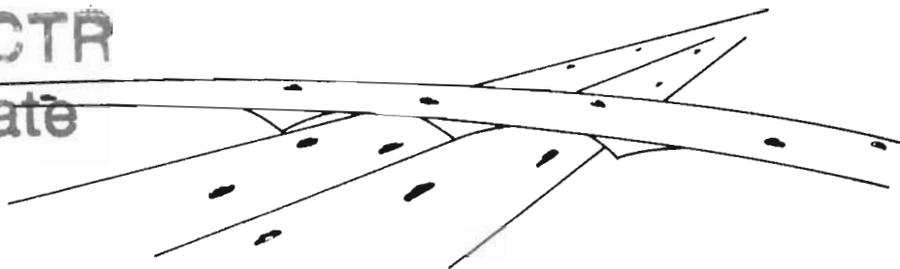


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DEPARTMENTAL RESEARCH

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AN EXTENSION OF RIGID PAVEMENT DESIGN METHODS

by

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and

B. F. McCullough

For Presentation
at the
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Highway Research Board

TEXAS HIGHWAY DEPARTMENT



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Research Section
Highway Design Division
TEXAS HIGHWAY DEPARTMENT

1964

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AN EXTENSION OF RIGID PAVEMENT DESIGN METHODS

by

W. R. Hudson and B. F. McCullough*

S Y N O P S I S

This paper verifies and extends certain developments in the AASHO Interim Pavement Design Guide. A choice of mathematical models is made based on studies of the AASHO and Maryland Road Tests' stress data as well as data from in-service pavements. Mathematical derivations and assumptions are presented in the appendix to provide a basis for future work on this subject.

As an extension of the initial work, the design thickness equation is expanded to include the concrete modulus of elasticity, total traffic, and pavement continuity (jointed or continuous). A nomograph is presented that allows a quick solution to the expanded equation.

New design charts are presented for design of the reinforcing steel in jointed and continuously reinforced pavements. In addition, a nomograph for solving bar spacing and bar size is included.

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AN EXTENSION OF RIGID PAVEMENT DESIGN METHODS

I. INTRODUCTION

Background

The first extensive concrete road system was constructed in Wayne County, Michigan, in 1909. Since that time civil engineers have been grappling with the many problems involved in the design of such pavements. A. I. Goldbeck and Clifford Older independently developed formulas for approximating the stresses in concrete pavements in early 1920. The best known of these formulas is generally called the "corner formula" and was the basis for rigid pavement design for many years. Results of the Bates Road Test in 1922-23 appeared to confirm the original corner formula. In 1926, Dr. H. M. Westergaard completed his treatise on the analysis of stresses in concrete pavements (Ref. 1). This analysis is concerned with the determination of maximum stresses in slabs of uniform thickness for three load conditions under several limiting assumptions (Ref. 2). The Westergaard equation for corner stresses has become the definitive design equation for portland cement concrete pavements. In this equation, Dr. Westergaard includes the following variables:

P = wheel load, in pounds

h = the thickness of the concrete slab, in inches

μ = Poisson's ratio for concrete

E = Young's modulus of elasticity for the concrete in pounds per square inch

k = subgrade modulus in pounds per cubic inch

a = radius of area of load contact, in inches.

Using this same general equation form, slightly different design equations have been developed by Spangler (Ref. 3), Kelly (Ref. 4), and Pickett (Ref. 5). These equations are all empirical or semi-empirical, but all retain the basic form of the Westergaard simplified theory.

It is important to note that all of these design equations are based upon static loading. This is necessary since very little theory exists to describe time dependent variables such as dynamic loads.

Road Test Results Used in Design

During the last fifty years, three large scale road tests have been conducted involving portland cement concrete pavement - the Bates Road Test, 1922; the Maryland Road Test, 1950; and the AASHO Road Test, 1958-61. All three of these full scale experiments have added valuable information to our knowledge of concrete pavement performance. However, only the AASHO Road Test was large enough to provide us with adequate information upon which to base dynamic design equations. In the early 1950's, the various highway departments formulated the AASHO Road Test to provide data on this problem. The first objective of the Road Test as outlined by the Road Test Advisory Committee (Ref. 6) was:

To determine the significant relationship between the number of repetitions of specified axle loads of different magnitude and arrangement and the performance of different thicknesses of uniformly designed and constructed asphaltic concrete, plain portland cement concrete, and reinforced portland concrete surfaces . . .

In addition to basic performance data, the AASHO Road Test also provided a tremendous opportunity to measure strains in concrete pavements under dynamic loads, and thus provide a mechanistic tie from these pavements to future designs.

Development of the AASHO Design Guide

As a correlary to the AASHO Road Test, the AASHO Operating Committee on Design appointed a working subcommittee on pavement design. The job of the subcommittee was to adapt the data from the AASHO Road Test to use in design procedures for asphaltic concrete pavements and portland cement concrete pavements. The committee's work on asphaltic concrete pavement is of no interest in this paper and will not be discussed further.

It was the unanimous opinion of the subcommittee that there are substantial factors to be considered in a design procedure which are not available as variables in the AASHO Road Test results. Four of these factors are:

- (1) The length of the time of the test relative to the normal life of the pavement being designed.
- (2) Climatic and geologic differences between the conditions at the Road Test site and other geographic regions.
- (3) The need for a guide in designing pavement types not included in the Road Test, such as continuously reinforced portland cement concrete pavements.
- (4) Expansion of the Road Test results to various other materials such as low modulus concrete and stabilized bases.

It was decided that the AASHO Road Test performance equations should form the basis for the AASHO Rigid Pavement Interim Design Guide to add these additional factors.

The Interim Design Guide was accepted by the AASHO Committee on Design in April 1962, and submitted to the states for one year of study.

The Interim Design Guide was developed as a guide for use in developing more exact design procedures.

The committee was very deliberate in its efforts to provide for future improvements in the work as additional information becomes available. The guide (Ref. 7) states that:

"The above design equations are based on fixed values for certain elements that are obviously important in the design of rigid pavement. These elements include thickness and quality of subbase, environmental effects, variations in the amount of load transfer at transverse joints, and the effects of joint elimination through continuous reinforcement. It is expected the design equations will be further modified in the future as experience is gained and these elements are evaluated."

The purpose of this paper is to show extensions to the Design Guide which have been developed for use by the Texas Highway Department in the design of concrete pavement.

II. PRESENTATION OF GUIDE

Scope of Guide

A complete and all encompassing design analysis of a rigid pavement structure would evolve into an elaborate study, to say the least, if all the parameters were considered. The term pavement structure as used here refers to the material placed on the subgrade to support the traffic load and distribute it to the roadbed as defined in the Interim Rigid Pavement Design Guide (Ref. 7). Therefore, the study of pavement structure design here is only a phase of the more complex problem of "highway design" that encompasses grade, alignment, etc.

Table 2.1 presents a detailed list of parameters that the authors feel should be incorporated into a rigid pavement structure analysis. The Rigid Guide presents a procedure that encompasses most of these parameters and allows the engineer to design the pavement structure from the subgrade up. Some of the design requirements are arrived at by formula while others are in the form of recommendations based on experience. Basically the Guide separates the design into four phases - slab dimensions, reinforcement, joints, and slab support control. The first two phases are handled by formula and will be discussed herein; whereas the latter two are handled in the form of recommendations and will not be covered.

Slab Dimensions - The Guide's approach to pavement structure design is a combination of theoretical and empirical relations. The design parameters covered by the various theoretical analyses discussed earlier are shown in column one of Table 2.1. Whereas, the final equation for the rigid pavement research phase of the AASHO Road Test encompassed the load application factor as well as the other parameter checked in column two of Table 2.1. In this latter case, the concrete properties, subgrade support and other design factors were fixed parameters and their effect cannot be evaluated by the AASHO Road Test equation.

TABLE 2.1

PARAMETERS OF PAVEMENT THICKNESS
COVERED BY VARIOUS METHODS OF DESIGN

Pavement Design Parameters	Theoretical Analysis	Road Test Equation	AASHO Design Equation
A. Loading Factor 1. Magnitude 2. Repetitions 3. Tire Pressure 4. Axle Type	X X	X X X	X X X X
B. Evaluation of Support Media 1. Strength 2. Volume Change 3. Quality	X		X
C. Concrete Properties 1. Strength 2. Modulus of Elasticity 3. Volume change properties 4. Poisson's Ratio	X X X		X X X
D. Continuity (Load Transfer)	X		X
E. Friction of Support Media			X
F. Regional Factors, i.e., Weather, Temperature, etc.			X

The AASHO Subcommittee for Rigid Pavement Design recognizing the merits of both the theoretical and the empirical, blended the two approaches into one equation. The parameters encompassed by the combined methods are checked in column three of Table 2.1. The development of this equation will be covered in a subsequent section.

Reinforcement - Steel reinforcement is placed in the slab for the purpose of holding any cracks that form in the pavement tightly closed, so that the pavement may perform as an integral structural unit. The Guide covers the design of two of the three basic types of reinforced concrete pavement, i.e., jointed reinforced and continuously reinforced (prestressed concrete pavement is not covered). Each of the two types requires an individual procedure.

The reinforcement for the jointed concrete pavement is determined by the application of the conventional "subgrade drag theory". In essence, the formula is based on the principle of balancing the slab's resistance to movement against the tensile strength of the steel.

The design method for continuously reinforced concrete pavement is based on the concept of balancing the internal concrete stresses developed by temperature and shrinkage against the tensile strength of steel (Ref. 10).

Development of Thickness Equation in Guide

Two general approaches were open for use in the Guide to combine the Road Test equation and theory. These being:

- (1) Use theoretical formulæ as the basic design form and modify by the load term in the final answer for repetitions.
- (2) Use the Road Test equation as a valid basis and add modifications from theory for variations in physical constants.

The second approach was selected as the more valid since it depends on the Road Test results for its starting

point and uses theory for determining variations in the basic equation. Also at the Road Test, failure was not defined as cracking (over-stress), but as a specific reduction in serviceability which usually did not occur until sometime after initial cracking. Such an approach is more realistic.

In order to evaluate the effect of variation in physical factors on the traffic life of the pavement, the Road Test performance data was used from a different angle. In addition, strain deflection, and condition survey data were also evaluated and utilized.

After a cursory examination of the information available, the Spangler equation was selected for use in the design equation because of its simplicity and because it showed a good correlation with Road Test measurements. It was stated in the Guide that, "one point of merit in this approach is that if a better stress equation is found, it can be incorporated into the design method with very little revision. . ."

After selecting the Spangler equation for modifying concrete properties, there were two possible choices for inserting it into the general AASHO equation: (1) obtaining a ratio of the selected concrete properties to those at the AASHO Road Test and making it an additive term to the AASHO equation, or (2) modifying the term in general equation to include various concrete properties. The committee selected the first alternative and derived the following equations:

When the terminal serviceability index (p) = 2.0:

$$\log W_t = 7.35 \log (D_2 + 1) + \frac{Gt}{\beta} - 0.06$$

$$+ 3.58 \log \left[\frac{S_c' (D_2^{0.75} - 1.132)}{690 \left(D_2^{0.75} - \frac{18.416}{20.25} \right)} \right]$$

When the terminal serviceability index (p) = 2.5:

$$\log W_t = 7.35 \log (D_2 + 1) + \frac{G_t}{\beta'} - 0.06$$

$$+ 3.42 \log \left[\frac{s_c (D_2^{0.75} - 1.132)}{690 \left(D_2^{0.75} - \frac{18.416}{z^{0.25}} \right)} \right]$$

Discussion of Design Charts

Design nomographs are presented in the Rigid Guide that solve for the thickness of jointed concrete pavement and the reinforcement requirements for both jointed and continuous concrete pavement.

Thickness - In deriving the nomograph for pavement thickness, the AASHO Road Test values for the modulus of elasticity and load transfer characteristics were fixed to solve the equation. This eliminates these factors as variables, hence the chart has variable scales only for traffic, working stress, and subgrade support. The chart, therefore, is not applicable to continuous concrete pavements or low modulus concrete pavements. Furthermore, the traffic scale is in terms of equivalent daily 18 kip single axle load applications for a 20 year traffic analysis. The daily traffic approach is restricting since the analysis is for fixed time period, and is difficult to use for other time periods or for evaluating the life of an existing pavement structure.

Reinforcement - The chart solution for reinforcement in jointed pavements is in graphic rather than nomographic form. The graphic solution has variable scales for pavement thickness, slab length, and working stress, but the graph is limited to the solution for a fixed friction factor.

The chart solution for reinforcement in continuously reinforced concrete pavement is flexible in that all the parameters involved in the design equation are included as variables on the nomograph.

III. DEVELOPMENT OF NEW EQUATION

The design equation developed for the AASHO Design Guide and discussed in Chapter II was a first attempt to utilize the AASHO Road Test data in pavement design. The equation is very cumbersome and several assumptions were made early in its development (Ref. 7). Furthermore, other refinements were omitted from that equation which would make it a more useable formula under actual conditions.

The purposes of this investigation are to:

- (1) Simplify the design equation if possible.
- (2) Investigate and clarify several of the assumptions made in the early development.
- (3) Include any additional refinements in the equation which can be developed from present data.

The equation developed herein has the following variations from the original equation presented in the Guide:

- (a) The Road Test stress data are used to verify the selection of a theoretical model.
- (b) Traffic is used as the total expected number of equivalent 18 kip application (ΣL) over the life of the pavement (design period).
- (c) The term for pavement continuity is evaluated and extended to continuously reinforced pavements.
- (d) The use of terms for both modulus of elasticity and subgrade modulus is encouraged.

Model Selection

In order to select a model for combining theory with Road Test performance data, the Road Test strain data

(Ref. 2) were compared with various modifications of Westergaard theory. The table in Figure 3.1 shows the equations which were examined and the correlation obtained. It can be seen that Spangler's equation does as good or better job of fitting the data than any of the more complicated equations.

It should be carefully noted at this point, that the data fitting does not support nor deny the theoretical formulation of \mathcal{L} (radius of relative stiffness), because none of the factors involved in the radius of relative stiffness, i.e., E, k, or μ , were varied at the Road Test in a manner allowing proper analysis.

After considering the fit of the data, the Spangler equation was selected because of its simplicity since it fit the data as well as any of the other equations. Figure 3.1 shows the correlation between Spangler, Westergaard, Pickett, and the Road Test stresses (calculated from corner load strains, Loop 1, AASHO Road Test (Ref. 2)). The following equation was selected as a result of the correlation.

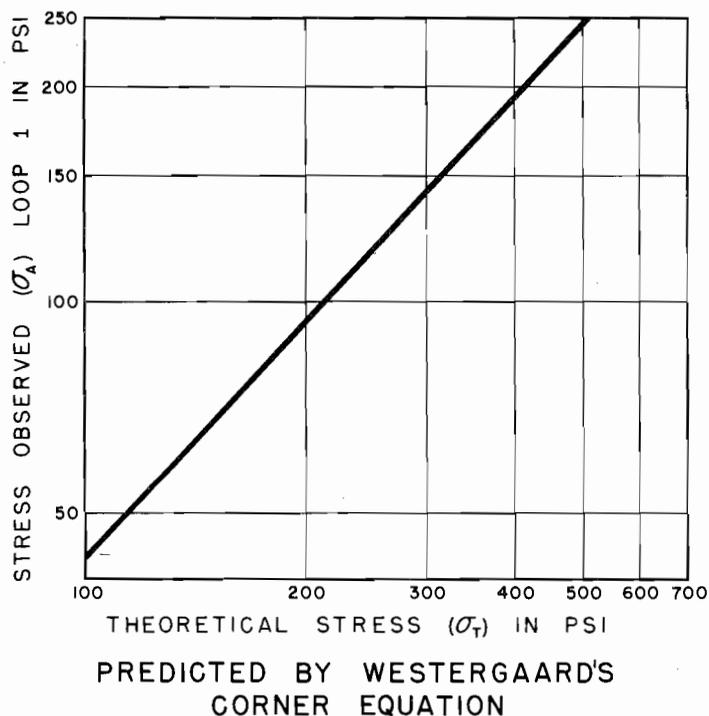
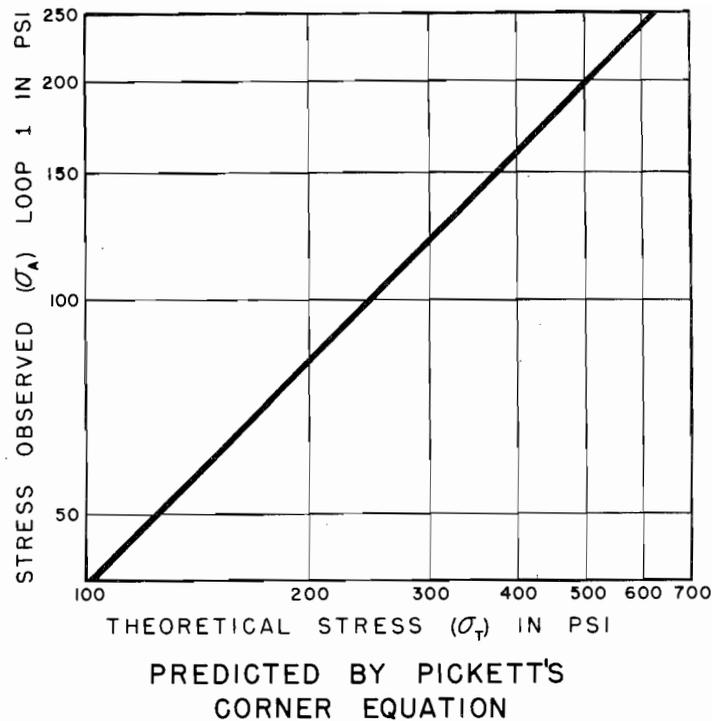
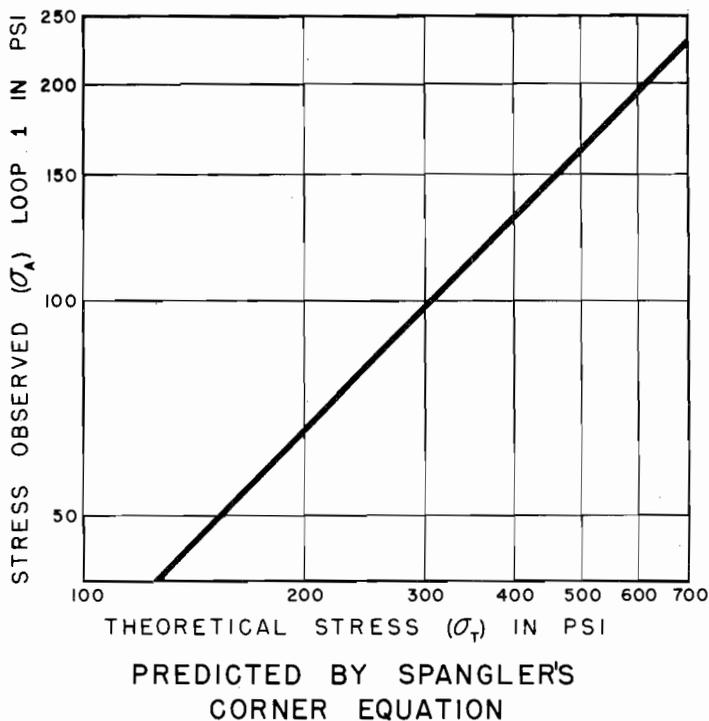
$$\text{Log } \sigma_{18} = 1.010 \log \sigma_{sp} - 0.521 \quad 3.1$$

where: σ_{18} = stress calculated from strains measured under an 18 kip single axle vibratory load on Loop 1, AASHO Road Test, psi.

σ_{sp} = stress predicted by the Spangler equation for a 9,000 pound wheel load (18 kip single axle), psi.

Modifying the Road Test Equation

After publication of the Road Test report, a study by Hudson and Scrivner (Ref. 8) showed excellent correlation between observed stresses at the Road Test, slab thickness and log W, i.e., the number of load applications carried. To extend the study and obtain a correlation of the form needed in this work, the writers correlated the



EQUATION FOR PREDICTING CORNER STRESS OBSERVED AT THE AASHO ROAD TEST IN TERMS OF A THEORETICAL STRESS:

$$\sigma_A = A\sigma_T^B$$

WHERE:

- σ_A = STRESSES OBSERVED LOOP 1 - AASHO ROAD TEST
- σ_T = THEORETICAL STRESSES
- A & B = CORRELATION CONSTANTS
- r^2 = CORRELATION COEFFICIENT

	A	B	r^2
SPANGLER'S EQUATION	.301	1.010	0.999
PICKETT'S EQUATION	.389	1.006	0.999
WESTERGAARD'S CORNER EQUATION	.309	1.078	0.999

COMPARING THEORETICAL STRESSES WITH THOSE OBSERVED ON LOOP 1 AT THE AASHO ROAD TEST

FIGURE 3.1

term (D + 1) with observed corner load stresses on the Road Test Loop 1 (Figure 3.2). The resulting equation 3.2 has a coefficient of determination (r^2) of 0.999.

$$\log (D + 1) = 1.995 - 0.517 \log \sigma_{18} \quad 3.2$$

Substituting equation 3.1 into equation 3.2 gives:

$$\log (D + 1) = 1.995 - 0.517 (1.010 \log \sigma_{sp} - 0.521)$$

combining terms,

$$\log (D + 1) = 2.264 - 0.522 \log \sigma_{sp} \quad 3.3$$

In Appendix B the Road Test equation is developed in terms of ΣL (accumulated equivalent 18 kip single axle loads).

The equation becomes:

$$\log \Sigma L = 7.35 \log (D + 1) - 0.06 + \frac{G}{\beta'} \quad 3.4$$

where: $G = \frac{4.5 - P_t}{3.0}$

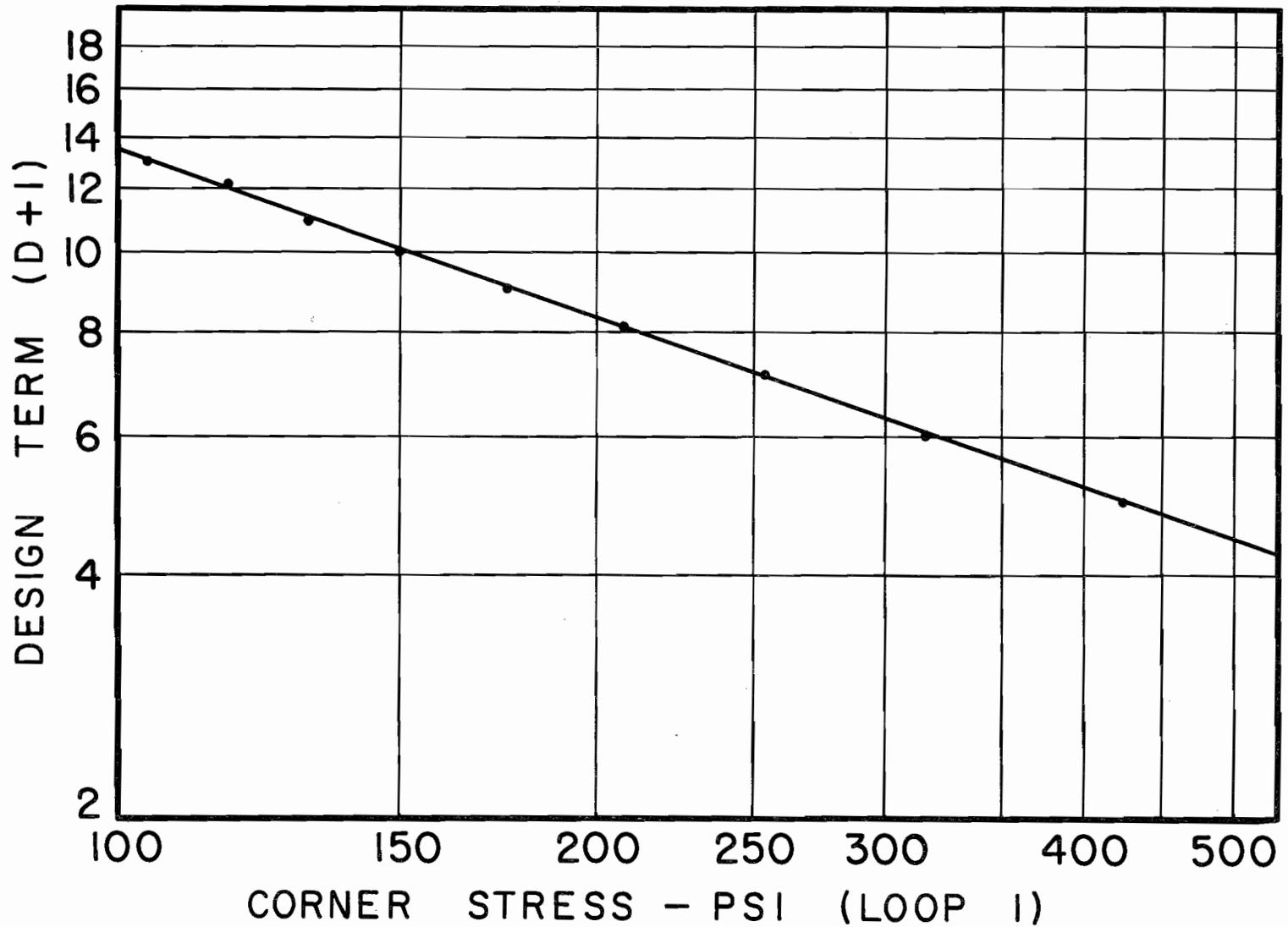
$$\beta' = 1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}} \quad 3.5$$

$$\log \rho = 7.35 \log (D + 1) - 0.06$$

P_t = serviceability at end of time, t.

In this equation β' is a curvature parameter, and ρ is a design function as shown when the equation is in the form:

$$G = \beta' [\log \Sigma L - \log \rho] \quad 3.6$$



CORRELATING DESIGN TERM (D+1)
WITH
CORNER LOAD STRESSES ON LOOP 1

FIGURE 3.2

This being the case, and because $(D + 1)$ exerts a large influence on $\log \Sigma L$ through the ρ term and only a weak influence through the β term, it was decided to substitute σ for $(D + 1)$ in the ρ term only. Therefore, substituting equation 3.3 into equation 3.4 we get:

$$\text{Log } \Sigma L = 7.35 \left[2.264 - .522 \log \sigma_{sp} \right] - 0.06 + \frac{G}{\beta} \quad 3.7$$

This equation obtains for pavements of a fixed strength, S_c , (28 day) for AASHO Road Test pavements was constant at 690 psi⁺ random variations. Previous design equations have relied on the σ/S_c ratio as the measure of adequate design. Work done for the AASHO Interim Rigid Pavement Design Guide related this ratio to pavement life in terms of $\log \Sigma L$. This can be stated as follows: "It can assumed that $\log \Sigma L$ is a function of the σ/S_c ratio; and that when an increased σ is matched by an increased S so that the ratio σ_x/S_x remains equal to the ratio σ/S_c , no change in ΣL would result. Therefore, the rate of change of ΣL as S_c changes is inversely proportional to the rate of change of $\log \Sigma L$ as σ changes. Inserting strength into equation 3.7 as such an inverse ratio with the fixed strength of the Road Test pavements (690 psi) we obtain:

$$\text{Log } \Sigma L = 7.35 \left[2.264 - .522 \log \left(\frac{\sigma_{sp} 690}{S_x} \right) \right] - 0.06 + \frac{G}{\beta} \quad 3.8$$

The Spangler equation for stress has the form,

$$\sigma_{sp} = \frac{J P}{D^2} \left(1 - \frac{a_1}{l} \right) \quad 3.9$$

Substituting the full Spangler equation, σ_{sp} , expanding and combining terms obtains:

$$\text{Log } \Sigma L = -9.483 - 3.837 \log \left(\frac{J}{S_x D^2} \left[1 - \frac{a_1}{l} \right] \right) + \frac{G}{\beta} \quad 3.10$$

where:

$$l = \left[\frac{Z D^3}{12 (1 - \nu^2)} \right]^{0.25} \quad 3.11$$

In order to simplify the design equation and without damage to the theory, Poisson's ration (ν) is fixed at a value of 0.20, resulting in a simplified form for the radius of relative stiffness:

$$l = \left[\frac{Z D^3}{11.52} \right]^{0.25} \quad 3.12$$

Taking $a_1 = a\sqrt{2}$ and substituting for l and a_1 , equation 3.10 becomes:

$$\text{Log } \Sigma L = -9.483 - 3.837 \log \left(\frac{J}{S_x D^2} \left[1 - \frac{2.61a}{Z^{1/4} D^{3/4}} \right] \right) + \frac{G}{\beta} \quad 3.13$$

ΣL = number of accumulated equivalent 18 kip single axle loads

J = a coefficient dependent upon load transfer characteristics or slab continuity

S_x = modulus of rupture of concrete at 28 days (psi)

D = nominal thickness of concrete pavement (inches)

$Z = E/k$

E = modulus of elasticity for concrete (psi)

k = modulus of subgrade reaction (psi/inch)

a = radius of equivalent loaded area = 7.15 for Road Test 18 kip axles

$$G = \frac{P_o - P_t}{3} = \frac{4.5 - P_t}{3}$$

$$\beta = 1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}}$$

At this point, a so-called life term must be inserted into the design equation. The life term will simply serve

to modify the life of a pavement section as predicted by the AASHO Road Test equation (a two year test). Studies of existing pavements in Texas and Illinois, among others, have established this fact. (It should be reiterated here that a substitution of the AASHO Road Test values for parameters in equation 3.13 would reduce it back to the basic Road Test equation.) Performance studies now being conducted in Texas have indicated that the logarithm of the predicted applications obtained by the Road Test equation must be reduced by a factor of 0.896. The AASHO Subcommittee on Rigid Pavement Design in effect reduced the logarithm of the predicted applications by a factor of 0.935 by using a safety factor (0.75 of the concrete strength for a working stress). Although the use of a safety factor to reduce the working stress is satisfactory, the use of a life term was adopted since future results of performance studies will undoubtedly provide a better estimate of the true factor and such values can be used to replace our trial value.

In determining the magnitude of the life factor both the Design Guide and the Texas performance studies were given equal consideration and an average factor of 0.9155 was selected.

Application of the life factor to the right side of equation 3.13 gives:

$$\text{Log } \Sigma L = -8.682 - 3.513 \log \left(\frac{J}{S_x D^2} \left[1 - \frac{2.61a}{Z^{1/4} D^{3/4}} \right] \right) + 0.9155 \frac{G}{\beta} \dots 3.14$$

where: all terms have been previously defined.

Only one term in equation 3.14 has not been evaluated adequately, the continuity or "J" term. The selection of a value, J, for design purposes must now be postulated on the basis of limited data. The J value for the jointed pavements on the AASHO Road Test is automatically fixed at the value of 3.2 which was used in all correlation work.

For the present time, this value shall be assumed to apply for all jointed concrete pavements with adequate load transfer. A J value of 2.2 was selected for continuously reinforced concrete pavements based on comparisons of previous design procedures and performance studies. This value also gives answers which are compatible with the recommendations in the AASHO Design Guide. Additional research is needed in this area.

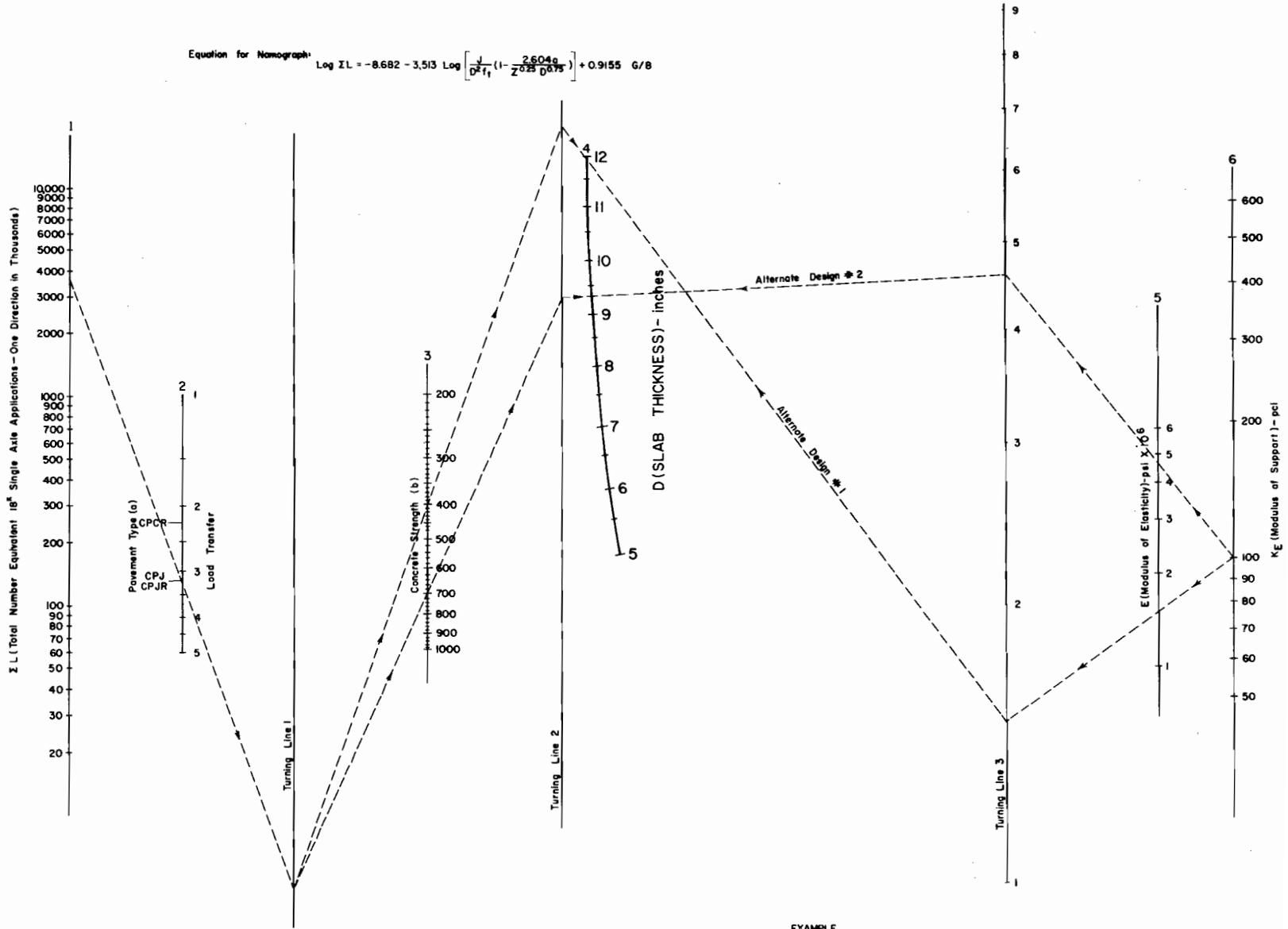
Graphical Solution

This equation is cumbersome; it being particularly hard to solve for thickness of concrete pavement (D). It is a very simple matter, however, to program this equation on a computer and solve for ΣL using all combinations of the other variables. The resulting output is useful in the form of tables (100 pages of computer typeout). These tables can be combined graphically into a very useful nomograph such as Figure 3.3. The nomograph is for a final serviceability level of 2.5. Evaluation of terminal serviceability throughout the United States has shown that an acceptable level for final or terminal condition of an Interstate pavement is 2.2 - 2.5. The Texas Highway Department has settled on 2.5 for use in design of such pavements. For design of lower class roads a terminal serviceability of 1.5 is felt to be satisfactory.

Use of the Nomograph

The examples on the chart show how typical design problems may be handled. Certain information is normally fixed by the conditions at the site or by arbitrary choice.

- (a) $\Sigma L = 3,650,000$ applications is an estimate of the traffic to be carried during the life of the proposed pavement. It should be established by statistical prediction procedures from study of past loadometer and traffic count data. (The methods used by the Texas Highway Department may be found in Reference 9.)
- (b) $k_E = 100$ pci is established by the existing subgrade plus some evaluation of the improvement that will be gained by the subbase (see Appendix A).



**MODIFIED
AASHO DESIGN CHART
RIGID PAVEMENTS
PRIMARY HIGHWAYS**
(p=2.5)
JANUARY
1964

EXAMPLE

Given:
 W = 3,650,000 applications
 Pavement Type—CPJ
 $K_E = 100$ pci

Alternate Design 1
 Concrete with: $f'_c = 400$ psi
 $E = 1.5 \times 10^6$ psi
 D required = 11.8 inches

Alternate Design 2
 Concrete with: $f'_c = 690$ psi
 $E = 4.5 \times 10^6$ psi
 D required = 9.3 inches

NOTE:

(a) Pavement Type:
 1. CPR—Concrete Pavement Continuously Reinforced
 2. CPJ—Concrete Pavement Jointed having adequate load transfer at all joints.
 3. CPJR—Concrete Pavement Jointed Reinforced having adequate load transfer at all joints.

(b) Based upon 28 day test, ASTM 78-59 (Third point loading)

(c) E to be determined at 28 days

FIGURE 3-3

- (c) Pavement type, CPJ, jointed plain concrete pavement with load transfer at the joints. This factor may be chosen by the designers and varied for several different designs. Often, however, the choice is dictated by other existing factors as assumed for this example.

With these factors provided it is appropriate to establish the value of each factor on its respective scale and proceed as follows:

- (1) Mark ΣL on scale #1.
- (2) Mark pavement type on scale #2.
- (3) Mark k_E value on scale #6.
- (4) Taking $E = 1.5$ in anticipation of using low modulus shell concrete for the first trial, mark 1.5 on scale #5 as shown.
- (5) Connect the points on scale #1 and scale #2 projecting the line to a point on turning line 1.
- (6) Select a trial concrete strength (400 psi) and mark it on scale #3.
- (7) Connect this point on scale #3 to the intersection on turning point 1 and extend it to a point on turning line 2.
- (8) Transferring over the scales #5 and #6, connect the points on these scales and project to turning line 3.
- (9) Connect turning points 2 and 3 to establish the required thickness $D = 11.8$ inches on scale #4.

It may often be desirable to check alternate designs. Another example is shown on the design chart, which using different concrete characteristics and following the same procedures yields $D = 9.3$ inches.

The choice of final design should rest on the economics involved among the possible choices. Which pavement provides the most performance for the lowest unit costs?

IV. MODIFICATION OF REINFORCEMENT DESIGN

In this chapter a simple procedure is presented whereby a designer can go from his design parameters of pavement thickness, friction factor, joint spacing, concrete strength, etc. to a required steel percentage and thence to the bar size and spacing to fulfill these requirements.

Jointed Reinforced Concrete Pavement

It was mentioned previously that friction factor was not included as a variable in the AASHO Design Guide nomograph for determining the reinforcement in jointed concrete pavement. The nomograph was solved for a fixed friction factor of 1.5. This was an adequate premise during the period when sand cushion blankets were used between the pavement and the subbase, but the current trend toward crushed stone or stabilized subbases emphasizes the need for considering friction factor in design. Experiments performed by the Texas Highway Department have shown friction factors in excess of two. If the Guide's nomograph was used to design the reinforcement for a high friction subbase, an inadequate design would result.

In addition to inserting friction factor into design, the authors feel the solution for the reinforcing requirements can best be expressed as a percentage in lieu of the current concept of using the area of steel per foot of slab width. The latter designation is satisfactory for estimating purposes, but is difficult to comprehend from a design standpoint. Furthermore, when the solution is expressed as a percentage, the values are comparable and compatible with the solutions obtained with continuously reinforced concrete pavement. By simply changing the designation of several expressions in the Guide, the answer for the "subgrade drag" would be in terms of percentage as follows (for the mathematical solution see Appendix C):

$$P_s = \frac{L F}{2 f_s} \times 100 \quad 4.1$$

where: P_s = required steel percentage, per cent
 L = length of slab between joints, feet
 F = friction factor of subbase
 f_s = allowable working stress in steel, psi.

Figure 4.1 presents a nomograph for solving equation 4.1. Note the flexibility provided in that the working stress can be varied between wide limits in addition to including friction factor as a variable. The inclusion of a complete scale for working stress in lieu of several fixed values allows the designer to apply any desired value.

In addition, the designer has added flexibility because he may use the two scales on the right to either select the steel type or grade and determine the resulting required steel percentage, or select an optimum steel percentage and determine the steel type.

Continuously Reinforced Concrete Pavement

The equation and nomograph presented in the Guide for the steel requirements of continuously reinforced concrete pavement have not been altered. For the purpose of convenience the nomograph is reproduced here as Figure 4.2.

Steel Size and Spacing Requirements

The solutions for both the jointed reinforced concrete pavement and continuously reinforced concrete pavement are expressed as a percentage. The next step in design after determining the steel percentage is to determine the bar spacing and size needed to fulfill the required percentage. The equation for solving for bar spacing is:

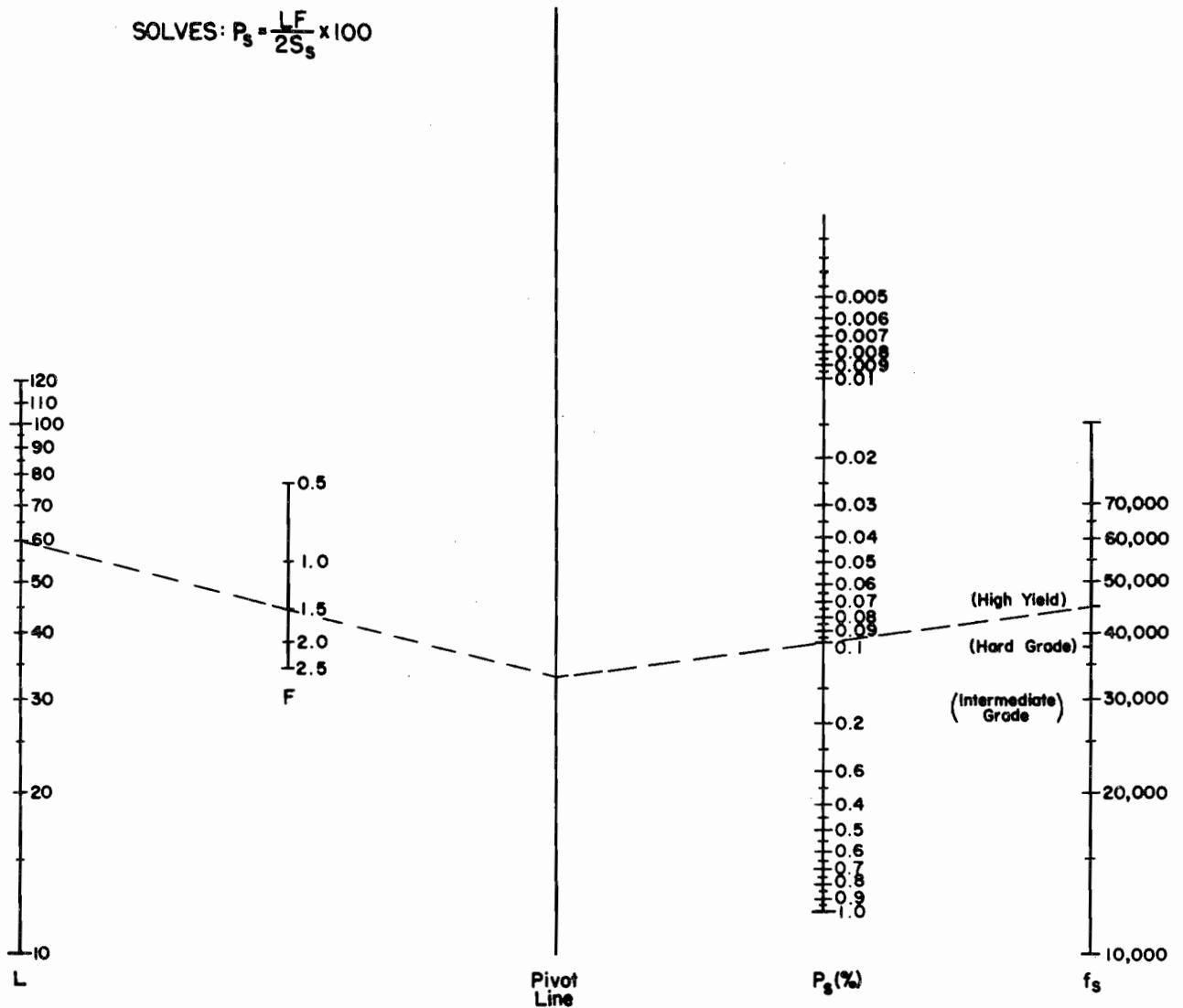
$$y = \frac{A_b}{D P_s} \times 100 \quad 4.2$$

where:

y = bar or wire spacing, center to center, inches

NOMOGRAPH

SOLVES: $P_s = \frac{LF}{2S_s} \times 100$



Example Problem:
 L = 60 ft
 F = 1.5
 $f_s = 45,000$ psi
 Answer: $P_s = 0.2\%$

Where:
 P_s = Required steel percentage — %.
 L = Length of slab between joints — feet.
 F = Friction factor of subbase.
 f_s = Allowable working stress in steel — psi.
 (0.75 of yield strength recommended,
 the equivalent of safety factor of 1.33)

FIGURE 4-1

DISTRIBUTED STEEL REQUIREMENT FOR LIGHT REINFORCED JOINTED CONCRETE PAVEMENTS

NOMOGRAPH

$$\text{SOLVES: } P_s = (1.3 - 0.2F) \frac{S'_c}{f_s - NS'_c} \times 100$$

ASSUMED: $E_s = 30,000,000$

EXAMPLE PROBLEM

$E_c = 4,000,000$

$S'_c = 260$

$f_s = 45,000$

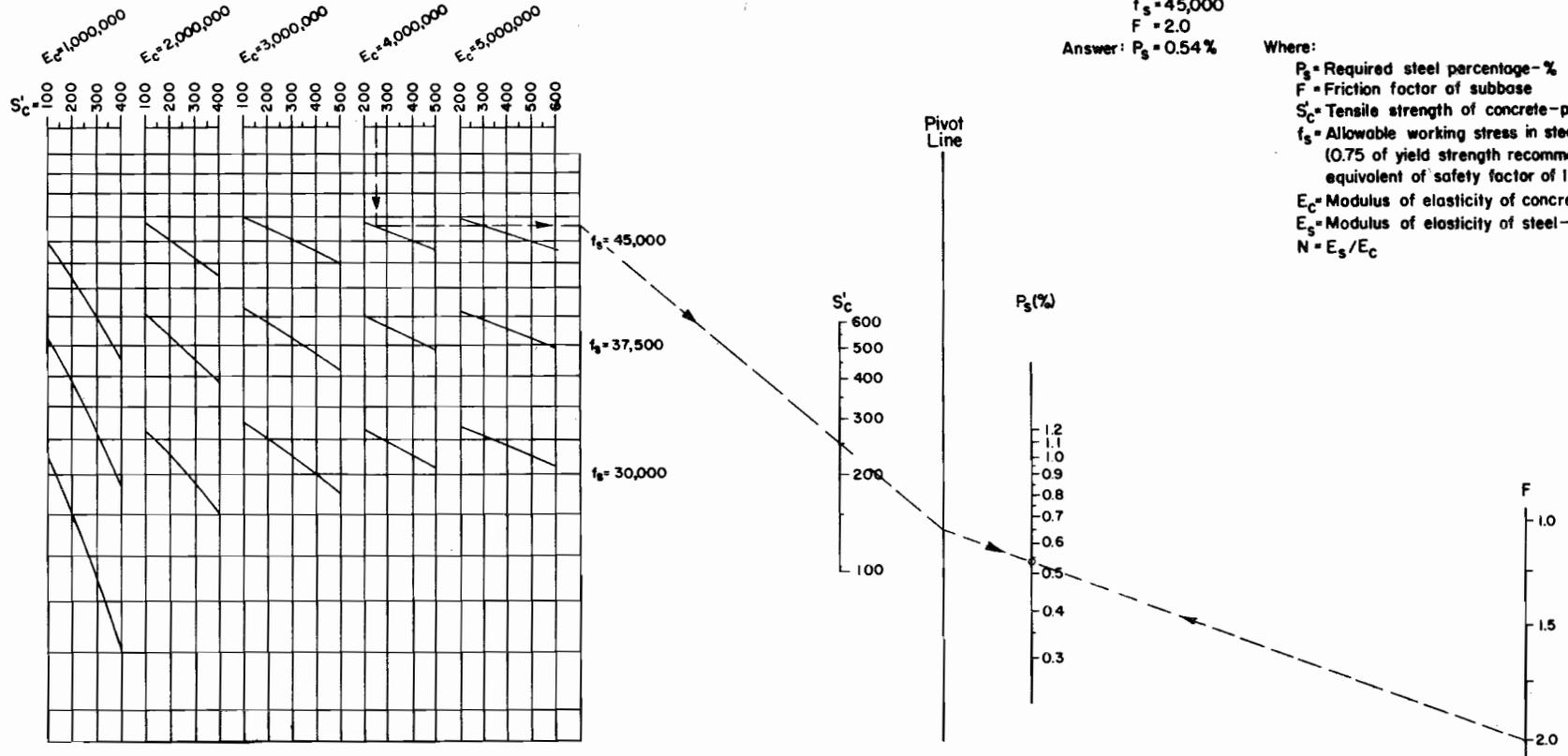
$F = 2.0$

Answer: $P_s = 0.54\%$

Where:

- P_s = Required steel percentage-%
- F = Friction factor of subbase
- S'_c = Tensile strength of concrete-psi
- f_s = Allowable working stress in steel-psi
(0.75 of yield strength recommended, the equivalent of safety factor of 1.33)
- E_c = Modulus of elasticity of concrete-psi
- E_s = Modulus of elasticity of steel-psi
- $N = E_s/E_c$

25



LONGITUDINAL STEEL REQUIREMENT FOR CONTINUOUSLY REINFORCED CONCRETE PAVEMENT

FIGURE 4.2

A_b = cross sectional area of bar or wire, square inches

D = pavement thickness, inches

P_s = required steel percentage, per cent

Figure 4.3 portrays a nomograph solution of this equation. By using the variable scales on the right side of the nomograph, the designer can readily obtain several combinations of bar spacings and sizes which meet the steel percentage requirements.

NOMOGRAPH

SOLVES: $Y = \frac{A_B}{D \cdot P_S} \times 100$

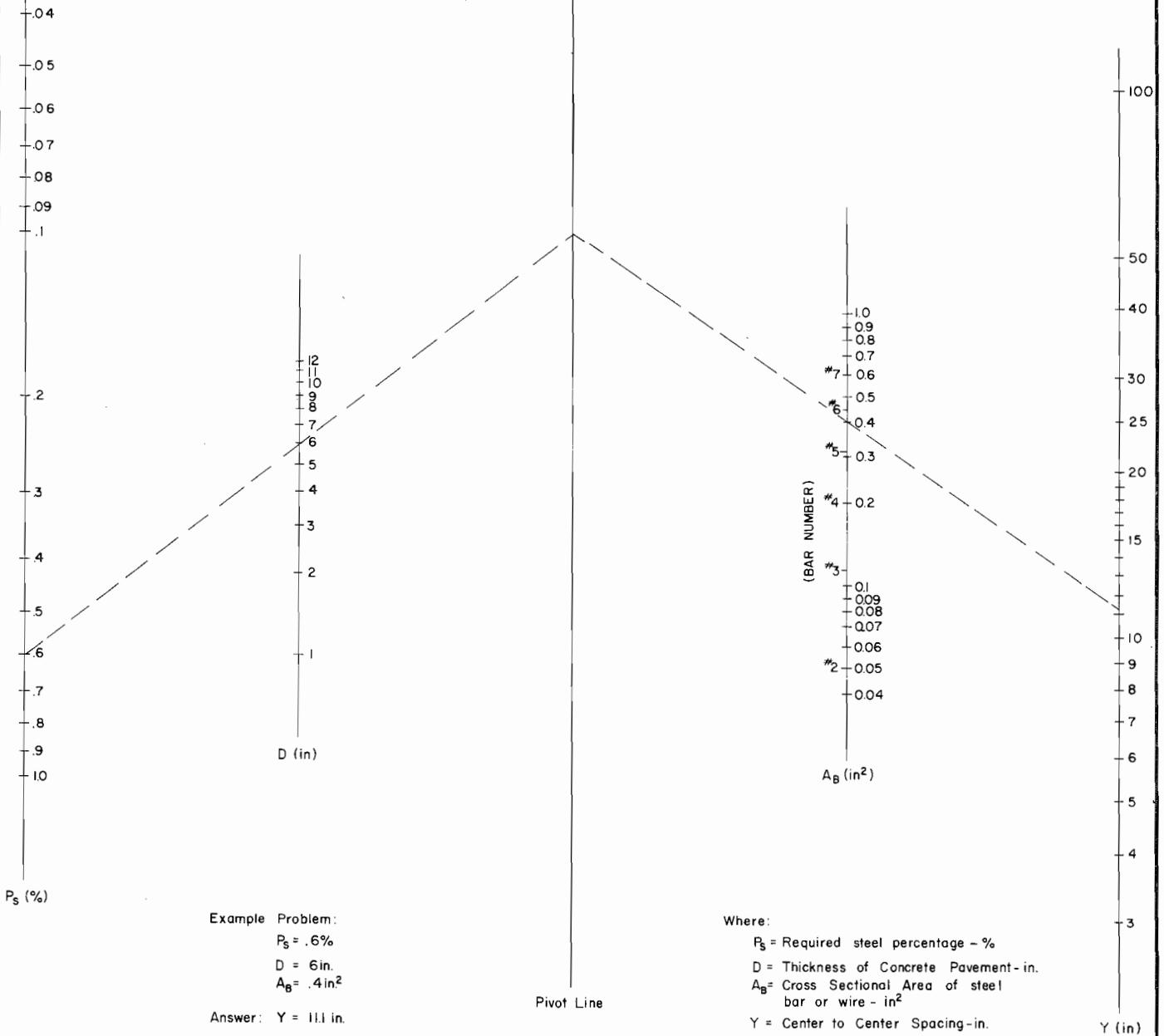


FIGURE 4.3

STEEL SPACING FOR REINFORCED CONCRETE PAVEMENT

V. SUMMARY

A. Conclusions

(1) Based on an analysis of stresses "observed" at the AASHO Road Test, the Spangler simplification of the Westergaard stress equations fits the Road Test pavements as well as the more complicated Westergaard or Pickett equations. The use of this equation as a stress model in design is therefore justified.

(2) A design equation relating load applications to pavement design factors including modulus of elasticity and slab continuity can be developed through the relationship of stress to slab thickness and load applications observed at the AASHO Road Test.

(3) The complicated design equation involving load applications, modulus of rupture, modulus of elasticity, slab continuity, modulus of subgrade reaction, thickness of slab, and pavement performance can be usefully displayed as a nomograph using general computer solutions of the equations.

(4) The evaluation of all variables and constants are reasonably well founded except for the value of the life term and slab continuity. Continued observations on existing pavements will help verify these effects.

(5) By use of a series of nomographs, the steel reinforcing requirements, i.e., bar size and spacing, for the design conditions of either jointed reinforced concrete pavement or continuously reinforced concrete pavement can be determined with several simple manipulations.

(6) The design charts developed herein allow the designer to consider numerous variables that were not accounted for in previous design methods. Hence, more flexibility is given the designer to arrive at the most economical design.

B. Needed Research

The design methods reported herein are intended to represent the best use of available knowledge concerning

portlant cement concrete pavements. They are not presented as anything more than empirical approximations of the true phenomena involved. The authors are continuing their research into this problem and hope that other will continue to investigate various aspects, some of which are discussed below.

Powerful computational techniques are becoming available with large computers. These methods along with the wealth of experimental data which is being accumulated should enable us to push back the frontier of knowledge of pavement performance. Specifically additional information is needed to evaluate a variable termed "subbase quality" (Q). This variable is related to the load carrying capacity, but must also evaluate the ability of the subgrade to maintain its integrity under repeated applications of the load. The search should also continue to develop a meaningful environment factor (\overline{RF}), a function of weather and other environmental conditions. This term would of course include the curling and warping effects of temperature and moisture differentials. A more complete design equation will be available after these terms can be added.

In addition to these two variables which are not included in the design equation developed herein, a great amount of work remains for the verification of the following parameters:

- (1) J, a function of slab continuity, load conditions, and jointing procedures.
- (2) ℓ , radius of relative stiffness, a function of E, K, and D. The present application of these factors is based on elastic theory. It can immediately be noted that K is far from elastic and additional study is warranted.
- (3) $\log \Sigma L$, several satellite studies designed to extend and verify the AASHO Road Test equations are in various stages of planning at the present time. Such studies are vital to the solution of this problem.

C. Method of Proposed Research

In addition to Road Test satellite studies which are considered to be vital to the solution of this problem, at least two other avenues of research must be exploited.

There is an immediate need for the development of more adequate and versatile methods of analysis which will permit the extension of the available solutions past the simplified special-case solutions developed by Westergaard in 1925. Particular attention is needed for dynamic loadings. Such research is presently in the planning stage.

A second need is that of developing additional information concerning the effects of dynamic loads on the so-called elastic material properties. For example, it would be desirable to study the true inter-relationships of dynamic load applications and other design variables, particularly those involving material properties, modulus of elasticity, modulus of subgrade reaction, Poisson's ratio, and flexural strength of concrete. There is sufficient proof available from the AASHO Road Test to indicate that such a study is both physically and economically feasible by employing a vibrating loader similar to that introduced at the Road Test (Ref. 2 & 8).

The Road Test strain-performance studies provide a basis for extending the Road Test performance equations to include additional design variables, for example:

- (1) modulus of elasticity of concrete (E)
- (2) flexural strength of concrete (S_c)
- (3) joint type and arrangement
- (4) subbase and subgrade characteristics (k)
- (5) slab loading conditions (continuity)

Studies could be set up to verify the theoretical design assumptions under dynamic load by building a set of small pavements of several thicknesses and designs to include the desired variables. These sections would be one or two slabs in length. The continuous section would have to be especially built to provide proper continuity conditions. The subbase characteristics could be studied

directly by including subbase as a variable. Loading for the test could be provided by a dynamic loader such as the one used on the Road Test.

In order to correlate these test sections with the Road Test results, control sections as nearly like the Road Test sections as possible would be constructed and instrumented with strain gages and deflectometers. The performance of these control sections would be compared directly with the Road Test performance equations. The control correlations could be developed from strains, deflections, and/or repetition histories of the reproduced test slabs.

These test slabs would meet the requirements for controlled satellite studies, but would not require expensive traffic, nor involve failures on the highway system. They would, however, provide us with an orthogonal, controlled experiment involving rigid pavements at a relatively low cost. All necessary analyses of the mechanics of this system should be done as a part of this project and the mathematics of the original theories could be tested.

The AASHO Road Test would serve as reference sections for pavement performance. Neither the research agency, nor the research location would be limited in these studies. The development of standard dynamic loading equipment and standard test procedure would enable nationwide correlation of data in this field. It would also provide a means to correlate experiments conducted with this general format anywhere in the world, as long as certain basic information concerning the test sections were available.

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APPENDIX A

Miscellaneous Data

APPENDIX A
 CHARACTERISTICS OF MATERIALS - RIGID PAVEMENT
 AASHO ROAD TEST

(1) Portland Cement Concrete

Item	Pavement Thickness	
	5 inches & Greater	2 1/2 & 3 1/2 Inches
Design Characteristics:		
Cement content ¹ , bags/cy	6.0	6.0
Water-cement ratio, gal/bag	4.8	4.9
Volume of sand, % total ag vol	32.1	34.1
Air content, per cent	3-6	3-6
Slump, inches	1 1/2--2 1/2	1 1/2--2 1/2
Maximum aggregate size ² , in.	2 1/2	1 1/2
Compressive Strength, psi:		
14 days	4,000	4,000
1 year	5,600	6,000
Flexural Strength, psi:		
14 days	640	670
1 year	790	880
Static Modulus of Elasticity: (10 ⁶ psi)		
	5.25	5.25
Dynamic Modulus of Elasticity: (10 ⁶ psi)		
	6.25	5.87

¹Type I cement was used.

²Uncrushed natural gravel.

(2) Subbase Materials

Item	Subbase
Aggregate gradation, Per Cent Passing:	
1 1/2 inch sieve	100
1 inch sieve	100
3/4 inch sieve	96
1/2 inch sieve	90
No. 4 sieve	71
No. 40 sieve	25
No. 200 sieve	7
Plasticity Index, minus No. 40 material	N.P.
Max. dry density, pcf	138
Field density, as Per Cent Compaction	102
Gross modulus of subbase reaction, k, psi/in.	60
Elastic modulus of subbase reaction, k, psi/in.	108

Formulae for converting principal strains to principal stresses are give below:

$$\sigma_x = \frac{E}{1 - \mu^2} (\epsilon_x + \mu\epsilon_y)$$

$$\sigma_y = \frac{E}{1 - \mu^2} (\epsilon_y + \mu\epsilon_x)$$

where:

σ = principal stress in direction designated, psi.

ϵ = strain in direction designated, in/in.

E = modulus of elasticity, psi.

μ = Poisson's ratio

References:

William M. Murray and Peter K. Stein, "Strain Gage Techniques", MIT, Cambridge, Mass., 1956, pp. 537-548.

S. Timoshenko and J. N. Goober, Theory of Elasticity, McGraw-Hill Book Co., 1951, pp. 24.

INFORMATION ON DETERMINATION OF K FOR USE WITH THE
RIGID PAVEMENT DESIGN EQUATIONS

K (modulus of support) as used on the AASHO Design Chart for Rigid Pavements is somewhat smaller in magnitude than the K value engineers have been accustomed to working with. The K normally used in rigid pavement design is usually the so-called elastic K. The K used as a basis for development of the AASHO Design Guide for Rigid Pavements is the "gross" K. The "gross" K is smaller than the "elastic" K because the total deflection of the plate is considered in the calculations.

The elastic K was used in this development since its values are generally in the range that engineers are familiar with and it comes closer to duplicating the original Westergaard assumptions. Therefore, when comparing the results of the design charts with the AASHO Design Charts, this difference in the K value should be taken into consideration. The studies at the AASHO Road Test showed the following correlation between the two K values:

$$K_E = 1.77 K_G$$

where:

K_E = elastic modulus of support, pci

K_G = gross modulus of support, pci

The problem of determining a K for use in rigid pavement design is compounded by other factors. The ability of a material to maintain its K or its integrity over the life of the pavement is also important in selecting the design value of K for the material. As an indication of what these values might be, we can look at the supporting materials used at the AASHO Road Test. The basic subgrade material was an A-6 clay Texas Triaxial Class 5.6. When

used directly this material had a gross K of 20 to 30. A subbase material was provided for most of the sections of the AASHO Road Test. This subbase material was a sandy gravel, Texas Triaxial Class 3.7. Six to nine inches of this material resulted in a gross K value of 50 to 75. The average was taken to be 60 pounds per cubic inch or 108 pounds per cubic inch for elastic K. Basically, the AASHO Road Test Design Guide is based on the performance of these sections. Those sections which had no subbase at all and thus had a gross K value of about 25 showed considerably poorer performance than those sections which had a subbase for protection.

From this information it appears that for use with the Guide, we might expect an elastic K of 100 to 200 pci from good granular subbases about six inches thick and an elastic K of 200 to 400 from stabilized material about six inches thick which have proven satisfactory in service.

Reference: "The AASHO Road Test - Pavement Research", Highway Research Board Special Report 61E, 1961.

PROCEDURE FOR DETERMINING THE MODULUS OF SUPPORT (k)

The following is a simple procedure for determining the modulus of subgrade reaction (k). It is the procedure used to determine k at the AASHO Road Test and is suggested for use with the Design Guide.

AASHO ROAD TEST - Plate Load Tests

A. Equipment

The basic equipment consists of: (1) a reaction trailer, (2) a hydraulic ram and a jack, (3) various heights of steel spacers for use where required by various depths of test, (4) a 12-inch diameter cylindrical steel loading frame cut out on two sides to allow the use of a center deflection dial, (5) a spherical bearing block, (6) a series of one inch thick steel plates that are 12, 18, 24, and 30 inches in diameter, and (7) a 16-foot aluminum reference beam. A schematic diagram of the apparatus is given in Figure 1.

A trailer of the flat-bed type, having no springs and four sets of dual wheels on the rear can be used as the reaction trailer. A cantilever beam protruding from the rear of the trailer is used as the reaction. The distance from the load to the rear wheels should be eight feet. A maximum reaction of about 12,000 lb. could be obtained with a 17,000-lb. loaded rear axle.

A standard hydraulic ram is used to apply the load. A calibration curve, which should be checked periodically, is used to convert gage pressures to load in pounds.

The load is applied to the plates through the 12 inch diameter steel loading frame and the spherical bearing block. The deflection is measured with a dial gage as shown in Figure A.1.

The weight of the loading frame and the plates is allowed to act as a seating load for which no correction should be made.

B. Test Procedure

Tests are made in areas about three to four feet wide. The procedure provides for the application and release of 5, 10, and 15 psi loads on a 30 inch plate and for measurement of the downward and upward movement of the plate. The loads are applied slowly with no provision for the deformation to come to equilibrium.

Basic steps in the procedure are:

- (1) The test area is covered with fine silica sand and leveled by rotating the plate.
- (2) The equipment is set in place (Figure A.1).
- (3) A seating pressure of 2 psi is applied and released. The dial gages are then set to zero.
- (4) The first increment of pressure is applied and held for fifteen seconds, then the dial gage is read.
- (5) The load is then released and the dial gage read at the end of a fifteen second period.
- (6) The load is reapplied and released in the same manner three times and readings are taken each time.
- (7) Steps 4 through 6 are repeated for the second (10 psi) and the third increment (17 psi) of load.
- (8) The gross and elastic deflections are computed from the dial gage readings.

C. Computation of Modulus of Support

- (a) The gross k value, k_g , equals the unit load divided by the maximum gross deflection obtained after three applications of a given unit load. The reported k is then an average of the computations for each of the unit loads.
- (b) The elastic k value, k_e , equals the unit load divided by the elastic deformation at each application of each incremental load. The reported k_e is an average of all nine of

these computations (3 loads x 3 applications each). The elastic deformation is equal to the difference between the maximum gross deflection and the final reading on the dial gage.

(c) $k_e = 1.77 k_g$ describes the relationship between the two k values as developed through correlation from numerous tests on the AASHO Road Test.

k values are reported as pounds per cubic inch.

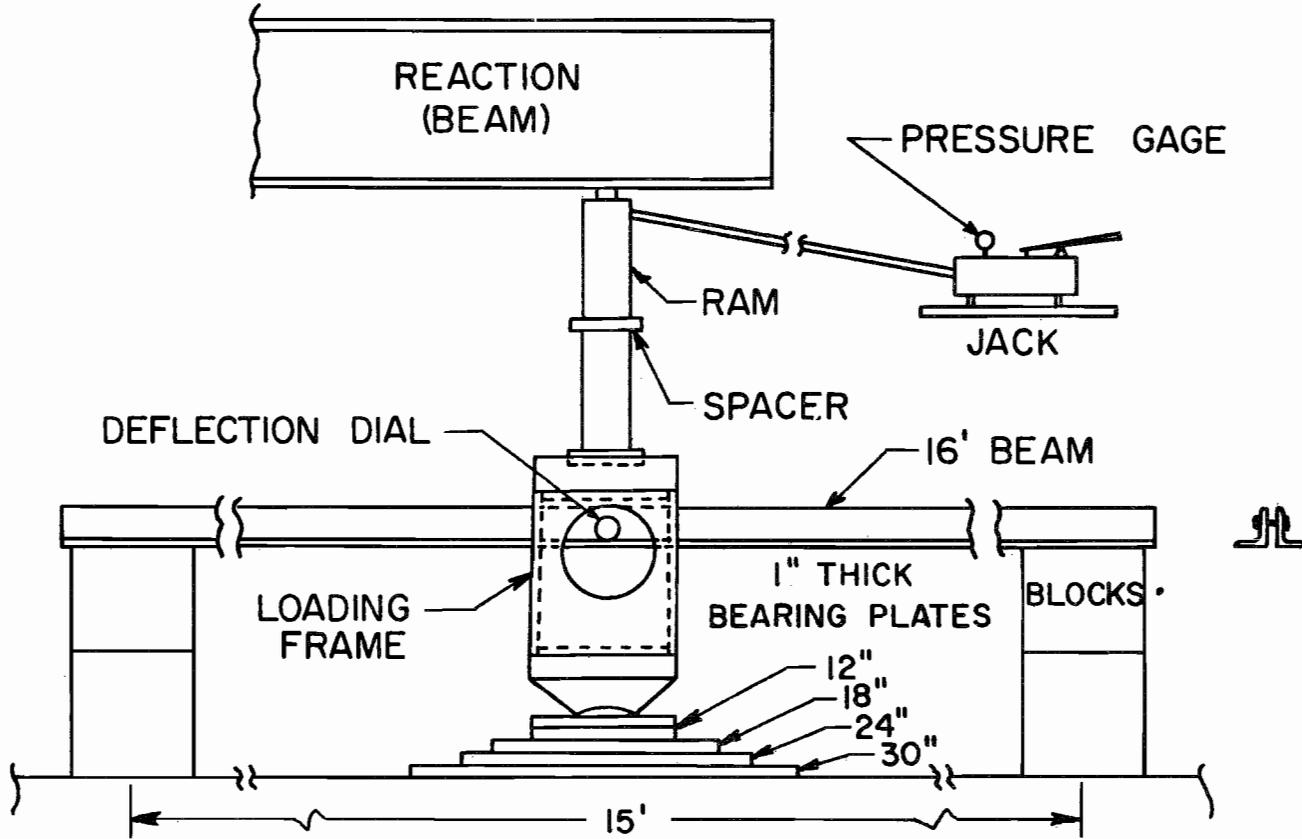


FIG. A.1 - APPARATUS FOR PLATE LOAD TEST

APPENDIX B

Derivation of Design Equations

APPENDIX B

AASHO ROAD TEST RIGID PAVEMENT PERFORMANCE EQUATION

Analyses were based on the following mathematical model:

$$\text{Log } W_t = \log \rho + \frac{G_t}{\beta}$$

where:

G_t = a function (the logarithm) of the ratio of loss in serviceability at time t to the total potential loss taken to the point where $p = 1.5$.

$$G_t = \log \frac{c_0 - p_t}{c_0 - 1.5}$$

p_t = present serviceability index value at time t.

c_0 = a constant related to initial condition of pavements in the Road Test.

$c_0 = 4.5$ for rigid pavements

$$c_1 = \text{a constant} = \frac{c_0 - 1.5}{c_0}$$

where 1.5 = level of serviceability when sections were removed from the test.

$c_1 = 0.667$ for rigid pavements

W_t = number of cumulative axle applications at time t.

ρ = a function of design variables and load variables that denotes the expected number of axle load applications to a serviceability index value of 1.5.

β = a function of design and load variables that influences the shape of the p vs W serviceability curve for a pavement.

Evaluation of the terms by the analysis techniques used at the Road Test gave the following equations:

$$\log \rho = 5.83 + 7.35 \log (D_2+1) - 4.62 \log (L_1+L_2) + 3.28 \log L_2$$

$$\beta = 1.00 + \frac{3.63 (L_1+L_2)^{5.20}}{(D_2+1)^{8.46} L_2^{3.52}}$$

Reference:

"Second Preliminary Report on the AASHO Road Test" to the Bureau of Public Roads, May 8, 1961. By AASHO Road Test staff, HRB, NAS-NRC.

CALCULATIONS OF β AND ρ

Where: $L_1 = 18$

$L_2 = 1$

$$\beta = 1 + \frac{3.63 (L_1 + L_2)^{5.20}}{(D_2 + 1)^{8.46} L_2^{3.52}}$$

$$\beta = 1 + \frac{3.63 (19)^{5.20}}{(D_2 + 1)^{8.46}}$$

$$\beta = 1 + \frac{(4.464) (3.63) \times 10^6}{(D_2 + 1)^{8.46}}$$

$$\beta = 1 + \frac{16.20 \times 10^6}{(D_2 + 1)^{8.46}}$$

$$\rho = \frac{10^{5.85} (D_2 + 1)^{7.35} L_2^{3.28}}{(L_1 + L_2)^{4.62}}$$

$$\text{Log } \rho = 5.85 + 7.35 \log (D_2+1) + 3.28 \log (1) - 4.62 \log 19$$

$$\text{Log } \rho = 5.85 + 7.35 \log (D_2+1) - 5.908$$

$$\text{Log } \rho = 7.35 \log (D_2+1) - 0.058$$

$$\text{Hence: } \text{Log } W = \log \varrho + \frac{\log 4.5 - P_t}{\beta}$$

becomes:

$$\text{Log } W = 7.35 \log (D_2+1) - 0.06 + \frac{G}{1 + \frac{16.20 \times 10^6}{(D_2 + 1)^{8.46}}}$$

APPENDIX C

Derivations for Reinforcement

EXPRESSING STEEL REQUIREMENTS FOR JOINTED CONCRETE
PAVEMENT AS A PERCENTAGE

The AASHO Rigid Pavement Design Guide uses the conventional subgrade drag theory for calculating the required steel percentage of reinforced jointed concrete pavements. This formula is expressed as follows:

$$A_s = \frac{F L W}{2 f_s} \quad \text{C.1}$$

A_s = cross sectional area of steel, sq. in/ft of slab width

F = friction coefficient of subbase

L = slab length, feet

f_s = allowable working stress of steel, psi.

W = weight of slab/sq. foot

The resulting answer for this analysis is in the units of square inches per foot of slab width. These units are not compatible with the generally current accepted practice of expressing the steel requirements as a percentage. Therefore, the following derivation was made in order to obtain the answer as a percentage.

The expression for percentage is:

$$P_s = \frac{A_s'}{A_c} \times 100 \quad \text{C.2}$$

where:

P_s = required steel percentage, %

A_s' = cross sectional area of steel, in²

A_c = cross sectional area of concrete, in²

Equation C.1 can be changed to the total required steel area by multiplying by width:

$$A_s' = \frac{F L W}{2 f_s} \cdot Z \quad C.3$$

The W term is simply a combination of the pavement thickness and concrete density:

$$W = D \cdot w \quad C.4$$

where:

D = pavement thickness, inches

w = concrete unit weight, #/cubic foot

Combining equations C.3 and C.4:

$$A_s' = \frac{F L D w Z}{2 f_s} \quad C.5$$

Rearranging terms:

$$\frac{A_s'}{D \cdot Z} = \frac{F L w}{2 f_s} \quad C.5$$

The left hand side of above term is the ratio of steel area to concrete area, therefore:

$$P_s = \frac{F L w}{2 f_s} \times 100 \quad C.6$$

To be dimensionally correct:

$$P_s = \frac{F L w}{2 \cdot 144 f_s} \times 100 \quad C.6$$

The unit weight of concrete is generally taken as 145 - 150 pounds/cubic foot; therefore, equation C.6 is approximately equal to:

$$P_s = \frac{F L}{2 f_s} \quad C.7$$