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of the splitting tensile te	st and the fle	exure beam test, for quality control and
design purposes.		
This report contains a	summary of th	e activities related to the development.
application and use of the	splitting tens	ile test and of the flexure beam test to
estimate the tensile streng	th of portland	cement concrete pavement. Information
related to the descriptive	statistics ass	ociated with each test is also summarized.
This includes mean strength	values, stand	ard deviations and coefficients of vari-
ation of concrete batches a	ctually tested	
Finally an empirical	equation relat	ed to transforming the tensile strength
measured by the flexure bea	m test to a te	esile strength value measured by the
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There was no invention or discovery conceived or first actually reduced to practice in the course of or under this contract, including any art, method, process, machine, manufacture, design or composition of matter, or any new and useful improvement thereof, or any variety of plant which is or may be patentable under the patent laws of the United States of America or any foreign country.

## PREFACE

This report is presented for Research Project 3-8-85-432, "An Evaluation of Tensile Strength Testing as a Means of Quality Control for Portland Cement Concrete Pavements." The work accomplished and summarized in this report has been subdivided into the following categories:

- (1) Flexure Beam Test,
- Splitting Tensile Test,
- (3) Testing Program, and
- (4) Relationship between the Splitting Tensile Test and the Flexure Beam Test.

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Appreciation is also extended to District personnel, who worked closely with the project; to the student employees, who provided the strong backbone for the labor intensive laboratory portion of this project; and to the staff of the Center for Transportation Research, whose assistance has been essential to the conduct of this study.

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# LIST OF REPORTS

Report No. 432-1F, "An Evaluation of Tensile Strength Testing," by L. Maureen Melis, A. H. Meyer, and D. W. Fowler, describes the analysis associated with the comparison of the splitting tensile test and the flexure beam test, for quality control and design purposes.

## ABSTRACT

This report summarizes the findings of Project 3-8-85-432, "An Evaluation of Tensile Strength Testing as a Means of Quality Control for Portland Cement Concrete Pavements," and describes a series of research activities related to the comparison of the splitting tensile test and the flexure beam test, for quality control and design purposes.

This report contains a summary of the activities related to the development, application and use of the splitting tensile test and of the flexure beam test to estimate the tensile strength of portland cement concrete pavement. Information related to the descriptive statistics associated with each test is also summarized. This includes mean strength values, standard deviations and coefficients of variation of concrete batches actually tested.

Finally, an empirical equation related to transforming the tensile strength measured by the flexure beam test to a tensile strength value measured by the splitting tensile test is provided.

KEYWORDS: Splitting tensile test, flexure beam test, tensile strength, portland cement concrete, pavements, indirect tensile test, concrete pavements, flexural strength, cylinders, test methods.

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## SUMMARY

The splitting tensile test is a practical and effective test for determining the tensile strength of portland cement concrete pavements. The test is widely accepted, both in the United States and Europe, and should be implemented as an alternative to the flexure beam test.

The design variables associated with concrete pavements vary significantly, depending upon the application. Information relating to the mean values of the measured tensile strength, as well as the descriptive statistics associated with the measured tensile strength values, is contained in this report.

Information and recommendations related to the above concepts are also summarized in this report.

#### IMPLEMENTATION STATEMENT

Steps should be taken to begin the implementation of the use of the splitting tensile test as an alternative to the flexure beam test. The use of the splitting tensile test, versus the flexure beam test, does not affect the reliability or variability of the test results. The necessary cylinders are easier to prepare than the flexure beams particularly the 4 inch x 8 inch cylinders. Current ASTM standards would mandate the use of 6 x 12 cylinders when 1-1/2 inch aggregate is used in the mix. However, it should be noted that 4 x 8 cylinders were used with 1-1/2 inch aggregates in this study with no apparent difficulties. The test is relatively easy to perform provided the necessary compresion testing equipment is available (60,000 pound capacity machine for  $4 \times 8$  cylinders and 120,000 pound capacity machine for 6 x 12 cylinders). Most SDHPT district laboratories do not, at present, have this type of equipment. The necessary equipment to implement this program on a state wide basis would be a major undertaking and would require a large expenditure of funds. Consideration was given to retrofitting the flexure beam test equipment, however none of these machines have the necessary capacity.

It is recommended that in those districts where the necessary compression testing equipment exists that for selected projects, both cylinder samples and beam samples be taken to (1) determine if indeed the cylinders are easier to prepare and handle than are beams and (2) to verify the transformation equation with additional field data.

Then, if the SDHPT ever adopts an end product specification or desires to perform strength tests on in-place concrete, the necessary correlation will be available. It should be noted that insitu cores (cylinders) are easier to obtain than insitu beams and cores are routinely taken now for thickness determination.

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## CHAPTER 1. INTRODUCTION

## BACKGROUND

Concrete pavements are typically designed based on the concrete(s tensile strength, as measured by the modulus of rupture. The modulus of rupture is determined by the flexure beam test. The two most commonly used flexure beam tests are the center point loading test and the third point loading test. The Texas State Department of Highways and Public Transportation (SDHPT) presently uses the center point loading test which involves midspan loading of a concrete beam with a square cross section and length equal to three times its depth. These specimens fail in tension when the ultimate flexural strength of the concrete has been reached. The modulus of rupture formula that is used to calculate the tensile stress is based on assumptions that do not hold at high stresses approaching failure; thus, the calculated modulus of rupture is fictitiously high when compared to the "true" tensile strength. The "true" tensile strength being the stress at rupture a uniaxial tension test, if one could be performed for concrete. Despite these shortcomings, the flexure test is a convenient measure for purposes of evaluation and is, therefore, often used (Ref 1).

The flexural beam testing apparatus used today has undergone only minor modifications since its inception in 1930. Positioning of the beam in the testing apparatus is often awkward. Each 6 x 6 x 20-inch concrete beam weighs approximately 60 pounds, and during casting the concrete is placed in a beam mold weighing from 10 to 35 pounds. Moving and lifting these specimens represents a significant burden to personnel and a potential safety hazard.

In the search for new methods for evaluating the tensile strength properties of concrete, the splitting tensile test was developed independently in the 1950[s by Carneiro (Ref 2) in Brazil and by Akazawa (Ref 3) in Japan. The splitting tensile test has been described under a series of names including: Brazilian split test, split test, diametral test, resilient modulus test, and Schmidt test, as well as indirect tension test.

1

The relationship between the splitting tensile strength and the flexural strength of concrete has been the subject of considerable study, including work by Narrow (Ref 4) and Greer (Ref 5). Both of these studies indicate a

correlation between flexural strength and splitting tensile strength for their tests.

Several highway agencies have already recommended the splitting tension test as an alternative to the flexure beam test. The Connecticut Department of Transportation found that the field test results correlating flexural and splitting tensile strengths deviated somewhat from the confidence bands established from their laboratory study (Ref 6). However, they did recommend the splitting tensile test as an alternative to beam breaks. In addition, Alabama has recommended representative values of the splitting tensile test for use in the "Alternative Procedures for Design of Rigid Pavement" (Ref 7). In light of this, Project 3-8-85-432, "An Evaluation of Tensile Strength Testing Versus Flexural Strength Testing as a Means of Quality Control for Portland Cement Concrete Pavements," was sponsored by the Texas State Department of Highways and Public Transportation (SDHPT) and the Federal Highway Administration (FHWA) and was conducted through the Center for Transportation Research at The University of Texas at Austin. The objective of this study was to determine the reliability of the splitting tensile test compared to the flexure beam test, using concrete mixes and variables typical of Texas pavement concrete.

## SCOPE OF PROJECT

Test data for this report were obtained from concrete batches designed in accordance with an allowable range of Texas SDHPT specifications. Altogether, 720 beams and 1,260 cylinders were tested for this study. The purpose of this report is to briefly summarize the activities, findings, and recommendations associated with this project.

#### CHAPTER 2. FLEXURE BEAM TEST

#### INTRODUCTION

The flexure beam test is an indirect method by which the tensile strength of concrete is estimated. In the present test method used by the SDHPT, Test Method Tex-420-A, the test is conducted by loading a beam specimen at length midspan, as shown in Fig 2.1. This loading configuration causes development of tensile stresses in the lower half of the beam and compression stresses in the upper half of the beam, perpendicular to the direction of the applied load.

## FLEXURE BEAM STRESS

The theoretical maximum tensile strength, or modulus of rupture, is calculated using the following formula:

$$\sigma_{\rm FB} = \frac{Mc}{I}$$
(2.1)

where

$\sigma_{FB}$	=	stress in the fiber farthest from the neutral axis, psi;
М	=	bending moment at the section, in1b;
I	=	moment of inertia of the cross section, in. $^4$ ; and
с	=	distance from the neutral axis to the farthest fiber, inch.

The shear and moment diagrams for a beam subjected to centerpoint loading are shown in Fig 2.2. As is evident from the moment diagram, the centerpoint loading configuration subjects only the center plane of the beam to the maximum moment. Since the weakest plane may or may not occur at the maximum

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Fig 2.1. Isometric of flexure beam test.



Fig 2.2. Shear and moment diagrams for center point flexure beam test.

moment, previous experimental evidence indicates that the calculated stress results tend to have more data scatter than do other types of tensile stress measuring tests (Ref 8).

Additionally, the flexure beam test tends to overestimate the "true" tensile strength, largely due to the fact that the stress distribution across the depth of the specimen is assumed to be linear. The assumed linearity of the stress distribution is inaccurate since concrete produces a nonlinear stress-strain curve. The actual stress distribution just before failure is more parabolic than triangular. Therefore the stress calculated using Eq 2.1 is actually greater than the true tensile strength. This is illustrated in Fig 2.3.

## TEST PROCEDURES

The beam specimens are cast, cured and tested in accordance with Test Method Tex-420-A (Ref 9). The load is generally applied at a constant rate of 125 psi/min. until failure. The tensile stress is then calculated using Eq 2.1.

The specimens are tested in a Reinhart beam tester, as shown in Fig 2.4. The test model shown represents those typically used in the field by the Texas SDHPT. Figures 2.5 and 2.6 represent a typical fracture plane of the concrete beam, tested according to Test Method Tex-420-A.

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Fig 2.3. Stress distribution in the flexure beam test.



Fig 2.4. Loading flexure beam specimen.



Fig 2.5. Typical fracture plane of beam specimen.



Fig 2.6. Close-up of typical fracture plane of beam specimen.

#### CHAPTER 3. SPLITTING TENSILE TEST

## INTRODUCTION

The splitting tensile test is conducted by loading in compression a cylinder placed on its side with a single load applied parallel to and along the vertical diametral plane (Fig 3.1). This loading configuration causes the development of relatively uniform tensile stresses perpendicular to the direction of the applied load and along the diametral plane. These stresses eventually exceed the ultimate tensile strength of the concrete, which causes the cylinder to split along the vertical diameter.

The distribution of the stresses loaded in such a configuration was developed mathematically by Frocht (Ref 10). The theory assumes a point load on a thin plate of the cylinder. Under such conditions, the specimen would fail near the load points due to compressive stresses and not in the center portion of the cylinder due to tensile stresses. However, the load is actually distributed over a loading strip of one inch. This not only reduces the vertical compressive stresses but changes the horizontal stresses along the vertical diameter from tension to compression near the points of the load In addition, biaxial stresses are developed within the application. If the compressive stress is at least three times the tensile specimen. strength, as it invariably is in concrete, the failure is in tension along the vertical diametral cross section. The stress distribution along the horizontal and vertical axis under such a loading is shown in Fig 3.2 (Ref 11).

## SPLITTING TENSILE TEST

The magnitude of the tensile stress is calculated using the following equation:



Fig 3.1. Isometric of splitting tensile test.

Ρ



Fig 3.2. Relative stress distributions and center element showing biaxial state of stress for the splitting tensile test (Ref 11).

$$\sigma_{\rm ST} = \frac{2P}{\pi LD}$$
(3.1)

### wher e

σst	=	tensile stress developed in center fibers, psi;
Р	=	applied load, 1bf;
L	=	length of specimen, in.; and
D	=	diameter of specimen, in.

TEST PROCEDURES

Specimens are prepared and tested in accordance with ASTM C496 (Ref 12). The test is performed on cylinders of standard size with lengths equal to twice the diameter. The bearing strips are 1-inch-wide x 1/8-inch-thick plywood. The specimen is aligned in the machine and the load is applied at a rate of 100 to 200 psi/min. until the specimen fails. The tensile stress is then calculated using Eq 3.1.

The specimens are tested in a 120-kip compression testing machine, as shown in Fig 3.3. Figure 3.4 represents a fracture plane in the vertical diameter, while Fig 3.5 shows a close-up of the same fracture plane.

i,



Fig 3.3. Loading 6 x 12-inch cylinder specimen.



Fig 3.4. Typical fracture place of splitting tensile test specimen.



Fig 3.5. Close-up of typical fracture plane of splitting tensile test specimen.

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## INTRODUCTION

The objective of this portion of the study was to examine the relationships between the splitting tensile test and the flexure beam test as influenced by several design variables. This objective was accomplished by comparing the splitting tensile test results with the flexure beam test results on a one to one basis.

## DESIGN MATRIX

In order to examine these relationships it was necessary to incorporate those variables that aftect the tensile strength of the concrete. After discussions with engineers from the Texas State Department of Highways and Public Transportation it was agreed to test eleven variables. Once these eleven variables were identified, they were prioritized by class and level, in order to test those variables of most interest. These eleven variable classes, ranked in order of testing priority, were:

Rank	Variable Class
1	Cement Factor
2	Cement Type
3	Air Content
4	Coarse Aggregate Type
5	Coarse Aggregate Size
6	Coarse Aggregate Factor
7	Admixtures
8	Water Cement Ratio
9	Age (Curing)
10	Slump
11	Temperature (Curing)

Each of these variable classes was subsequently divided into two or three levels. Each level represented an acceptable design value for pavement concrete conforming to Texas State Department of Highways and Public Transportation (SDHPT) Standard Specifications. The combination of the two or three levels represented an acceptable design range for concrete pavements. The variable classes and levels are shown in Table 4.1.

A full eleven by eleven variable matrix was developed, as shown in Table 4.2. Each block design consists of two test variables, i.e., a test variable from the column of variable classes and a test variable from the row of variable classes. This is most easily explained with an example:

From Table 4.2

	Cement <sup>1</sup>
	Factor
	(1)
Cement <sup>2</sup>	[ 211 ] <sup>3</sup>
Туре	212
(2)	213
	214
	215

where

1 =	test	variable	from	the	column	of	variables	classes,
-----	------	----------	------	-----	--------	----	-----------	----------

- 2 = test variable from the row of variable classes,
- 3 = block of batch design numbers of which both cement type and cement factor varied.

The batch number associated with each design block conformed to the following numbering convention:

Batch number representation "RCO"

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```
(1) Cement Factor<sup>*</sup> (CF)<sup>**</sup>
    1. 5 SKS/CY***
    2. 6 SKS/CY
(2) Cement Type (CT)
    1. Type I
    2. Type II
    3. Type III
(3) Air Content (AC)
    1. Medium (4-6%)
    2. High (8-10%)
    3. Low (1-3%)
(4) Coarse Aggregate Type (CAT)
    1. Silicious
    2. Limestone
(5) Coarse Aggregate Size (CAS)
    1. Grade No. 2 (1-1/2 in nominal size)
     2. Grade No. 5 (3/4 in nominal size)
(6) Coarse Aggregate Factor (CAF)
    1. Medium (0.75)
    2. High (0.80)
    3. Low (0.65)
(7) Admixtures (ADM)
    1. None (Except Air Entrainment)
     2. Water Reducer and Air Entrainment
     3. Accelerator and Air Entrainment
```

\*Variable Class (continued) \*\* Variable Class Abbreviation \*\*\* Variable Class Level

# TABLE 4.1. (CONTINUED)

```
(8) Water Cement Ratio (W/C)

Medium 0.50 (5.6 GALS/SCK)
High 0.554 (6.25 GALS/SCK)
Low 0.40 (4.5 GALS/SCK)

(9) Age (A) (Curing)

Standard (4, 7 days)
Long (28, 56 days)

(10) Slump (S)

Medium (1-3 in.)
High (5-7 in.)
Low (0-1 in.)

(11) Temperature (T)

Room (75'F)
High (100'F)
Low (50'F)
```
	C <del>em</del> ent Factor	Cement Type	Air Content	Coarse Aggregate Type	Coarse Aggregate Size	Coarse Aggregate Factor	Admixtures	Water Cement Ratio	Age	Slump	Temperature
¥ariables*	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Cement Factor (1)	-										
Cement Type (2)	211 *** 212 213 214 215										
Air Content (3)	311 312 314 315	321 323									
Coarse Aggregate Type (4)	411 412	422									
Coarse Aggregate Size (5)	511 512	521 523	531 532								
Coarse Aggregate Factor (6)	611 612 613 614				652 653 654						

## TABLE 4.2. MATRIX OF VARIABLES

\*see Table 4.1 \*\* Batch Number

(continued)

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	Cement Factor	C <b>em</b> ent Type	Air Content	Coarse Aggregate Type	Coarse Aggregate Size	Coarse Aggregate Factor	Admixtures	Water Cement Ratio	Age	Slump	Temperature
Variables*	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Admixture (7)	711 712 713 714	721 722 723	733 735 736		753 754				_		
Water Cement Ratio (8)	811 812 813	82 3 82 4	83 3 83 4 83 5	843	853 854						
Age (9)	912	922 923	931 932 933				971 972	981			
Slump (10)	1011 1012		1031 1032				1072				
Temperature (11)	1111 1112 1113 1114								1192 1193		
Number of Batcher	31	12	13	1	7		3	1	2	• ;	70

\* see Table 4.1
\*\* Batch Number

R = test variable from the row of variable classes,
C = test variable from the column of variable classes, and
0 = test order.

An example of this numbering convention is:

Batch number "211"

#### where

where

2	3	cement type variable class,
1	2	cement factor variable class, and
1	=	level one; in this case cement type I and cement factor 5.

## BATCH DESIGN

Each batch was designed to test two variables. The expanded factorial of batch design numbers is shown in Table 4.3. From this table it is evident which two variables were tested with each batch.

The batch design calculations were performed using the Concrete Design Work Sheet as used by the Texas State Department of Highways and Public Transportation (SDHPT). The reader is referred to Ref 13 for instruction in the use of the work sheet. Each batch was designed to yield a volume of 6.75 cubic feet.

#### STANDARD BATCH

A standard batch was designed as a reference to represent a typical batch of concrete pavement. The standard batch variable classes and their associated levels included:

## RR432-1F/04

		Combina				
<b>a</b> riables	Batch Number	Variable Class Levels		Batch** Number	Batch Reference	
67-65	21.1		055		Standand CT I	
	211				Stanuaru ti I	
	212				Chandrand CT II	
	213		055		Standard CI II	
	214			422		
	215	CT III CT III	CF6	422		
AC ~C F	311	High	CF5			
		Medium	CF 5	211		
	312	Low	CF5			
	314	High	CF6			
		Medium	CF 6	212		
	315	Low	CF6			
AC -C T		High	CTI	31 1		
		Medium	CTI	211		
		Low	CTI	31 2		
	321	High	CTII			
		Medium	CTII	21 3		
	323	Low	CTII			
	*	High	CTIII			
		Medium	CTIII	422		
	*	Low	CTIII			
CAT-CF	411	Limestone	CF 5			
		Silicious	CF 5	211		
	412	Limestone	CF 6			
		Silicious	CF 6	212		
CAT-CT		Limestone	CT I	41 1		
		Silicious	CT I	211		
	*	Lime	CT II			
		Silicious	CT II	21 3		
	*	Lime	CT III			
	422	Silicious	CT III		Standard CT III	

# TABLE 4.3. BATCH DESIGN VARIABLES

\*Combination not tested.

\*\* Variable class level already tested.

		Combi			
Variables	Batch Number	Variable Leve	Class Is	Batch** Number	Batch Reference
CAS-CF		1-1/2"	CF 5	211	
		1-1/2"	CF6	212	
	511	3/4"	CF5		
	512	3/4"	CF6		
T1-241		1-1/2"	CT I	211	
		3/4"		511	
		1-1/2"		213	
	521	3/4"		215	
	521	1-1/2"		422	
	523	3/4"	CT III	422	
CAS-AC		1-1/2"	High	311	
		1-1/2"	Medium	211	
		1-1/2"	Low	312	
	531	3/4"	High		
		3/4"	Medium	511	
	532	3/4"	Low		
CAE-CE	611	High	CE 5		
	011	Medium	CES	211	
	612		0.65	211	
	613	High	CF6		
	•••	Medium	CF6	212	
	614	Low	CF6		
CAF-CAS		High	1-1/2"	611	
		Medium	1-1/2"	211	
		Low	1-1/2"	612	
	653	High	3/4"		
		Medium	3/4"	511	
	654	Low	3/4"		

\*\* Variable class level already tested.

Variables	Batch Number	Variable Class Levels		Batch** Number	Batch Reference	
ADM-CF		None	CF 5	211		
	711	WR	CF5			
	712	ACC	CF5			
		None	CF6	212		
	713	WR	CF6			
	714	ACC	CF 6			
ADM-CT		None	CT I	211		
		WR	CT I	711		
		ACC	CT I	712		
		None	CT II	213		
	721	WR	CT II			
	722	ACC	CT II			
		None	CT III	422		
	723	WR	CT III			
	*	ACC	CT III			
ADM-AC		None	High	311		
		None	Medium	211		
		None	Low	312		
	733	WR	High			
		WR	Medium	711		
	*	WR	Low			
	735	ACC	High			
		ACC	Medium	712		
	736	ACC	Low			
ADM-CAS		None	1-1/2"	211		
		WR	1-1/2"	711		
		ACC	1-1/2"	712		
		None	3/4 •	511		
	753	WR	3/4"			
	754	ACC	3/4"			

\*\* Variable class level already tested.

Variables	Batch Number	Varia L	ble Class evels	Batch** Number	Batch Reference	
W/C-CF	811	High	CF5			
		Medium	CF5	211		
	812	Low	CF5			
	813	High	CF6			
		Medium	CF6	212		
	*	Low	CF6			
W/C-CT		High	CT I	811		
		Medium	CT I	211		
		Low	СТ І	812		
	823	High	CT II			
		Medium	CT II	812		
	824	Low	CT II			
	*	High	CT III			
		Medium	CT III	422		
	*	Low	CT III			
W/C-AC	834	High	High			
		Medium	High	311		
	833	Low	High			
		High	Medium	811		
		Medium	Medium	211		
		Low	Medium	812		
	835	High	Low			
		Medium	Low	312		
	*	Low	Low			
W/C-CAT		High	Silicious	811		
		Medium	Silicious	211		
		Low	Silicious	812		
	843	High	Limestone			
		Medium	Limestone	411		
	*	Low	Limestone			

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\*\* Variable class level already tested.

## TABLE 4.3. (CONTINUED)

		Coml				
Variables	Batch Number	Variab Le	le Class vels	Batch** Number	Batch Reference	
W/C-CAS		1-1/2"	Hiah	811		
		1-1/2"	Medium	211		
		1-1/2"	Low	812		
	853	3/4"	Hiah			
		3/4"	Medium	511		
	854	3/4"	Low			
A-CF		Standard	CF 5	211		
		Standard	CF6	212		
		Long	CF 5	932		
	912	Long	CF6			
A-CT		Standard	CT I	211		
		Standard	CT II	213		
		Standard	CT III	422		
		Long	CT I	932		
	922	Long	CT 11			
	923	Long	CT III			
A-AC		Standard	High	311		
		Standard	Medium	211		
		Standard	Low	312		
	931	Long	High			
	932	Long	Medium			
	933	Long	LOW			
A-ADM		Standard	None	211		
		Standard	WR	711		
		Standard	ACC	/12		
		Long	None	932		
	971	Long	WR			
	972	Long	ACC			

\*Combination not tested.

**\*\*** Variable class level already tested.

		Com				
Variables	Batch Number	Variab Le	le Class vels	Batch** Number	Batch Reference	
A-W/C		Standard	High	911		
A-W/C		Standard	niyn Modium	211		
		Standard	Meurum	211		
	0.01	Stanuaru	LUW	812		
	981	Long	nign Maddum	020		
		Long	Mealum	932		
		Long	LOW			
S-CF	1011	Hiah	CF 5			
		Medium	CF5	211		
		Low	CF 5			
	1012	Hiah	CF6			
		Medium	CF6	212		
	*	Low	CF6			
S-AC	1022	Uiab	Uiah			
3-AC	1032	n iyn Modfum	n iyn Nigh	21.1		
	*	Medium	n ign High	311		
	1021		пign Modium			
	1031	Hign Maddum	Medium	21.1		
	+	Meatum	Medium	211		
	*		Medium			
	•	High	LOW	21.0		
	+	Medium	LOW	312		
	-	LOW	LOW			
S-ADM		High	None	1031		
		Medium	None			
	*	Low	None	211		
	1072	High	WR			
		Medium	WR	711		
	*	Low	WR			
	*	High	ACC			
		Medium	ACC	712		
	*	Low	ACC			

**\*\*** Variable class level already tested.

Variables	Batch Number	Va	riable Class Levels	Batch** Number	Batch Reference
T-CF	1111	High	CF 5		
	1112	High	CF 6		
		Room	CF5	211	
		Room	CF6	212	
	1113	Low	CF 5		
	1114	Low	CF6		
T-A		High	Standard	1111	
		Room	Standard	211	
	1192	Low	Standard	1113	
	1193	High	Long		
		Room	Long	932	
	*	Low	Long		

\*\* Variable class level already tested.

	Standard Batch*							
	Variable Class	Level						
1.	Cement Type	Туре І						
2.	Coarse Aggregate Size	l-1/2 inch nominal size						
3.	Cement Factor	5						
4.	Admixture	Air Entrainment						
5.	Air Content	5% <u>+</u> 1%						
6.	Water Cement Ratio	5.0 (5.6 Gal/Sck)						
7.	Slump	1-3 in.						
8.	Temperature (Curing)	Room						
9.	Coarse Aggregate Factor	0.75						
10.	Coarse Aggregate Type	Silicious						
11.	Age (Curing)	4 and 7 days						

\*Batch number 211

## MIXING AND CASTING

Mixing was performed in a tilting drum type mixer with a 9-cubic-foot capacity rating. From each batch, eight 6 x 6 x 20-inch beams, eight 6 x 12-inch cylinders and four 4 x 8-inch cylinders were cast. The number of 4 x 8-inch cylinders was increased to eight midway in the project. The slump, air content, unit weight, and workability were recorded for each batch. The beams and cylinders were mixed, cast and cured according to Tex-420-A and Tex-418-A standards, respectively.

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## CHAPTER 5. FIELD TESTS

Field samples were obtained from four pavement concrete and one structural concrete construction projects in progress in four Districts around the state. Figure 5.1 shows the geographical distribution of the Texas State Department of Highways and Public Transportation Research Districts from which concrete samples were obtained. This includes locations in Houston, Dallas, Plainview, and Port Arthur. Researchers collected a total of twelve samples at each of five job site location, including four beam samples and four cylinder samples of both 12-inch and 8-inch length. A typical field sampling layout is shown in Fig 5.2. The specimens were cured overnight at the job site before being transported. Once these specimens were returned to the laboratory and cured, the tensile properties of the concrete field samples were estimated using the flexure beam procedure and the splitting tensile test procedure as summarized in Chapters 2 and 3, respectively.

In addition to the twelve specimens made at each location, two cores were taken from the hardened concrete at three of the paving locations. These cores were obtained during the routine coring operation procedure typically used by the Texas State Department of Highways and Public Transportation to determine the pavement thickness of newly constructed pavements. These cores were obtained with a 4-inch inside diameter core barrel, for the nominal depth of the concrete pavement.

Before the specimens were tested, the length was adjusted to 8 inches according to the standard ASTM-C42. All of the dimensions were then measured and recorded. The tensile property of the core samples was then estimated using the splitting tensile test procedure as summarized in Chapter 3.

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Fig 5.1. Districts (SDHPT) from which cores were obtained.



Fig 5.2. Typical field sampling layout.

#### CHAPTER 6. ANALYSIS AND DISCUSSION OF RESULTS

#### INTRODUCTION

The primary objective of this study was to determine if the splitting tensile test is a more reliable indicator of the tensile stress of concrete compared to the flexure beam test for quality control and design purposes.

The majority of this analysis was performed using SAS, a computer software system for data analysis. SAS runs on an IBM 370 computer under the VM/CMS host operating system. The batch computer program written for this report is available through the Center for Transportation Research (Ref 14).

## INDIVIDUAL TEST RESULTS

## Splitting Tensile Test

The descriptive statistics associated with the splitting tensile test 6 x 12-inch cylinders, are summarized in Table 6.1. A more detailed summary is available in Ref 15. The same values are shown for the 4 x 8-inch cylinders in Table 6.2, and in more detailed form in Ref 16.

Comparison of Table 6.2 with Table 6.1 reveals that the average value of the 6 x 12-inch cylinders for all test ages is appreciably smaller than the average value of the 4 x 8-inch cylinders for a given test age, such that the mean strength of the 6 x 12-inch cylinders, is on the average, 87 percent of the mean strength of the 4 x 8-inch cylinders. Also, the standard deviation and coefficient of variation are greater for all cases with the 4 x 8-inch cylinders than with the 6 x 12-inch cylinders. This conforms to results of earlier tests by Wright (Ref 8) that show, in general, that an increase in specimen size is accompanied by a reduction in the observed strength and a decrease in the variability of the results. Additionally, each value represented by the 4 x 8-inch cylinder results was the average of either two or four tests. Those tests with only two test points typically showed an

## TABLE 6.1. SPLITTING TENSILE STRESSES, 6 x 12-INCH CYLINDERS

Age (days)	Number of Tests <sup>1</sup>	Mean Strength <sup>2</sup> (ps1)	Standard Deviation (psi)	Coefficent of Variation (psi/psi)
4	76	390	25	6
7	76	41 5	26	6
28	12	470	27	6
56	12	510	25	5
Total <sup>3</sup>	176	415	26	6

 $^{1}$ Each value is the average of 4 tests.  $^{2}$ To the nearest 5 psi.  $^{3}$ All variables.

Note: The mean, standard deviation and coefficient of variation are overall averages for respective individual batches.

# TABLE 6.2.SPLITTING TENSILE STRESSES,4 x 8-INCH CYLINDERS

Age (days)	Number of Tests <sup>1</sup>	Mean Strength <sup>2</sup> (psi)	Standard Deviation (psi)	Coefficent of Variation (psi/psi)
4	74	440	38	9
7	74	480	37	8
28	12	550	46	8
56	12	580	54	9
Total <sup>3</sup>	172	475	39	8

 $^{1}\text{Each}$  value is the average of either 2 tests or 4 tests.  $^{2}\text{To}$  the nearest 5 psi.  $^{3}\text{All}$  variables.

increase in the value of the standard deviation, and this also caused an increase in the coefficient of variation.

## Flexure Beam Test

The descriptive statistics associated with the flexure beam test are shown in Table 6.3, and in more detailed form in Ref 17.

### DISCUSSION

Comparison of Table 6.3 with Table 6.1 indicates that the variability of the 6 x 12-inch cylinders, as measured by the coefficient of variation is not significantly different than the variability of the results of the flexure beam test. This is in agreement with earlier research performed by Wright (Ref 8) in which he compared the variability of these two tests. His experiment involved preparing eight batches of concrete and then performing the tests when they were 28 days old. His results are summarized in Table 6.4.

It should be considered that these test results were obtained from carefully performed laboratory tests, and the control of these laboratory specimens is completely different than the actual control at a job site. It is well known that the flexure beam strength will be remarkably affected by the surface drying and shrinkage of the test specimens, but that the effect on the splitting tensile strength is much smaller under similar circumstances (Ref 8). This phenomena can be better understood after realizing that the maximum tensile stress occurs in the inner fibers of the splitting tensile specimen (Fig 3.2) and in the outer fibers in the flexure beam test (Fig 2.3). In the case of the flexure beam test, the dry surface will reduce the flexure beam strength considerably. This drying out effect will more likely occur in the field than in the laboratory. This could result in a greater dispersion of strengths of field test data in the flexure beam test than in

# TABLE 6.3. FLEXURE BEAM STRESS<sup>1</sup>

Age (days)	Number of Tests <sup>2</sup>	Mean Strength <sup>3</sup> (psi)	Standard Deviation (psi)	Coefficent of Variation (psi/psi)
4	76	630	40	6
7	76	670	40	6
28	12	770	42	6
56	12	780	37	5
 Total <sup>4</sup>	176	665	40	6

<sup>1</sup>Center point loading. <sup>2</sup>Each value is the average of 4 tests. <sup>3</sup>To the nearest 5 psi. <sup>4</sup>All variables.

Test	Mean Strength (ps1)	Standard Deviation (psi)	Coefficient of Variation
Splitting Tensile Test <sup>1</sup>	405	20	5
Flexure Beam Test	605	36	6

<sup>1</sup>6 x 12-inch specimens. (Ref 8) the splitting tensile test. These considerations will be discussed in the field test portion of this study.

#### DISPERSION OF TEST RESULTS

The results of the flexure beam test and of the splitting tensile test of the 6 x 12-inch cylinders are compared to determine if the dispersion for the splitting tensile test is significantly different than the dispersion for the flexure beam test. This comparison is made by using the F-test (Ref 18). Based on the F-test calculations, there is no significant difference between the dispersion results of the flexure beam test and dispersion results of the splitting tensile test of the 6 x 12-inch specimens, at any test age.

The same calculation was performed for a comparison of the 4 x 8-inchcylinder splitting tensile test and the flexure beam test, and it was determined that the dispersion for the 4 x 8-inch cylinders is significantly different from the dispersion for the flexure beam test. Therefore, it can be said that the 4 x 8-inch cylinders will not produce results that are as reliable as the flexure beam test results. However it should be remembered that the 4 x 8-inch cylinder values were the average of two specimens in many cases. Thus the variability might have been less if four specimens could have been used in all cases.

#### **RELATIONSHIPS BETWEEN TESTS**

Approximate linear relationships were found between the splitting tensile stress results and the flexure beam stress results for the concrete, for a variety of batch designs. The relationships are shown for all class level combinations tested for the 6 x 12-inch cylinder strengths versus the flexure beam strengths and the 4 x 8-inch cylinder strengths versus the flexure beam strengths, in Figs 6.1 and 6.2, respectively.

As observed in Fig 6.1 and Fig 6.2, the splitting tensile stress generally increases linearly as the flexure beam stress increases. Additionally, both the splitting tensile strength and the flexure beam



Fig 6.1. Relationship between splitting tensile strength and flexural strength for 6 x 12-inch cylinders.



Fig 6.2. Relationship between splitting tensile strength and flexural strength for 4 x 8-inch cylinders.

strength increase with age, although not at the same rate as discussed and illustrated in the next section. This is also indicated by the different symbols in Fig 6.1 and Fig 6.2 representing the test age of the concrete.

#### RATIO OF TWO TESTS

The splitting tensile strength and the flexural beam strength ratios for similar concretes are summarized in Tables 6.5 and 6.6. The tables show that the ratios of the results of the two tests increase as the concrete strength increases. This is in accordance with the observations of other authors (Refs 8 and 19). The data indicate a good correlation between the splitting tensile test and the flexure beam test within a single test series. Results by several other investigators concerning the ratio of the splitting tensile stress and the flexure beam stress are summarized in Table 6.7. The results shown in Table 6.7, indicate results similar to those obtained from experiments associated with this report (Table 6.5).

The main reason for the difference between the stress values of the two tests seems to be the application of Hooke's law in the usual calculation of the flexural strength of concrete. Concrete deformations do not follow Hooke's law, particularly in the tensile zone of the concrete beam. Additionally the theory assumes there is no bending of the beam during the tests, while in actuality bending does occur. The distribution of this ratio is shown in histogram form (Fig 6.3) and cumulative form (Fig 6.4). Similarly, the distribution of the 4 x 8-inch cylinders is shown in Figs 6.5 and 6.6 for the histogram form and cumulative form, respectively.

### TRANSFORMATION EQUATION

The splitting tensile strength for the 6 x 12-inch cylinders has the same coefficient of variation as the flexural strength. Hence, using the mean value of either test would be equally likely to accept a bad batch or reject a good batch.

## TABLE 6.5. SPLITTING TENSILE VERSUS FLEXURAL STRENGTH RATIOS FOR 6 x 12-INCH CYLINDERS

Age	Number of Tests	σ <sub>st</sub> /σ <sub>fb</sub> <sup>1</sup>
4	76	0.62
7	76	0.62
28	12	0.62
56	12	0.66
Total	176	0.62

<sup>1</sup>Mean ratio of all samples by age of test.  $\sigma_{ST}$  = splitting tensile strength for  $6 \times 12$ -inch cylinders.  $\sigma_{FB}$  = flexure strength for center point loading.

## TABLE 6.6. SPLITTING TENSILE VERSUS FLEXURE BEAM STRENGTH RATIOS FOR 4 x 8-INCH CYLINDERS

Age	Number of Tests	σ <sub>st</sub> /σ <sub>fb</sub> <sup>1</sup>
4	74	0.71
7	74	0.72
28	12	0.73
56	12	0.75
Total	172	0.72

<sup>1</sup>Mean ratio of all samples by age of test.
<sup>σ</sup> ST = splitting tensile strength for 6 x 12-inch cylinders.
<sup>σ</sup> FB = flexure strength for center point loading.

σ <sub>st</sub> /σ <sub>fb</sub>	Authority	Remarks
0.8	Sen and Bharara	f <sub>fl</sub> varies from 214 to 630 psi; tested at various ages
0.39 to 0.74	Akazawa	Recommended average: 0.47
0.45 to 0.53	Ramesh and Chopra	f <sub>fl</sub> varies from 34 to 68 kg/cm <sup>2</sup>
0.67 to 0.91	Efsen and Glarbo	f <sub>fl</sub> varies from 16 to 24 kg/cm <sup>2</sup> ; ratio decreases with decreasing strength
0.62 to 0.90	Walker and Bloem	f <sub>fl</sub> varies from 800 to 300 psi; with aggregates of different maximum sizes
0.63 to 0.83	Rusch and Vigerust	f <sub>fl</sub> = about 45 kg/cm <sup>2</sup> ; ratio decreases with decreasing strength
0.72 to 0.77	Kaplan	f <sub>fl</sub> varies 850 to 550 psi
0.55 to 0.71	Narrow and Ullberg	f <sub>fl</sub> varies from 550 to 850 psi; with different aggregates
0.65 to 0.89	Grieb and Werner	f <sub>fl</sub> varies from 350 to 955 psi; crushed stone with 1-1/2 inch maximum size
0.51 to 0.78	Grieb and Werner	f <sub>fl</sub> varies from 250 to 790 psi; sand and gravel with 1-1/2 inch maximum size
0.57 to 0.88	Grieb and Werner	f <sub>fl</sub> varies from 430 to 750 psi; lightweight aggregate concrete
0.6 + 100/f <sub>fl</sub>	Popovics	f <sub>fl</sub> varies from 490 to 750 psi; with different aggregates
0.67	Wright	f <sub>sp</sub> = 405 psi at the age of 28 days
0.63	Sell	f <sub>fl</sub> = about 45 kg/cm <sup>2</sup> at the age of 14 days
0.66	McNeely and Lash	f <sub>fl</sub> = 690 psi; coarse aggregate is crushed gravel

# TABLE 6.7.SPLITTING TENSILE VERSUS FLEXURE BEAM RATIOS<br/>(FROM POPOVICS, REF 20)



Fig 6.3. Distribution of splitting tensile versus flexural stress ratios for 6 x 12-inch cylinders, histogram form.



Fig 6.4. Distribution of splitting tensile versus flexural stress ratios for 6 x 12-inch cylinders, cumulative form.



Fig 6.5. Distribution of splitting tensile versus flexural stress ratios for 4 x 8-inch cylinders, histogram form.



Fig 6.6. Distribution of splitting tensile versus flexural stress ratios for 4 x 8-inch cylinders, cumulative form.

Thus, the transformation equation developed from this research is

$$\sigma_{ST} = k \sigma_{FB}$$

where

k	=	constant from Table 6.5,
σ <sub>st</sub>	2	value of tensile stress measured from the splitting tensile
		test, and
σ <sub>fb</sub>	-	value of tensile stress measured from the flexure beam test.

For example, at a test age of 7 days and a flexure beam strength requirement of 650 psi, the required stress measured by the splitting tensile test for 6 x 12-inch cylinders would be equal to:

$$\sigma_{ST} = k \sigma_{FB}$$
  
= (0.62) (650 psi)  
= 400 psi.

The splitting tensile strength for the 4 x 8 cylinders has an 8 percent coefficient of variation for the flexural strength. Hence, the use of the mean for the 4 x 8 cylinders would be less conservative than the mean of the flexural strength. However, using the near values plus one standard deviation would be more conservative than the flexural strength. One standard deviation is approximately 40 psi at the early ages (Table 6.2).

Hence, for the 4 x 8-inch cylinders, the calculations would be equal to

$$\sigma_{ST} = k \sigma_{FB} + 1 \text{ std. dev.}$$
  
= (0.72) (650 psi) + 40  
= 500 psi

where

k = constant from Table 6.6.

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DISCUSSION OF  $\sigma_{ST} / \sigma_{FB}$ 

Examination of the  $\sigma_{\rm ST}^{\sigma}/\sigma_{\rm FB}$  ratio indicates that the deviation of the flexure beam stress varies with the strength of the concrete. This can be explained by the different plastic and elastic properties of the concrete at different strengths. Low strength concretes exhibit plastic properties (Ref 21). A possible approximation of the stress distribution of low strength concrete in the cross section of the beam is a hyperbolic sine function as shown in Fig 6.7.

Consider the following:

$$y = hyperbolic sine x$$
 (6.1)

where

$$x = \left[ \ln \left( y + \sqrt{y^2 + 1} \right) \right]$$
(6.2)

Substituting y = 3 inches in Eq 6.2, where 3 inches is the distance from the neutral axis of the beam to the extreme fiber, leads to

$$\mathbf{x} = \left[ \ln \left(3 + \sqrt{3^2 + 1}\right) \right]$$
  
$$\sigma_{\text{ST}} = 0.55 \,\sigma_{\text{FB}}$$

This analysis assumes no effects of bending. If bending were considered, the above  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio would be increased by approximately 1 to 10 percent (Ref 22), depending upon how much bending occurred. The  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio would then closely approximate those values presented in Table 6.5. This

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Fig 6.7. Change in stress distribution in the flexure beam specimen as the strength of the concrete increases.
approximation, of course, is valid only for concrete with low strengths, i.e., those concretes tested at an early age. As the concrete gains strength, the concrete exhibits elastic properties, almost to failure (Ref 21). Thus the ratio  $\sigma_{\rm ST} / \sigma_{\rm FB}$  approaches unity. From this analysis it is evident that the ratio  $\sigma_{\rm ST} / \sigma_{\rm FB}$  is not a constant value for all strengths of concrete, but rather, the ratio increases as the strength of the concrete increases.

The analysis presented in this section assumes that the tensile stress measured from the splitting tensile test of the concrete cylinder is equal to the true tensile strength of the concrete. An indirect proof of this was discussed by Abeles (Ref 21) for high strength concretes, from shear tests with prestressed concrete beams. Abeles also reported, that, for lower strength concretes, the tensile splitting strength deviates from the true tensile strength, although not appreciably.

### ANALYSIS OF VARIANCE

The splitting tensile stress and flexure beam stress ratio was analyzed to determine the variation in the strength ratio as a result of the different variable classes. This analysis was performed using an analysis of variance procedure. This test revealed various sources of significant influence at a level of five percent. The theoretical background associated with the analysis of variance test is beyond the scope of this report. References 23 and 24 contain further explanation of this statistical tool.

Results of the analysis are summarized in Tables 6.8, 6.9 and 6.10. Inspection of these tables leads to the conclusion that the following main affects are significant, at a confidence interval of 90 percent, for the ratio of the splitting tensile stress of 6 x 12-inch cylinders to the modulus of rupture:

- (1) cement factor,
- (2) cement type,
- (3) admixture,

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### TABLE 6.8. ANALYSIS OF VARIANCE FOR THE RATIO OF THE SPLITTING TENSILE STRESS AND THE MODULUS OF RUPTURE

Class Level Information<sup>1</sup>

Class	Level	Values				
Age	4	4, 7, 28, 56				
Cement Factor	2	5,6				
Cement Type	3	I, II, III				
Air Content	3	High, Medium, Low				
Coarse Aggregate Type	2	Limestone, Silicious				
Coarse Aggregate Size	2	1-1/2, 3/4				
Coarse Aggregate Factor	3	1, 2, 3				
Admixture	3	None, Water Reducer, Accelerator				
Water Cement Ratio	3	High, Medium, Low				
Slump	3	High, Medium, Low				
Temperature	3	High, Medium, Low				

 $^{1}\mathrm{Applies}$  to both 6 x 12-inch cylinders and 4 x 8-inch cylinders.

# TABLE 6.9. ANALYSIS OF VARIANCE FOR RATIO OF THE SPLITTING TENSILE STRESS OF 6 $\times$ 12-INCH CYLINDERS AND OF THE MODULUS OF RUPTURE

.

	Degrees of	Sum of	F	Probability F (Percent)	
Source of Variance	Freedom	34001 63	Tarue		
Model	63	0.4206	2.60	0.01	
Error	112	0.2879			
Total	175	0.7085			
				Probability	
	Degrees of	Sum of	F	F	
Model	Freedom	Squares	Value	(Percent)	
Age	3	0.0100	1.31	27.4	
Cement Factor	1	0.0191	7.45	0.7*	
Cement Type	2	0.0131	2.55	8.3**	
Air Content	2	0.0150	2.93	5.8	
Coarse Aggregate Type	1	0.0065	2.52	11.6	
Coarse Aggregate Size	1	0.0074	2.86	9.3	
Coarse Aggregate Factor	2	0.0020	0.39	67.6	
Admixture	2	0.0353	6.86	0.1*	
Water Cement Ratio	2	0.0092	1.79	17.2	
Slump	2	0.0129	2.51	8.6**	
Temperature	2	0.1699	33.04	0.01*	
Cement Factor * Cement Type	2	0.0006	0.12	89.1	
Cement Factor * Air Content	2	0.0130	2.53	8.5	
Cement Type * Air Content	3	0.0055	0.71	54.9	
Cement Factor * Coarse Aggregate Type	1	0.0023	0.91	34.3	
Cement Type * Coarse Aggregate Type	1	0.0003	0.13	71.9	
Cement Factor * Coarse Aggregate Size	1	0.0010	0.40	53.0	
Cement Type * Coarse Aggregate Size	2	0.0196	3.82	2.5*	
Air Content * Coarse Aggregate Size	2	0.0090	1.75	17.8	
Cement Factor * Coarse Aggregate Factor	2	0.0098	1.90	15.5	
Coarse Aggregate Size * Coarse Aggregate Factor	1	0.0016	0.63	42.8	
Cement Factor * Admixture	2	0.0004	0.07	93.3	
Cement Type * Admixture	3	0.0031	0.40	75.9	
Air Content * Admixture	4	0.0019	0.18	94.8	
Coarse Aggregate Size * Admixture	1	0.0024	0.92	33.9	

### TABLE 6. 9. (CONTINUED)

Mode 1	Degrees of Freedom	Sum of Squares	F Value	Probability F (Percent)
Cement Factor * Water Cement Ratio	2	0.0009	0.17	84.0
Cement Type * Water Cement Ratio	2	0.0059	1.14	32.4
Air Content * Water Cement Ratio	4	0.0080	0.78	54.2
Coarse Aggregate Type * Water Cement Ratio	1	0.0001	0.04	84.8
Coarse Aggregate Size * Water Cement Ratio	1	0.0031	1.19	27.7
Cement Factor * Slump	1	0.0003	0.12	73.1
Air Content * Slump	3	0.0115	1.49	22.0
Cement Factor * Temperature	2	0.0199	3.87	2.4*

\*Significant level of influence at a 95 percent confidence interval. \*\*Significant level of influence at a 90 percent confidence interval.

# TABLE 6.10. ANALYSIS OF VARIANCE FOR RATIO OF THE SPLITTING TENSILE STRESS OF 4 $\times$ 8-INCH CYLINDERS AND OF THE MODULUS OF RUPTURE

Source of Variance	Degrees of Freedom	Sum of Squares	F Value	Probability F (Percent)
Mode 1	61	0.9649	2.22	0.01
Error	110	0.7821		
Total	171	1.7470	_	
Mode 1	Degrees of Freedom	Sum of Squares	F Value	Probability F (Percent)
Age	3	0.0195	0.92	43.8
Cement Factor	1	0.0003	0.04	83.8
Cement Type	2	0.0319	2.25	11.1
Air Content	2	0.0246	1.73	18.2
Coarse Aggregate Type	1	0.0199	2.80	9./**
Loarse Aggregate Size	1	0.0496	6.98	0.9*
Loarse Aggregate Factor	2	0.0241	1.70	18.8
Admixture	2	0.0869	0.11	0.3* 5.4**
Slump	2	0.0427	1 91	16.8
Temperature	2	0.0255	21 41	0.01*
Coment Factor * Coment Type	2	0.0019	0 13	87 5
Cement Factor * Air Content	2	0.0504	3.54	3.2*
Cement Type * Air Content	3	0.0041	0.19	89.9
Cement Factor * Coarse Aggregate Type	1	0.0001	0.02	88.2
Cement Type * Coarse Aggregate Type	1	0.0005	0.06	80.1
Cement Factor * Coarse Aggregate Size	1	0.0020	0.28	60.1
Cement Type * Coarse Aggregate Size	2	0.0027	0.19	83.0
Air Content * Coarse Aggregate Size	2	0.0445	3.13	4.8*
Cement Factor * Coarse Aggregate Factor	2	0.0279	1.96	14.6
Coarse Aggregate Size * Coarse Aggregate Factor	1	0.0516	7.25	0.8*
Cement Factor * Admixture	2	0.0070	0.49	61.1
Cement Type * Admixture	3	0.0058	0.27	84.6
Air Content * Admixture	4	0.0392	1.38	24.6
Coarse Aggregate Size * Admixture	1	0.0002	0.03	86.5

### TABLE 6.10. (CONTINUED)

Model	Degrees of Freedom	Sum of Squares	F Value	Probability F (Percent)
Cement Factor * Water Cement Ratio	2	0.0175	1.23	29.5
Cement Type * Water Cement Ratio	2	0.0081	0.57	56.8
Air Content * Water Cement Ratio	4	0.0165	0.58	67.7
Coarse Aggregate Type * Water Cement Ratio	1	0.0025	0.35	55.7
Coarse Aggregate Size * Water Cement Ratio	1	0.0006	0.09	76.5
Cement Factor * Slump	1	0.0011	0.16	69.3
Air Content * Slump	1	0.0001	0.01	98.7
Cement Factor * Temperature	2	0.0508	3.57	3.1*

\*Significant level of influence at a 95 percent confidence interval. \*\*Significant level of influence at a 90 percent confidence interval.

- (4) slump, and
- (5) temperature.

This is evident by inspection of Table 6.9.

The following main affects were significant, at a level of 90 percent, for the ratio of the splitting tensile stress of 4 x 8-inch cylinders to the modulus of rupture:

- (1) coarse aggregate type,
- (2) coarse aggregate size,
- (3) admixture,
- (4) water content, and
- (5) temperature.

This is evident by inspection of Table 6.10.

The differences are then examined more closely and are shown in Tables 6.11 and 6.12. From the analysis, the use of the design variables of a 6 sack/cubic yard cement factor and a Type III cement type will cause a reduction in the value of  $\sigma_{\rm ST}/\sigma_{\rm FB}$ , and the design variables of an accelerator, a low slump and a high curing temperature will cause an increase in the value of  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio for the splitting tensile test involved with the 6 x 12-inch specimens.

Concerning the  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio for the 4 x 8-inch cylinders, design variables of a high air content, a 3/4-inch coarse aggregate size, and a high water content will cause a decrease in the  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio, while the design variables of an accelerator and a high curing temperature will cause an increase in the  $\sigma_{\rm ST}/\sigma_{\rm FB}$  ratio.

FIELD TEST RESULTS

Table 6.13 shows the number of field samples obtained from four concrete pavement and one structural concrete (bridge deck) construction projects. All specimens, except the cores, were cast at the project site and cured

Class Variable	Level	σ <sub>ST</sub> /σ <sub>FB</sub>
Cement Factor	5 sck/cu.yd. 6 sck/cu.yd.	0.63 0.61*
Cement Type	I I I	0.62
Admixture	III None	0.59* 0.62
	Accelerator Water Reducer	0.67* 0.61
Slump	Low Medium High	0.69* 0.62 0.61
Temperature	High Room Low	0.71* 0.62 0.59

## TABLE 6.11. COMPARISON OF MEANS OF $\sigma_{ST}^{\prime}/\sigma_{FB}^{\prime}$ FOR 6 x 12-INCH CYLINDERS

\*Significantly different at a 95 percent confidence interval.

Class Variable	Level	σ <sub>st</sub> /σ <sub>f</sub>		
Air Content	Low	0.74		
	Medium	0.72		
	High	0.69*		
Coarse Aggregate Size	1-1/2 inch	0,72		
	3/4 inch	0.67*		
Admixture	None	0.71		
	Accelerator	0.79*		
	Water Reducer	0.72		
Water Cement Ratio	Low	0.69		
	Medium	0.73		
	High	0.67*		
Temperature	High	0.85*		
	Room	0.70		
	Low	0.67		

## TABLE 6.12. COMPARISON OF MEANS OF $\sigma_{ST}^{}/\sigma_{FB}^{}$ For 4 x 8-INCH CYLINDERS

\*Significantly different at a 95 percent confidence interval.

TABLE 6.13.	NUMBER OF S	SPECIMENS	OBTAINED	FROM	THE	FIELD	FOR	STRENGTH	DETERMINATION
	AT VARIOUS	CURING TI	IMES.						

		7-Day	28-Day		280-Day				
Project No.	FB	ST 6 in x 12 in <sup>1</sup>	ST 4 in x 8 in	ST <sup>2</sup> ST 4 in x 8 in	FB	ST 6 in x 12 in	ST 4 in x 8 in	ST <sup>2</sup> 4 in x 8 in	
1-0	4	4	4	2					
2-H	4	4	4					2	
3-H	2	2	2		2	1	2		
4-P	2	2	2		2	2	2	2	
5-P <sup>3</sup>	4	4	4						

Size of specimens
 Cores from pavements
 Bridge deck



Fig 6.8. Comparison between lab and field splitting tensile and flexural strength for 6 x 12-inch cylinders.



Fig 6.9. Comparison between lab and field splitting tensile and flexural strength for 4 x 8-inch cylinders.

overnight before being transported to CTR. These specimens were cured for seven days before being tested. Cores were taken from the hardened concrete at the same location from which the other specimens were taken. Cores form Projects 2-H, and 4-P were tested at the age of 280 days, while those from Project 1-D were tested at the age of 28 days. Cores were taken by the District from project 3-H, however it could not be verified that the cores were from the section of interest to this project and hence were not included No cores were taken from Project 5-P because coring is in this report. prohibited for structural concrete. Average concrete strengths and ratios of the splitting tensile strength to the flexural strength are shown in Table 6.14. Statistical analysis of these values will not be attempted because of limited data available. However, comparisons will be made between values and conclusions will be drawn. Strength ratios of the 6 x 12-inch 7-day specimens ranged from 0.58 to 0.75 for an average of 0.65. The mean ratio of all the laboratory data was previously found to be 0.62. For the 4 x 8-inch 7-day specimens, the field strength ratios ranged from 0.63 to 0.77 for an average value of 0.72. The mean laboratory ratio was 0.72. It is therefore indicated that for both specimen sizes that the mean field ratio from all the projects is very close to the mean laboratory value. This concludes that the 7-day field strength values can safely be predicted by the transformation equation developed using the laboratory data. For example, for Project No. 1-D, the flexural strength would be predicted to be 350 0.62 = 565 psiwhich is slightly higher than 550 psi, the actual value.

The average 280-day field strength ratio for the 6 x 12-inch and 4 x 8inch specimens was found to be 0.59 and 0.64, respectively. These values are lower than the respective 7-day strength ratio values from the field data, but are close to the mean laboratory values. Hence, based on these results, the 280-day flexural strength could be predicted by the transformation equation. It is important to note that the equation was developed using specimens of up to 56 days of age. The applicability of the equation to specimens of 280 days of age means that the relative increase of the tensile strength to the flexural strength at 280 days is similar to the increase in strength at up to 56 days.

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		7 <b>-</b> Day		28-Day		280-Day				
Project	FB	ST	ST	ST 2	FB	ST	ST	ST <sup>2</sup>		
No.	(psi)	6 in x 12 in <sup>1</sup> (psi)	4 in x 8 in (psi)	4 in x 8 in (psi)	(psi)	6 in x 12 in (psi)	4 in x 8 in (psi)	4 in x 8 in (psi)		
1-D ST <sup>/</sup> FB	550	350 0.64	395 0.72	405 0.74						
2-H ST <sup>/</sup> FB	750	435 0.58	575 0.77					510		
3-H ST <sup>/</sup> FB	695	430 0.62	435 0.63		820	435 0.53	450 0.55			
4-P St <sup>/</sup> FB	515	385 0.75	370 0.72		760	485 0.64	555 0.73	460 0.61		
5-P ST <sup>/</sup> FB	695	455 0.65	520 0.75							
Average ST <sup>/</sup> FB	640	410 0.65	455 0.72	405 0.74	790	460 0.59	505 0.64	485 0.67		

TABLE 6.14. MEANS STRENGTHS AND  $\sigma_{\rm ST}/\sigma_{\rm FB}$  RATIOS OF SPECIMENS OBTAINED FROM THE FIELD AND TESTED AT VARIOUS CURING TIMES.

1 Size of specimens
2 Cores from pavements

Table 6.14 shows also the tensile strength of cores obtained from the hardened pavements. The 28-day core strength from Project 1-D is only 1.03 times bigger than the 7-day field strength, while the respective laboratory value taken from Table 6.2, is 1.15. The 280-day core strength from Project 2-H is lower than the 7-day field value, and the 280-day core strength from Project 4-P is much less than the field specimen of the same age. The above indicate that cores exhibit lower strengths than field specimens obtained during construction of pavements. The strength ratio of cores from Project 4-P is 0.61, which is 0.10 less than the average laboratory ratio. Therefore, the transformation equation would predict conservative strengths.

For a better understanding of the nature of the field strengths, the data obtained in the field were added to Figs 6.1 and 6.2 and are shown in Figs 6.8 and 6.9. As observed, most  $\sigma_{\rm ST}^{}/\sigma_{\rm FB}^{}$  ratios fall close to the 50 percent confidence level (mean value).

### CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

### SUMMARY

Concrete pavement designs are typically based on the tensile strength of the concrete, as estimated by the flexure beam test. The tensile stress formula that is used to calculate the stresses at the extreme fibers at failure is based on a linear stress distribution at the failure plane. Since the stress distribution is not linear, the calculated modulus of rupture is high compared to the "true" tensile strength.

The splitting tensile test was developed to estimate the tensile strength of the concrete by loading a cylinder in compression, on its side. Several agencies have recommended the splitting tensile test as an alternative to the flexure beam test (Refs 6 and 7).

This report presents the results of the laboratory tests comparing these two tests. A total of 88 concrete batches were designed, batched and tested for use in comparing the splitting tensile test and the flexure beam test. Concrete test variables used in the design included cement type, coarse aggregate size, cement factor, admixtures, air content, water cement ratio, slump, temperature (curing), coarse aggregate factor, coarse aggregate type and age (curing). Tests were performed according to ASTM and Texas test methods, using 6 x 12-inch cylinders, 4 x 8-inch cylinders, and 6 x 6 x 20-inch beams.

### CONCLUSIONS AND FINDINGS

The major findings and conclusions from the test results are summarized below:

(1) The splitting tensile test is an effective test method for estimating the tensile strength of concrete. The cylinders associated with the splitting tensile test are easier to handle

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than those specimens used for the flexure beam test. The cylinders require less material and hence weigh less and the cylinder molds that are required are the same as those required for compressive strength tests.

- (2) The measured tensile strength of the concrete varied significantly depending upon the test method. The flexure beam test stress results were typically higher than those from either the 6 x 12inch-cylinder or the 4 x 8-inch-cylinder specimens of the splitting tensile test. This is attributed to the fact that the modulus of rupture calculation assumes a linear stress distribution. These results are in agreement with other researchers' results (Ref 20).
- (3) The variability and dispersion of the results from the 6 x 12-inchcylinder specimens tested by the splitting tensile test method did not differ significantly from the variability and dispersion of those from the flexure beam test method. However, the variability and dispersion of the stress results of the 4 x 8-inch cylinder specimens tested by the splitting tensile test method, were significantly greater than the variability and dispersion with the flexure beam test method. Similar results were obtained by Wright (Ref 8).
- (4) Approximate linear relationships were found between the splitting tensile stress results and the flexure beam test results for the concrete, for a variety of batch designs. Test results indicated that both the splitting tensile strength and the flexure beam strength increase with age, although not at the same rate. This is explained by the different plastic and elastic properties of the concrete at different strengths.
- (5) A transformation equation was developed to convert a tensile stress value measured with the flexure beam test to the theoretical tensile stress value that could be expected if the same concrete were tested by the splitting tensile test method. The data were further evaluated to determine a value of "k" in the transformation equation. The value of "k" for both the four and seven day tests represented a wide range of test variables, ranging to both

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extremes of the Texas SDHPT specifications. However, the "k" value for the 28 and 56-day test results were based on only 12 tests and should therefore be used discriminantly.

(6) An analysis of variance test was performed on the ratio of the splitting tensile test results to the flexure beam test results to determine the variation in the strength ratio as a result of the different variable classes. Results of the calculations reveal that the class level design variables of a 6 sack/yd<sup>3</sup> cement factor, and a Type III cement type will cause a reduction in the value of  $\sigma_{\rm ST} / \sigma_{\rm FB}$ , and the design variables of an accelerator, a low slump, and a high curing temperature will cause an increase in the value of  $\sigma_{\rm ST} / \sigma_{\rm FB}$  ratio for the splitting tensile test involved with 6 x 12-inch specimens.

Concerning the  $\sigma_{\rm ST}^{\ / \sigma}_{\ \rm FB}$  ratio for the 4 x 8-inch cylinders, design variables of a high air content, a 3/4-inch coarse aggregate size, and a high water cement ratio will cause a decrease in the  $\sigma_{\rm ST}^{\ / \ \sigma}_{\rm FB}$  ratio, while the design variables of an accelerator and a high curing temperature will cause an increase in the  $\sigma_{\rm ST}^{\ / \ \sigma}_{\rm FB}$  ratio.

#### RECOMMENDATIONS

Based on the test results and conclusions from this report, the following actions are recommended:

- (1) The use of the splitting tensile test on 6 x 12-inch cylinders as an alternative to the flexure beam test is recommended to measure the tensile stress of concrete.
- (2) The transformation equation for 6 x 12-inch cylinders,

$$\sigma = k \sigma_{FB},$$

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is recommended for converting from the flexure beam test to the splitting tensile test, or vice versa. A k value of 0.62 is recommended for converting the flexure beam test to the splitting tensile test of 6 x 12-inch cylinders. This yields a value for the splitting tensile stress of 400 psi as equivalent to a flexure beam text of 650 psi. The transformation equation for 4 x 8-inch cylinders is

 $\sigma_{ST} = k \sigma_{FB} + 40 \text{ psi}$ 

Similarly, a k value of 0.72 is recommended to convert the flexure beam test to the splitting tensile test of 4 x 8-inch cylinders for the variables tested. This yields a value of 500 psi as equivalent to a flexure beam test of 650 psi.

### CONTINUING RESEARCH

Although a large amount of research has been completed on the relationship between the splitting tensile test and the flexure beam test, there is a need for continuing research in the following areas.

- (1) The relationship between the splitting tensile test and the flexure beam test for field test data should be examined further. Although four tests were researched as part of this project, a much broader and larger scope project is recommended in order to fully understand this relationship.
- (2) The relationship between the splitting tensile test with 4 x 8-inch cylinders and the flexure beam test should continue to be examined. This report considered both of these tests, but the 4 x 8-inchcylinder sample size was two versus a sample size of four for all the other tests. It is realized, that the smaller sample size may have influenced the descriptive statistics of the stress values.

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