LRFD Art. 5.9.4.1]

$$f'_{ci\text{ -reqd.}} = \frac{3535}{0.60} = 5892 \text{ psi} \quad \text{(controls)}$$

The required concrete strengths are updated based on the above results as follows.

Concrete strength at release, f'_{ci} = 5892 psi

Concrete strength at service, f'_c is greater of 4973 psi and f'_{ci}

f'_c = 5892 psi

The difference in the required concrete strengths at release and at service obtained from Iterations 2 and 3 is almost 20 psi. Hence, the concrete strengths have sufficiently converged, and another iteration is not required.

Therefore, provide:

f'_{ci} = 5892 psi (as compared to 5455 psi obtained for the Standard design example, an increase of 8 percent)

f'_c = 5892 psi (as compared to 5583 psi obtained for the Standard design example, an increase of 5.5 percent)

54 – 0.5 in. diameter, 10 draped at the end, GR 270 low-relaxation strands (as compared to 50 strands obtained for the Standard design example, an increase of 8 percent).

The final strand patterns at the midspan section and at the girder ends are shown in Figures A.2.7.1 and A.2.7.2. The longitudinal strand profile is shown in Figure A.2.7.3.

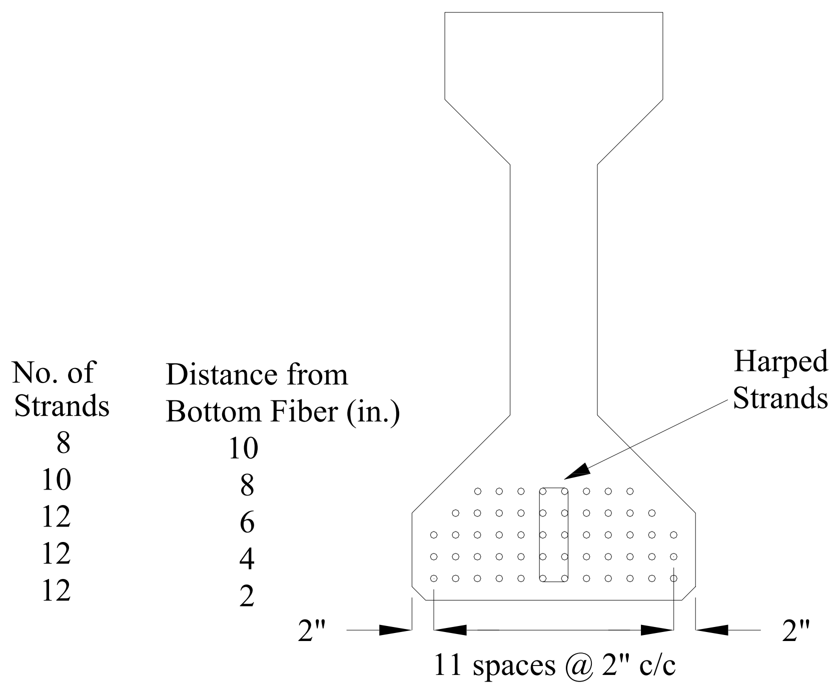

Figure A.2.7.1. Final Strand Pattern at Midspan Section.

No. of Strands	Distance from Bottom Fiber (in.)
2	52
2	50
2	48
2	46
2	44

No. of Strands	Distance from Bottom Fiber (in.)
6	10
8	8
10	6
10	4
10	2

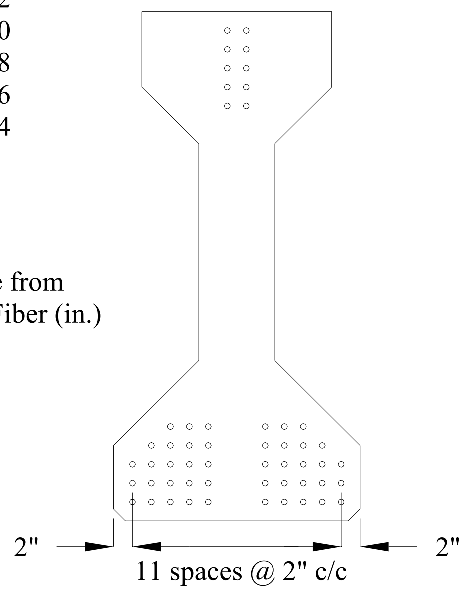

Figure A.2.7.2. Final Strand Pattern at Girder End.

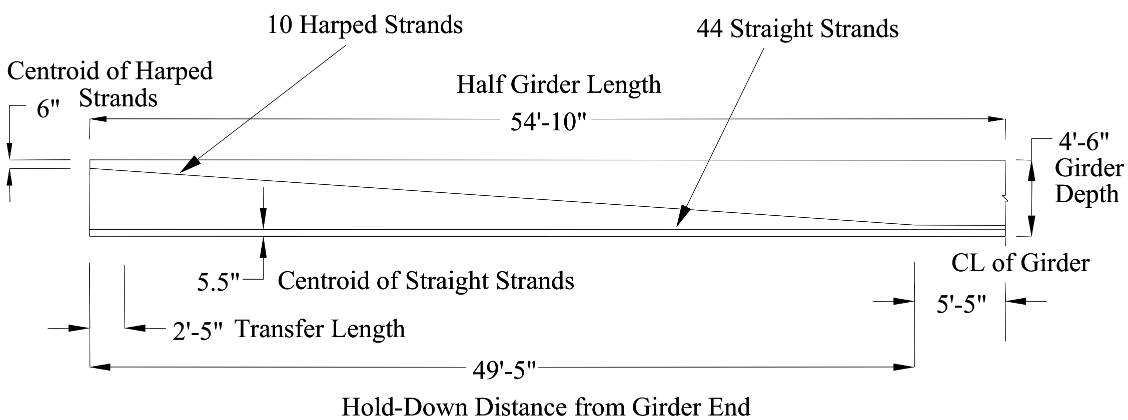

Figure A.2.7.3. Longitudinal Strand Profile (half of the girder length is shown).

The distance between the centroid of the 10 harped strands and the top fiber of the girder at the girder end

$$= \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \text{ in.}$$

The distance between the centroid of the 10 harped strands and the bottom fiber of the girder at the harp points

$$= \frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10)}{10} = 6 \text{ in.}$$

Transfer length distance from girder end = 60(strand diameter)

[LRFD Art. 5.8.2.3]

Transfer length = 60(0.50) = 30 in. = 2 ft.-6 in.

The distance between the centroid of 10 harped strands and the top of the girder at the transfer length section

$$= 6 \text{ in.} + \frac{(54 \text{ in.} - 6 \text{ in.} - 6 \text{ in.})}{49.4 \text{ ft.}}(2.5 \text{ ft.}) = 8.13 \text{ in.}$$

The distance between the centroid of the 44 straight strands and the bottom fiber of the girder at all locations

$$= \frac{10(2) + 10(4) + 10(6) + 8(8) + 6(10)}{44} = 5.55 \text{ in.}$$

A.2.8 STRESS SUMMARY

A.2.8.1 Concrete Stresses at Transfer

A.2.8.1.1 Allowable Stress Limits

[LRFD Art. 5.9.4]

The allowable stress limits at transfer for fully prestressed components, specified by the LRFD Specifications, are as follows.

Compression: $0.6\,f'_{ci} = 0.6(5892) = +3535$ psi = +3.535 ksi

Tension: The maximum allowable tensile stress for fully prestressed components is specified as follows:

- In areas other than the precompressed tensile zone and without bonded reinforcement: $0.0948\sqrt{f'_{ci}} \le 0.2$ ksi

  $0.0948\sqrt{f'_{ci}} = 0.0948\sqrt{5.892} = 0.23$ ksi > 0.2 ksi

  Allowable tension without bonded reinforcement = – 0.2 ksi

- In areas with bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of $0.5f_y$, not to exceed 30 ksi (see LRFD C 5.9.4.1.2):

  $0.24\sqrt{f'_{ci}} = 0.24\sqrt{5.892} = -0.582$ ksi (tension)

A.2.8.1.2 Stresses at Girder Ends

Stresses at the girder ends are checked only at transfer, because it almost always governs.

The eccentricity of the prestressing strands at the girder end when 10 web strands are harped to the topmost location (centroid of the topmost row of harped strands is at a distance of 2 inches from the top fiber of the girder) is calculated as follows (see Fig. A.2.7.2).

$$e_e = 24.75 - \frac{10(2+4+6) + 8(8) + 6(10) + 2(52+50+48+46+44)}{54}$$

$$= 11.34 \text{ in.}$$

Prestressing force after allowing for initial prestress loss

P_i = (number of strands)(area of strand)(effective initial prestress)

= 54(0.153)(182.35) = 1506.58 kips

(Effective initial prestress calculations are presented in Section A.2.7.3.5.)

Concrete stress at the top fiber of the girder at the girder end at transfer stage:

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e_e}{S_t}$$

$$= \frac{1506.58}{788.4} - \frac{1506.58(11.34)}{8902.67} = 1.911 - 1.919 = -0.008 \text{ ksi}$$

Allowable tension without additional bonded reinforcement is – 0.20 ksi < – 0.008 ksi (reqd.). (O.K.)

(The additional bonded reinforcement is not required in this case, but where necessary, required area of reinforcement can be calculated using LRFD C 5.9.4.1.2.)

Concrete stress at the bottom fiber of the girder at the girder end at transfer stage:

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_e}{S_b}$$

$$= \frac{1506.58}{788.4} + \frac{1506.58\,(11.34)}{10{,}521.33} = 1.911 + 1.624 = +3.535 \text{ ksi}$$

Allowable compression: +3.535 ksi = +3.535 ksi (reqd.) (O.K.)

A.2.8.1.3 Stresses at Transfer Length Section

Stresses at transfer length are checked only at release, because it almost always governs.

Transfer length = 60(strand diameter) [LRFD Art. 5.8.2.3]
= 60(0.5) = 30 in. = 2 ft.-6 in.

The transfer length section is located at a distance of 2 ft.-6 in. from the end of the girder or at a point 1 ft.-11.5 in. from the centerline of the bearing support, as the girder extends 6.5 in. beyond the bearing centerline. Overall girder length of 109 ft.-8 in. is considered for the calculation of bending moment at the transfer length section.

Moment due to girder self-weight, $M_g = 0.5wx(L - x)$

where:

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Transfer length distance from girder end = 2.5 ft.

$M_g = 0.5(0.821)(2.5)(109.67 - 2.5) = 109.98$ k–ft.

Eccentricity of prestressing strands at transfer length section

$$e_t = e_c - (e_c - e_e)\,\frac{(49.404 - x)}{49.404}$$

where:

e_c = Eccentricity of prestressing strands at midspan = 19.12 in.

e_e = Eccentricity of prestressing strands at girder end = 11.34 in.

x = Distance of transfer length section from girder end = 2.5 ft.

$$e_t = 19.12 - (19.12 - 11.34)\,\frac{(49.404 - 2.5)}{49.404} = 11.73 \text{ in.}$$

Initial concrete stress at top fiber of the girder at the transfer length section due to self-weight of the girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_t}{S_t} + \frac{M_g}{S_t}$$

$$= \frac{1506.58}{788.4} - \frac{1506.58\,(11.73)}{8902.67} + \frac{109.98\,(12\text{ in./ft.})}{8902.67}$$

= 1.911 – 1.985 + 0.148 = +0.074 ksi

Allowable compression: +3.535 ksi >> 0.074 ksi (reqd.) (O.K.)

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_t}{S_b} - \frac{M_g}{S_b}$$

$$= \frac{1506.58}{788.4} + \frac{1506.58\,(11.73)}{10{,}521.33} - \frac{109.98\,(12\text{ in./ft.})}{10{,}521.33}$$

= 1.911 + 1.680 – 0.125 = 3.466 ksi

Allowable compression: +3.535 ksi > 3.466 ksi (reqd.) (O.K.)

A.2.8.1.4 Stresses at Hold-Down Points

The eccentricity of the prestressing strands at the harp points is the same as at midspan.

$e_{harp} = e_c$ = 19.12 in.

Initial concrete stress at top fiber of the girder at hold-down point due to self-weight of the girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_{harp}}{S_t} + \frac{M_g}{S_t}$$

where:

M_g = Moment due to girder self-weight at hold-down point based on overall girder length of 109 ft.-8 in. = 1222.22 k-ft. (see Section A.2.7.1.8)

$$f_{ti} = \frac{1506.58}{788.4} - \frac{1506.58\,(19.12)}{8902.67} + \frac{1222.22\,(12\text{ in./ft.})}{8902.67}$$

= 1.911 – 3.236 + 1.647 = 0.322 ksi

Allowable compression: +3.535 ksi >> 0.322 ksi (reqd.) (O.K.)

Initial concrete stress at bottom fiber of the girder at hold-down point due to self-weight of the girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_{harp}}{S_b} - \frac{M_g}{S_b}$$

$$= \frac{1506.58}{788.4} + \frac{1506.58\,(19.12)}{10{,}521.33} - \frac{1222.22\,(12\text{ in./ft.})}{10{,}521.33}$$

= 1.911 + 2.738 – 1.394 = 3.255 ksi

Allowable compression: +3.535 ksi > 3.255 ksi (reqd.) (O.K.)

A.2.8.1.5 Stresses at Midspan

Bending moment due to girder self-weight at midspan section based on overall girder length of 109 ft.-8 in.

$$M_g = 0.5wx(L - x)$$

where:

w = Self-weight of the girder = 0.821 kips/ft.

L = Overall girder length = 109.67 ft.

x = Half the girder length = 54.84 ft.

M_g = 0.5(0.821)(54.84)(109.67 – 54.84) = 1234.32 k-ft.

Initial concrete stress at top fiber of the girder at midspan section due to self-weight of girder and effective initial prestress

$$f_{ti} = \frac{P_i}{A} - \frac{P_i\, e_c}{S_t} + \frac{M_g}{S_t}$$

$$= \frac{1506.58}{788.4} - \frac{1506.58\,(19.12)}{8902.67} + \frac{1234.32\,(12\text{ in./ft.})}{8902.67}$$

= 1.911 – 3.236 + 1.664 = +0.339 ksi

Allowable compression: +3.535 ksi >> +0.339 ksi (reqd.) (O.K.)

Initial concrete stress at bottom fiber of the girder at midspan section due to self-weight of the girder and effective initial prestress

$$f_{bi} = \frac{P_i}{A} + \frac{P_i\, e_c}{S_b} - \frac{M_g}{S_b}$$

$$= \frac{1506.58}{788.4} + \frac{1506.58\,(19.12)}{10{,}521.33} - \frac{1234.32\,(12\text{ in./ft.})}{10{,}521.33}$$

= 1.911 + 2.738 – 1.408 = 3.241 ksi

Allowable compression: +3.535 ksi > 3.241 ksi (reqd.) (O.K.)

A.2.8.1.6 Stress Summary at Transfer

Allowable Stress Limits:

Compression: + 3.535 ksi

Tension: – 0.20 ksi without additional bonded reinforcement

– 0.582 ksi with additional bonded reinforcement

Stresses due to effective initial prestress and self-weight of the girder:

Location	Top of girder f_t (ksi)	Bottom of girder f_b (ksi)
Girder end	–0.008	+3.535
Transfer length section	+0.074	+3.466
Hold-down points	+0.322	+3.255
Midspan	+0.339	+3.241

A.2.8.2 Concrete Stresses at Service Loads

A.2.8.2.1 Allowable Stress Limits

[LRFD Art. 5.9.4.2]

The allowable stress limits at service load after losses have occurred, specified by the LRFD Specifications, are presented as follows.

Compression:

Case (I): For stresses due to sum of effective prestress and permanent loads

$0.45\,f_c' = 0.45(5892)/1000 = +2.651$ ksi (for precast girder)

$0.45\,f_c' = 0.45(4000)/1000 = +1.800$ ksi (for slab)

(Note that the allowable stress limit for this case is specified as $0.40\,f_c'$ in Standard Specifications.)

Case (II): For stresses due to live load and one-half the sum of effective prestress and permanent loads

$0.40\,f_c' = 0.40(5892)/1000 = +2.356$ ksi (for precast girder)

$0.40\,f_c' = 0.40(4000)/1000 = +1.600$ ksi (for slab)

Case (III): For stresses due to sum of effective prestress, permanent loads, and transient loads

$$0.60\,f_c' = 0.60(5892)/1000 = +3.535 \text{ ksi (for precast girder)}$$

$$0.60\,f_c' = 0.60(4000)/1000 = +2.400 \text{ ksi (for slab)}$$

Tension: For components with bonded prestressing tendons that are subjected to not worse than moderate corrosion conditions, for stresses due to load combination Service III

$$0.19\sqrt{f_c'} = 0.19\sqrt{5.892} = -\,0.461 \text{ ksi}$$

A.2.8.2.2 Final Stresses at Midspan

Effective prestressing force after allowing for final prestress loss

P_{pe} = (number of strands)(area of each strand)(f_{pe})

= 54(0.153)(145.80) = 1204.60 kips

(Calculations for effective final prestress (f_{pe}) are shown in Section A.2.7.3.6.)

Concrete stresses at the top fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress will be investigated for the following three cases using Service I limit state shown as follows.

Case (I): Concrete stress at the top fiber of the girder at the midspan section due to the sum of effective final prestress and permanent loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}$$

where:

f_{tf} = Concrete stress at the top fiber of the girder, ksi

M_{DCN} = Moment due to non-composite dead loads, k-ft.
= $M_g + M_S$

M_g = Moment due to girder self-weight = 1209.98 k-ft.

M_S = Moment due to slab weight = 1179.03 k-ft.

M_{DCN} = 1209.98 + 1179.03 = 2389.01 k-ft.

M_{DCC} = Moment due to composite dead loads except wearing surface load, k-ft. = M_{barr}

M_{barr} = Moment due to barrier weight = 160.64 k-ft.

M_{DCC} = 160.64 k-ft.

M_{DW} = Moment due to wearing surface load = 188.64 k-ft.

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder = 8902.67 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder = 54,083.9 in.^{3}

$$f_{tf} = \frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12 \text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12 \text{ in./ft.})}{54,083.9}$$

= 1.528 – 2.587 + 3.220 + 0.077 = +2.238 ksi

Allowable compression: +2.651 ksi > +2.238 ksi (reqd.) (O.K.)

Case (II): Concrete stress at the top fiber of the girder at the midspan section due to the live load and one-half the sum of effective final prestress and permanent loads

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW}}{S_{tg}}\right)$$

where:

M_{LT} = Distributed moment due to HS 20-44 truck load including dynamic load allowance = 1423.00 k-ft.

M_{LL} = Distributed moment due to lane load = 602.72 k-ft.

$$f_{tf} = \frac{(1423.00 + 602.72)(12 \text{ in./ft.})}{54,083.9} + 0.5\left\{\frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12 \text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64)(12 \text{ in./ft.})}{54,083.9}\right\}$$

= 0.449 + 0.5(1.528 – 2.587 + 3.220 + 0.077) = 1.568 ksi

Allowable compression: +2.356 ksi > +1.568 ksi (reqd.) (O.K.)

Case (III): Concrete stress at the top fiber of the girder at the midspan section due to the sum of effective final prestress, permanent loads, and transient loads

$$f_{tf} = \frac{P_{pe}}{A} - \frac{P_{pe}\,e_c}{S_t} + \frac{M_{DCN}}{S_t} + \frac{M_{DCC} + M_{DW} + M_{LT} + M_{LL}}{S_{tg}}$$

$$= \frac{1204.60}{788.4} - \frac{1204.60(19.12)}{8902.67} + \frac{(2389.01)(12\text{ in./ft.})}{8902.67} + \frac{(160.64 + 188.64 + 1423.00 + 602.72)(12\text{ in./ft.})}{54{,}083.9}$$

= 1.528 – 2.587 + 3.220 + 0.527 = 2.688 ksi

Allowable compression: +3.535 ksi > 2.688 ksi (reqd.) (O.K.)

Concrete stresses at the bottom fiber of the girder at the midspan section due to transient loads, permanent loads, and effective final prestress is investigated using Service III limit state as follows.

$$f_{bf} = \frac{P_{pe}}{A} + \frac{P_{pe}\,e_c}{S_b} - \frac{M_{DCN}}{S_b} - \frac{M_{DCC} + M_{DW} + 0.8(M_{LT} + M_{LL})}{S_{bc}}$$

where:

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder = 10,521.33 in.^{3}

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder = 16,876.83 in.^{3}

$$f_{bf} = \frac{1204.60}{788.4} + \frac{1204.60(19.12)}{10{,}521.33} - \frac{(2389.01)(12\text{ in./ft.})}{10{,}521.33} - \frac{[160.64 + 188.64 + 0.8(1423.00 + 602.72)](12\text{ in./ft.})}{16{,}876.83}$$

= 1.528 + 2.189 – 2.725 – 1.401 = – 0.409 ksi

Allowable tension: – 0.461 ksi < – 0.409 ksi (reqd.) (O.K.)

Superimposed dead loads and live loads contribute to the stresses at the top of the slab calculated as follows.

Case (I): Superimposed dead load effect

Concrete stress at the top fiber of the slab at midspan section due to superimposed dead loads

$$f_t = \frac{M_{DCC} + M_{DW}}{S_{tc}}$$

$$= \frac{(160.64 + 188.64)(12 \text{ in./ft.})}{33{,}325.31} = 0.126 \text{ ksi}$$

Allowable compression: +1.800 ksi >> +0.126 ksi (reqd.) (O.K.)

Case (II): Live load + 0.5(superimposed dead loads)

Concrete stress at the top fiber of the slab at midspan section due to sum of live loads and one-half the superimposed dead loads

$$f_t = \frac{M_{LT} + M_{LL} + 0.5(M_{DCC} + M_{DW})}{S_{tc}}$$

$$= \frac{[1423.00 + 602.72 + 0.5(160.64 + 188.64)](12 \text{ in./ft.})}{33{,}325.31}$$

$$= +0.792 \text{ ksi}$$

Allowable compression: +1.600 ksi > +0.792 ksi (reqd.) (O.K.)

Case (III): Superimposed dead loads + Live load

Concrete stress at the top fiber of the slab at midspan section due to sum of permanent loads and live load

$$f_t = \frac{M_{LT} + M_{LL} + M_{DCC} + M_{DW}}{S_{tc}}$$

$$= \frac{[1423.00 + 602.72 + 160.64 + 188.64](12 \text{ in./ft.})}{33{,}325.31} = +0.855 \text{ ksi}$$

Allowable compression: +2.400 ksi > +0.855 ksi (reqd.) (O.K.)

A.2.8.2.3
Summary of Stresses at Service Loads

The final stresses at the top and bottom fiber of the girder and at the top fiber of the slab at service conditions for the cases defined in Section A.2.8.2.2 are summarized as follows.

At Midspan	Top of slab f_t (ksi)	Top of Girder f_t (ksi)	Bottom of girder f_b (ksi)
Case I	+0.126	+2.238	–
Case II	+0.792	+1.568	–
Case III	+0.855	+2.688	– 0 .409

A.2.8.2.4
Composite Section Properties

The composite section properties calculated in Section A.2.4.2.3 were based on the modular ratio value of 1. But as the actual concrete strength is now selected, the actual modular ratio can be determined, and the corresponding composite section properties can be evaluated. The updated composite section properties are presented in Table A.2.8.1.

Modular ratio between slab and girder concrete

$$n = \left(\frac{E_{cs}}{E_{cp}}\right)$$

where:

n = Modular ratio between slab and girder concrete

E_{cs} = Modulus of elasticity of slab concrete, ksi
= $33{,}000(w_c)^{1.5}\sqrt{f'_{cs}}$ [LRFD Eq. 5.4.2.4-1]

w_c = Unit weight of concrete = (must be between 0.09 and 0.155 kcf for LRFD Eq. 5.4.2.4-1 to be applicable)
= 0.150 kcf

f'_{cs} = Compressive strength of slab concrete at service
= 4.0 ksi

E_{cs} = $[33{,}000(0.150)^{1.5}\sqrt{4}\,] = 3834.25$ ksi

E_{cp} = Modulus of elasticity of girder concrete at service, ksi
= $33{,}000(w_c)^{1.5}\sqrt{f'_c}$

f'_c = Compressive strength of precast girder concrete at service
= 5.892 ksi

$E_{cp} = [33{,}000(0.150)^{1.5}\sqrt{5.892}\,] = 4653.53$ ksi

$n = \dfrac{3834.25}{4653.53} = 0.824$

Transformed flange width, $b_{tf} = n \times$ (effective flange width)

Effective flange width = 96 in. (see Section A.2.4.2)

$b_{tf} = 0.824(96) = 79.10$ in.

Transformed flange area, $A_{tf} = n \times$ (effective flange width)(t_s)

t_s = Slab thickness = 8 in.

$A_{tf} = 0.824(96)(8) = 632.83$ in.^{2}

Table A.2.8.1. Properties of Composite Section.

	Transformed Area A (in.^{2})	y_b (in.)	Ay_b (in.^{3})	$A(y_{bc} - y_b)^2$	I (in.^{4})	$I + A(y_{bc} - y_b)^2$ (in.^{4})
Girder	788.40	24.75	19,512.9	172,924.58	260,403.0	433,327.6
Slab	632.83	58.00	36,704.1	215,183.46	3374.9	218,558.4
∑	1421.23		56,217.0			651,886.0

A_c = Total area of composite section = 1421.23 in.^{2}

h_c = Total height of composite section = 54 in. + 8 in. = 62 in.

I_c = Moment of inertia of composite section = 651,886.0 in^4

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder, in.
= 56,217.0/1421.23 = 39.56 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder, in.
= 54 – 39.56 = 14.44 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 39.56 = 22.44 in.

S_{bc} = Section modulus of composite section referenced to the extreme bottom fiber of the precast girder, in.^{3}
= I_c/y_{bc} = 651,886.0/39.56 = 16,478.41 in.^{3}

S_{tg} = Section modulus of composite section referenced to the top fiber of the precast girder, in.^{3}
= I_c/y_{tg} = 651,886.0/14.44 = 45,144.46 in.^{3}

S_{tc} = Section modulus of composite section referenced to the top fiber of the slab, in.^{3}
= I_c/y_{tc} = 651,886.0/22.44 = 29,050.18 in.^{3}

A.2.9 CHECK FOR LIVE LOAD MOMENT DISTRIBUTION FACTOR

The live load moment distribution factor calculation involves a parameter for longitudinal stiffness, K_g. This parameter depends on the modular ratio between the girder and the slab concrete. The live load moment distribution factor calculated in Section A.2.5.2.2.1 is based on the assumption that the modular ratio between the girder and slab concrete is 1. However, as the actual concrete strength is now chosen, the live load moment distribution factor based on the actual modular ratio needs to be calculated and compared to the distribution factor calculated in Section A.2.5.2.2.1. If the difference between the two is found to be large, the bending moments have to be updated based on the calculated live load moment distribution factor.

$K_g = n(I + A\,e_g^2)$ [LRFD Art. 3.6.1.1.1]

where:

n = Modular ratio between girder and slab concrete

$$= \frac{E_c \text{ for girder concrete}}{E_c \text{ for slab concrete}} = \left(\frac{E_{cp}}{E_{cs}}\right)$$

(Note that this ratio is the inverse of the one defined for composite section properties in Section A.2.8.2.4.)

E_{cs} = Modulus of elasticity of slab concrete, ksi

$$= 33{,}000(w_c)^{1.5}\sqrt{f'_{cs}}$$ [LRFD Eq. 5.4.2.4-1]

w_c = Unit weight of concrete = (must be between 0.09 and 0.155 kcf for LRFD Eq. 5.4.2.4-1 to be applicable)
= 0.150 kcf

f'_{cs} = Compressive strength of slab concrete at service
= 4.0 ksi

$$E_{cs} = [33{,}000(0.150)^{1.5}\sqrt{4}\,] = 3834.25 \text{ ksi}$$

E_{cp} = Modulus of elasticity of girder concrete at service, ksi

$$= 33{,}000(w_c)^{1.5}\sqrt{f'_c}$$

f'_c = Compressive strength of precast girder concrete at service = 5.892 ksi

E_{cp} = $[33{,}000(0.150)^{1.5}\sqrt{5.892}\,]$ = 4653.53 ksi

$n = \dfrac{4653.53}{3834.25} = 1.214$

A = Area of girder cross section (non-composite section) = 788.4 in.^{2}

I = Moment of inertia about the centroid of the non-composite precast girder = 260,403 in.^{4}

e_g = Distance between centers of gravity of the girder and slab, in.
= $(t_s/2 + y_t)$ = (8/2 + 29.25) = 33.25 in.

$K_g = (1.214)[260{,}403 + 788.4\,(33.25)^2] = 1{,}374{,}282.6$ in.^{4}

The approximate live load moment distribution factors for Type k bridge girders, specified by LRFD Table 4.6.2.2.2b-1, are applicable if the following condition for K_g is satisfied (other requirements are provided in section A.2.5.2.2.1).

$10{,}000 \le K_g \le 7{,}000{,}000$

$10{,}000 \le 1{,}374{,}282.6 \le 7{,}000{,}000$ (O.K.)

For one design lane loaded:

$$DFM = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0\,L\,t_s^3}\right)^{0.1}$$

where:

DFM = Live load moment distribution factor for interior girders

S = Spacing of adjacent girders = 8 ft.

L = Design span length = 108.583 ft.

t_s = Thickness of slab = 8 in.

$$DFM = 0.06 + \left(\frac{8}{14}\right)^{0.4}\left(\frac{8}{108.583}\right)^{0.3}\left(\frac{1{,}374{,}282.6}{12.0(108.583)(8)^3}\right)^{0.1}$$

DFM = 0.06 + (0.8)(0.457)(1.075) = 0.453 lanes/girder

For two or more lanes loaded:

$$DFM = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0\,L\,t_s^3}\right)^{0.1}$$

$$DFM = 0.075 + \left(\frac{8}{9.5}\right)^{0.6}\left(\frac{8}{108.583}\right)^{0.2}\left(\frac{1{,}374{,}282.6}{12.0(108.583)(8)^3}\right)^{0.1}$$

= 0.075 + (0.902)(0.593)(1.075) = 0.650 lanes/girder

The greater of the above two distribution factors governs. Thus, the case of two or more lanes loaded controls.

DFM = 0.650 lanes/girder

The live load moment distribution factor from Section A.2.5.2.2.1 is DFM = 0.639 lanes/girder.

$$\text{Percent difference in } DFM = \left(\frac{0.650 - 0.639}{0.650}\right)100 = 1.69 \text{ percent}$$

The difference in the live load moment distribution factors is negligible, and its impact on the live load moments will also be negligible. Hence, the live load moments obtained using the distribution factor from Section A.2.5.2.2.1 can be used for the ultimate flexural strength design.

A.2.10 FATIGUE LIMIT STATE

LRFD Art. 5.5.3 specifies that the check for fatigue of the prestressing strands is not required for fully prestressed components that are designed to have extreme fiber tensile stress due to the Service III limit state within the specified limit of $0.19\sqrt{f'_c}$.

The AASHTO Type IV girder in this design example is designed as a fully prestressed member, and the tensile stress due to Service III limit state is less than $0.19\sqrt{f'_c}$, as shown in Section A.2.8.2.2. Hence, the fatigue check for the prestressing strands is not required.

A.2.11 FLEXURAL STRENGTH LIMIT STATE

[LRFD Art. 5.7.3]

The flexural strength limit state is investigated for the Strength I load combination specified by LRFD Table 3.4.1-1 as follows.

$M_u = 1.25(M_{DC}) + 1.5(M_{DW}) + 1.75(M_{LL+IM})$

where:

M_u = Factored ultimate moment at the midspan, k-ft.

M_{DC} = Moment at the midspan due to dead load of structural components and non-structural attachments, k-ft.
$= M_g + M_S + M_{barr}$

M_g = Moment at the midspan due to girder self-weight
= 1209.98 k-ft.

M_S = Moment at the midspan due to slab weight
= 1179.03 k-ft.

M_{barr} = Moment at the midspan due to barrier weight
= 160.64 k-ft.

M_{DC} = 1209.98 + 1179.03 + 160.64 = 2549.65 k-ft.

M_{DW} = Moment at the midspan due to wearing surface load
= 188.64 k-ft.

M_{LL+IM} = Moment at the midspan due to vehicular live load including dynamic allowance, k-ft.
$= M_{LT} + M_{LL}$

M_{LT} = Distributed moment due to HS 20-44 truck load including dynamic load allowance = 1423.00 k-ft.

M_{LL} = Distributed moment due to lane load = 602.72 k-ft.

M_{LL+IM} = 1423.00 + 602.72 = 2025.72 k-ft.

The factored ultimate bending moment at midspan

M_u = 1.25(2549.65) + 1.5(188.64) + 1.75(2025.72)
= 7015.03 k-ft.

[LRFD Art. 5.7.3.1.1]

The average stress in the prestressing steel, f_{ps}, for rectangular or flanged sections subjected to flexure about one axis for which $f_{pe} \geq 0.5 f_{pu}$, is given as:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$ [LRFD Eq. 5.7.3.1.1-1]

where:

f_{ps} = Average stress in the prestressing steel, ksi

f_{pu} = Specified tensile strength of prestressing steel = 270 ksi

f_{pe} = Effective prestress after final losses = $f_{pj} - \Delta f_{pT}$

f_{pj} = Jacking stress in the prestressing strands = 202.5 ksi

Δf_{pT} = Total final loss in prestress = 56.70 ksi (Section A.2.7.3.6)

f_{pe} = 202.5 – 56.70 = 145.80 ksi > $0.5 f_{pu}$ = 0.5(270) = 135 ksi
Therefore, the equation for f_{ps} shown above is applicable.

k = $2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$ [LRFD Eq. 5.7.3.1.1-2]
= 0.28 for low-relaxation prestressing strands
[LRFD Table C5.7.3.1.1-1]

d_p = Distance from the extreme compression fiber to the centroid of the prestressing tendons, in.
= $h_c - y_{bs}$

h_c = Total height of the composite section = 54 + 8 = 62 in.

y_{bs} = Distance from centroid of the prestressing strands at midspan to the bottom fiber of the girder = 5.63 in. (see Section A.2.7.3.3)

d_p = 62 – 5.63 = 56.37 in.

c = Distance between neutral axis and the compressive face of the section, in.

The depth of the neutral axis from the compressive face, c, is computed assuming rectangular section behavior. A check is made to confirm that the neutral axis is lying in the cast-in-place slab; otherwise, the neutral axis will be calculated based on the flanged section behavior. [LRFD C5.7.3.2.2]

For rectangular section behavior,

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A_s' f_s'}{0.85 f_c' \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}}$$ [LRFD Eq. 5.7.3.1.1.-4]

A_{ps} = Area of prestressing steel, in.^{2}
= (number of strands)(area of each strand)
= 54(0.153) = 8.262 in.^{2}

f_{pu} = Specified tensile strength of prestressing steel = 270 ksi

A_s = Area of mild steel tension reinforcement = 0 in.^{2}

A_s' = Area of compression reinforcement = 0 in.^{2}

f_c' = Compressive strength of deck concrete = 4.0 ksi

f_y = Yield strength of tension reinforcement, ksi

f_y' = Yield strength of compression reinforcement, ksi

β_1 = Stress factor for compression block [LRFD Art. 5.7.2.2]
= 0.85 for $f_c' \leq 4.0$ ksi

b = Effective width of compression flange = 96 in. (based on non-transformed section)

Depth of neutral axis from compressive face

$$c = \frac{8.262(270) + 0 - 0}{0.85(4.0)(0.85)(96) + 0.28(8.262)\left(\dfrac{270}{56.37}\right)}$$

= 7.73 in. < t_s = 8.0 in. (O.K.)

The neutral axis lies in the slab; therefore, the assumption of rectangular section behavior is valid.

The average stress in prestressing steel

$$f_{ps} = 270\left(1 - 0.28\frac{7.73}{56.37}\right) = 259.63 \text{ ksi}$$

For prestressed concrete members having rectangular section behavior, the nominal flexural resistance is given as:

[LRFD Art. 5.7.3.2.3]

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)$$ [LRFD Eq. 5.7.3.2.2-1]

The above equation is a simplified form of LRFD Equation 5.7.3.2.2-1 because no compression reinforcement or mild tension reinforcement is provided.

a = Depth of the equivalent rectangular compression block, in.
= $\beta_1 c$

β_1 = Stress factor for compression block = 0.85 for $f'_c \leq 4.0$ ksi

a = 0.85(7.73) = 6.57 in.

Nominal flexural resistance

$$M_n = (8.262)(259.63)\left(56.37 - \frac{6.57}{2}\right)$$

= 113,870.67 k-in. = 9489.22 k-ft.

Factored flexural resistance

$$M_r = \phi M_n$$ [LRFD Eq. 5.7.3.2.1-1]

where:

ϕ = Resistance factor [LRFD Art. 5.5.4.2.1]
= 1.0 for flexure and tension of prestressed concrete members

M_r = 1.0×(9489.22) = 9489.22 k-ft. > M_u = 7015.03 k-ft. (O.K.)

A.2.12 LIMITS FOR REINFORCEMENT

[LRFD Art. 5.7.3.3]

A.2.12.1 Maximum Reinforcement

[LRFD Art. 5.7.3.3.1]

The maximum amount of the prestressed and non-prestressed reinforcement should be such that

$$\frac{c}{d_e} \leq 0.42$$ [LRFD Eq. 5.7.3.3.1-1]

in which:

$$d_e = \frac{A_{ps}f_{ps}d_p + A_s f_y d_s}{A_{ps}f_{ps} + A_s f_y}$$ [LRFD Eq. 5.7.3.3.1-2]

c = Distance from the extreme compression fiber to the neutral axis = 7.73 in.

d_e = The corresponding effective depth from the extreme fiber to the centroid of the tensile force in the tensile reinforcement, in.
= d_p, if mild steel tension reinforcement is not used

d_p = Distance from the extreme compression fiber to the centroid of the prestressing tendons = 56.37 in.

Therefore d_e = 56.37 in.

$$\frac{c}{d_e} = \frac{7.73}{56.37} = 0.137 << 0.42 \qquad \text{(O.K.)}$$

A.2.12.2 Minimum Reinforcement

[LRFD Art. 5.7.3.3.2]

At any section of a flexural component, the amount of prestressed and non-prestressed tensile reinforcement should be adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of:

- 1.2 times the cracking moment, M_{cr}, determined on the basis of elastic stress distribution and the modulus of rupture of concrete, f_r; and
- 1.33 times the factored moment required by the applicable strength load combination.

The above requirements are checked at the midspan section in this design example. Similar calculations can be performed at any section along the girder span to check these requirements.

The cracking moment is given as:

$$M_{cr} = S_c(f_r + f_{cpe}) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \leq S_c f_r \qquad \text{[LRFD Eq. 5.7.3.3.2-1]}$$

where:

f_r = Modulus of rupture, ksi
= $0.24\sqrt{f'_c}$ for normal weight concrete [LRFD Art. 5.4.2.6]

f'_c = Compressive strength of girder concrete at service
= 5.892 ksi

f_r = $0.24\sqrt{5.892}$ = 0.582 ksi

f_{cpe} = Compressive stress in concrete due to effective prestress force at extreme fiber of the section where tensile stress is caused by externally applied loads, ksi

$$= \frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b}$$

P_{pe} = Effective prestressing force after allowing for final prestress loss, kips
= (number of strands)(area of each strand)(f_{pe})
= 54(0.153)(145.80) = 1204.60 kips

(Calculations for effective final prestress (f_{pe}) are shown in Section A.2.7.3.6.)

e_c = Eccentricity of prestressing strands at the midspan
= 19.12 in.

A = Area of girder cross section = 788.4 in.^{2}

S_b = Section modulus of the precast girder referenced to the extreme bottom fiber of the non-composite precast girder
= 10,521.33 in.^{3}

$$f_{cpe} = \frac{1204.60}{788.4} + \frac{1204.60(19.12)}{10{,}521.33}$$

= 1.528 + 2.189 = 3.717 ksi

M_{dnc} = Total unfactored dead load moment acting on the non-composite section
= $M_g + M_S$

M_g = Moment at the midspan due to girder self-weight
= 1209.98 k-ft.

M_S = Moment at the midspan due to slab weight
= 1179.03 k-ft.

M_{dnc} = 1209.98 + 1179.03 = 2389.01 k-ft. = 28,668.12 k-in.

S_{nc} = Section modulus of the non-composite section referenced to the extreme fiber where the tensile stress is caused by externally applied loads = 10,521.33 in.^{3}

S_c = Section modulus of the composite section referenced to the extreme fiber where the tensile stress is caused by externally applied loads = 16,478.41 in.^{3} (based on updated composite section properties)

The cracking moment is:

$$M_{cr} = (16{,}478.41)(0.582 + 3.717) - (28{,}668.12)\left(\frac{16{,}478.41}{10{,}521.33} - 1\right)$$

$$= 70{,}840.68 - 16{,}231.62 = 54{,}609.06 \text{ k-in.} = 4{,}550.76 \text{ k-ft.}$$

$S_c f_r$ = (16,478.41)(0.582) = 9,590.43 k-in.
= 799.20 k-ft. < 4,550.76 k-ft.

Therefore, use M_{cr} = 799.20 k-ft.

1.2 M_{cr} = 1.2(799.20) = 959.04 k-ft.

Factored moment required by Strength I load combination at midspan
M_u = 7015.03 k-ft.

1.33 M_u = 1.33(7,015.03 k-ft.) = 9330 k-ft.

Since 1.2 M_{cr} < 1.33 M_u, the 1.2M_{cr} requirement controls.

M_r = 9489.22 k-ft >> 1.2 M_{cr} = 959.04 (O.K.)

A.2.13 TRANSVERSE SHEAR DESIGN

The area and spacing of shear reinforcement must be determined at regular intervals along the entire span length of the girder. In this design example, transverse shear design procedures are demonstrated below by determining these values at the critical section near the supports. Similar calculations can be performed to determine shear reinforcement requirements at any selected section.

LRFD Art. 5.8.2.4 specifies that the transverse shear reinforcement is required when:

$$V_u > 0.5\,\phi\,(V_c + V_p)$$ [LRFD Art. 5.8.2.4-1]

where:

V_u = Total factored shear force at the section, kips

V_c = Nominal shear resistance of the concrete, kips

V_p = Component of the effective prestressing force in the direction of the applied shear, kips

ϕ = Resistance factor = 0.90 for shear in prestressed concrete members [LRFD Art. 5.5.4.2.1]

A.2.13.1 Critical Section

Critical section near the supports is the greater of:

[LRFD Art. 5.8.3.2]

$$0.5\ d_v \cot\theta \text{ or } d_v$$

where:

d_v = Effective shear depth, in.
= Distance between the resultants of tensile and compressive forces, (d_e – a/2), but not less than the greater of (0.9d_e) or (0.72h) [LRFD Art. 5.8.2.9]

d_e = Corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement [LRFD Art. 5.7.3.3.1]

a = Depth of compression block = 6.57 in. at midspan (see Section A.2.11)

h = Height of composite section = 62 in.

A.2.13.1.1 Angle of Diagonal Compressive Stresses

The angle of inclination of the diagonal compressive stresses is calculated using an iterative method. As an initial estimate θ is taken as 23 degrees.

A.2.13.1.2 Effective Shear Depth

The shear design at any section depends on the angle of diagonal compressive stresses at the section. Shear design is an iterative process that begins with assuming a value for θ.

Because some of the strands are harped at the girder end, the effective depth, d_e, varies from point to point. However, d_e must be calculated at the critical section for shear, which is not yet known. Therefore, for the first iteration, d_e is calculated based on the center of gravity of the straight strand group at the end of the girder, y_{bsend}. This methodology is given in *PCI Bridge Design Manual* (PCI 2003).

Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement

$d_e = h - y_{bsend}$ = 62.0 – 5.55 = 56.45 in. (see Sec. A.2.7.3.9 for y_{bsend})

Effective shear depth

$$d_v = d_e - 0.5(a) = 56.45 - 0.5(6.57) = 53.17 \text{ in.} \quad \text{(controls)}$$
$$\geq 0.9d_e = 0.9(56.45) = 50.80 \text{ in.}$$
$$\geq 0.72h = 0.72(62) = 44.64 \text{ in.} \quad \text{(O.K.)}$$

Therefore, d_v = 53.17 in.

A.2.13.1.3 Calculation of Critical Section

[LRFD Art. 5.8.3.2]

The critical section near the support is greater of:

d_v = 53.17 in. and

0.5 d_v cot θ = 0.5(53.17)(cot 23°) = 62.63 in. from the support face (controls)

Add half the bearing width (3.5 in., standard pad size for prestressed girders is 7 in. × 22 in.) to the critical section distance from the face of the support to get the distance of the critical section from the centerline of bearing.

Critical section for shear

x = 62.63 + 3.5 = 66.13 in. = 5.51 ft. (0.051L) from the centerline of the bearing, where L is the design span length.

The value of d_e is calculated at the girder end, which can be refined based on the critical section location. However, it is conservative not to refine the value of d_e based on the critical section 0.051L. The value, if refined, will have a small difference (PCI 2003).

A.2.13.2 Contribution of Concrete to Nominal Shear Resistance

[LRFD Art. 5.8.3.3]

The contribution of the concrete to the nominal shear resistance is given as:

$$V_c = 0.0316\beta\sqrt{f'_c}\, b_v d_v \qquad \text{[LRFD Eq. 5.8.3.3-3]}$$

where:

β = A factor indicating the ability of diagonally cracked concrete to transmit tension

f'_c = Compressive strength of concrete at service = 5.892 ksi

b_v = Effective web width taken as the minimum web width within the depth d_v, in. = 8 in. (see Figure A.2.4.1)

d_v = Effective shear depth = 53.17 in.

A.2.13.2.1
Strain in Flexural Tension Reinforcement

[LRFD Art. 5.8.3.4.2]

The θ and β values are determined based on the strain in the flexural tension reinforcement. The strain in the reinforcement, ε_x, is determined assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Art. 5.8.2.5.

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_sA_s + E_pA_{ps})} \leq 0.001$$

[LRFD Eq. 5.8.3.4.2-1]

where:

V_u = Applied factored shear force at the specified section, $0.051L$
= 1.25(40.04 + 39.02 + 5.36) +1.50(6.15) + 1.75(67.28 + 25.48) = 277.08 kips

M_u = Applied factored moment at the specified section, 0.051L $> V_ud_v$
= 1.25(233.54 + 227.56 + 31.29) + 1.50(35.84) + 1.75(291.58 + 116.33)
= 1383.09 k-ft. > 277.08(53.17/12) = 1227.69 k-ft. (O.K.)

N_u = Applied factored normal force at the specified section, $0.051L$ = 0 kips

f_{po} = Parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For pretensioned members, LRFD Art. C5.8.3.4.2 indicates that f_{po} can be taken as the stress in strands when the concrete is cast around them, which is jacking stress f_{pj}, or f_{pu}.
= 0.75(270.0) = 202.5 ksi

V_p = Component of the effective prestressing force in the direction of the applied shear, kips
= (force per strand)(number of harped strands)(sinΨ)

$$\Psi = \tan^{-1}\left(\frac{42.45}{49.4(12\text{in./ft.})}\right) = 0.072 \text{ rad.}$$ (see Figure A.2.7.3)

V_p = 22.82(10) sin (0.072) = 16.42 kips

$$\varepsilon_x = \frac{\dfrac{1383.09(12\text{ in./ft.})}{53.17} + 0.5(277.08 - 16.42)\cot 23^{o} - 44(0.153)(202.5)}{2\left[28{,}000(0.0) + 28{,}500(44)(0.153)\right]}$$

$\varepsilon_x = -0.00194$

Since this value is negative, LRFD Eq. 5.8.3.4.2-3 should be used to calculate ε_x.

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})}$$

where:

A_c = Area of the concrete on the flexural tension side below $h/2$ = 473 in.^{2}

E_c = Modulus fo elasticity of girder concrete, ksi
= $33{,}000(w_c)^{1.5}\sqrt{f'_c}$
= $[33{,}000(0.150)^{1.5}\sqrt{5.892}\,]$ = 4653.53 ksi

Strain in the flexural tension reinforcement is

$$\varepsilon_x = \frac{\dfrac{1383.09(12\text{ in./ft.})}{53.17} + 0.5(277.08 - 16.42)\cot 23^{o} - 44(0.153)(202.5)}{2\left[4653.53(473) + 28{,}000(0.0) + 28{,}500(44)(0.153)\right]}$$

$\varepsilon_x = -0.000155$

Shear stress in the concrete is given as:

$$v_u = \frac{V_u - \phi V_p}{\phi b_v d_v}$$ [LRFD Eq. 5.8.3.4.2-1]

where:

ϕ = Resistance factor = 0.9 for shear in prestressed concrete members [LRFD Art. 5.5.4.2.1]

$$v_u = \frac{277.08\ -\ 0.9(16.42)}{0.9(8.0)(53.17)} = 0.685\text{ ksi}$$

v_u / f'_c = 0.685/5.892 = 0.12

A.2.13.2.2
Values of β and θ

The values of β and θ are determined using LRFD Table 5.8.3.4.2-1. Linear interpolation is allowed if the values lie between two rows, as shown in Table A.2.13.1.

Table A.2.13.1. Interpolation for θ and β Values.

υ_u / f'_c	$\varepsilon_x \times 1000$		
	≤ –0.200	–0.155	≤ –0.100
≤ 0.100	18.100		20.400
	3.790		3.380
0.12	19.540	20.47	21.600
	3.302	3.20	3.068
≤ 0.125	19.900		21.900
	3.180		2.990

θ = 20.47° > 23° (assumed)

Another iteration is made with θ = 20.65° to arrive at the correct value of β and θ.

d_e = Effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement = 56.45 in.

d_v = Effective shear depth = 53.17 in.

The critical section near the support is greater of:
d_v = 53.17 in. and

$0.5d_v\cot\theta$ = 0.5(53.17)(cot 20.47°) = 71.2 in. from the face of the support (controls)

Add half the bearing width (3.5 in.) to the critical section distance from the face of the support to get the distance of the critical section from the centerline of bearing.

Critical section for shear

x = 71.2 + 3.5 = 74.7 in. = 6.22 ft. (0.057L) from the centerline of bearing

Assuming the strain will be negative again, LRFD Eq. 5.8.3.4.2-3 will be used to calculate ε_x.

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})}$$

The shear forces and bending moments will be updated based on the updated critical section location.

V_u = Applied factored shear force at the specified section, 0.057L
= 1.25(39.49 + 38.48 + 5.29) +1.50(6.06) + 1.75(66.81 + 25.15) = 274.10 kips

M_u = Applied factored moment at the specified section, 0.057L $> V_u d_v$
= 1.25(260.18 + 253.53 + 34.86) + 1.50(39.93) + 1.75(324.63 + 129.60)
= 1540.50 k-ft. > 274.10(53.17/12) = 1222.03 k-ft. (O.K.)

$$\varepsilon_x = \frac{\dfrac{1540.50(12\text{ in./ft.})}{53.17} + 0.5(274.10 - 16.42)\cot 20.47^{\circ} - 44(0.153)(202.5)}{2\left[4653.53(473) + 28{,}000(0.0) + 28{,}500(44)(0.153)\right]}$$

$\varepsilon_x = -0.000140$

Shear stress in concrete

$$v_u = \frac{V_u - \phi V_p}{\phi b_v d_v} = \frac{274.10 - 0.9(16.42)}{0.9(8)(53.17)} = 0.677 \text{ ksi}$$

[LRFD Eq. 5.8.3.4.2-1]

v_u / f'_c = 0.677/5.892 = 0.115

Table A.2.13.2 shows the values of β and θ obtained using linear interpolation.

Table A.2.13.2. Interpolation for θ and β Values.

v_u / f'_c	$\varepsilon_x \times 1000$		
	≤ –0.200	–0.140	≤ –0.100
≤ 0.100	18.100		20.40
	3.790		3.380
0.115	18.59	20.22	21.30
	3.424	3.26	3.146
≤ 0.125	19.90		21.900
	3.180		2.990

θ = 20.22° ≈ 20.47° (from first iteration)

Therefore, no further iteration is needed.

β = 3.26

A.2.13.2.3 Computation of Concrete Contribution

The contribution of the concrete to the nominal shear resistance is given as:

$$V_c = 0.0316\beta\sqrt{f_c'}\, b_v d_v \qquad \text{[LRFD Eq. 5.8.3.3-3]}$$

where:

β = A factor indicating the ability of diagonally cracked concrete to transmit tension = 3.26

f_c' = Compressive strength of concrete at service = 5.892 ksi

b_v = Effective web width taken as the minimum web width within the depth d_v, in. = 8 in. (see Figure A.2.4.1)

d_v = Effective shear depth = 53.17 in.

V_c = $0.0316(3.26)(\sqrt{5.892}\ (8.0)(53.17)$ = 106.36 kips

A.2.13.3 Contribution of Reinforcement to Nominal Shear Resistance

A.2.13.3.1 Requirement for Reinforcement

Check if $V_u > 0.5\,\phi\,(V_c + V_p)$ [LRFD Eq. 5.8.2.4-1]

V_u = 274.10 kips > 0.5(0.9)(106.36 + 16.42) = 55.25 kips

Therefore, transverse shear reinforcement should be provided.

A.2.13.3.2 Required Area of Reinforcement

The required area of transverse shear reinforcement is:

$$\frac{V_u}{\phi} \le V_n = (V_c + V_s + V_p) \qquad \text{[LRFD Eq. 5.8.3.3-1]}$$

where:

V_s = Shear force carried by transverse reinforcement

$$= \frac{V_u}{\phi} - V_c - V_p = \left(\frac{274.10}{0.9} - 106.36 - 16.42\right) = 181.77 \text{ kips}$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$ [LRFD Eq. 5.8.3.3-4]

where:

A_v = Area of shear reinforcement within a distance s, in.^{2}

s = Spacing of stirrups, in.

f_y = Yield strength of shear reinforcement, ksi

α = Angle of inclination of transverse reinforcement to longitudinal axis = 90° for vertical stirrups

Therefore, area of shear reinforcement within a distance s is:

$A_v = (sV_s)/ f_y d_v (\cot\theta + \cot\alpha) \sin\alpha$

$= (s)(181.77)/(60)(53.17)(\cot 20.22° + \cot 90°)(\sin 90°) = 0.021(s)$

If s = 12 in., required A_v = 0.252 in.^{2}/ft.

A.2.13.3.3 Determine Spacing of Reinforcement

Check for maximum spacing of transverse reinforcement [LRFD Art. 5.8.2.7]

Check if $v_u < 0.125 f'_c$ [LRFD Eq. 5.8.2.7-1]

or if $v_u \geq 0.125 f'_c$ [LRFD Eq. 5.8.2.7-2]

$0.125 f'_c = 0.125(5.892) = 0.74$ ksi

$v_u = 0.677$ ksi

$v_u < 0.125 f'_c$, therefore, $s \leq 24$ in. [LRFD Eq. 5.8.2.7-2]

$s \leq 0.8 d_v = 0.8(53.17) = 42.54$ in.

Therefore, maximum s = 24.0 in. > s provided (O.K.)

Use #4 bar double-legged stirrups at 12 in. c/c,

$A_v = 2(0.20) = 0.40$ in.^{2}/ft. > 0.252 in.^{2}/ft.

$$V_s = \frac{0.4(60)(53.17)(\cot 20.47^\circ)}{12} = 283.9 \text{ kips}$$

A.2.13.3.4 Minimum Reinforcement Requirement

The area of transverse reinforcement should not be less than:

[LRFD Art. 5.8.2.5]

$$0.0316\sqrt{f_c'}\frac{b_v s}{f_y}$$ [LRFD Eq. 5.8.2.5-1]

$$= 0.0316\sqrt{5.892}\frac{(8)(12)}{60} = 0.12 < A_v \text{ provided} \qquad \text{(O.K.)}$$

A.2.13.4 Maximum Nominal Shear Resistance

In order to ensure that the concrete in the web of the girder will not crush prior to yielding of the transverse reinforcement, the LRFD Specifications give an upper limit for V_n as follows:

$$V_n = 0.25\ f_c'\ b_v d_v + V_p$$ [LRFD Eq. 5.8.3.3-2]

Comparing the above equation with LRFD Eq. 5.8.3.3-1

$$V_c + V_s \leq 0.25\ f_c'\ b_v d_v$$

$$106.36 + 283.9 = 390.26 \text{ kips} \leq 0.25(5.892)(8)(53.17)$$
$$= 626.55 \text{ kips} \qquad \text{O.K.}$$

This is a sample calculation for determining the transverse reinforcement requirement at the critical section. This procedure can be followed to find the transverse reinforcement requirement at increments along the length of the girder.

A.2.14 INTERFACE SHEAR TRANSFER

A.2.14.1 Factored Horizontal Shear

[LRFD Art. 5.8.4]

At the strength limit state, the horizontal shear at a section can be calculated as follows:

$$V_h = \frac{V_u}{d_v}$$ [LRFD Eq. C5.8.4.1-1]

where:

V_h = Horizontal shear per unit length of the girder, kips

V_u = Factored shear force at specified section due to superimposed loads, kips

d_v = Distance between resultants of tensile and compressive forces (d_e - a/2), in.

The LRFD Specifications do not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.057L.

Using the Strength I load combination:

V_u = 1.25(5.29) +1.50(6.06) + 1.75(66.81 + 25.15) = 176.63 kips

d_v = 53.17 in.

Therefore, the applied factored horizontal shear is:

$V_h = \frac{176.63}{53.17}$ = 3.30 kips/in.

Required $V_n = V_h / \phi$ = 3.30/0.9 = 3.67 kips/in.

A.2.14.2 Required Nominal Resistance

The nominal shear resistance of the interface surface is:

$$V_n = cA_{cv} + \mu\, [A_{vf}\, f_y + P_c]$$ [LRFD Eq. 5.8.4.1-1]

where:

c = Cohesion factor [LRFD Art. 5.8.4.2]

μ = Friction factor [LRFD Art. 5.8.4.2]

A_{cv} = Area of concrete engaged in shear transfer, in.^{2}

A_{vf} = Area of shear reinforcement crossing the shear plane, in.^{2}

P_c = Permanent net compressive force normal to the shear plane, kips

f_y = Shear reinforcement yield strength, ksi

A.2.14.3 Required Interface Shear Reinforcement

For concrete placed against clean, hardened concrete and free of laitance, but not an intentionally roughened surface:

[LRFD Art. 5.8.4.2]

$c = 0.075$ ksi

$\mu = 0.6\ \lambda$, where $\lambda = 1.0$ for normal weight concrete, and therefore,

$\mu = 0.6$

The actual contact width, b_v, between the slab and the girder is 20 in.

A_{cv} = (20 in.)(1 in.) = 20 in.^{2}

The LRFD Eq. 5.8.4.1-1 can be solved for A_{vf} as follows:

$3.67 = (0.075)(20) + 0.6(A_{vf}(60) + 0)$

Solving for A_{vf} = 0.06 in.^{2}/in. or 0.72 in.^{2} / ft.

2 #4 double-legged bars per ft. are provided.

Area of steel provided = 2 (0.40) = 0.80 in.^{2} / ft.

Provide 2 #4 double-legged bars at 6 in. c/c

The web reinforcement shall be provided at 6 in. c/c and can be extended into the cast-in-place slab as interface shear reinforcement.

A.2.14.3.1 Minimum Interface Shear Reinforcement

Minimum $A_{vf} \geq (0.05 b_v)/f_y$ [LRFD Eq. 5.8.4.1-4]

where b_v = width of the interface

A_{vf} = 0.80 in.^{2}/ft. > [0.05(20)/60](12 in./ft) = 0.2 in.^{2}/ft. O.K.

$$V_n \text{ provided} = 0.075(20) + 0.6\left(\frac{0.80}{12}(60) + 0\right) = 3.9 \text{ kips/in.}$$

$0.2\ f'_c A_{cv} = 0.2(4.0)(20) = 16$ kips/in.

$0.8A_{cv} = 0.8(20) = 16$ kips/in.

Provided $V_n \leq 0.2\, f'_c\, A_{cv}$ O.K. [LRFD Eq. 5.8.4.1-2]

$\leq 0.8\, A_{cv}$ O.K. [LRFD Eq. 5.8.4.1-3]

A.2.15 MINIMUM LONGITUDINAL REINFORCEMENT REQUIREMENT

[LRFD Art. 5.8.3.5]

Longitudinal reinforcement should be proportioned so that at each section the following equation is satisfied:

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M_u}{d_v \phi} + 0.5\frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta$$

[LRFD Eq. 5.8.3.5-1]

where:

A_s = Area of nonprestressed tension reinforcement, in.^{2}

f_y = Specified minimum yield strength of reinforcing bars, ksi

A_{ps} = Area of prestressing steel at the tension side of the section, in.^{2}

f_{ps} = Average stress in prestressing steel at the time for which the nominal resistance is required, ksi

M_u = Factored moment at the section corresponding to the factored shear force, kip-ft.

N_u = Applied factored axial force, kips

V_u = Factored shear force at the section, kips

V_s = Shear resistance provided by shear reinforcement, kips

V_p = Component in the direction of the applied shear of the effective prestressing force, kips

d_v = Effective shear depth, in.

θ = Angle of inclination of diagonal compressive stresses

A.2.15.1 Required Reinforcement at Face of Bearing

[LRFD Art. 5.8.3.5]

Width of bearing = 7.0 in.

Distance of section = 7/2 = 3.5 in. = 0.291 ft.

Shear forces and bending moment are calculated at this section

V_u = 1.25(44.35 + 43.22 + 5.94) + 1.50(6.81) + 1.75(71.05 + 28.14)

= 300.69 kips

M_u = 1.25(12.04 + 11.73 + 1.61) + 1.50(1.85) + 1.75(15.11 + 6.00)

= 71.44 kip-ft.

$$\frac{M_u}{d_v \phi} + 0.5\frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta$$

=

$$\frac{71.44(12 \text{ in./ft.})}{53.17(0.9)} + 0 + \left(\frac{300.69}{0.90} - 0.5(283.9) - 16.42\right)\cot 20.47^\circ$$

= 484.09 kips

The crack plane crosses the centroid of the 44 straight strands at a distance of 6 + 5.33 cot 20.47° = 20.14 in. from the girder end.

Because the transfer length is 30 in., the available prestress from 44 straight strands is a fraction of the effective prestress, f_{pe}, in these strands. The 10 harped strands do not contribute to the tensile capacity since they are not on the flexural tension side of the member.

Therefore, the available prestress force is:

$$A_s f_y + A_{ps} f_{ps} = 0 + 44(0.153)\left(149.18\frac{20.33}{30}\right) = 680.57 \text{ kips}$$

$A_s f_y + A_{ps} f_{ps}$ = 649.63 kips > 484.09 kips

Therefore, additional longitudinal reinforcement is not required.

A.2.16 PRETENSIONED ANCHORAGE ZONE

[LRFD Art. 5.10.10]

A.2.16.1 Minimum Vertical Reinforcement

[LRFD Art. 5.10.10.1]

Design of the anchorage zone reinforcement is computed using the force in the strands just prior to transfer:

Force in the strands at transfer
F_{pi} = 54(0.153)(202.5) = 1673.06 kips

The bursting resistance, P_r, should not be less than 4 percent of F_{pi}.
[LRFD Arts. 5.10.10.1 and C3.4.3]

$P_r = f_s A_s \geq 0.04F_{pi}$ = 0.04(1673.06) = 66.90 kips

where:

A_s = Total area of vertical reinforcement located within a distance of h/4 from the end of the girder, in.^{2}

f_s = Stress in steel not exceeding 20 ksi.

Solving for required area of steel A_s= 66.90/20 = 3.35 in.^{2}

At least 3.35 in.^{2} of vertical transverse reinforcement should be provided within a distance of (h/4 = 62 / 4 = 15.5 in.) from the end of the girder.

Use 6 #5 double-legged bars at 2 in. spacing starting at 2 in. from the end of the girder.

The provided A_s = 6(2)0.31 = 3.72 in.^{2} > 3.35 in.^{2} (O.K.)

A.2.16.2 Confinement Reinforcement

[LRFD Art. 5.10.10.2]

For a distance of 1.5d = 1.5(54) = 81 in. from the end of the girder, reinforcement is placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be of a shape that will confine (enclose) the strands.

A.2.17 CAMBER AND DEFLECTIONS

A.2.17.1 Maximum Camber

The LRFD Specifications do not provide guidelines for the determination camber of prestressed concrete members. The Hyperbolic Functions Method (Furr et al. 1968, Sinno 1968, Furr and Sinno 1970) for the calculation of maximum camber is used by TxDOT's prestressed concrete bridge design software, PSTRS14 (TxDOT 2004). The following steps illustrate the Hyperbolic Functions Method for the estimation of maximum camber.

Step 1: The total prestressing force after initial prestress loss due to elastic shortening has occurred

$$P = \frac{P_i}{\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)} + \frac{M_D\, e_c\, A_s\, n}{I\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)}$$

where:

P_i = Anchor force in prestressing steel
= (number of strands)(area of strand)(f_{si})

P_i = 54(0.153)(202.5) = 1673.06 kips

f_{pi} = Before transfer, $\leq$ 0.75 f_{pu} = 202,500 psi

[LRFD Table 5.9.3-1]

f_{pu} = Ultimate strength of prestressing strands = 270 ksi

f_{pi} = 0.75(270) = 202.5 ksi

I = Moment of inertia of the non-composite precast girder
= 260,403 in.^{4}

e_c = Eccentricity of prestressing strands at the midspan
= 19.12 in.

M_D = Moment due to self-weight of the girder at midspan
= 1209.98 k-ft.

A_s = Area of prestressing steel
= (number of strands)(area of strand)
= 54(0.153) = 8.262 in.^{2}

p = A_s/A

A = Area of girder cross section = 788.4 in.^{2}

$$p = \frac{8.262}{788.4} = 0.0105$$

n = Modular ratio between prestressing steel and the girder concrete at release = E_s/E_{ci}

E_{ci} = Modulus of elasticity of the girder concrete at release
= $33(w_c)^{3/2}\sqrt{f'_{ci}}$ [STD Eq. 9-8]

w_c = Unit weight of concrete = 150 pcf

f'_{ci} = Compressive strength of precast girder concrete at release = 5892 psi

$$E_{ci} = [33(150)^{3/2}\sqrt{5892}\,]\left(\frac{1}{1{,}000}\right) = 4653.53 \text{ ksi}$$

E_s = Modulus of elasticity of prestressing strands
= 28,000 ksi

n = 28,500/4653.53 = 6.12

$$\left(1 + pn + \frac{e_c^2 A_s n}{I}\right) = 1 + (0.0105)(6.\ 12) + \frac{(19.12^2)(8.262)(6.12)}{260,403}$$
$$= 1.135$$

$$P = \frac{1673.06}{1.135} + \frac{(1209.98)(12 \text{ in./ft.})(19.12)(8.262)(6.12)}{260{,}403(1.135)}$$

= 1474.06 + 47.49 = 1521.55 kips

Initial prestress loss is defined as

$$PL_i = \frac{P_i - P}{P_i} = \frac{1673.06 - 1521.55}{1673.06} = 0.091 = 9.1\%$$

The stress in the concrete at the level of the centroid of the prestressing steel immediately after transfer is determined as follows.

$$f_{ci}^s = P\left(\frac{1}{A} + \frac{e_c^2}{I}\right) - f_c^s$$

where:

f_c^s = Concrete stress at the level of centroid of prestressing steel due to dead loads, ksi

$$= \frac{M_D\, e_c}{I} = \frac{(1209.98)(12 \text{ in./ft.})(19.12)}{260{,}403} = 1.066 \text{ ksi}$$

$$f_{ci}^s = 1521.55\left(\frac{1}{788.4} + \frac{19.12^2}{260{,}403}\right) - 1.066 = 3.0 \text{ ksi}$$

The ultimate time dependent prestress loss is dependent on the ultimate creep and shrinkage strains. As the creep strains vary with the concrete stress, the following steps are used to evaluate the concrete stresses and adjust the strains to arrive at the ultimate prestress loss. It is assumed that the creep strain is proportional to the concrete stress, and the shrinkage stress is independent of concrete stress.

Step 2: Initial estimate of total strain at steel level assuming constant sustained stress immediately after transfer

$$\varepsilon_{c1}^s = \varepsilon_{cr}^\infty f_{ci}^s + \varepsilon_{sh}^\infty$$

where:

ε_{cr}^∞ = Ultimate unit creep strain = 0.00034 in./in. [this value is prescribed by Furr and Sinno (1970)].

$\varepsilon_{sh}^{\infty}$ = Ultimate unit shrinkage strain = 0.000175 in./in. [this value is prescribed by Furr and Sinno (1970)].

ε_{c1}^{s} = 0.00034(3.0) + 0.000175 = 0.001195 in./in.

Step 3: The total strain obtained in Step 2 is adjusted by subtracting the elastic strain rebound as follows

$$\varepsilon_{c2}^{s} = \varepsilon_{c1}^{s} - \varepsilon_{c1}^{s} E_s \frac{A_s}{E_{ci}}\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\varepsilon_{c2}^{s} = 0.001195 - 0.001195\ (28{,}500)\frac{8.262}{4{,}653.53}\left(\frac{1}{788.4} + \frac{19.12^2}{260{,}403}\right)$$

= 0.001033 in./in.

Step 4: The change in concrete stress at the level of centroid of prestressing steel is computed as follows:

$$\Delta f_c^s = \varepsilon_{c2}^{s} E_s A_s \left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\Delta f_c^s = 0.001033\ (28{,}500)(8.262)\left(\frac{1}{788.4} + \frac{19.12^2}{260{,}403}\right) = 0.648 \text{ ksi}$$

Step 5: The total strain computed in Step 2 needs to be corrected for the change in the concrete stress due to creep and shrinkage strains.

$$\varepsilon_{c4}^{s} = \varepsilon_{cr}^{\infty}\left(f_{ci}^{s} - \frac{\Delta f_c^s}{2}\right) + \varepsilon_{sh}^{\infty}$$

$$\varepsilon_{c4}^{s} = 0.00034\left(3.0 - \frac{0.648}{2}\right) + 0.000175 = 0.001085 \text{ in./in.}$$

Step 6: The total strain obtained in Step 5 is adjusted by subtracting the elastic strain rebound as follows

$$\varepsilon_{c5}^{s} = \varepsilon_{c4}^{s} - \varepsilon_{c4}^{s} E_s \frac{A_s}{E_{ci}}\left(\frac{1}{A} + \frac{e_c^2}{I}\right)$$

$$\varepsilon_{c5}^{s} = 0.001085 - 0.001085(28500)\ \frac{8.262}{4653.53}\left(\frac{1}{788.4} + \frac{19.12^2}{260403}\right)$$

= 0.000938 in./in

Furr and Sinno (1970) recommend stopping the updating of stresses and adjustment process after Step 6. However, as the difference between the strains obtained in Steps 3 and 6 is not negligible, this process is carried on until the total strain value converges.

Step 7: The change in concrete stress at the level of centroid of prestressing steel is computed as follows:

$$\Delta f_{c1}^{s} = \varepsilon_{c5}^{s} E_s A_s \left(\frac{1}{A} + \frac{e_c^2}{I} \right)$$

$$\Delta f_{c1}^{s} = 0.000938(28{,}500)(8.262)\left(\frac{1}{788.4} + \frac{19.12^2}{260{,}403} \right) = 0.5902 \text{ ksi}$$

Step 8: The total strain computed in Step 5 needs to be corrected for the change in the concrete stress due to creep and shrinkage strains.

$$\varepsilon_{c6}^{s} = \varepsilon_{cr}^{\infty} \left(f_{ci}^{s} - \frac{\Delta f_{c1}^{s}}{2} \right) + \varepsilon_{sh}^{\infty}$$

$$\varepsilon_{c6}^{s} = 0.00034\left(3.0 - \frac{0.5902}{2} \right) + 0.000175 = 0.001095 \text{ in./in.}$$

Step 9: The total strain obtained in Step 8 is adjusted by subtracting the elastic strain rebound as follows:

$$\varepsilon_{c7}^{s} = \varepsilon_{c6}^{s} - \varepsilon_{c6}^{s} E_s \frac{A_s}{E_{ci}} \left(\frac{1}{A} + \frac{e_c^2}{I} \right)$$

$$\varepsilon_{c7}^{s} = 0.001095 - 0.001095(28{,}500)\ \frac{8.262}{4{,}653.53}\left(\frac{1}{788.4} + \frac{19.12^2}{260{,}403} \right)$$

$$= 0.000947 \text{ in./in}$$

The strains have sufficiently converged, and no more adjustments are needed.

Step 10: Computation of final prestress loss

Time dependent loss in prestress due to creep and shrinkage strains is given as:

$$PL^{\infty} = \frac{\varepsilon_{c7}^{s} E_s A_s}{P_i} = \frac{0.000947(28{,}500)(8.262)}{1{,}673.06} = 0.133 = 13.3\%$$

Total final prestress loss is the sum of initial prestress loss and the time dependent prestress loss expressed as follows:

$$PL = PL_i + PL^{\infty}$$

where:

PL = Total final prestress loss percent

PL_i = Initial prestress loss percent = 9.1 percent

PL^{∞}= Time dependent prestress loss percent = 13.3 percent

$PL = 9.1 + 13.3 = 22.4\%$

Step 11: The initial deflection of the girder under self-weight is calculated using the elastic analysis as follows:

$$C_{DL} = \frac{5\,w\,L^4}{384 E_{ci} I}$$

where:

C_{DL} = Initial deflection of the girder under self-weight, ft.

w = Self-weight of the girder = 0.821 kips/ft.

L = Total girder length = 109.67 ft.

E_{ci} = Modulus of elasticity of the girder concrete at release = 4653.53 ksi = 670,108.32 k/ft.^{2}

I = Moment of inertia of the non-composite precast girder = 260,403 in.^{4} = 12.558 ft.^{4}

$$C_{DL} = \frac{5(0.821)(109.67^4)}{384(670{,}108.32)(12.558)} = 0.184 \text{ ft.} = 2.208 \text{ in.}$$

Step 12: Initial camber due to prestress is calculated using the moment area method. The following expression is obtained from the M/EI diagram to compute the camber resulting from the initial prestress.

$$C_{pi} = \frac{M_{pi}}{E_{ci} I}$$

where:

M_{pi} = $[0.5(P)\,(e_e)\,(0.5L)^2 + 0.5(P)\,(e_c - e_e)\,(0.67)\,(HD)^2 + 0.5P\,(e_c - e_e)\,(HD_{dis})\,(0.5L + HD)]/(E_{ci})(I)$

P = Total prestressing force after initial prestress loss due to elastic shortening have occurred = 1521.55 kips

HD = Hold-down distance from girder end
= 49.404 ft. = 592.85 in. (see Figure A.1.7.3)

HD_{dis} = Hold-down distance from the center of the girder span
= 0.5(109.67) – 49.404 = 5.431 ft. = 65.17 in.

e_e = Eccentricity of prestressing strands at girder end
= 11.34 in.

e_c = Eccentricity of prestressing strands at midspan
= 19.12 in.

L = Overall girder length = 109.67 ft. = 1316.04 in.

$$M_{pi} = \{0.5(1521.55)\,(11.34)\,[(0.5)\,(1316.04)]^2 + 0.5(1521.55)\,(19.12 - 11.34)\,(0.67)\,(592.85)^2 + 0.5(1521.55)\,(19.12 - 11.34)\,(65.17)[0.5(1316.04) + 592.85]\}$$

$$M_{pi} = 3.736 \text{ x } 10^9 + 1.394 \text{ x } 10^9 + 0.483 \text{ x } 10^9 = 5.613 \text{ x } 10^9$$

$$C_{pi} = \frac{5.613 \times 10^9}{(4653.53)(260{,}403)} = 4.63 \text{ in.} = 0.386 \text{ ft.}$$

Step 13: The initial camber, C_I, is the difference between the upward camber due to initial prestressing and the downward deflection due to self-weight of the girder.

$$C_i = C_{pi} - C_{DL} = 4.63 - 2.208 = 2.422 \text{ in.} = 0.202 \text{ ft.}$$

Step 14: The ultimate time-dependent camber is evaluated using the following expression.

$$\text{Ultimate camber } C_t = C_i\,(1 - PL^{\infty})\frac{\varepsilon_{cr}^{\infty}\left(f_{ci}^{s} - \dfrac{\Delta f_{c1}^{s}}{2}\right) + \varepsilon_{e}^{s}}{\varepsilon_{e}^{s}}$$

where:

$$\varepsilon_e^s = \frac{f_{ci}^s}{E_{ci}} = \frac{3.0}{4{,}653.53} = 0.000619 \text{ in./in.}$$

$$C_t = 2.422(1 - 0.133)\frac{0.00034\left(3.0 - \dfrac{0.5902}{2}\right) + 0.000645}{0.000645}$$

C_t = 5.094 in. = 0.425 ft. ↑

A.2.17.2 Deflection due to Slab Weight

The deflection due to the slab weight is calculated using an elastic analysis as follows.

Deflection of the girder at midspan

$$\Delta_{slab1} = \frac{5\,w_s\,L^4}{384\,E_c\,I}$$

where:

w_s = Weight of the slab = 0.80 kips/ft.

E_c = Modulus of elasticity of girder concrete at service
$= 33(w_c)^{3/2}\sqrt{f'_c}$
$= 33(150)^{1.5}\sqrt{5{,}892}\left(\dfrac{1}{1{,}000}\right) = 4{,}653.53$ ksi

I = Moment of inertia of the non-composite girder section
= 260,403 in.^{4}

L = Design span length of girder (center-to-center bearing)
= 108.583 ft.

$$\Delta_{slab1} = \frac{5\left(\dfrac{0.80}{12 \text{ in./ft.}}\right)[(108.583)(12 \text{ in./ft.})]^4}{384(4653.53)(260{,}403)}$$

= 2.06 in. = 0.172 ft. ↓

Deflection at quarter span due to slab weight

$$\Delta_{slab2} = \frac{57\ w_s\ L^4}{6144\ E_c\ I}$$

$$\Delta_{slab2} = \frac{57\left(\dfrac{0.80}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{6144(4653.53)(260,403)}$$

= 1.471 in. = 0.123 ft. ↓

A.2.17.3 Deflections due to Superimposed Dead Loads

Deflection due to barrier weight at midspan

$$\Delta_{barr1} = \frac{5\ w_{barr}\ L^4}{384\ E_c\ I_c}$$

where:

w_{barr} = Weight of the barrier = 0.109 kips/ft.

I_c = Moment of inertia of composite section = 651,886.0 in^4

$$\Delta_{barr1} = \frac{5\left(\dfrac{0.109}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{384(4653.53)(651,886.0\ )}$$

= 0.141 in. = 0.0118 ft. ↓

Deflection at quarter span due to barrier weight

$$\Delta_{barr2} = \frac{57\ w_{barr}\ L^4}{6144\ E_c\ I_c}$$

$$\Delta_{barr2} = \frac{57\left(\dfrac{0.109}{12\text{ in./ft.}}\right)[(108.583)(12\text{ in./ft.})]^4}{6144(4653.53)(651,886.0)}$$

= 0.08 in. = 0.0067 ft. ↓

Deflection due to wearing surface weight at midspan

$$\Delta_{ws1} = \frac{5\ w_{ws}\ L^4}{384\ E_c\ I_c}$$

where:

w_{ws} = Weight of wearing surface = 0.128 kips/ft.

$$\Delta_{ws1} = \frac{5\left(\frac{0.128}{12 \text{ in./ft.}}\right)[(108.583)(12 \text{ in./ft.})]^4}{384(4653.53)(651,886.0\ )}$$

$$= 0.132 \text{ in.} = 0.011 \text{ ft.} \downarrow$$

Deflection at quarter span due to wearing surface

$$\Delta_{ws2} = \frac{57\ w_{ws}\ L^4}{6144\ E_c\ I}$$

$$\Delta_{ws2} = \frac{57\left(\frac{0.128}{12 \text{ in./ft.}}\right)[(108.583)(12 \text{ in./ft.})]^4}{6144(4529.66)(657,658.4)}$$

$$= 0.094 \text{ in.} = 0.0078 \text{ ft.} \downarrow$$

A.2.17.4 Total Deflection due to Dead Loads

The total deflection at midspan due to slab weight and superimposed loads is:

$$\Delta_{T1} = \Delta_{slab1} + \Delta_{barr1} + \Delta_{ws1}$$

$$= 0.172 + 0.0118 + 0.011 = 0.1948 \text{ ft.} \downarrow$$

The total deflection at quarter span due to slab weight and superimposed loads is:

$$\Delta_{T2} = \Delta_{slab2} + \Delta_{barr2} + \Delta_{ws2}$$

$$= 0.123 + 0.0067 + 0.0078 = 0.1375 \text{ ft.} \downarrow$$

The deflections due to live loads are not calculated in this example as they are not a design factor for TxDOT bridges.

A.2.18 REFERENCES

AASHTO (2004), *AASHTO LRFD Bridge Design Specifications*, 3rd Ed., American Association of State Highway and Transportation Officials (AASHTO), Customary U.S. Units, Washington, D.C.

Furr, H.L., R. Sinno and L.L. Ingram (1968). "Prestress Loss and Creep Camber in a Highway Bridge with Reinforced Concrete Slab on Prestressed Concrete Beams," *Texas Transportation Institute Report*, Texas A&M University, College Station.

Furr, H.L. and R. Sinno (1970) "Hyperbolic Functions for Prestress Loss and Camber," *Journal of the Structural Division*, Vol. 96, No. 4, pp. 803-821.

PCI (2003). "Precast Prestressed Concrete Bridge Design Manual," 2nd Ed., Precast/Prestressed Concrete Institute, Chicago, Illinois.

Sinno, R. (1968). "The Time-Dependent Deflections of Prestressed Concrete Bridge Beams," *Ph.D. Dissertation*, Texas A&M University, College Station.

TxDOT (2001). "TxDOT Bridge Design Manual," Bridge Division, Texas Department of Transportation.

TxDOT (2004). "Prestressed Concrete Beam Design/Analysis Program," User Guide, Version 4.00, Bridge Division, Texas Department of Transportation.

Appendix B.1

Design Example for Interior Texas U54 Girder using AASHTO Standard Specifications

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

TABLE Page

B.1 Design Example for Interior Texas U54 Girder using AASHTO Standard Specifications

B.1.1 INTRODUCTION

The following detailed example shows sample calculations for the design of a typical interior Texas precast, prestressed concrete U54 girder supporting a single span bridge. The design is based on the *AASHTO Standard Specifications for Highway Bridges*, 17th Edition (AASHTO 2002). The recommendations provided by the TxDOT Bridge Design Manual (TxDOT 2001) are considered in the design. The number of strands and concrete strength at release and at service are optimized using the TxDOT methodology.

B.1.2 DESIGN PARAMETERS

The bridge considered for design example has a span length of 110 ft. (c/c abutment distance), a total width of 46 ft., and total roadway width of 44 ft. The bridge superstructure consists of four Texas U54 girders spaced 11.5 ft. center-to-center and designed to act compositely with an 8 in. thick cast-in-place concrete deck as shown in Figure B.1.2.1. The wearing surface thickness is 1.5 in., which includes the thickness of any future wearing surface. T501 type rails are used. AASHTO HS20 is the design live load. A relative humidity of 60 percent is considered in the design. The bridge cross section is shown in Figure B.1.2.1.

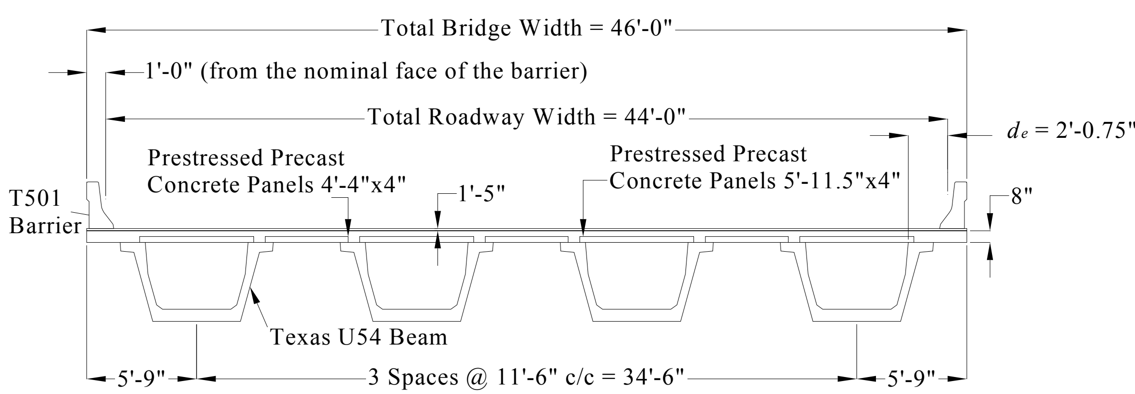

Figure B.1.2.1. Bridge Cross-Section Details.

The design span and overall girder length are based on the following calculations. Figure B.1.2.2 shows the girder end details for Texas U54 girders. It is clear that the distance between the centerline of the interior bent and end of the girder is 3 in., and the distance between the centerline of the interior bent and the centerline of the bearings is 9.5 in.

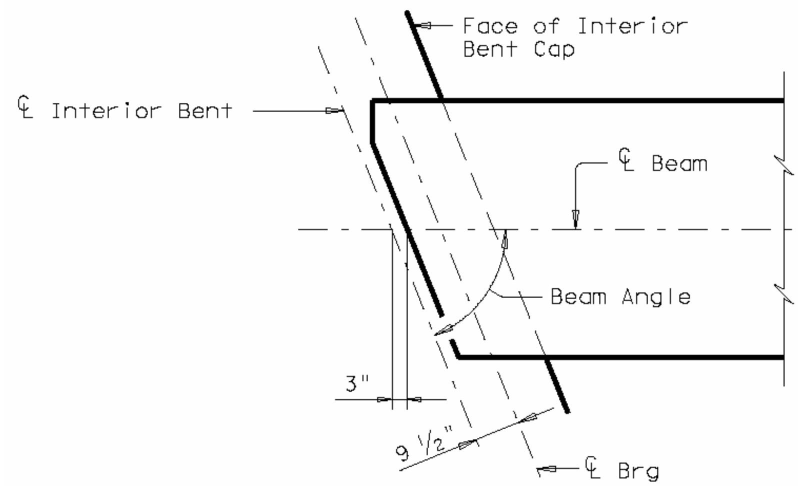

Figure B.1.2.2. Girder End Detail for Texas U54 Girders (TxDOT 2001).

Span length (c/c abutments) = 110 ft.-0 in.

From Figure B1.2.2.:

Overall girder length = 110 ft. – 2(3 in.) = 109 ft.-6 in.

Design span = 110 ft. – 2(9.5 in.) = 108 ft.-5 in.

= 108.417 ft. (c/c of bearing)

B.1.3 MATERIAL PROPERTIES

Cast-in-place slab:

Thickness t_s = 8.0 in.

Concrete strength at 28 days, f'_c = 4000 psi

Unit weight of concrete = 150 pcf

Wearing surface:

Thickness of asphalt wearing surface (including any future wearing surfaces), t_w = 1.5 in.

Unit weight of asphalt wearing surface = 140 pcf

[TxDOT recommendation]

Precast girders: Texas U54 girder

Concrete strength at release, f'_{ci} = 4000 psi*

Concrete strength at 28 days, f'_c = 5000 psi*

Concrete unit weight, w_c = 150 pcf

*This value is taken as an initial estimate and will be finalized based on the optimum design.

Prestressing strands: 0.5 in. diameter: seven wire low-relaxation

Area of one strand = 0.153 in.^{2}

Ultimate stress, f'_s = 270,000 psi

Yield strength, f_y = 0.9 f'_s = 243,000 psi [STD Art. 9.1.2]

Initial pretensioning, f_{si} = 0.75 f'_s

= 202,500 psi [STD Art. 9.15.1]

Modulus of elasticity, E_s = 28,000 ksi [STD Art. 16.2.1.2]

Non-prestressed reinforcement:

Yield strength, f_y = 60,000 psi

Traffic barrier:

T501 type barrier weight = 326 plf /side

B.1.4 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR GIRDER

B.1.4.1 Non-Composite Section

The section properties of a Texas U54 girder as described in the TxDOT Bridge Design Manual (TxDOT 2001) are provided in Table B.1.4.1. The strand pattern and section geometry is shown in Figure B.1.4.1.

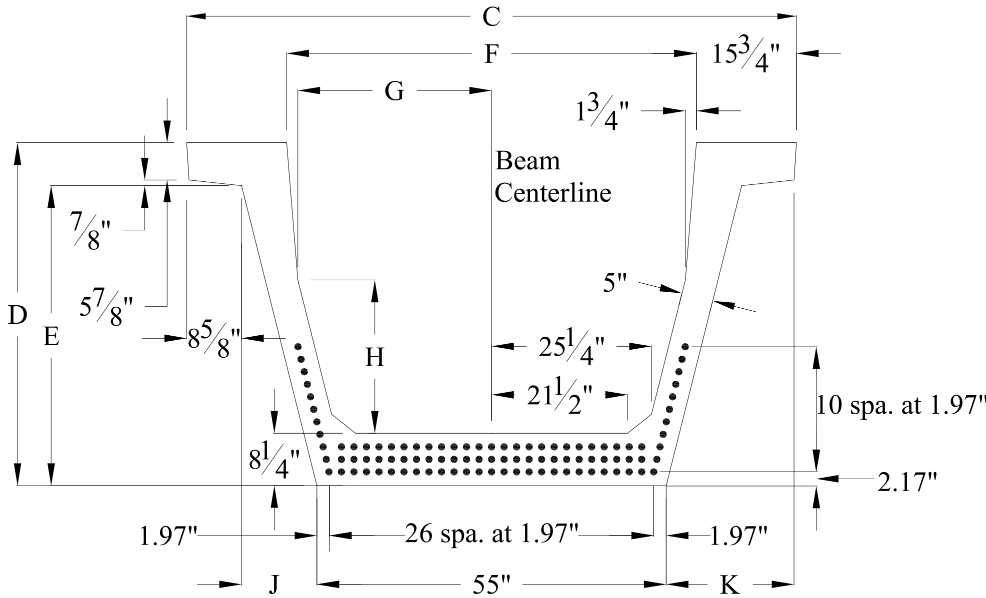

Figure B.1.4.1. Typical Section and Strand Pattern of Texas U54 Girders (TxDOT 2001).

Table B.1.4.1. Section Properties of Texas U54 girders [adapted from TxDOT Bridge Design Manual (TxDOT 2001)].

C	D	E	F	G	H	J	K	y_t	y_b	Area	I	Weight
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.^{2}	in.^{4}	plf
96	54	47.25	64.5	30.5	24.125	11.875	20.5	31.58	22.36	1120	403,020	1167

Note: Notations as used in Figure B.1.2.3.

where:

I = Moment of inertia about the centroid of the non-composite precast girder, in.^{4}

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder, in.

y_t = Distance from centroid to the extreme top fiber of the non-composite precast girder, in.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder, in.^{3}
$= I/\ y_b = 403{,}020/22.36 = 18{,}024.15$ in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder, in.^{3}
$= I/y_t = 403{,}020/31.58 = 12{,}761.88$ in.^{3}

B.1.4.2 Composite Section

B.1.4.2.1 Effective Flange Width

[STD Art. 9.8.3]

The Standard Specifications do not give specific guidelines regarding the calculation of effective flange width for open box sections. Following the LRFD recommendations, the effective flange width is determined as though each web is an individual supporting element. Thus, the effective flange width will be calculated according to guidelines of the Standard Specifications Art. 9.8.3 as below, and Figure B.1.4.2 shows the application of this assumption.

[STD Art. 9.8.3.1]

The effective web width of the precast girder is lesser of:

b_e = Top flange width = 15.75 in. (controls)

or, $b_e = 6\times$(flange thickness) + web thickness + fillets

$= 6\times(5.875 \text{ in.} + 0.875 \text{ in.}) + 5.00 \text{ in.} + 0 \text{ in.} = 45.5 \text{ in.}$

The effective flange width is lesser of: [STD Art. 9.8.3.2]

- $0.25\times$ effective girder span length
  $= \dfrac{108.417 \text{ ft. } (12 \text{ in./ft.})}{4} = 325.25 \text{ in.}$

- $6\times$(slab thickness on each side of the effective web width) + effective girder web width
  $= 6\times(8.0 \text{ in.} + 8.0 \text{ in.}) + 15.75 \text{ in.} = 111.75 \text{ in.}$

- One-half the clear distance on each side of the effective web width plus the effective web width:
  $= 0.5\times(4.0625 \text{ ft.} + 4.8125 \text{ ft.}) + 1.3125 \text{ ft.}$
  = 69 in. = 5.75 ft. (controls)

For the entire U54 girder, the effective flange width is

$2\times(5.75 \text{ ft.}\times 12) = 138 \text{ in.} = 11.5 \text{ ft.}$

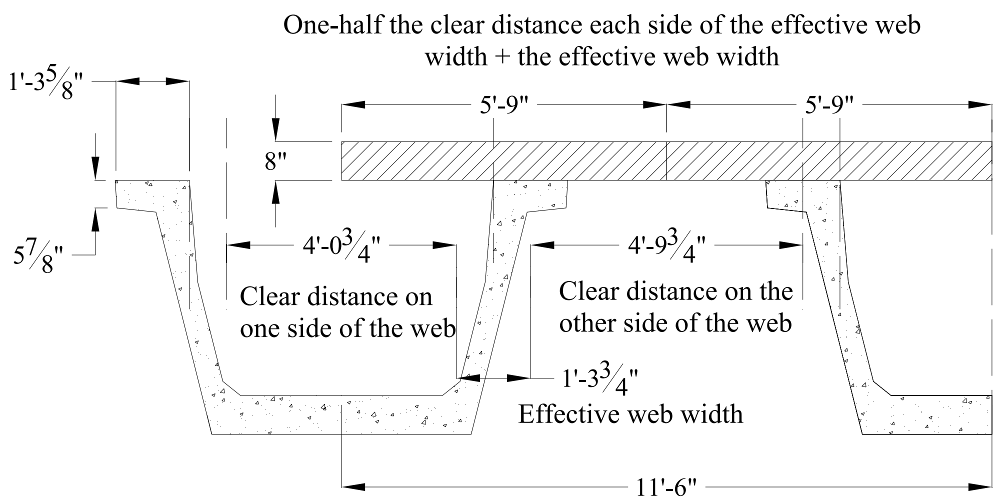

Figure B.1.4.2. Effective Flange Width Calculation.

B.1.4.2.2 Modular Ratio between Slab and Girder Concrete

Following the TxDOT design recommendation, the modular ratio between the slab and girder materials is taken as 1.

$$n = \left(\frac{E_c \text{ for slab}}{E_c \text{ for beam}} \right) = 1$$

where:

n = Modular ratio

E_c = Modulus of elasticity of concrete (ksi)

B.1.4.2.3 Transformed Section Properties

Figure B.1.4.3 shows the composite section dimensions, and Table B.1.4.2 shows the calculations for the transformed composite section.

Transformed flange width = n × (effective flange width)
= 1(138 in.) = 138 in.

Transformed flange area = n × (effective flange width) (t_s)
= 1 (138 in.) (8 in.) = 1104 in.^{2}

Table B.1.4.2. Properties of Composite Section.

	Transformed Area in.^{2}	y_b in.	$A\,y_b$ in.	$A(y_{bc} - y_b)^2$ in.^{4}	I in.^{4}	$I+A(y_{bc}- y_b)^2$ in.^{4}
Girder	1120	22.36	25,043.2	350,488	403,020	753,508
Slab	1104	58	64,032	355,712	5888	361,600
$\sum$	2224		89,075.2			1,115,108

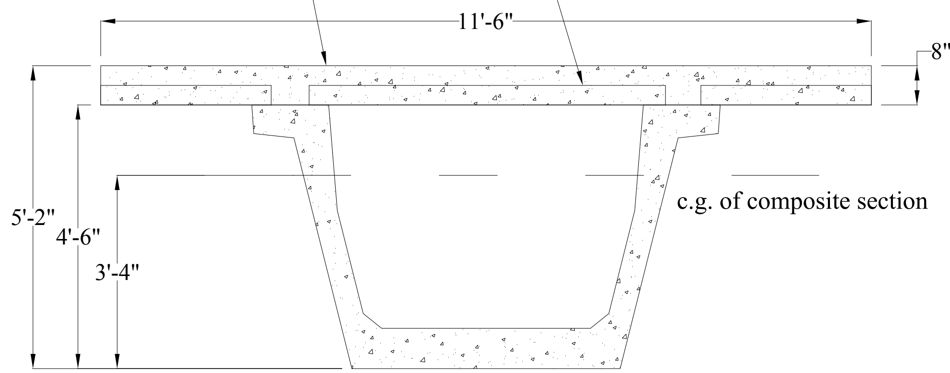

Figure B.1.4.3. Composite Section.

A_c = Total area of composite section = 2224 in.^{2}

h_c = Total height of composite section = 62 in.

I_c = Moment of inertia about the centroid of the composite section = 1,115,107.99 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder
= 89,075.2/ 2224 = 40.05 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder
= 54 – 40.05 = 13.95 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 40.05 = 21.95 in.

S_{bc} = Composite section modulus referenced to the extreme bottom fiber of the precast girder = I_c/y_{bc}
= 1,115,107.99 / 40.05 = 27,842.9in.^{3}

S_{tg} = Composite section modulus referenced to the top fiber of the precast girder = I_c/y_{tg}
= 1,115,107.99 / 13.95 = 79,936.06 in.^{3}

S_{tc} = Composite section modulus referenced to the top fiber of the slab = I_c/y_{tc} = 1,115,107.99 / 21.95 = 50,802.19 in.^{3}

B.1.5 SHEAR FORCES AND BENDING MOMENTS

B.1.5.1 Shear Forces and Bending Moments due to Dead Loads

The self-weight of the girder and the weight of slab act on the non-composite simple span structure, while the weight of barriers, future wearing surface, and live load plus impact act on the composite simple span structure.

B.1.5.1.1 Dead Loads

B.1.5.1.1.1 Due to Girder Self-Weight

Self-weight of the girder = 1.167 kips/ft.
[TxDOT Bridge Design Manual (TxDOT 2001)]

B.1.5.1.1.2 Due to Deck Slab

Weight of the CIP deck and precast panels on each girder

$$= (0.150 \text{ kcf})\left(\frac{8 \text{ in.}}{12 \text{ in./ft.}}\right)\left(\frac{138 \text{ in.}}{12 \text{ in./ft.}}\right) = 1.15 \text{ kips/ft.}$$

B.1.5.1.1.3 Due to Diaphragm

The TxDOT Bridge Design Manual (TxDOT 2001) requires two interior diaphragms for U54 girders, located as close as 10 ft. from the midspan of the girder. Shear forces and bending moment values in the interior girder can be calculated using the following equations. The arrangement of diaphragms is shown in Figure B.1.5.1.

For $x = 0$ ft. – 44.21 ft.

$$V_x = 3 \text{ kips} \qquad M_x = 3x \text{ kips}$$

For $x = 44.21$ ft. – 54.21 ft.

$$V_x = 0 \text{ kips} \qquad M_x = 3x - 3(x - 44.21) \text{ kips}$$

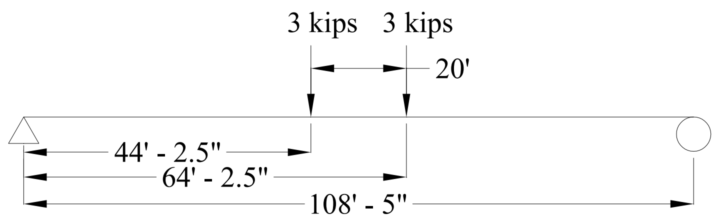

Figure B.1.5.1. Location of Interior Diaphragms on a Simply Supported Bridge Girder.

B.1.5.1.1.4 Due to Haunch

For U54 bridge girder design, TxDOT Bridge Design Manual (TxDOT 2001) accounts for haunches in designs that require special geometry and where the haunch will be large enough to have a significant impact on the overall girder. Because this project is for typical bridges, a haunch will not be included for U54 girders for composite properties of the section and additional dead load considerations.

B.1.5.1.2 Superimposed Dead Load

The TxDOT Bridge Design Manual (TxDOT 2001) recommends that one-third of the rail dead load should be used for an interior girder adjacent to the exterior girder.

Weight of T501 rails or barriers on each interior girder =

$$\left(\frac{326 \text{ plf}/1000}{3}\right) = 0.109 \text{ kips/ft./interior girder}$$

The dead loads placed on the composite structure are distributed equally among all girders [STD Art. 3.23.2.3.1.1 & TxDOT Bridge Design Manual (TxDOT 2001)].

$$\text{Weight of 1.5 in. wearing surface} = \frac{(0.140 \text{ pcf})\left(\frac{1.5 \text{ in.}}{12 \text{ in./ft.}}\right)(44 \text{ ft.})}{4 \text{ beams}}$$

$$= 0.193 \text{ kips/ft.}$$

Total superimposed dead load = 0.109 + 0.193 = 0.302 kip/ft.

B.1.5.1.3 Unfactored Shear Forces and Bending Moments

Shear forces and bending moments in the girder due to dead loads, superimposed dead loads at every tenth of the span, and at critical sections (midspan and $h/2$) are shown in this section. The bending moment and shear force due to dead loads and superimposed dead loads at any section at a distance x are calculated using the following expressions.

$$M = 0.5\, w\, x\, (L - x)$$

$$V = w\,(0.5L - x)$$

Critical section for shear is located at a distance $h/2 = 62/2 = 31$ in. $= 2.583$ ft.

The shear forces and bending moments due to dead loads and superimposed dead loads are shown in Tables B.1.5.1 and B.1.5.2.

Table B.1.5.1. Shear Forces due to Dead Loads.

Distance	Section	Non-Composite Dead Load			Superimposed Dead Loads		Total Dead Load Shear Force
		Girder Weight	Slab Weight	Diaphram Weight	Barrier Weight	Wearing Surface Weight	
x	x/L	V_g	V_{slab}	V_{dia}	V_b	V_{ws}	
ft.		kips	kips	kips	kips	kips	kips
0.000	0.000	63.26	62.34	3.00	5.91	10.46	144.97
2.583	0.024	60.25	59.37	3.00	5.63	9.96	138.21
10.842	0.100	50.61	49.87	3.00	4.73	8.37	116.58
21.683	0.200	37.96	37.40	3.00	3.55	6.28	88.19
32.525	0.300	25.30	24.94	3.00	2.36	4.18	59.78
43.367	0.400	12.65	12.47	3.00	1.18	2.09	31.39
54.209	0.500	0.00	0.00	0.00	0.00	0.00	0.00

Table B.1.5.2. Bending Moments due to Dead Loads.

Distance	Section	Non-Composite Dead Load			Superimposed Dead Loads		Total Dead Load Bending Moment
		Girder Weight	Slab Weight	Diaphram Weight	Barrier Weight	Wearing Surface Weight	
x	x/L	M_g	M_{slab}	M_{dia}	M_b	M_{ws}	
ft.		k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
0.000	0.000	0.00	0.00	0.00	0.00	0.00	0.00
2.583	0.024	159.51	157.19	7.75	14.90	26.38	365.73
10.842	0.100	617.29	608.30	32.53	57.66	102.09	1417.87
21.683	0.200	1097.36	1081.38	65.05	102.50	181.48	2527.77
32.525	0.300	1440.30	1419.32	97.58	134.53	238.20	3329.93
43.367	0.400	1646.07	1622.09	130.10	153.75	272.23	3824.24
54.209	0.500	1714.65	1689.67	132.63	160.15	283.57	3980.67

B.1.5.2 Shear Forces and Bending Moments due to Live Load

B.1.5.2.1 Due to Truck Load, V_{LT} and M_{LT}

[STD Art. 3.7.1.1]

The AASHTO Standard Specifications requires the live load to be taken as either HS20 truck loading or lane loading, whichever yields greater moments. The maximum shear force, V_T, and bending moment, M_T, due to HS20 truck load on a per-lane-basis are calculated using the following equations as given in the *PCI Design Manual* (PCI 2003).

Maximum undistributed bending moment,
For $x/L = 0 - 0.333$

$$M_T = \frac{72(x)[(L - x) - 9.33]}{L}$$

For $x/L = 0.333 - 0.5$

$$M_T = \frac{72(x)[(L - x) - 4.67]}{L} - 112$$

Maximum undistributed shear force,
For $x/L = 0 - 0.5$

$$V_T = \frac{72[(L - x) - 9.33]}{L}$$

where:

x = Distance from the center of the bearing to the section at which bending moment or shear force is calculated, ft.

L = Design span length = 108.417 ft.

M_T = Maximum undistributed bending moment due to HS-20 truck loading

V_T = Maximum undistributed shear force due to HS-20 truck loading

The maximum undistributed bending moments and maximum undistributed shear forces due to HS-20 truck load are calculated at every tength of the span and at critical section for shear. Table B.1.5.3 presents the values.

B.1.5.2.2 Due to Lane Load, V_L and M_L

The maximum bending moments and shear forces due to uniformly distributed lane load of 0.64 kip/ft. are calculated using the following equations as given in the *PCI Design Manual* (PCI 2003).

Maximum undistributed bending moment,

$$M_L = \frac{P(x)(L - x)}{L} + 0.5(w)(x)(L-x)$$

Maximum undistributed shear force,

$$V_L = \frac{Q(L - x)}{L} + (w)(\frac{L}{2} - x)$$

where:

x = Section at which bending moment or shear force is calculated

L = Span length = 108.417 ft.

P = Concentrated load for moment = 18 kips

Q = Concentrated load for shear = 26 kips

w = Uniform load per linear foot of load lane = 0.64 klf

The maximum undistributed bending moments and maximum undistributed shear forces due to HS-20 lane loading are calculated at every tenth of the span and at critical section for shear. The values are presented in Table B.1.5.3.

B.1.5.3 Distributed Live Load Bending and Shear

Distributed live load shear and bending moments are calculated by multiplying the distribution factor and the impact factor as follows:

Distributed bending moment, M_{LL+I}

$$M_{LL+I} = (\text{bending moment per lane})\,(DF)\,(1+I)$$

Distributed shear force, V_{LL+I}

$$V_{LL+I} = (\text{shear force per lane})\,(DF)\,(1+I)$$

where:

DF = Distribution factor

I = Live load impact factor

B.1.5.3.1 Live Load Distribution Factor for a Typical Interior Girder

As per recommendation of the TxDOT Bridge Design Manual (TxDOT 2001), the live load distribution factor for moment for a precast, prestressed concrete U54 interior girder is given by the following expression.

$$DF_{mom} = \frac{S}{11} = \frac{11.5}{11} = 1.045 \text{ per truck/lane} \quad \text{[TxDOT 2001]}$$

where:

S = Average interior girder spacing measured between girder centerlines (ft.)

The minimum value of DF_{mom} is limited to 0.9.

For simplicity of calculation and because there is no significant difference, the distribution factor for moment is used also for shear as recommended by the TxDOT Bridge Design Manual (TxDOT 2001).

The maximum distributed bending moments and maximum distributed shear forces due to HS-20 truck and HS-20 lane loading are calculated at every tenth of the span and at critical section for shear. The values are presented in Table B.1.5.3.

B.1.5.3.2
Live Load Impact Factor

The live load impact factor is given by the following expression:

$$I = \frac{50}{L + 125}$$

[STD Eq. 3-1]

where:

I = Impact fraction to a maximum of 30 percent

L = Span length (ft.) = 108.417 ft. [STD Art. 3.8.2.2]

$$I = \frac{50}{108.417 + 125} = 0.214$$

Impact for shear varies along the span according to the location of the truck but the impact factor computed above is used for simplicity.

Table B.1.5.3. Shear Forces and Bending Moments due to Live Loads.

Distance	Section	Live Load + Impact							
		HS 20 Truck Loading (controls)				HS20 Lane Loading			
		Undistributed		Distributed		Undistributed		Distributed	
x	x/L	Shear	Moment	Shear	Moment	Shear	Moment	Shear	Moment
ft.		kips	k-ft.	kips	k-ft.	kips	k-ft.	kips	k-ft.
0.000	0.000	65.80	0.00	83.52	0.00	34.69	0.00	36.27	0.00
2.583	0.024	64.09	165.54	81.34	210.10	33.06	87.48	34.56	91.45
10.842	0.100	58.60	635.38	74.38	806.41	28.10	338.53	29.38	353.92
21.683	0.200	51.40	1114.60	65.24	1414.62	22.20	601.81	23.21	629.16
32.525	0.300	44.20	1437.73	56.10	1824.74	17.00	789.88	17.77	825.78
43.370	0.400	37.00	1626.98	46.96	2064.93	12.49	902.73	13.06	943.76
54.210	0.500	29.80	1671.37	37.83	2121.27	8.67	940.34	9.07	983.08

B.1.5.4 Load Combinations

[STD Table 3.22.1A]

For service load design (Group I): 1.00 D + 1.00(L+I)

where:

D = Dead load

L = Live load

I = Impact factor

For load factor design (Group I): 1.3[1.00D + 1.67(L+I)]

B.1.6 ESTIMATION OF REQUIRED PRESTRESS

B.1.6.1 Service Load Stresses at Midspan

The preliminary estimate of the required prestress and number of strands is based on the stresses at midspan.

Bottom tensile stresses at midspan due to applied loads

$$f_b = \frac{M_g + M_S}{S_b} + \frac{M_{SDL} + M_{LL+I}}{S_{bc}}$$

Top tensile stresses at midspan due to applied loads

$$f_t = \frac{M_g + M_S}{S_t} + \frac{M_{SDL} + M_{LL+I}}{S_{tg}}$$

where:

f_b = Concrete stress at the bottom fiber of the girder (ksi)

f_t = Concrete stress at the top fiber of the girder (ksi)

M_g = Unfactored bending moment due to girder self-weight (k-ft.)

M_S = Unfactored bending moment due to slab, diaphragm weight (k-ft.)

M_{SDL} = Unfactored bending moment due to super imposed dead load (k-ft.)

M_{LL+I} = Factored bending moment due to superimposed dead load (k-ft.)

Substituting the bending moments and section modulus values, the bottom tensile stress at midspan is:

$$f_b = \frac{(1714.64 + 1689.66 + 132.63)(12)}{18024.15} + \frac{(443.72 + 2121.27)(12)}{27842.9}$$
$$= 3.46 \text{ ksi}$$

$$f_t = \frac{(1714.64 + 1689.66 + 132.63)(12)}{12761.88} + \frac{(443.72 + 2121.27)(12)}{79936.06}$$
$$= 3.71 \text{ ksi}$$

B.1.6.2 Allowable Stress Limit

At service load conditions, allowable tensile stress is:

$$F_b = 6\sqrt{f_c'} = 6\sqrt{5000}\left(\frac{1}{1000}\right) = 0.424 \text{ ksi} \qquad \text{[STD Art. 9.15.2.2]}$$

B.1.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – allowable tensile stress at final = $f_b - F_b$
= 3.46 – 0.424 = 3.036 ksi

Assuming the distance from the center of gravity of strands to the bottom fiber of the girder is equal to y_{bs} = 2 in.

Strand eccentricity at midspan:

$$e_c = y_b - y_{bs} = 22.36 - 2 = 20.36 \text{ in.}$$

Bottom fiber stress due to prestress after losses:

$$f_b = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b}$$

where:

P_{se} = Effective pretension force after all losses

$$3.036 = \frac{P_{se}}{1120} + \frac{20.36\ P_{se}}{18024.15}$$

Solving for P_{se}:

P_{se} = 1501.148 kips

Assuming final losses = 20 percent of f_{si}

Assumed final losses = 0.2(202.5 ksi) = 40.5 ksi

The prestress force per strand after losses:

P_{se} = (cross-sectional area of one strand) [f_{si} – losses]

P_{se} = 0.153(202.5 – 40.5) = 24.786 kips

Number of strands required = 1500.159/24.786 = 60.56

Try 62 – 0.5 in. diameter, 270 ksi strands.

Strand eccentricity at midspan after strand arrangement

$$e_c = 22.36 - \frac{27(2.17)+27(4.14)+8(6.11)}{62} = 18.934 \text{ in.}$$

P_{se} = 62(24.786) = 1536.732 kips

$$f_b = \frac{1536.732}{1120} + \frac{18.934(1536.732)}{18024.15}$$

$$= 1.372 + 1.614 = 2.986 \text{ ksi} < f_b \text{ reqd.} = 3.034 \text{ ksi}$$

Try 64 – 0.5 in. diameter, 270 ksi strands

Strand eccentricity at midspan after strand arrangement

$$e_c = 22.36 - \frac{27(2.17)+27(4.14)+10(6.11)}{64} = 18.743 \text{ in.}$$

P_{se} = 64(24.786) = 1586.304 kips

$$f_b = \frac{1586.304}{1120} + \frac{18.743(1586.304)}{18024.15}$$

$$= 1.416 + 1.650 = 3.066 \text{ ksi} > f_b \text{ reqd.} = 3.036 \text{ ksi}$$

Therefore, use 64 strands as shown in Figure B.1.6.1.

Number of Strands	Distance from Bottom (in.)
27	2.17
27	4.14
10	6.11

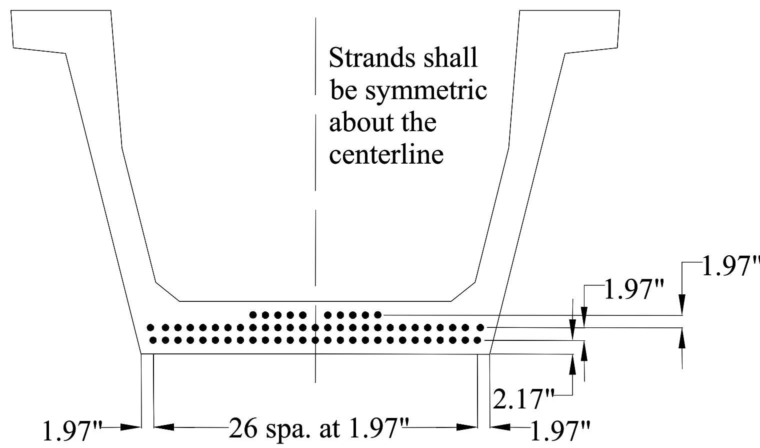

Figure B.1.6.1. Initial Strand Pattern.

B.1.7 PRESTRESS LOSSES

[STD Art. 9.16.2]

Total prestress losses = $SH + ES + CR_C + CR_S$ [STD Eq. 9-3]

where:

SH = Loss of prestress due to concrete shrinkage

EC = Loss of prestress due to elastic shortening

CR_C = Loss of prestress due to creep of concrete

CR_S = Loss of prestress due to relaxation of prestressing steel.

Number of strands = 64

A number of iterations will be performed to arrive at the optimum values of f'_c and f'_{ci}.

B.1.7.1 Iteration 1

B.1.7.1.1 Shrinkage

[STD Art. 9.16.2.1.1]

SH = 17,000 – 150 RH [STD Eq. 9-4]

where RH is the relative humidity = 60 percent

$$SH = [17000 - 150(60)]\frac{1}{1000} = 8 \text{ ksi}$$

B.1.7.1.2 Elastic Shortening

[STD Art. 9.16.2.1.2]

$$ES = \frac{E_s}{E_{ci}} f_{cir} \qquad \text{[STD Eq. 9-6]}$$

where:

f_{cir} = Average concrete stress at the center of gravity of the prestressing steel due to pretensioning force and dead load of girder immediately after transfer

$$= \frac{P_{si}}{A} + \frac{P_{si}\, e_c^2}{I} - \frac{(M_g)e_c}{I}$$

P_{si} = Pretensioning force after allowing for the initial losses, assuming 8 percent initial losses = $A_{ps}\,[0.92(0.75\, f_s')]$
= 64(0.153)(0.92)(0.75)(270) = 1824.25 kips

M_g =Unfactored bending moment due to girder self weight
= 1714.64 k-ft.

e_c = Eccentricity of the strand at the midspan = 18.743 in.

$$f_{ci} = \frac{1824.25}{1120} + \frac{1824.25\,(18.743)^2}{403{,}020} - \frac{1714.64(12)(18.743)}{403{,}020}$$

= 1.629 + 1.590 – 0.957 = 2.262 ksi

Assuming f'_{ci} = 4000 psi

$$E_{ci} = (150)1.5(33)\sqrt{4000}\;\frac{1}{1000} = 3834.254 \text{ ksi} \qquad \text{[STD Eq. 9-8]}$$

$$ES = \frac{28000}{3834.254}\,(2.262) = 16.518 \text{ ksi}$$

B.1.7.1.3
Creep of Concrete

[STD Art. 9.16.2.1.3]

$CR_C = 12\, f_{cir} - 7\, f_{cds}$ [STD Eq. 9-9]

where:

f_{cds} = Concrete stress at the center of gravity of the prestressing steel due to all dead loads except the dead load present at the time the pretensioning force is applied (ksi)

$$= \frac{M_S\, e_c}{I} + \frac{M_{SDL}\,(y_{bc} - y_{bs})}{I_c}$$

where:

M_S = Moment due to slab and diaphragm = 1822.29 k-ft.

M_{SDL} = Superimposed dead load moment = 443.72 k-ft.

y_{bc} = 40.05 in.

y_{bs} = Distance from center of gravity of the strand at midspan to the bottom of the girder
= 22.36 – 18.743 = 3.617 in.

I = Moment of inertia of the non-composite section
= 403,020 in.^{4}

I_c = Moment of inertia of composite section
= 1,115,107.99 in.^{4}

$$f_{cds} = \frac{1822.29(12)(18.743)}{403020} + \frac{(443.72)(12)(40.05 - 3.617)}{1115107.99}$$

= 1.017 + 0.174 = 1.191 ksi

CR_C = 12(2.262) – 7(1.191) = 18.807 ksi

B.1.7.1.4
Relaxation of Prestressing Steel

[STD Art. 9.16.2.1.4]

For pretensioned members with 270 ksi low-relaxation strand,

$CR_S = 5000 - 0.10\, ES - 0.05(SH + CR_C)$ [STD Eq. 9-10A]

$$= [5000 - 0.10(16518) - 0.05(8000 + 18{,}807)]\left(\frac{1}{1000}\right)$$

= 2.008 ksi

The PCI Bridge Design Manual (PCI 2003) considers only the elastic shortening loss in the calculation of total initial prestress loss. Whereas, the TxDOT Bridge Design Manual (TxDOT 2001) recommends that 50 percent of the final steel relaxation loss shall also be considered for calculation of total initial prestress loss given as [elastic shortening loss + 0.50 (total steel relaxation loss)]. Based on the TxDOT Bridge Design Manual (TxDOT 2001) recommendations, the initial prestress loss is calculated as follows.

$$\text{Initial prestress loss} = \frac{(ES + 0.5CRs)100}{0.75 f_s'}$$

$$= \frac{[16.518+0.5(2.008)]100}{0.75(270)}$$

= 8.653 % > 8 % (assumed initial prestress losses)

Therefore, another trial is required assuming 8.653 percent initial losses.

The change in initial prestress loss will not affect the prestress loss due to concrete shrinkage. Therefore, the next trials will involve updating the losses due to elastic shortening, steel relaxation, and creep of concrete.

Loss in prestress due to elastic shortening

$$ES = \frac{E_s}{E_{ci}} f_{cir} \qquad \text{[STD Eq. 9-6]}$$

where:

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}\, e_c^2}{I} - \frac{(M_g)\, e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, assuming 8.653 percent initial losses

$= A_{ps}[0.9135(0.75\, f_s')]$

= 64(0.153)(0.9135)(0.75)(270) = 1811.3 kips

A_{ps} = Total area of prestressing steel, in^2.

M_g = Unfactored bending moment due to girder self-weight = 1714.64 k-ft.

e_c = Ecentricity of the strand at the midspan = 18.743 in.

$$f_{cir} = \frac{1811.3}{1120} + \frac{1811.3(18.743)^2}{403020} - \frac{1714.64(12)(18.743)}{403{,}020}$$

= 1.617 + 1.579 – 0.957 = 2.239 ksi

Assuming $f_{ci}' = 4000$ psi

$$E_{ci} = (150)^{1.5}(33)\sqrt{4000}\ \frac{1}{1000} = 3834.254 \text{ ksi}$$

$$ES = \frac{28000}{3834.254}(2.239) = 16.351 \text{ ksi}$$

Loss in prestress due to creep of concrete

$$CR_C = 12\, f_{cir} - 7\, f_{cds}$$

The value of f_{cds} is independent of the initial prestressing force value and will be the same as calculated in Section B.1.7.1.3.

Therefore, f_{cds} = 1.191 ksi

CR_C = 12(2.239) – 7(1.191) = 18.531 ksi.

Loss in prestress due to relaxation of steel

$$CR_S = 5000 - 0.10\, ES - 0.05(SH + CR_C)$$

$$= [5000 - 0.10(16351) - 0.05(8000 + 18531)]\left(\frac{1}{1000}\right)$$

$$= 2.038 \text{ ksi}$$

$$\text{Initial prestress loss} = \frac{(ES + 0.5CRs)100}{0.75 f_s'}$$

$$= \frac{[16.351+0.5(2.038)]100}{0.75(270)}$$

= 8.578 percent < 8.653 percent (assumed initial prestress losses)

Therefore, next trial is required assuming 8.580 percent initial losses,

Loss in prestress due to elastic shortening

$$ES = \frac{E_s}{E_{ci}} f_{cir}$$

where:

$$f_{cir} = \frac{P_{si}}{A} + \frac{P_{si}\, e_c^2}{I} - \frac{(M_g)\, e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, assuming 8.580 percent initial losses

$= A_{ps}$ [0.9142 (0.75 f'_s)]

= 64(0.153)(0.9142)(0.75)(270) = 1812.75 kips

$$f_{cir} = \frac{1812.75}{1120} + \frac{1812.75(18.743)^2}{403{,}020} - \frac{1714.64(12)(18.743)}{403{,}020}$$

= 1.619 + 1.580 – 0.957 = 2.242 ksi

Assuming f'_{ci} = 4000 psi

$$E_{ci} = (150)^{1.5}(33)\sqrt{4000}\ \frac{1}{1000} = 3834.254 \text{ ksi}$$ [STD Eq. 9-8]

$$ES = \frac{28000}{3834.254}(2.242) = 16.372 \text{ ksi}$$

Loss in prestress due to creep of concrete

$CR_C = 12\,f_{cir} - 7\,f_{cds}$

f_{cds} = 1.191 ksi

CR_C = 12(2.242) – 7(1.191) = 18.567 ksi

Loss in prestress due to relaxation of steel

$CR_S = 5000 - 0.10\,ES - 0.05(SH + CR_C)$

$$= [5000 - 0.10(16{,}372) - 0.05(8000 + 18{,}567)]\left(\frac{1}{1000}\right)$$

= 2.034 ksi

$$\text{Initial prestress loss} = \frac{(ES + 0.5CRs)100}{0.75 f'_s}$$

$$= \frac{[16.372+0.5(2.034)]100}{0.75(270)} = 8.587 \text{ percent} \approx 8.580 \text{ percent}$$

(assumed initial prestress losses)

B.1.7.1.5 Total Losses at Transfer

Total initial losses = $(ES + 0.5CR_S)$ = [16.372+0.5(2.034)] = 17.389 ksi

f_{si} = Effective initial prestress = 202.5 – 17.389 = 185.111 ksi

P_{si} = Effective pretension force after allowing for the initial losses

= 64(0.153)(185.111) = 1812.607 kips

B.1.7.1.6
Total Losses at Service Loads

SH = 8 ksi

ES = 16.372 ksi

CR_C = 18.587 ksi

CR_S = 2.034 ksi

Total final losses = 8 + 16.372 + 18.587 + 2.034 = 44.973 ksi

or $\dfrac{44.973(100)}{0.75(270)}$ =22.21 percent

f_{se} = Effective final prestress = 0.75(270) – 44.973 = 157.527 ksi

P_{se} = 64(0.153)(157.527) = 1542.504 kips

B.1.7.1.7
Final Stresses at Midspan

Final stress in the bottom fiber at midspan:

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1542.504}{1120} + \frac{18.743(1542.504\ )}{18024.15} - 3.458$$

= 1.334 + 1.554 – 3.458 = –0.57 ksi > –0.424 ksi (N.G.)

Therefore, try 66 strands,

$$e_c = 22.36 - \frac{27(2.17) + 27(4.14) + 12(6.11)}{66} = 18.67 \text{ in.}$$

P_{se} = 66(0.153)(157.527) = 1590.708 kips

$$f_{bf} = \frac{1590.708}{1120} + \frac{18.67(1590.708\ )}{18024.15} - 3.458$$

= 1.42 + 1.648 – 3.458 = -0.39 ksi < -0.424 ksi (O.K.)

Therefore, use 66 strands.

Final concrete stress at the top fiber of the girder at midspan,

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + f_t = \frac{1590.708}{1120} - \frac{18.67(1590.708\ )}{12761.88} + 3.71$$

$= 1.42 - 2.327 + 3.71 = 2.803$ ksi

B.1.7.1.8
Initial Stresses at End

Initial concrete stress at top fiber of the girder at girder end

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\ e_c}{S_t} + \frac{M_g}{S_t}$$

where:

P_{si} = 66(0.153)(185.111) = 1869.251 kips

M_g = Moment due to girder self-weight at girder end = 0 k-ft.

$$f_{ti} = \frac{1869.251}{1120} - \frac{18.67(1869.251)}{12,761.88}$$

$= 1.669 - 2.735 = -1.066$ ksi

Tension stress limit at transfer is $7.5\sqrt{f'_{ci}}$. [STD Art. 9.15.2.1]

Therefore, $f'_{ci\ \text{reqd.}} = \left(\frac{1066}{7.5}\right)^2 = 20{,}202$ psi

Initial concrete stress at bottom fiber of the girder at girder end

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1869.251}{1120} + \frac{18.67(1869.251)}{18024.15}$$

$=1.669 + 1.936 = 3.605$ ksi

Compression stress limit at transfer is $0.6\ f'_{ci}$. [STD Art. 9.15.2.1]

Therefore, $f'_{ci\ \text{reqd.}} = \frac{3605}{0.6} = 6009$ psi

B.1.7.1.9 Debonding of Strands and Debonding Length

The calculation for initial stresses at the girder end shows that the preliminary estimate of $f'_{ci} = 4000$ psi is not adequate to keep the tensile and compressive stresses at transfer within allowable stress limits as per STD Art. 9.15.2.1. Therefore, debonding of strands is required to keep the stresses within allowable stress limits.

To be consistent with the TxDOT design procedures, the debonding of strands is carried out in accordance with the procedure followed in PSTRS14 (TxDOT 2004). Two strands are debonded at a time at each section located at uniform increments of 3 ft. along the span length, beginning at the end of the girder. The debonding is started at the end of the girder because due to relatively higher initial stresses at the end, a greater number of strands are required to be debonded and the debonding requirement, in terms of number of strands, reduces as the section moves away from the end of the girder. To make the most efficient use of debonding the debonding, at each section begins at the bottommost row where the eccentricity is largest and moves up. Debonding at a particular section will continue until the initial stresses are within the allowable stress limits or until a debonding limit is reached. When the debonding limit is reached, the initial concrete strength is increased and the design cycles to convergence. As per TxDOT Bridge Design Manual (TxDOT 2001) the limits of debonding for partially debonded strands are described as follows:

1. Maximum percentage of debonded strands per row and per section
    a. TxDOT Bridge Design Manual (TxDOT 2001) recommends a maximum percentage of debonded strands per row should not exceed 75 percent.
    b. TxDOT Bridge Design Manual (TxDOT 2001) recommends a maximum percentage of debonded strands per section should not exceed 75 percent.

2. Maximum length of debonding
    a. TxDOT Bridge Design Manual (TxDOT 2001) recommends to use the maximum debonding length to be the lesser of the following:
        i. 15 ft.,
        ii. 0.2 times the span length, or
        iii. half the span length minus the maximum development length as specified in the 1996 AASHTO Standard Specifications for Highway Bridges, Section 9.28.

B.1.7.1.10 Maximum Debonding Length

As per the TxDOT Bridge Design Manual (TxDOT 2001), the maximum debonding length is the lesser of the following:

- 15 ft.,
- 0.2 (L), or
- 0.5 (L) – l_d

where, l_d is the development length calculated based on AASHTO STD Art. 9.28.1 as follows:

$$l_d \geq \left(f_{su}^* - \frac{2}{3} f_{se} \right) D \qquad \text{[STD Eq. 9.42]}$$

where:

l_d = Development length (in.)

f_{se} = Effective stress in the prestressing steel after losses = 157.527 (ksi)

D = Nominal strand diameter = 0.5 in.

f_{su}^* = Average stress in the prestressing steel at the ultimate load (ksi)

$$f_{su}^* = f_s' \left[1 - \left(\frac{\gamma^*}{\beta_1} \right) \left(\frac{\rho^* f_s'}{f_c'} \right) \right] \qquad \text{[STD Eq. 9.17]}$$

where:

f_s' = Ultimate stress of prestressing steel (ksi)

γ^* = Factor based on type of prestressing steel = 0.28 for low-relaxation steel

f_c' = Compressive strength of concrete at 28 days (psi)

$$\rho^* = \frac{A_s^*}{bd} = \text{ratio of prestressing steel}$$

$$= \frac{0.153 \times 66}{138 \times 8.67 \times 12} = 0.00033$$

β_1 = Factor for concrete strength

$$\beta_1 = 0.85 - 0.05 \frac{(f_c' - 4000)}{1000} \qquad \text{[STD Art. 8.16.2.7]}$$

$$= 0.85 - 0.05 \frac{(5000 - 4000)}{1000} = 0.80$$

$$f_{su}^{*} = 270\left[1-\left(\frac{0.28}{0.80}\right)\left(\frac{0.00033\times 270}{5}\right)\right] = 268.32 \text{ ksi}$$

The development length is calculated as:

$$l_d \geq \left(268.32-\frac{2}{3}157.527\right)\times 0.5$$

$$l_d = 6.8 \text{ ft.}$$

As per STD Art. 9.28.3, the development length calculated above should be doubled.

$$l_d = 13.6 \text{ ft.}$$

Hence, the debonding length is the lesser of the following:

- 15 ft.
- 0.2 × 108.417 = 21.68 ft., or
- 0.5 × 108.417 - 13.6 = 40.6 ft.

Hence, the maximum debonding length to which the strands can be debonded is 15 ft.

In Table B.1.7.1, the calculation of initial stresses at the extreme fibers and corresponding requirement of f'_{ci} suggests that the preliminary estimate of f'_{ci} to be 4000 psi is inadequate.

Table B.1.7.1 Calculation of Initial Stresses at Extreme Fibers and Corresponding Required Initial Concrete Strengths.

	Location of the Debonding Section (ft. from end)						
	End	3	6	9	12	15	Midspan
Row No. 1 (bottom row)	27	27	27	27	27	27	27
Row No. 2	27	27	27	27	27	27	27
Row No. 3	12	12	12	12	12	12	12
No. of Strands	66	66	66	66	66	66	66
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1869.25	1869.25	1869.25	1869.25	1869.25	1869.25	1869.25
e_c (in.)	18.67	18.67	18.67	18.67	18.67	18.67	18.67
Top Fiber Stresses (ksi)	-1.066	-0.892	-0.728	-0.575	-0.431	-0.297	0.547
Corresponding $f'_{ci\ reqd}$ (psi)	20202	14145	9422	5878	3302	1568	912
Bottom Fiber Stresses (ksi)	3.605	3.482	3.366	3.258	3.156	3.061	2.464
Corresponding $f'_{ci\ reqd}$ (psi)	6009	5804	5611	5429	5260	5101	4106

Because the strands cannot be debonded beyond the section located at 15 ft. from the end of the girder, f'_{ci} is increased from 4000 psi to 5101 psi and at all other sections debonding can be done. The strands are debonded to bring the required f'_{ci} below 5101 psi. Table B.1.7.2 shows the debonding schedule based on the procedure described earlier.

Table B.1.7.2. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	End	3	6	9	12	15	Midspan
Row No. 1 (bottom row)	7	7	15	23	25	27	27
Row No. 2	17	27	27	27	27	27	27
Row No. 3	12	12	12	12	12	12	12
No. of Strands	36	46	54	62	64	66	66
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1019.59	1302.81	1529.39	1755.96	1812.61	1869.25	1869.25
e_c (in.)	17.95	18.01	18.33	18.57	18.62	18.67	18.67
Top Fiber Stresses (ksi)	-0.524	-0.502	-0.494	-0.496	-0.391	-0.297	0.547
Corresponding $f'_{ci\ reqd}$ (psi)	4881	4480	4338	4374	2718	1568	912
Bottom Fiber Stresses (ksi)	1.926	2.342	2.682	3.029	3.041	3.061	2.464
Corresponding $f'_{ci\ reqd}$ (psi)	3210	3904	4470	5049	5069	5101	4106

B.1.7.2 Iteration 2

Following the procedure in Iteration 1, another iteration is required to calculate prestress losses based on the new value of f'_{ci} = 5101 psi. The results of this second iteration are shown in Table B.1.7.3.

Table B.1.7.3. Results of Iteration 2.

	Trial #1	Trial # 2	Trial # 3	Units
No. of Strands	66	66	66	
e_c	18.67	18.67	18.67	in.
SR	8	8	8	ksi
Assumed Initial Prestress Loss	8.587	7.967	8.031	percent
P_{si}	1869.19	1881.87	1880.64	kips
M_g	1714.65	1714.65	1714.65	k-ft.
f_{cir}	2.332	2.354	2.352	ksi
f_{ci}	5101	5101	5101	psi
E_{ci}	4329.91	4329.91	4329.91	ksi
ES	15.08	15.22	15.21	ksi
f_{cds}	1.187	1.187	1.187	ksi
CRc	19.68	19.94	19.92	ksi
CRs	2.11	2.08	2.08	ksi
Calculated Initial Prestress Loss	7.967	8.031	8.025	percent
Total Prestress Loss	44.86	45.24	45.21	ksi

B.1.7.2.1
Total Losses at Transfer

Total initial losses = $(ES + 0.5CRs)$ = [15.21 + 0.5(2.08)] = 16.25 ksi

f_{si} = Effective initial prestress = 202.5 – 16.25 = 186.248 ksi

P_{si} = Effective pretension force after allowing for the initial losses

P_{si} = 66(0.153)(186.248) = 1880.732 kips

B.1.7.2.2
Total Losses at Service Loads

SH = 8 ksi

ES = 15.21 ksi

CR_C = 19.92 ksi

CR_S = 2.08 ksi

Total final losses = 8 + 15.21 + 19.92 + 2.08 = 45.21 ksi

or $\frac{45.21(100)}{0.75(270)}$ = 22.32 percent

f_{se} = Effective final prestress = 0.75(270) – 45.21 = 157.29 ksi

P_{se} = 66(0.153)(157.29) = 1588.34 kips

B.1.7.2.3
Final Stresses at Midspan

Top fiber stress in concrete at midspan at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + f_t = \frac{1588.34}{1120} - \frac{18.67(1588.34)}{12761.88} + 3.71$$

$$= 1.418 - 2.323 + 3.71 = 2.805 \text{ ksi}$$

Allowable compression stress for all load combinations = 0.6 f_c'

$f'_{c\ reqd}$ = 2805/0.6 = 4675 psi

Top fiber stress in concrete at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \frac{1588.34}{1120} - \frac{18.67(1588.34)}{12761.88} + \frac{(1714.64 + 1822.29)(12)}{12761.88} + \frac{443.72(12)}{79936.06}$$

= 1.418 – 2.323 + 3.326 + 0.067 = 2.49 ksi

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f_c' [STD Art. 9.15.2.2]

$f'_{c\ reqd}$ = 2490/0.4 = 6225 psi (controls)

Top fiber stress in concrete at midspan due to live load + 0.5 (effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{2121.27(12)}{79936.06} + 0.5\left(\begin{matrix}\dfrac{1588.34}{1120} - \dfrac{18.67(1588.34)}{12761.88} \\ + \dfrac{(1714.64 + 1822.29)(12)}{12761.88} + \dfrac{443.72(12)}{79936.06}\end{matrix}\right)$$

= 0.318 + 0.5(1.418 – 2.323 + 3.326 + 0.067) = 1.629 ksi

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f_c' [STD Art. 9.15.2.2]

$f'_{c\ reqd}$ = 1562/0.4 = 3905 psi

Bottom fiber stress in concrete at midspan at service load

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1588.34}{1120} + \frac{18.67(1588.34)}{18024.15} - 3.46$$

= 1.418 + 1.633 – 3.46 = -0.397 ksi

Allowable tension in concrete = $6\sqrt{f_c'}$ [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \left(\frac{3970}{6}\right)^2 = 4366 \text{ psi}$$

B.1.7.2.4
Initial Stresses at Debonding Locations

With the same number of debonded strands as was determined in the previous iteration, the top and bottom fiber stresses with their corresponding initial concrete strengths are calculated. It can be observed that at the 15-ft. location, the f'_{ci} value is updated to 5138 psi. The results are shown in Table B.1.7.4.

Table B.1.7.4. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	0	3	6	9	12	15	54.2
Row No. 1 (bottom row)	7	7	15	23	25	27	27
Row No. 2	17	27	27	27	27	27	27
Row No. 3	12	12	12	12	12	12	12
No. of Strands	36	46	54	62	64	66	66
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1025.85	1310.81	1538.78	1766.75	1823.74	1880.73	1880.73
e_c (in.)	17.95	18.01	18.33	18.57	18.62	18.67	18.67
Top Fiber Stresses (ksi)	-0.527	-0.506	-0.499	-0.502	-0.398	-0.303	0.540
Corresponding $f'_{ci\ reqd}$ (psi)	4937	4552	4427	4480	2816	1632	900
Bottom Fiber Stresses (ksi)	1.938	2.357	2.700	3.050	3.063	3.083	2.486
Corresponding $f'_{ci\ reqd}$ (psi)	3229	3929	4500	5084	5105	5138	4143

B.1.7.3
Iteration 3

Following the procedure in iteration 1, a third iteration is required to calculate prestress losses based on the new value of f'_{ci} = 5138 psi. The results of this second iteration are shown in Table B.1.7.5.

Table B.1.7.5. Results of Iteration 3.

	Trial #1	Trial # 2	Units
No. of Strands	66	66	
e_c	18.67	18.67	in.
SR	8	8	ksi
Assumed Initial Prestress Loss	8.025	8.000	percent
P_{si}	1880.85	1881.26	kips
M_g	1714.65	1714.65	k-ft.
f_{cir}	2.352	2.354	ksi
f_{ci}	5138	5138	psi
E_{ci}	4346	4346	ksi
ES	15.16	15.17	ksi
f_{cds}	1.187	1.187	ksi
CRc	19.92	19.94	ksi
CRs	2.09	2.09	ksi
Calculated Initial Prestress Loss	8.000	8.005	percent
Total Prestress Loss	45.16	45.19	ksi

B.1.7.3.1
Total Losses at Transfer

Total initial losses $= (ES + 0.5CRs)$ = [15.17+0.5(2.09)]
= 16.211 ksi

f_{si} = Effective initial prestress = 202.5 – 16.211 = 186.289 ksi

P_{si} = Effective pretension force after allowing for the initial losses
= 66(0.153)(186.289) = 1881.146 kips

B.1.7.3.2
Total Losses at Service Loads

SH = 8 ksi

ES = 15.17 ksi

CR_C = 19.94 ksi

CR_S = 2.09 ksi

Total final losses = 8 + 15.17 + 19.94 + 2.09 = 45.193 ksi

or $\frac{45.193\ (100)}{0.75(270)}$ = 22.32 percent

f_{se} = Effective final prestress = 0.75(270) – 45.193 = 157.307 ksi

P_{se} = 66(0.153)(157.307) = 1588.486 kips

B.1.7.3.3
Final Stresses at Midspan

Top fiber stress in concrete at midspan at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + f_t = \frac{1588.486}{1120} - \frac{18.67(1588.486)}{12761.88} + 3.71$$

= 1.418 – 2.323 + 3.71 = 2.805 ksi

Allowable compression stress limit for all load combinations = $0.6\ f'_c$

$f'_{c\ reqd}$ = 2805/0.6 = 4675 psi

Top fiber stress in concrete at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$= \left(\frac{1588.486}{1120} - \frac{18.67(1588.486)}{12761.88} + \frac{(1714.64 + 1822.29)(12)}{12761.88} + \frac{443.72(12)}{79936.06} \right)$$

= 1.418 – 2.323 + 3.326 + 0.067 = 2.49 ksi

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f'_c [STD Art. 9.15.2.2]

$f'_{c\ reqd}$ = 2490/0.4 = 6225 psi (controls)

Top fiber stress in concrete at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{2121.27(12)}{79936.06} + 0.5\left(\frac{1588.486}{1120} - \frac{18.67(1588.486)}{12761.88} + \frac{(1714.64 + 1822.29)(12)}{12761.88} + \frac{443.72(12)}{79936.06}\right)$$

$$= 0.318 + 0.5(1.418 - 2.323 + 3.326 + 0.067) = 1.562 \text{ ksi}$$

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f'_c [STD Art. 9.15.2.2]

$f'_{c\ reqd}$ = 1562/0.4 = 3905 psi

Bottom fiber stress in concrete at midspan at service load

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1588.486}{1120} + \frac{18.67(1588.486)}{18024.15} - 3.458$$

$$= 1.418 + 1.645 - 3.46 = -0.397 \text{ ksi}$$

Allowable tension in concrete = $6\sqrt{f'_c}$ [STD Art. 9.15.2.2]

$$f'_{c\ reqd} = \left(\frac{3970}{6}\right)^2 = 4366 \text{ psi}$$

B.1.7.3.4 Initial Stresses at Debonding Location

With the same number of debonded strands, as was determined in the previous iteration, the top and bottom fiber stresses with their corresponding initial concrete strengths are calculated. It can be observed that at the 15-ft. location, the f'_{ci} value is updated to 5140 psi. The results are shown in Table B.1.7.6.

Table B.1.7.6. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	0	3	6	9	12	15	54.2
Row No. 1 (bottom row)	7	7	15	23	25	27	27
Row No. 2	17	27	27	27	27	27	27
Row No. 3	12	12	12	12	12	12	12
No. of Strands	36	46	54	62	64	66	66
M_g (k-ft.)	0	185	359	522	675	818	1,715
P_{si} (kips)	1026.08	1311.10	1539.12	1767.14	1824.14	1881.15	1881.15
e_c (in.)	17.95	18.01	18.33	18.57	18.62	18.67	18.67
Top Fiber Stresses (ksi)	-0.527	-0.506	-0.499	-0.503	-0.398	-0.304	0.540
Corresponding $f'_{ci\ reqd}$ (psi)	4937	4552	4427	4498	2816	1643	900
Bottom Fiber Stresses (ksi)	1.938	2.358	2.701	3.051	3.064	3.084	2.487
Corresponding $f'_{ci\ reqd}$ (psi)	3230	3930	4501	5085	5106	5140	4144

The actual initial losses are 8.005 percent, as compared to the previously assumed 8.0 percent, and f'_{ci} = 5140 psi, as compared to the previously calculated f'_{ci} = 5138 psi. These values are sufficiently converged, so no further iteration will be required. The optimized value of f'_c required is 6225 psi. AASHTO Standard Article 9.23 requires f'_{ci} to be at least 4000 psi for pretensioned members.

Use $f'_c =$ 6225 psi and $f'_{ci} =$ 5140 psi.

B.1.8 STRESS SUMMARY

B.1.8.1 Concrete Stresses at Transfer

B.1.8.1.1 Allowable Stress Limits

[STD Art. 9.15.2.1]

The allowable stress limits at transfer are as follows:

Compression: 0.6 f'_{ci} = 0.6(5140) = +3084 psi = 3.084 ksi

Tension: The maximum allowable tensile stress is the smaller of

$3\sqrt{f'_{ci}} = 3\sqrt{5140} = 215.1$ psi and 200 psi (controls)

or

$7.5\sqrt{f'_{ci}} = 7.5\sqrt{5140} = 537.71$ psi (tension) > 200 psi

Bonded reinforcement should be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section to allow a tensile stress of 537.71 psi in the concrete.

B.1.8.1.2 Stresses at Girder End and at Transfer Length Section

The stresses at the girder end and at the transfer length section need only be checked at release, because losses with time will reduce the concrete stresses making them less critical.

B.1.8.1.2.1 Stresses at Transfer Length Section

Transfer length = 50 (strand diameter)
= 50 (0.5) = 25 in. = 2.083 ft. [STD Art. 9.20.2.4]

Transfer length section is located at a distance of 2.083 ft. from the end of the girder. Overall girder length of 109.5 ft. is considered for the calculation of bending moment at transfer length. As shown in Table B.1.7.6, the number of strands at this location, after debonding of strands, is 36.

Moment due to girder self-weight

M_g = 0.5(1.167)(2.083)(109.5 – 2.083)
= 130.558 k-ft.

Concrete stress at top fiber of the girder

$$f_t = \frac{P_{si}}{A} - \frac{P_{si}\,e_t}{S_t} + \frac{M_g}{S_t}$$

P_{si} = 36(0.153)(185.946) = 1024.19 kips

Strand eccentricity at transfer section, e_c = 17.95 in.

$$f_t = \frac{1024.19}{1120} - \frac{17.95\,(1024.19)}{12761.88} + \frac{130.558(12)}{12761.88}$$

$$= 0.915 - 1.44 + 0.123 = -0.403 \text{ ksi}$$

Allowable tension (with bonded reinforcement)
= 537.71 psi > 403 psi (O.K.)

Compute stress limit for concrete at the bottom fiber of the girder

Concrete stress at the bottom fiber of the girder

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1024.19}{1120} + \frac{17.95\,(1024.19)}{18024.15} - \frac{130.558(12)}{18024.15}$$

$$= 0.915 + 1.02 - 0.087 = 1.848 \text{ ksi}$$

Allowable compression = 3.084 ksi > 1.848 ksi (reqd.) (O.K.)

B.1.8.1.2.2 Stresses at Girder End

Strand eccentricity at end of girder is:

$$e_c = 22.36 - \frac{7(2.17)+17(4.14)+12(6.11)}{36} = 17.95 \text{ in.}$$

P_{si} = 36 (0.153) (185.946) = 1024.19 kips

Concrete stress at the top fiber of the girder

$$f_t = \frac{1024.19}{1120} - \frac{17.95\,(1024.19)}{12761.88} = 0.915 - 1.44 = -0.526 \text{ ksi}$$

Allowable tension (with bonded reinforcement) =
537.71 psi > 526 psi (O.K.)

Concrete stress at the bottom fiber of the girder

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_b = \frac{1021.701}{1120} + \frac{17.95\,(1021.701)}{18024.15} = 0.915 + 1.02 = 1.935 \text{ ksi}$$

Allowable compression = 3.084 ksi > 1.935 ksi (O.K.)

B.1.8.1.3 Stresses at Midspan

Bending moment at midspan due to girder self-weight based on overall length.

M_g = 0.5(1.167)(54.21)(109.5 – 54.21) = 1748.908 k-ft.

Concrete stress at top fiber of the girder at midspan

$$f_t = \frac{P_{si}}{A} - \frac{P_{si}\,e_c}{S_t} + \frac{M_g}{S_t}$$

$$f_t = \frac{1881.15}{1120} - \frac{17.95\,(1881.15)}{12761.88} + \frac{1748.908\,(12)}{12761.88}$$

$$= 1.68 - 2.64 + 1.644 = 0.684 \text{ ksi}$$

Allowable compression: 3.084 ksi >> 0.684 ksi (reqd.)

Concrete stresses in bottom fiber of the girder at midspan

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\,e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_b = \frac{1881.15}{1120} + \frac{17.95(1881.15)}{18024.15} - \frac{1748.908(12)}{18024.15}$$

$$= 1.68 + 1.87 - 1.164 = 2.386 \text{ ksi}$$

Allowable compression: 3.084 ksi > 2.386 ksi (reqd.) (O.K.)

B.1.8.1.4 Stress Summary at Transfer

	Top of Girder f_t (ksi)	Bottom of Girder f_b (ksi)
At End	-0.526	+1.935
At Transfer Length Section from End	-0.403	+1.848
At Midspan	+0.684	+2.386

B.1.8.2 Concrete Stresses at Service Loads

B.1.8.2.1 Allowable Stress Limits

[STD Art. 9.15.2.2]

The allowable stress limits at service are as follows:

Compression

Case (I): for all load combinations

$0.60\, f_c' = 0.60(6225)/1000 = +3.74$ ksi (for precast girder)

$0.60\, f_c' = 0.60(4000)/1000 = +2.4$ ksi (for slab)

Case (II): for effective pretension force + permanent dead loads

$0.40\, f_c' = 0.40(6225)/1000 = +2.493$ ksi (for precast girder)

$0.40\, f_c' = 0.40(4000)/1000 = +1.6$ ksi (for slab)

Case (III): for live load +0.5(effective pretension force + dead loads)

$0.40\, f_c' = 0.40(6225)/1000 = +2.493$ ksi (for precast girder)

$0.40\, f_c' = 0.40(4000)/1000 = +1.6$ ksi (for slab)

Tension: $6\sqrt{f_c'} = 6\sqrt{6225}\left(\frac{1}{1000}\right) = -0.4737$ ksi

B.1.8.2.2 Stresses at Midspan

$P_{se} = 66(0.153)(157.307) = 1588.49$ kips

Concrete stresses at top fiber of the girder at service loads

$$f_t = \frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL} + M_{LL+I}}{S_{tg}}$$

Case (I):

$$f_t = \left(\begin{array}{l} \dfrac{1588.49}{1120} - \dfrac{18.67(1588.49)}{12761.88} \\ + \dfrac{(1714.64+1822.29)(12)}{12761.88} + \dfrac{(443.72+2121.278)(12)}{79936.06} \end{array} \right)$$

f_t = 1.418 – 2.323 + 3.326 + 0.385 = 2.805 ksi (O.K.)

Allowable compression: 3.84 ksi > 2.805 ksi (reqd.)

Case (II): Effective pretension force + permanent dead loads

$$f_t = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}$$

$$f_t = \frac{1588.49}{1120} - \frac{18.67(1588.49)}{12761.88} + \frac{(1714.64+1822.29)(12)}{12761.88} + \frac{(443.72)(12)}{79936.06}$$

f_t =1.418 – 2.323 + 3.326 + 0.067 = 2.49 ksi

Allowable compression: +2.493 ksi > +2.49 ksi (reqd.) (O.K.)

Case (III): Live load + 0.5 (Pretensioning force + dead loads)

$$f_t = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_S}{S_t} + \frac{M_{SDL}}{S_{tg}}\right)$$

$$= \frac{2121.27(12)}{79936.06} + 0.5\left(\begin{array}{l}\frac{1588.49}{1120} - \frac{18.67(1588.49)}{12761.88} + \\ \frac{(1714.64+1822.29)(12)}{12761.88} + \frac{(443.72)(12)}{79936.06}\end{array}\right)$$

f_t = 0.318 + 0.5(1.418 – 2.323 + 3.326 + 0.067) = 1.563 ksi

Allowable compression: 2.493 ksi > 1.563 ksi (reqd.) (O.K.)

Concrete stresses at bottom fiber of the girder:

$$f_b = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - \frac{M_g + M_S}{S_b} - \frac{M_{SDL} + M_{LL+I}}{S_{bc}}$$

$$f_b = \left(\frac{1588.49}{1120} - \frac{18.67(1588.49)}{18024.15} - \frac{(1714.64+1822.29)(12)}{18024.15} - \frac{(443.72+2121.27)(12)}{27842.9} \right)$$

f_b = 1.418 + 1.645 – 2.36 – 1.098 = – 0.397 ksi

Allowable Tension: 473.7 ksi > 397 psi (O.K.)

Stresses at the top of the slab

Case (I):

$$f_t = \frac{M_{SDL} + M_{LL+I}}{S_{tc}} = \frac{(443.72+2121.27)(12)}{50802.19} = +0.604 \text{ ksi}$$

Allowable compression: +2.4 ksi > +0.604 ksi (reqd.) (O.K.)

Case (II):

$$f_t = \frac{M_{SDL}}{S_{tc}} = \frac{(443.72)(12)}{50802.19} = 0.103 \text{ ksi}$$

Allowable compression: +1.6 ksi > +0.103 ksi (reqd.) (O.K.)

Case (III):

$$f_t = \frac{M_{LL+I} + 0.5(M_{SDL})}{S_{tc}} = \frac{(2121.27)(12) + 0.5(443.72)(12)}{50802.19}$$
$$= 0.553 \text{ ksi}$$

Allowable compression: +1.6 ksi > +0.553 ksi (reqd.) (O.K.)

B.1.8.2.3 Summary of Stresses at Service Loads

		Top of Slab f_t (ksi)	Top of Girder f_t (ksi)	Bottom of Girder f_b (ksi)
At Midspan	CASE I	+0.604	+2.805	
	CASE II	+0.103	+2.490	-0.397
	CASE III	+0.553	+1.563	

B.1.8.3 Actual Modular Ratio and Transformed Section Properties for Strength Limit State and Deflection Calculations

Up to this point, a modular ratio equal to 1 has been used for the service limit state design. For the evaluation of strength limit state and for deflection calculations, the actual modular ratio will be calculated, and the transformed section properties will be used. Table B.1.8.1 shows the calculations for the transformed composite section.

$$n = \frac{E_c \text{ for slab}}{E_c \text{ for beam}} = \left(\frac{3834.25}{4531.48}\right) = 0.883$$

Transformed flange width = n (effective flange width)
= 0.883(138 in.) = 121.85 in.

Transformed flange area = n (effective flange width) (t_s)
= 1(121.85 in.)(8 in.) = 974.8 in.^{2}

Table B.1.8.1. Properties of Composite Section.

	Transformed Area in.^{2}	y_b in.	$A\,y_b$ in.	$A(y_{bc}-y_b)^2$	I in.^{4}	$I{+}A(y_{bc}-y_b)^2$ in.^{4}
Girder	1120	22.36	25,043.20	307,883.97	403,020	710,903.97
Slab	974.8	58	56,538.40	354,128.85	41,591	395,720.32
∑	2094.8		81,581.60			1,106,624.29

A_c = Total area of composite section = 2094.8 in.^{2}

h_c = Total height of composite section = 62 in.

I_c = Moment of inertia of composite section = 1,106,624.29 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 81,581.6 / 2094.8 = 38.94 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder = 54 – 38.94 = 15.06 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 38.94 = 23.06 in.

S_{bc} = Composite section modulus with reference to the extreme bottom fiber of the precast girder = I_c / y_{bc} = 1,106,624.29 / 38.94 = 28,418.7 in.^{3}

S_{tg} = Composite section modulus with reference to the top fiber of the precast girder = I_c / y_{tg}= 1,106,624.29 / 15.06 = 73,418.03 in.^{3}

S_{tc} = Composite section modulus with reference to the top fiber of the slab = I_c / y_{tc} = 1,106,624.29 / 23.06 = 47,988.91 in.^{3}

B.1.9 FLEXURAL STRENGTH

[STD Art. 9.17]

Group I load factor design loading combination

$M_u = 1.3[M_g + M_s + M_{SDL} + 1.67(M_{LL+I})]$ [STD Table 3.22.1A]

$= 1.3[1714.64 + 1822.29 + 443.72 + 1.67(2121.27)] = 9780.12$ k-ft.

Average stress in pretensioning steel at ultimate load

$$f_{su}^* = f_s'\left(1 - \frac{\gamma^*}{\beta_1}\rho^*\frac{f_s'}{f_c'}\right)$$ [STD Eq. 9-17]

where:

f_{su}^* = Average stress in prestressing steel at ultimate load

γ^* = 0.28 for low-relaxation strand [STD Art. 9.1.2]

$$\beta_1 = 0.85 - 0.05\frac{(f_c' - 4000)}{1000}$$ [STD Art. 8.16.2.7]

$$= 0.85 - 0.05\frac{(4000 - 4000)}{1000} = 0.85$$

$$\rho^* = \frac{A_s^*}{bd}$$

where:

A_s^* = Area of pretensioned reinforcement = 66(0.153) = 10.1 in.^{2}

b = Transformed effective flange width = 121.85 in.

y_{bs} = Distance from center of gravity of the strands to the bottom fiber of the girder = 22.36 – 18.67 = 3.69 in.

d = Distance from top of slab to centroid of pretensioning strands

= Girder depth (h) + slab thickness – y_{bs}

= 54 + 8 – 3.69 = 58.31 in.

$$\rho^* = \frac{10.1}{121.85(58.31)} = 0.00142$$

$$f_{su}^* = 270\left[1 - \left(\frac{0.28}{0.85}\right)(0.00142)\left(\frac{270}{4}\right)\right] = 261.48 \text{ ksi}$$

Depth of compression block [STD Art. 9.17.2]

$$a = \frac{A_s^* f_{su}^*}{0.85 f_c' \, b} = \frac{10.1(261.48\ )}{0.85(4)(121.85)} = 6.375 \text{ in.} < 8.0 \text{ in.}$$

The depth of compression block is less than the flange thickness; hence, the section is designed as rectangular section.

Design flexural strength:

$$\phi M_n = \phi A_s^* f_{su}^* d \left(1 - 0.6 \frac{\rho^* f_{su}^*}{f_c'}\right)$$ [STD Eq. 9-13]

where:

ϕ = Strength reduction factor = 1.0 [STD Art. 9.14]

M_n = Nominal moment strength of a section

$$\phi M_n = 1.0(10.1)(261.48)\frac{(58.31)}{12}\left(1 - 0.6\ \frac{0.00142(261.48)}{4}\right)$$

= 12118.1 k-ft. > 9780.12 k-ft. (O.K.)

B.1.10 DUCTILITY LIMITS

B.1.10.1 Maximum Reinforcement

[STD Art. 9.18.1]

To ensure that steel is yielding as the ultimate capacity is approached, the reinforcement index for a rectangular section shall be such that:

$$\frac{\rho^* f_{su}^*}{f_c'} < 0.36\, \beta_1$$ [STD Eq. 9-20]

$$0.00142\left(\frac{261.48}{4}\right) = 0.093 < 0.36(0.85) = 0.306$$ (O.K.)

B.1.10.2 Minimum Reinforcement

[STD Art. 9.18.2]

The ultimate moment at the critical section developed by the pretensioned and non-pretensioned reinforcement shall be at least 1.2 times the cracking moment, M_{cr}.

[STD Art. 9.18.2.1]

$\phi M_n \geq 1.2\ M_{cr}$

Cracking moment $M_{cr} = (f_r + f_{pe})\ S_{bc} - M_{d\text{-}nc}\left(\frac{S_{bc}}{S_b} - 1\right)$

where:

[STD Art. 9.15.2.3]

f_r = Modulus of rupture (ksi)

$$= 7.5\sqrt{f'_c} = 7.5\sqrt{6225}\left(\frac{1}{1000}\right) = 0.592 \text{ ksi}$$

f_{pe} = Compressive stress in concrete due to effective prestress forces at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$$f_{pe} = \frac{P_{se}}{A} + \frac{P_{se}\,e_c}{S_b}$$

where:

P_{se} = Effective prestress force after losses = 1583.791 kips

e_c = 18.67 in.

$$f_{pe} = \frac{1588.49}{1120} + \frac{1588.49\ (18.67)}{18024.15} = 1.418 + 1.641$$

= 3.055 ksi

$M_{d\text{-}nc}$ = Non-composite dead load moment at midspan due to self-weight of girder and weight of slab

= 1714.64 + 1822.29 = 3536.93 k-ft.

$$M_{cr} = (0.592 + 3.055)(28418.7) - 3536.93\left(\frac{28{,}418.7}{18{,}024.15} - 1\right)$$

= 8636.92 – 2039.75 = 6597.165 k-ft.

$1.2\ M_{cr}$ = 1.2(6597.165) = 7916.6 k-ft. ≤ ϕM_n = 12,118.1 k-ft.

(O.K.)

B.1.11 TRANSVERSE SHEAR DESIGN

[STD Art. 9.20]

Members subject to shear shall be designed so that:

$V_u \leq \phi(V_c + V_s)$ [STD Eq. 9-26]

where:

V_u = The factored shear force at the section considered

V_c = The nominal shear strength provided by concrete

V_s = The nominal shear strength provided by web reinforcement

ϕ = Strength reduction factor for shear = 0.90

The critical section for shear is located at a distance *h*/2 from the face of the support; however, the critical section for shear is conservatively calculated from the centerline of the support.

$h/2 = \dfrac{62}{2(12)} = 2.583$ ft. [STD Art. 9.20.1.4]

From Tables B.1.5.1 and B.1.5.2, the shear forces at the critical section are as follows:

V_d = Shear force due to total dead loads at section considered
= 144.75 kips

V_{LL+I} = Shear force due to live load and impact at critical section
= 81.34 kips

V_u = $1.3(V_d + 1.67V_{LL+I})$ = 1.3(144.75 + 1.67(81.34)
= 364.764 kips

Computation of V_{ci}

$$V_{ci} = 0.6\sqrt{f'_c}b'd + V_d + \frac{V_i M_{cr}}{M_{max}}$$ [STD Eq. 9-27]

where:

b' = Width of web of a flanged member = 5 in.

f'_c = Compressive strength of girder concrete at 28 days
= 6225 psi

M_d = Bending moment at section due to unfactored dead load
= 365.18 k-ft.

M_{LL+I} = Bending moment at section due to live load and impact
= 210.1 k-ft.

M_u = Factored bending moment at the section
= $1.3(M_d + 1.67M_{LL+I})$ = 1.3[365.18 + 1.67(210.1)]
= 930.861 k-ft.

V_{mu} = Factored shear force occurring simultaneously with M_u conservatively taken as maximum shear load at the section = 364.764 kips

M_{max} = Maximum factored moment at the section due to externally applied loads = $M_u - M_d$ = 930.861 – 365.18
= 565.681 k-ft.

V_i = Factored shear force at the section due to externally applied loads occurring simultaneously with
$M_{max} = V_{mu} - V_d$ = 364.764 – 144.75 = 220.014 kips

f_{pe} = Compressive stress in concrete due to effective pretension forces at extreme fiber of section where tensile stress is caused by externally applied loads, i.e., bottom of the girder in present case

$$f_{pe} = \frac{P_{se}}{A} + \frac{P_{se}\,e}{S_b}$$

Eccentricity of the strands at $h_c/2$

$e_{h/2}$ = 18.046 in.

P_{se} = 36(0.153)(157.307) = 866.45 kips

$$f_{pe} = \frac{866.45}{1120} + \frac{866.45(17.95)}{18024.15} = 0.77 + 0.86 = 1.63 \text{ ksi}$$

f_d = Stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads

$$= \left[\frac{M_g + M_S}{S_b} + \frac{M_{SDL}}{S_{bc}}\right]$$

$$= \left[\frac{(159.51 + 157.19{+}7.75)(12)}{18{,}024.15} + \frac{41.28(12)}{28{,}418.70}\right] = 0.234 \text{ ksi}$$

M_{cr} = Moment causing flexural cracking of section due to externally applied loads

$= (6\,f_c' + f_{pe} - f_d)\,S_{bc}$ [STD Eq. 9-28]

$$= \left(\frac{6\sqrt{6225}}{1000} + 1.631 - 0.234\right)\frac{28{,}418.70}{12} = 4429.5 \text{ k-ft.}$$

d = Distance from extreme compressive fiber to centroid of pretensioned reinforcement, but not less than $0.8h_c$ = 49.6 in. = 62 – 4.41 = 57.59 in. > 49.96 in.

Therefore, use = 57.59 in.

$$V_{ci} = 0.6\sqrt{f_c'}\,b'd + V_d + \frac{V_i M_{cr}}{M_{\max}}$$ [STD Eq. 9-27]

$$= \frac{0.6\sqrt{6225}(2\times 5)(57.59\ )}{1000} + 144.75 + \frac{220.014(4429.5)}{565.681}$$

= 1894.81 kips

This value should not be less than

Minimum $V_{ci} = 1.7\sqrt{f_c'}\,b'd$ [STD Art. 9.20.2.2]

$$= \frac{1.7\sqrt{6225}(2\times5)(57.59)}{1000} = 77.24 \text{ kips} < V_{ci} \qquad \text{(O.K.)}$$

Computation of V_{cw} [STD Art. 9.20.2.3]

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3 f_{pc})b'd + V_p \qquad \text{[STD Eq. 9-29]}$$

where:

f_{pc} = Compressive stress in concrete at centroid of cross section (since the centroid of the composite section does not lie within the flange of the cross section) resisting externally applied loads. For a non-composite section,

$$f_{pc} = \frac{P_{se}}{A} - \frac{P_{se}\,e(y_{bc} - y_b)}{I} + \frac{M_D(y_{bc} - y_b)}{I}$$

M_D = Moment due to unfactored non-composite dead loads = 324.45 k-ft.

$$f_{pc} = \left(\frac{863.89}{1120} - \frac{863.89\,(17.95)(38.94-22.36)}{403020} + \frac{324.45(12)(38.94-22.36)}{403020}\right)$$

$= 0.771 - 0.638 + 0.160 = 0.293$ psi

$V_p = 0$

$$V_{cw} = \left(\frac{3.5\sqrt{6225}}{1000} + 0.3(0.293)\right)(2\times5)(57.59)$$

$= 209.65$ kips (controls)

The allowable nominal shear strength provided by concrete should be the lesser of V_{ci} = 1894.81 kips and V_{cw} = 209.65 kips.

Therefore, V_c = 209.65 kips

$$V_u \leq \phi(V_c + V_s)$$

where:

ϕ = Strength reduction factor for shear = 0.90

$$\text{Required } V_s = \frac{V_u}{\phi} - V_c = \frac{364.764}{0.9} - 209.65 = 195.643 \text{ kips}$$

Maximum shear force that can be carried by reinforcement

$$V_{s\,max} = 8\sqrt{f'_c}\,b'd \qquad \text{[STD Art. 9.20.3.1]}$$

$$= 8\sqrt{6225}\,\frac{(2\times5)(57.59)}{1000}$$

$$= 363.502 \text{ kips} > \text{required } V_s = 195.643 \text{ kips} \quad \text{(O.K.)}$$

Area of shear steel required [STD Art. 9.20.3.1]

$$V_s = \frac{A_v f_y d}{s} \qquad \text{[STD Eq. 9-30]}$$

or

$$A_v = \frac{V_s s}{f_y d}$$

where:

A_v = Area of web reinforcement, in.^{2}

s = Longitudinal spacing of the web reinforcement, in.

Setting s = 12 in. to have units of in.^{2}/ft. for A_v

$$A_v = \frac{(195.643)(12)}{(60)(57.59)} = 0.6794 \text{ in.}^2\text{/ft.}$$

Minimum shear reinforcement [STD Art. 9.20.3.3]

$$A_{v-min} = \frac{50\,b'\,s}{f_y} = \frac{(50)(2\times5)(12)}{60{,}000} = 0.1 \text{ in.}^2\text{/ft.} \qquad \text{[STD Eq. 9-31]}$$

The required shear reinforcement is the maximum of A_v = 0.6794 in.^{2}/ft. and A_{v-min} = 0.10 in.^{2}/ft.

Try 1 #4 double-legged stirrup with A_v = 0. 40 in.^{2} / ft. The required spacing can be calculated as

$$s = \frac{f_y\,d\,A_v}{V_s} = \frac{60\times57.59\times0.40}{195.643} = 7.06 \text{ in.}$$

[STD Art. 9.20.3.2]

Maximum spacing of web reinforcement is 0.75 h_c or 24 in., unless

$$V_s = 195.643 \text{ kips} > 4\sqrt{f'_c}\,b'\,d = 4\sqrt{6225}\,\frac{(2\times5)(57.59)}{1000}$$

$$= 181.751 \text{ kips}$$

Since V_s is greater than the limit,
Maximum spacing = 0.5(0.75 h) = 0.5[0.75(54 + 8 + 1.5)]

= 47.63 in., or 0.5(24 in.) = 12 in.

Therefore, maximum s = 12 in.

Use #4, double-legged stirrups at 7 in. spacing.

B.1.12 HORIZONTAL SHEAR DESIGN

The critical section for horizontal shear is at a distance of $h_c/2$ from the centerline of the support.

V_u = 364.764 kips

$V_u \leq V_{nh}$ [STD Eq. 9-31a]

where:

V_{nh} = Nominal horizontal shear strength, kips

$$V_{nh} \geq \frac{V_u}{\phi} = \frac{364.764}{0.9} = 405.293 \text{ kips}$$

Case (a & b): Contact surface is roughened or when minimum ties are used.

Allowable shear force: [STD Art. 9.20.4.3]

V_{nh} = $80 b_v\, d$

where:

b_v = Width of cross section at the contact surface being investigated = 2×15.75 = 31.5 in.

d = Distance from extreme compressive fiber to centroid of the pretensioning force = 54 – 4.41 = 49.59 in.

$$V_{nh} = \frac{80(31.5)(49.59)}{1000} = 124.97 \text{ kips} < 405.293 \text{ kips} \quad \text{(N.G.)}$$

Case(c): Minimum ties provided and contact surface roughened

Allowable shear force: [STD Art. 9.20.4.3]

V_{nh} = $350 b_v\, d$

$$= \frac{350(31.5)(49.59)}{1000} = 546.73 \text{ kips} > 405.293 \text{ kips} \quad \text{(O.K.)}$$

Required number of stirrups for horizontal shear

[STD Art. 9.20.4.5]

Minimum $A_{vh} = 50\dfrac{b_v\, s}{f_y} = 50\dfrac{(31.5)(6.5)}{60{,}000} = 0.171$ in.^{2}/ft.

Therefore, extend every alternate web reinforcement into the cast-in-place slab to satisfy the horizontal shear requirements (provided $A_{vh} = 0.34$ in.^{2}/ft.).

Maximum spacing = $4b = 4(2 \times 15.75) = 126$ in. or = 24 in.

[STD Art. 9.20.4.5.a]

Maximum spacing = 24 in. $>$ $s_{provided}$ = 14 in.

B.1.13 PRETENSIONED ANCHORAGE ZONE

B.1.13.1 Minimum Vertical Reinforcement

[STD Art. 9.22]

In a pretensioned girder, vertical stirrups acting at a unit stress of 20,000 psi to resist at least 4 percent of the total pretensioning force must be placed within the distance of $d/4$ of the girder end.

[STD Art. 9.22.1]

Minimum stirrups at the each end of the girder:

P_s = Prestress force before initial losses

= 36(0.153)[(0.75)(270)] = 1,115.37 kips

4 percent of P_s = 0.04(1115.37) = 44.62 kips

Required $A_v = \dfrac{44.62}{20} = 2.231$ in.^{2}

$$\frac{d}{4} = \frac{57.59}{4} = 14.4 \text{ in.}$$

At least 2.31 in.^{2} of vertical transverse reinforcement should be provided within a distance of ($d/4$ = 14.4 in.) from the end of the girder.

[STD Art. 9.22.2]

STD Art. 9.22.2 specifies that nominal reinforcement must be placed to enclose the prestressing steel in the bottom flange for a distance d from the end of the girder.

B.1.14 DEFLECTION AND CAMBER

B.1.14.1 Maximum Camber Calculations using Hyperbolic Functions Method

The Standard Specifications do not provide guidelines for the determination camber of prestressed concrete members. The Hyperbolic Functions Method (Furr et al. 1968, Sinno 1968, Furr and Sinno 1970) for the calculation of maximum camber is used by TxDOT's prestressed concrete bridge design software, PSTRS14 (TxDOT 2004). The following steps illustrate the Hyperbolic Functions Method for the estimation of maximum camber.

Step 1: Total prestress after release

$$P = \frac{P_{si}}{\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)} + \frac{M_D e_c A_s n}{I\left(1 + pn + \dfrac{e_c^2 A_s n}{I}\right)}$$

where:

P_{si} = Total prestressing force = 1881.146 kips

I = Moment of inertia of non-composite section = 403,020 in.^{4}

e_c = Eccentricity of pretensioning force at the midspan = 18.67 in.

M_D = Moment due to self-weight of the girder at midspan = 1714.64 k-ft.

A_s = Area of strands = number of strands (area of each strand) = 66(0.153) = 10.098 in.^{2}

p = Reinforcement ratio = A_s/A

where:

A = Area of cross section of girder = 1120 in.^{2}

p = 10.098/1120 = 0.009016

E_c = Modulus of elasticity of the girder concrete at release, ksi

$= 33(w_c)^{3/2}\sqrt{f'_c}$ [STD Eq. 9-8]

$= 33(150)^{1.5}\sqrt{5140}\ \dfrac{1}{1000} = 4346.43$ ksi

E_s = Modulus of elasticity of prestressing strands = 28000 ksi

n = E_s/E_c = 28000/4346.43 = 6.45

$$\left(1 + pn + \frac{e_c^2 A_s n}{I}\right) = 1 + (0.009016)(6.45) + \frac{(18.67^2)(10.098)(6.45)}{403020}$$

$$= 1.115$$

$$P = \frac{P_{si}}{\left(1 + pn + \dfrac{e_c^{\,2} A_s n}{I}\right)} + \frac{M_D e_c A_s n}{I\left(1 + pn + \dfrac{e_c^{\,2} A_s n}{I}\right)}$$

$$= \frac{1881.15}{1.115} + \frac{(1714.64)(12 \text{ in./ft.})(18.67)(10.098)(6.45)}{403020(1.115)}$$

$$= 1687.13 + 55.68 = 1742.81 \text{ kips}$$

Concrete stress at steel level immediately after transfer

$$f_{ci}^{s} = P\left(\frac{1}{A} + \frac{e_c^{\,2}}{I}\right) - f_c^{s}$$

where:

f_c^s = Concrete stress at steel level due to dead loads

$$= \frac{M_D e_c}{I} = \frac{(1714.64)(12 \text{ in./ft.})(18.67)}{403020} = 0.953 \text{ ksi}$$

$$f_{ci}^{s} = 1742.81\left(\frac{1}{1120} + \frac{18.67^2}{403020}\right) - 0.953 = 2.105 \text{ ksi}$$

Step 2: ultimate time-dependent strain at steel level

$$\varepsilon_{c1}^{s} = \varepsilon_{cr}^{\infty} f_{ci}^{s} + \varepsilon_{sh}^{\infty}$$

where:

$\varepsilon_{cr}^{\infty}$ = Ultimate unit creep strain = 0.00034 in./in. [This value is prescribed by Furr and Sinno (1970).]

$\varepsilon_{sh}^{\infty}$ = Ultimate unit creep strain = 0.000175 in./in. [This value is prescribed by Furr and Sinno (1970).]

$\varepsilon_{c1}^{\infty}$ = 0.00034(2.105) + 0.000175 = 0.0008907 in./in.

Step 3: Adjustment of total strain in step 2

$$\varepsilon_{c2}^{s} = \varepsilon_{c1}^{s} - \varepsilon_{c1}^{s} E_{ps} \frac{A_s}{E_{ci}}\left(\frac{1}{A_n} + \frac{e_c^{\,2}}{I}\right)$$

$$= 0.0008907 - 0.0008907(28000)\frac{10.098}{4346.43}\left(\frac{1}{1120} + \frac{18.67^2}{403020}\right)$$

$$= 0.000993 \text{ in./in.}$$

Step 4: Change in concrete stress at steel level

$$\Delta f_c^s = \varepsilon_{c2}^s \; E_{ps} A_s \left(\frac{1}{A_n} + \frac{e_c^{\,2}}{I} \right)$$

$$\Delta f_c^s = 0.000993\ (28000)(10.098) \left(\frac{1}{1120} + \frac{18.67^2}{403020} \right)$$

$$= 0.494 \text{ ksi}$$

Step 5: Correction of the total strain from Step 2

$$\varepsilon_{c4}^s = \varepsilon_{\text{cr}}^{\infty} + \left(f_{ci}^s - \frac{\Delta f_c^s}{2} \right) + \varepsilon_{\text{sh}}^{\infty}$$

$$\varepsilon_{\text{c4}}^s = 0.00034 \left(2.105 - \frac{0.494}{2} \right) + 0.000175 = 0.000807 \text{ in./in.}$$

Step 6: Adjustment in total strain from Step 5

$$\varepsilon_{c5}^s = \varepsilon_{c4}^s - \varepsilon_{c4}^s \; E_{ps} \frac{A_s}{E_c} \left(\frac{1}{A_n} + \frac{e_c^{\,2}}{I} \right)$$

$$= 0.000807 - 0.000807(28000) \frac{10.098}{4346.43} \left(\frac{1}{1120} + \frac{18.67^2}{403020} \right)$$

$$= 0.000715 \text{ in./in.}$$

Step 7: Change in concrete stress at steel level

$$\Delta f_{c1}^s = \varepsilon_{c5}^s \; E_{ps} \; A_s \left(\frac{1}{A_n} + \frac{e_c^{\,2}}{I} \right)$$

$$= 0.000715\ (28000)(10.098) \left(\frac{1}{1120} + \frac{18.67^2}{403020} \right)$$

$$= 0.36 \text{ ksi}$$

Step 8: Correction of the total strain from Step 5

$$\varepsilon_{\text{c6}}^s = \varepsilon_{\text{cr}}^{\infty} + \left(f_{ci}^s - \frac{\Delta f_{c1}^s}{2} \right) + \varepsilon_{\text{sh}}^{\infty}$$

$$\varepsilon_{\text{c6}}^s = 0.00034 \left(2.105 - \frac{0.36}{2} \right) + 0.000175 = 0.00083 \text{ in./in.}$$

Step 9: Adjustment in total strain from Step 8

$$\varepsilon_{c7}^{s} = \varepsilon_{c6}^{s} - \varepsilon_{c6}^{s} E_{ps} \frac{A_s}{E_{ci}} \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.00083 - 0.00083\ (28000)\ \frac{10.098}{4346.43} \left(\frac{1}{1120} + \frac{18.67^2}{403020} \right)$$

$$= 0.000735 \text{ in./in.}$$

Step 10: Computation of initial prestress loss

$$PL_i = \frac{P_{si} - P}{P_{si}} = \frac{1877.68 - 1742.81}{1877.68} = 0.0735$$

Step 11: Computation of final prestress loss

$$PL^{\infty} = \frac{\varepsilon_{c7}^{\infty}\ E_{ps}\, A_s}{P_{si}} = \frac{0.000735(28000)(10.098)}{1877.68} = 0.111$$

Total prestress loss

$PL = PL_i + PL^{\infty} = 100(0.0735 + 0.111) = 18.45$ percent

Step 12: Initial deflection due to dead load

$$C_{DL} = \frac{5\ w\ L^4}{384\ E_c\ I}$$

where:

w = Weight of girder = 1.167 kips/ft.

L = Span length = 108.417 ft.

$$C_{DL} = \frac{5\left(\frac{1.167}{12 \text{ in./ft.}} \right)[(108.417)(12 \text{ in./ft.})]^4}{384(4346.43)(403020)} = 2.073 \text{ in.}$$

Step 13: Initial camber due to prestress

The initial camber due to prestress is calculated using the moment area method. The diagram for the moment caused by the initial prestressing, is shown in Figure B.1.14.1. Due to debonding of strands, the number of strands vary at each debonding section location. Strands that are bonded achieve their effective prestress level at the end of transfer length. Points 1 through 6 show the end of transfer length for the preceding section. The following expression is obtained from the moment (*M/EI*) diagram shown. The *M/EI* values are calculated as:

$$\frac{M}{EI} = \frac{P_{si} \times e_c}{E_c I}$$

The *M/EI* values are calculated for each point 1 through 6 and are shown in Table B.1.14.1. The initial camber due to prestress, C_{pi}, can be calculated using the Moment Area Method by taking the moment of the *M/EI* diagram about the end of the girder.

C_{pi} = 4.06 in.

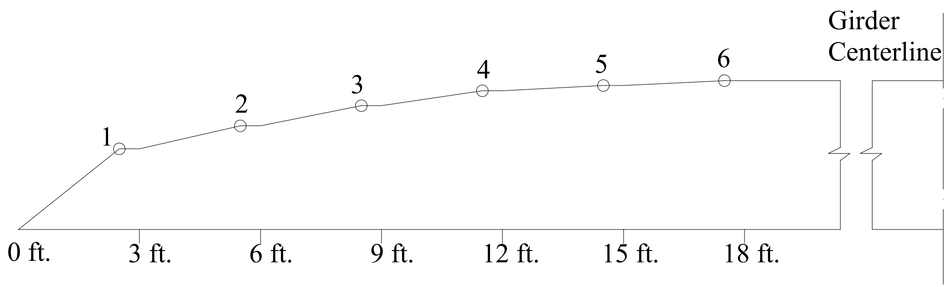

Figure B.1.14.1. M/EI Diagram to Calculate the Initial Camber due to Prestress.

Table B.1.14.1. M/EI Values at the End of Transfer Length.

Identifier for the End of Transfer Length	P_{si} (kips)	e_c (in.)	*M/EI* (in.^{3})
1	1024.19	17.95	1.026E-08
2	1308.69	18.01	1.029E-08
3	1536.29	18.33	1.048E-08
4	1763.88	18.57	1.061E-08
5	1820.78	18.62	1.064E-08
6	1877.68	18.67	1.067E-08

Step 14: Initial camber

$C_i = C_{pi} - C_{DL}$ = 4.06 – 2.073 = 1.987 in.

Step 15: Ultimate time dependent camber

Ultimate strain $\varepsilon_e^s = \frac{f_{ci}^s}{E_c}$ = 2.105/4346.43 = 0.00049 in./in.

$$\text{Ultimate camber } C_t = C_i\left(1 - PL^{\infty}\right)\frac{\varepsilon_{cr}^{\infty}\left(f_{ci}^{s} - \dfrac{\Delta f_{c1}^{s}}{2}\right) + \varepsilon_e^{s}}{\varepsilon_e^{s}}$$

$$= 1.987(1 - 0.111)\frac{0.00034\left(2.105 - \dfrac{0.494}{2}\right) + 0.00049}{0.00049}$$

$$C_t = 4.044 \text{ in.} = 0.34 \text{ ft.} \uparrow$$

B.1.14.2 Deflection due to Girder Self-Weight

Deflection due to girder self-weight at transfer

$$\Delta_{girder} = \frac{5w_g L^4}{384 E_{ci} I}$$

where:

w_g = Girder weight = 1.167 kips/ft.

$$\Delta_{girder} = \frac{5(1.167/12)[(109.5)(12)]^4}{384(4346.43)(403020)} = 2.16 \text{ in.} \downarrow$$

Deflection due to girder self-weight used to compute deflection at erection:

$$\Delta_{girder} = \frac{5(1.167/12)[(108.4167)(12)]^4}{384(4783.22)(403020)} = 1.88 \text{ in.}\downarrow$$

B.1.14.3 Deflection due to Slab and Diaphragm Weight

$$\Delta_{slab} = \frac{5w_s L^4}{384 E_c I} + \frac{w_{dia} b}{24 E_c I}\left(3l^2 - 4b^2\right)$$

where:

w_s = Slab weight = 1.15 kips/ft.

E_c = Modulus of elasticity of girder concrete at service = 4783.22 ksi

$$\Delta_{slab} = \left(\begin{array}{l}\dfrac{5(1.15/12)[(108.4167)(12)]^4}{384(4783.22)(403020)} + \\ \dfrac{(3)(44.2083\times 12)}{(24\times 4783.22\times 403020)}\left(3(108.4167\times 12)^2 - 4(44.2083\times 12)^2\right)\end{array}\right)$$

$$= 1.99 \text{ in.} \downarrow$$

B.1.14.4 Deflection due to Superimposed Loads

$$\Delta_{SDL} = \frac{5 w_{SDL} L^4}{384 E_c I_c}$$

where:

w_{SDL} = Superimposed dead load = 0.31 kips/ft.

I_c = Moment of inertia of composite section = 1,106,624.29 in.^{4}

$$\Delta_{SDL} = \frac{5(0.302/12)[(108.4167)(12)]^4}{384(4783.22)(1106624.29)} = 0.18 \text{ in.} \downarrow$$

Total deflection at service due to all dead loads = 1.88 + 1.99 + 0.18 = 4.05 in. = 0.34 ft.

B.1.14.5 Deflection due to Live Loads

The deflections due to live loads are not calculated in this example as they are not a design factor for TxDOT bridges.

B.1.15 COMPARISON OF RESULTS

To results of this detailed design example, the results are compared with that of PSTRS14 (TxDOT 2004). The summary of comparison is shown in Table B.1.15. In the service limit state design, the results of this example matches those of PSTRS14 with very insignificant differences. A difference of 20.5 percent in transverse shear stirrup spacing is observed. This difference may be because PSTRS14 calculates the spacing according to the 1989 AASHTO Standard Specifications, while in this detailed design example all calculations were performed according to the 2002 AASHTO Standard Specifications. There is a difference of 15.3 percent in the camber calculation, which may be because PSTRS14 uses a single-step hyperbolic functions method, whereas a multi-step approach is used in this detailed design example.

Table B.1.15.1. Comparison of Results for the AASHTO Standard Specifications (PSTRS14 versus Detailed Design Example).

Design Parameters		PSTRS14	Detailed Design Example	Percent Diff. with respect to PSTRS14
Prestress Losses (percent)	Initial	8.00	8.01	-0.1
	Final	22.32	22.32	0.0
Required Concrete Strengths (psi)	f'_{ci}	5140	5140	0.0
	f'_c	6223	6225	0.0
At Transfer (ends) (psi)	Top	-530	-526	0.8
	Bottom	1938	1935	0.2
At Service (midspan) (psi)	Top	-402	-397	1.2
	Bottom	2810	2805	0.2
Number of Strands		66	66	0.0
Number of Debonded Strands		(20+10)	(20+10)	0.0
M_u (kip–ft.)		9801	9780	0.3
ϕM_n (kip–ft.)		12,086	12,118.1	-0.3
Transverse Shear Stirrup (#4 bar) Spacing (in.)		8.8	7.0	20.5
Maximum Camber (ft.)		0.295	0.34	-15.3

B.1.16 REFERENCES

AASHTO (2002), *Standard Specifications for Highway Bridges*, 17th Ed., American Association of Highway and Transportation Officials (AASHTO), Inc., Washington, D.C.

Furr, H.L., R. Sinno and L.L. Ingram (1968). “Prestress Loss and Creep Camber in a Highway Bridge with Reinforced Concrete Slab on Prestressed Concrete Beams,” *Texas Transportation Institute Report*, Texas A&M University, College Station.

Furr, H.L. and R. Sinno (1970) "Hyperbolic Functions for Prestress Loss and Camber,” *Journal of the Structural Division*, Vol. 96, No. 4, pp. 803-821.

PCI (2003). “Precast Prestressed Concrete Bridge Design Manual,” 2nd Ed., Precast/Prestressed Concrete Institute, Chicago, Illinois.

Sinno, R. (1968). “The Time-Dependent Deflections of Prestressed Concrete Bridge Beams,” *Ph.D. Dissertation*, Texas A&M University, College Station.

TxDOT (2001). "TxDOT Bridge Design Manual," Bridge Division, Texas Department of Transportation.

TxDOT (2004). "Prestressed Concrete Beam Design/Analysis Program," User Guide, Version 4.00, Bridge Division, Texas Department of Transportation.

Appendix B.2

Design Example for Interior Texas U54 Girder using AASHTO LRFD Specifications

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

B.2 Design Example for Interior Texas U54 Girder using AASHTO LRFD Specifications

B.2.1 INTRODUCTION

The following detailed example shows sample calculations for the design of a typical interior Texas precast, prestressed concrete U54 girder supporting a single span bridge. The design is based on the *AASHTO LRFD Bridge Design Specifications*, 3rd Edition (AASHTO 2004). The recommendations provided by the TxDOT Bridge Design Manual (TxDOT 2001) are considered in the design. The number of strands and concrete strength at release and at service are optimized using the TxDOT methodology.

B.2.2 DESIGN PARAMETERS

The bridge considered for design has a span length of 110 ft. (c/c abutment distance), a total width of 46 ft., and total roadway width of 44 ft. The bridge superstructure consists of four Texas U54 girders spaced 11.5 ft. center-to-center and designed to act compositely with an 8 in. thick cast-in-place concrete deck as shown in Figure B.2.2.1. The wearing surface thickness is 1.5 in., which includes the thickness of any future wearing surface. T501 type rails are used. AASHTO LRFD HL-93 is the design live load. A relative humidity of 60 percent is considered in the design. The bridge cross-section is shown in Figure B.2.2.1.

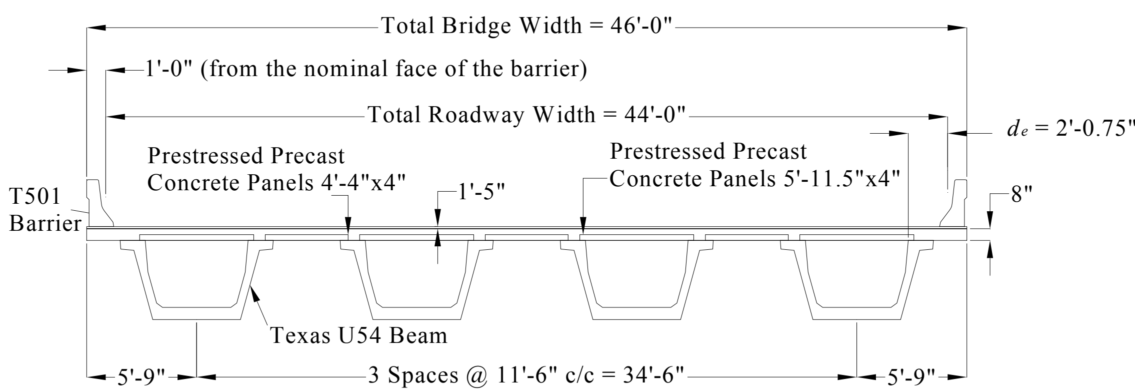

Figure B.2.2.1. Bridge Cross-Section Details.

The design span and overall girder length are based on the following calculations. Figure B.2.2.2 shows the girder end details for Texas U54 girders. It is clear that the distance between the centerline of the interior bent and end of the girder is 3 in., and the distance between the centerline of the interior bent and the centerline of the bearings is 9.5 in.

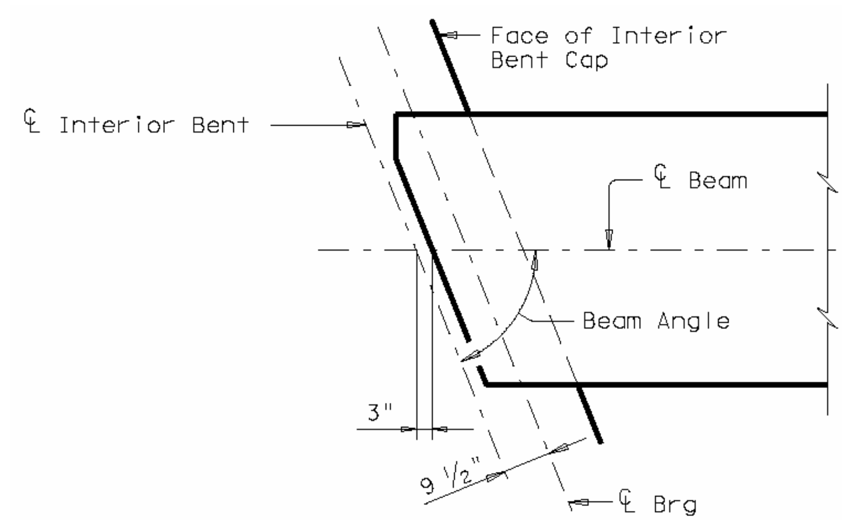

Figure B.2.2.2. Girder End Detail for Texas U54 Girders (TxDOT Standard Drawing 2001).

Span length (c/c interior bents) = 110 ft. - 0 in.
From Figure B.2.2.2:

Overall girder length = 110 ft. – 2(3 in.) = 109 ft. - 6 in.

Design span = 110 ft. – 2(9.5 in.) = 108 ft. - 5 in.

= 108.417 ft. (c/c of bearing)

B.2.3 MATERIAL PROPERTIES

Cast-in-place slab:

Thickness t_s = 8.0 in.

Concrete strength at 28 days, f'_c = 4000 psi

Unit weight of concrete = 150 pcf

Wearing surface:

Thickness of asphalt wearing surface (including any future wearing surfaces), t_w = 1.5 in.

Unit weight of asphalt wearing surface = 140 pcf

Precast girders: Texas U54 girder

Concrete strength at release, f'_{ci} = 4000 psi*

Concrete strength at 28 days, f'_c = 5000 psi*

Concrete unit weight = 150 pcf

*This value is taken as an initial estimate and will be updated based on the optimum design.

Prestressing strands: 0.5 in. diameter, seven wire low-relaxation

Area of one strand = 0.153 in.^{2}

Ultimate tensile strength, f_{pu} = 270,000 psi

[LRFD Table 5.4.4.1-1]

Yield strength, $f_{py} = 0.9\ f_{pu}$ = 243,000 psi

[LRFD Table 5.4.4.1-1]

Modulus of elasticity, E_s = 28,500 ksi [LRFD Art. 5.4.4.2]

Stress limits for prestressing strands: [LRFD Table 5.9.3-1]

before transfer, $f_{pi} \leq 0.75\ f_{pu}$ = 202,500 psi

at service limit state (after all losses)

$f_{pe} \leq 0.80\ f_{py}$ = 194,400 psi

Non-prestressed reinforcement:

Yield strength, f_y = 60,000 psi

Modulus of elasticity, E_s = 29,000 ksi [LRFD Art. 5.4.3.2]

Traffic barrier:

T501 type barrier weight = 326 plf /side

B.2.4 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR GIRDER

The section properties of a Texas U54 girder as described in the TxDOT Bridge Design Manual (TxDOT 2001) are provided in Table B.2.4.1. The strand pattern and section geometry are shown in Figure B.2.4.1.

B.2.4.1 Non-Composite Section

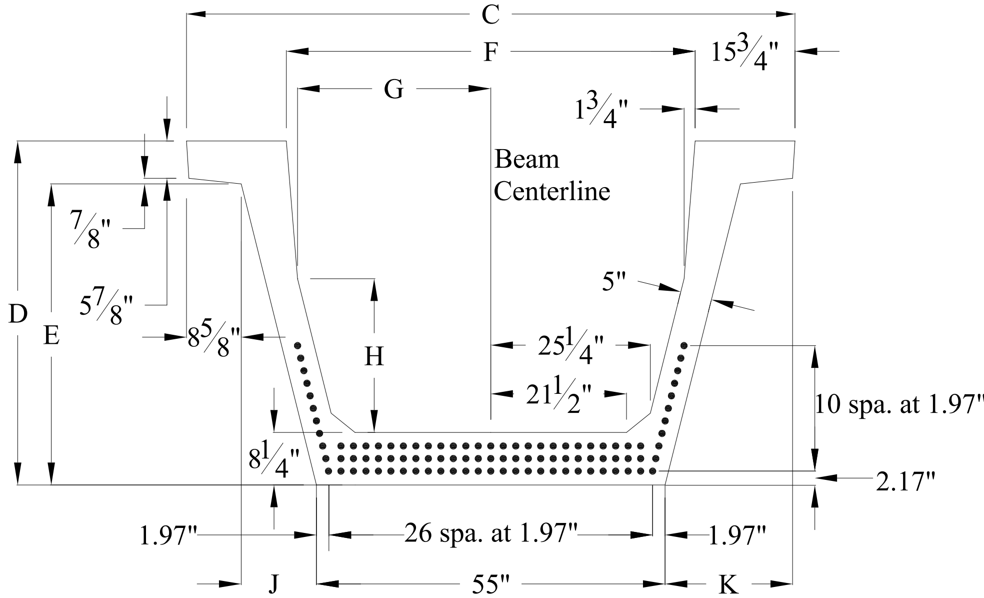

Figure B.2.4.1. Typical Section and Strand Pattern of Texas U54 Girders (TxDOT 2001).

Table B.2.4.1. Section Properties of Texas U54 Girders [Adapted from TxDOT Bridge Design Manual (TxDOT 2001)].

C	D	E	F	G	H	J	K	y_t	y_b	Area	I	Weight
in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.^{2}	in.^{4}	plf
96	54	47.25	64.5	30.5	24.125	11.875	20.5	31.58	22.36	1120	403,020	1167

Note: Notations as used in Figure B.2.4.1.

where:

I = Moment of inertia about the centroid of the non-composite precast girder, in^4.

y_b = Distance from centroid to the extreme bottom fiber of the non-composite precast girder, in.

y_t = Distance from centroid to the extreme top fiber of the non-composite precast girder, in.

S_b = Section modulus referenced to the extreme bottom fiber of the non-composite precast girder, in.^{3}
$= I/\ y_b = 403{,}020/22.36 = 18{,}024.15$ in.^{3}

S_t = Section modulus referenced to the extreme top fiber of the non-composite precast girder, in.^{3}
$= I/\ y_t = 403{,}020/31.58 = 12{,}761.88$ in.^{3}

B.2.4.2 Composite Section

B.2.4.2.1 Effective Flange Width

According to LRFD Art. C4.6.2.6.1, the effective flange width of the U54 girder is determined as though each web is an individual supporting element. Figure B.2.4.2 shows the application of this assumption, and the cross-hatched area of the deck slab shows the combined effective flange width for the two individual webs of adjacent U54 girders.

[LRFD Art. 4.6.2.6.1]

The effective flange width of each web may be taken as the least of:

- 0.25×(effective girder span length):
  $= \frac{108.417 \text{ ft. } (12 \text{ in./ft.})}{4} = 325.25 \text{ in.}$
- 12×(average depth of slab) + greater of (web thickness or one-half the width of the top flange of the girder [web, in this case]) = 12 × (8.0 in.) + greater of (5 in. or 15.75 in./2)
  = 103.875 in.
- The average spacing of the adjacent girders (webs, in this case) = 69 in. = 5.75 ft. (controls)

  For the entire U54 girder the effective flange width is
  = 2×(5.75 ft. ×12) = 138 in.

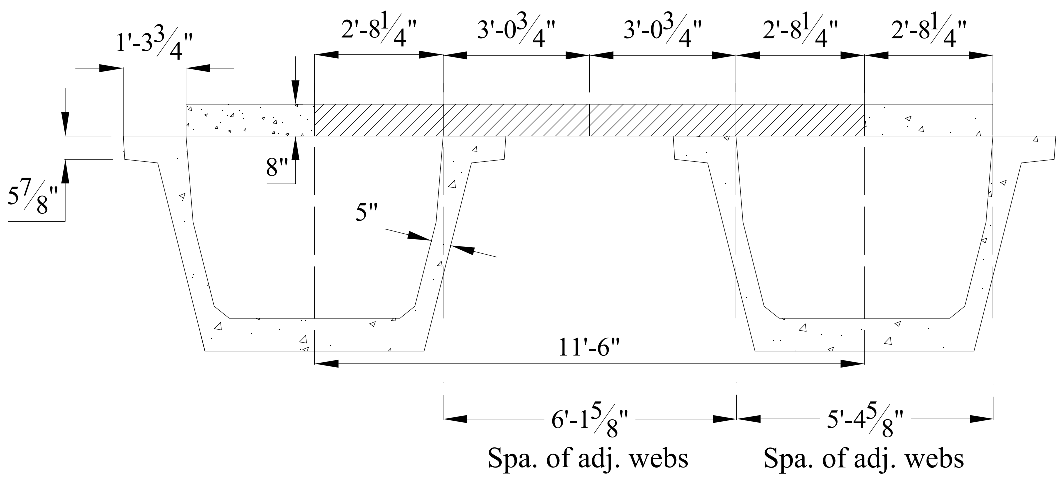

Figure B.2.4.2. Effective Flange Width Calculations.

B.2.4.2.2 Modular Ratio between Slab and Girder Concrete

Following the TxDOT Bridge Design Manual (TxDOT 2001) recommendation, the modular ratio between the slab and girder concrete is taken as 1. This assumption is used for service load design calculations. For the flexural strength limit design, shear design, and deflection calculations, the actual modular ratio based on optimized concrete strengths is used.

$$n = \left(\frac{E_c \text{ for slab}}{E_c \text{ for beam}}\right) = 1$$

where:

n = Modular ratio
E_c = Modulus of elasticity, ksi

B.2.4.2.3 Transformed Section Properties

Figure B.2.4.3 shows the composite section dimensions, and Table B.2.4.2 shows the calculations for the transformed composite section.

Transformed flange width = n × (effective flange width)
= 1 (138 in.) = 138 in.

Transformed flange area = n × (effective flange width) (t_s)
= 1 (138 in.) (8 in.) = 1104 in.^{2}

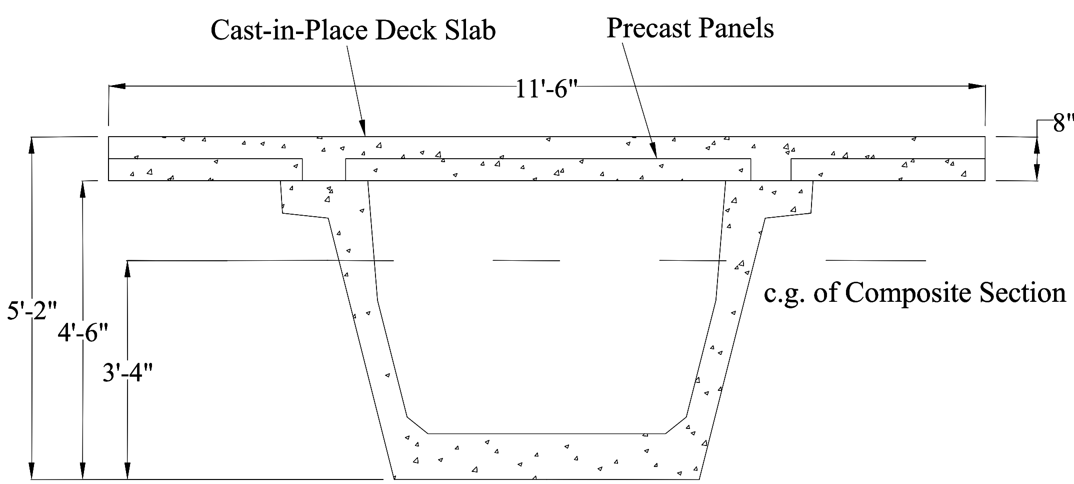

Figure B.2.4.3. Composite Section.

Table B.2.4.2. Properties of Composite Section.

	Transformed Area in.^{2}	y_b in.	$A\,y_b$ in.	$A(y_{bc} - y_b)^2$ in.^{4}	I in.^{4}	$I + A(y_{bc} - y_b)^2$ in.^{4}
Girder	1120	22.36	25,043.2	350,488	403,020	753,508
Slab	1104	58	64,032	355,712	5888	361,600
∑	2224		89,075.2			1,115,108

A_c = Total area of composite section = 2224 in.^{2}

h_c = Total height of composite section = 62 in.

I_c = Moment of inertia about the centroid of the composite section = 1,115,107.99 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder = 89,075.2 / 2224 = 40.05 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder = 54 – 40.05 = 13.95 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 40.05 = 21.95 in.

S_{bc} = Composite section modulus for extreme bottom fiber of the precast girder = I_c/y_{bc} = 1,115,107.99 / 40.05 = 27,842.9 in.^{3}

S_{tg} = Composite section modulus for top fiber of the precast girder = I_c/y_{tg} = 1,115,107.99 / 13.95 = 79,936.06 in.^{3}

S_{tc} = Composite section modulus for top fiber of the slab = I_c/y_{tc} = 1,115,107.99 / 21.95 = 50,802.19 in.^{3}

B.2.5 SHEAR FORCES AND BENDING MOMENTS

B.2.5.1 Shear Force and Bending Moments due to Dead Loads

B.2.5.1.1 Dead Loads

Self-weight of the girder = 1.167 kips/ft.

[TxDOT Bridge Design Manual (TxDOT 2001)]

Weight of CIP deck and precast panels on each girder

$$= (0.150 \text{ pcf})\left(\frac{8 \text{ in.}}{12 \text{ in./ft.}}\right)\left(\frac{138 \text{ in.}}{12 \text{ in./ft.}}\right) = 1.15 \text{ kips/ft.}$$

B.2.5.1.2 Superimposed Dead Loads

Superimposed dead loads are the dead loads assumed to act after the composite action between girders and deck slab is developed. LRFD Art. 4.6.2.2.1 states that permanent loads (rail, sidewalks, and future wearing surface) may be distributed uniformly among all girders if the following conditions are met:

1. Width of the deck is constant. (O.K.)
2. Number of girders, N_b, is not less than four ($N_b = 4$) (O.K.)
3. The roadway part of the overhang, $d_e \leq 3.0$ ft.
   (see Figure B.2.5.1)
   $d_e = 5.75 - 1.0 - 27.5/12 - 4.75/12 = 2.063$ ft. (O.K.)

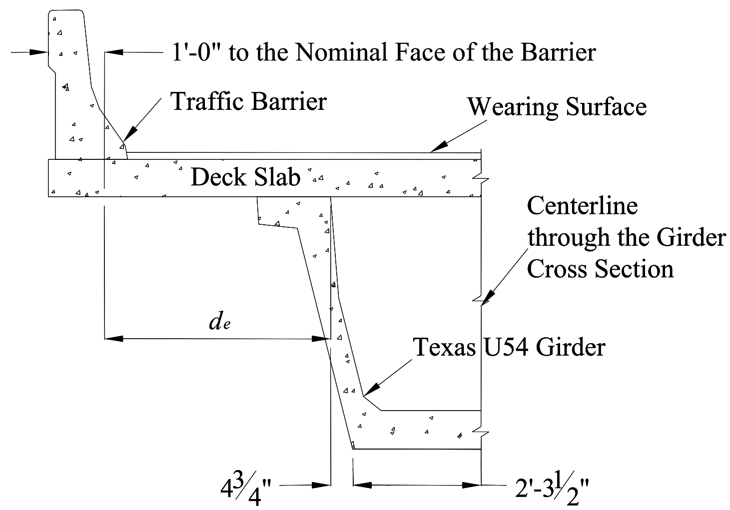

Figure B.2.5.1. Illustration of d_e Calculation.

4. Curvature in plan is less than 4 degrees (curvature is 0 degrees). (O.K.)
5. Cross section of the bridge is consistent with one of the cross sections given in Table 4.6.2.2.1-1 of the LRFD Specifications; the girder type is (c) – spread box beams. (O.K.)

Because these criteria are satisfied, the barrier and wearing surface loads are equally distributed among the four girders.

B.2.5.1.2.1 Due to Diaphragm

The TxDOT Bridge Design Manual (TxDOT 2001) requires two interior diaphragms for U54 girder, located as close as 10 ft. from the midspan of the girder. Shear forces and bending moment values in the interior girder can be calculated using the following equations. Figure B.2.5.2 shows the placement of the diaphragms.

For $x = 0$ ft. – 44.21 ft.

$$V_x = 3 \text{ kips} \qquad M_x = 3x \text{ kips}$$

For $x = 44.21$ ft. – 54.21 ft.

$$V_x = 0 \text{ kips} \qquad M_x = 3x - 3(x - 44.21) \text{ kips}$$

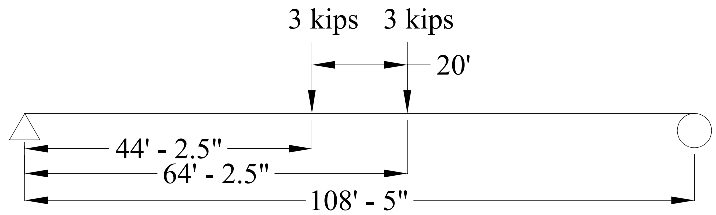

Figure B.2.5.2. Location of Interior Diaphragms on a Simply Supported Bridge Girder.

B.2.5.1.2.2 Due to Haunch

For a U54 girder bridge design, TxDOT accounts for haunches in designs that require special geometry and where the haunch will be large enough to have a significant impact on the overall girder. Because this project is for typical bridges, a haunch will not be included for U54 girders for composite properties of the section and additional dead load considerations.

B.2.5.1.2.3 Due toT501 Rail

The TxDOT Bridge Design Manual recommends (TxDOT 2001, Chap. 7 Sec. 24) that one-third of the rail dead load should be used for an interior girder adjacent to the exterior girder.

Weight of T501 rails or barriers on each interior girder

$$= \left(\frac{326 \text{ plf}/1000}{3}\right) = 0.109 \text{ kips/ft./interior girder}$$

B.2.5.1.2.4 Due to Wearing Surface

Weight of 1.5 in. wearing surface

$$= \frac{(0.140 \text{ pcf})\left(\frac{1.5 \text{ in.}}{12 \text{ in./ft.}}\right)(44 \text{ ft.})}{4 \text{ beams}} = 0.193 \text{ kips/ft.}$$

Total superimposed dead load = 0.109 + 0.193 = 0.302 kips/ft.

B.2.5.1.3 Unfactored Shear Forces and Bending Moments

Shear forces and bending moments in the girder due to dead loads, superimposed dead loads at every tenth of the design span, and at critical sections (midspan and critical section for shear) are provided in this section. The critical section for shear design is determined by an iterative procedure later in the example. The bending moment and shear force due to uniform dead loads and uniform superimposed dead loads at any section at a distance x are calculated using the following expressions, where the uniform dead load is denoted as w.

$$M = 0.5wx\,(L - x)$$

$$V = w\,(0.5L - x)$$

The shear forces and bending moments due to dead loads and superimposed dead loads are shown in Tables B.2.5.1 and B.2.5.2 respectively.

Table B.2.5.1. Shear Forces due to Dead Loads.

Distance	Section	Non-Composite Dead Loads			Superimposed Dead Loads		Total Dead Load Shear Force
		Girder Weight	Slab Weight	Diaphram Weight	Barrier Weight	Wearing Surface Weight	
x	x/L	V_g	V_{slab}	V_{dia}	V_b	V_{ws}	
ft.		kips	kips	kips	kips	kips	kips
0.375	0.003	62.82	61.91	3.00	5.87	10.39	143.99
5.503	0.051	56.84	56.01	3.00	5.31	9.40	130.56
10.842	0.100	50.61	49.87	3.00	4.73	8.37	116.58
21.683	0.200	37.96	37.40	3.00	3.55	6.28	88.19
32.525	0.300	25.30	24.94	3.00	2.36	4.18	59.78
43.367	0.400	12.65	12.47	3.00	1.18	2.09	31.39
54.209	0.500	0.00	0.00	0.00	0.00	0.00	0.00

Table B.2.5.2. Bending Moments due to Dead Loads.

Distance	Section	Non-Composite Dead Loads			Superimposed Dead Loads		Total Dead Load Moment
		Girder Weight	Slab Weight	Diaphram Weight	Barrier Weight	Wearing Surface Weight	
x	x/L	M_g	M_{slab}	M_{dia}	M_b	M_{ws}	
ft.		k-ft.	k-ft.	k-ft.	k-ft.	k-ft.	k-ft.
0.375	0.003	23.64	23.30	1.13	2.21	3.91	54.19
5.503	0.051	330.46	325.64	16.51	30.87	54.65	758.13
10.842	0.100	617.29	608.30	32.53	57.66	102.09	1417.87
21.683	0.200	1097.36	1081.38	65.05	102.50	181.48	2527.77
32.525	0.300	1440.30	1419.32	97.58	134.53	238.20	3329.93
43.367	0.400	1646.07	1622.09	130.10	153.75	272.23	3824.24
54.209	0.500	1714.65	1689.67	132.63	160.15	283.57	3980.67

B.2.5.2 Shear Forces and Bending Moments due to Live Load

B.2.5.2.1 Live Load

[LRFD Art. 3.6.1.2.1]

The LRFD Specifications specify a different live load as compared to the Standard Specifications. The LRFD design live load is designated as HL-93, which consists of a combination of:

- design truck with dynamic allowance or design tandem with dynamic allowance, whichever produces greater moments and shears, and
- design lane load without dynamic allowance.

[LRFD Art. 3.6.1.2.2]

The design truck consists of an 8-kips front axle and two 32-kip rear axles. The distance between the axles is constant at 14 ft.

[LRFD Art. 3.6.1.2.3]

The design tandem consists of a pair of 25-kip axles spaced 4.0 ft. apart. However, the tandem loading governs for shorter spans (i.e., spans less than 40 ft.).

[LRFD Art. 3.6.1.2.4]

The lane load consists of a load of 0.64 klf uniformly distributed in the longitudinal direction.

B.2.5.2.2 Live Load Distribution Factor for Typical Interior Girder

[LRFD Art. 4.6.2.2]

The bending moments and shear forces due to vehicular live load can be distributed to individual girders using the simplified approximate distribution factor formulas specified by the LRFD Specifications. However, the simplified live load distribution factor formulas can be used only if the following conditions are met:

1. Width of the slab is constant. (O.K.)
2. Number of girders, N_b, is not less than four ($N_b = 4$). (O.K.)
3. Girders are parallel and of the same stiffness. (O.K.)
4. The roadway part of the overhang, $d_e \leq 3.0$ ft.
$d_e = 5.75 - 1.0 - 27.5/12 - 4.75/12 = 2.063$ ft. (O.K.)
5. Curvature in plan is less than 4 degrees (curvature is 0 degrees). (O.K.)
6. Cross section of the bridge girder is consistent with one of the cross sections given in LRFD Table 4.6.2.2.1-1; the girder type is (c) – spread box beams. (O.K.)

The number of design lanes is computed as:

[LRFD Art. 3.6.1.1.1]

Number of design lanes = the integer part of the ratio of (w/12), where w is the clear roadway width (ft.), between curbs/or barriers.

w = 44 ft.

Number of design lanes = integer part of (44 ft./12) = 3 lanes

B.2.5.2.3 Distribution Factor for Bending Moment

The LRFD Table 4.6.2.2.2b-1 specifies the approximate vehicular live load moment distribution factors for interior girders.

For two or more design lanes loaded:

$$DFM = \left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125} \qquad \text{[LRFD Table 4.6.2.2.2b-1]}$$

Provided that:	$6.0 \leq S \leq 18.0$;	$S = 11.5$ ft.	(O.K.)
	$20 \leq L \leq 140$;	$L = 108.417$ ft.	(O.K.)
	$18 \leq d \leq 65$;	$d = 54$ in.	(O.K.)
	$N_b \geq 3$;	$N_b = 4$	(O.K.)

where:

DFM = Live load moment distribution factor for interior girder

S = Girder spacing, ft.

L = Girder span, ft.

D = Depth of the girder, ft.

N_b = Number of girders

$$DFM = \left(\frac{11.5}{6.3}\right)^{0.6}\left(\frac{11.5 \times 54}{12.0 \times (108.417)^2}\right)^{0.125} = 0.728 \text{ lanes/girder}$$

For one design lane loaded:

$$DFM = \left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25} \qquad \text{[LRFD Table 4.6.2.2.2b-1]}$$

$$DFM = \left(\frac{11.5}{3.0}\right)^{0.35}\left(\frac{11.5\times 54}{12.0\times(108.417)^2}\right)^{0.25} = 0.412 \text{ lanes/girder}$$

Thus, the case for two or more lanes loaded controls and DFM = 0.728 lanes/girder.

B.2.5.2.4 Distribution Factor for Shear Force

LRFD Table 4.6.2.2.3a-1 specifies the approximate vehicular live load shear distribution factors for interior girders.

For two or more design lanes loaded:

$$DFV = \left(\frac{S}{7.4}\right)^{0.8}\left(\frac{d}{12.0L}\right)^{0.1} \qquad \text{[LRFD Table 4.6.2.2.3a-1]}$$

Provided that:	$6.0 \le S \le 18.0$;	S = 11.5 ft.	(O.K.)
	$20 \le L \le 140$;	L = 108.417 ft.	(O.K.)
	$18 \le d \le 65$;	d = 54 in.	(O.K.)
	$N_b \ge 3$;	N_b = 4	(O.K.)

where:

DFV = Live load shear distribution factor for interior girder

S = Girder spacing, ft.

L = Girder span, ft.

D = Depth of the girder, ft.

N_b = Number of girders

$$DFV = \left(\frac{11.5}{7.4}\right)^{0.8}\left(\frac{54}{12.0\times 108.417}\right)^{0.1} = 1.035 \text{ lanes/girder}$$

For one design lane loaded:

$$DFV = \left(\frac{S}{10}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1} \qquad \text{[LRFD Table 4.6.2.2.3a-1]}$$

$$DFV = \left(\frac{11.5}{10}\right)^{0.6}\left(\frac{54}{12.0\times 108.417}\right)^{0.1} = 0.791 \text{ lanes/girder}$$

Thus, the case for two or more lanes loaded controls and DFV = 1.035 lanes/girder.

B.2.5.2.5
Skew Correction

LRFD Article 4.6.2.2.2e specifies the skew correction factors for load distribution factors for bending moment in longitudinal girders on skewed supports. LRFD Table 4.6.2.2.2e-1 presents the skew correction factor formulas for Type C girders (spread box beams).

For Type C girders the skew correction factor is given by the following formula:

For $0^\circ \leq \theta \leq 60^\circ$,

$$\text{Skew Correction} = 1.05 - 0.25 \tan\theta \leq 1.0$$
If $\theta > 60^\circ$, use $\theta = 60^\circ$

The LRFD Specifications specify a skew correction for shear in the obtuse corner of the skewed bridge plan. This design example considers only the interior girders, which are not in the obtuse corner of a skewed bridge. Therefore, the distribution factors for shear are not reduced for skew.

B.2.5.2.6
Dynamic Allowance

The LRFD Specifications specify the dynamic load effects as a percentage of the static live load effects. LRFD Table 3.6.2.1.-1 specifies the dynamic allowance to be taken as 33 percent of the static load effects for all limit states except the fatigue limit state and 15 percent for the fatigue limit state. The factor to be applied to the static load shall be taken as:

$$(1 + IM/100)$$

where:

IM = Dynamic load allowance, applied to truck load only

IM = 33 percent

B.2.5.2.7
Undistributed Shear Forces and Bending Moments

B.2.5.2.7.1
Due to Truck Load, V_{LT} and M_{LT}

The maximum shear force, V_T, and bending moment, M_T, due to the HS-20 truck loading for all limit states, except for the fatigue limit state, on a per-lane basis are calculated using the following equations given in the *PCI Bridge Design Manual* (PCI 2003):

Maximum undistributed bending moment,

For $x/L = 0 - 0.333$

$$M_T = \frac{72(x)[(L - x) - 9.33]}{L}$$

For $x/L = 0.333 - 0.5$

$$M_T = \frac{72(x)[(L - x) - 4.67]}{L} - 112$$

Maximum undistributed shear force,
For $x/L = 0 - 0.5$

$$V_T = \frac{72[(L - x) - 9.33]}{L}$$

where:

x = Distance from the center of the bearing to the section at which bending moment or shear force is calculated, ft.

L = Design span length = 108.417 ft.

M_T = Maximum undistributed bending moment due to HS-20 truck loading

V_T = Maximum undistributed shear force due to HS-20 truck loading

Distributed bending moment due to truck load including dynamic load allowance (M_{LT}) is calculated as follows:

$$\begin{aligned} M_{LT} &= (\mathrm{M_T})\,(DFM)\,(1+IM/100) \\ &= (M_T)\,(0.728)\,(1+0.33) \\ &= (M_T)\,(0.968) \text{ k-ft.} \end{aligned}$$

Distributed shear force due to truck load including dynamic load allowance (V_{LT}) is calculated as follows:

$$\begin{aligned} V_{LT} &= (V_T)\,(DFV)\,(1+IM/100) \\ &= (V_T)\,(1.035)\,(1+0.33) \\ &= (V_T)\,(1.378) \text{ kips} \end{aligned}$$

where:

DFM = Live load moment distribution factor for interior girders

DFV = Live load shear distribution factor for interior girders

The maximum bending moments and shear forces due to HS-20 truck load are calculated at every tenth of the span and at critical section for shear. The values are presented in Table B.2.5.3.

B.2.5.2.7.2 Due to Tandem Load, V_{TA} and M_{TA}

The maximum shear forces, V_{TA}, and bending moments, M_{TA}, for all limit states, except for the fatigue limit state, on a per-lane basis due to HL-93 tandem loadings are calculated using the following equations:

Maximum undistributed bending moment,
For $x/L = 0 - 0.5$

$$M_{TA} = 50(x)\left(\frac{L-x-2}{L}\right)$$

Maximum undistributed shear force,
For $x/L = 0 - 0.5$

$$V_{TA} = 50\left(\frac{L-x-2}{L}\right)$$

The distributed bending moment, M_{TA}, and distributed shear forces, V_{TA}, are calculated in the same way as for the HL-93 truck loading, as shown in the previous section.

B.2.5.2.7.3 Due to Lane Load, V_L and M_L

The maximum bending moments, M_L, and maximum shear forces, V_L, due to uniformly distributed lane load of 0.64 kip/ft. are calculated using the following expressions.

Maximum undistributed bending moment,
$M_L = 0.5(w)(x)(L-x)$

Maximum undistributed shear force,

$$V_L = \frac{0.32\times(L-x)^2}{L} \quad for \quad x \le 0.5L$$

where:

M_L = Maximum undistributed bending moment due to HL-93 lane loading (k-ft.)

V_L = Maximum undistributed shear force due to HL-93 lane loading (kips)

w = Uniform load per linear foot of load lane = 0.64 klf

Note that maximum shear force at a section is calculated at a section by placing the uniform load on the right of the section considered, as described in the *PCI Bridge Design Manual* (PCI 2003). This method yields a conservative estimate of the shear force as compared to the shear force at a section under uniform load placed on the entire span length. The critical load placement for shear due to lane loading is shown in Figure B.2.5.3.

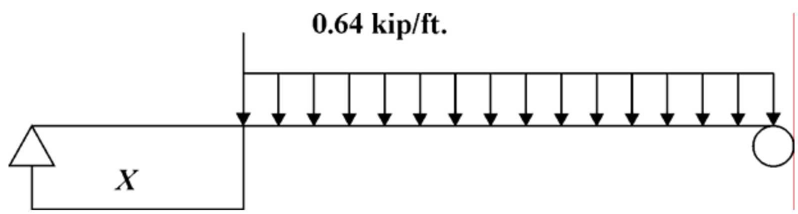

Figure B.2.5.3. Design Lane Loading Placement for Undistributed Shear Calculation.

Distributed bending moment due to lane load (M_{LL}) is calculated as follows:

$$M_{LL} = (M_L)\,(DFM) = (M_L)\,(0.728) \text{ k-ft.}$$

Distributed shear force due to lane load (V_{LL}) is calculated as follows:

$$V_{LL} = (V_L)\,(DFV) = (V_L)\,(1.035) \text{ kips}$$

The maximum bending moments and maximum shear forces due to HL-93 lane loading are calculated at every tenth of the span and at the critical section for shear. The values are presented in Table B.2.5.3.

Table B.2.5.3. Shear Forces and Bending Moments due to Live Loads.

Distance from Bearing Centerline	Section	HS-20 Truck Load with Impact (controls)		Lane Load		Tandem Load with Impact	
		V_{LT}	M_{LT}	V_L	M_L	V_{TA}	M_{TA}
x	x/L	Shear	Moment	Shear	Moment	Shear	Moment
ft.		kips	k-ft.	kips	k-ft.	kips	k-ft.
0.375	0.000	90.24	23.81	35.66	9.44	67.32	17.76
6.000	0.055	85.10	359.14	32.04	143.15	64.06	247.97
10.842	0.100	80.67	615.45	29.08	246.55	60.67	462.71
21.683	0.200	70.76	1079.64	22.98	438.30	53.79	820.41
32.525	0.300	60.85	1392.64	17.59	575.27	46.91	1073.17
43.370	0.400	50.93	1575.96	12.93	657.47	40.03	1220.96
54.210	0.500	41.03	1618.96	8.98	684.85	33.14	1263.76

B.2.5.3 Load Combinations

LRFD Art. 3.4.1 specifies load factors and load combinations. The total factored load effect is specified to be taken as:

$$Q = \sum \eta_i\, \gamma_i\, Q_i \qquad \text{[LRFD Eq. 3.4.1-1]}$$

where:

Q = Factored force effects

Q_i = Unfactored force effects

γ_i = Load factor, a statistically determined multiplier applied to force effects specified in LRFD Table 3.4.1-1

η_i = Load modifier, a factor related to ductility, redundancy, and operational importance

= $\eta_D \, \eta_R \, \eta_I \geq 0.95$, for loads for which a maximum value of γ_i is appropriate [LRFD Eq. 1.3.2.1-2]

= $1/(\eta_D \;\; \eta_R \;\; \eta_I \;) \leq 1.0$, for loads for which a minimum value of γ_i is appropriate [LRFD Eq. 1.3.2.1-3]

η_D = factor relating to ductility
= 1.00 for all limit states except strength limit state

For the strength limit state:

η_D ≥ 1.05 for non-ductile components and connections

η_D = 1.00 for conventional design and details complying with the LRFD Specifications

η_D ≤ 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by the LRFD Specifications

η_D = 1.00 is used in this example for strength and service limit states as this design is considered to be conventional and complying with the LRFD Specifications.

η_R = A factor relating to redundancy
= 1.00 for all limit states except strength limit state

For the strength limit state:

η_R ≥ 1.05 for nonredundant members

η_R = 1.00 for conventional levels of redundancy

η_R ≤ 0.95 for exceptional levels of redundancy

η_R = 1.00 is used in this example for strength and service limit states.

η_D = A factor relating to operational importance
= 1.00 for all limit states except strength limit state

For the strength limit state:

η_I ≥ 1.05 for important bridges

η_I = 1.00 for typical bridges

η_I ≤ 0.95 for relatively less important bridges

η_I = 1.00 is used in this example for strength and service limit states as this example illustrates the design of a typical bridge.

η_i = $\eta_D\ \eta_R\ \eta_I$ = 1.00 for this example

LRFD Art. 3.4.1 specifies load combinations for various limit states. The load combinations pertinent to this design example are shown in the following.

Service I: Check compressive stresses in prestressed concrete components:

$Q = 1.00(DC + DW) + 1.00(LL + IM)$ [LRFD Table 3.4.1-1]

Service III: Check tensile stresses in prestressed concrete components:

$Q = 1.00(DC + DW) + 0.80(LL + IM)$ [LRFD Table 3.4.1-1]

Strength I: Check ultimate strength: [LRFD Table 3.4.1-1 & 2]

Maximum $Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$

Minimum $Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM)$

where:

DC = Dead load of structural components and non-structural attachments

DW = Dead load of wearing surface and utilities

LL = Vehicular live load

IM = Vehicular dynamic load allowance

B.2.6 ESTIMATION OF REQUIRED PRESTRESS

B.2.6.1 Service Load Stresses at Midspan

The preliminary estimate of the required prestress and number of strands is based on the stresses at midspan.

Bottom fiber tensile stresses (Service III) at midspan due to applied loads

$$f_b = \frac{M_g + M_S}{S_b} + \frac{M_b + M_{ws} + 0.8(M_{LT} + M_{LL})}{S_{bc}}$$

Top fiber compressive stresses (Service I) at midspan due to applied loads

$$f_t = \frac{M_g + M_S}{S_t} + \frac{M_b + M_{ws} + M_{LT} + M_{LL}}{S_{tg}}$$

where:

f_b = Concrete stress at the bottom fiber of the girder, ksi

f_t = Concrete stress at the top fiber of the girder, ksi

M_g = Unfactored bending moment due to girder self-weight, k-ft.

M_S = Unfactored bending moment due to slab and diaphragm weight, k-ft.

M_b = Unfactored bending moment due to barrier weight, k-ft.

M_{ws} = Unfactored bending moment due to wearing surface, k-ft.

M_{LT} = Factored bending moment due to truck load, k-ft.

M_{LL} = Factored bending moment due to lane load, k-ft.

Substituting the bending moments and section modulus values, the bottom fiber tensile stress at midspan is:

$$f_b = \left(\frac{(1714.65+1689.67+132.63)(12)}{18,024.15} + \frac{(160.15+283.57+0.8\times(1618.3+684.57))(12)}{27,842.9} \right)$$

$= 3.34$ ksi

The top fiber compressive stress at midspan is:

$$f_t = \left(\frac{(1714.65+1689.67+132.63)(12)}{12,761.88} + \frac{(160.15+283.57+1618.3+684.57)(12)}{79,936.06} \right)$$

$= 3.738$ ksi

B.2.6.2
Allowable Stress Limit

At service load conditions, the allowable tensile stress limit is:

f'_c = specified 28-day concrete strength of girder
= 5000 psi (initial estimate)

$$F_b = 0.19\sqrt{f'_c(\text{ksi})} = 0.19\sqrt{5} = 0.425 \text{ ksi}$$

[LRFD Table. 5.9.4.2.2-1]

B.2.6.3 Required Number of Strands

Required precompressive stress in the bottom fiber after losses:

Bottom tensile stress – allowable tensile stress at final

$= f_b - F_b$

= 3.34 – 0.425 = 2.915 ksi

Assuming the distance from the center of gravity of strands to the bottom fiber of the girder is equal to y_{bs} = 2 in.

Strand eccentricity at midspan:

$$e_c = y_b - y_{bs} = 22.36 - 2 = 20.36 \text{ in.}$$

Bottom fiber stress due to prestress after losses:

$$f_b = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b}$$

where P_{se} = effective pretension force after all losses

$$2.915 = \frac{P_{se}}{1120} + \frac{20.36\ P_{se}}{18{,}024.15}$$

Solving for P_{se}:

P_{se} = 1441.319 kips

Assuming final losses = 20 percent of f_{pi}

Assumed final losses = 0.2(202.5 ksi) = 40.5 ksi

The prestress force per strand after losses

= (cross-sectional area of one strand) (f_{pe})

= 0.153 × (202.5-40.5) = 24.786 kips

Number of strands required = 1441.319 /24.786 = 58.151

Try 60 – 0.5 in. diameter, 270 ksi strands.

Strand eccentricity at midspan after strand arrangement

$$e_c = 22.36 - \frac{27(2.17) + 27(4.14) + 6(6.11)}{60} = 18.91 \text{ in.}$$

P_{se} = 60(24.786) = 1487.16 kips

$$f_b = \frac{1487.16}{1120} + \frac{18.91(1487.16\ )}{18024.15}$$

$$= 1.328 + 1.56 = 2.888 \text{ ksi} < 2.915 \text{ ksi} \qquad \text{(N.G.)}$$

Try 62 – 0.5 in. diameter, 270 ksi strands.
Strand eccentricity at midspan after strand arrangement

$$e_c = 22.36 - \frac{27(2.17)+27(4.14)+8(6.11)}{62} = 18.824 \text{ in.}$$

$$P_{se} = 62(24.786) = 1536.732 \text{ kips}$$

$$f_b = \frac{1536.732}{1120} + \frac{18.824(1536.732)}{18024.15}$$

$$= 1.372 + 1.605 = 2.977 \text{ ksi} > 2.915 \text{ ksi (O.K.)}$$

Therefore, use 62 strands with the pattern shown in Figure B.2.6.1.

Number of Strands	Distance from bottom (in.)
27	2.17
27	4.14
8	6.11

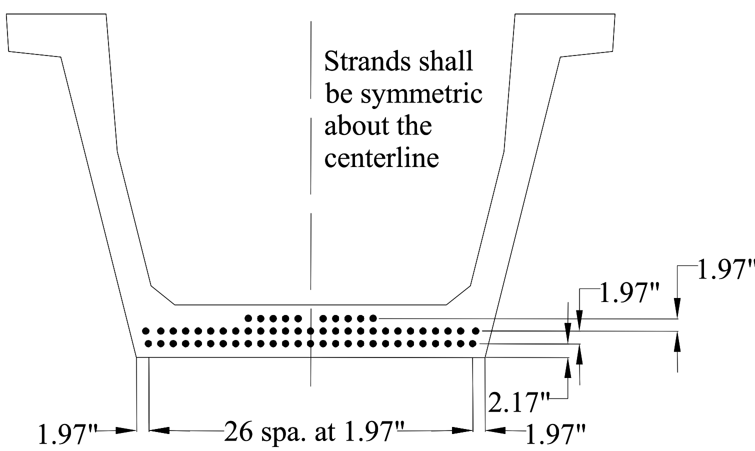

Figure B.2.6.1. Initial Strand Pattern.

B.2.7 PRESTRESS LOSSES

Total prestress losses = $\Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$

[LRFD Eq. 5.9.5.1-1]

where:

Δf_{pSR} = Loss of prestress due to concrete shrinkage

Δf_{pES} = Loss of prestress due to elastic shortening

Δf_{pCR} = Loss of prestress due to creep of concrete

Δf_{pR2} = Loss of prestress due to relaxation of prestressing steel after transfer

Number of strands = 62

A number of iterations will be performed to arrive at the optimum f'_c and f'_{ci}.

B.2.7.1 Iteration 1

B.2.7.1.1 Concrete Shrinkage

$\Delta f_{pSR} = (17.0 - 0.15\,H)$ [LRFD Eq. 5.9.5.4.2-1]

where:

H = Relative humidity = 60 percent

$$\Delta f_{pSR} = [17.0 - 0.150(60)]\frac{1}{1000} = 8 \text{ ksi}$$

B.2.7.1.2 Elastic Shortening

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Eq. 5.9.5.2.3a-1]

where:

$$f_{cgp} = \frac{P_{si}}{A} + \frac{P_{si}\,e_c^2}{I} - \frac{(M_g)e_c}{I}$$

The LRFD Specifications, Art. 5.9.5.2.3a, states that f_{cgp} can be calculated on the basis of prestressing steel stress assumed to be $0.7f_{pu}$ for low-relaxation strands. However, the initial loss as a percentage of initial prestress is assumed before release, f_{pi}. The assumed initial losses shall be checked, and if different from the assumed value, a second iteration will be carried on. Moreover, iterations may also be required if the f'_{ci} value does not match the value calculated in a previous step.

where:

f_{cgp} = Sum of the concrete stresses at the center of gravity of the prestressing tendons due to prestressing force and the self-weight of the member at the sections of the maximum moment (ksi)

P_{si} = Pretension force after allowing for the initial losses (kips)

As the initial losses are unknown at this point, 8 percent initial loss in prestress is assumed as a first estimate.

P_{si} = (number of strands)(area of each strand)[0.92(0.75 f_{pu})]
= 62(0.153)(0.92)(0.75)(270) = 1767.242 kips

M_g = Unfactored bending moment due to girder self-weight
= 1714.64 k-ft.

e_c = Eccentricity of the strand at the midspan = 18.824 in.

$$f_{cgp} = \frac{1767.242}{1120} + \frac{1767.242(18.824)^2}{403020} - \frac{1714.64(12)(18.824)}{403020}$$

= 1.578 + 1.554 – 0.961 = 2.171 ksi

Initial estimate for concrete strength at release, f'_{ci} = 4000 psi

E_{ci} = $[33{,}000(0.150)^{1.5}\sqrt{5.870}\,]$ = 3834.254 ksi

$$\Delta f_{pES} = \frac{28500}{3834.254}(2.171) = 16.137 \text{ ksi}$$

B.2.7.1.3 Creep of Concrete

Losses due to creep are computed as follows.

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

where:

Δf_{cdp} = Change in the concrete stress at the center of gravity of prestressing steel resulting from permanent loads, with the exception of the load acting at the time the prestressing force is applied. Values of Δf_{cdp} should be calculated at the same section or at sections for which f_{cgp} is calculated (ksi).

$$\Delta f_{cdp} = \frac{(M_{slab} + M_{dia})e_c}{I} + \frac{(M_b + M_{ws})(y_{bc} - y_{bs})}{I_c}$$

where:

y_{bc} = 40.05 in.

y_{bs} = The distance from center of gravity of the strand at midspan to the bottom of the girder = 22.36 – 18.824 = 3.536 in.

I = Moment of inertia of the non-composite section = 403,020 in.^{4}

I_c = Moment of inertia of composite section = 1,115,107.99 in.^{4}

$$f_{cd} = \left(\frac{(1689.67+132.63)(12)(18.824)}{403{,}020} + \frac{(160.15+283.57)(12)(37.54-3.536)}{1{,}115{,}107.99} \right)$$

= 1.021 + 0.174 = 1.195 ksi

Δf_{pCR} = 12(2.171) – 7(1.195) = 17.687 ksi

B.2.7.1.4 Relaxation of Prestressing Steel

For pretensioned members with 270 ksi low-relaxation strands conforming to AASHTO M 203: [LRFD Art. 5.9.5.4.4c]

Relaxation loss after transfer:

[LRFD Eq. 5.9.5.4.4c-1]

Δf_{pR2} = 0.3 [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})]

= 0.3 [20.0 – 0.4(16.137) – 0.2(8 + 17.687)] = 2.522 ksi

Relaxation loss before transfer:

Initial relaxation loss, Δf_{pR1}, is generally determined and accounted for by the fabricator. However, Δf_{pR1} is calculated and included in the losses calculations for demonstration purposes, and alternatively, it can be assumed to be zero. A period of 0.5 days is assumed between stressing of strands and initial transfer of prestress force. As per LRFD Commentary C.5.9.5.4.4, f_{pj} is assumed to be $0.8 \times f_{pu}$ for this example.

$$\Delta f_{pR1} = \frac{\log(24.0 \times t)}{40.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj} \qquad \text{[LRFD Eq. 5.9.5.4.4b-2]}$$

$$= \frac{\log(24.0 \times 0.5 \text{ day})}{40.0}\left[\frac{216}{243} - 0.55\right] 216 = 1.975 \text{ ksi}$$

Δf_{pR1} will remain constant for all iterations, and Δf_{pR1} = 1.975 ksi will be used throughout the loss calculation procedure.

Total initial prestress loss = $\Delta f_{pES} + \Delta f_{pR1}$

= 16.137 + 1.975 = 18.663 ksi

Initial prestress loss = $\dfrac{(\Delta f_{ES} + \Delta f_{pR1}) \times 100}{0.75 f_{pu}} = \dfrac{(16.137+1.975)100}{0.75(270)}$

= 8.944 percent > 8 percent (assumed initial prestress loss)

Therefore, another trial is required assuming 8.944 percent initial losses.

Δf_{pSR} = 8 ksi [LRFD Eq. 5.9.5.4.2-1]

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Eq. 5.9.5.2.3a-1]

where:

$$f_{cgp} = \frac{P_{si}}{A} + \frac{P_{si}\, e_c^2}{I} - \frac{(M_g) e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, assuming 8.944 percent initial losses

= $A_{ps}[0.9106(0.75\, f_{pu})]$

= 62(0.153)(0.9106)(0.75)(270) = 1749.185 kips

$$f_{cgp} = \frac{1749.185}{1120} + \frac{1749.185\,(18.824)^2}{403{,}020} - \frac{1714.65(12)(18.824)}{403{,}020}$$

= 1.562 + 1.538 – 0.961 = 2.139 ksi

$$\Delta f_{pES} = \frac{28500}{3834.254}(2.139) = 15.899 \text{ ksi}$$

$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp}$ [LRFD Eq. 5.9.5.4.3-1]

Δf_{cdp} is the same as calculated in the previous trial.

Δf_{cdp} = 1.195 ksi

Δf_{pCR} = 12(2.139) – 7(1.195) = 17.303 ksi.

For pretensioned members with 270 ksi low-relaxation strand conforming to AASHTO M 203 [LRFD Art. 5.9.5.4.4c]

Δf_{pR2} = 0.3 [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})
= 0.3 [20.0 – 0.4(15.899) – 0.2(8 + 17.303)] = 2.574 ksi

Total initial prestress loss = Δf_{pES} + Δf_{pR1}
= 15.899 + 1.975 = 17.874 ksi

Initial prestress loss = $\dfrac{(\Delta f_{ES} + \Delta f_{pR1}) \times 100}{0.75 f_{pu}} = \dfrac{[15.899 + 1.975]100}{0.75(270)}$
= 8.827 percent < 8.944 percent (assumed initial prestress losses)

Therefore, another trial is required assuming 8.827 percent initial losses.

Δf_{pSR} = 8 ksi [LRFD Eq. 5.9.5.4.2-1]

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ [LRFD Eq. 5.9.5.2.3a-1]

where:

$$f_{cgp} = \frac{P_{si}}{A} + \frac{P_{si}\, e_c^2}{I} - \frac{(M_g)\, e_c}{I}$$

P_{si} = Pretension force after allowing for the initial losses, assuming 8.827 percent initial losses
= (number of strands)(area of each strand)[0.9117(0.75 f_{pu})]
= 62(0.153)(0.9117)(0.75)(270) = 1,751.298 kips

$$f_{cgp} = \frac{1751.298}{1120} + \frac{1751.298(18.824)^2}{403{,}020} - \frac{1714.65(12)(18.824)}{403{,}020}$$
= 1.564 + 1.54 – 0.961 = 2.143 ksi

Assuming f'_{ci} = 4000 psi

$$\Delta f_{pES} = \frac{28500}{3834.254}(2.143) = 15.929 \text{ ksi}$$

Δf_{pCR} = 12 f_{cgp} – 7Δf_{cdp} [LRFD Eq. 5.9.5.4.3-1]

Δf_{cdp} is the same as calculated in the previous trial.

Δf_{cdp} = 1.193 ksi

Δf_{pCR} = 12(2.143) – 7(1.193) = 17.351 ksi.

For pretensioned members with 270 ksi low-relaxation strand conforming to AASHTO M 203 [LRFD Art. 5.9.5.4.4c]

Δf_{pR2} = 0.3 [20.0 – 0.4 Δf_{pES} – 0.2(Δf_{pSR} + Δf_{pCR})

= 0.3[20.0 – 0.4(15.929) – 0.2(8 + 17.351)] = 2.567 ksi

B.2.7.1.5
Total Losses at Transfer

Total initial prestress loss = Δf_{pES} + Δf_{pR1}

= 15.929 + 1.975= 17.904 ksi

$$\text{Initial prestress loss} = \frac{\left(\Delta f_{ES} + \Delta f_{pR1}\right)\times 100}{0.75 f_{pu}} = \frac{[15.929+2.526]100}{0.75(270)}$$

= 8.841 percent ≈ 8.827 percent (assumed initial prestress losses)

B.2.7.1.6
Total Losses at Service Loads

Total initial losses = Δf_{pi} = 15.929 + 1.975 = 17.904 ksi

f_{si} = Effective initial prestress = 202.5 – 17.904 = 184.596 ksi
P_{si} = Effective pretension force after allowing for the initial losses
= 62(0.153)(184.596) = 1751.078 kips

Δf_{SR} = 8 ksi
Δf_{ES} = 15.929 ksi
Δf_{R2} = 2.567 ksi
Δf_{CR} = 17.351 ksi

Total final losses = Δf_{pT} = 8+15.929+2.567+17.351 = 45.822 ksi

$$\text{or } \frac{45.822\ (100)}{0.75(270)} = 22.63 \text{ percent}$$

B.2.7.1.7
Final Stresses at Midspan

f_{se} = Effective final prestress = 0.75(270) – 45.822 = 156.678 ksi
P_{se} = 62(0.153)(156.678) = 1486.248 kips

Bottom fiber stress in concrete at midspan at service load

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1486.248}{1120} + \frac{18.824(1486.248)}{18024.15} - 3.34 = 1.327 + 1.552 - 3.34$$

= -0.461 ksi < -0.425 ksi (allowable) (N.G.)

This shows that 62 strands are not adequate. Therefore, try 64 strands.

$$e_c = 22.36 - \frac{27(2.17)+27(4.14)+10(6.11)}{62} = 18.743 \text{ in}$$

P_{se}= 64(0.153)(156.678) = 1534.191 kips

$$f_{bf} = \frac{1534.191}{1120} + \frac{18.743(1534.191)}{18024.15} - 3.34 = 1.370 + 1.595 - 3.34$$

$$= -0.375 \text{ ksi} > -0.425 \text{ ksi (allowable)} \qquad \text{(O.K.)}$$

Therefore, use 64 strands.

Allowable tension in concrete = $0.19\sqrt{f_c'(\text{ksi})}$

$$f_{c\ reqd.}' = \left(\frac{0.375}{0.19}\right)^2 \times 1000 = 3896 \text{ psi}$$

Top fiber stress in concrete at midspan at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + f_t = \frac{1534.191}{1120} - \frac{18.743(1534.191)}{12761.88} + 3.737$$

$$= 1.370 - 2.253 + 3.737 = 2.854 \text{ ksi}$$

Allowable compression stress limit for all load combinations
= $0.6\ f_c'$

$$f_{c\ reqd}' = 2854/0.6 = 4757 \text{ psi}$$

Top fiber stress in concrete at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}$$

$$= \left(\begin{array}{l} \dfrac{1534.191}{1120} - \dfrac{18.743(1534.191)}{12761.88} \\ + \dfrac{(1714.65 + 1689.67 + 132.63)(12)}{12761.88} \\ + \dfrac{(160.15 + 283.57)(12)}{79936.06} \end{array} \right)$$

$$= 1.370 - 2.253 + 3.326 + 0.067 = 2.510 \text{ ksi}$$

Allowable compression stress limit for effective pretension force + permanent dead loads = $0.45\ f_c'$

$$f_{c\ reqd.}' = 2510/0.45 = 5578 \text{ psi} \qquad \text{(controls)}$$

Top fiber stress in concrete at midspan due to live load + 0.5 (effective prestress + dead loads)

$$f_{tf} = \frac{M_{LL+I}}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}\right)$$

$$= \frac{(1618.3+684.57)(12)}{79936.06} + 0.5\left(\begin{array}{l}\frac{1534.191}{1120} - \frac{18.743(1534.191)}{12761.88} \\ + \frac{(1714.65 + 1689.67+132.63)(12)}{12761.88} \\ + \frac{(160.15+283.57)(12)}{79936.06}\end{array}\right)$$

= 0.346 + 0.5(1.370 – 2. 253 + 3.326 + 0.067) = 1.601 ksi

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f'_c

$f'_{c\ reqd.}$ = 1601/0.4 = 4003 psi

B.2.7.1.8
Initial Stresses at End

P_{si} = 64 (0.153) (184.596) = 1807.564 kips

Initial concrete stress at top fiber of the girder at midspan

$$f_{ti} = \frac{P_{si}}{A} - \frac{P_{si}\ e_c}{S_t} + \frac{M_g}{S_t}$$

where M_g = Moment due to girder self-weight at girder end = 0 k-ft.

$$f_{ti} = \frac{1807.564}{1120} - \frac{18.743(1807.564)}{12761.88} = 1.614 - 2.655 = -1.041 \text{ ksi}$$

Tension stress limit at transfer = $0.24\sqrt{f'_{ci}(\text{ksi})}$

Therefore, $f'_{ci\ reqd.} = \left(\frac{1.041}{0.24}\right)^2 \times 1000$ = 18,814 psi

$$f_{bi} = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1807.564}{1120} + \frac{18.743(1807.564)}{18024.15}$$

= 1.614 + 1.88 = 3.494 ksi

Compression stress limit at transfer = 0.6 f'_{ci}

Therefore, $f'_{ci\ reqd} = \frac{3494}{0.6}$ = 5823 psi

The calculation for initial stresses at the girder end shows that the preliminary estimate of $f'_{ci} = 4000$ psi is not adequate to keep the tensile and compressive stresses at transfer within allowable stress limits as per LRFD Art. 5.9.4.1. Therefore, debonding of strands is required to meet the allowable stress limits.

B.2.7.1.9 Debonding of Strands and Debonding Length

To be consistent with the TxDOT design procedures, the debonding of strands is carried out in accordance with the procedure followed in PSTRS14 (TxDOT 2004).

Two strands are debonded at a time at each section located at uniform increments of 3 ft. along the span length, beginning at the end of the girder. The debonding begins at the girder end because due to relatively higher initial stresses at the end, a greater number of strands are required to be debonded, and the debonding requirement reduces as the section moves away from the end of the girder. To make the most efficient use of debonding, the debonding at each section begins at the bottommost row where the eccentricity is largest and moves up. Debonding at a particular section will continue until the initial stresses are within the allowable stress limits or until a debonding limit is reached. When the debonding limit is reached, the initial concrete strength is increased, and the design cycles to convergence. As per TxDOT Bridge Design Manual (TxDOT 2001) and AASHTO LRFD Art. 5.11.4.3, the limits of debonding for partially debonded strands are described as follows:

1. Maximum percentage of debonded strands per row
   a. TxDOT Bridge Design Manual (TxDOT 2001) recommends the maximum percentage of debonded strands per row should not exceed 75 percent.
   b. AASHTO LRFD recommends the maximum percentage of debonded strands per row should not exceed 40 percent.
2. Maximum percentage of debonded strands per section
   a. TxDOT Bridge Design Manual (TxDOT 2001) recommends the maximum percentage of debonded strands per section should not exceed 75 percent.
   b. AASHTO LRFD recommends the maximum percentage of debonded strands per section should not exceed 25 percent.

3. AASHTO LRFD requires that not more than 40 percent of the debonded strands or four strands, whichever is greater, shall have debonding terminated at any section.

4. Maximum length of debonding

    a. TxDOT Bridge Design Manual (TxDOT 2001) recommends that the maximum debonding length chosen to be the lesser of the following:

        i. 15 ft.,

        ii. 0.2 times the span length, or

        iii. Half the span length minus the maximum development length as specified in the 1996 AASHTO Standard Specifications for Highway Bridges, Section 9.28. However, for demonstration purposes, the maximum development length will be calculated as specified in AASHTO LRFD Art. 5.11.4.2 and Art. 5.11.4.3.

    b. AASHTO LRFD recommends, "the length of debonding of any strand shall be such that all limit states are satisfied with consideration of the total developed resistance at any section being investigated."

5. AASHTO LRFD further recommends, "Debonded strands shall be symmetrically distributed about the center line of the member. Debonded lengths of pairs of strands that are symmetrically positioned about the centerline of the member shall be equal. Exterior strands in each horizontal row shall be fully bonded."

The recommendations of TxDOT Bridge Design Manual regarding the debonding percentage per section per row and maximum debonding length as described above are followed in this detailed design example.

B.2.7.1.10 Maximum Debonding Length

As per TxDOT Bridge Design Manual (TxDOT 2001), the maximum debonding length is the lesser of the following:

a. 15 ft.,
b. 0.2 (L), or
c. 0.5 (L) – l_d.

where:

l_d = Development length calculated based on AASHTO LRFD Art. 5.11.4.2 and Art. 5.11.4.3, as follows:

$$l_d \geq \kappa\left(f_{ps} - \frac{2}{3}f_{pe}\right)d_b \quad \text{[LRFD Eq. 5.11.4.2-1]}$$

where:

l_d = Development length (in.)

κ = 2.0 for pretensioned strands [LRFD Art. 5.11.4.3]

f_{pe} = Effective stress in the prestressing steel after losses
= 156.276 ksi

d_b = Nominal strand diameter = 0.5 in.

f_{ps} = Average stress in the prestressing steel at the time for which the nominal resistance of the member is required, calculated in the following (ksi)

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \quad \text{[LRFD Eq. 5.7.3.1.1-1]}$$

k = 0.28 for low-relaxation strand [LRFD Table C5.7.3.1.1-1]

For rectangular section behavior,

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}} \quad \text{[LRFD Eq. 5.7.3.1.1-4]}$$

$d_p = h - y_{bs}$ = 62 – 3.617= 58.383 in.

β_1 = 0.85 for f'c $f'_c \leq 4.0$ ksi [LRFD Art. 5.7.2.2]

= 0.85 – 0.05(f'_c – 4.0) ≤ 0.65 for $f'_c \geq 4.0$ ksi

= 0.85

k = 0.28

For rectangular section behavior,

$$c = \frac{(64)(0.153)(270)}{(0.85)(4)(0.85)(138) + (0.28)(64)(0.153)\dfrac{270}{58.383}} = 6.425 \text{ in.}$$

a = 0.85 × 6.425 = 5.461 in. < 8 in.

Thus, the assumption of a rectangular section behavior is correct.

$$f_{ps} = 270\left(1 - 0.28\frac{6.425}{58.383}\right) = 261.68 \text{ ksi}$$

The development length is calculated as:

$$l_d \geq 2.0\left(261.68 - \frac{2}{3}(156.28)\right)(0.5) = 157.5 \text{ in.}$$

l_d = 13.12 ft.

Hence, the debonding length is the lesser of the following:

a. 15 ft., (controls)
b. 0.2 × 108.417 = 21.68 ft., or
c. 0.5 × 108.417 - 13.12 = 41 ft.

Therefore, the maximum debonding length is 15 ft.

Table B.2.7.1 summarizes the initial stresses and corresponding initial concrete strength requirements within the first 15 ft. from the girder end and at midspan.

Table B.2.7.1. Calculation of Initial Stresses at Extreme Fibers and Corresponding Required Initial Concrete Strengths.

	Location of the Debonding Section (ft. from end)						
	End	3	6	9	12	15	Midspan
Row No. 1 (bottom row)	27	27	27	27	27	27	27
Row No. 2	27	27	27	27	27	27	27
Row No. 3	10	10	10	10	10	10	10
No. of Strands	64	64	64	64	64	64	64
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1807.56	1807.56	1807.56	1807.56	1807.56	1807.56	1807.56
e_c (in.)	18.743	18.743	18.743	18.743	18.743	18.743	18.743
Top Fiber Stresses (ksi)	-1.041	-0.867	-0.704	-0.550	-0.406	-0.272	0.571
Corresponding $f'_{ci\ reqd}$ (psi)	18,814	13,050	8604	5252	2862	1284	5660
Bottom Fiber Stresses (ksi)	3.494	3.371	3.255	3.146	3.044	2.949	2.352
Corresponding $f'_{ci\ reqd}$ (psi)	5823	5618	5425	5243	5074	4915	3920

The values in Table B.2.7.1 suggest that the preliminary estimate of 4000 psi for f'_{ci} is inadequate. Because strands cannot be debonded beyond the section located at 15 ft. from the end of the girder, f'_{ci} is increased from 4000 psi to 4915 psi. For all other sections where debonding can be done, the strands are debonded to bring the required f'_{ci} below 4915 psi. Table B.2.7.2 shows the debonding schedule based on the procedure described earlier.

Table B.2.7.2. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	End	3	6	9	12	15	Midspan
Row No. 1 (bottom row)	7	9	17	23	25	27	27
Row No. 2	19	27	27	27	27	27	27
Row No. 3	10	10	10	10	10	10	10
No. of Strands	36	46	54	60	62	64	64
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1016.76	1299.19	1525.13	1694.591	1751.08	1807.56	1807.56
e_c (in.)	18.056	18.177	18.475	18.647	18.697	18.743	18.743
Top Fiber Stresses (ksi)	-0.531	-0.517	-0.509	-0.472	-0.367	-0.272	0.571
Corresponding $f'_{ci\ reqd}$ (psi)	4895	4640	4498	3868	2338	1284	5660
Bottom Fiber Stresses (ksi)	1.926	2.347	2.686	2.919	2.930	2.949	2.352
Corresponding $f'_{ci\ reqd}$ (psi)	3211	3912	4477	4864	4884	4915	3920

B.2.7.2 Iteration 2

Following the procedure in Iteration 1, another iteration is required to calculate prestress losses based on the new value of f'_{ci} = 4915 psi. The results of this second iteration are shown in Table B.2.7.3. Table B.2.7.3 shows the results of this second iteration.

Table B.2.7.3. Results of Iteration 2.

	Trial #1	Trial # 2	Trial # 3	Units
No. of Strands	64	64	64	
e_c	18.743	18.743	18.743	in
Δf_{pSR}	8	8	8	ksi
Assumed Initial Prestress Loss	8.841	8.369	8.423	percent
P_{si}	1807.59	1816.91	1815.92	kips
M_g	1714.65	1714.65	1714.65	k - ft.
f_{cgp}	2.233	2.249	2.247	ksi
f_{ci}	4915	4915	4915	psi
E_{ci}	4250	4250	4250	ksi
Δf_{pES}	14.973	15.081	15.067	ksi
f_{cdp}	1.191	1.191	1.191	ksi
Δf_{pCR}	18.459	18.651	18.627	ksi
Δf_{pR1}	1.975	1.975	1.975	ksi
Δf_{pR2}	2.616	2.591	2.594	ksi
Calculated Initial Prestress Loss	8.369	8.423	8.416	percent
Total Prestress Loss	46.023	46.298	46.263	ksi

B.2.7.2.1
Total Losses at Transfer

Total initial losses = $\Delta f_{ES} + \Delta f_{R1}$ = 15.067 + 1.975 = 17.042 ksi

f_{si} = Effective initial prestress = 202.5 – 17.042 = 185.458 ksi

P_{si} = Effective pretension force after allowing for the initial losses

= 64(0.153)(185.458) = 1816.005 kips

B.2.7.2.2
Total Losses at Service Loads

Δf_{SH} = 8 ksi

Δf_{ES} = 15.067 ksi

Δf_{R2} = 2.594 ksi

Δf_{R1} = 1.975 ksi

Δf_{CR} = 18.519 ksi

Total final losses = 8 + 15.067 + 2.594 + 1.975 + 18.627
= 46.263 ksi

or $\dfrac{46.263(100)}{0.75(270)}$ = 22.85 percent

f_{se} = Effective final prestress = 0.75(270) – 46.263 = 156.237 ksi

P_{se} = 64 (0.153) (156.237) = 1529.873 kips

B.2.7.2.3
Final Stresses at Midspan

Top fiber stress in concrete at midspan at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + f_t = \frac{1529.873}{1120} - \frac{18.743(1529.873)}{12761.88} + 3.737$$

= 1.366 – 2.247 + 3.737 = 2.856 ksi

Allowable compression stress limit for all load combinations
= 0.6 f'_c

$f'_{c\ reqd.}$ = 2856/0.6 = 4760 psi

Top fiber stress in concrete at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}$$

$$= \left(\frac{1529.873}{1120} - \frac{18.743(1529.873)}{12761.88} + \frac{(1714.65 + 1689.67 + 132.63)(12)}{12761.88} + \frac{(160.15 + 283.57)(12)}{79936.06} \right)$$

= 1.366 – 2.247 + 3.326 + 0.067 = 2.512 ksi

The allowable compressive stress limit for the effective pretension force + permanent dead loads = 0.45 f'_c

$f'_{c\ reqd.}$ = 2512/0.45 = 5582 psi (controls)

Top fiber stress in concrete at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}\right)$$

$$= \frac{(1618.3+684.57)(12)}{79936.06} + 0.5\left(\frac{1529.873}{1120} - \frac{18.743(1529.873)}{12761.88} + \frac{(1714.65 + 1689.67+132.63)(12)}{12761.88} + \frac{(160.15+283.57)(12)}{79936.06}\right)$$

$$= 0.346 + 0.5(1.366 - 2.247 + 3.326 + 0.067) = 1.602 \text{ ksi}$$

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f'_c

$f'_{c\ reqd.}$ = 1602/0.4 = 4005 psi

Bottom fiber stress in concrete at midspan at service load

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1529.873}{1120} + \frac{18.743(1529.873)}{18024.15} - 3.34$$

$$= 1.366 + 1.591 - 3.34 = -0.383 \text{ ksi}$$

Allowable tension in concrete = $0.19\sqrt{f'_c(\text{ksi})}$

$$f'_{c\ reqd.} = \left(\frac{383}{0.19}\right)^2 \times 1000 = 4063 \text{ psi}$$

B.2.7.2.4
Initial Stresses at Debonding Locations

With the same number of debonded strands as was determined in the previous iteration, the top and bottom fiber stresses with their corresponding initial concrete strengths are calculated, and results are presented in Table B.2.7.4. It can be observed that at the 15 ft. location, the f'_{ci} value is updated to 4943 psi.

Table B.2.7.4. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	0	3	6	9	12	15	54.2
Row No. 1 (bottom row)	7	9	17	23	25	27	27
Row No. 2	19	27	27	27	27	27	27
Row No. 3	10	10	10	10	10	10	10
No. of Strands	36	46	54	60	62	64	64
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1021.50	1305.25	1532.25	1702.50	1759.26	1816.01	1816.01
e_c (in.)	18.056	18.177	18.475	18.647	18.697	18.743	18.743
Top Fiber Stresses (ksi)	-0.533	-0.520	-0.513	-0.477	-0.372	-0.277	0.567
Corresponding $f'_{ci\ reqd}$ (psi)	4932	4694	4569	3950	2403	1332	5581
Bottom Fiber Stresses (ksi)	1.935	2.359	2.700	2.934	2.946	2.966	2.368
Corresponding $f'_{ci\ reqd}$ (psi)	3226	3931	4500	4890	4910	4943	3947

B.2.7.3
Iteration 3

Following the procedure in Iteration 1, a third iteration is required to calculate prestress losses based on the new value of f'_{ci} = 4943 psi. Table B.2.7.5 shows the results of this third iteration.

Table B.2.7.5. Results of Iteration 3.

	Trial #1	Trial #2	Units
No. of Strands	64	64	
e_c	18.743	18.743	in.
Δf_{pSR}	8	8	ksi
Assumed Initial Prestress Loss	8.416	8.395	percent
P_{si}	1815	1816	kips
M_g	1714.65	1714.65	k-ft.
f_{cgp}	2.247	2.248	ksi
f_{ci}	4943	4943	psi
E_{ci}	4262	4262	ksi
Δf_{pES}	15.025	15.031	ksi
f_{cdp}	1.191	1.191	ksi
Δf_{pCR}	18.627	18.639	ksi
Δf_{pR1}	1.975	1.975	ksi
Δf_{pR2}	2.599	2.598	ksi
Corresponding Initial Prestress Loss	8.395	8.398	percent
Total Prestress Loss	46.226	46.243	ksi

B.2.7.3.1
Total Losses at Transfer

Total initial losses = $\Delta f_{ES} + \Delta f_{R1}$ = 15.031 + 1.975 = 17.006 ksi

f_{si} = Effective initial prestress = 202.5 – 17.006 = 185.494 ksi

P_{si} = Effective pretension force after allowing for the initial losses

= 64(0.153)(185.494) = 1816.357 kips

B.2.7.3.2
Total Losses at Service Loads

Δf_{SH} = 8 ksi

Δf_{ES} = 15.031 ksi

Δf_{R2} = 2.598 ksi

Δf_{R1} = 1.975 ksi

Δf_{CR} = 18.639 ksi

Total final losses = 8 + 15.031 + 2.598 + 1.975 + 18.639 = 46.243 ksi

or $\dfrac{46.243\ (100)}{0.75(270)}$ = 22.84 percent

f_{se} = Effective final prestress = 0.75(270) – 46.243 = 156.257 ksi

P_{se} = 64(0.153)(156.257) = 1530.069 kips

B.2.7.3.3
Final Stresses at Midspan

Top fiber stress in concrete at midspan at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + f_t = \frac{1530.069}{1120} - \frac{18.743(1530.069)}{12761.88} + 3.737$$

= 1.366 – 2.247 + 3.737 = 2.856 ksi

Allowable compression stress limit = 0.6 f'_c

$f'_{c\ reqd.}$ = 2856/0.6 = 4760 psi

Top fiber stress in concrete at midspan due to effective prestress + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\, e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}$$

$$= \left(\begin{array}{l} \dfrac{1530.069}{1120} - \dfrac{1530.069(18.743)}{12761.88} + \dfrac{(1714.65 + 1689.67 + 132.63)(12)}{12{,}761.88} \\ + \dfrac{(160.15 + 283.57)(12)}{79{,}936.06} \end{array} \right)$$

= 1.366 – 2.247 + 3.326 + 0.067 = 2.512 ksi

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.45 f'_c

$f'_{c\ reqd}$ = 2512/0.45 = 5582 psi (controls)

Top fiber stress in concrete at midspan due to live load + 0.5(effective prestress + dead loads)

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}\right)$$

$$= \frac{(1618.3+684.57)(12)}{79{,}936.06} + 0.5\left(\begin{array}{l}\dfrac{1530.069}{1120} - \dfrac{1530.069(18.743)}{12{,}761.88} \\ + \dfrac{(1714.65 + 1689.67+132.63)(12)}{12{,}761.88} \\ + \dfrac{(160.15+283.57)(12)}{79{,}936.06}\end{array}\right)$$

$$= 0.346 + 0.5(1.366 - 2.247 + 3.326 + 0.067) = 1.602 \text{ ksi}$$

Allowable compression stress limit for effective pretension force + permanent dead loads = 0.4 f'_c

$f'_{c\ reqd}$ = 1602/0.4 = 4005 psi

Bottom fiber stress in concrete at midspan at service load

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\ e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1530.069}{1120} + \frac{18.743(1530.069)}{18{,}024.15} - 3.34 = 1.366 + 1.591 - 3.34$$

= -0.383 ksi

Allowable tension in concrete = 0.19 $\sqrt{f'_c \text{(ksi)}}$

$$f'_{c\ reqd} = \left(\frac{383}{0.19}\right)^2 \times 1000 = 4063 \text{ psi}$$

B.2.7.3.4 Initial Stresses at Debonding Location

With the same number of debonded strands, as was determined in the previous iteration, the top and bottom fiber stresses with their corresponding initial concrete strengths are calculated, and results are presented in Table B.2.7.6. It can be observed that at the 15-ft. location, the f'_{ci} value is updated to 4944 psi.

Table B.2.7.6. Debonding of Strands at Each Section.

	Location of the Debonding Section (ft. from end)						
	0	3	6	9	12	15	54.2
Row No. 1 (bottom row)	7	9	17	23	25	27	27
Row No. 2	19	27	27	27	27	27	27
Row No. 3	10	10	10	10	10	10	10
No. of Strands	36	46	54	60	62	64	64
M_g (k-ft.)	0	185	359	522	675	818	1715
P_{si} (kips)	1021.70	1305.51	1532.55	1702.84	1759.60	1816.36	1816.36
e_c (in.)	18.056	18.177	18.475	18.647	18.697	18.743	18.743
Top Fiber Stresses (ksi)	-0.533	-0.520	-0.513	-0.477	-0.372	-0.277	0.566
Corresponding $f'_{ci\ reqd}$ (psi)	4932	4694	4569	3950	2403	1332	5562
Bottom Fiber Stresses (ksi)	1.936	2.359	2.701	2.934	2.947	2.966	2.369
Corresponding $f'_{ci\ reqd}$ (psi)	3226	3932	4501	4891	4911	4944	3948

Since in the last iteration, actual initial losses are 8.398 percent as compared to previously assumed 8.395 percent and f'_{ci} = 4944 psi as compared to previously assumed f'_{ci} = 4943 psi. These values are close enough, so no further iteration will be required.
Use f'_c = 5582 psi and f'_{ci} = 4944 psi.

B.2.8 STRESS SUMMARY

B.2.8.1 Concrete Stresses at Transfer

Compression: $0.6 f'_{ci} = 0.6(4944) = 2966.4$ psi
$= 2.966$ ksi (compression)

B.2.8.1.1 Allowable Stress Limits

Tension: The maximum allowable tensile stress with bonded reinforcement (precompressed tensile zone) is:
$0.24\sqrt{f'_{ci}} = 0.24\sqrt{4.944} = 0.534$ ksi

The maximum allowable tensile stress without bonded reinforcement (non-precompressed tensile zone) is:
$-0.0948\sqrt{f'_{ci}} = -0.0948\times\sqrt{4.944} = 0.211$ ksi > 0.2 ksi

Allowable tensile stress without bonded reinforcement = 0.2 ksi

B.2.8.1.2 Stresses at Girder End and at Transfer Length Section

Stresses at girder end and transfer length section need only be checked at release, because losses with time will reduce the concrete stresses making them less critical.

Transfer length = 60 (strand diameter) [LRFD Art. 5.8.2.3]
= 60 (0.5) = 30 in. = 2.5 ft.

B.2.8.1.2.1 Stresses at Transfer Length Section

Transfer length section is located at a distance of 2.5 ft. from the girder end. An overall girder length of 109.5 ft. is considered for the calculation of the bending moment at transfer length. As shown in Table B.2.7.6, the number of strands at this location, after debonding of strands, is 36.

Moment due to girder self-weight and diaphragm

M_g = 0.5(1.167) (2.5) (109.5 – 2.5) = 156.086 k-ft.

M_{dia} = 3(2.5) = 7.5 k-ft.

Concrete stress at top fiber of the girder

$$f_t = \frac{P_{si}}{A} - \frac{P_{si}\,e_t}{S_t} + \frac{M_g + M_{dia}}{S_t}$$

P_{si} = 36 (0.153) (185.494) = 1021.701 kips

Strand eccentricity at transfer section, e_c = 18.056 in.

$$f_t = \frac{1021.701}{1120} - \frac{18.056(1021.701)}{12{,}761.88} + \frac{(156.086 + 7.5)(12)}{12{,}761.88}$$

= 0.912 – 1.445 + 0.154 = -0.379 ksi (tension)

Allowable tension (with bonded reinforcement)
= 534 psi > 379 psi (O.K.)

Concrete stress at the bottom fiber of the girder

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\,e_c}{S_b} - \frac{M_g + M_{dia}}{S_b}$$

$$f_{bi} = \frac{1021.701}{1120} + \frac{18.056(1021.701)}{18{,}024.15} - \frac{(156.086 + 7.5)(12)}{18{,}024.15}$$

= 0.912 + 1.024 – 0.109 = 1.827 ksi

Allowable compression = 2.966 ksi > 1.827 ksi (reqd.) (O.K.)

B.2.8.1.2.2 Stresses at Girder End

The strand eccentricity at end of girder is:

$$e_c = 22.36 - \frac{7(2.17) + 17(4.14) + 8(6.11)}{36} = 18.056 \text{ in.}$$

P_{si} = 36 (0.153) (185.494) = 1021.701 kips

Concrete stress at the top fiber of the girder

$$f_t = \frac{1021.701}{1120} - \frac{18.056(1021.701)}{12761.88} = 0.912 - 1.445 = -0.533 \text{ ksi}$$

Allowable tension (with bonded reinforcement)
= 0.534 ksi > 0.533 ksi (O.K.)

Concrete stress at the bottom fiber of the girder

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_{bi} = \frac{1021.701}{1120} + \frac{18.056(1021.701)}{18024.15} = 0.912 + 1.024 = 1.936 \text{ ksi}$$

Allowable compression = 2.966 ksi > 1.936 ksi (reqd.) (O.K.)

B.2.8.1.3 Stresses at Midspan

Bending moment at midspan due to girder self-weight based on overall length

M_g = 0.5(1.167)(54.21)(109.5 – 54.21) = 1749.078 k-ft.

P_{si} = 64 (0.153) (185.494) = 1816.357 kips

Concrete stress at top fiber of the girder at midspan

$$f_t = \frac{P_{si}}{A} - \frac{P_{si}\ e_c}{S_t} + \frac{M_g}{S_t}$$

$$f_t = \frac{1816.357}{1120} - \frac{18.743(1816.357)}{12{,}761.88} + \frac{1749.078(12)}{12{,}761.88}$$

$$= 1.622 - 2.668 + 1.769 = 0.723 \text{ ksi}$$

Allowable compression: 2.966 ksi >> 0.723 ksi (reqd.) (O.K.)

Concrete stresses in bottom fibers of the girder at midspan

$$f_b = \frac{P_{si}}{A} + \frac{P_{si}\ e_c}{S_b} - \frac{M_g}{S_b}$$

$$f_b = \frac{1816.357}{1120} + \frac{18.743(1816.357)}{18{,}024.15} - \frac{1749.078\ (12)}{18{,}024.15}$$

$$= 1.622 + 1.889 - 1.253 = 2.258 \text{ ksi}$$

Allowable compression: 2.966 ksi > 2.258 ksi (reqd.) (O.K.)

B.2.8.1.4 Stress Summary at Transfer

	Top of Girder f_t (ksi)	Bottom of Girder f_b (ksi)
At End	-0.533	+1.936
At Transfer Length Section	-0.379	+1.827
At Midspan	+0.723	+2.258

B.2.8.2 Concrete Stresses at Service Loads

B.2.8.2.1 Allowable Stress Limits

Compression:
Case (I): for all load combinations

$0.60\ f_c' = 0.60(5582)/1000 = +3.349$ ksi (for precast girder)

$0.60\ f_c' = 0.60(4000)/1000 = +2.4$ ksi (for slab)

Case (II): for effective pretension force + permanent dead loads

$0.45\ f_c' = 0.45(5582)/1000 = +2.512$ ksi (for precast girder)

$0.45\ f_c' = 0.45(4000)/1000 = +1.8$ ksi (for slab)

Case (III): for live load + 0.5 (effective pretension force + dead loads)

$0.40\ f_c' = 0.40(5582)/1000 = +2.233$ ksi (for precast girder)

$0.40\ f_c' = 0.40(4000)/1000 = +1.6$ ksi (for slab)

Tension: $0.19\sqrt{f_c'} = 0.19\sqrt{5.582} = -0.449$ ksi

$P_{se} = 64(0.153)(156.257) = 1530.069$ kips

B.2.8.2.2 Stresses at Midspan

Case (I): Concrete stresses at top fiber of the girder at service loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + f_t = \frac{1530.069}{1120} - \frac{18.743(1530.069)}{12761.88} + 3.737$$

$$= 1.366 - 2.247 + 3.737 = 2.856 \text{ ksi}$$

Allowable compression: +3.349 ksi > +2.856 ksi (reqd.) (O.K.)

Case (II): Effective pretension force + permanent dead loads

$$f_{tf} = \frac{P_{se}}{A} - \frac{P_{se}\ e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}}$$

$$= \left(\frac{1530.069}{1120} - \frac{18.743(1530.069)}{12{,}761.88} + \frac{(1714.65 + 1689.67 + 132.63)(12)}{12{,}761.88} + \frac{(160.15+283.57)(12)}{79{,}936.06} \right)$$

= 1.366 – 2.247 + 2.326 + 0.067 = 1.512 ksi

Allowable compression: +2.512 ksi > +1.512 ksi (reqd.) (O.K.)

Case (III): Live load + 0.5(Pretensioning force + Dead loads)

$$f_{tf} = \frac{(M_{LT} + M_{LL})}{S_{tg}} + 0.5\left(\frac{P_{se}}{A} - \frac{P_{se}\,e_c}{S_t} + \frac{M_g + M_b + M_{dia}}{S_t} + \frac{M_b + M_{ws}}{S_{tg}} \right)$$

$$= \frac{(1618.3+684.57)(12)}{79{,}936.06} + 0.5\left(\frac{1525.956}{1120} - \frac{18.743(1525.956)}{12{,}761.88} + \frac{(1714.65 + 1689.67 + 132.63)(12)}{12{,}761.88} + \frac{(160.15+283.57)(12)}{79{,}936.06} \right)$$

= 0.346 + 0.5(1.366 – 2.247 + 2.326 + 0.067) = 1.602 ksi

Allowable compression: +2.233 ksi > +1.602 ksi (reqd.) (O.K.)

Concrete stresses at bottom fiber of the girder

$$f_{bf} = \frac{P_{se}}{A} + \frac{P_{se}\,e_c}{S_b} - f_b$$

$$f_{bf} = \frac{1530.069}{1120} + \frac{18.743(1530.069)}{18{,}024.15} - 3.34 = 1.366 + 1.591 - 3.338$$

= -0.383 ksi

Allowable tension: 0.449 ksi > 0.383 ksi (reqd.) (O.K.)

B.2.8.2.3 Stresses at the Top of the Deck Slab

Stresses at the top of the slab

Case (I):

$$f_t = \frac{M_b + M_{ws} + M_{LT} M_{LL}}{S_{tc}} = \frac{(1618.3+684.57+160.15+283.57)(12)}{50{,}802.19}$$

= +0.649 ksi

Allowable compression: +2.4 ksi > +0.649 ksi (reqd.) (O.K.)

Case (II):

$$f_t = \frac{M_b + M_{ws}}{S_{tc}} = \frac{(160.15+283.57)(12)}{50{,}802.19} = 0.105 \text{ ksi}$$

Allowable compression: +1.8 ksi > +0.105 ksi (reqd.) (O.K.)

Case (III):

$$f_t = \frac{0.5(M_b + M_{ws}) + M_{LT} M_{LL}}{S_{tc}}$$

$$= \frac{(1618.3+684.57+0.5(160.15+283.57))(12)}{50{,}802.19} = 0.596 \text{ ksi}$$

Allowable compression: +1.6 ksi > +0.596 ksi (reqd.) (O.K.)

B.2.8.2.4 Summary of Stresses at Service Loads

		Top of Slab f_t (ksi)	Top of Girder f_t (ksi)	Bottom of Girder f_b (ksi)
At Midspan	CASE I	+ 0.649	+2.856	
	CASE II	+ 0.105	+1.512	-0.383
	CASE III	+0.596	+1.602	

B.2.8.3 Fatigue Stress Limit

According to LRFD Art. 5.5.3, the fatigue of the reinforcement need not be checked for fully prestressed components designed to have extreme fiber tensile stress due to the Service III limit state within the tensile stress limit. In this example, the U54 girder is being designed as a fully prestressed component and the extreme fiber tensile stress due to Service III limit state is within the allowable tensile stress limits, so no fatigue check is required.

B.2.8.4 Actual Modular Ratio and Transformed Section Properties for Strength Limit State and Deflection Calculations

Up to this point, a modular ratio equal to 1 has been used for the service limit state design. For the evaluation of the strength limit state and deflection calculations, the actual modular ratio will be calculated and the transformed section properties will be used (see Table B.2.8.1).

$$n = \frac{E_c \text{ for slab}}{E_c \text{ for beam}} = \left(\frac{3834.25}{4341.78}\right) = 0.846$$

Transformed flange width = n (effective flange width)
= 0.846(138 in.) = 116.75 in.

Transformed flange area = n (effective flange width) (t_s)
= 1(116.75 in.)(8 in.) = 934 in.^{2}

Table B.2.8.1. Properties of Composite Section.

	Transformed Area in.^{2}	y_b in.	$A\ y_b$ in.	$A(y_{bc} - y_b)^2$	I in.^{4}	$I+A(y_{bc} - y_b)^2$ in.^{4}
Girder	1120	22.36	25,043.20	294,295.79	403,020	697,315.79
Slab	934	58	54,172.00	352,608.26	4981	357,589.59
∑	2054		79,215.20			1,054,905.38

where:

A_c = Total area of composite section = 2054 in.^{2}

h_c = Total height of composite section = 62 in.

I_c = Moment of inertia of composite section
= 1,054,905.38 in.^{4}

y_{bc} = Distance from the centroid of the composite section to extreme bottom fiber of the precast girder
= 79,215.20 / 2054 = 38.57 in.

y_{tg} = Distance from the centroid of the composite section to extreme top fiber of the precast girder
= 54 – 38.57 = 15.43 in.

y_{tc} = Distance from the centroid of the composite section to extreme top fiber of the slab = 62 – 38.57 = 23.43 in.

S_{bc} = Composite section modulus for extreme bottom fiber of the precast girder
= I_c/y_{bc} = 1,054,905.38 / 38.57 = 27,350.41 in.^{3}

S_{tg} = Composite section modulus for top fiber of the precast girder
= I_c/y_{tg} = 1,054,905.38 / 15.43 = 68,367.17 in.^{3}

S_{tc} = Composite section modulus for top fiber of the slab
= I_c/y_{tc} = 1,054,905.38 / 23.43 = 45,023.7 in.^{3}

B.2.9 STRENGTH LIMIT STATE

Total ultimate moment from Strength I is:

$$M_u = 1.25(DC) + 1.5(DW) + 1.75(LL + IM)$$

$$M_u = 1.25(1714.65 + 1689.67 + 132.63 + 160.15) + 1.5(283.57) + 1.75(1618.3 + 684.57) = 9076.73 \text{ k-ft}$$

Average stress in prestressing steel when:

$f_{pe} \geq 0.5 f_{pu}$ [f_{pe} = 156.257 ksi > 0.5(270)= 135 ksi]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$ [LRFD Eq. 5.7.3.1.1-1]

k = 0.28 for low-relaxation strand [LRFD Table C5.7.3.1.1-1]

For rectangular section behavior,

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}}$$ [LRFD Eq. 5.7.3.1.1-4]

$d_p = h - y_{bs}$ = 62 – 3.617= 58.383 in.

β_1 = 0.85 for $f'_c \leq 4.0$ ksi [LRFD Art. 5.7.2.2]
= 0.85 – 0.05(f'_c – 4.0) ≤ 0.65 for $f'_c \geq 4.0$ ksi
= 0.85

k = 0.28

For rectangular section behavior,

$$c = \frac{64(0.153)(270)}{0.85(5.587)(0.85)(116.75) + (0.28)64(0.153)\dfrac{270}{(58.383)}} = 5.463 \text{ in.}$$

a = 0.85 x 5.463 = 4.64 in. < 8 in. = t_s (O.K.)

The assumption of rectangular section behavior is valid.

$$f_{ps} = 270\left(1 - 0.28\frac{5.463}{(58.383)}\right) = 262.93 \text{ ksi}$$

Nominal flexural resistance [LRFD Art. 5.7.3.2.3]

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)$$ [LRFD Eq. 5.7.3.2.2-1]

The equation above is a simplified form of LRFD Equation 5.7.3.2.2-1 because no compression reinforcement or mild tension reinforcement is considered, and the section behaves as a rectangular section.

$$M_n = 64(0.153)(262.93)\left(58.383 - \frac{4.64}{2}\right)$$

$$= 144,340.39 \text{ k-in.} = 12,028.37 \text{ k-ft.}$$

Factored flexural resistance

$M_r = \phi M_n$ [LRFD Eq. 5.7.3.2.1-1]

where:

ϕ = Resistance factor [LRFD Eq. 5.5.4.2.1]

= 1.00, for flexure and tension of prestressed concrete

M_r = 12,028.37 k-ft. > M_u = 9076.73 k-ft. (O.K.)

B.2.9.1 Limits of Reinforcment

B.2.9.1.1 Maximum Reinforcement

[LRFD Eq. 5.7.3.3]

The amount of prestressed and non-prestressed reinforcement should be such that:

$$\frac{c}{d_e} \le 0.42$$ [LRFD Eq. 5.7.3.3.1-1]

where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$ [LRFD Eq. 5.7.3.3.1-2]

Since A_s = 0, $d_e = d_p$ = 58.383 in.

$$\frac{c}{d_e} = \frac{5.463}{58.383} = 0.094 \le 0.42$$ (O.K.)

B.2.9.1.2 Minimum Reinforcement

[LRFD Art. 5.7.3.3.2]

At any section, the amount of prestressed and nonprestressed tensile reinforcement should be adequate to develop a factored flexural resistant, M_r, equal to the lesser of:

- 1.2 times the cracking moment strength determined on the basis of elastic stress distribution and the modulus of rupture, and
- 1.33 times the factored moment required by the applicable strength load combination.

Check at the midspan: [LRFD Eq. 5.7.3.3.2-1]

$$M_{cr} = S_c(f_r + f_{cpe}) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \le S_c f_r$$

f_{cpe} = Compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$$f_{cpe} = \frac{P_{se}}{A} + \frac{P_{se}e_c}{S_b} = \frac{1530.069}{1120} + \frac{1530.069(18.743)}{18,024.15}$$

$$= 1.366 + 1.591 = 2.957 \text{ ksi}$$

M_{dnc} = Total unfactored dead load moment acting on the monolithic or noncomposite section (kip-ft.)

= $M_g + M_{slab} + M_{dia}$

= 1714.65+1689.67+132.63 = 3536.95 kip-ft.

$S_c = S_{bc}$

$S_{nc} = S_b$

f_r = $f_r = 0.24\sqrt{f_c'} = 0.24(\sqrt{5.587}) = 0.567$ ksi [LRFD Art. 5.4.6.2]

$$M_{cr} = \frac{27{,}350.41}{12}(0.567 + 2.957) - 3536.95\left(\frac{27{,}350.41}{18{,}024.15} - 1\right)$$

$$\leq \frac{27{,}350.41}{12}(0.567)$$

M_{cr} = 6,183.54 k-ft. ≤ 1292.31 k-ft.

so use M_{cr} = 1292.31 k-ft.

1.2M_{cr} =1550.772 k-ft.

where M_u =9076.73 k-ft.

1.33 M_u = 12,097.684 k-ft.

Since 1.2M_{cr} < 1.33 M_u, the 1.2 M_{cr} requirement controls.

M_r = 12,028.37 k-ft. > 1.2M_{cr} = 1550.772 k-ft. (O.K.)

LRFD Art. 5.7.3.3.2 requires that this criterion be met at every section.

B.2.10 TRANSVERSE SHEAR DESIGN

The area and spacing of shear reinforcement must be determined at regular intervals along the entire length of the girder. In this design example, transverse shear design procedures are demonstrated below for the critical section near the supports.

Transverse shear reinforcement is provided when:

$V_u > 0.5\ \phi\ (V_c + V_p)$ [LRFD Art. 5.8.2.4-1]

where:

V_u = Factored shear force at the section considered

V_c = Nominal shear strength provided by concrete

V_p = Component of prestressing force in direction of shear force

ϕ = Strength reduction factor for shear = 0.90 [LRFD Art. 5.5.4.2.1]

B.2.10.1 Critical Section

Critical section near the supports is the greater of: [LRFD Art. 5.8.3.2]

$$0.5\ d_v \cot\theta \text{ or } d_v$$

where:

d_v = Effective shear depth, in.
= Distance between the resultants of tensile and compressive forces, (d_e – a/2), but not less than the greater of (0.9d_e) or (0.72h) [LRFD Art. 5.8.2.9]

d_e = Corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement [LRFD Art. 5.7.3.3.1]

B.2.10.1.1 Angle of Diagonal Compressive Stresses

The angle of inclination of the diagonal compressive stresses is calculated using an iterative method. As an initial estimate θ is taken as 23 degrees.

B.2.10.1.2 Effective Shear Depth

$d_v = d_e - a/2 = 58.383 - 4.64/2 = 56.063$ in.
$\geq 0.9\ d_e = 0.9\ (58.383) = 52.545$ in.
$\geq 0.72h = 0.72\times 62 = 44.64$ in. (O.K.)

B.2.10.1.3 Calculation of Critical Section

The critical section near the support is the greater of:

d_v = 56.063 in. or

$0.5 d_v \cot\theta = 0.5\times(56.063)\times\cot(23^\circ) = 66.04$ in.=5.503 ft. (controls)

B.2.10.2 Contribution of Concrete to Nominal Shear Resistance

The contribution of the concrete to the nominal shear resistance is:

$$V_c = 0.0316\beta\sqrt{f_c'}b_v d_v$$ [LRFD Eq. 5.8.3.3-3]

B.2.10.2.1 Strain in Flexural Tension Reinforcement

Calculate the strain in the reinforcement on the flexural tension side. Assuming that the section contains at least the minimum transverse reinforcement as specified in LRFD Art. 5.8.2.5:

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_s A_s + E_p A_{ps})} \le 0.001$$

[LRFD Eq. 5.8.3.3-1]

If LRFD Eq. 5.8.3.3-1 yields a negative value, then LRFD Eq. 5.8.3.3-3 should be used:

$$\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2(E_c A_c + E_s A_s + E_p A_{ps})}$$ [LRFD Eq. 5.8.3.3-3]

where:

V_u = Factored shear force at the critical section, taken as positive quantity
= 1.25(56.84 + 56.01 + 3.00 + 5.31) + 1.50(9.40) + 1.75(85.55 + 32.36) = 371.893 kips

M_u = Factored moment, taken as positive quantity
= 1.25(330.46+325.64+16.51+30.87)+1.5(54.65) +1.75(331.15+131.93)

M_u = 1771.715 k-ft. > $V_u d_v$
= 1771.715 k-ft. > 371.893×56.063/12 = 1737.45 kip-ft. (O.K.)

V_p = Component of prestressing force in direction of shear force
= 0 (because no harped strands are used)

N_u = Applied factored normal force at the specified section = 0

A_c = Area of the concrete (in.^{2}) on the flexural tension side below $h/2$ = 714 in.^{2}

$$v_u = \frac{V_u - \phi V_p}{\phi b_v d_v} = \frac{371.893}{0.9 \times 10 \times 56.063} = 0.737 \text{ ksi}$$ [LRFD Eq. 5.8.2.9-1]

where b_v = 2 × 5 = 10 in.

v_u / f'_c = 0.737 / 5.587 = 0.132

As per LRFD Art. 5.8.3.4.2, if the section is within the transfer length of any strands, then calculate the effective value of f_{po}; else assume $f_{po} = 0.7f_{pu}$. The transfer length of the bonded strands at the section located 3 ft. from the girder end extends from 3 ft. to 5.5 ft. from the girder end, and the critical section for shear is 5.47 ft. from the support centerline. The support centerline is 6.5 in. away from the girder end. The critical section for shear will be 5.47 + 6.5/12 = 6.00 ft. from the girder end, so the critical section does not fall within the transfer length of the strands that are bonded from the section located at 3 ft. from the end of the girder. Thus, detailed calculations for f_{po} are not required.

f_{po} = Parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)
= Approximately equal to 0.7 f_{pu} [LRFD Fig. C5.8.3.4.2-5]
= 0.70 f_{pu} = 0.70 × 270 = 189 ksi

Or f_{po} can be conservatively taken as the effective stress in the prestressing steel, f_{pe}

$$f_{po} = f_{pe} + f_{pc}\left(\frac{E_{ps}}{E_c}\right)$$

where:

f_{pc} = Compressive stress in concrete after all prestress losses have occurred either at the centroid of the crosssection resisting live load or at the junction of the web and flange when the centroid lies in the flange (ksi); in a composite section, it is the resultant compressive stress at the centroid of the composite section or at the junction of the web and flange when the centroid lies within the flange that results from both prestress and the bending moments resisted by the precast member acting alone (ksi).

$$f_{pc} = \frac{P_{se}}{A_n} - \frac{P_{se}e_c\left(y_{bc} - y_b\right)}{I} + \frac{\left(M_g + M_{slab}\right)\left(y_{bc} - y_b\right)}{I}$$

The number of strands at the critical section location is 46 and the corresponding eccentricity is 18.177 in., as calculated in Table B.2.7.6.

$$P_{se} = 46 \times 0.153 \times 155.837 = 1096.781 \text{ kips}$$

$$f_{pc} = \left(\begin{array}{c} \dfrac{1096.781}{1120} - \dfrac{1096.781 \times 18.177\left(40.05 - 22.36\right)}{403020} \\ + \dfrac{12 \times \left(328.58 + 323.79\right)\left(40.05 - 22.36\right)}{403020} \end{array}\right) = 0.492 \text{ ksi}$$

$$f_{po} = 155.837 + 0.492\left(\frac{28500}{4531.48}\right) = 158.93 \text{ ksi}$$

$$\varepsilon_x = \frac{\left(\begin{array}{l}\dfrac{1771.715\times12}{56.063}+0.5(0.0)+0.5(371.893-0.0)\cot 23^{\circ}\\ -46\times0.153\times158.93\end{array}\right)}{2(28000\times0.0+28500\times46\times0.153)} \leq 0.001$$

ε_x = -0.000751

Since this value is negative, LRFD Eq. 5.8.3.4.2-3 should be used to calculate ε_x.

$$\varepsilon_x = \frac{\dfrac{1771.715\times12}{56.063}+0.5(0.0)+0.5(371.893-0.0)\cot 23^{\circ}-46\times0.153\times158.93}{2[(4531.48)(714)+(28,500)(46)(0.153)]}$$

$\varepsilon_x = -4.384 \text{ x } 10^{-5}$

B.2.10.2.2
Values of β and θ

The values of β and θ are taken from LRFD Table 5.8.3.4.2-1, and after interpolation, the final values are determined, as shown in Table B.2.10.1. Because θ = 23.3 degrees is close to the 23 degrees assumed, no further iterations are required.

Table B.2.10.1. Interpolation for β *and* θ*.*

v_u/f'_c	$\varepsilon_x \times 1000$		
	-0.05	-0.04384	0
0.15	24.2		25
	2.776		2.72
0.132	23.19	θ = 23.3	24.06
	2.895	β = 2.89	2.83
0.125	22.8		23.7
	2.941		2.87

B.2.10.2.3
Concrete Contribution

The nominal shear resisted by the concrete is:

$$V_c = 0.0316\beta\sqrt{f_c'(\text{ksi})}\, b_v d_v \quad \text{[LRFD Eq. 5.8.3.3-3]}$$

$$V_c = 0.0316(2.89)\sqrt{5.587}(10)(56.063) = 121.02 \text{ kips}$$

B.2.10.3
Contribution of Reinforcement to Nominal Shear Resistance

B.2.10.3.1
Requirement for Reinforcement

Check if $V_u > 0.5\ \phi(V_c + V_p)$ [LRFD Eq. 5.8.2.4-1]

V_u = 371.893 > 0.5×0.9×(121.02+0) = 54.46 kips

Therefore, transverse shear reinforcement should be provided.

$$\frac{V_u}{\phi} \le V_n = V_c + V_s + V_p \quad \text{[LRFD Eq. 5.8.3.3-1]}$$

V_s (reqd.) = Shear force carried by transverse reinforcement

$$= \frac{V_u}{\phi} - V_c - V_p = \left(\frac{371.893}{0.9} - 121.02 - 0\right) = 292.19 \text{ kips}$$

$$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s} \quad \text{[LRFD Eq. 5.8.3.3-4]}$$

where:

s = Spacing of stirrups, in.

α = Angle of inclination of transverse reinforcement to longitudinal axis = 90 degrees

B.2.10.3.2
Required Area of Reinforcement

Therefore, the required area of shear reinforcement within a spacing *s* is:

A_v (reqd.) = (s V_s)/(f_y d_v cotθ)

= (s × 292.19)/[60 × 56.063 × cot(23)] = 0.0369 × s

B.2.10.3.3
Spacing of Reinforcement

If s = 12 in., then A_v = 0.443 in.^{2} / ft.

Maximum spacing of transverse reinforcement may not exceed the following: [LRFD Art. 5.8.2.7]

Since v_u =0.737 ksi > 0.125× f_c' = 0.125×5.587 = 0.689 ksi

So, s_{max} = 0.4 × 56.063 = 22.43 in. < 24.0 in.

Use s_{max} = 22.43 in.

Use 1 #4 transverse bar per web: A_v = 0.20(2 webs) = 0.40 in.^{2} / ft.; the required spacing can be calculated as:

$$s = \frac{A_v}{0.0369} = \frac{0.40}{0.0369} = 10.8 \text{ in.} \quad (\text{try } s = 10 \text{ in.})$$

$$V_s = \frac{0.40(60)(56.063)(\cot 23)}{10}$$

$$= 316.98 \text{ kips} > V_s(\text{reqd.}) = 292.19 \text{ kips}$$

[LRFD Art. 5.8.2.5]

B.2.10.3.4 Minimum Reinforcement Requirement

The area of transverse reinforcement should be less than:

$$A_v \geq 0.0316\sqrt{f_c'(\text{ksi})}\,\frac{b_v s}{f_y} \qquad \text{[LRFD Eq. 5.8.2.5-1]}$$

$$A_v \geq 0.0316\sqrt{5.587}\,\frac{10\times 10}{60} = 0.125 \text{ in.}^2 \qquad \text{(O.K.)}$$

B.2.10.3.5 Maximum Nominal Shear Reinforcement

To ensure that the concrete in the girder web will not crush prior to yield of the transverse reinforcement, the LRFD Specifications give an upper limit for V_n as follows:

$$V_n = 0.25 f_c' b_v d_v + V_p \qquad \text{[LRFD Eq. 5.8.3.3-2]}$$

$$V_c + V_s \leq 0.25 f_c' b_v d_v + V_p$$

$$(121.02 + 316.98) < (0.25\times 5.587\times 10\times 56.063 + 0)$$

$$438.00 \text{ kips} < 783.06 \text{ kips} \quad \text{(O.K.)}$$

B.2.10.4 Minimum Longitudinal Reinforcement Requirement

Longitudinal reinforcement should be proportioned so that at each section the following LRFD Equation 5.8.3.5-1 is satisfied:

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M_u}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\frac{V_u}{\phi_v} + 0.5V_s - V_p\right)\cot\theta$$

Using the Strength I load combination, the factored shear force and bending moment at the bearing face is:

V_u = 1.25(62.82+61.91+3+5.87) + 1.5(10.39) + 1.75(90.24+35.66)
= 402.91 kips

M_u= 1.25(23.64+23.3+1.13+2.2) + 1.5(3.91) + 1.75(23.81+9.44)
= 126.885 k-ft.

$$46\times 0.153\times 262.93 \geq \frac{126.885\times 12}{56.063\times 1.0} + 0.0 + \left(\frac{402.91}{0.9} + 0.5\times 310.643 - 0.0\right)(\cot 23)$$

$$1850.5 \text{ kips} \geq 1448.074 \text{ kips} \qquad \text{(O.K.)}$$

B.2.11 INTERFACE SHEAR TRANSFER

[LRFD Art. 5.8.4]

According to the guidance given by the LRFD Specifications for computing the factored horizontal shear:

B.2.11.1 Factored Horizontal Shear

$$V_h = \frac{V_u}{d_e}$$ [LRFD Eq. C5.8.4.1-1]

V_h = Horizontal shear per unit length of girder, kips

V_u = Factored vertical shear, kips

d_e = The distance between the centroid of the steel in the tension side of the girder to the center of the compression blocks in the deck ($d_e - a/2$), in.

The LRFD Specifications do not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear (i.e., 5.503 ft. from the support centerline).

V_u = 1.25(5.31) + 1.50(9.40) + 1.75(85.55+32.36) = 227.08 kips
d_e = 58.383 – 4.64/2 = 56.063 in.

$$V_h = \frac{227.08}{56.063} = 4.05 \text{ kips/in.}$$

B.2.11.2 Required Nominal Resistance

V_n (reqd.) = V_h / ϕ = 4.05 / 0.9 = 4.5 kip / in.

B.2.11.3 Required Interface Shear Reinforcement

The nominal shear resistance of the interface surface is:

$$V_n = c\,A_{cv} + \mu\left[A_{vf} f_y + P_c\right]$$ [LRFD Eq. 5.8.4.1-1]

c = Cohesion factor [LRFD Art. 5.8.4.2]

μ = Friction factor [LRFD Art. 5.8.4.2]

A_{cv} = Area of concrete engaged in shear transfer, in.^{2}

A_{vf} = Area of shear reinforcement crossing the shear plane, in.^{2}

P_c = Permanent net compressive force normal to the shear plane, kips

f_y = Shear reinforcement yield strength, ksi [LRFD Art. 5.8.4.2]

For concrete placed against clean, hardened concrete and free of laitance, but not an intentionally roughened surface:

c = 0.075 ksi

$\mu = 0.6\lambda$, where λ = 1.0 for normal weight concrete, and therefore,

μ = 0.6

The actual contact width, b_v, between the slab and the girder = 2(15.75) = 31.5 in.

A_{cv} = (31.5in.)(1in.) = 31.5 in.^{2}/in.

The LRFD Eq. 5.8.4.1-1 can be solved for A_{vf} as follows:

$$4.5 = 0.075 \times 31.5 + 0.6\left[A_{vf}(60) + 0.0\right]$$

Solving for A_{vf} = 0.0594 in.^{2}/in. = 0.713 in.^{2} / ft.

The #4 transverse reinforcing bar provided in each web will be bent 180 degrees to double the available interface shear reinforcement. For the required A_{vf} = 0.713 in.^{2} / ft., the required spacing can be calculated as:

$$s = \frac{A_v \times 12}{A_{vf}} = \frac{(0.20)(2)(2) \times 12}{0.713} = 13.46 \text{ in.} > 10 \text{ in.} = s \text{ (provided) (O.K.)}$$

Ultimate horizontal shear stress between slab and top of girder can be calculated:

$$V_{ult} = \frac{V_n \times 1000}{b_f} = \frac{4.5 \times 1000}{31.5} = 143.86 \text{ psi}$$

B.2.12 PRETENSIONED ANCHORAGE ZONE

[LRFD Art. 5.10.10.1]

Design of the anchorage zone reinforcement is based on the force in the strands just at transfer.

B.2.12.1 Anchorage Zone Reinforcement

Force in the strands at transfer:

F_{pi} = 64 (0.153)(202.5) = 1982.88 kips

The bursting resistance, P_r, should not be less than 4 percent of F_{pi}.

$$P_r = f_s A_s \geq 0.04 F_{pi} = 0.04(1982.88) = 79.32 \text{ kips}$$

where:

A_s = Total area of vertical reinforcement located within a distance of h/4 from the end of the girder, in.^{2}

f_s = Stress in steel not exceeding 20 ksi

Solving for required area of steel A_s= 79.32 /20 = 3.97 in.^{2}

At least 3.97 in.^{2} of vertical transverse reinforcement should be provided within a distance of (h/4 = 62 / 4 = 15.5 in.) from the end of the girder.

B.2.12.2 Confinement Reinforcement

For a distance of 1.5d from the girder end, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be of a shape that will confine (enclose) the strands. For box beams, transverse reinforcement shall be provided and anchored by extending the leg of the stirrup into the web of the girder.

B.2.13 DEFLECTION AND CAMBER

B.2.13.1 Maximum Camber Calculations using Hyperbolic Functions Method

The LRFD Specifications do not provide guidelines for the determination camber of prestressed concrete members. The Hyperbolic Functions Method (Furr et al. 1968, Sinno 1968, Furr and Sinno 1970) for the calculation of maximum camber is used by TxDOT's prestressed concrete bridge design software, PSTRS14 (TxDOT 2004). The following steps illustrate the Hyperbolic Functions Method for the estimation of maximum camber.

Step 1: Total prestress after release

$$P = \frac{P_{si}}{\left(1 + p\,n + \dfrac{e_c^2 A_s\, n}{I}\right)} + \frac{M_D\, e_c\, A_s\, n}{I\left(1 + p\,n + \dfrac{e_c^2 A_s\, n}{I}\right)}$$

where:

P_{si} = Total prestressing force = 1811.295 kips
I = Moment of inertia of non-composite section = 403,020 in.^{4}
e_c = Eccentricity of pretensioning force at the midspan
= 18.743 in.
M_D = Moment due to self-weight of the girder at midspan
= 1714.65 k-ft.
A_s = Area of strands = number of strands (area of each strand)
= 64(0.153) = 9.792 in.^{2}
p = A_s / A_n

where:
A_n = Area of cross-section of girder = 1120 in.^{2}
p = 9.972/1120 = 0.009

PSTRS14 uses final concrete strength to calculate E_c.
E_c = Modulus of elasticity of the girder concrete, ksi

$$= 33(w_c)^{3/2}\sqrt{f'_c} = 33(150)^{1.5}\sqrt{5587}\ \frac{1}{1000} = 4531.48 \text{ ksi}$$

[LRFD Art. 5.10.10.2]

E_{ps} = Modulus of elasticity of prestressing strands = 28,500 ksi

n = E_{ps}/E_c = 28,500/4531.48= 6.29

$$\left(1 + pn + \frac{e_c^2 A_s n}{I}\right) = 1 + (0.009)(6.29) + \frac{(18.743^2)(9.792)(6.29)}{403,020}$$

$$= 1.109$$

$$P = \frac{P_{si}}{\left(1 + p\,n + \dfrac{e_c^2 A_s\,n}{I}\right)} + \frac{M_D\,e_c\,A_s\,n}{I\left(1 + p\,n + \dfrac{e_c^2 A_s\,n}{I}\right)}$$

$$= \frac{1811.295}{1.109} + \frac{(1714.65)(12\text{ in./ft.})(18.743)(9.792)(6.29)}{403{,}020(1.109)}$$

= 1632.68 + 53.13 = 1685.81 kips

Concrete stress at steel level immediately after transfer

$$f_{ci}^s = P\left(\frac{1}{A} + \frac{e_c^2}{I}\right) - f_c^s$$

where:

f_c^s = Concrete stress at steel level due to dead loads

$$= \frac{M_D\,e_c}{I} = \frac{(1714.65)(12\text{ in./ft.})(18.743)}{403{,}020} = 0.957\text{ ksi}$$

$$f_{ci}^s = 1685.81\left(\frac{1}{1120} + \frac{18.743^2}{403{,}020}\right) - 0.957 = 2.018\text{ ksi}$$

Step 2: Ultimate time-dependent strain at steel level

$$\varepsilon_{c1}^s = \varepsilon_{cr}^\infty f_{ci}^s + \varepsilon_{sh}^\infty$$

where:

ε_{cr}^∞ = Ultimate unit creep strain = 0.00034 in./in. [This value is prescribed by Furr and Sinno (1970).]

ε_{sh}^∞ = Ultimate unit creep strain = 0.000175 in./in. [This value is prescribed by Furr and Sinno (1970).]

ε_{c1}^∞ = 0.00034(2.018) + 0.000175 = 0.0008611 in./in.

Step 3: Adjustment of total strain in Step 2

$$\varepsilon_{c2}^{s} = \varepsilon_{c1}^{s} - \varepsilon_{c1}^{s} E_{ps} \frac{A_s}{E_{ci}} \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.0008611 - 0.0008611\ (28500) \frac{9.792}{4531.48} \left(\frac{1}{1120} + \frac{18.743^2}{403020} \right)$$

$$= 0.000768 \text{ in./in.}$$

Step 4: Change in concrete stress at steel level

$$\Delta f_c^s = \varepsilon_{c2}^{s} E_{ps} A_s \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.000768\ (28{,}500)(9.792) \left(\frac{1}{1120} + \frac{18.743^2}{403{,}020} \right)$$

$$\Delta f_c^s = 0.375 \text{ ksi}$$

Step 5: Correction of the total strain from Step 2

$$\varepsilon_{c4}^{s} = \varepsilon_{\text{cr}}^{\infty} + \left(f_{ci}^{s} - \frac{\Delta f_c^s}{2} \right) + \varepsilon_{\text{sh}}^{\infty}$$

$$\varepsilon_{c4}^{s} = 0.00034 \left(2.018 - \frac{0.375}{2} \right) + 0.000175 = 0.0007974 \text{ in./in.}$$

Step 6: Adjustment in total strain from Step 5

$$\varepsilon_{c5}^{s} = \varepsilon_{c4}^{s} - \varepsilon_{c4}^{s} E_{ps} \frac{A_s}{E_c} \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.0007974 - 0.0007974\ (28{,}500)\ \frac{9.792}{4531.48} \left(\frac{1}{1120} + \frac{18.743^2}{403{,}020} \right)$$

$$= 0.000711 \text{ in./in.}$$

Step 7: Change in concrete stress at steel level

$$\Delta f_{c1}^s = \varepsilon_{c5}^{s} E_{ps} A_s \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.000711(28{,}500)(9.792) \left(\frac{1}{1120} + \frac{18.743^2}{403{,}020} \right)$$

$$\Delta f_{c1}^s = 0.350 \text{ ksi}$$

Step 8: Correction of the total strain from Step 5

$$\varepsilon_{c6}^{s} = \varepsilon_{cr}^{\infty} + \left(f_{ci}^{s} - \frac{\Delta f_{c1}^{s}}{2} \right) + \varepsilon_{sh}^{\infty}$$

$$\varepsilon_{c6}^{s} = 0.00034\left(2.018 - \frac{0.350}{2} \right) + 0.000175 = 0.000802 \text{ in./in.}$$

Step 9: Adjustment in total strain from Step 8

$$\varepsilon_{c7}^{s} = \varepsilon_{c6}^{s} - \varepsilon_{c6}^{s} E_{ps} \frac{A_s}{E_{ci}} \left(\frac{1}{A_n} + \frac{e_c^2}{I} \right)$$

$$= 0.000802 - 0.000802\ (28,500)\ \frac{9.792}{4531.48} \left(\frac{1}{1120} + \frac{18.743^2}{403,020} \right)$$

$$= 0.000715 \text{ in./in.}$$

Step 10: Computation of initial prestress loss

$$PL_i = \frac{P_{si} - P}{P_{si}} = \frac{1811.295 - 1685.81}{1811.295} = 0.0693$$

Step 11: Computation of final prestress loss

$$PL^{\infty} = \frac{\varepsilon_{c7}^{\infty} E_{ps} A_s}{P_{si}} = \frac{0.000715(28,500)(9.792)}{1811.295} = 0.109$$

Total prestress loss

$$PL = PL_i + PL^{\infty} = 100(0.0693 + 0.109) = 17.83 \text{ percent}$$

Step 12: Initial deflection due to dead load

$$C_{DL} = \frac{5\ w\ L^4}{384\ E_c\ I}$$

where:

w = Weight of girder = 1.167 kips/ft.

L = Span length = 108.417 ft.

$$C_{DL} = \frac{5\left(\frac{1.167}{12 \text{ in./ft.}} \right)[(108.417)(12 \text{ in./ft.})]^4}{384(4531.48)(403,020)} = 1.986 \text{ in.}$$

Step 13: Initial camber due to prestress

M/EI diagram is drawn for the moment caused by the initial prestressing and is shown in Figure B.2.13.1. Due to debonding of strands, the number of strands vary at each debonding section location. Strands that are bonded, achieve their effective prestress level at the end of transfer length. Points 1 through 6 show the end of transfer length for the preceding section. The *M/EI* values are calculated as:

$$\frac{M}{EI} = \frac{P_{si} \times e_c}{E_c I}$$

The *M/EI* values are calculated for each point 1 through 6 and are shown in Table B.2.13.1. The initial camber due to prestress, C_{pi}, can be calculated by the Moment Area Method, by taking the moment of the *M/EI* diagram about the end of the girder.

C_{pi} = 3.88 in.

Table B.2.13.1. M/EI Values at the End of Transfer Length.

Identifier for the End of Transfer Length	P_{si} (kips)	e_c (in.)	*M/EI* (in.^{3})
1	1018.864	18.056	1.01E-05
2	1301.882	18.177	1.30E-05
3	1528.296	18.475	1.55E-05
4	1698.107	18.647	1.73E-05
5	1754.711	18.697	1.80E-05
6	1811.314	18.743	1.86E-05

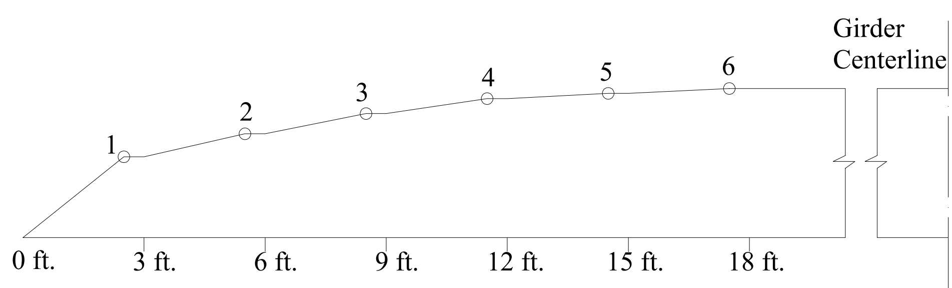

Figure B.2.13.1. M/EI Diagram to Calculate the Initial Camber due to Prestress.

Step 14: Initial camber

$C_i = C_{pi} - C_{DL} = 3.88 - 1.986 = 1.894$ in.

Step 15: Ultimate time dependent camber

Ultimate strain $\varepsilon_e^s = \dfrac{f_{ci}^s}{E_c}$ = 2.018/4531.48 = 0.000445 in./in.

Ultimate camber

$$C_t = C_i\left(1 - PL^{\infty}\right)\left(\frac{\varepsilon_{cr}^{\infty}\left(f_{ci}^s - \dfrac{\Delta f_{c1}^s}{2}\right) + \varepsilon_e^s}{\varepsilon_e^s}\right)$$

$$= 1.894(1 - 0.109)\left(\frac{0.00034\left(2.018 - \dfrac{0.347}{2}\right) + 0.000445}{0.000445}\right)$$

C_t = 4.06 in. = 0.34 ft. ↑

B.2.13.2 Deflection due to Girder Self-Weight

$$\Delta_{girder} = \frac{5 w_g L^4}{384 E_{ci} I}$$

where w_g = girder weight = 1.167 kips/ft.

Deflection due to girder self-weight at transfer

$$\Delta_{girder} = \frac{5(1.167/12)[(109.5)(12)]^4}{384(4262.75)(403{,}020)} = 0.186 \text{ ft.} \downarrow$$

Deflection due to girder self-weight used to compute deflection at erection

$$\Delta_{girder} = \frac{5(1.167/12)[(108.417)(12)]^4}{384(4262.75)(403{,}020)} = 0.165 \text{ ft.} \downarrow$$

B.2.13.3 Deflection due to Slab and Diaphragm Weight

$$\Delta_{slab} = \frac{5 w_s L^4}{384 E_c I} + \frac{w_{dia} b}{24 E_c I}\left(3l^2 - 4b^2\right)$$

where:

w_s = Slab weight = 1.15 kips/ft.

E_c = Modulus of elasticity of girder concrete at service = 4529.45 ksi

$$\Delta_{slab} = \left(\begin{array}{l} \dfrac{5(1.15/12)[(108.417)(12)]^4}{384(4529.45)(403,020)} + \\ \dfrac{(3)(44.21\times12)}{(24\times4529.45\times403,020)}\left(3(108.417\times12)^2 - 4(44.21\times12)^2\right) \end{array} \right)$$

$= 0.163$ ft.↓

B.2.13.4 Deflection due to Superimposed Loads

$$\Delta_{SDL} = \frac{5 w_{SDL} L^4}{384 E_c I_c}$$

where:

w_{SDL} = Superimposed dead load = 0.302 kips/ft.

I_c = Moment of inertia of composite section = 1,054,905.38 in.^{4}

$$\Delta_{SDL} = \frac{5(0.302/12)[(108.417)(12)]^4}{384(4529.45)(1,054,905.38)} = 0.0155 \text{ ft.} \downarrow$$

Total deflection at service for all dead loads = 0.165 + 0.163 + 0.0155 = 0.34 ft. ↓

B.2.13.5 Deflection due to Live Load and Impact

The deflections due to live loads are not calculated in this example as they are not a design factor for TxDOT bridges.

B.2.14 COMPARISON OF RESULTS

To measure the level of accuracy in this detailed design example, the results are compared with that of PSTRS14 (TxDOT 2004). A summary is shown in Table B.2.14.1 In the service limit state design, the results of this example matches those of PSTRS14 with very insignificant differences. A difference up to 5.9 percent was found for the top and bottom fiber stress calculation at transfer. This is due to the difference in top fiber section modulus values and the number of debonded strands in the end zone, respectively. There is a significant difference of 24.5 percent in camber calculation, which may be due to the fact that PSTRS14 uses a single-step hyperbolic functions method, whereas a multi-step approach is used in this detailed design example.

Table B.2.14.1. Comparison of Results for the AASHTO LRFD Specifications (PSTRS14 versus Detailed Design Example).

Design Parameters		PSTRS14	Detailed Design Example	Percent Difference with respect to PSTRS14
Prestress Losses (percent)	Initial	8.41	8.398	0.1
	Final	22.85	22.84	0.0
Required Concrete Strengths (psi)	f'_{ci}	4944	4944	0.0
	f'_c	5586	5582	0.1
At Transfer (ends) (psi)	Top	-506	-533	-5.4
	Bottom	1828	1936	-5.9
At Service (midspan) (psi)	Top	2860	2856	0.1
	Bottom	-384	-383	0.3
Number of Strands		64	64	0.0
Number of Debonded Strands		(20+10)	(20+8)	2
M_u (kip-ft.)		9082	9077	-0.1
ϕM_n (kip-ft.)		11,888	12,028	-1.2
Ultimate Horizontal Shear Stress at Critical Section (psi)		143.3	143.9	0.0
Transverse Shear Reinforcement (#4 bar) Spacing (in.)		10.3	10	2.9
Maximum Camber (ft.)		0.281	0.35	-24.6

B.2.15 REFERENCES

AASHTO (2004), *AASHTO LRFD Bridge Design Specifications*, 3rd Ed., American Association of State Highway and Transportation Officials (AASHTO), Customary U.S. Units, Washington, D.C.

Furr, H.L., R. Sinno and L.L. Ingram (1968). "Prestress Loss and Creep Camber in a Highway Bridge with Reinforced Concrete Slab on Prestressed Concrete Beams," *Texas Transportation Institute Report*, Texas A&M University, College Station.

Furr, H.L. and R. Sinno (1970) "Hyperbolic Functions for Prestress Loss and Camber," *Journal of the Structural Division*, Vol. 96, No. 4, pp. 803-821.

PCI (2003). "Precast Prestressed Concrete Bridge Design Manual," 2nd Ed., Precast/Prestressed Concrete Institute, Chicago, Illinois.

Sinno, R. (1968). "The Time-Dependent Deflections of Prestressed Concrete Bridge Beams," *Ph.D. Dissertation*, Texas A&M University, College Station.

TxDOT (2001). "TxDOT Bridge Design Manual," Bridge Division, Texas Department of Transportation.

TxDOT (2004). "Prestressed Concrete Beam Design/Analysis Program," User Guide, Version 4.00, Bridge Division, Texas Department of Transportation.