

DEPARTMENTAL RESEARCH

Report Number: 46-1

DEVELOPMENT OF EQUIPMENT AND TECHNIQUES

FOR A

STATEWIDE RIGID PAVEMENT DEFLECTION STUDY

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by

B. F. McCullough

Research Project I-8-63-46

HIGHWAY DESIGN DIVISION
TEXAS HIGHWAY DEPARTMENT



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STATEWIDE RIGID PAVEMENT DEFLECTION STUDY

By

B. F. McCullough
Supervising Design Engineer

Research Report Number 46-1

Performance Study of Continuously
Reinforced Concrete Pavement
Research Project 1-8-63-46



Conducted by

Highway Design Division, Research Section
The Texas Highway Department
In Cooperation with the
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INTRODUCTION

In the early part of 1963, the Texas Highway Department initiated a research project entitled "A Performance Study of Continuously Reinforced Concrete Pavement". One phase of this project was to study the factors influencing deflection and their resulting effect on the performance of continuously reinforced concrete pavement.

The AASHO Road Test techniques and findings provided an excellent guideline for procedures to use in a deflection study, as well as the parameters to consider. Due to the dispersion of the project's test sections over an extensive area of the state and the inclusion of parameters not previously considered, the initial stages of the deflection study required the development of several new pieces of equipment and a refinement of test procedures. This work entailed: (1) a method of obtaining the slab temperature differential between the top and bottom of the concrete pavement slab, (2) the pavement's radius of curvature as it deflects under a given wheel load, (3) a loaded truck that was capable of moving about the State with a minimum of difficulty, and (4) development of testing techniques.

The scope of this report is to present these development procedures. For ease in presentation and assimilation, each of the development areas discussed above will be presented as a separate section in this report.

Section I

**A METHOD OF OBTAINING THE TEMPERATURE
DIFFERENTIAL FOR CONCRETE PAVEMENTS**

Background

The amount of deflection of a highway pavement under load can be used as a measure of the support given by the base material to that pavement, provided the effect of a number of variables can be accounted for. The AASHO Road Test indicated that the difference in temperature between the top and bottom of the slab has an important effect on pavement deflection. Formulas were presented in the final report on pavement research which allows for correlation of deflection data obtained at any known temperature differential.¹ To make this correction, it is necessary to determine the temperature differential at the time and location that the deflection data is taken. The method used to make this measurement is unimportant but accuracy of results is desirable. Since the Texas project covers an extensive portion of the State, the method should be portable and employ the least amount of equipment necessary to obtain the desired results.

Theoretical Approach To Temperature Differential

An investigation was conducted into the possibility of measuring the air temperature and, through theoretical means, projecting this data to give the desired temperature differentials. Reports of several men who had worked in this field were examined, but these were found to be of little value for the requirements

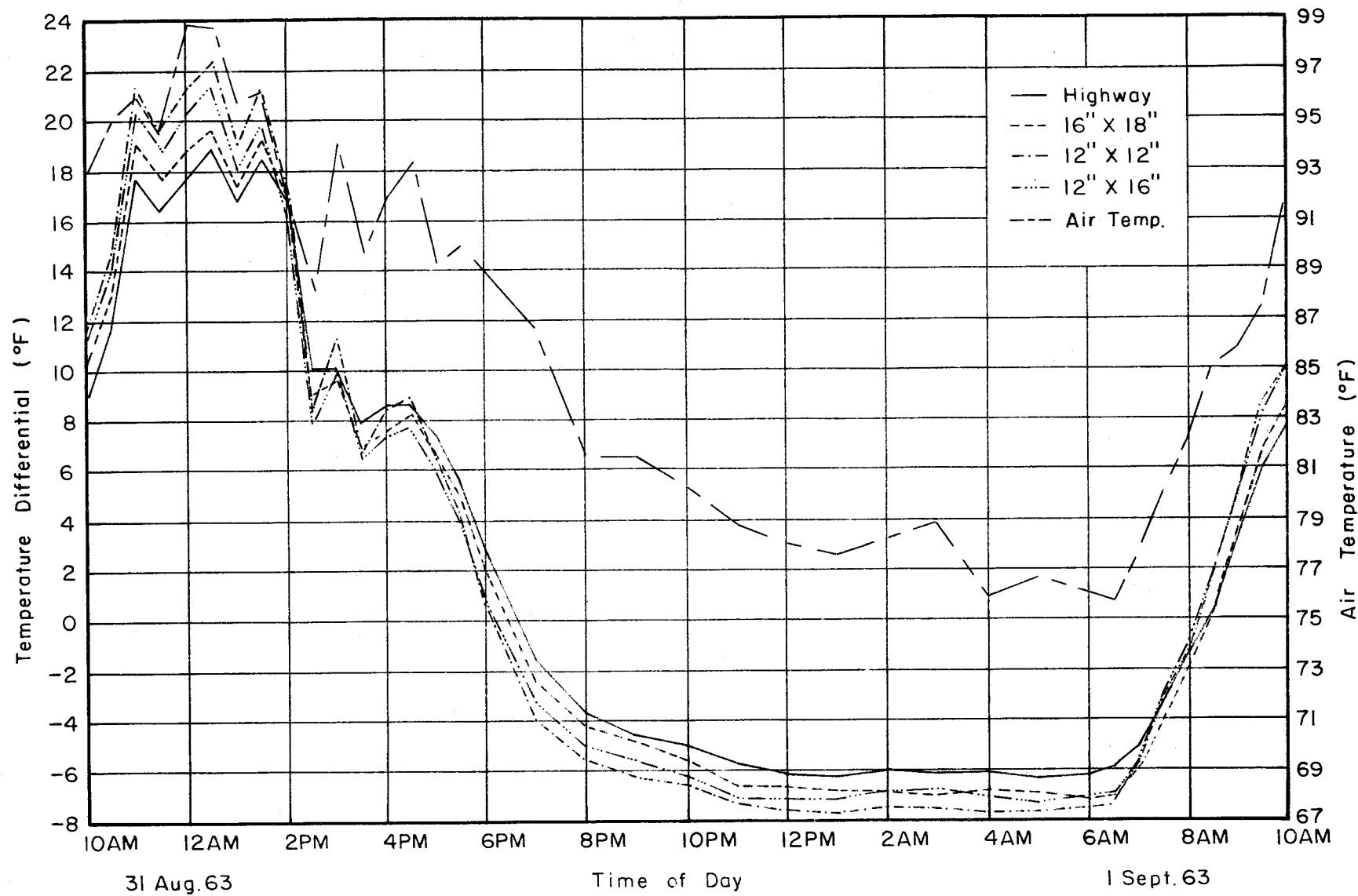
of the proposed research project.^{2,3,4} The methods presented in these reports appear quite adequate to predict the maximum and minimum differentials for design work in a particular season, but they have very little validity in predicting the differentials as they change from minute to minute.

Material was presented to show that when there was direct solar radiation the temperature curve resembled a sine wave, but when this direct radiation was absent the temperature curve became erratic, thus making an accurate instantaneous prediction of temperature differential impossible by means of air temperature alone. (The erratic tendencies of the temperature curve were later verified by direct measurement (Figure 1.1) in the field experiment.) The theoretical material showed that there was a definite need for a more positive method of obtaining the temperature differential of the highway slab.

Field Experiment

Objective. The purpose of the field experiment was to investigate the feasibility of measuring the temperature differential of a small test slab with resistance thermometers cast near its top and bottom surfaces and correlating these measured differentials with those of an in-place highway slab.

In order to investigate several factors which might influence the results obtained on a statewide application, the experiment



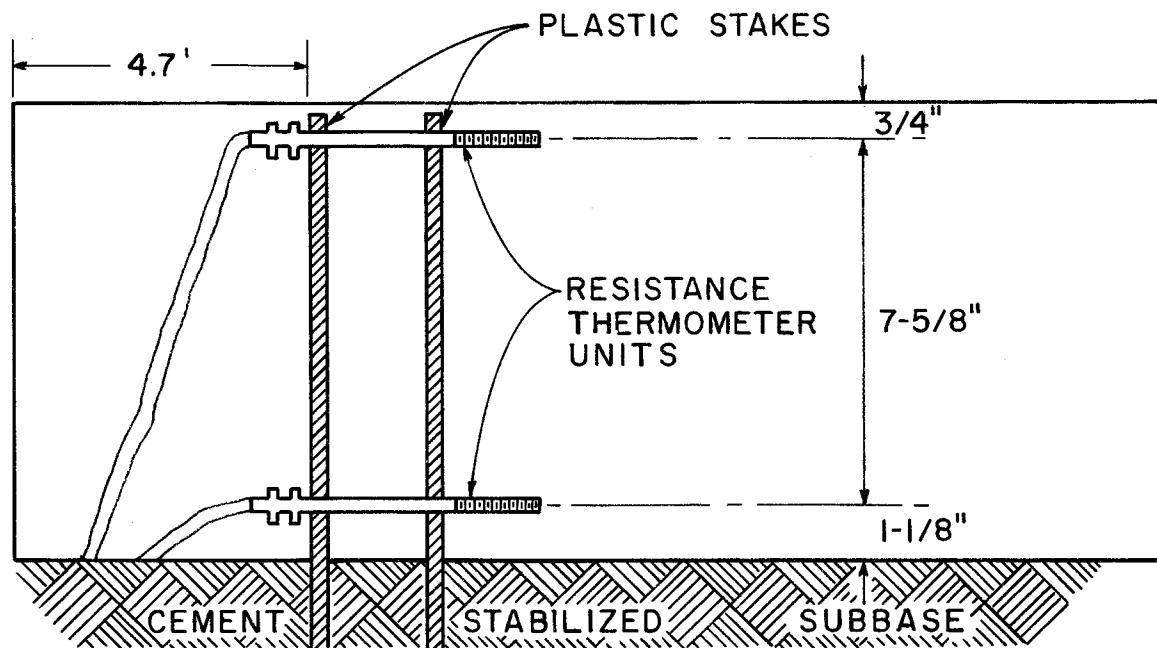
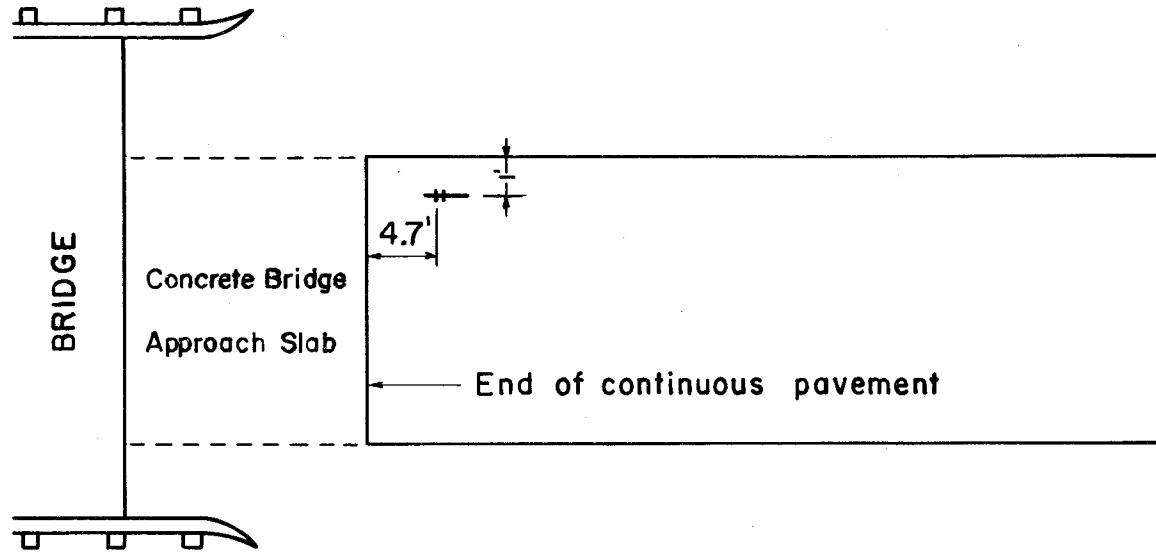
TYPICAL DAILY CYCLE (TEMPERATURE VS. TIME)

FIGURE 1.1

was conducted in two phases. The first phase was conducted to determine the accuracy of results of the test slabs and all variables were kept the same as for the highway pavement. The second phase was conducted to determine the influence that conditions of the base materials would have on the results and to determine whether or not it was necessary to bank earth against the edges of the test slab to simulate the highway shoulders.

Test Site and Equipment. The test site selected was on Interstate Highway 10 near the State Highway 71 overpass in Colorado County. Initial work began with the installation of resistance thermometers near the top and bottom surfaces of the south edge of the east-bound lane of Interstate Highway 10 during the regular paving operation. The thermometers were maintained at their proper spacing and position by means of two plastic stakes previously drilled and driven into the highway base. Electrical leads from the thermometers were carried under the highway shoulder by means of a 3/4 inch flexible vinyl conduit and terminated in a junction box attached to the guard rail post. This phase of the test preparations was complete on July 9, 1963 (See Figure 1.2 and Appendix).

The portable test slabs were cast of the same mix as the highway section to minimize variations due to mix design. Size was primarily determined on the basis of having a maximum amount of concrete between the sensing element and all exposed edges, but small enough for handling by a two man crew. Exposed edges

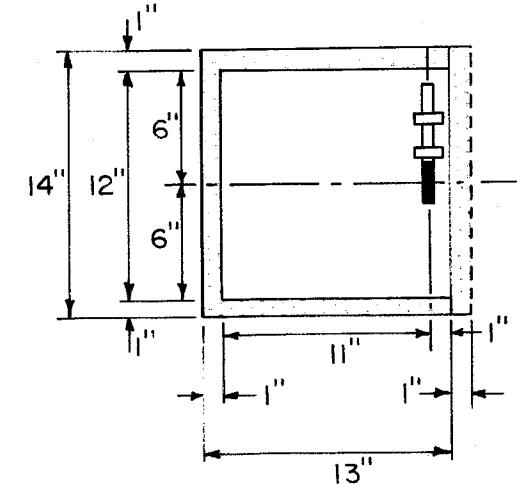
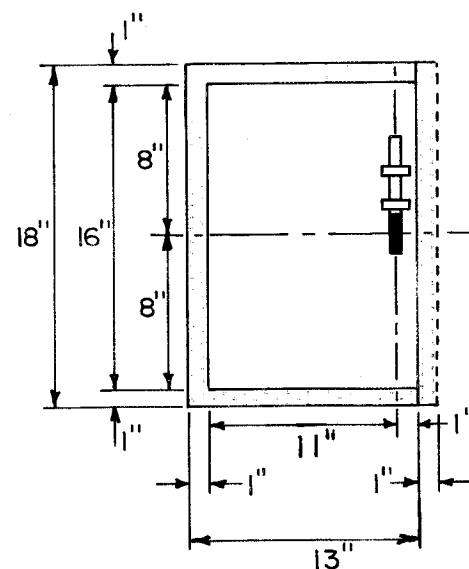
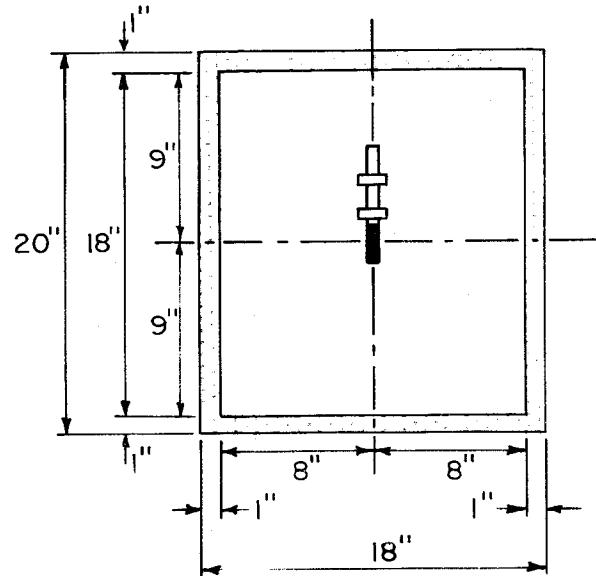


DETAILS OF INITIAL INSTALLATION
FIGURE I.2

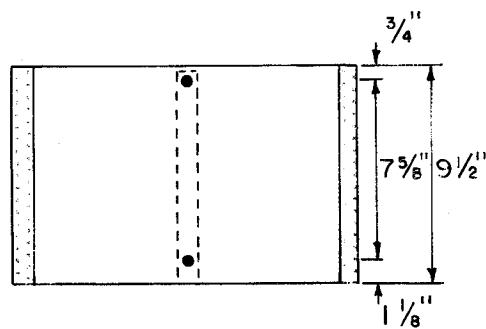
were insulated with one-inch of styrafoam to minimize heat flow into or out of the edge of the test section.

All test slabs were $9\frac{1}{2}$ inches deep, the depth of the highway at the point of the thermometer installation. The vertical locations of the thermometers in these slabs were identical to those in the highway test section. Plastic spacers similar to those used in the highway were screwed to the bottom of the test slab forms to assure the proper position of the thermometer bulbs. The temperature sensing element of the thermometer bulb is approximately two inches in length and its location in the bulb is in the end opposite the wire leads. This sensing element was centered on the transverse axis of the test slab in each case. The weights and pertinent dimensions of each slab are presented in Figure 1.3.

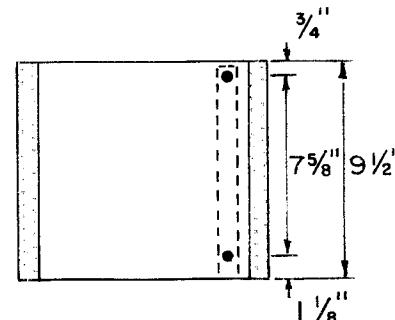
All four edges of slab #1 were insulated and the thermometers were located along the longitudinal axis of the slab. The purpose of the off-centered location of the smaller slabs was to obtain a maximum path for heat to travel from the exposed edge to the thermometer bulbs. Two lengths of slab were tried in an effort to find out if a longer heat path from the two ends would increase the accuracy of data. These latter two slabs were insulated on the three sides of greatest edge distance from the thermometers. The fourth side of these two slabs was left without insulation.



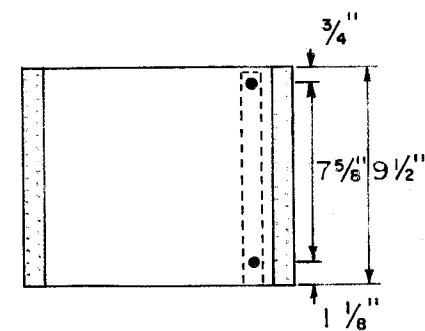
PLAN VIEWS



Weight: 238 lb.



Weight: 158 lb.



Weight: 119 lb.

SECTION VIEWS

DETAILS OF TEMPERATURE SLABS

FIGURE 1.3

The thermometers used were Honeywell High Speed Resistance Thermometer Bulbs, Model Number 921A3, six inches in length with a stainless steel tube. These thermometers have a range of calibrated accuracy from -110°F to 300°F with an accuracy of $\pm 0.5^{\circ}\text{F}$ for the range of temperatures encountered in the test. Due to the high resistance of the sensing element, the length of the lead wire can be varied considerably with a minimum of error being introduced.

The temperatures were recorded on a Honeywell Universal ElectroniK 15, Multipoint Recorder. This instrument has a twelve point recording capability with fixed cycle printing. One point was recorded every 15 seconds, hence, each point was recorded once every three minutes. This rate of recording gave a continuous feed of data, and the rapidity of response to atmospheric changes could be checked quite closely.

Testing Procedures

Phase I. The three test slabs were located on the shoulder base as shown in Figure 1.4. The two smaller slabs were placed with their uninsulated edges adjacent to the edge of the highway with a 3/8 inch joint between the test slab and the highway. This joint was filled with concrete grout in an effort to achieve a continuity between the highway and the test slab, to compensate for the lack of concrete and insulation of the edge in which the thermometers were located. The large test slab was placed on the

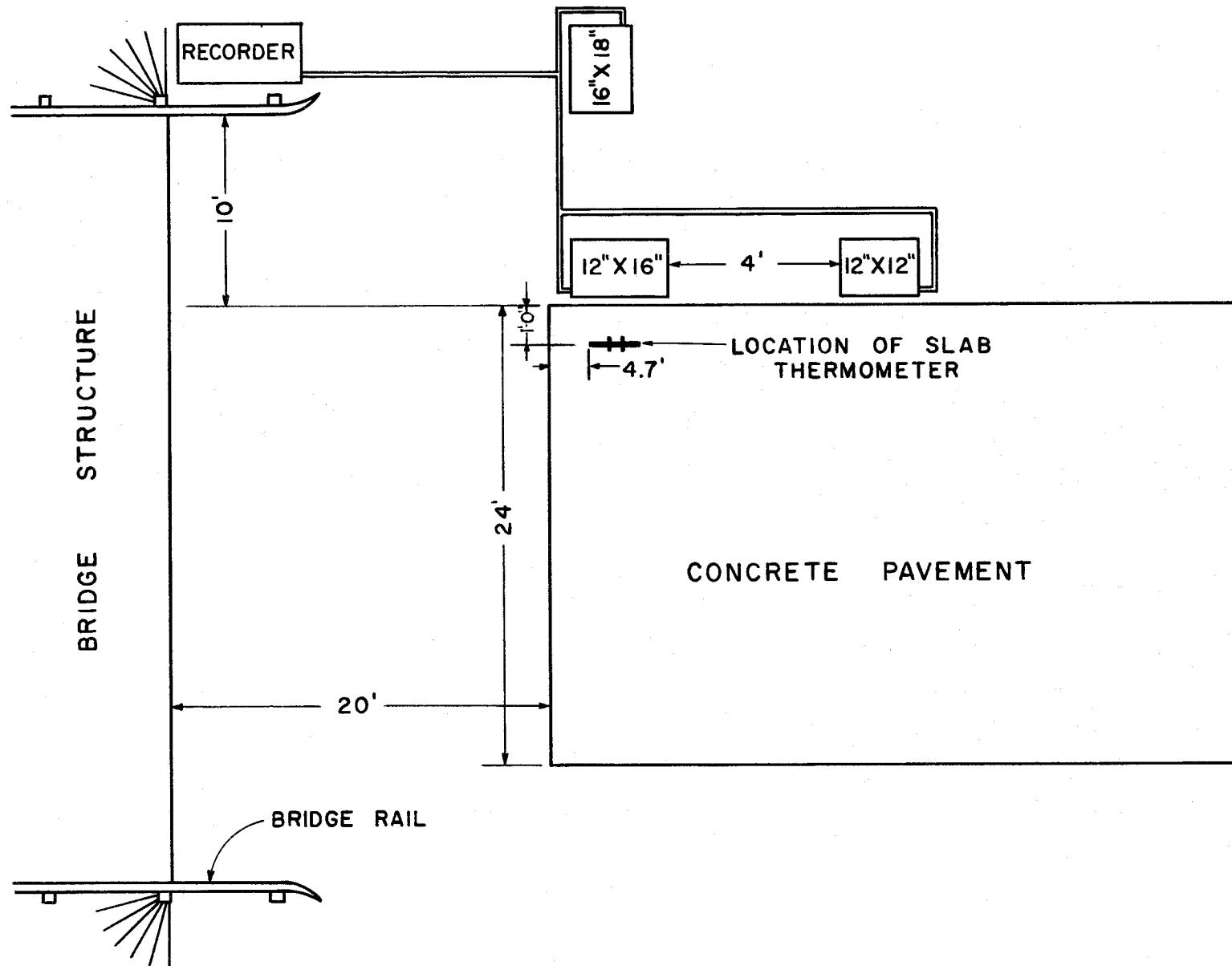


FIGURE 1.4

cement stabilized shoulder base. Dirt was banked against the exposed edges of all slabs (including the highway pavement for a distance of four feet beyond the test slabs) in an effort to simulate the action of the shoulder material which would be in place on any highway after completion but not in place during the test period.

In addition to the eight thermometers located in the three test slabs and the highway, one thermometer was located under the overpass to record air temperature and another was located in the recorder housing to keep track of the temperature of the recording device. Temperatures were recorded for a period of eleven days, at which time Phase I was concluded and the data was evaluated.

Phase II. The two test slabs grouted to the pavement edge were separated and the fourth side of each was insulated. All three test slabs were placed on the stabilized soil and earth was banked against edges of the 12 inch X 12 inch slab. After allowing one day for temperatures to equalize, the temperatures were recorded for a period of 51 hours. At this time the slabs were moved to non-stabilized soil. The earth banked against the edges of the 12 inch x 12 inch slab was deleted, and another day was allowed for temperatures to reach equalization. Temperatures were again recorded for a period of 51 hours and the experiment was concluded.

Presentation of Results

Phase I. The three test slabs all correlated very well to the conditions measured in the highway slab. All four slabs reacted in a similar manner to changes in cloud cover as well as atmospheric conditions. High and low peaks were reached simultaneously and the changeover from the top being warmer than the bottom and vice versa occurred at the same time in all slabs.

Baring unusual climatic conditions, the peak temperature differential of 20°F occurred around 2 P.M. under these seasonal conditions with the night differential remaining fairly constant at a negative 7°F. For purposes of this investigation, positive temperature differential is defined as the top of the slab being warmer than the bottom of the slab; negative temperature differential is defined as the top of the slab being cooler than the bottom of the slab. The change from a positive to a negative temperature differential consistently occurred shortly after 6 P.M.; the reverse occurred about 8 A.M. throughout the eleven day period of investigation. This was true for all three test slabs and the highway pavement. The magnitude and time of occurrence of the maximum positive and negative temperature differential will change with the time of the year, but the pattern itself represents a typical daily cycle.

The temperature differentials of each slab were plotted against the corresponding measured temperature differential of the highway pavement, and a linear regression analysis was run for each 24 hour cycle. The linear regression analysis produced an equation of the form:

$$T_H = B_0 + m T_S$$

where: T_H = Actual highway temperature differential, $^{\circ}$ F
 B_0 = Intercept along T_H axis
 m = Slope of regression line
 T_S = Measured temperature differential of test slab, $^{\circ}$ F

This equation can be used to predict the highway temperature differential based on the measured temperature differential of a portable slab. While the intercept and slope fell in a narrow range band, they did not remain constant from day to day and the figures obtained depended on the number of data points used. There was some variance in results when all positive, all negative or a mixture of positive and negative numbers were used within a 24 hour cycle.

Results of the linear regression are tabulated in Table 1.1. Cycles 8-30(D) and 8-31(D) compare daylight results, 8-30(N) and 8-31(N) compare night results, and 9-1 through 9-8 compare 24 hour results respectively. Each comparison group is based on a common number of data points.

The correlation of the reactions of the four slabs to atmospheric conditions is shown quite clearly in the 24 hour

TABLE I.I
TABULATION OF LINEAR REGRESSION RESULTS FOR PHASE I

CYCLE	12" x 12" SLAB			12" x 16" SLAB			16" x 18" SLAB		
	INTERCEPT (°F)	SLOPE	COEFFICIENT OF CORRELATION	INTERCEPT (°F)	SLOPE	COEFFICIENT OF CORRELATION	INTERCEPT (°F)	SLOPE	COEFFICIENT OF CORRELATION
8 - 30(D)*	1.49	0.80	.9878	1.26	0.85	.9901	0.45	0.92	.9982
8 - 30(N)	- 0.50	0.73	.9951	- 0.55	0.78	.9965	- 0.19	0.85	.9968
8 - 31(D)	1.58	0.78	.9908	1.55	0.83	.9889	0.72	0.92	.9961
8 - 31(N)	- 0.54	0.73	.9988	- 0.59	0.78	.9983	- 0.05	0.87	.9989
9 - 1	0.59	0.84	.9891	0.41	0.92	.9926	0.44	0.92	.9978
9 - 2	1.28	0.82	.9720	1.25	0.88	.9712	1.08	0.89	.9714
9 - 3	0.87	0.83	.9877	1.21	0.86	.9904	0.69	0.88	.9959
9 - 4	0.34	0.84	.9901	0.43	0.93	.9898	0.12	0.88	.9967
9 - 5	0.40	0.83	.9905	0.43	0.93	.9934	0.07	0.86	.9974
9 - 6	0.23	0.82	.9929	0.28	0.94	.9965	0.00	0.83	.9985
9 - 7	0.69	0.86	.9939	0.64	0.95	.9959	0.24	0.88	.9985
9 - 8	0.42	0.80	.9953	0.55	0.95	.9953	0.36	0.88	.9984

* 8-30 August 30th

(D) Day

(N) Night

cycle from 10 A.M., August 31, 1963, to 10 A. M. September 1, 1963 (See Figure 1.1). The resulting pattern of this typical cycle indicates that this method of approach offers a solution to the problem of measuring the temperature differential.

The temperature differential of the slab was not a direct function of the air temperature. The differential was more sensitive to direct solar radiation than it was to the air temperature and, thus, was much more affected by cloud formations and rain than it was by air temperature alone. A method of direct measurement appears to offer a solution of reasonable accuracy while a solution based on air temperature does not appear to offer a solution of sufficient accuracy to warrant its use.

Phase II. Once again the intercepts and slopes of the correlation between test slab and pavement temperatures were not consistent, but did remain within a fairly narrow band (Table 1.2). The slopes showed very little variance, whether on stabilized or non-stabilized base. The variation in intercept was slightly greater when only the positive data was used, but even this remained within a 1.4 degree maximum band. Results for the complete cycle of data were very close. There was no appreciable variation between results of phase one and phase two, even though there had been a lapse of almost

TABLE I.2

TABULATION OF LINEAR REGRESSION RESULTS FOR PHASE II

SLAB SIZE	CYCLE TYPE	STABILIZED			NON - STABILIZED		
		INTERCEPT (°F)	SLOPE	COEFFICIENT OF CORRELATION	INTERCEPT (°F)	SLOPE	COEFFICIENT OF CORRELATION
12" x 12" SLAB	POSITIVE	-0.34	0.82	.9604	0.63	0.77	.9360
	NEGATIVE	0.22	0.89	.9643	0.38	0.89	.8876
	COMPLETE	-0.33	0.83	.9937	0.17	0.83	.9866
12" x 16" SLAB	POSITIVE	-0.40	0.99	.9711	0.42	0.91	.9597
	NEGATIVE	0.92	1.05	.9477	0.24	0.97	.9585
	COMPLETE	0.10	0.96	.9943	0.28	0.95	.9912
16" x 18" SLAB	POSTIVE	-0.03	1.01	.9830	0.41	0.95	.9754
	NEGATIVE	1.14	1.12	.9792	1.06	1.10	.9451
	COMPLETE	0.20	0.99	.9966	0.44	0.97	.9943

one month. There was no recognizable variation due to the test slab being on stabilized or non-stabilized soil, and banking soil around the test slab appears to be an unnecessary precaution.

Conclusions and Recommendations

Examination of a typical 24-hour cycle, 10 A. M. September 6 to 10 A. M. September 7, 1963 (See Tables 1.3 & 1.4) shows that by making predictions by means of the correlation formulas for that cycle (Corrected Data) gives a standard error of estimate of 1.0°F for the 12 inch x 12 inch slab, 0.8°F for the 12 inch x 16 inch slab, and 0.5°F for the 16 inch x 18 inch slab. If the temperatures are used as measured by the small slabs (Uncorrected Data), the standard error of estimate from the highway readings would be 2.2°F , 0.8°F , and 2.0°F , respectively.

The AASHO Road Test formula for temperature correction of pavement deflections can be written as:

$$d_e = K 10^{-0.0075T} \quad (\text{See Appendix})$$

A typical deflection reading of 0.014 of an inch at a 14°F differential would result in a corrected reading of 0.0178 of an inch. Assuming the method of measurement is off by its standard error of estimate based on values corrected by a correlation formula, the 12 inch x 12 inch slab would yield

TABLE 1.3

TABULATION OF ACTUAL AND PREDICTED TEMPERATURE DIFFERENTIALS FOR HIGHWAY AND THREE TEST SLABS COVERING PERIOD FROM 10 AM 6 SEPTEMBER 1963 TO 10 AM 7 SEPTEMBER 1963

ACTUAL HIGHWAY TEMP. DIFF. (°F)	12" x 12" TEST SLAB				12" x 16" TEST SLAB				16" x 18" TEST SLAB			
	TEMPERATURE DIFFERENTIAL(°F)		ERROR FROM HIGHWAY (°F)		TEMPERATURE DIFFERENTIAL(°F)		ERROR FROM HIGHWAY (°F)		TEMPERATURE DIFFERENTIAL(°F)		ERROR FROM HIGHWAY (°F)	
	ACTUAL	PREDICTED	UN-CORRECTED	CORRECTED	ACTUAL	PREDICTED	UN-CORRECTED	CORRECTED	ACTUAL	PREDICTED	UN-CORRECTED	CORRECTED
6.2	9.8	8.2	3.6	2.0	7.9	7.7	1.7	1.5	8.5	7.1	2.3	0.9
15.0	18.7	15.5	3.7	0.5	15.5	14.9	0.5	-0.1	18.8	15.6	3.8	0.6
16.8	19.0	15.8	2.2	-1.0	17.0	16.3	0.2	-0.5	19.7	16.4	2.9	-0.4
10.2	11.5	9.6	1.3	-0.6	10.5	10.2	0.3	0.0	12.7	10.5	2.5	0.3
2.4	0.4	0.6	-2.0	-1.8	0.7	0.9	-1.5	-1.5	1.7	1.4	-0.7	-1.0
-4.8	-6.4	-5.5	1.6	0.7	-5.5	-5.5	0.7	0.7	-5.8	-4.8	1.0	0.0
-5.5	-6.9	-5.9	1.4	0.4	-5.9	-5.9	0.4	0.4	-6.7	-5.6	1.2	0.1
-6.0	-7.5	-6.4	1.5	0.4	-6.5	-6.4	0.5	0.4	-7.2	-6.0	1.2	0.0
-5.8	-7.2	-6.1	1.4	0.3	-6.5	-6.4	0.7	0.6	-6.8	-5.6	1.0	-0.2
-1.2	-0.8	-0.9	-0.4	-0.3	-1.2	-1.4	0.0	0.2	-1.4	-1.2	0.2	0.0

TABLE 1.4

RESULTS OF LINEAR REGRESSION FOR DATA IN TABLE 1.3

SIZE OF SLAB	CORRECTION FACTORS				STANDARD COEFFICIENT OF CORRELATION	ERROR OF ESTIMATE (°F)	
	INTERCEPT (°F)	SLOPE	FORMULA	UNCORRECTED DATA		CORRECTED DATA	
12" x 12"	0.23	0.817	TH = 0.23 + .817ts	.9929	2.142	0.992	
12" x 16"	0.28	0.944	TH = 0.28 + .944ts	.9965	0.819	0.773	
16" x 18"	0.00	0.830	TH = 0.00 + .830ts	.9985	1.995	0.497	

an error in deflection of ± 0.00031 of an inch, the error for the 12 inch x 16 inch slab would be ± 0.00024 of an inch, and the error of the 16 inch x 18 inch slab would be ± 0.00015 of an inch. If the standard error of estimate of uncorrected values is used as a measurement of accuracy, the error in deflection would be ± 0.00066 , ± 0.00024 , and ± 0.00063 of an inch, respectively. The latter method is well within the accuracy required by the capability of the Benkelman Beam.

The method of direct measurement of temperature differential in a small slab promises to be an accurate, effective method of obtaining the necessary data. Using direct measurement, rather than a correlation formula, is well within the necessary accuracy, with little extra effort involved in taking and using the data. Use of one of the smaller slabs increases the portability of the system, so an effective method to be used in the field is that of direct measurement of the 12 inch x 16 inch slab.

Like any accelerated research project, there is room for additional research, because many other variables remain to be examined. Things such as mix design, aggregate type, surface color, and length of time prior to starting measurements may all enter into the results. As a by-product of other research studies, a six-inch lightweight section with thermometer bulbs placed at similar spacings from the respective surfaces was

found to measure the same temperature differentials as the portable 9½" test slab placed along side of it. A period of twelve hours is always allowed for temperature equalization by positioning the slab the night before the taking of data is to begin.

Section II
DEVELOPMENT OF THE BASIN BEAM

Background

During the formative stages of the research project, the need for measuring the pavement radius of curvature while deflecting under a wheel load was recognized. To arrive at any pavement stress comparison without an extensive strain gage system, a relative measure of the radius of curvature is required, since the pavement flexural stress is inversely proportional to the radius of curvature. The importance of measuring basin deflection characteristics quickly became evident on some of the initial deflection measurements. Figure 2.1 portrays two different deflection basins obtained through the use of a series of Benkelman Beams. In this particular case, the maximum deflection for both basins was in the neighborhood of 0.015 of an inch. It is evident that the deflection characteristics are different; hence, so are the stress characteristics. The deflection basin of the pavement with subbase A is much sharper in curvature relative to the one with subbase B. Since the flexural stress is an inverse function of the radius of curvature, the stress in the pavement on subbase A will be higher than in the pavement on subbase B.

The shape of the entire deflection basin as portrayed in Figure 2.1 is of interest, but the major point of comparison is the minimum magnitude of the radius of curvature. Referring

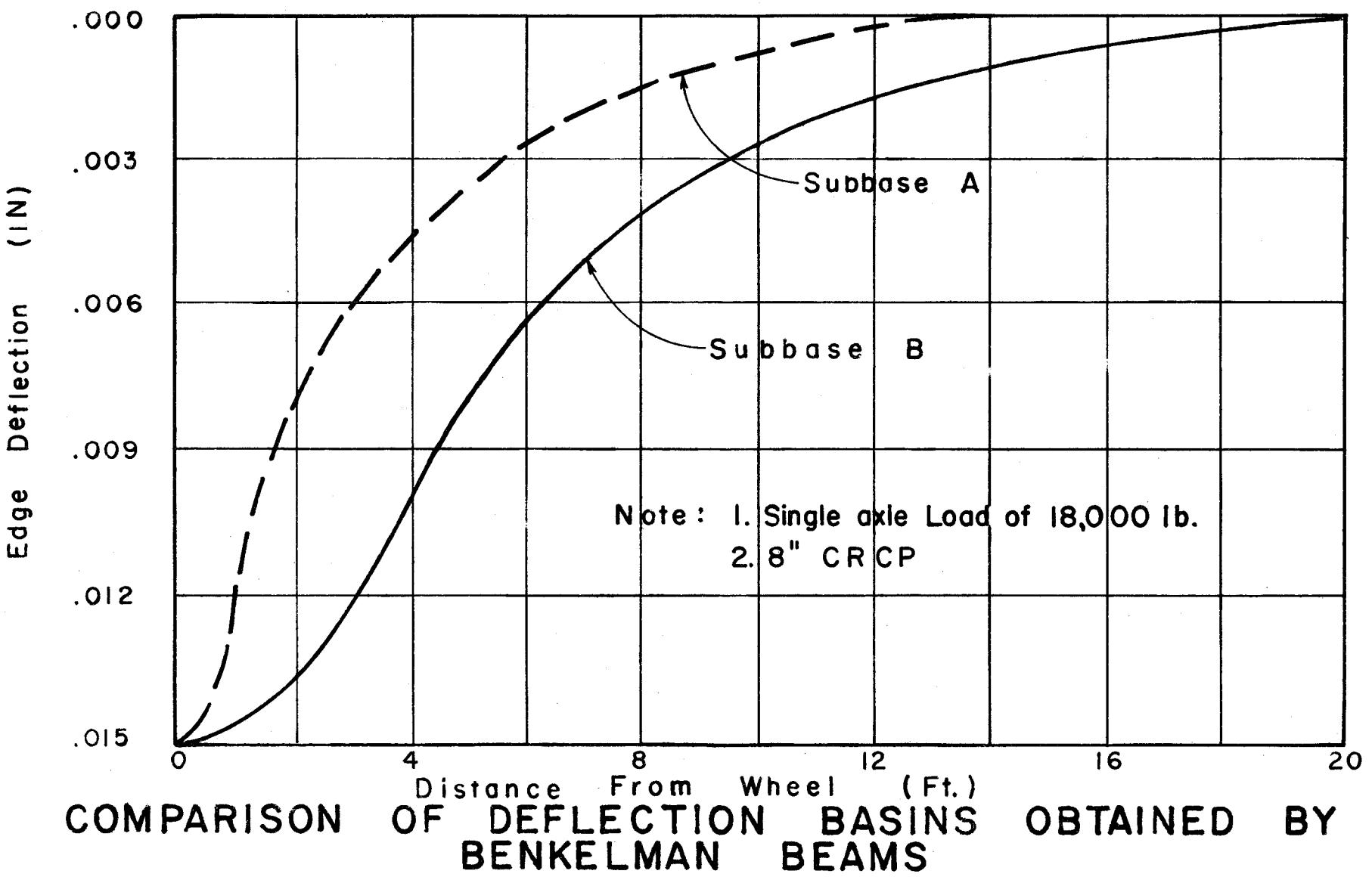


FIGURE 2.1

again to Figure 2.1, it is evident that the minimum radius of curvature, i.e. maximum stress, occurs directly beneath the wheel load. Therefore, if the simple method of measurement is obtained for determining the radius beneath the wheel load, a basis is obtained for comparing the stress characteristics of the two deflection basins.

Development of a Formula for Radius of Curvature

From basic geometry, it may be recalled that any time the coordinates of three points are known, a curve may be passed through these points (See Figure 2.2). In the Appendix, a basic expression has been derived for calculating the radius of a curvature of a circle that passes through the known coordinates of three points. The simplified version of the mathematical expression is as follows:

$$R = 1/2 E \dots \quad (1)$$

where: $E = \frac{(x_2^2 - x_1^2) z - (x_3^2 - x_1^2) + (y_2^2 - y_1^2) z - (y_3^2 - y_1^2)}{(y_3 - y_1) - z(y_2 - y_1)}$

$$z = \frac{x_3 - x_1}{x_2 - x_1}$$

x_n, y_n = Cartesian coordinates of points

R = Radius of curvature

In a preliminary feasibility study, two methods of obtaining the data for the above equation were considered. One method

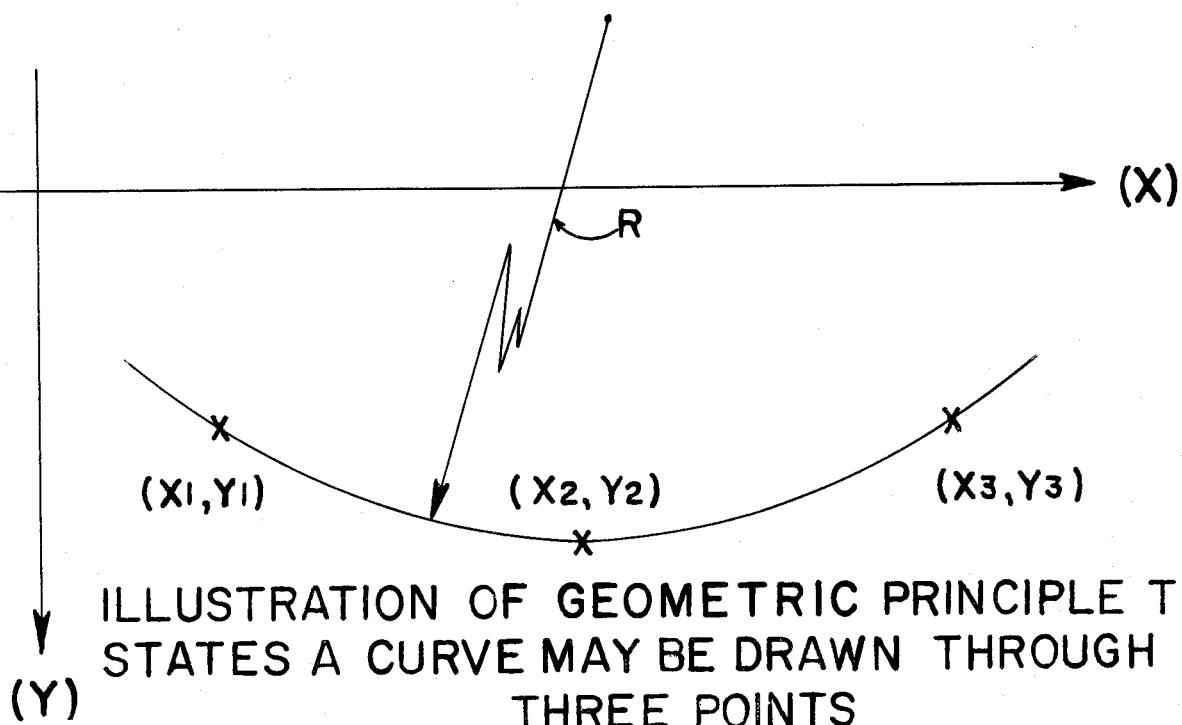
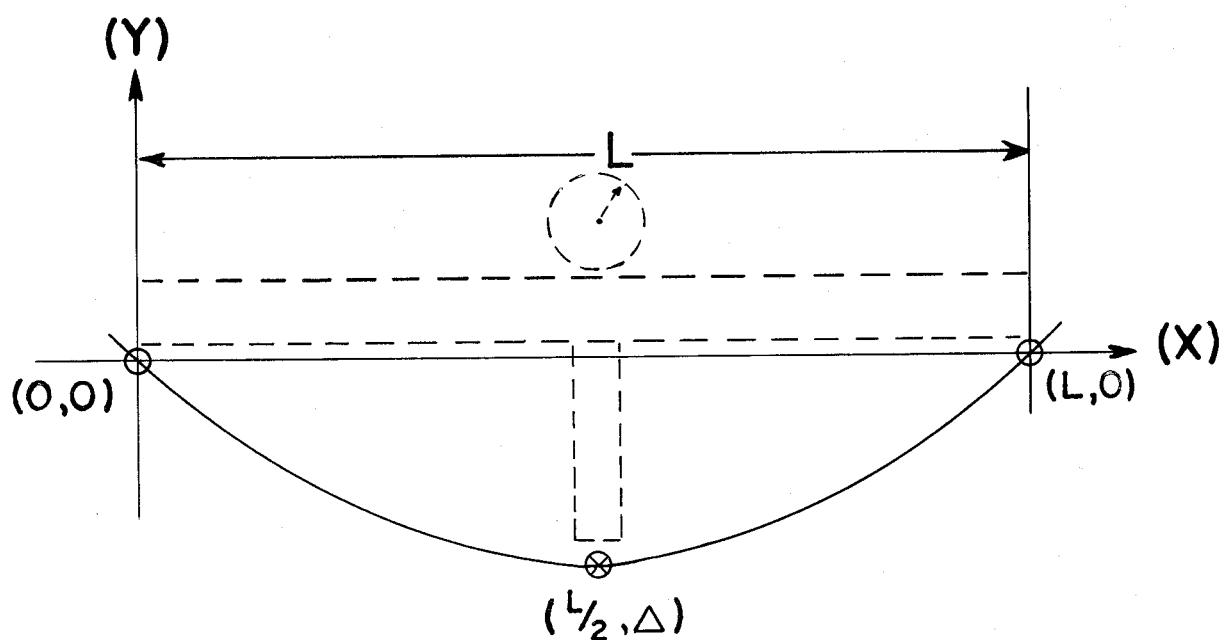


FIGURE 2.2



COORDINATE SYSTEM FOR CALCULATING RADIUS OF CURVATURE WITH DATA OBTAINED FROM THE BASIN BEAM

FIGURE 2.3

was to use three Benkelman Beams placed at close intervals. The other method considered was a Radius of Curvature Meter as presently being used in South Africa to evaluate flexible pavements.⁵

Limited measurements of radius of curvature with the three Benkelman Beams indicated that this method was not feasible in a state-wide survey. In the first place, the resolution of the beams (± 0.002 of an inch) was not sufficient to establish a uniform reproducibility of the radius of curvature. Furthermore, the beams were too bulky for rapid movement, and the spacing of the beams to the accuracy required was too time consuming.

Therefore, on the basis of this experience, it was decided to measure the radius of curvature through the use of a Radius of Curvature Meter (hereafter referred to as the Basin Beam) that would meet the following prerequisites: (1) Be a minimum weight and durable. (2) The setup and reading time should not exceed that of a Benkelman Beam which was being used to obtain the total magnitude of deflection. (3) The cost should be at a minimum.

Development of the Basin Beam

If the length of the Basin Beam is fixed, and only one reading gage is used at the center of the beam, equation (1)

for predicting radius of curvature may be simplified considerably. Referring to Figure 2.3, the origin of the coordinate system is placed at one end of the beam and the X-axis is passed through the other end of the beam. Using the coordinates as shown on the graph, and substituting into equation (1), the formula for radius of curvature is reduced to the following:

$$R = \frac{L^2 - 4\Delta^2}{8\Delta} \quad \dots \quad (2)$$

where:

L = Length of Basin Beam, inches.

Δ = The radial distance from the mid-point of the circle segment to the chord of the segment, inches.

From a practical standpoint, the numerator in equation (2) can be simplified further. For beam length of ten inches or greater, the second term in the equation will be so small relative to the first term that it may be deleted. This simplification along with converting the radius of curvature to feet results in the following expression:

$$R = \frac{L^2}{96\Delta} \quad \dots \quad (3)$$

The Basin Beam will always be within the deflection basin, but this is irrelevant since it is not being used to measure the total magnitude of deflection. The Basin Beam simply gives the relative coordinates of three points within the deflection basin.

Selection of Length. In theory, the length of the Basin Beam should be at the minimum possible, since the shorter the chord length the nearer the approximation of the true radius of curvature at the point in question. If the deflection basin corresponded to a circle, the length of the chord would be irrelevant; but since the deflection basin is a higher order curve than a circle, the chord length has a definite effect upon the magnitude of the radius.

Therefore, the question of Basin Beam length resolves to one of reducing the length to a minimum that will provide a magnitude of elevation difference at the center point of the chord that is within the capabilities of the measuring system. After considering the factors of economy, dependability, and simplicity, it was decided to use a normal dial gage for measuring the difference of elevation at the center of the beam.

To select a length compatible with the accuracy of the dial gage measuring system, deflection basins obtained with Benkelman Beams were studied on three projects that represented extremes in deflection characteristics. The 10 inch length used by other investigators on flexible pavements was feasible on pavements with non-stabilized subbases, but completely incompatible with the dial gage measuring system on pavements with stabilized subbases. After considering the deflection

basins on the three projects, a distance of 40 inches was found to be long enough to provide sufficient dial readings on stabilized subbases and short enough to obtain realistic values on pavements with non-stabilized subbases.

Using a length of 40 inches, in equation 3, the radius of curvature in feet may be calculated as follows:

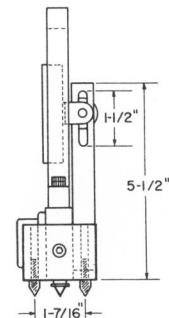
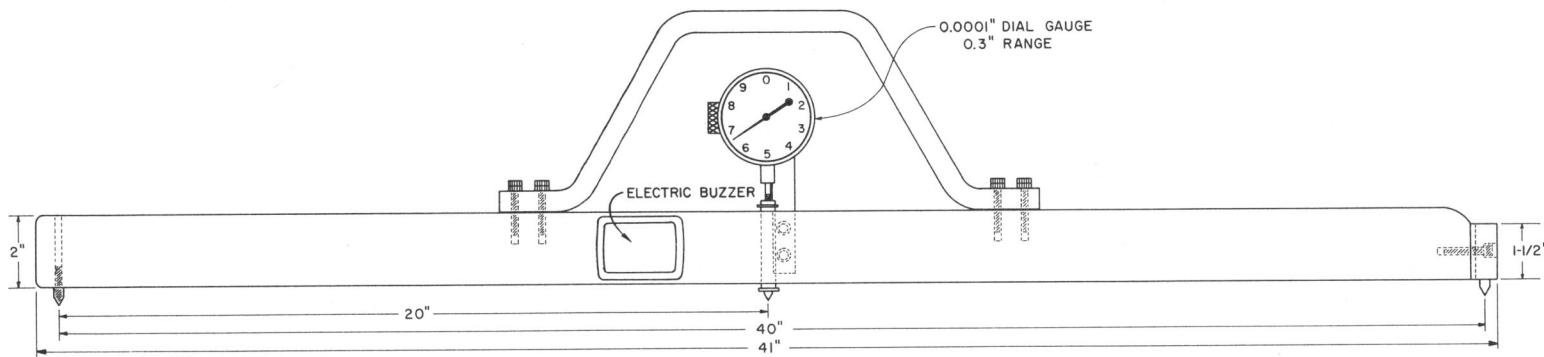
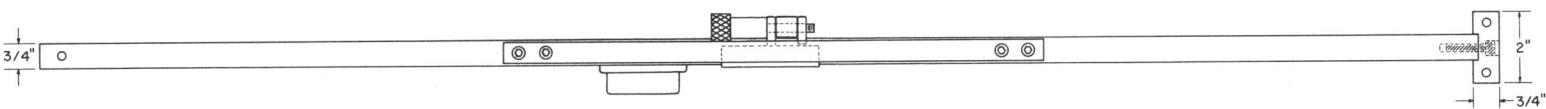
$$R = \frac{16.67}{\Delta} \quad \dots \dots \dots \dots \dots \dots \dots \dots \quad (4)$$

Description of Basin Beam. Figure 2.4 is a photograph of the Basin Beam in operation, and Figure 2.5 is a shop drawing showing the pertinent assembly details. The dial gage mounted on the beam has a sensitivity of 0.0001 of an inch. Two support points were placed at one end of the beam in order for it to stand upright without being supported by an operator. This eliminates the possibility of the operator applying any external force that would cause erroneous readings on the dial gage.

Since minute length changes will not significantly affect the radius of curvature calculation, the beam was constructed of aluminum with the exception of the dial gage. This reduced the weight to a practical minimum. The beam weighs 8.25 pounds and can be handled by one operator with a minimum of effort. The beam was constructed in the Texas Highway Department shops, and the total cost including the carrying case was \$295.00.



Figure 2.4 Photograph Of The Basin Beam



DRAWING OF BASIN BEAM

FIGURE 2.5

Evaluation of Basin Beam. In this section typical data obtained with the Basin Beam will be analyzed, and in addition, the accuracy of the Basin Beam will be discussed. The procedure for using the Basin Beam as presently followed by the Texas Highway Department is outlined in the Appendix.

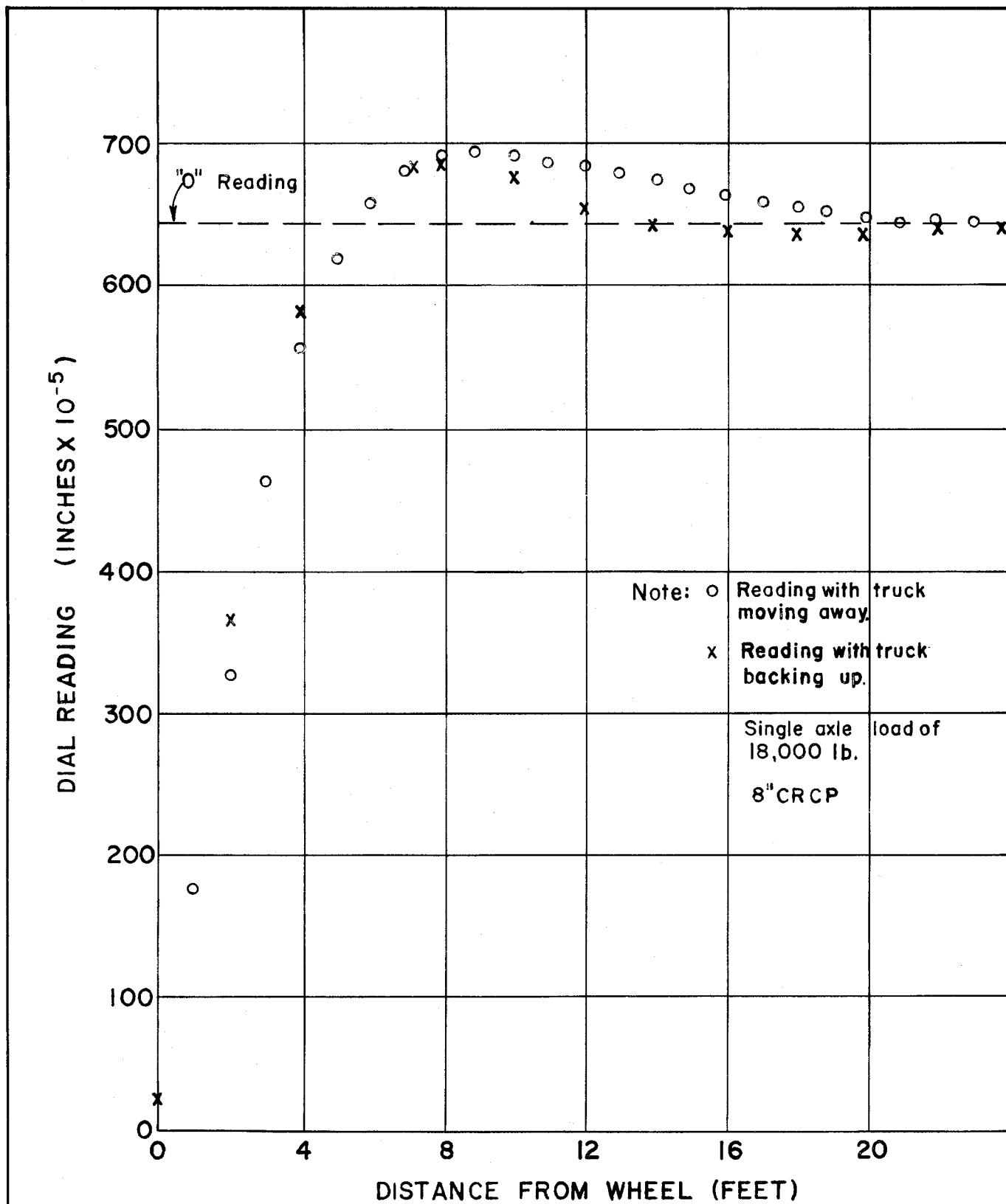
Analysis of Typical Data. Figure 2.6 is a typical data plot obtained through use of the Basin Beam. This plot is simply a record of the dial readings at a given location as a static wheel load is moved longitudinally down the pavement. The data shows a very smooth trend for any given direction of truck movement. In addition, the data shows the deflection basin to be approximately 21 feet long since there was no variation in dial reading past this point. The point at which the dial readings cease to vary with movement of the truck was taken as a zero reading. The following formula is used to obtain the delta value for use in equation (4) to calculate the radius of curvature:

where:

DR_n = The dial reading where there is no variation in reading as the truck continues to move away.

DR_x = The dial reading at the point where the radius of curvature is desired.

A plus sign indicates a tensile stress in the bottom of the slab, and a minus sign indicates a compressive stress at



TYPICAL DATA PLOT SHOWING THE DIAL READING AS THE TRUCK MOVES AWAY FROM THE BASIN BEAM

FIGURE 2.6

the bottom of the slab.

Using the delta values calculated from Figure 2.6 in connection with equation (4), the radii of curvature at the various points were calculated. The change in radius of curvature as the wheel load moves along the slab is shown in Figure 2.7. In examining the graph, note that the minimum radius of curvature occurs with the wheel load at the beam, as would be expected, and gradually increases and approaches infinity at a distance of approximately five feet. (An infinite radius of curvature is a straight line.) Passing the point of inflection the radius of curvature now having a minus sign, decreases from infinity to another minimum value at approximately nine feet. This change in sign in the radius of curvature simply indicates there is a reverse curve in the deflection basin. The minimum radius of curvature in the second curve does not approach the minimum obtained in the first curve.

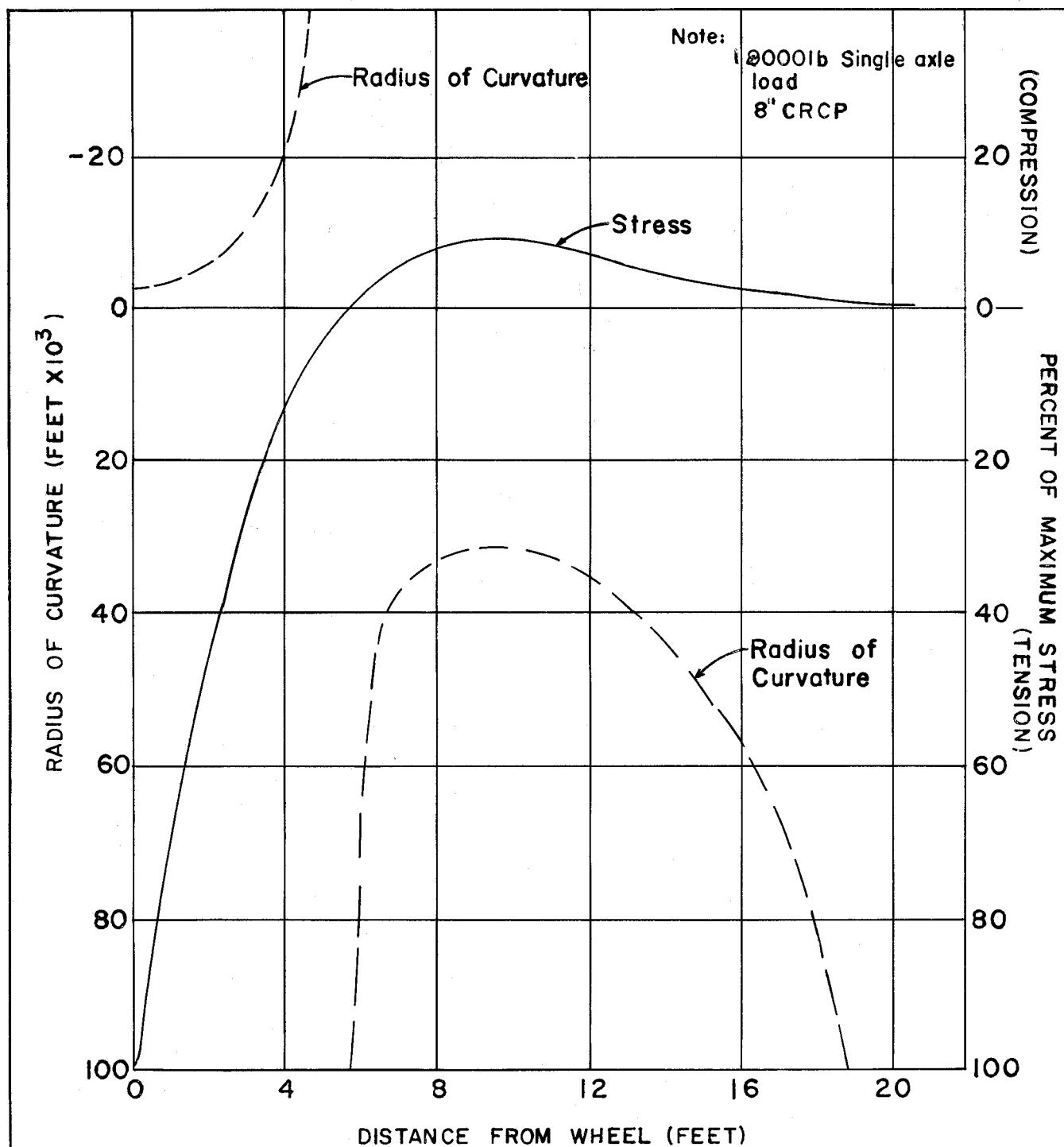
The radius of curvature is an excellent indicator of stress in the slab since the fundamental flexure formula states that stress is inversely proportional to the radius of curvature as follows:

$$S_f = \frac{E_n}{12R} \quad \dots \quad (6)$$

where:

s_f = Flexural stress in the slab, in psi. (the same sign convention applies)

E = Concrete modulus of elasticity, psi



THE RADIUS OF CURVATURE AND THE RELATIVE STRESS LEVEL AT THE BOTTOM OF THE SLAB AS THE WHEEL LOAD MOVES AWAY FROM A GIVEN POINT

FIGURE 2.7

n = Distance from neutral axis to extreme fiber in inches.

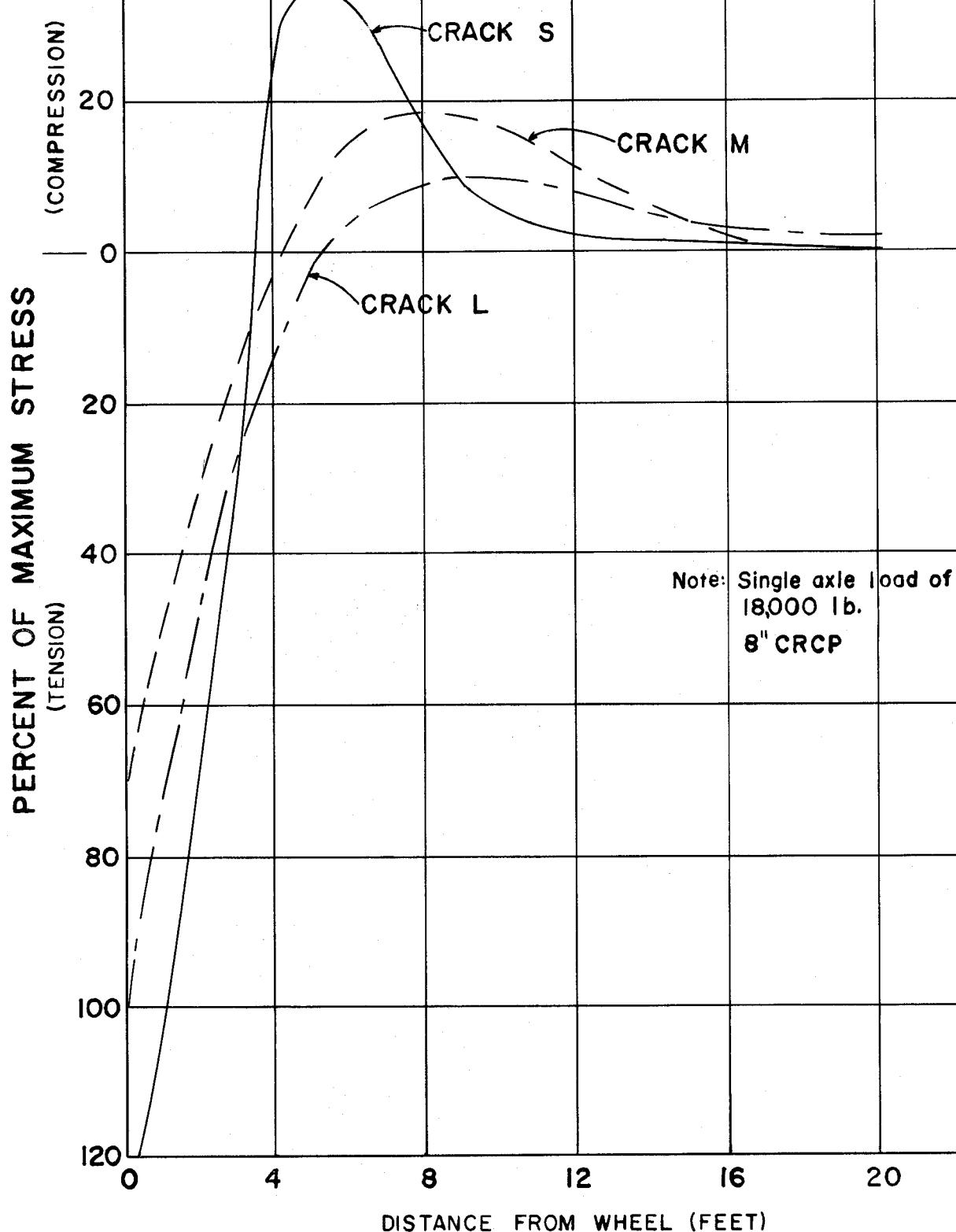
To obtain the correct stress with the above equation, the actual value of the radius of curvature must be used. The Basin Beam gives only a relative value of the radius of curvature; therefore, equation (6) must be multiplied by a correlation constant. Since the Texas Highway Department places longitudinal steel at mid-depth, it is logical to assume that the neutral axis at mid-span is at this location. Incorporating these two changes and making the necessary dimension changes, the following equation is derived for predicting stress:

D = Pavement thickness in inches.

K = Correlation constant.

Using the above expression and the maximum stress as unity, a relative indication of the stress distribution may be obtained. This relative stress distribution is shown by the solid line on Figure 2.7. Note the maximum tensile stress is obtained when the wheel load is at the beam and decreases to zero at the point where the bottom of the slab goes into compression. A maximum compressive stress is reached at a point approximately nine feet from the wheel, and from there gradually decreases to zero.

A sample comparison of data obtained with the Basin Beam is shown in Figure 2.8, which presents the relative stress dis-



RELATIVE STRESS DISTRIBUTION AT THE BOTTOM OF
A SLAB FOR THREE DIFFERENT CRACK SPACINGS

FIGURE 2.8

distribution at the bottom of the slab for three different crack spacings, i.e. small, medium, and large. In this Figure, crack spacing is the only variable since subbase, subgrade, etc. are all equal. The percent stresses are based upon minimum radius of curvature obtained with crack L; hence, the others are percentages of the stress that would be obtained at this crack. Examining the figure, it may be noted that the smallest crack spacing results in the maximum stress, whereas, the medium crack spacing has the smallest stress. Another point of interest is that in all three cases, the maximum total deflection was approximately equal, but due to the character of the deflection distribution the stresses varied considerably.

Accuracy of Basin Beam. If equation (4) is differentiated in terms of the delta term, the following expression is obtained for predicting the error in the radius of curvature due to an error in the delta term:

$$dR = -\frac{16.67}{\Delta^2} d\Delta \dots \dots \dots \dots \dots \dots \dots \quad (8)$$

A limited study was conducted at nine different locations to evaluate the instrument error of the Basin Beam. The results of this investigation are presented in Table 2.1. At these various locations, the truck and beam were placed in accordance with the procedure outlined in the Appendix, then the truck was moved down the pavement until

TABLE 2.1

EVALUATING THE ACCURACY OF THE BASIN
BEAM FOR PREDICTING RADIUS OF CURVATURE

Δ (inches) $\times 10^{-3}$	Difference in Initial & Final Dial Gauge Reading $\times 10^{-5}$	Replication Error (feet)	Instrument Error (feet)
4.10	10	99	50
2.70	20	457	115
3.21	10	162	81
2.69	20	460	115
2.00	10	417	209
3.50	30	408	68
6.40	28	114	20
4.93	5	34	35
7.68	30	84	14
Averages		248	79

no change was noted in the dial reading. After the dial reading stabilized, the truck was backed to its original location, and a final reading was taken on the dial gage. This difference was considered to be the error in reproducibility or replication. These values were then entered in equation 8 to obtain the effect on radius of curvature. The instrument error was obtained by using the same delta value and multiplying by the resolution of the dial gage (a value of 0.00005 of an inch was used since readings can be estimated to one-half of a division).

The data shows the instrument error would affect the radius of curvature by an average value of \pm 79 feet. Using a radius of curvature of 5,000 feet which is relatively small, the instrument error is less than two percent. The average replication error of the instrument affects the radius of curvature by a value of \pm 248 feet. Using the same 5,000 feet for radius of curvature, the replication error is less than five percent.

Conclusions

This study warrants the following conclusions:

1. The Basin Beam gives an additional factor to use in comparing the deflection characteristics of concrete pavement. The data points out that pavements with the

same maximum deflection can have marked differences in stress distribution.

2. The data obtained with the Basin Beam is a measure of the stress characteristics of the pavement.
3. The Basin Beam gives excellent and reproducible results.

Section III

DEFLECTION TRUCK

Truck and Equipment

The truck is a large stake body type rated at three tons. It is equipped with a tool box near the cab, a large box for dead load, and a water tank for additional weight. (See Figures 3.1 and 3.2).

After much discussion it was decided to assemble the load on the truck in two ways: (1) A dead load of a low cost material with a high specific gravity. (2) A load which could be quickly and easily removed for easy travel from test site to test site. The removable load should be such that small variations could be made to account for additional weight placed on or taken off after the initial calibration of weight. It was thought that by this manner additional equipment and loss of gasoline could be accounted for.

Stationary Load

The dead load, which was placed in the large box, consisted of sign blanks and steel shot. The sign blanks are 1/8 inch steel plate which are obsolete highway signs. The steel shot, which is more easily placed, was used to balance the load so that an equal load was on each rear wheel. The shot was also obsolete having been previously used in core drilling rigs.

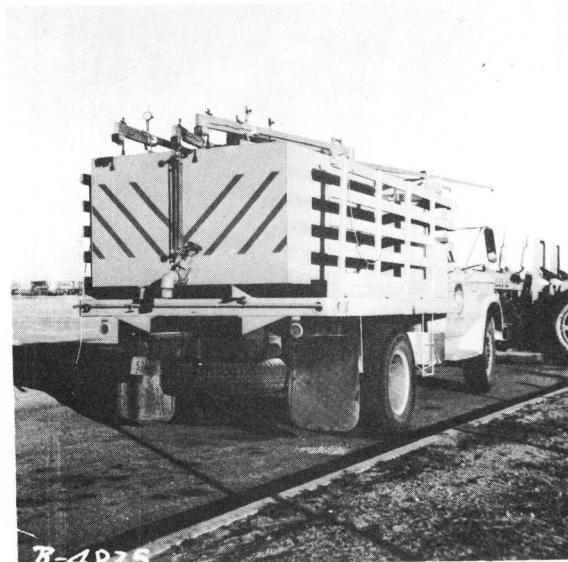


Figure 3.1 Completed Truck Carrying Benkelman Beams On Location

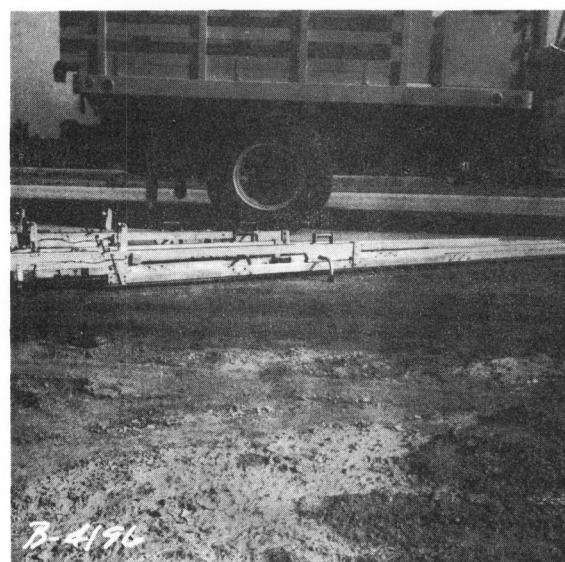


Figure 3.2 From Left to Right The Water Tank, Shot Box, and Tool Box

Removable Load

Water was used as the removable load with a steel tank as the container. The tank was equipped with glass sight tubes front and rear for easy control of water level in adjusting for weight. A figure of plus or minus one percent of the rear axle weight was selected as a point in which a weight change would be made. The truck was weighted by two methods (public scales and loadometer scales) after the addition of each individual piece of equipment.

The finished truck was one in which the load could be reduced for travel and also produce a dead load ranging from 11,000 to 23,000 pounds on the rear axle, as may be required by present or future need.

Section IV

DEVELOPMENT OF TESTING PROCEDURES

Background

At the outset of the statewide deflection study, operational procedures and equipment limitations were somewhat unfamiliar and it was deemed necessary that the test procedures be continually refined to assure that the data taken would be representative and uniform. Procedures used at the AASHO Road Test were to be followed insofar as possible^{1,8} and were to be modified to fit the needs of the Texas program. Several studies were conducted to develop and check these test procedures prior to the start of data taking connected with the main factorial experiment. This experimental stage was designated as a part of Phase I of the primary study. Most Phase I studies were conducted in connection with overnight data runs on continuous pavements in Colorado, Jefferson, and Smith Counties. The selected study areas permitted a comparison of results obtained under various climatic and subgrade conditions.

Procedures Used In Deflection Studies

The AASHO Road Test had shown that in order to obtain a replication of results it was necessary to make several passes with the deflection truck to "iron" out the site. Deflection readings were taken after each pass and it was found that by the third pass, replication of results was

possible. Present procedure calls for ironing each point with three passes of the deflection truck prior to taking a deflection reading.

Since the deflection basin of a rigid pavement usually extends a considerable distance behind the truck, the Benkelman Beam was placed on the shoulder as was the case at the Road Test in order to keep the support feet out of the deflection basin. In investigating various placement angles, one beam was placed in position to take deflections and a second beam was placed so that it would check the deflection present at the feet of the first beam. A range of angles was found to be satisfactory; therefore, the 30 degree angle with the pavement edge used at the AASHO Road Test was selected.

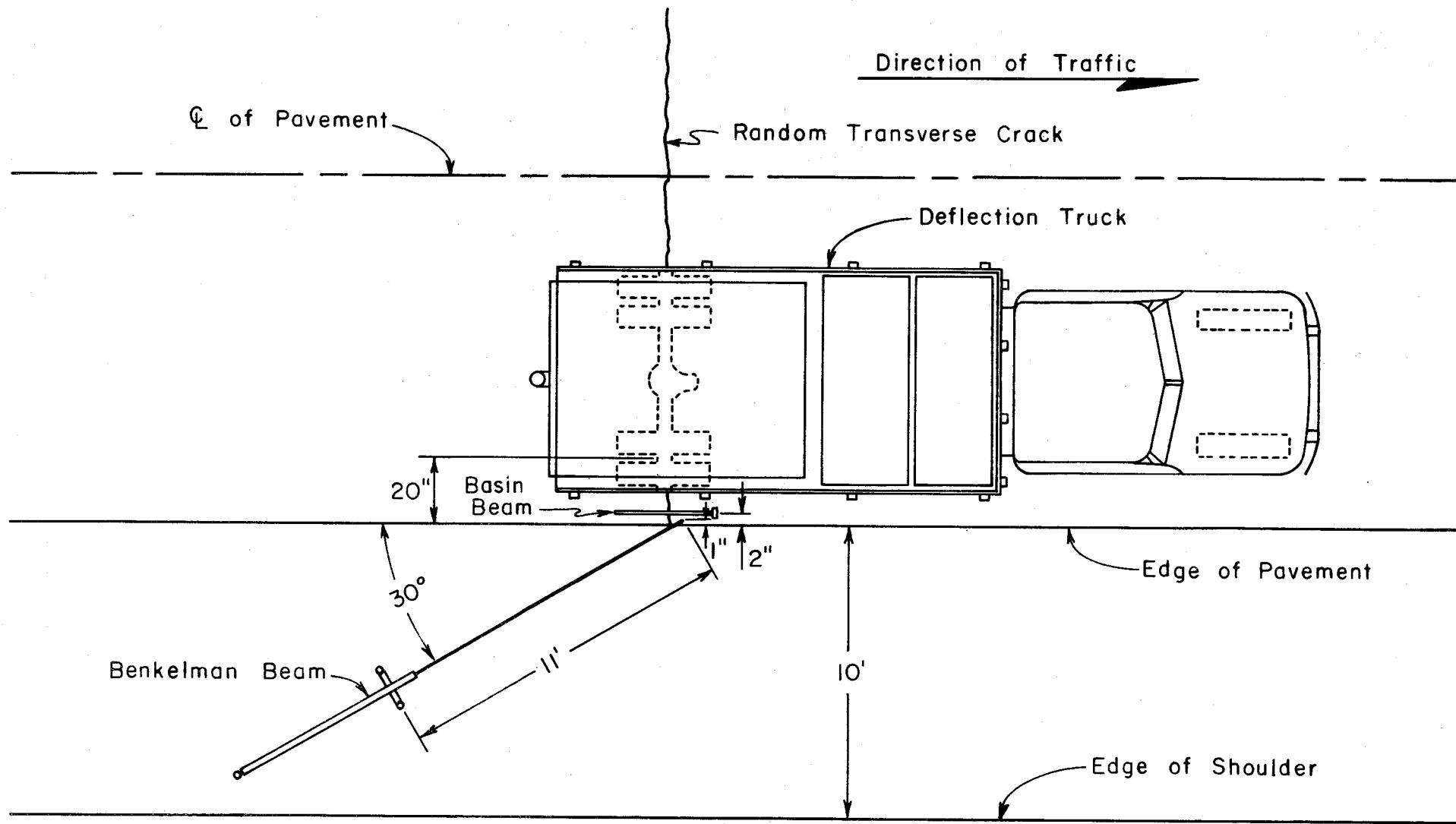
In the AASHO Road Test procedure, the wheel was placed so that the load was on the slab just in front of the joint. Due to the presence of volume change cracks in continuous pavements in lieu of joints, the procedure was revised. Tests were conducted with the wheel load placed in front of the crack, behind the crack, and directly over the crack, but due to the continuity of the slab, there was very little difference in deflection with placement variation provided the end of the Benkelman Beam was centered with the load. Lateral placement affected the deflection as shown by

readings taken with the load placed at the edge and at various distances in from the edge in two inch increments.

For purposes of taking deflection readings in the main factorial, the procedure was standardized as follows: (See Figure 4.1)

1. The wheel load is centered directly over the crack with the center of the dual wheels located twenty inches from the edge. A two inch variance is allowed in the transverse placement.
2. The Benkelman Beam is placed on the shoulder behind the truck at a 30 degree angle to the roadway with the probe pointing towards the load. The end of the probe is placed on the pavement one inch from the edge and centered on the load.

The overnight studies showed that deflection is influenced by crack width, but a practical method of obtaining these measurements is not presently available. In the earlier stages of study, crack width was taken with a microscope, but subsequent examination of this data has proved of little value. The measurement obtained was only the surface crack and was greatly influenced by traffic wear and not representative of the true crack. Present procedure does not attempt to take crack measurements.



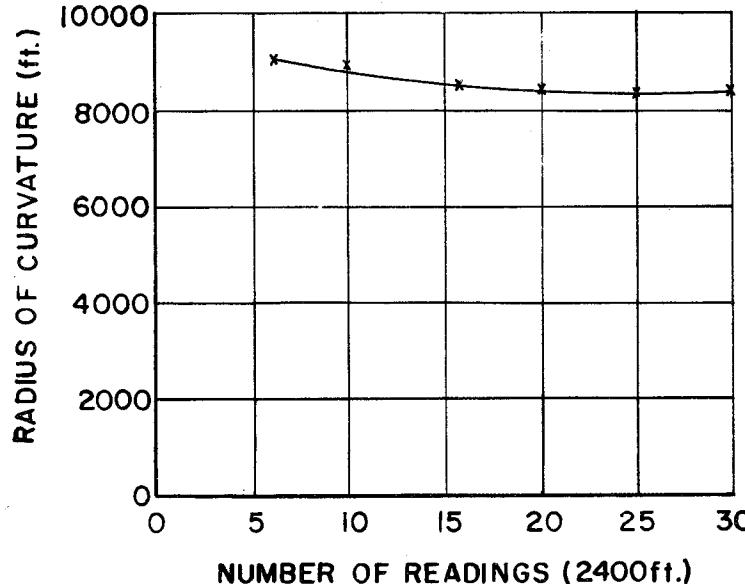
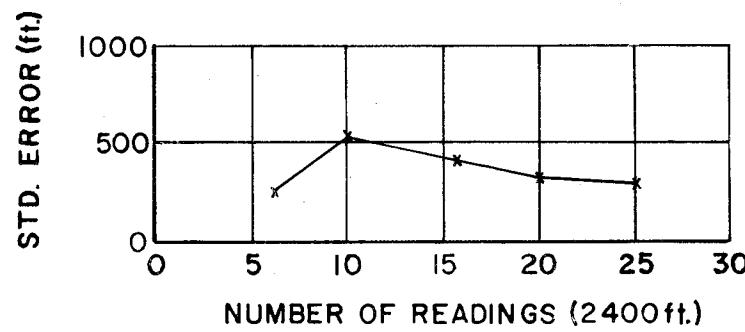
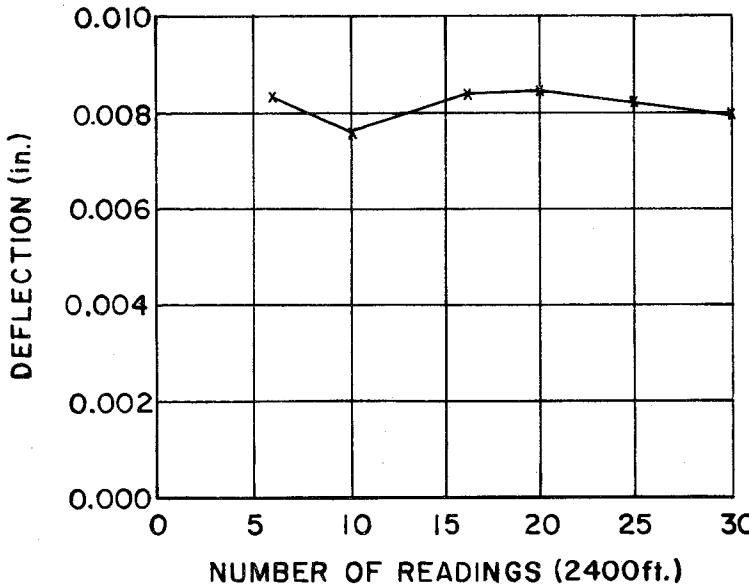
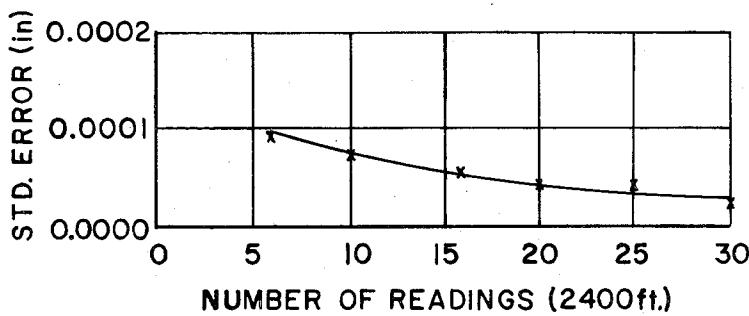
PLAN VIEW OF EQUIPMENT ARRANGEMENT TAKING DATA

Figure 4.1

During the earlier stages of data taking, deflection readings were taken at thirty cracks and mid-way between cracks at thirty locations within each 1200 foot section. In the interest of conserving time, this was later lowered to twenty then to fifteen and later to ten points each for a 1200 foot section. In order to find the minimum number of points necessary to obtain good data, a test was conducted on sections in the Waco District in which 30 points were taken in a 2400 foot section. The data was then analyzed several times using a different number of points each time the data was analyzed. The readings in each group were selected by means of a table of random numbers, and the results of each group were plotted against number of points. From these curves (A typical curve is shown in Figure 4.2), Mean Deflection vs. Number of Points and Standard Error vs. Number of Points, it was decided that fourteen data points per 2400 feet would give representative data. Present procedure calls for taking data at fourteen cracks and fourteen "mid-spans" in each 2400 foot section, thus giving an approximate 200 foot interval between points of the same type.

Procedures Used In Basin Studies

After the Basin Beam was developed, it was incorporated into the data collecting procedures for the main factorial experiment.



TYPICAL PLOTS OF AVERAGE DATA VALUES VS. NUMBER OF READINGS TAKEN

The Basin Beam is placed parallel to the edge of the roadway between the probe of the Benkelman Beam and the truck tire (about 2 inches in from the edge) and located so that the center, or moveable point, is directly in front of the probe. When taking readings at the crack, care must be taken to insure that the moveable point of the Basin Beam is not actually in the crack but just to one side of it. In this manner, a measurement of the radius of curvature can be obtained at the same time the deflection is obtained with the Benkelman Beam.

To establish the length of the deflection basin, the Basin Beam is set in place at a crack with the wheel load located on the crack. The pavement is then marked off in two foot intervals and a reading taken at the crack; the truck is then moved two feet forward and another reading taken with the Basin Beam remaining at the crack. This process is continued until the reading becomes constant. The length of the deflection basin is then taken as twice the distance from the crack to the point where the truck is located when the reading at the crack becomes constant. Tests conducted on in-service sections showed that one such test in 1200 feet was representative of this section. Present procedure calls for one such test within each 1200 foot section.

Miscellaneous Checks

Various other minor checks are made to insure uniformity of the data. The load was measured on both loadometer and public scales and this load is maintained or varied by manipulation of the water level in the water tank. Tire pressure is maintained at 75 psi to keep the contact area uniform. Calibration of the Benkelman Beams is checked periodically.

Conclusion

Use of the equipment involved in the main factorial experiment and the procedures used have been an evolution of a need for a rapidity in data taking due to the size of the experiment and a need for sufficient data to satisfy the statistical methods used in the analysis of this data. These procedures are reviewed periodically with both these objectives in mind.

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A P P E N D I X

A P P E N D I X

Selection of Thermometer Bulb Spacing

Due to the fairly large diameter (5/16 inch) of the thermometer bulbs, it was considered advisable to have sufficient concrete cover, (3/4 inch), to prevent spawling from occurring above the element. The lower thermometer bulb was placed at a depth to most nearly measure a differential that might occur in an eight-inch pavement slab. This left a slightly greater dimension from the bottom thermometer to the subbase but it was felt that this would not be significant in view of the more gradual temperature changes occurring on the "earth" side of the pavement.

**Derivation of the Exponential Term Used
for Temperature Correction**

Report 5, the AASHO Road Test, in its findings for jointed reinforced concrete pavement presents the following formulas for deflection as a function of design, load, and temperature under single axle loading:

Static Edge Deflection:

$$\frac{d'e}{L_1} = \frac{0.00883}{10^{0.0075T} D_2^{1.178}} \quad (\text{Page } 193)$$

Static Corner Deflection:

$$\frac{d'c}{L_1} = \frac{0.013}{10^{0.015T} D_2^{1.18}} \quad (\text{Page } 194)$$

where:

$d'e, c$ = Static rebound deflection, inches

L_1 = Single axle load, kips

T = Temperature differential, °F

D_2 = Pavement depth, inches

By holding the load and pavement depth constant, these formulas can be rewritten in the following form:

Static Edge Deflection:

$$d'e = \frac{0.00883 L_1}{D_2^{1.178}} \cdot \frac{1}{10^{0.0075T}} = K_1 \frac{1}{10^{0.0075T}}$$

$$d'e = K_1 10^{-0.0075T} \quad \text{where: } K_1 = \frac{0.00883 L_1}{D_2^{1.178}}$$

Static Corner Deflection:

$$d'_c = \frac{0.013 L_1}{D_2 1.18} \quad \frac{1}{10^{0.015T}} = K_2 \frac{1}{100.015T}$$

$$d'_c = K_2 10^{-0.015T} \quad \text{where: } K_2 = \frac{0.013L_1}{D_2 1.18}$$

Since no continuous pavements were studied at the Road Test, no specific formula was given for temperature correction for this pavement type, but it was felt that a continuous pavement would more nearly act like the edge condition due to the continuity of the steel and better granular interlock of the concrete. Initial plots of deflection taken midway between cracks vs. deflection taken at a crack tended to bear out the continuity obtained across the crack so that this correction factor was accepted until such time that a more specific exponential term could be developed.

Basin Beam

Operating Procedure

The procedure for operating the Basin Beam is as follows:

1. The Basin Beam is placed two inches from, and parallel to the edge of the pavement. For measurement of the radius of curvature over the crack, the center point is placed just far enough from the crack to insure a good bearing. A slight hand pressure is placed on both ends of the Basin Beam to be sure that the outer points are firmly set.

Midspan measurements are made by placing the center point midway between two cracks that are at least 42 inches apart and repeating the procedure described above.

2. With the basin beam in place, the pavement is "ironed" by slowly running the load wheels of the truck back and forth a minimum of three times in the lane at the point at which the measurement is to be made. This "ironing" process is necessary to obtain consistent results. At the completion of the ironing process, the load wheel of the truck is placed parallel to the Basin Beam with the center of the tire load located 20 inches from the pavement edge and centered on the center point of the beam. The electric vibrator switch is then turned on and the dial gage set to zero.

3. The truck is moved forward until the load is outside the zone of influence, a minimum of 50 feet, and the

dial gage read to the nearest ten thousandth of an inch.

4. The final operation is performed by turning off the vibrator switch and proceeding with the next measurement, alternating between crack and midspan measurements as described above.

Derivation of a Formula

for

Radius of Curvature

With three points given on a circle a radius can be calculated using the following equations:

$$a. \quad x^2 + y^2 + Dx + Ey + F = 0$$

$$b. \quad r = 1/2 \sqrt{D^2 + E^2 - 4F} \text{ (in inches)}$$

(Page 71 & 72, New Analytic Geometry by Smith, Gale, Neelley)

By substituting the points in for x and y in equation (a), three equations may be formed.

$$\text{Point 1} - (x_1 \quad y_1)$$

$$\text{Point 2} - (x_2 \quad y_2)$$

$$\text{Point 3} - (x_3 \quad y_3)$$

$$(1) \quad x_1^2 + y_1^2 + Dx_1 + Ey_1 + F = 0$$

$$(2) \quad x_2^2 + y_2^2 + Dx_2 + Ey_2 + F = 0$$

$$(3) \quad x_3^2 + y_3^2 + Dx_3 + Ey_3 + F = 0$$

Solving these equations simultaneously the following may be

obtained:

$$E = \frac{(x_2^2 - x_1^2)z - (x_3^2 - x_1^2) + (y_2^2 - y_1^2)z - (y_3^2 - y_1^2)}{(y_3 - y_1) - z(y_2 - y_1)}$$

$$\text{Where : } z = \frac{(x_3 - x_1)}{(x_2 - x_1)}$$

$$D = - \frac{(x_3^2 - x_1^2) + (y_3^2 - y_1^2) + (y_3 - y_1) E}{(x_3 - x_1)}$$

$$F = - (x_1^2 + y_1^2 + Dx_1 + Ey_1)$$

It was decided after several trial calculations to delete D and F since in this application the quantities which they represent are minute when compared to E.

Substituting into equation:

$$r = 1/2 \sqrt{D^2 + E^2 - 4F} \quad (\text{in inches})$$

we obtain

$$r = 1/2 \sqrt{E^2} \quad \text{or} \quad r = 1/2 E \quad (\text{in inches})$$