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16. Abstract

At signalized intersections, the common treatment for improving left-turn performance is to increase left-turn capacity by installing a left-turn bay or a separate left-turn phase. However, in a given traffic condition and geometric configuration, there have been no universal guidelines for ascertaining the need for a left-turn treatment. In this research, the TEXAS Simulation Model is employed to study the capacity and performance for left-turn movements at signalized intersections in order to explore warrants for left-turn treatments. Since left-turn performance is germane to left-turn capacity, existing methods for estimating left-turn capacity are thoroughly reviewed. After recognizing inadequacies in existing methods, a new method which can yield reasonable estimates for left-turn capacity under general conditions of left-turn movements is proposed. Furthermore, different measures of effectiveness are used to evaluate the performance of left-turn movements under various traffic conditions. With a set of delay criteria, critical conditions of left-turn movements are identified. Finally, a new capacity warrant is derived based on the relation between the critical left-turn volume and left-turn capacity.

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GUIDELINES FOR USE OF LEFT-TURN LANES AND SIGNAL PHASES

by

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Clyde E. Lee
Robert Herman

Research Report Number 258-1

Warrants for Left-Turn Lanes and Signal Phases (Actuated Controllers)

Research Project 3-18-80-258

conducted for

Texas
State Department of Highways and Public Transportation

in cooperation with the
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by the

Center for Transportation Research
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The University of Texas at Austin

January 1984

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ABSTRACT

At signalized intersections, the common treatment for improving left-turn performance is to increase left-turn capacity by installing a left-turn bay or a separate left-turn phase. However, in a given traffic condition and geometric configuration, there have been no universal guidelines for ascertaining the need for a left-turn treatment. In this research, the TEXAS Simulation Model is employed to study the capacity and performance for left-turn movements at signalized intersections in order to explore warrants for left-turn treatments. Since left-turn performance is germane to left-turn capacity, existing methods for estimating left-turn capacity are thoroughly reviewed. After recognizing inadequacies in existing methods, a new method which can yield reasonable estimates for left-turn capacity under general conditions of left-turn movements is proposed. Furthermore, different measures of effectiveness are used to evaluate the performance of left-turn movements under various traffic conditions. With a set of delay criteria, critical conditions of left-turn movements are identified. Finally, a new capacity warrant is derived based on the relation between the critical left-turn volume and left-turn capacity.

Key Words: Left Turn, Actuated Controller, Vehicular Delay, Transparency, Separate Phase, Left-Turn Bay

SUMMARY

A comprehensive analysis of left-turning operations in at-grade intersections has been conducted. A methodology for estimating the maximum possible number of unprotected left-turn maneuvers across opposing traffic streams has been developed. This "capacity" estimation procedure permits the user to account for the effects of various numbers of opposing traffic streams, the presence of trucks in opposing and left-turn streams, and the effects of signalization.

Guidelines have been developed for installation of protected left-turn signal phases as well as left-turn bays. These guidelines are based upon commonly accepted measures of vehicular delay. Like the capacity analysis procedures, they allow explicit accounting for effects of a fairly complete range of operating conditions including intersection geometry, traffic stream composition, and signalization. Guidelines for installation of left-turn bays are complemented by a methodology for estimating the required bay length.

IMPLEMENTATION STATEMENT

Guidelines for left-turn capacity analyses, protected signal phase, and bay installations contained within this interim report will be of immediate use to state and municipal traffic engineers. The techniques presented here will provide a greatly enhanced ability to make decisions regarding potential left-turn treatments. Example problem solutions are presented to encourage field application and testing of the methodologies.

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CHAPTER 1. INTRODUCTION

At signalized intersections, the presence of left-turning vehicles tends to lower the intersection capacity, cause excessive delay, and increase the accident potential as traffic volume increases. In order to ameliorate or preclude operational difficulties incurred by left turns, left-turn treatments such as adding a left-turn bay, or a separate left-turn phase, or both are frequently used. While the choice of a left-turn treatment depends on traffic conditions and geometric configurations, no unique set of guidelines exists for such a treatment. Highway agencies and research institutions have developed various guidelines in terms of delay, volume, capacity, conflicts, and accidents. However, these guidelines are sometimes conflicting and in some cases are not sufficiently rigorous. It has been pointed out that left-turn treatments utilized when unneeded will not help left-turners but may cause more delay to other drivers [Refs 1 and 2]. Thus, it is important to have lucid and effective guidelines for left-turn treatments. The objective of this study will be to explore reasonable answers to the following questions:

(1) When should a left-turn bay be provided?

(2) How long should the left-turn bay be?

(3) When should a separate left-turn phase be added?

Using the TEXAS Simulation Model, left-turn operations at signalized intersections with a pre-timed signal will be analyzed. From simulation output, a special statistical method, called replications analysis, will be used to obtain reliable, stable statistics. In addition to presenting the simulation results, analytical models will be constructed to show explicitly the interactions among important variables. These analytical models not only give a clear description of left-turn operations but also facilitate sensitivity studies of changes in traffic conditions and signal-timing schemes.

Since left-turn performance is believed to be germane to left-turn capacity, methods for estimating left-turn capacity will be throughly reviewed. After recognizing inadequacies in existing methods, a new method which can yield a more reasonable estimate of left-turn capacity will be developed. The effects of cycle length, cycle split, left-turn bays, the number of opposing lanes, the headway distributions, and trucks on the left-turn capacity will be addressed.

Left-turn operations under various traffic conditions and geometric configurations will be evaluated with different performance measures. The four criteria employed for identifying critical conditions are: (1) 35 seconds of average left-turn delay, (2) 73 seconds of ninety-percentile left-turn delay, (3) five percent of left turners being delayed more than two cycles, and (4) four left turners in one hour being delayed more than two cycles. A left-turn bay will be recommended if the above four criteria and an additional criterion regarding through delay are satisfied. The bay length required can be determined with the aid of charts provided. For signalized intersections with adequate length of bay, a separate left-turn phase will be recommended if the above four criteria are met. A new type of left-turn warrant will be proposed and compared with other warrants. With the charts and tables provided in this study, traffic engineers will be able to evaluate the need for a left-turn treatment with field volume counts.

CHAPTER 2. PROBLEM DEFINITION AND ANALYTICAL APPROACH

PROBLEM DEFINITION

At signalized intersections, unprotected left turners have to wait for acceptable gaps or amber time to clear the intersection. The maximum number of left-turns that can be made in one hour, called the left-turn capacity, depends primarily on the opposing traffic, driver behavior, vehicular characteristics, intersection geometric configuration, and the signal-timing scheme. It can be recognized that as the left-turn demand approaches capacity, more left turners will queue up and incur excessive delay. If a left-turn bay is not adequately long or not provided, the left-turning queue will also impede through movements. When drivers suffer long delays, they may become impatient and make hazardous maneuvers. Although repeated accidents and complaints may indicate the need for some form of left-turn treatment, such as a left-turn bay, a separate left-turn phase, or both, traffic engineers need lucid guidelines for justifying a left-turn treatment without having to experience such situations. The problem is how to identify the critical condition of left-turn operations so that the needed left-turn treatment can be implemented.

Some researchers have tried to relate the critical condition to left-turn capacity. However, procedures for determining left-turn capacity have been debated heatedly for decades. Until the controversy surrounding left-turn capacity is cleared up, left-turn warrants based on the concept of practical capacity can not be utilized effectively.

Different methodologies and criteria have been used to generate various types of left-turn warrants. Unfortunately, these warrants are not all consistent. Where the inconsistencies originate and how a reasonable left-turn warrant can be reached are key issues of this study.

ANALYTICAL APPROACH

There are three basic techniques for studying traffic operations. They are field observation, mathematical analysis, and computer simulation. Although field studies provide direct information about traffic operations, they are usually time consuming and costly. Furthermore, it is difficult to generalize field results since each traffic condition observed is essentially unique. Hence, it is always desirable to formulate traffic operations as a model and use field observations for validation only. However, a mathematical formulation of traffic operations under various flow conditions, geometric configurations, and signal phase sequences compounded with the stochastic nature of traffic parameters becomes mathematically intractable. Computer simulation offers many of the advantages of the other two techniques. Among the several traffic simulation models that are available, the TEXAS Model, developed at The University of Texas at Austin, was chosen for this study.

The TEXAS Model is a microscopic traffic simulation package consisting of a geometry processor, a driver-vehicle processor, and a traffic simulation processor. The geometry processor calculates the paths that vehicles will follow, intersection conflicts, and available sight distance. The driver-vehicle processor generates random descriptors of the driver-vehicle units that are used in the simulation. The traffic simulation processor processes each driver-vehicle unit through the intersection system while gathering a large selection of performance statistics as output. The TEXAS Model allows assessment of the effects of changes in road geometry, driver and vehicle characteristics, flow conditions, intersection control, lane control, and signal timing schemes upon traffic operations. With minor modifications, the model has been programmed to reveal important information regarding left-turn operations such as opposing headway distributions, gap acceptance criteria, left-turn delay distribution, number of vehicles incurring delay greater than a specified value, and the queue built-up process. This flexibility makes the TEXAS Model particularly useful and powerful for studying left-turn operations. One limitation of the TEXAS Model is that it deals exclusively with one intersection instead of a network. In urban areas, it is a common practice to coordinate traffic signals in a network. Consequently, traffic conditions at one intersection may be affected by those at intersections in the neighborhood. Since most network simulation models are macroscopic, they are inadequate for studying left-turn operations at a single intersection. The NETSIM model is an excellent microscopic network model but it utilizes certain stochastic processes for turn maneuvers and does not provide the desired level of detail in output information for turn maneuvers. In order to understand basic characteristics of left-turn operations, the TEXAS Model seems to be most appealing.

Using the TEXAS Model, left-turn capacity and left-turn delay at intersections with pretimed signals will be studied. The statistical method for obtaining reliable statistics from simulation output is addressed in the next section. With selected delay criteria, warrants for left-turn bays or phases will be developed.

STATISTICAL ANALYSIS OF SIMULATION OUTPUT

Along with the increasing applications of simulation models in recent years, methods for obtaining reliable statistics from simulation output have been studied by so many statisticians that this area has now become a new branch of statistics [Refs 3 and 4]. One might ask the following questions: What is the difference between the statistical methods used for analyzing simulation output and those used for other purposes? Why are they needed? Some answers will be demonstrated by the following example. Suppose that a traffic simulation model, starting with an empty system, is run for an hour. During the simulation, N vehicles have been observed and the delay for each vehicle has been recorded. Let X_i be a random variable denoting the delay for the i-th vehicle. From elementary statistics, one would estimate the average value, the variance, and the corresponding (1-α) percent confidence interval of delay as follows:

$$\text{Average:}\quad X = \sum_{i=1}^{N} X_i / N \tag{2-1}$$

$$\text{Variance:}\quad S^2 = \sum_{i=1}^{N} (X_i - X)^2 / (N - 1) \tag{2-2}$$

(1 - α)% confidence

$$\text{interval of average delay:}\quad \overline{X} \pm Z_{1-\alpha/2} (S/\sqrt{N}) \tag{2-3}$$

or,

$$\overline{X} \pm t_{1-\alpha/2,N-1} (S/\sqrt{N}) \tag{2-4}$$

There are at least two things wrong with these estimations. First, because the simulation starts with an empty system, delay statistics collected during the initial transient period are unstable data. Unless corrections are made on data from the transient period, Eq 2-1 will yield a biased estimate for the average delay under stable conditions. That means the confidence intervals computed from Eq 2-3 and 2-4 are not centered at the true mean. Second, the estimate of variance from Eq 2-2 is unbiased only when data are independent. Since vehicular delays in the traffic simulation are correlated to some degree, the variance of average delay will be larger than that predicted by Eq 2-2 due to nonzero covariance terms.

To cope with these problems, three methods are usually used. They are the regenerative method, the batch-mean method, and the replications analysis. Details about the regenerative method and the batch-mean method can be found in Refs 5 and 6. In this study, the replications analysis is adopted because of its relative application ease. Procedures for carrying out replications analysis are as follows:

(1) Determine the start-up time after which statistics are to be collected

(2) Determine the simulation time for each run

(3) Make k independent runs of simulation. Let X_{ij} be the j-th statistics observed in the i-th simulation run. $1 \leq i \leq k$, and $1 \leq j \leq m$

(4) Compute the average statistics for the i-th run.

$$\overline{X}_i = (1/m)\sum_{j=1}^{m} X_{ij} \tag{2-5}$$

(5) Compute the grand average over k runs.

$$\overline{\overline{X}} = \sum_{i=1}^{k} \overline{X}_i / k \tag{2-6}$$

(6) Compute the variance.

$$S^2 = \sum_{i=1}^{k} (\overline{X}_i - \overline{\overline{X}})^2 / (k - 1) \tag{2-7}$$

(7) Compute the $(1-\alpha)\%$ confidence interval of $\bar{\bar{X}}$.

$$\bar{\bar{X}} \pm t_{1-\alpha/2,\ k-1} (S/\sqrt{k}) \tag{2-8}$$

The start-up time, total simulation time, and number of replications must be determined before applying replications analysis to the output of the TEXAS Simulation Model. More importantly, the existence of a stable state of the statistics studied is essential for making statistical inference. Figure 2-1 shows unstable states of average left-turn delay when the left-turn traffic is over-saturated. Stability of the left-turn capacity over time is as shown in Fig 2-2. As to the start-up time, the rule of thumb in simulation is one quarter of the total simulation time. This could also mean quite a lot of wasted data. Figures 2-3 through 2-6 show the variations of average left-turn delay over time for independent replications under various traffic conditions. It was found that although the average left-turn delay of each replication may have wide variations over time, the average value over eight replications will stabilize after running the simulation for about 40 minutes including 5 minutes of start-up time (1000 ft of approach). In this study, all cases will be simulated for 50 minutes including 5 minutes of start-up time. The number of replications depends on the precision desired. In practice, a number of replications from 5 to 20 are usually recommended. Although more replications will improve the accuracy of statistical inference, due to budget constraints eight independent replications are employed here.

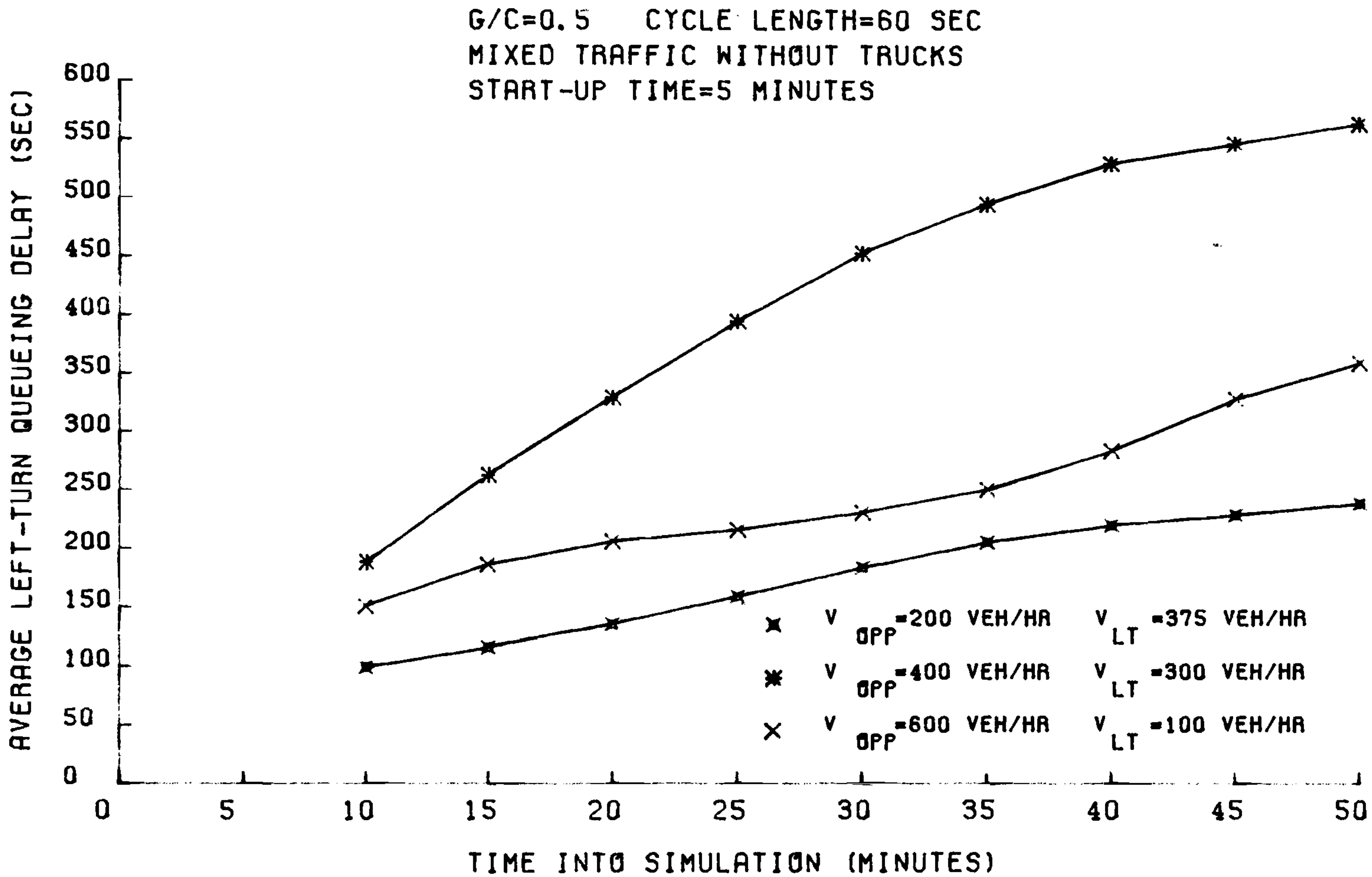


Fig 2-1. Unstable behavior of average left-turn delay statistics under oversaturated conditions.

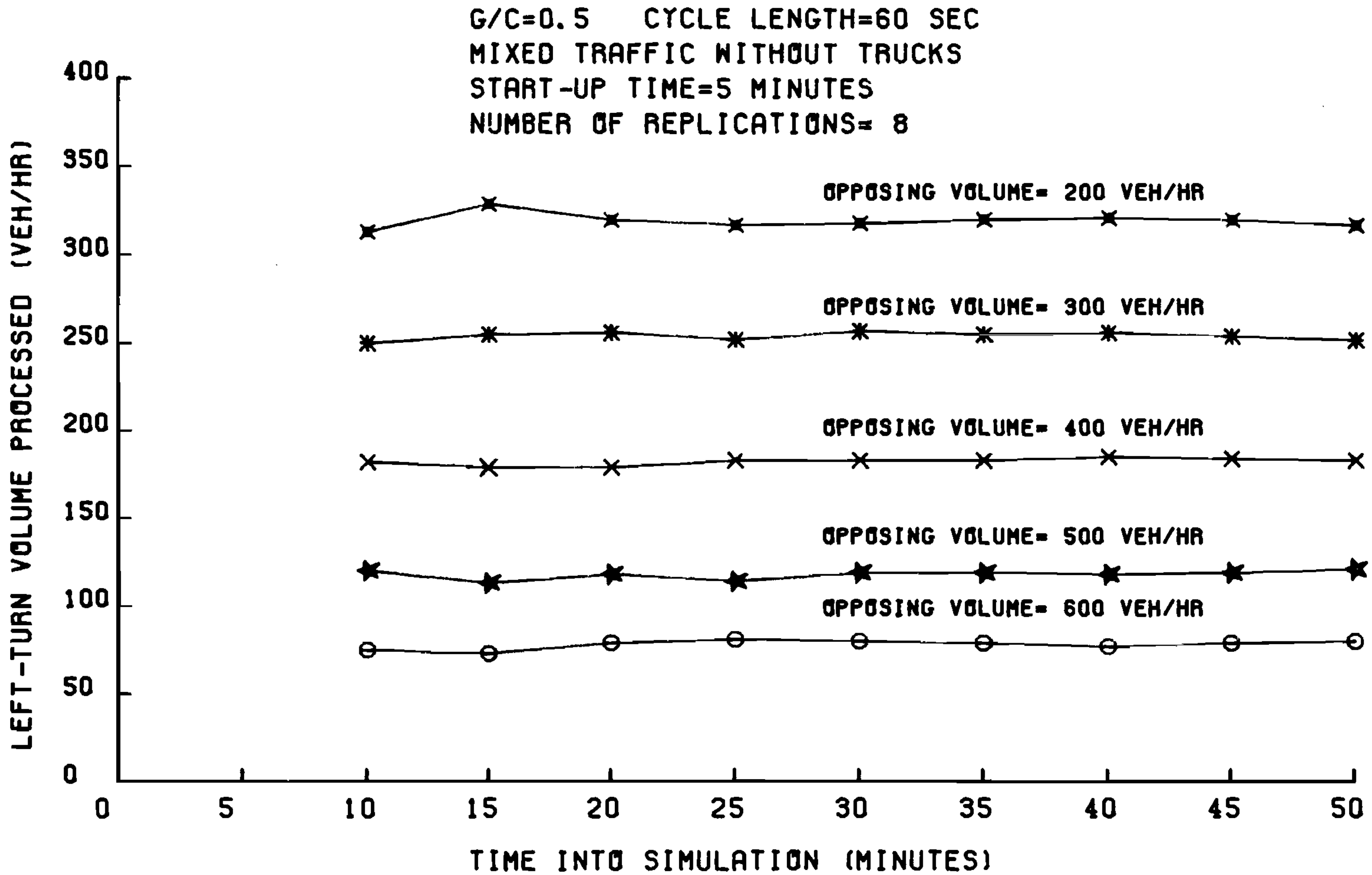


Fig 2-2. Stable behavior of left-turn flow under oversaturated conditions.

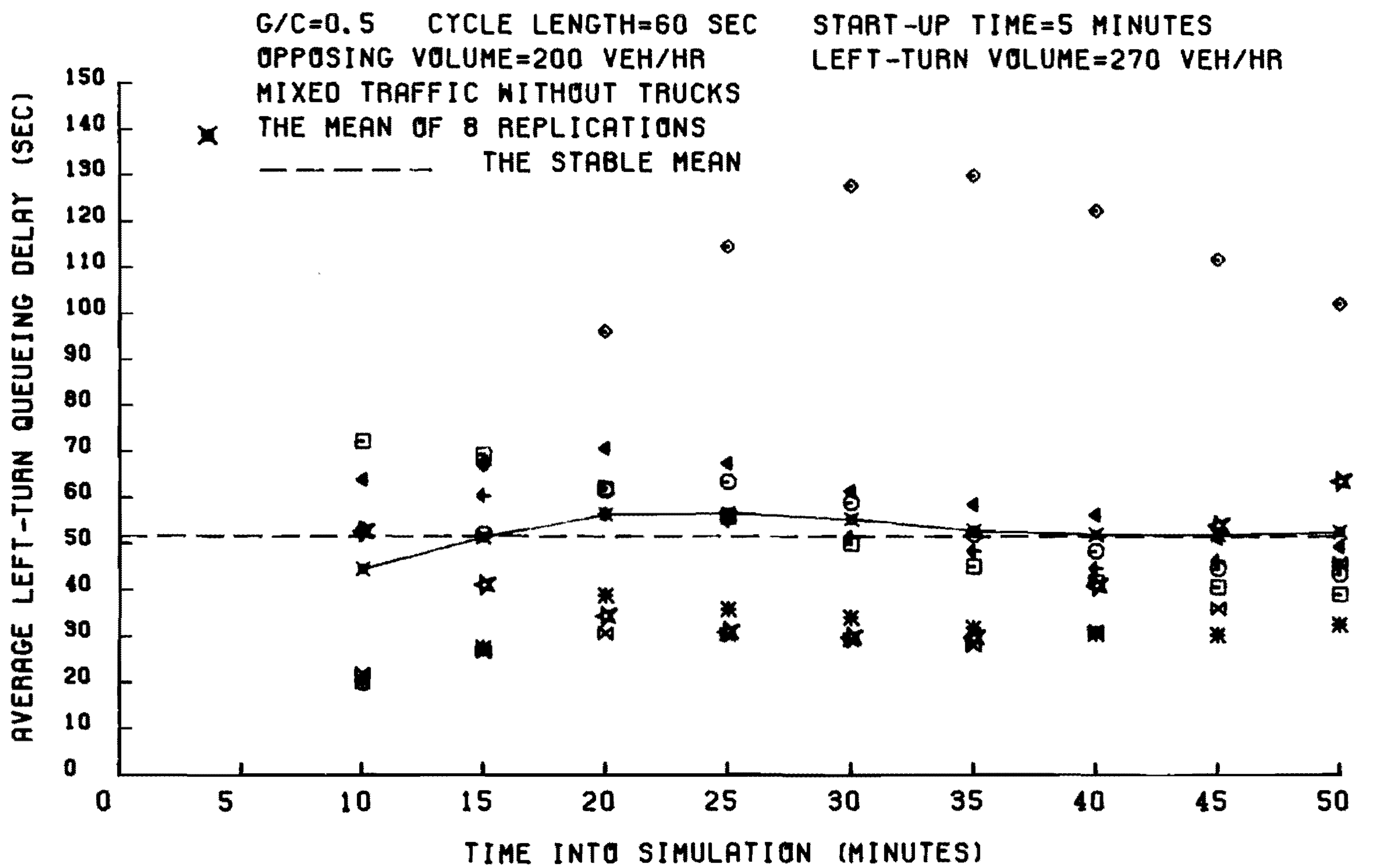


Fig 2-3. Stable behavior of the mean of the average left-turn delay over eight independent replications for opposing volume 200 vph.

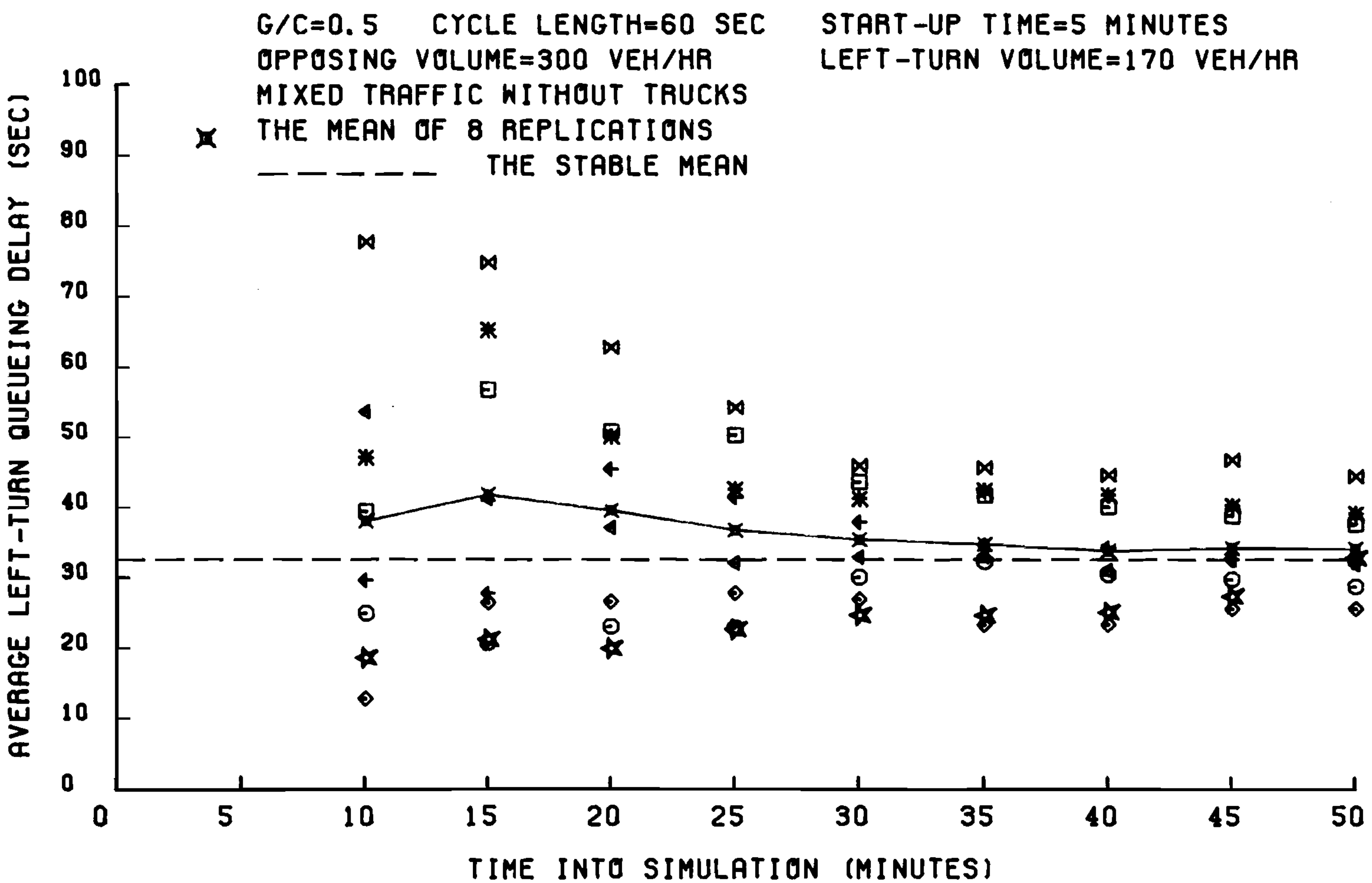


Fig 2-4. Stable behavior of the mean of the average left-turn delay over eight independent replications for opposing volume 300 vph.

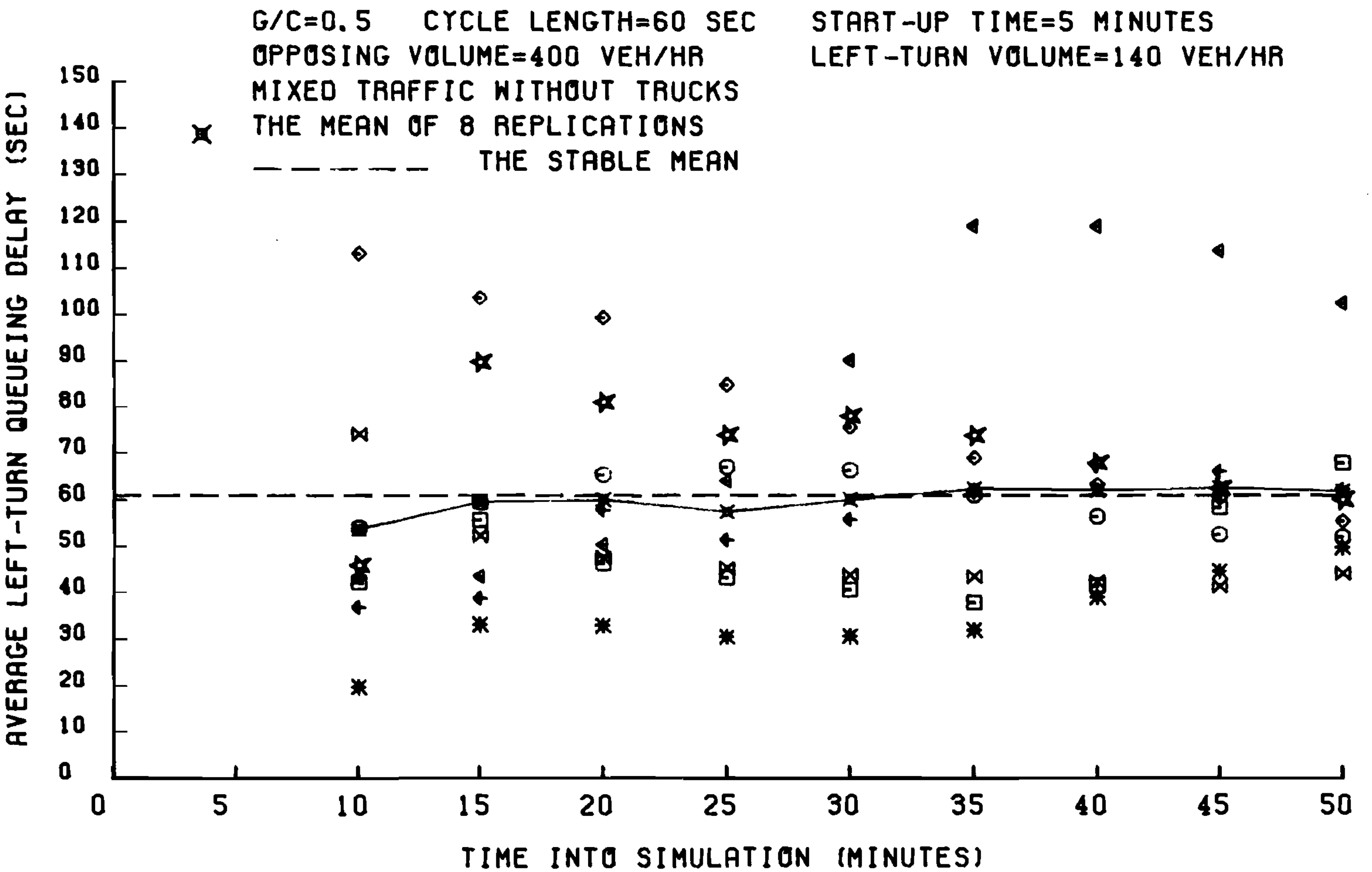


Fig 2-5. Stable behavior of the mean of the average left-turn delay over eight independent replications for opposing volume 400 vph.

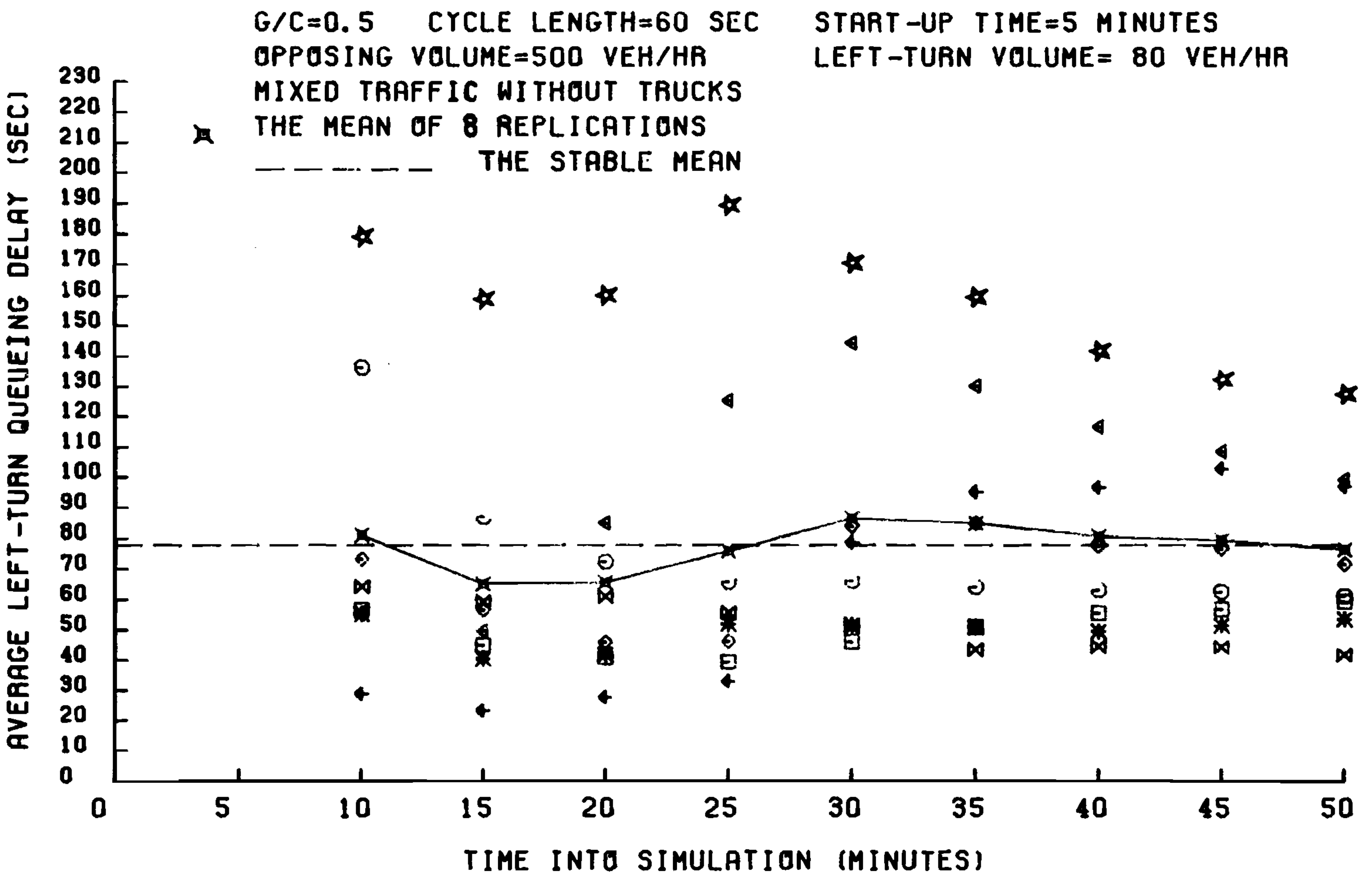


Fig 2-6. Stable behavior of the mean of the average left-turn delay over eight independent replications for opposing volume 500 vph.

CHAPTER 3. LEFT-TURN OPERATIONS STUDY

Before using a simulation model to study left-turn performance, it is desirable to know how well the model simulates traffic in the real world and to evaluate critically the features that pertain to left-turn operations. Moreover, in order to draw appropriate conclusions from simulation results, the basic traffic and geometric conditions under which simulation studies are carried out have to be specified. In this chapter, vehicular behavior in the TEXAS Model will be briefly illustrated; specifications for simulation runs will be outlined; and finally, factors important to left-turn operations such as opposing headways at the stop line, discharging headway, turning headway, and gap acceptance criteria will be examined.

VEHICULAR BEHAVIOR IN THE TEXAS MODEL

In the TEXAS Model, each driver-vehicle unit is randomly logged into an inbound lane at some specified distance (say 1000 feet) from the intersection according to some type of headway distribution. Each injected driver-vehicle unit is labelled as follows: (1) time of log-in, (2) vehicle class, (3) driver class, (4) desired velocity, (5) outbound log, (6) inbound approach, and (7) inbound lane. Up to five classes of drivers and 15 classes of vehicles covering a wide range of driver and vehicular characteristics are provided. The prevailing conditions for each driver-vehicle unit, which are updated at selected time intervals (say one second), may consist of the following information: (1) desired velocity, (2) destination, (3) current position, (4) velocity, (5) magnitude of acceleration/deceleration, (6) rate of change of acceleration/deceleration, (7) relative position and velocity of adjacent vehicles, (8) critical stopping distance, (9) sight restrictions, and (10) the location and status of traffic control devices. In response to these prevailing conditions, the simulated driver-vehicle unit may maintain speed, accelerate, decelerate, or maneuver to change lanes while traveling on an approach path. The basic premise for the response of the driver-vehicle unit is that the driver wants to sustain a desired speed while obeying the laws and maintaining safety and comfort. How a driver-vehicle unit behaves on the approach path and how a left-turn driver-vehicle unit chooses an acceptable gap in the opposing traffic will be briefly illustrated in this section. More details about the model can be found in Refs 7 and 8.

The first step toward understanding the TEXAS Model, perhaps, is to know how a driver-vehicle unit responds to prevailing conditions. A logical binary network as shown in Fig 3-1 generates a unique response for each driver-vehicle unit at every selected time interval. Typical responses are:

(1) accelerating to desired velocity when entering the intersection,

(2) accelerating to lead vehicle velocity when moving forward from the stopping status,

(3) following the vehicle ahead,

(4) initializing deceleration for stop to avoid a potential collision,

(5) continuing deceleration to a stop, and

(6) remaining stopped.

Except for the action of following a vehicle ahead, all other responses are straightforward. How a driver-vehicle unit behaves when following a vehicle ahead will be explained in more detail. An acceptable car-following distance is defined in the TEXAS Model, and the behavior of a following driver-vehicle unit depends on how the relative position (distance between the lead and following vehicles) compares with the acceptable car-following

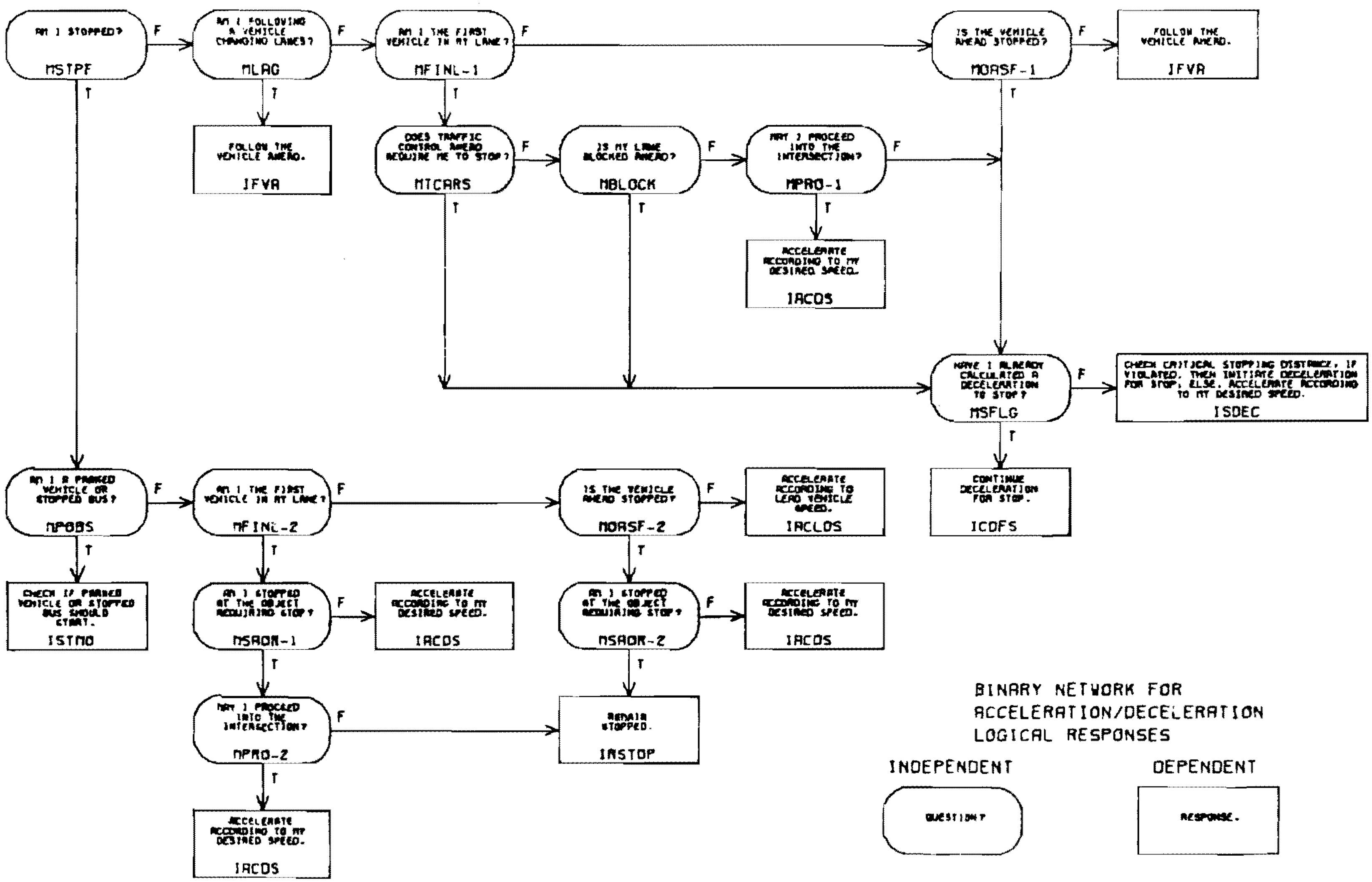


Fig 3-1. Logical binary network for acceleration and deceleration responses.

distance. There can be two usual conditions for a driver-vehicle unit traveling behind another. The lead vehicle is going faster than the following vehicle (positive relative velocity) or the lead vehicle is slower (negative relative velocity). In the first condition, the acceptable car-following distance is defined as follows:

$$S_a = 1.7v_p/D_f \tag{3-1}$$

where

S_a = acceptable car-following distance, ft;

v_p = velocity of the lead driver-vehicle unit, ft/sec;

D_f = driver operational factor,

1.1 for aggressive drivers,
1.0 for average drivers, and
0.85 for slow drivers.

For example, if the velocity of the lead vehicle is 30 mph and the driver of the following vehicle is an average driver, the acceptable car-following distance would be 75 ft. When the lead driver-vehicle unit is going faster than the following driver-vehicle unit, the response for the following driver-vehicle unit is as shown in Fig 3-2a.

On the other hand, when the lead driver-vehicle unit is going slower than the following driver-vehicle unit, the acceptable car-following distance is defined as follows:

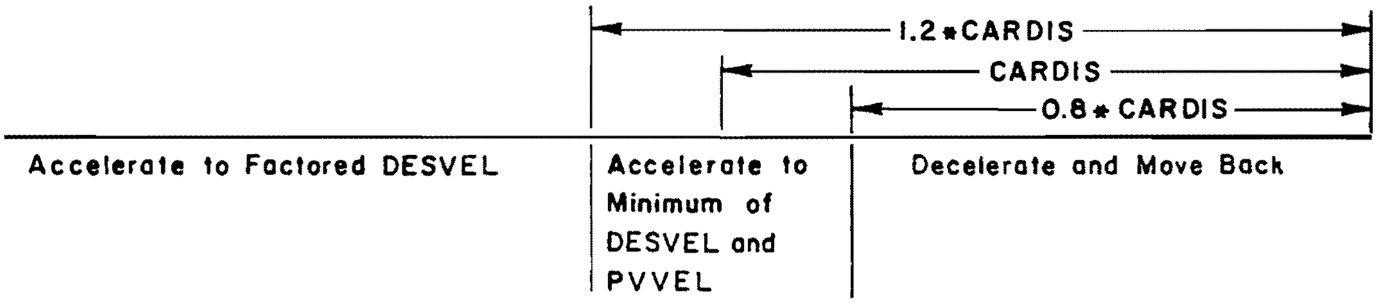
$$S_a = (1.7v_p + 4.0v_r^2)/D_f \tag{3-2}$$

where

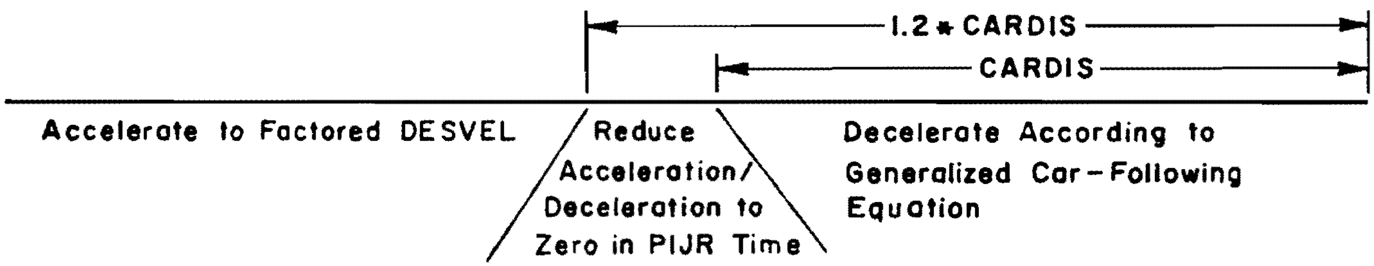
S_a = acceptable car-following distance, ft;

v_r = relative velocity, ft/sec; and

v_p and D_f are as defined previously.



(a) Leading driver-vehicle unit is going faster than the following driver-vehicle unit.



(b) Following driver-vehicle unit is going faster than the leading driver-vehicle unit.

Fig 3-2. Conceptualization of car-following states.

For example, if the velocity for the lead and following vehicles are 30 mph and 35 mph, respectively, and the driver in the following vehicle is an average driver, then the acceptable car-following distance in this case will be about 290 ft. When the lead driver-vehicle unit is going slower than the following driver-vehicle unit, the following driver-vehicle unit will behave as shown in Fig 3-2b.

Since the response for a driver-vehicle unit involves either acceleration or deceleration, it would be of interest to know how the model simulates accelerating and decelerating behavior. The basic idea is to find the appropriate rate of change of acceleration or deceleration (jerk) for each driver-vehicle unit in every time increment. With the assumption of linear variation of acceleration or deceleration over the time increment, the acceleration or deceleration, velocity, and position of each driver-vehicle unit can be updated by the following equations:

$$a = a_o + b\Delta t \qquad (3\text{-}3)$$

$$v = v_o + a_o\Delta t + 0.5b\Delta t^2 \qquad (3\text{-}4)$$

$$x = x_o + v_o\Delta t + 0.5a_o\Delta t^2 + 0.167b\Delta t^3 \qquad (3\text{-}5)$$

where

a = acceleration at the end of Δt, ft/sec^2;

a_o = acceleration at the beginning of Δt, ft/sec^2;

v = velocity at the end of Δt, ft/sec;

v_o = velocity at the beginning of Δt, ft/sec;

x = position at the end of Δt, ft;

x_o = position at the beginning of Δt, ft;

b = rate of change of acceleration (jerk), ft/sec^3; and

Δt = time increment, sec.

The rate of change of acceleration (deceleration), or jerk, is a function of current acceleration (deceleration), current velocity, desired velocity, driver characteristics, and vehicular characteristics. Details about how to obtain the appropriate jerk for acceleration or deceleration can be found in Ref 9. Typical acceleration behaviors for different initial velocities are shown in Fig 3-3.

Sometimes a driver-vehicle unit needs to change lanes in order to make turns in the proper lane or to experience less delay. The former is forced while the latter is optional. For forced lane-change, a driver-vehicle unit must check for an acceptable gap in the alternate lane. While checking for an acceptable gap, a driver-vehicle unit may either move slowly or stop, depending on whether it is a right-turning or a left-turning vehicle. If a lane-change is not completed after the end of the alternate lane is passed, then the unit is forced to abandon its original destination. When the lane-change is optional an expected delay is computed for the driver-vehicle's current lane as well as for alternate lanes. If less delay can be expected by changing lanes, the alternate lane is checked for the presence of acceptable gaps. The expected delay is determined by the equivalent number of driver-vehicle units in front of the driver-vehicle unit being checked. A penalty is added to the actual number of vehicles in the queue based on the turn code of the last driver-vehicle unit in the queue and the unit being checked.

When executing unprotected left turns, driver-vehicle units must choose acceptable gaps in the opposing traffic stream. Generally, there are three behavioral models of gap-acceptance decision making: (1) all drivers have an identical minimum acceptable gap, (2) each driver has his own minimum acceptable gap, and (3) each driver has an identical probability distribution

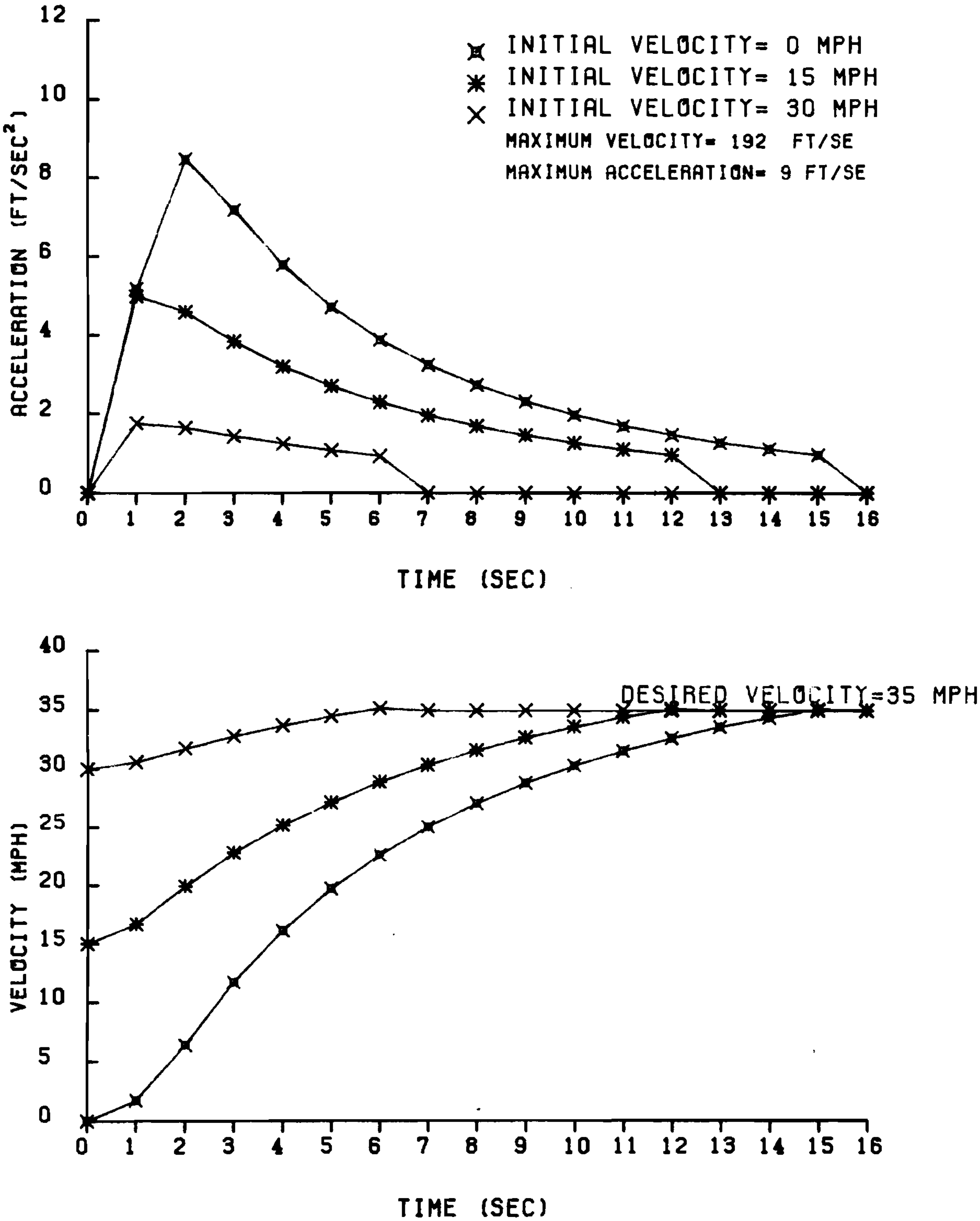


Fig 3-3. Accelerating behavior under different levels of initial velocity in reaching a desired velocity of 35 mph.

of gap acceptance. The actual situation is probably somewhere between the second and the third model, with each driver having his own gap-acceptance probability function. However, modeling the actual gap-acceptance behavior is very difficult. In the TEXAS Model, drivers check the presented gaps against a required safety zone. This safety zone is made up of the time for the gap-accepting vehicle to pass through the point of intersection conflict, plus a lead safety zone, plus a lag safety zone, plus perception-reaction time, plus a time for judgement error as depicted in Fig 3-4. The safety zone will be a random variable, since each driver-vehicle unit has been randomly assigned a desired velocity, driver's class, and vehicular class, Typical distributions of acceptable headways will be shown later in this chapter.

SPECIFICATIONS FOR SIMULATION RUNS

The input variables of the TEXAS Model generally can be classified in the following groups:

(1) geometric configuration,

(2) driver characteristics,

(3) vehicular characteristics,

(4) traffic,

(5) traffic control,

(6) traffic actuation, and

(7) detector.

Each group of variables consists of a large amount of detailed information. In order to make the experimental design feasible, unless otherwise specified, input variables of TEXAS Model will be set to the following values:

⟨ME				⟨HIM				⟨ME
TPASSM	$\frac{\text{ERRJUD}}{2}$	TLEAD minus APIJR	PIJR	TPASSH	TLAG minus APIJR	PIJR	$\frac{\text{ERRJUD}}{2}$	

where:

TPASSM	is the time for ME (the turning vehicle) to pass through the point of intersection conflict
ERRJUD	is the judgment error
TLEAD	is the time for the lead safety zone
APIJR	is the average perception-reaction time for all drivers
PIJR	is the perception-reaction time for the turning driver, ME
TPASSH	is the time for HIM (the opposing vehicle) to pass through the point of intersection conflict
TLAG	is the time for the lag safety zone

⟨HIM 1				⟨ME				⟨HIM 2
	TLAG minus APIJR	PIJR	$\frac{\text{ERRJUD}}{2}$	TPASSM	$\frac{\text{ERRJUD}}{2}$	TLEAD minus APIJR	PIJR	

|←——— MINIMUM GAP ———→|

Fig 3-4. Intersection conflict checking safety zones.

(1) Geometric Configuration Variables:

(a) lane width: 12 ft,

(b) inbound approach length: 1000 ft,

(c) outbound lane length: 400 ft,

(d) curb return radius: 25 ft,

(e) sight distance restrictions: none,

(f) angle of intersection: right angle, and

(g) length of left-turn lane: 1000 ft.

(2) Driver Characteristics Variables: See Table 3-1.

(3) Vehicular Characteristics Variables: See Table 3-1.

(4) Traffic Variables:

(a) type of injected headway distribution: shifted-negative exponential,

(b) minimum injected headway: 1.7 sec,

(c) mean speed: 30 mph,

(d) 85 percentile speed: 35 mph,

(e) speed limit: 35 mph,

(f) percentage of right turns: zero, and

(g) percentage of left turns in opposing traffic: zero.

(5) Traffic Control Variable:

(a) cycle length: 60 seconds,

(b) cycle split: 50 percent, and

(c) yellow signal interval: 3 seconds.

(6) Traffic Actuation Variables: Not applicable.

(7) Detector Variables: Not applicable.

TABLE 3-1. DRIVER AND VEHICULAR CHARACTERISTICS

Item	Vehicle Class and Type				
	1 Small Car	2 Medium Car	3 Large Cars	4 Vans, Mini-bus	5 Sports Car
Length, ft	15	17	19	25	14
Operating Characteristic Factor	100	110	110	100	115
Maximum Deceleration, ft/sec^2	16	16	16	16	16
Maximum Acceleration, ft/sec^2	8	9	11	8	14
Maximum Velocity, ft/sec	150	192	200	150	205
Minumum Turning Radius, ft	20	22	24	28	20
Percentage Aggressive Drivers, percent	30	35	20	25	50
Percentage Average Drivers, percent	40	35	40	50	40
Percentage Slow Drivers, percent	30	30	40	25	10
Percentage in Traffic Stream, percent	20	33.5	30	15	1.5

Driver Class and Type	1 Aggressive	2 Average	3 Slow
Driver Operational Factor	110	100	85
Perception Reaction Time, sec	0.5	1.0	1.5

OPPOSING HEADWAYS AT THE STOP LINE

At signalized intersections without an exclusive left-turn phase, left turns can be made only through opposing gaps during the remaining green time after the opposing queue has been discharged and during yellow intervals. Consequently, the distribution of opposing headways at the stop line after the opposing queue has been discharged plays a key role in left-turn delay and left-turn capacity. Although many theoretical probability distributions such as the shifted negative-exponential, Erlang, and Gamma distributions are found to describe certain headway patterns very well, few studies have been made on headways at the stop line at signalized intersections. Due to car-following and signalization, it is doubtful that headways at the stop line after queue dissipation have any relation to the headway distribution far upstream of the intersection. From the TEXAS Model, histograms of headways at the stop line after the queue discharged under different levels of traffic volumes are obtained as shown in Figs 3-5 through 3-8. It can be seen that these headway histograms deviate from the shifted negative-exponential headway distributions injected 1000 feet upstream. As the traffic volumes increase, more headways will be concentrated in the range of two to three seconds which is approximately the discharging headway. The effects of car-following and signalization on the average headway are as shown in Fig 3-9. It can be seen that the average headway at the stop line after the queue has been discharged is about half of that at 1000 feet upstream of the intersection provided the cycle split is 50 percent.

It has been shown that car-following and signalization will cause the headways at the stop line after the queue has been discharged to be different from that 1000 ft upstream. The question is whether different types of headway distributions at 1000 feet from the stop line will influence the

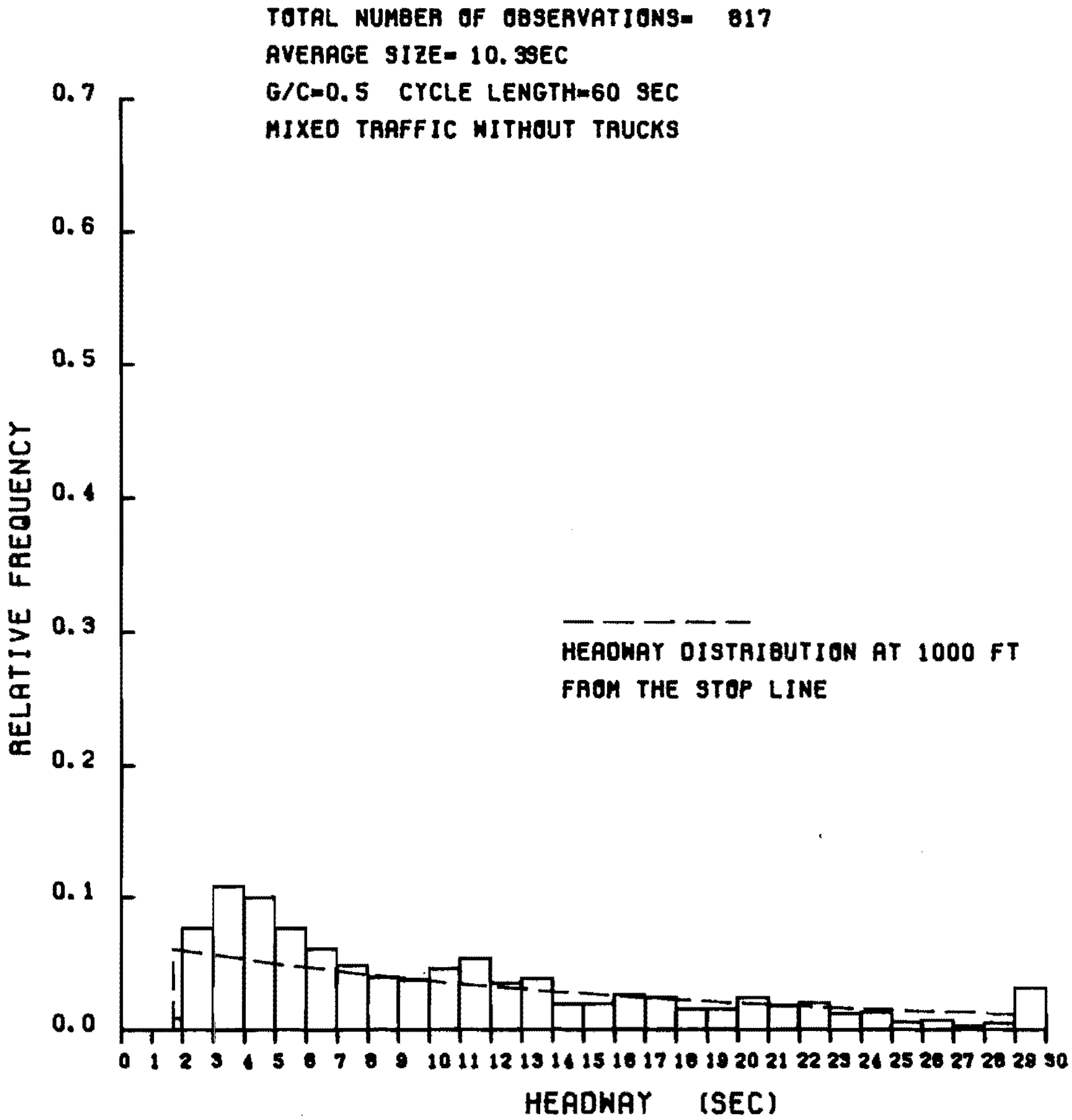


Fig 3-5. Histogram of the opposing headways at the stop line after queue dissipation when the opposing volume is 200 vph.

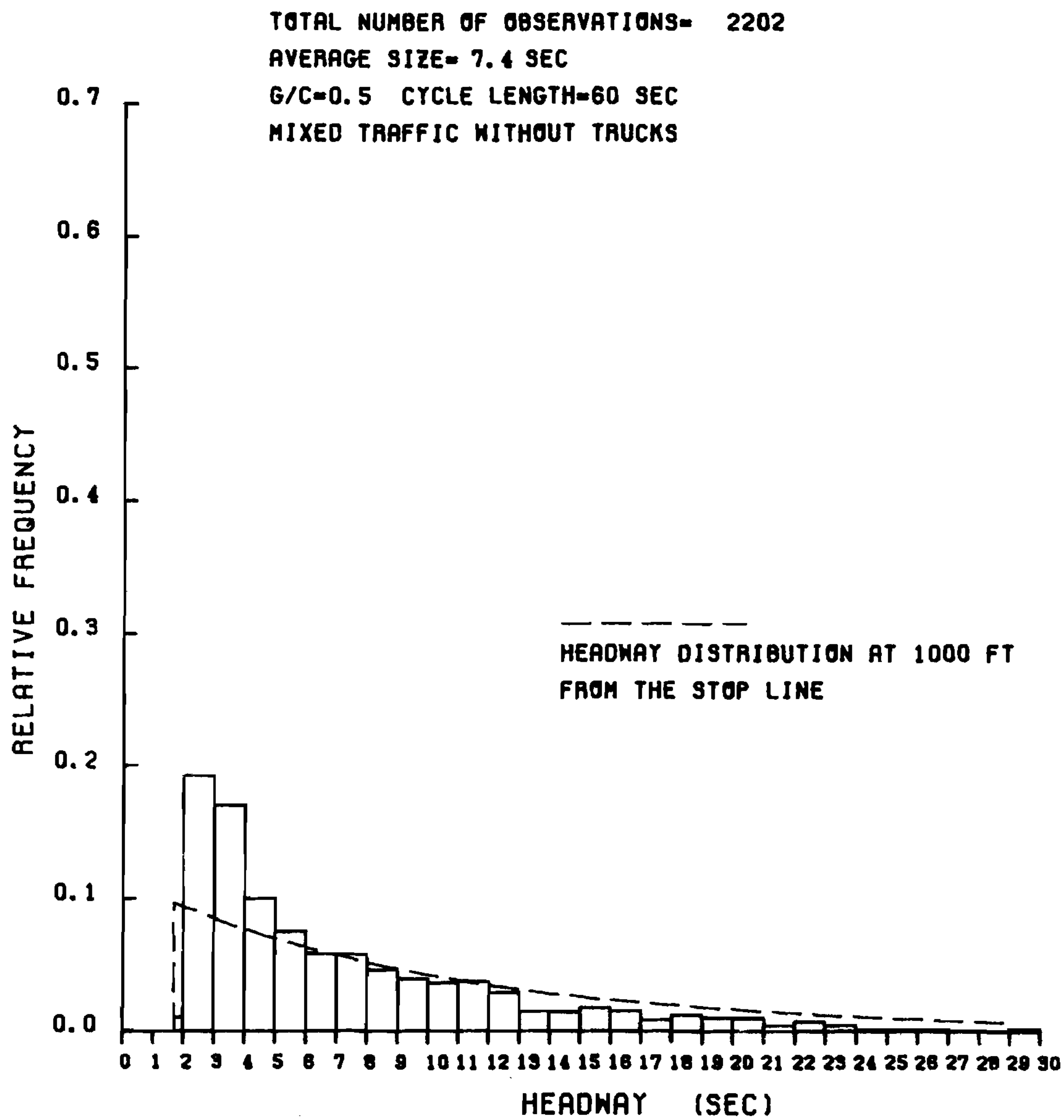


Fig 3-6. Histogram of the opposing headways at the stop line after queue dissipation when the opposing volume is 300 vph.

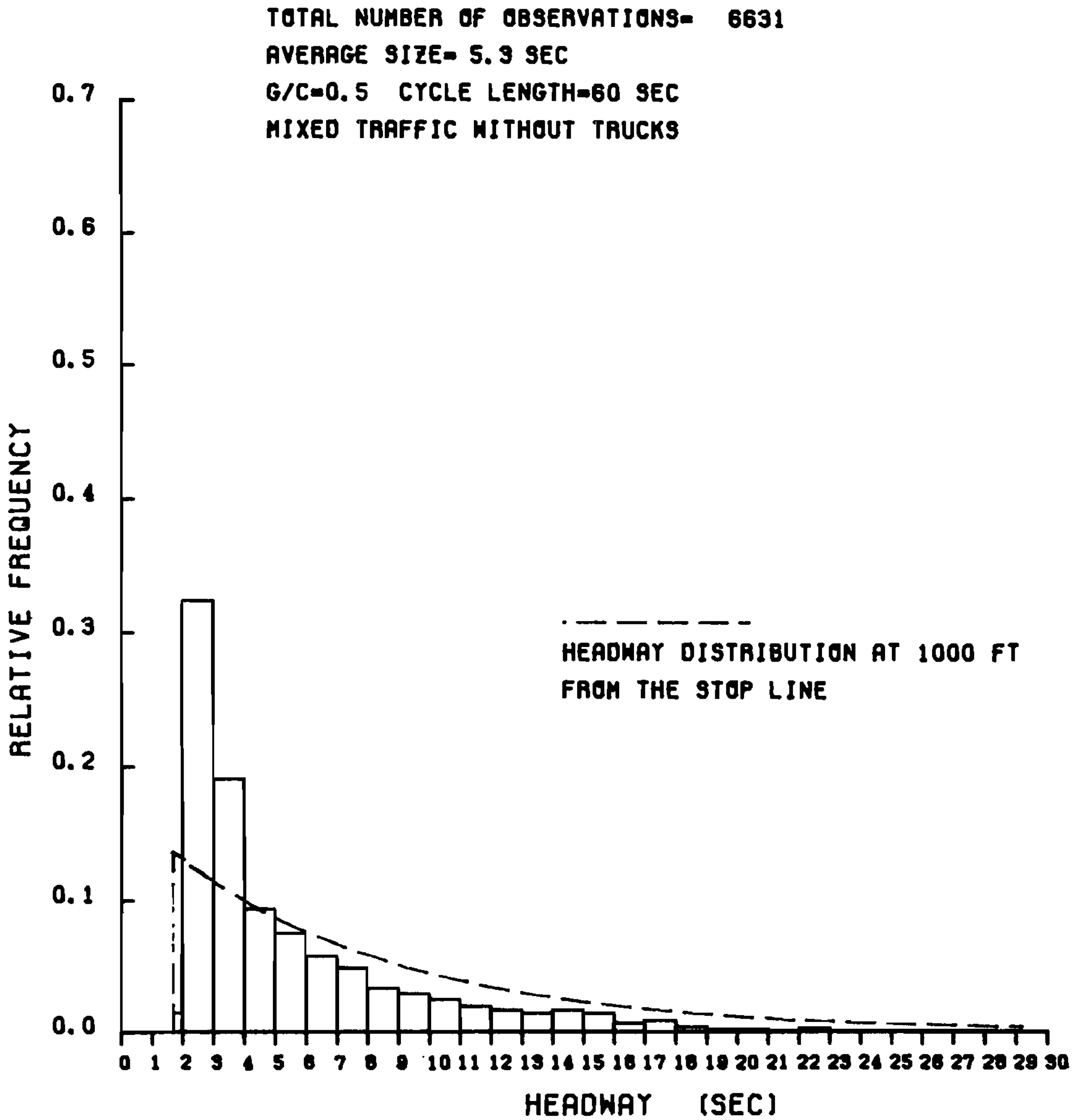


Fig 3-7. Histogram of the opposing headways at the stop line after queue dissipation when the opposing volume is 400 vph.

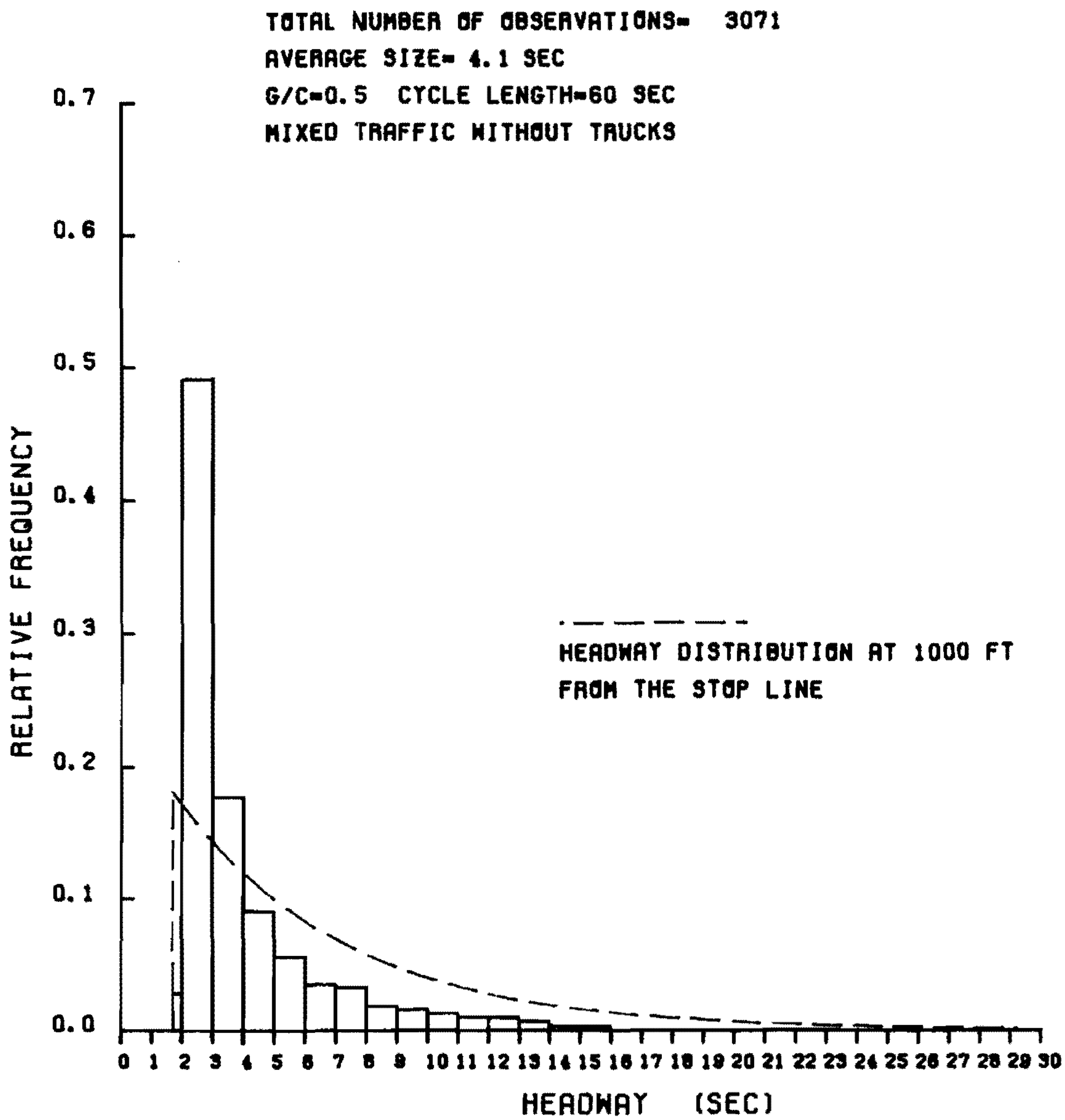


Fig 3-8. Histogram of opposing headways at the stop line after queue dissipation when the opposing volume is 500 vph.

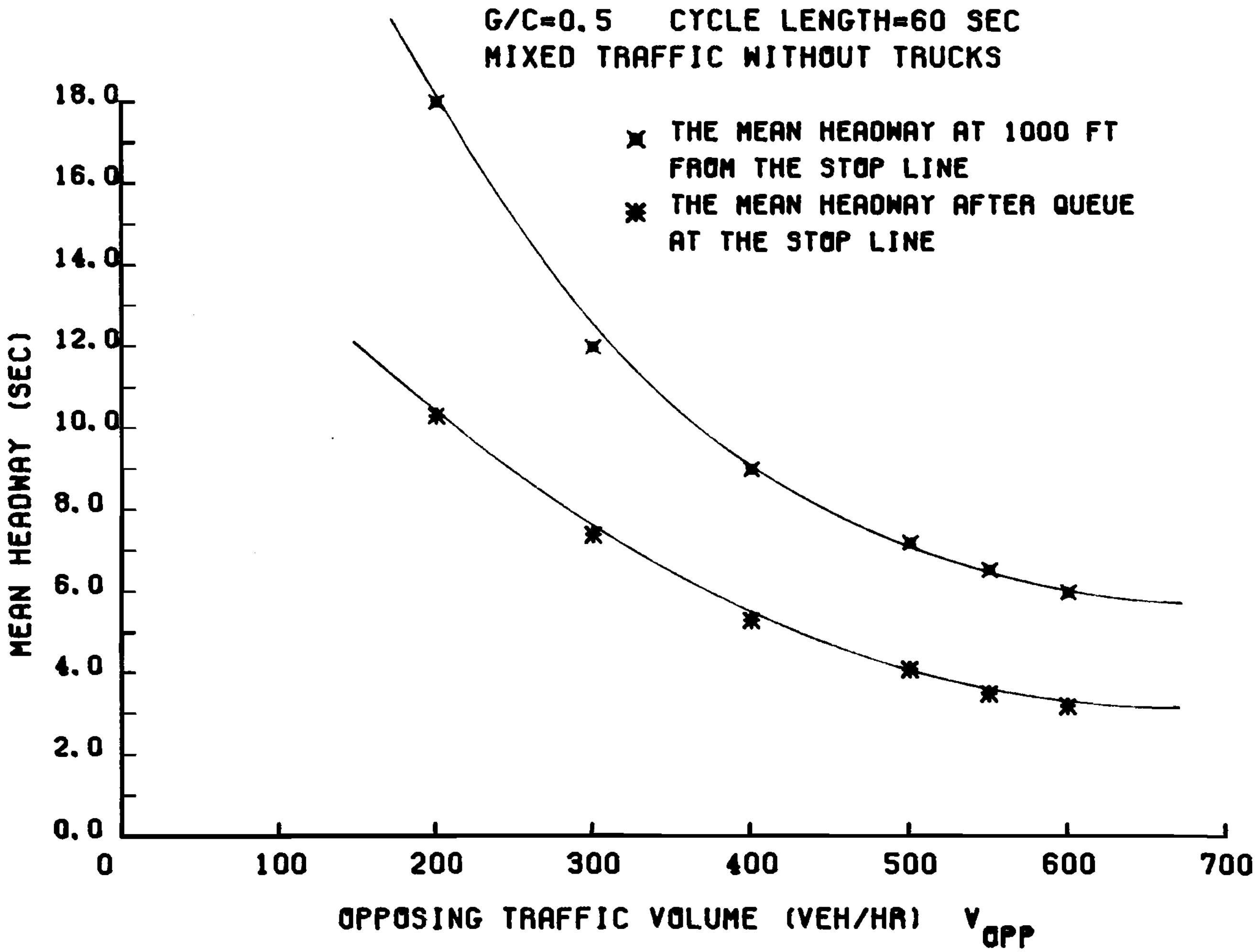


Fig 3-9. Comparison between the average opposing headway at the stop line and at 1000 feet upstream.

headway distribution at the stop line after the queue has been discharged. After changing the injected headway distribution from the shifted negative-exponential (coefficient of variation = 1.0) to an Erlang with coefficient of variation equal to 0.5, headways at the stop line after the queue had been discharged were computed by the TEXAS Model and in Figs 3-10 through 3-13. Histograms of these headways are quite similar to those in Figs 3-5 through 3-8 except that they are less peaked in the range of 2 to 4 seconds. It can therefore be said that changing the coefficient of variation from 1.0 (shifted negative-exponential distribution to 0.5 (Erlang distribution) is not critical to analysis of left-turn operations. Left-turn capacity is not significantly effected by the choice of injected headway distribution. This observation is discussed in more detail in Chapter 4

DISCHARGING HEADWAY

Since an unprotected left turn can basically be made only after the opposing queue has discharged, the discharging headways of an opposing queue are of importance to left-turn operations. Greenshields [Ref 10] found the discharging headway for passenger cars after the first five or six vehicles to be 2.1 seconds and proposed a simplified equation: $3.7 + 2.1N$ for computing the total seconds of green time needed for discharging N passenger cars in a queue. Regression analysis on the outputs of the TEXAS Model reveals an average discharge headway of 2.5 seconds (see Fig 3-14). The model assumes that the lead vehicle is stopped at the stop line and enters the intersection immediately after perception-reaction time for the driver has elapsed. This means that the saturated lane flow is 1400 vehicles per green hour. The time required for discharging the queue under different levels of traffic volumes at signalized intersections with cycle split 0.5 and cycle length 60 seconds is found from the TEXAS Model and is shown in

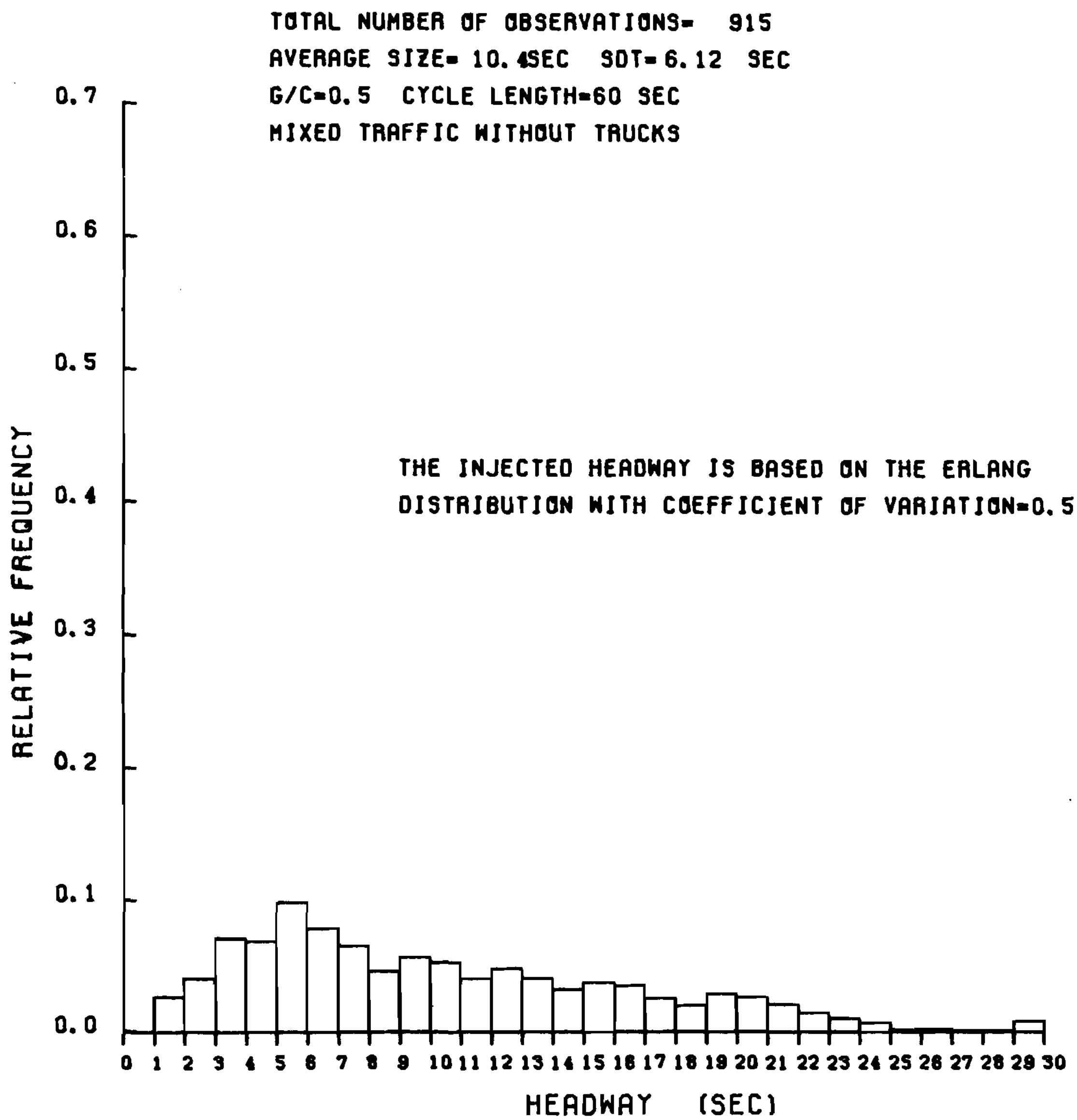


Fig 3-10. Histogram of opposing headways at the stop line after queue dissipation when the vehicles are injected at 1000 feet upstream according to an Erlang distribution with c.v. = 0.5, and mean = 18 sec.

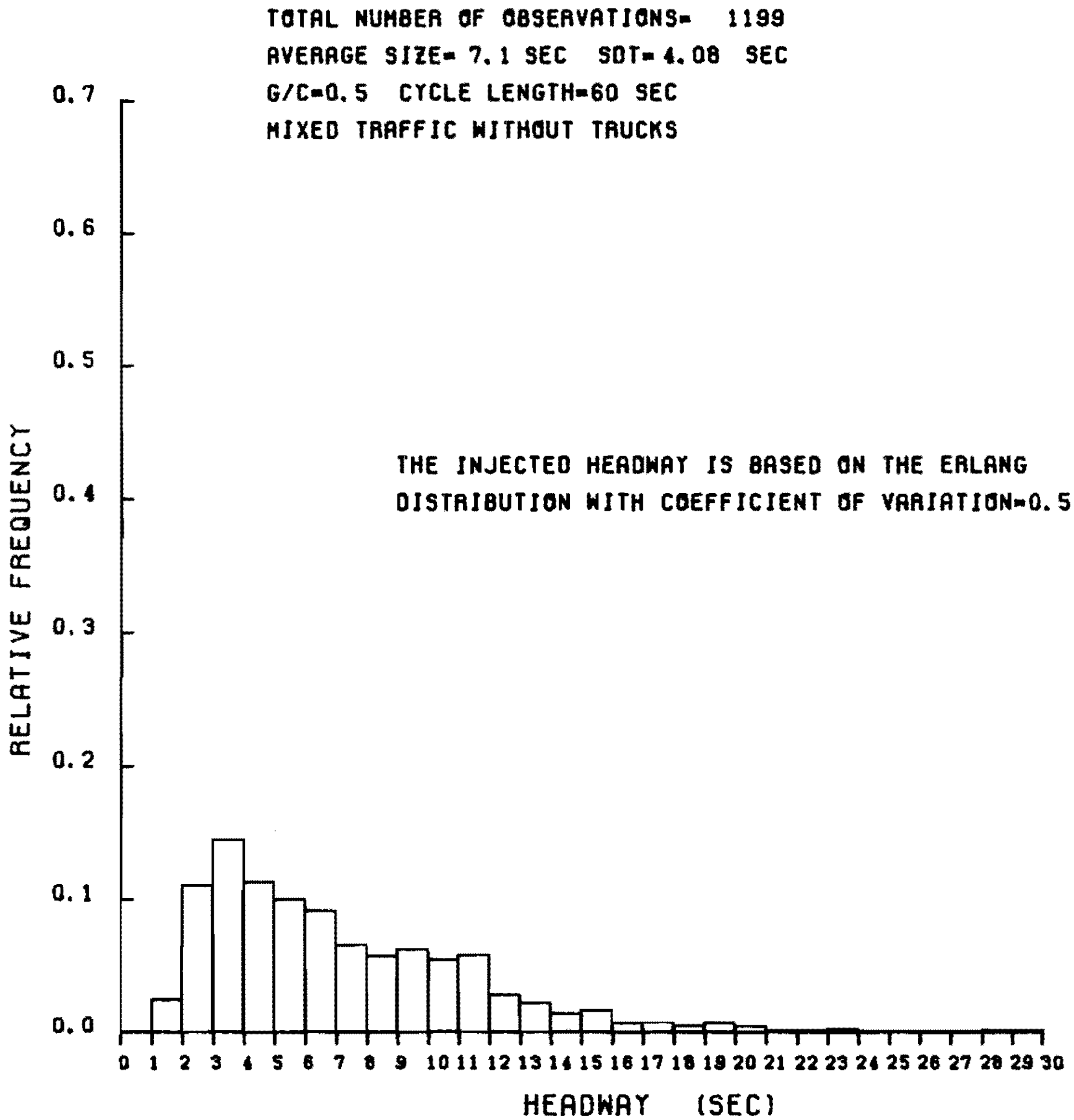


Fig 3-11. Histogram of opposing headways at the stop line after queue dissipation when the vehicles are injected at 1000 feet upstream according to an Erlang distribution with c.v. = 0.5, and mean = 12 sec.

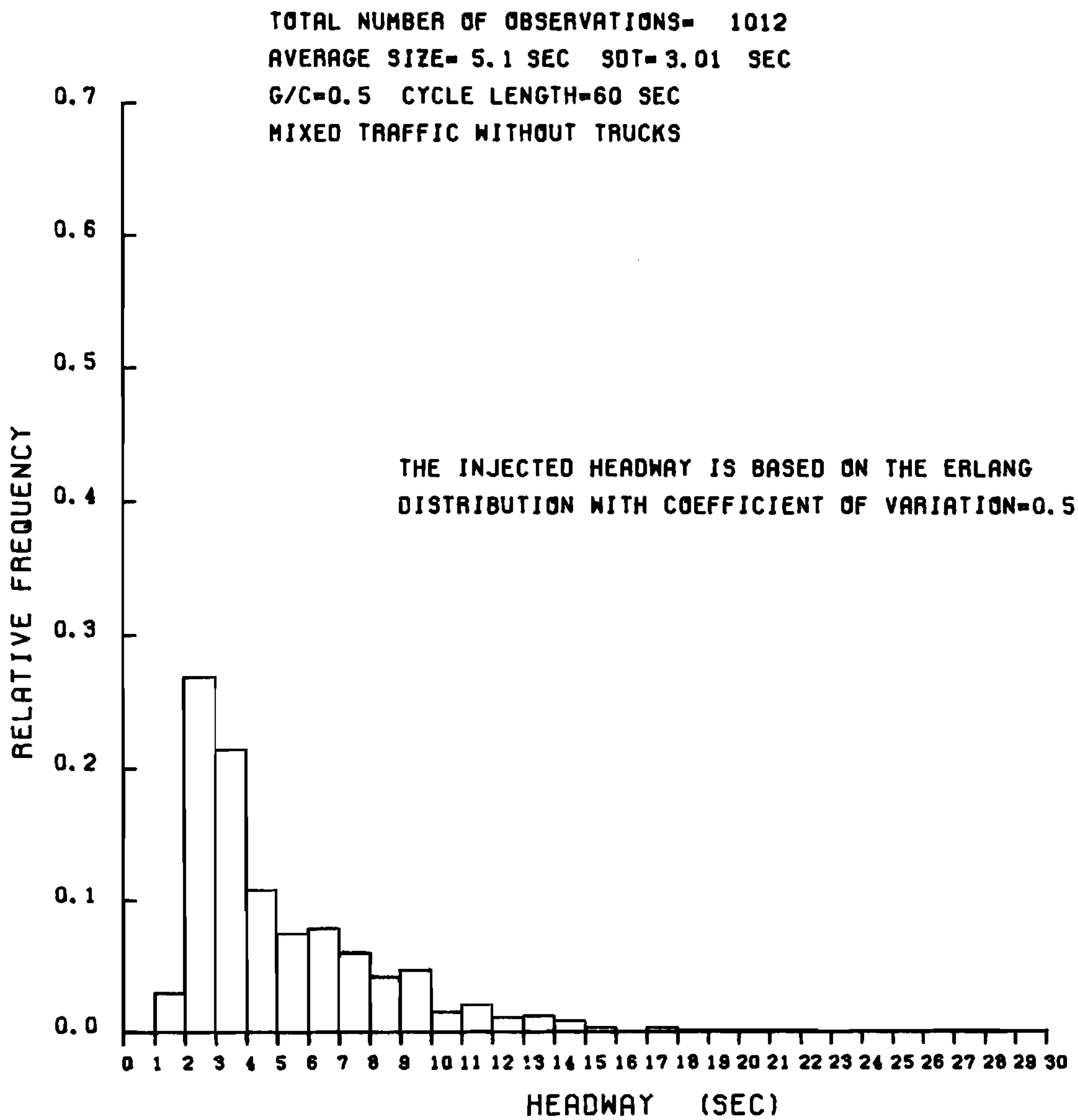


Fig 3-12. Histogram of opposing headways at the stop line after queue dissipation when the vehicles are injected at 1000 feet upstream according to an Erlang distribution with c.v. = 0.5, and mean = 7.2 sec.

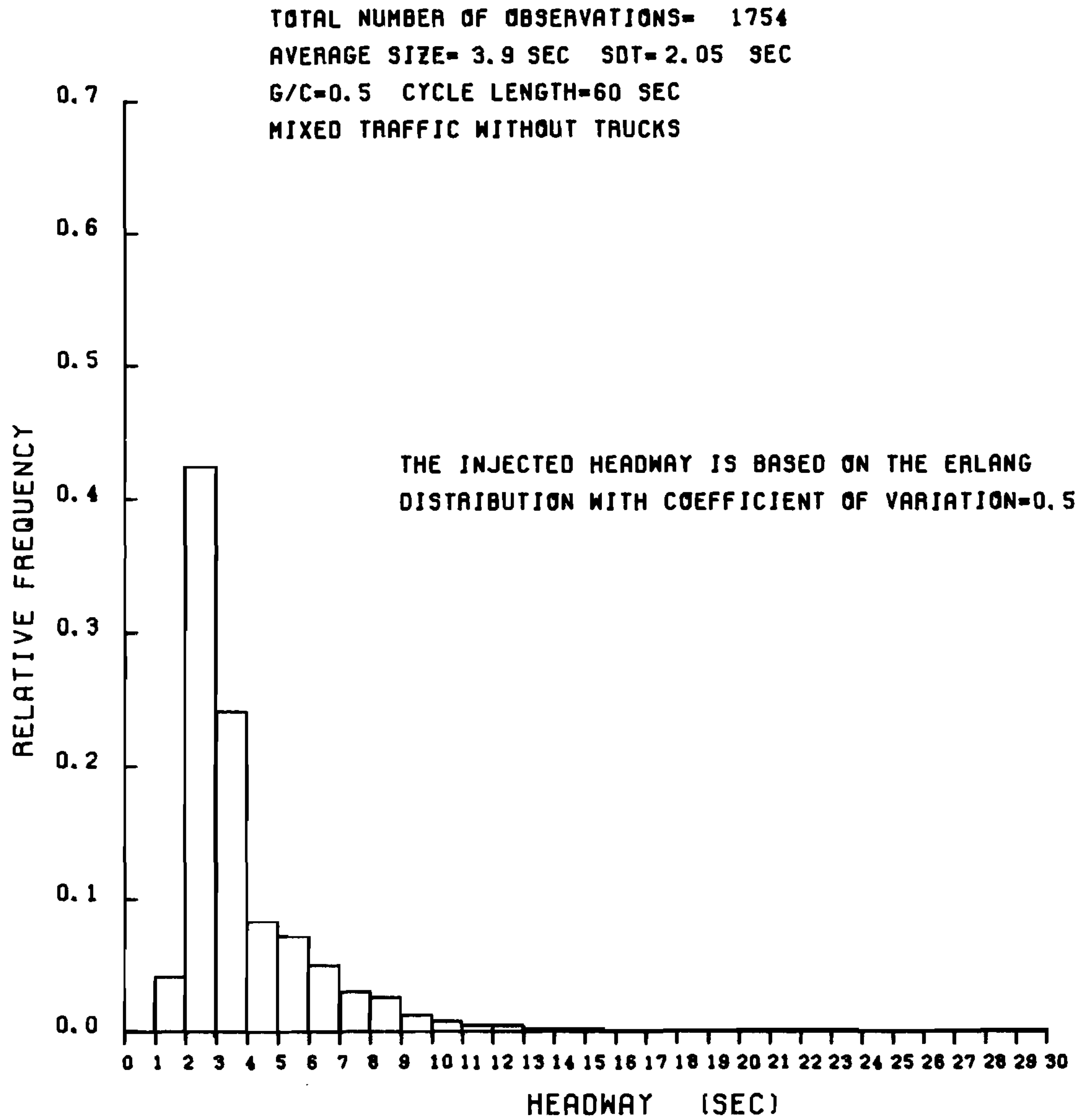


Fig 3-13. Histogram of opposing headways at the stop line after queue dissipation when the vehicles are injected at 1000 feet upstream according to an Erlang distribution with c.v. = 0.5, and mean = 7.2 sec.

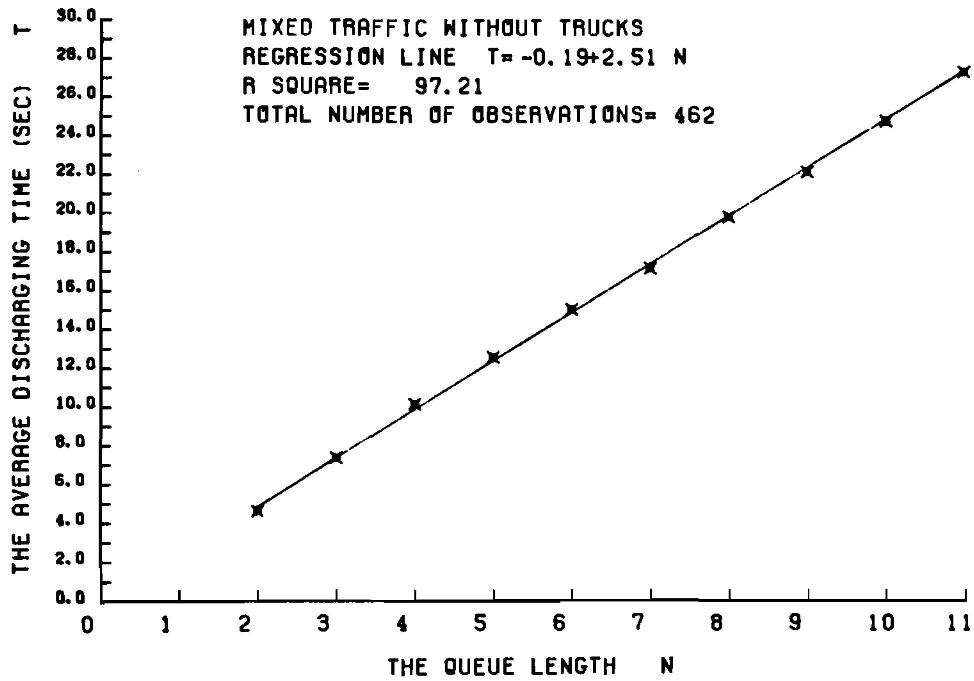


Fig 3-14. The average discharging headway at signalized intersection.

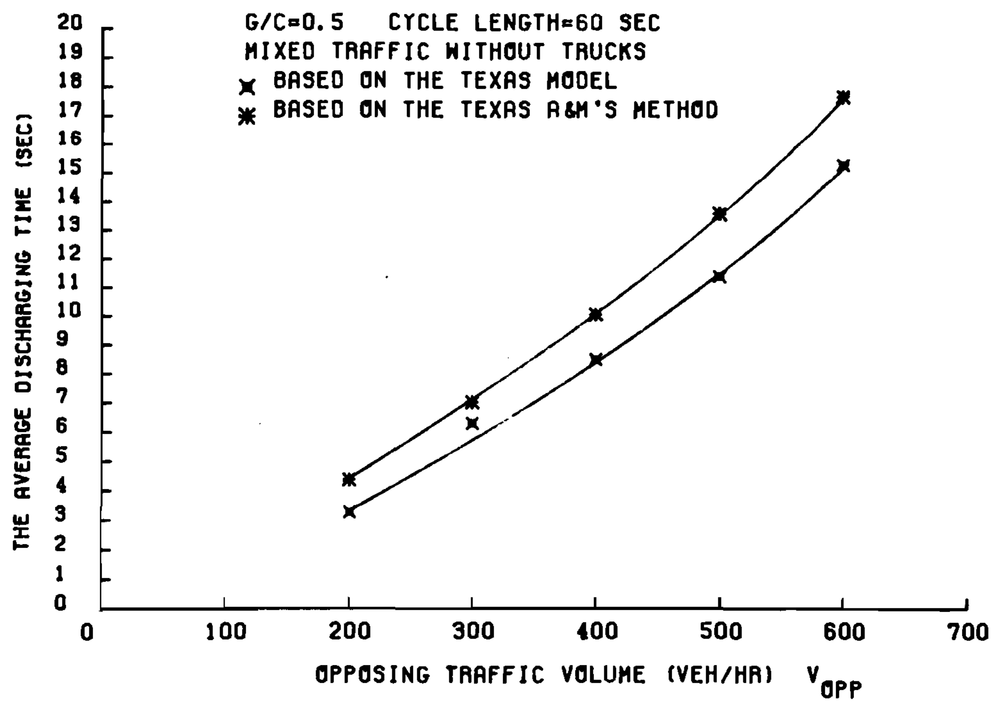


Fig 3-15. The time required for discharging the opposing queue under different levels of opposing volume.

Fig 3-15. Texas Transportation Institute [Ref 11] also proposed an equation for computing the time required for discharging the queue. This equation predicts queue discharging time about two to three seconds longer than that from the TEXAS Model as Fig 3-15 shows.

TURNING HEADWAY

If an opposing gap is large enough to accommodate more than one left turn, the headway between successive left-turn vehicles has a significant effect upon how many left turns can go through it. Fambro and Messer [Ref 11] observed turning headways as follows: (1) no separate phase and left-turn lane: 2.6 seconds, (2) No separate left-turn phase, with left-turn lane: 2.5 seconds, and (3) separate left-turn phase with left-turn lane: 2.1 seconds. Regression analysis on outputs of the TEXAS Model shows an average headway between successive left-turning vehicles to be 3.6 seconds (see Fig 3-16).

GAP-ACCEPTANCE CRITERIA

A gap may be measured by time between the rear of the lead vehicle and the front of the following vehicle. Gap acceptance criteria are often characterized by the "critical" gap, which is a gap of duration just sufficient for left turners to have an equal probability of accepting or rejecting it. Agent [Ref 12] found a critical gap of 4.2 seconds at signalized intersections with a left-turn lane and an unprotected signal phase. Other values of critical gap ranging from 3.8 to 5.8 seconds are reported in the literature. A critical headway (not gap) of 5.4 seconds was obtained from the TEXAS Model as shown in Fig 3-17. If an average speed of 30 mph and an average car length of 20 ft are taken into account, this critical headway is equivalent to a critical gap of approximately 5.0

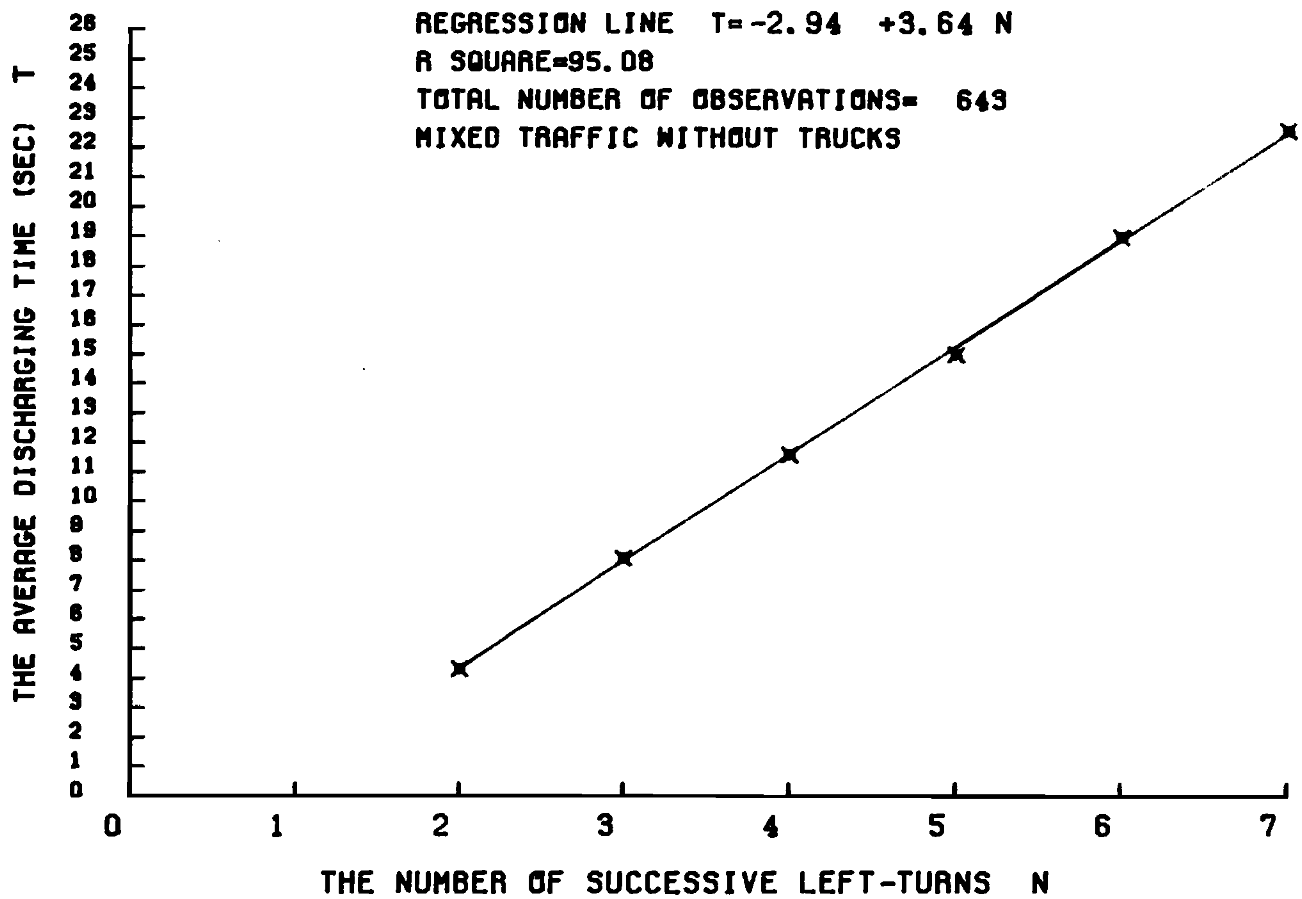


Fig 3-16. The average time required for completing N successive left turns.

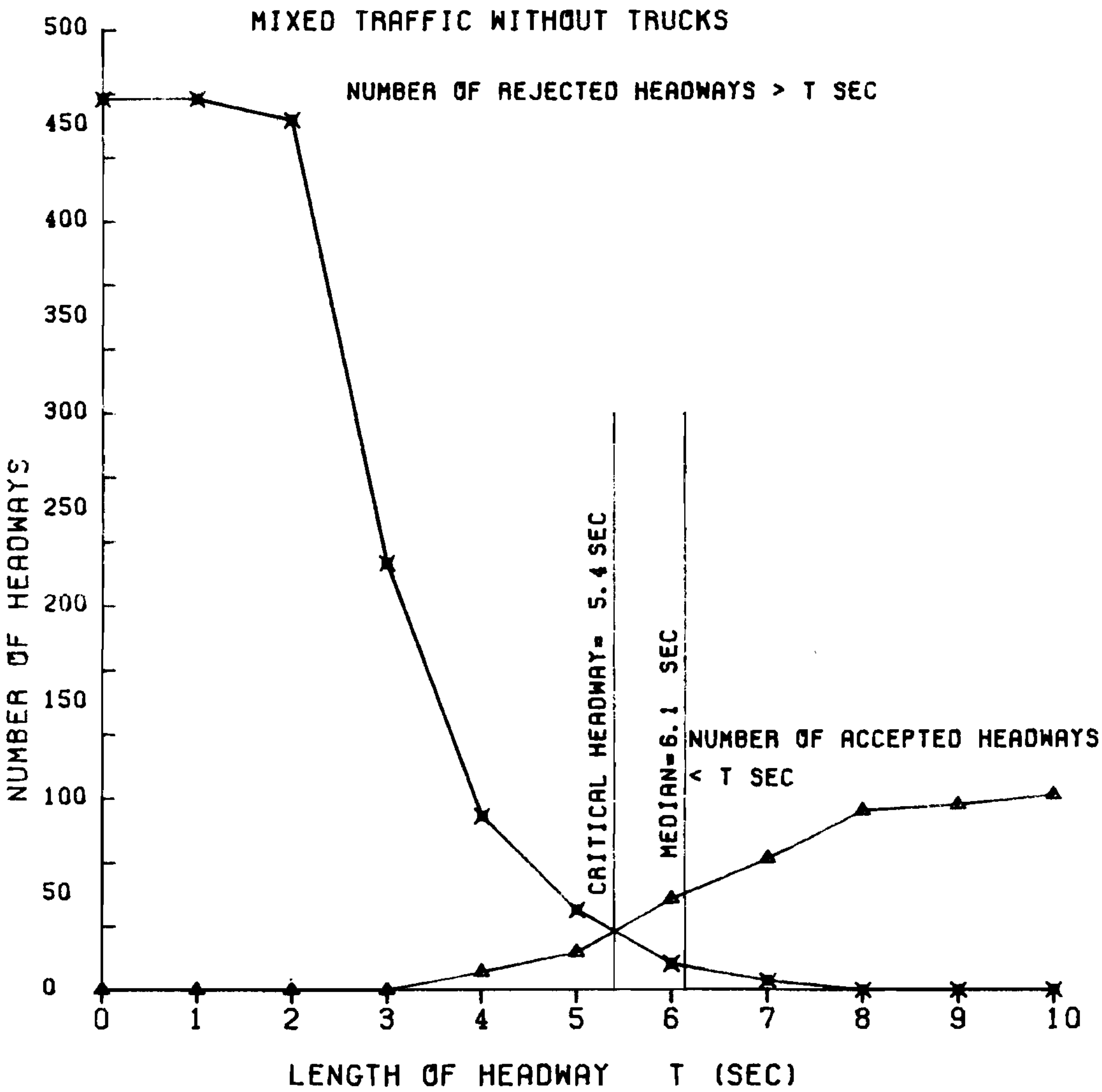
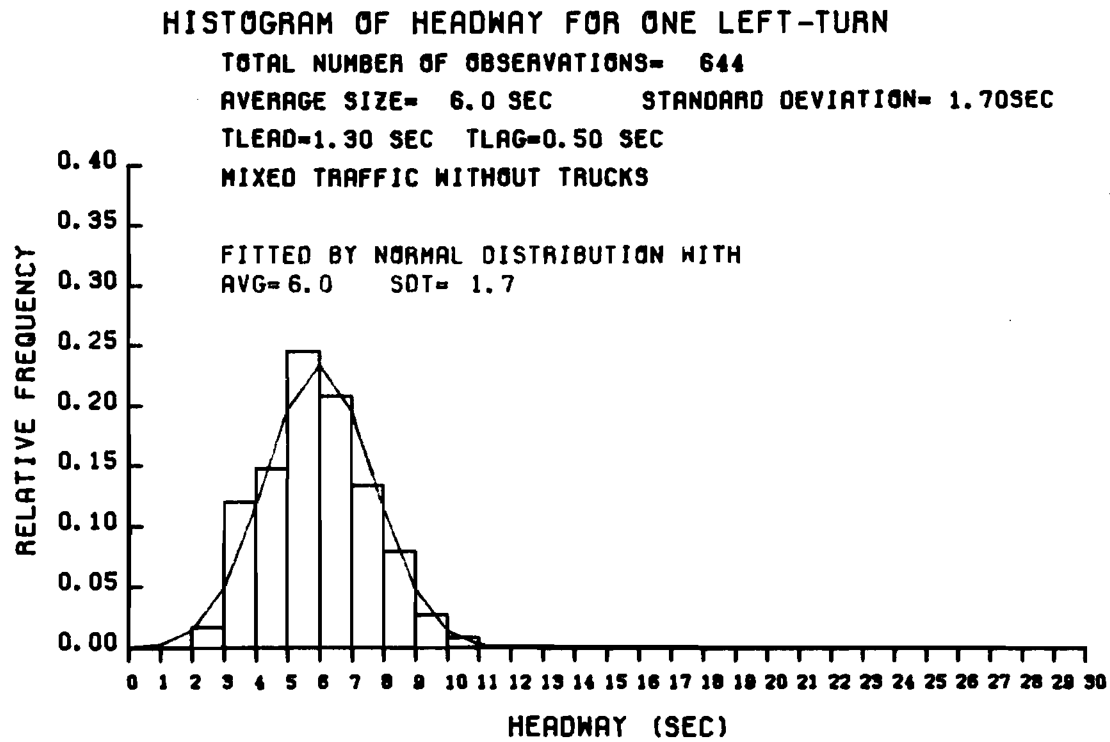


Fig 3-17. The critical headway and the median headway in opposing traffic obtained from the TEXAS Model.

seconds. Histograms of headways which allow no left turns, one, two, three, and four left turns in the TEXAS Model are shown in Figs 3-18 through 3-22.



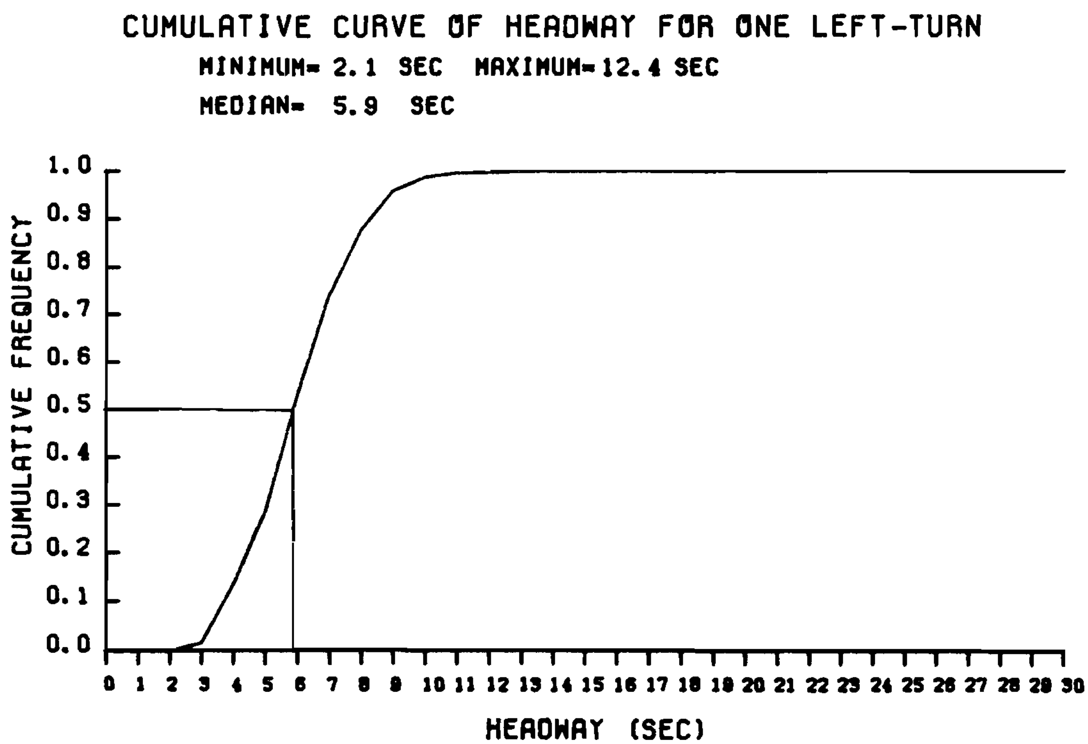
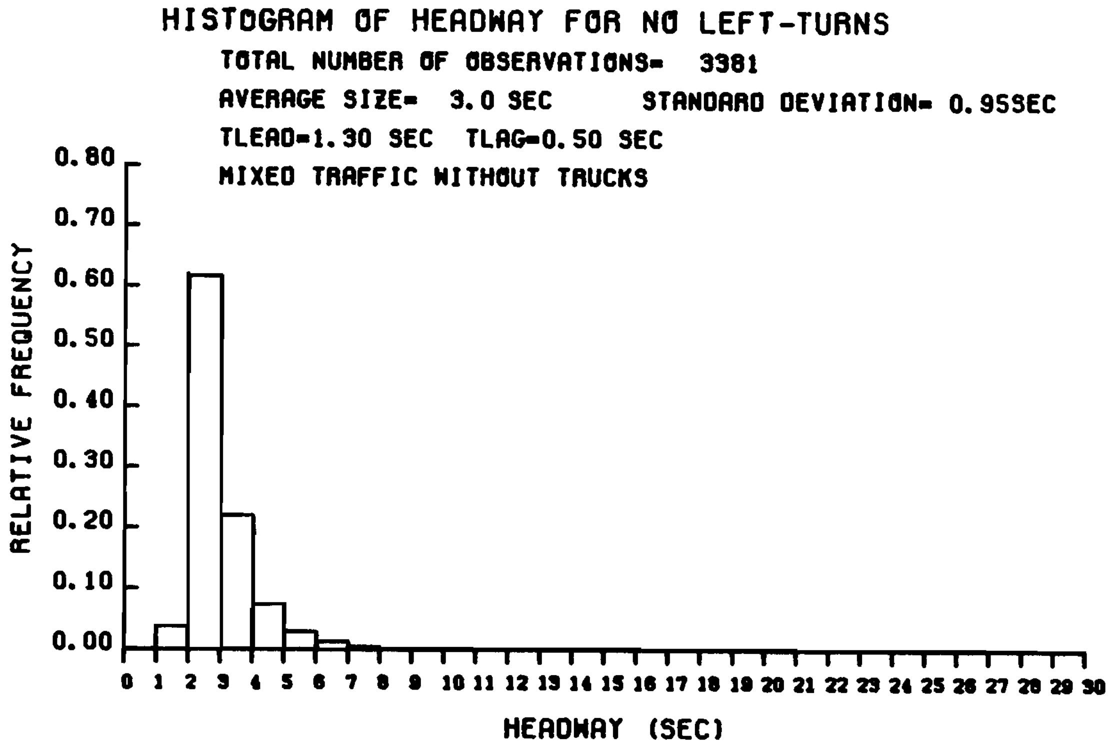


Fig 3-18. Histogram and cumulative frequency of headways which allowed no left turns.



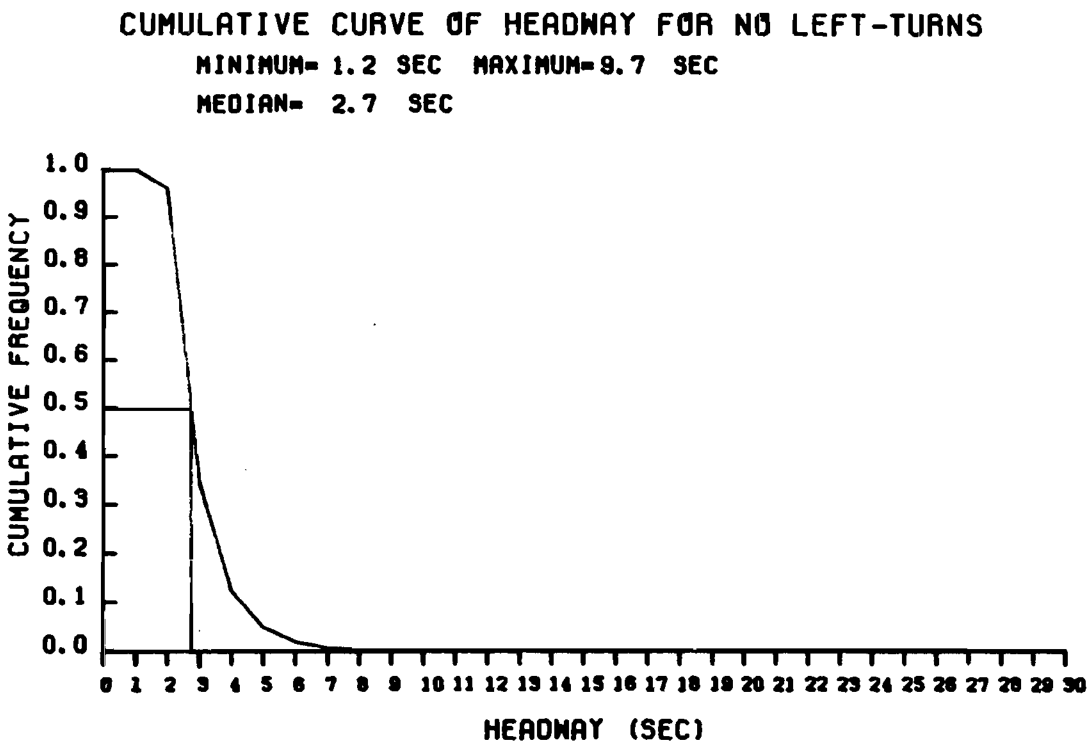
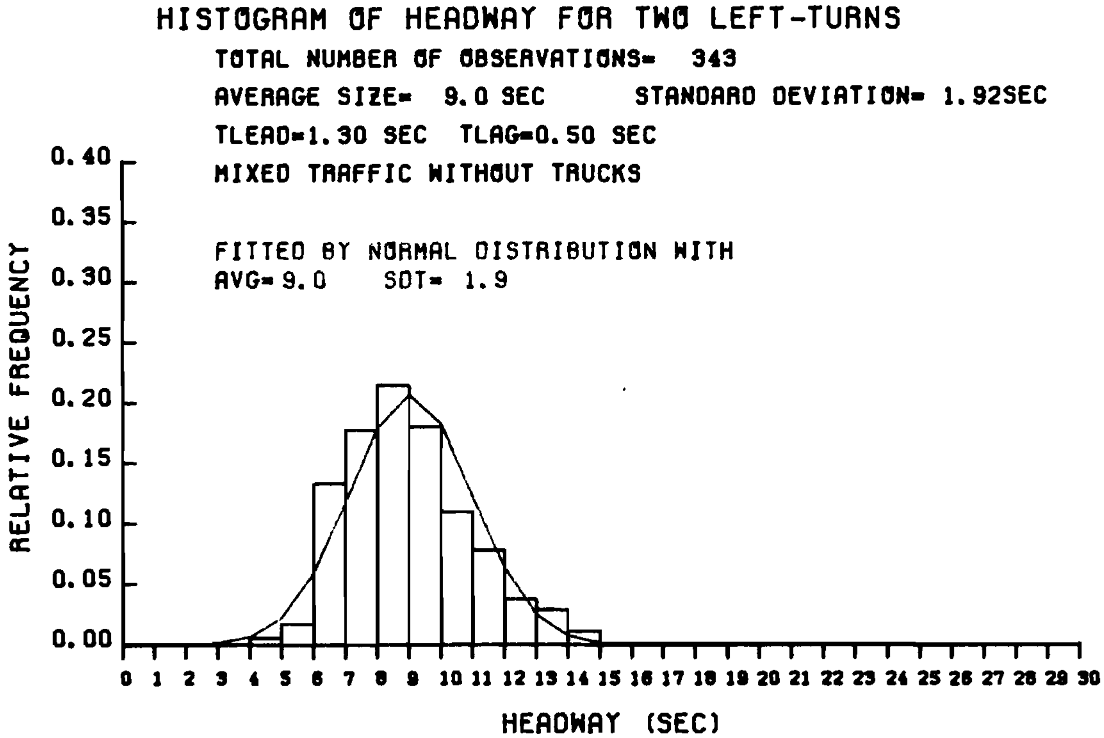


Fig 3-19. Histogram and cumulative frequency of headways in opposing traffic for accommodating one left turn.



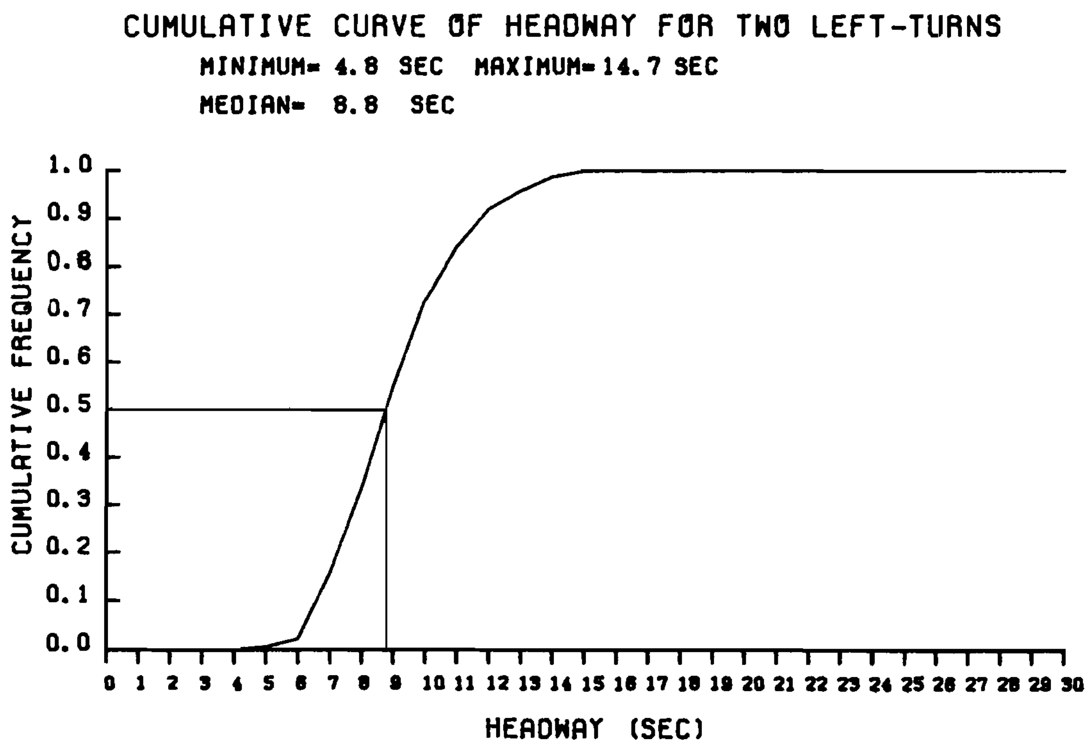
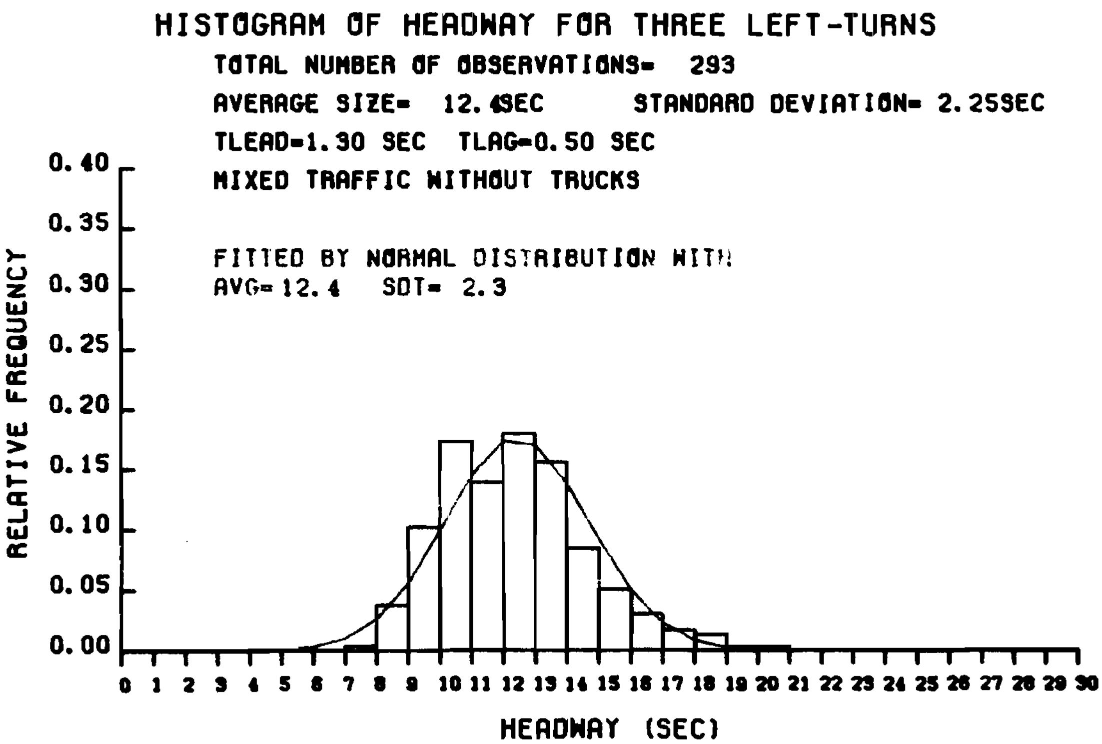


Fig 3-20. Histogram and cumulative frequency of headways in opposing traffic for accommodating two left-turns.



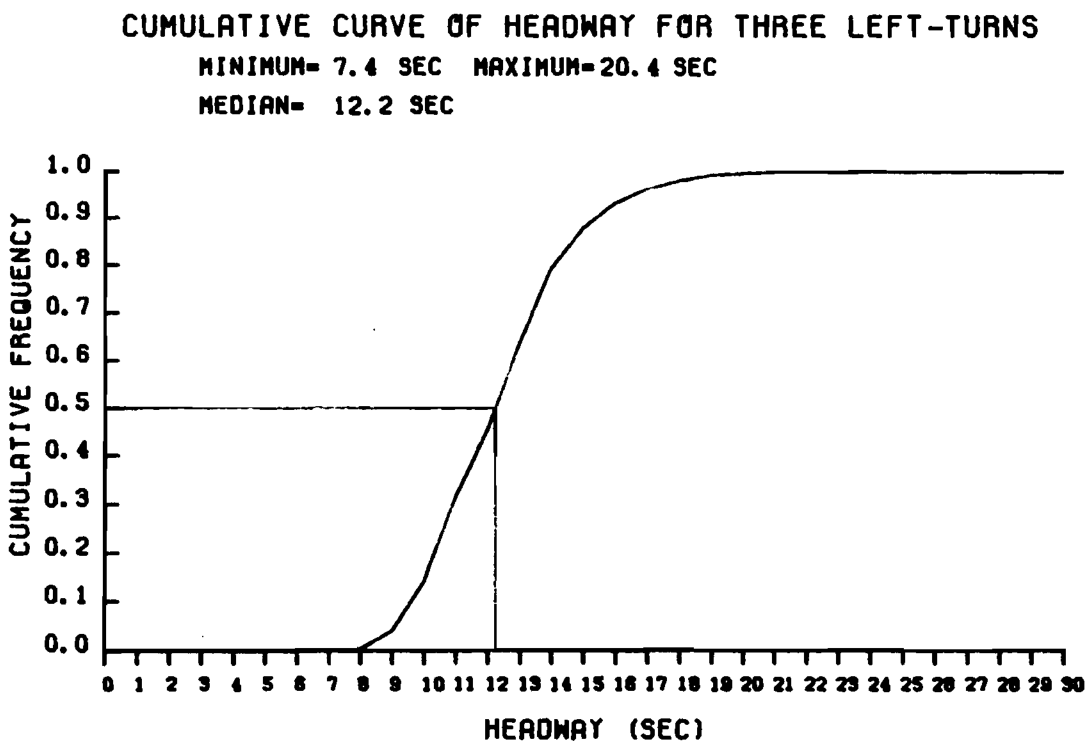
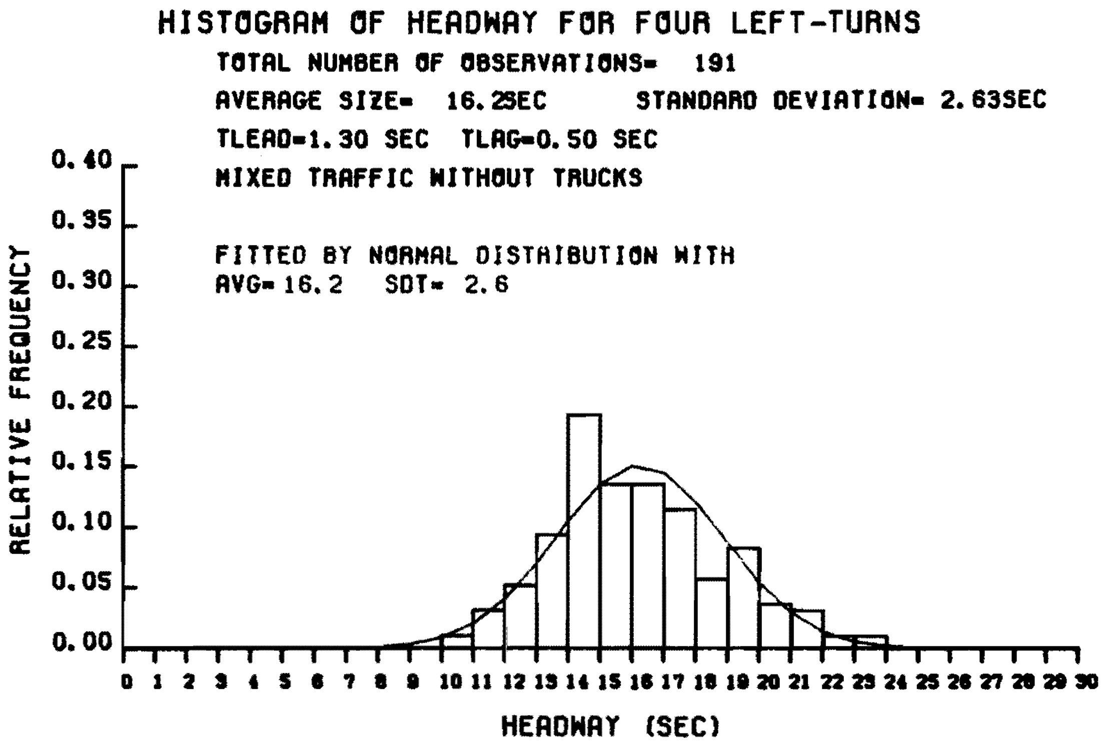


Fig 3-21. Histogram and cumulative frequency of headways in opposing traffic for accommodating three left turns.



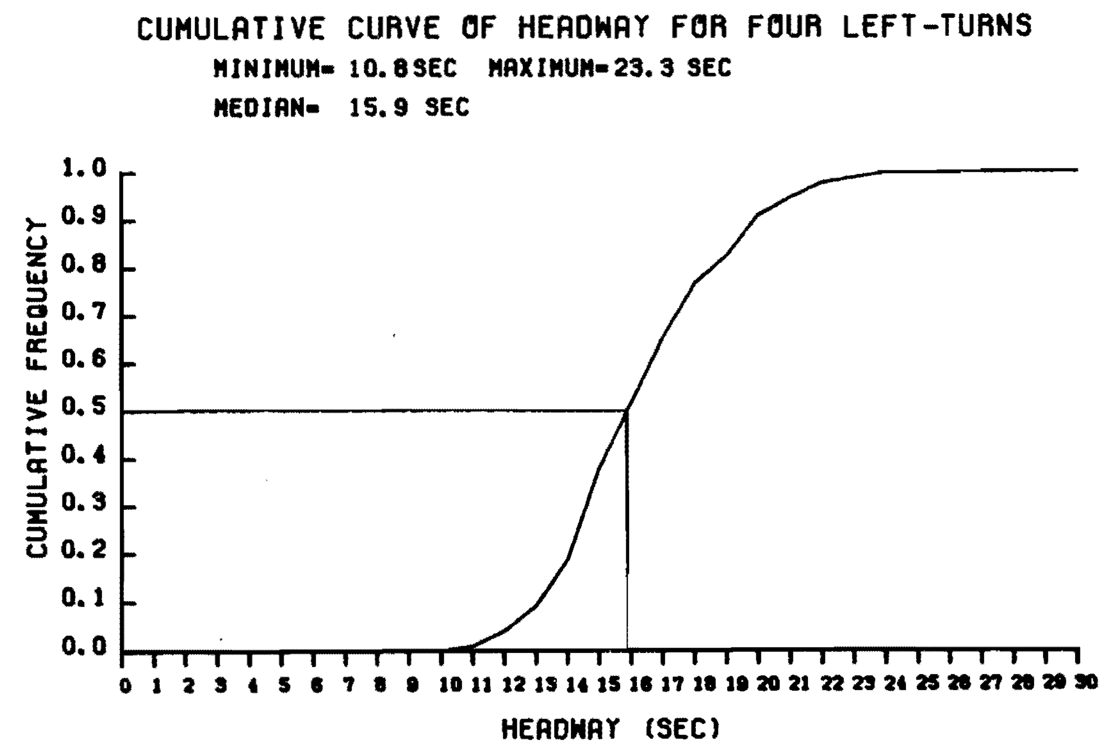


Fig 3-22. Histogram and cumulative frequency of headways in opposing traffic for accommodating four left-turns.

CHAPTER 4. LEFT-TURN CAPACITY STUDY

When left-turn volume approaches capacity, left turners may incur excessive delay, may block the through traffic if the left-turn bay length is not adequate, and may attempt to make hazardous maneuvers. Therefore, traffic control and geometric features for left turns must be designed to preclude near capacity operation during most if not all time periods. Should a high left-turn demand exist, some type of left-turn treatment must be implemented to assure adequate left-turn capacity. Knowledge of left-turn capacity is thus of extreme importance in determining left-turn strategies. Left-turn capacity at signalized intersections will first be defined for conditions where there is an adequate bay length and no separate left-turn phase. Within this discussion, unless otherwise specified, a bay of adequate length and no separate left-turn phase are always assumed. Moreover, the term, left-turn saturation flow, refers to the maximum number of left turns that can be made through the uninterrupted opposing traffic in one hour. The term, left-turn capacity, is reserved for use in defining the maximum number of unprotected left turns which can be accomplished at signalized intersections.

Different semi-empirical and theoretical equations have been proposed for estimating the left-turn saturation flow. These equations can be modified to determine left-turn capacity at signalized intersections. For example, the Highway Capacity Manual [Ref 13] and the Australian Road Capacity Guide [Ref 14] provide simple semi-empirical formulas for computing left-turn capacity. Tanner [Ref 15] used the technique of regeneration points to model the average delay to vehicles on the minor road at an intersection where traffic on the major road has absolute priority. He then obtained the greatest flow that can pass on the minor road by the limiting case of infinite delay. Webster and Cobbe [Ref 16] applied Tanner's model to left-turn operations and further combined it with the signal timing to get the left-turn capacity at signalized intersections. Drew [Ref 17] employed assumptions of the negative-exponential distribution for opposing headways and a step-function type of gap acceptance criteria to derive left-turn saturation flow. It will be shown later that Drew's result is a special case of Tanner's model. Fambro et al [Ref 11] incorporated signal effects in Drew's equation to predict left-turn capacity at signalized intersections. Michalopoulos et al [Ref 18] tested the ability of the above methods for estimating the unprotected left-turn capacity at both signalized and nonsignalized intersections in upstate New York and concluded that none of the methods was satisfactory in all cases. Consequently, he proposed a multiple regression model based on his observed data.

Generally speaking, unprotected left-turn capacity depends primarily on the gap-acceptance behavior of drivers, the signal timing, the gaps in opposing traffic presented at the stop line, and the intersection geometry. Any model that fails to take all these factors into account can not yield consistently reasonable estimates for left-turn capacity. On the other hand, it has also been recognized that mathematical models of left-turn capacity suffer from either implausibility or infeasibility. In view of this, a traffic simulation model, the TEXAS Model, is considered to be a suitable tool for studying left-turn capacity. In this chapter, the existing methods and the method employing the TEXAS Model for estimating left-turn capacity will be individually introduced and discussed.

HIGHWAY CAPACITY MANUAL METHOD (HCM)

The Highway Capacity Manual [Ref 13] states that the capacity of a left-turn lane is equal to "the difference between 1200 vehicles and the total opposing volume in terms of passenger car per hour of green, but not less than two vehicles per signal cycle." More specifically, the unprotected left-turn capacity at signalized intersections can be estimated from the following equation:

$$Q_L = 2100(G/C) - Q_o \tag{4-1}$$

if

$$Q_L > 7200/C$$

otherwise

$$= 7200/C \qquad \text{(i.e., minimum of 2 pcu/cycle)}$$

where

Q_L = unprotected left-turn capacity, pcu/hr;
Q_0 = opposing volume, pcu/hr;
G = green phase duration, sec; and
C = cycle length, sec.

It has been stated [Ref 19] that Eq 4-1 is adequate for small volume opposing flows, but it underestimates left-turn capacity for opposing flows over 600 vph if the opposing traffic is distributed across two or three lanes. This underestimation stems from the fact that for the same opposing volume, the average gap size for each lane will be larger if the volume is distributed on two or three lanes rather than on one lane. The HCM method fails to account for this effect. In addition, the HCM method claims that any change in the opposing volume will affect an equal change in the left-turn capacity. This is not quite true because the left-turn capacity is somewhat less sensitive to opposing vehicles than the one-to-one correspondence assumed by the HCM.

AUSTRALIAN ROAD CAPACITY GUIDE (ARCG)

The Australian Road Capacity Guide estimates the left-turn saturation flow by discounting 1200 vph with a factor depending on the opposing volume. Thus, left-turn capacity at signalized intersections is:

$$Q_L = 1200f(G/C) \qquad (4\text{-}2)$$

where

Q_L = left-turn capacity, veh/hr;
G = green phase duration, sec;
C = cycle length, sec; and
f = left-turn equivalency factor, depending on opposing volume Q_o.

Q_o (veh/hr)	0	200	400	600	800
f	1.0	0.81	0.65	0.54	0.45

This method, like the HCM method, does not take into account the number of opposing lanes. Moreover, as the opposing volume is increased from 0 to 800 veh/hr, the left-turn capacity is reduced by only 55 percent. This seems to overestimate the left-turn capacity for the case of a single opposing flow.

THE WEBSTER METHOD

Webster basically adopted Tanner's model. When applied to left-turn operations, Tanner's model predicts the left-turn saturation flow as follows:

For single opposing flow:

$$S_L = Q_o(1-q_o\tau)/\exp[q_o(t_c-\tau)]\{1-\exp[-q_o H]\} \qquad (4\text{-}3)$$

τ = 3 sec, t_c = 5 sec, and H = 2.5 sec

For two opposing flows:

$$S_L = 2Q_o(1-q_o\tau)/\exp[2q_o(t_c-0.5\tau)]\{1-\exp[-2q_o H]\} \tag{4-4}$$

$$\tau = 1 \text{ sec}, t_c = 6 \text{ sec, and } H = 2.5 \text{ sec}$$

where

S_L = left-turn saturation flow, veh/hr;

Q_o = opposing approach volume, veh/hr;

q_o = opposing flow rate, veh/sec/lane;

τ = minimum headway of opposing flow, sec;

H = average turning headway, sec; and

t_c = critical gap, sec.

If the constraint on the minimum opposing headway is removed, i.e., $\tau=0$, then Eq 4-3 becomes:

$$S_L = Q_o \exp[-q_o t_c]/\{1-\exp[-q_o H]\} \tag{4-5}$$

It will be seen later that Eq 4-5 is exactly Drew's equation derived through a different approach.

The basic notion of Webster method is that after the queue has been discharged, the opposing traffic will resume a free-flowing state which is exactly the same as the uninterrupted flow at nonsignalized intersections. In this sense, Webster regarded the left-turn capacity at signalized intersections as proportional to the left-turn saturation flow:

$$Q_L = (T_A/C)S_L \tag{4-6}$$

where

Q_L = left-turn capacity, vph;

T_A = remaining green time after queue dissipation, sec;

C = cycle length, sec; and
S_L = left-turn saturation flow, vph, as defined in Eq 4-3 or 4-4.

The remaining green time after queue dissipation, T_A, can be determined in the following way. Referring to Fig 4-1, let T_D, r, g, q_o, and s be defined as follows:

T_D = average time for discharging the opposing queue, sec;
r = effective red time, sec;
g = effective green time, sec;
q_o = opposing flow rate, veh/sec; and
s = saturated opposing flow rate, 1750 veh/3600 sec.

The total number of vehicles queued per cycle is $(T_D+r)q_o$. Since the average discharging headway is $1/s$, the average time for discharging the opposing queue is

$$T_D = (T_D+r)Q_o/s \tag{4-7}$$

After rearranging Eq 4-7, T_D will become

$$\begin{aligned} T_D &= rq_o/(s-q_o) \\ &= (C-g)q_o/(s-q_o) \end{aligned} \tag{4-8}$$

So,

$$\begin{aligned} T_A &= g - T_D \\ &= (gs-q_oC)/(s-q_o) \end{aligned} \tag{4-9}$$

Substituting Eq 4-9 into Eq 4-6, the left-turn capacity will be

$$Q_L = (gs-q_oC)S_L/[C(s-q_o)] \tag{4-10}$$

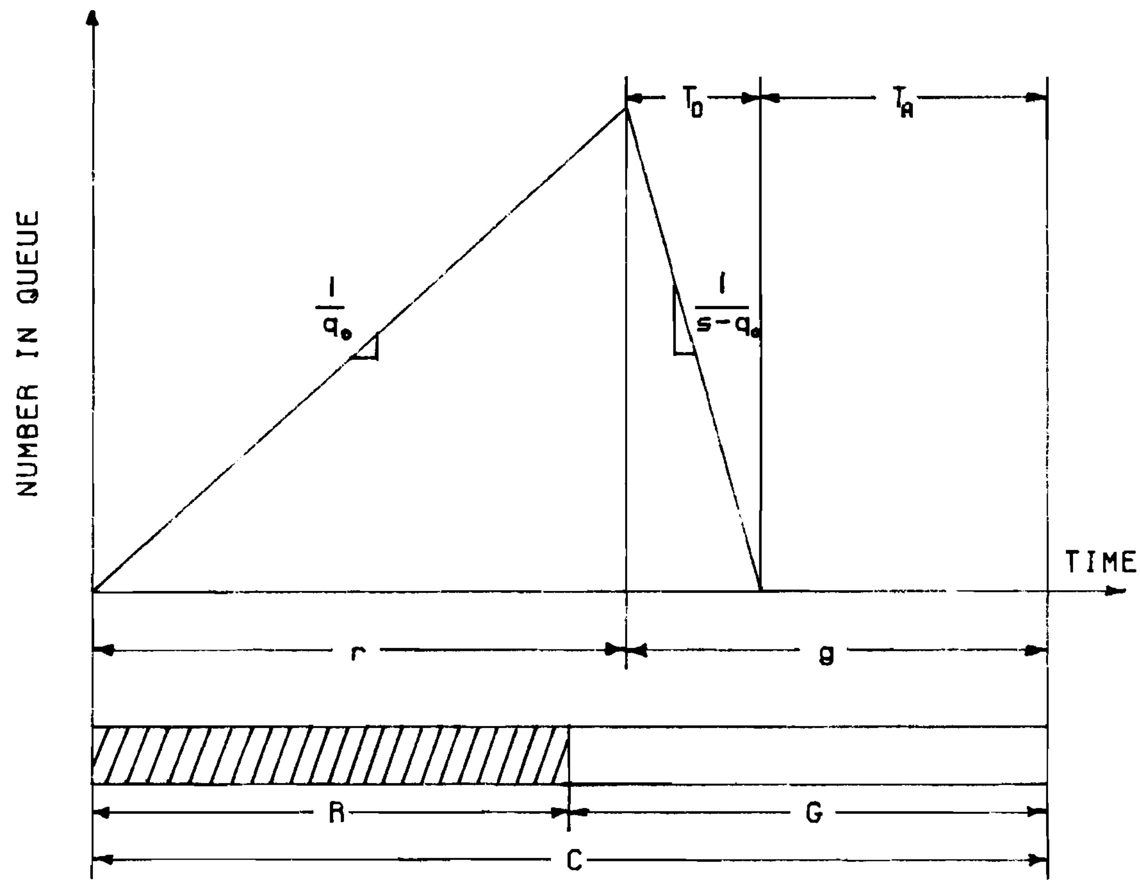


Fig 4-1. The free-flowing time after queue dissipation at signalized intersections.

Tanner assumed the negative-exponential distribution for opposing headways and a step-function of gap-acceptance criterion in deriving left-turn saturation flow. As a result, Webster's method sustains the weakness of these two assumptions. Moreover, Tanner regarded that two opposing flows each with flow rate q_o is equivalent to a single stream with flow rate $2q_o$. This tends to underestimate the left-turn saturation flow in the case of two opposing flows. More seriously, at signalized intersections the opposing flow after queue dissipation is not the same as an uninterrupted flow. This issue will be fully addressed in the discussion of Fambro's method.

FAMBRO'S METHOD

Fambro et al applied Drew's model of left-turn saturation flow to signalized intersections. For the purpose of this discussion, it is necessary to briefly introduce Drew's model. Drew used the following assumptions to model left-turn saturation flow:

(1) The opposing traffic is an uninterrupted flow with flow rate q_o veh/sec, and its headways are negative-exponentially distributed.

(2) There is a continuous left-turn queue.

(3) If a gap is less than t_c, then no left-turning vehicle will accept the gap. If a gap is larger than or equal to t_c but less than t_c+H, then one left turn can be made. If a gap is larger than or equal to t_c+H but less than t_c+2H, then two left-turning vehicles can go through, etc. In general, if there is a gap t, and $t_c + iH \le t < t_c + (i+1)H$, then (i+1) left-turning vehicles can go through the gap.

With these assumptions, Drew obtained the left-turn saturation flow as follows:

$$S_L = Q_o \sum_{i=o}^{\infty} (i+1)\Pr\{t_c + iH \leq t < t_c + (i+1)H\}$$

$$= Q_o \sum_{i=o}^{\infty} (i+1)\{\exp[-q_o(t_c+iH)]-\exp[-q_o(t_c+(i+1)H)]\}$$

$$= Q_o \exp[-q_o t_c] \sum_{i=o}^{\infty} \exp[-q_o(iH)]]$$

$$= Q_o \exp[-q_o t_c]/\{1-\exp[-q_o H]\} \qquad (4\text{-}11)$$

where

S_L = left-turn saturation flow, veh/hr;
Q_o = opposing traffic volume, veh/hr;
q_o = opposing flow rate, veh/sec;
t_c = critical gap, sec, use 4.5 sec; and
H = turning headway, sec (use 2.5 sec if there is a bay; otherwise, use 2.6 sec).

By comparing Eq 4-11 with Eq 4-5, it can be found that Drew's equation is a special case of Tanner's equation, and therefore, Drew's equation has the same weakness as Tanner's. Similar to the Webster method, Fambro et al compute left-turn capacity as follows:

$$Q_L = (T_A/C)S_L$$

$$= (T_A/C)Q_o\{\exp[-q_o t_c]/(1-\exp[-q_o H])\} \qquad (4\text{-}12)$$

where

$T_A = g + Y - L_1 - L_2 - T_D;$ (4-13)
$T_D = PQ_o(L_1+R+L_2)/(S_T-PQ_o)$ (4-14)
= time for the longest opposing queue to clear, sec;
L_1 = portion of yellow time not used by through traffic, sec;
R = length of red phase of cycle, sec;
L_2 = initial lost time at the beginning of green interval, sec;

S_T = saturation flow of opposing queue, 1750 veh/hr/lane;

P = percentage traffic in highest volume lane (in decimals)
one-lane approach: $P=1.00$
two-lane approach: $P=0.55+0.45\exp[-0.18m]$
three-lane approach: $P=0.40+0.60\exp[-0.13m]$;

m = average number of arrivals per cycle, $= Q_o C/3600$; and

$L_1 + L_2$ = 4 seconds recommended.

It can be seen that Webster's method and Fambro's method share the same notion. Both methods assume that at signalized intersections the opposing flow after queue dissipation is exactly the same as the uninterrupted flow at nonsignalized intersections. Consequently, in both equations the opposing traffic volumes are given in terms of vehicles per hour instead of vehicles per "hour of green." This is very deceptive at first glance. It will be shown here that due to the effects of signalization, the opposing traffic after queue dissipation is not the same as the uninterrupted flow at nonsignalized intersections. First, the maximum opposing gap at signalized intersections is constrained by the duration of the green phase. The end of each green phase is the absolute end of any opposing gap. Second, left turners face opposing traffic which flows only during green time. For example, at signalized intersections with cycle split 0.5, if the opposing volume is 400 vph, then left turners will, in fact, observe opposing traffic of 400 vehicles in half an hour instead of 400 vehicles spread over one hour. Therefore, the average opposing headway is 4.5 seconds instead of 9 seconds as it would be for an uninterrupted flow. In general, for an opposing volume Q_o vph at a signalized intersection, the average opposing headway $\overline{h}'$ is:

$$\overline{h}' = \frac{3600(G/C)}{Q_o} = \frac{3600}{Q_o/(G/C)}$$

$$= 3600/Q'_o \qquad (4\text{-}15)$$

In contrast, the average headway of an uninterrupted opposing flow is $\overline{h}$:

$$\overline{h} = 3600/Q_o \qquad (4\text{-}16)$$

Conceptual illustrations of headways at nonsignalized and signalized intersections with a flow of 400 vph are shown in Fig 4-2a and 4-2b. Generally speaking, the average opposing headway after queue dissipation is larger than the average discharging headway. However, for small opposing volumes there are few vehicles in the queue, while for large opposing volumes the difference between the average headway after queue dissipation and the discharging headway is small. Hence, the average opposing headway after queue dissipation in most cases is very close to the average headway of opposing traffic in green time (see Fig 4-3). Therefore, when applying Drew's equation to signalized intersections, it seems more appropriate to have the opposing volume and the opposing flow rate in terms of vehicles per hour of green and vehicles per second of green, respectively. This statement can also be supported by examining the physical meaning of Drew's equation.

Theoretically speaking, given a headway distribution function F(t) and a gap-acceptance function a(t), the integral of a(t) over dF(t) is the average number of left turns per gap. That means, in general:

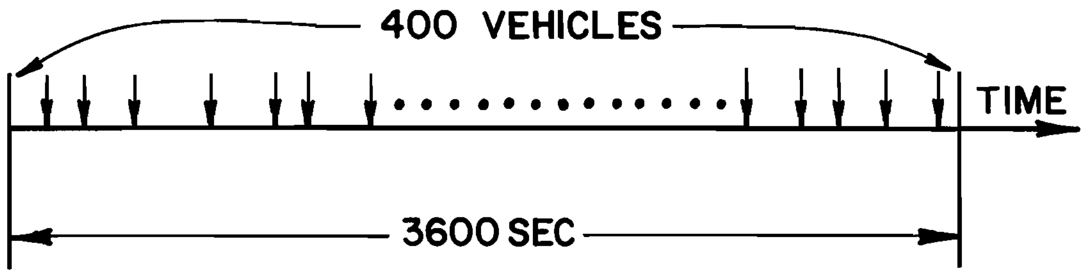


Fig 4-2a. The uninterrupted flow.

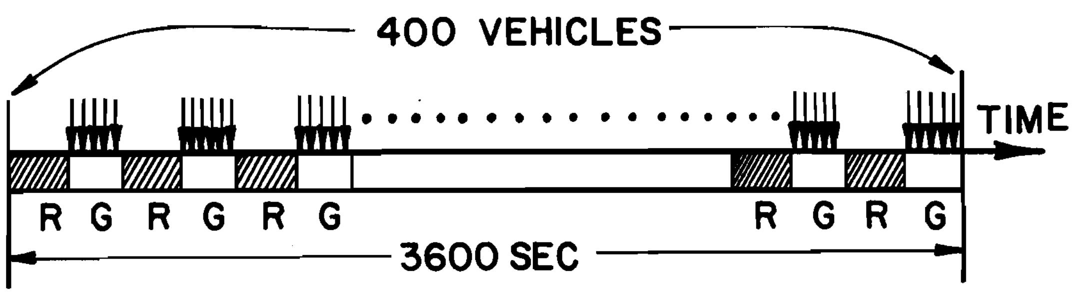


Fig 4-2b. The interrupted flow.

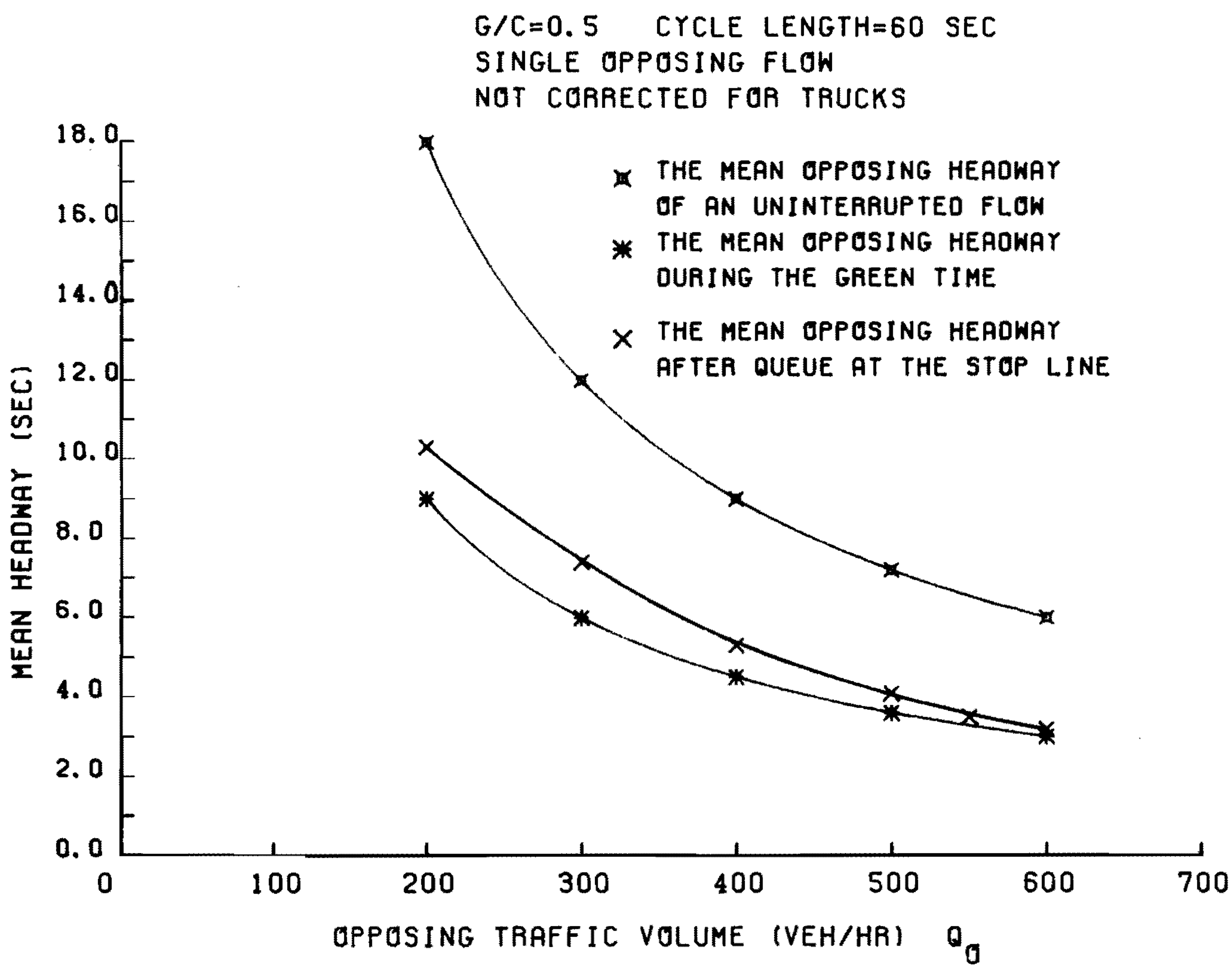


Fig 4-3. The comparison between the average headway of an uninterrupted flow and that of an interrupted flow at signalized intersections with G/C = 0.5, and C = 60 sec.

$$m = \text{average number of left-turns per gap}$$
$$= \int a(t)dF(t) \qquad (4\text{-}17)$$

According to Drew's model

$$a(t) = 0 \qquad \text{if} \quad 0 \leq t < t_c$$
$$= 1 \qquad \text{if} \quad t_c \leq t < t_c + H$$
$$= 2 \qquad \text{if} \quad t_c + H \leq t < t_c + 2H$$
$$\vdots$$
$$= (i+1) \qquad \text{if} \quad t_c + iH \leq t < t_c + (i+1)H \qquad (4\text{-}18)$$

and

$$F(t) = 1 - \exp[-q_o t] \qquad \text{if } t \geq 0 \qquad (4\text{-}19)$$

It follows that

$$m = \sum_{i=0}^{\infty} (i+1)\Pr\{t_c + iH \leq t < t_c + (i+1)H\}$$
$$= \sum_{i=0}^{\infty} (i+1)\{\exp[-q_o(t_c + iH)] - \exp[-q_o(t_c + (i+1)H]\}$$
$$= \exp[-q_o t_c]/\{1-\exp[-q_o H]\} \qquad (4\text{-}20)$$

Let

$$N_A = \text{total number of gaps in one hour}$$
$$= 3600/\overline{h} = Q_o \qquad (4\text{-}21)$$

Then

$$S_L = mN_A = mQ_o \qquad (4\text{-}22)$$
$$= Q_o\{\exp[-q_o t_c]/(1-\exp[-q_o H])\} \qquad (4\text{-}23)$$

Drew's equation indicates that the left-turn saturation flow is the average number of left turns per gap multiplied by the total number of opposing gaps in one hour. Since the average opposing headway at signalized intersections is $\overline{h}'$ instead of $\overline{h}$, when applying Drew's model to signalized intersections the average number of left-turns per gap should be:

$$m' = \exp[-q'_o t_c]/(1-\exp[-q'_o H]) \tag{4-24}$$

where

$$q'_o = Q_o/[3600(G/C)] = q_o/(G/C)$$

The total opposing gaps after queue dissipation will be

$$\begin{aligned} N'_A &= (T_A/C)(3600/h') \\ &= (T_A/C)Q'_o \end{aligned} \tag{4-25}$$

Consequently, the left-turn capacity at signalized intersections can be computed as follows:

Modified Method I:

$$\begin{aligned} Q_L &= m'N'_A \\ &= (T_A/C)Q'_o\{\exp[-q'_o t_c](1-\exp[-q'_o H])\} \end{aligned} \tag{4-26}$$

Equation 4-26 shows Fambro's equation after necessary corrections for the physical meaning of Drew's equation. However, after more careful thought concerning the derivation of Drew's equation, it appears unreasonable to allow an infinite number of left turns through a gap at a signalized intersection. At signalized intersections, the maximum gap size is constrained by the green duration, so the maximum number of left turns through any gap will be finite. If the condition of a finite number of left turns is imposed in Drew's model (i.e., the infinite series in the model is replaced by a finite series), then the average number of left turns per gap becomes:

$$m'' = \frac{\exp[-q'_o t_c]\{1-\exp[-q'_o(n-1)H]\}}{1 - \exp[-q'_o H]} - (n)\exp[-q'_o(t_c + nH)] \quad (4\text{-}27)$$

where

n = maximum number of left-turns that can go through an opposing gap under a given level of opposing volume Q_o

Q_o (vph)	200	300	400	500	600
n	8	7	6	5	4

So, the left-turn capacity can be further modified as:

Modified Method II:

$$Q_L = (T_A/C)Q'_o m'' \quad (4\text{-}28)$$

A comparison of Fambro's method, Modified Method I, and Modified Method II is presented in Table 4-1. On the average, it is found that Fambro's method underestimates the number of gaps after the queue by 50 percent while overestimating the average number of left turns per gap by 360 percent. As a whole, Fambro's method overestimates the left-turn capacity by about 180 percent due to ignoring the effects of signalization on the gap size after queue dissipation and ignoring the maximum number of left turns through a gap (Table 4-2). After corrections have been made on the opposing volume and the opposing flow rate, Fambro's method overestimates by 26 percent. If further corrections are made on the maximum number of left turns through a gap, then the results differ only by about 10 percent from that of the TEXAS simulation model. It is remarkable that Drew's model with several simplifying assumptions can predict the left-turn capacity across a single opposing flow as accurately as a sophisticated simulation model.

TABLE 4-1. COMPARISONS BETWEEN LEFT-TURN CAPACITY COMPUTED FROM FAMBRO'S METHOD AND MODIFIED METHODS

Opposing Volume (vph)	T_A/C	Number of Gaps After Opposing Volume			Average Number of Left Turns Per Gap			Left-Turn Capacity (vph)		
		Fambro's Method	Modified Method I	Modified Method II	Fambro's Method	Modified Method I	Modified Method II	Fambro's Method	Modified Method I	Modified Method II
200	0.4188	84	168	168	6.000	2.500	1.636	503	419	274
300	0.3786	114	228	228	3.654	1.386	1.103	415	315	250
400	0.3330	133	266	266	2.500	0.863	0.736	333	230	196
500	0.285	142	284	284	1.825	0.572	0.497	260	163	142
600	0.218	131	262	262	1.386	0.395	0.335	181	103	88

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks

TABLE 4-2. THE EFFECTS OF CORRECTIONS ON FAMBRO'S METHOD

Opposing Volume (vph)	Number of Gaps After Opposing Queue		Average Number of Left Turns Per Gap		Left-Turn Capacity (vph)		
	Fambro's / Modified II	Modified I / Modified II	Fambro's / Modified II	Modified I / Modified II	Fambro's / Modified II	Modified I / Modified II	Modified II / TEXAS Model
200	0.5	1.0	3.66	1.52	1.83	1.53	0.86
300	0.5	1.0	3.31	1.26	1.65	1.25	1.00
400	0.5	1.0	3.39	1.17	1.70	1.17	1.07
500	0.5	1.0	3.67	1.15	1.83	1.15	1.18
600	0.5	1.0	4.13	1.18	2.06	1.17	1.10
Average	0.5	1.0	3.63	1.26	1.81	1.26	1.04

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks

With respect to the left-turn capacity for multiple opposing flows, Drew concentrates opposing flows into a single stream. This treatment disregards the stagger of gaps if that flow occurs on several lanes. A more reasonable way to tackle the case of multiple opposing flows will be discussed later.

MICHALOPOULOS'S METHOD

Using field data, Michalopoulos et al constructed a multiple regression model for predicting left-turn saturation flow as follows:

$$S_L = -0.233 Q_o t_c + 0.000015 Q_o^2 t_c^2 + 126X + 103Y + 995 \qquad (4\text{-}29)$$

where

S_L = left-turn saturation flow: vehicles per hour for nonsignalized intersections, and vehicles per green hour for signalized intersections;

Q_o = opposing approach volume, veh/hr;

t_c = critical gap, sec;

X = 0 if there is one opposing lane;
X = 1 if there are two opposing lanes;
Y = 0 if the intersection is unsignalized; and
Y = 1 if the intersection is signalized.

The left-turn capacity at signalized intersections will be:

$$Q_L = [(gs - q_o C) S_L / C(s - q_o)] + 3600(K/C) \qquad (4\text{-}30)$$

where

S_L = left-turn saturation flow as defined in Eq 4-29; and

K = number of left turns during amber and red periods.

The first term in Eq 4-30 is similar to that of Webster's equation and Drew's equation. The second term is a correction factor for left turns made prior to the green and during amber period. Michalopoulos's method would be useful if the regression model were validated over many conditions.

A METHOD EMPLOYING THE TEXAS MODEL

Mathematical models can be used as tools for studying left-turn operations. However, to make the mathematics tractable for such a complex system many simplifying assumptions must often be made which may result in unrealistic answers. For this reason, simulation models are frequently utilized. The TEXAS Model, a microscopic traffic simulation package, was chosen for studying left-turn operations. In order to explain left-turn capacity conceptually, it is instructive to introduce the concepts of "transparency" and "average left-turn processing time". The effects of cycle length, cycle split, multiple opposing lanes, left-turn bays, headway distributions, and trucks on left-turn capacity will be addressed.

Transparency. Transparency, a term first adopted by Herman and Weiss [Ref 20] in studying the highway crossing problem, is defined as the ratio of the total unblocked time gap to the total time gap. A gap is unblocked if it can be used by drivers; otherwise, it is blocked. At signalized intersections, unprotected left turns are blocked by the red phase, opposing queues, or unacceptable gaps among opposing vehicles after queues. In this sense, transparency can be expressed as the ratio of the total acceptable gap time to the total observed time. Roughly speaking, transparency characterizes the over all impedance of the opposing traffic and the signalization to left turns.

In order to study the transparency or the left-turn capacity, the left-turn traffic must be over-saturated so that a continuous left-turn queue exists. Thus, whenever there is an acceptable gap, left turners are ready to utilize it. Table 4-3 is an illustration of results from the TEXAS Model for study of transparency for a simple case of single opposing flow, cycle length of 60 seconds, and 50 percent cycle split. Figure 4-4 depicts for different

TABLE 4-3. TRANSPARENCY STUDY FOR CYCLE SPLIT 0.5 AND CYCLE LENGTH 60 SECONDS

Statistics	Opposing Traffic Volume, vph							
	0	100	200	300	400	500	550	600
Total observed time, sec	2700	2700	2700	2700	2700	2700	2700	2700
Total time after the opposing queue is cleared, sec	1350	1294	1200	1065	966	841	735	663
Total acceptable gap time, sec	1350	1220	1035	828	597	393	281	213
Percent of time after the opposing queue is cleared, percent	50.0	47.9	44.4	39.4	35.8	31.1	27.2	24.5
Transparency, percent	50.0	45.2	38.3	30.7	22.1	15.2	10.4	7.0
Total number of left-turns through gaps during simulation time	319	271	222	172	117	67	41	26
Total number of left-turns made in amber periods during simulation	10	11	16	17	20	25	30	34
The left-turn capacity, vph	437	376	317	252	183	121	95	80
The average left-turn processing time, sec	4.12	4.33	4.38	4.38	4.36	4.33	3.97	3.58

G/C = 0.5 C = 60 sec

Single opposing flow

Not corrected for trucks

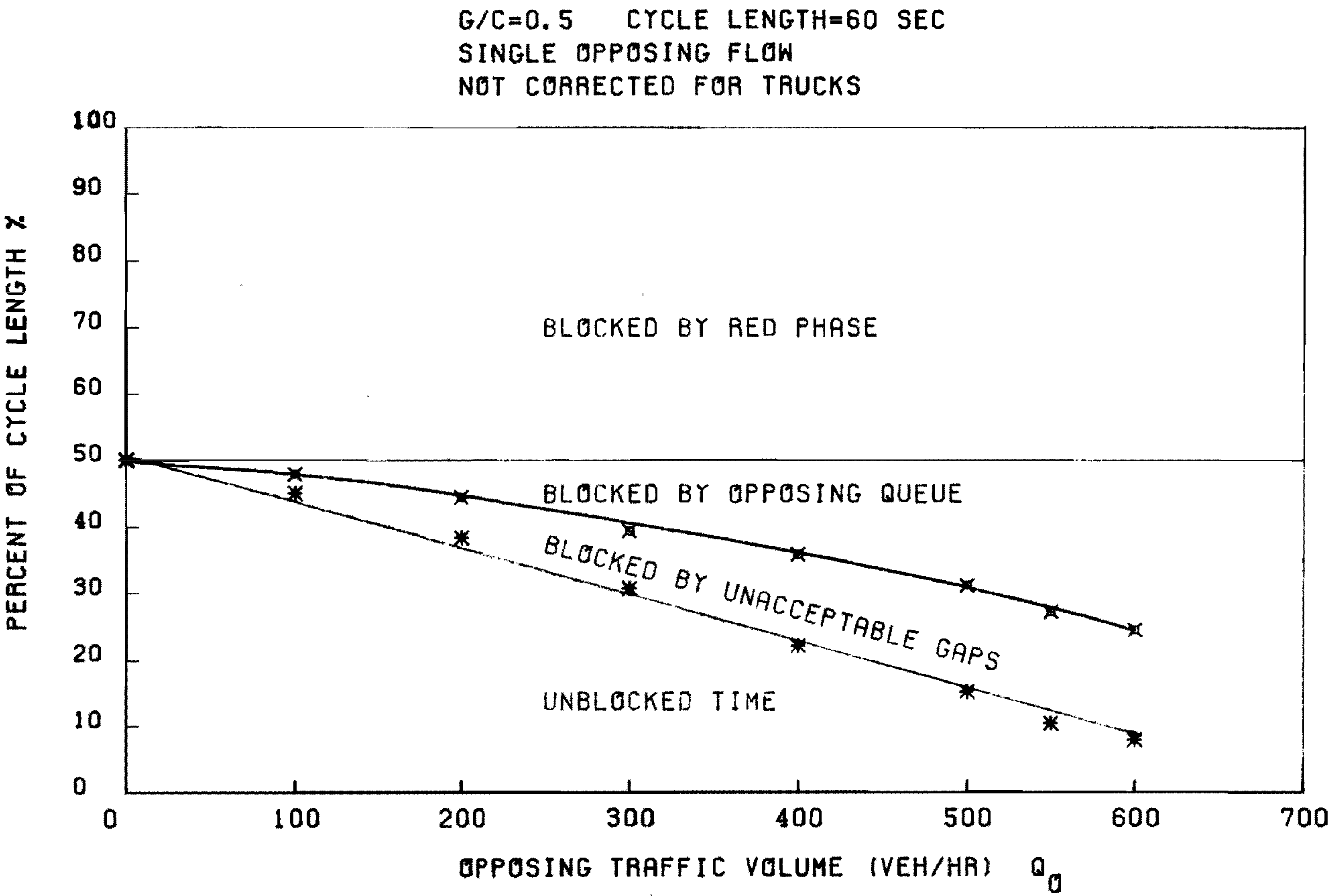


Fig 4-4. The blocked and unblocked durations for left-turning movements under different levels of opposing volume.

levels of opposing volume the average percentage of cycle time that left turns are blocked by the red phase, the opposing queue, and unacceptable opposing gaps after queue dissipation. From this study, the following conclusions can be drawn:

(1) Cycle split is the major factor which influences the transparency. For example, if cycle split is changed from 0.5 to 0.7, then the percentage of cycle time that left turns are blocked by the red phase will be reduced from 50 percent to 30 percent.

(2) The percentage of cycle time that left turns are blocked by the opposing queue increases nonlinearly from 2.1 percent to 26.1 percent as the opposing volume increases from 100 vph to 600 vph. This percentage will decrease as cycle split increases.

(3) The percentage of cycle time that left turns are blocked by unacceptable gaps after queue dissipation increases nonlinearly from 2.7 percent to 17.5 percent as the opposing volume increases from 100 vph to 600 vph. This portion is also affected by cycle split.

(4) From Fig 4-5 it can be seen that the transparency is not changed drastically until the average opposing headway after queue dissipation is less than seven seconds.

(5) The transparency changes linearly for opposing volume from 100 to 600 vph (see Fig 4-6). The linear relation between transparency, T, and opposing volume Q_o can be expressed as follows:

$$T = 0.5322 - 0.0007675Q_o \tag{4-31}$$

(6) The total seconds available for left-turns in one hour, , under different levels of opposing volumes is:

$$\begin{aligned} \Gamma &= 3600T \\ &= 1916 - 2.763Q_o \end{aligned} \tag{4-32}$$

<u>Average Left-turn Processing Time</u>. Left turns at signalized intersections usually can be divided into two types: those through opposing gaps and those completed during yellow and red times. Figure 4-7 shows the maximum number of these two types of left turns that can be made in one hour under different levels of opposing volumes. As the opposing traffic flow

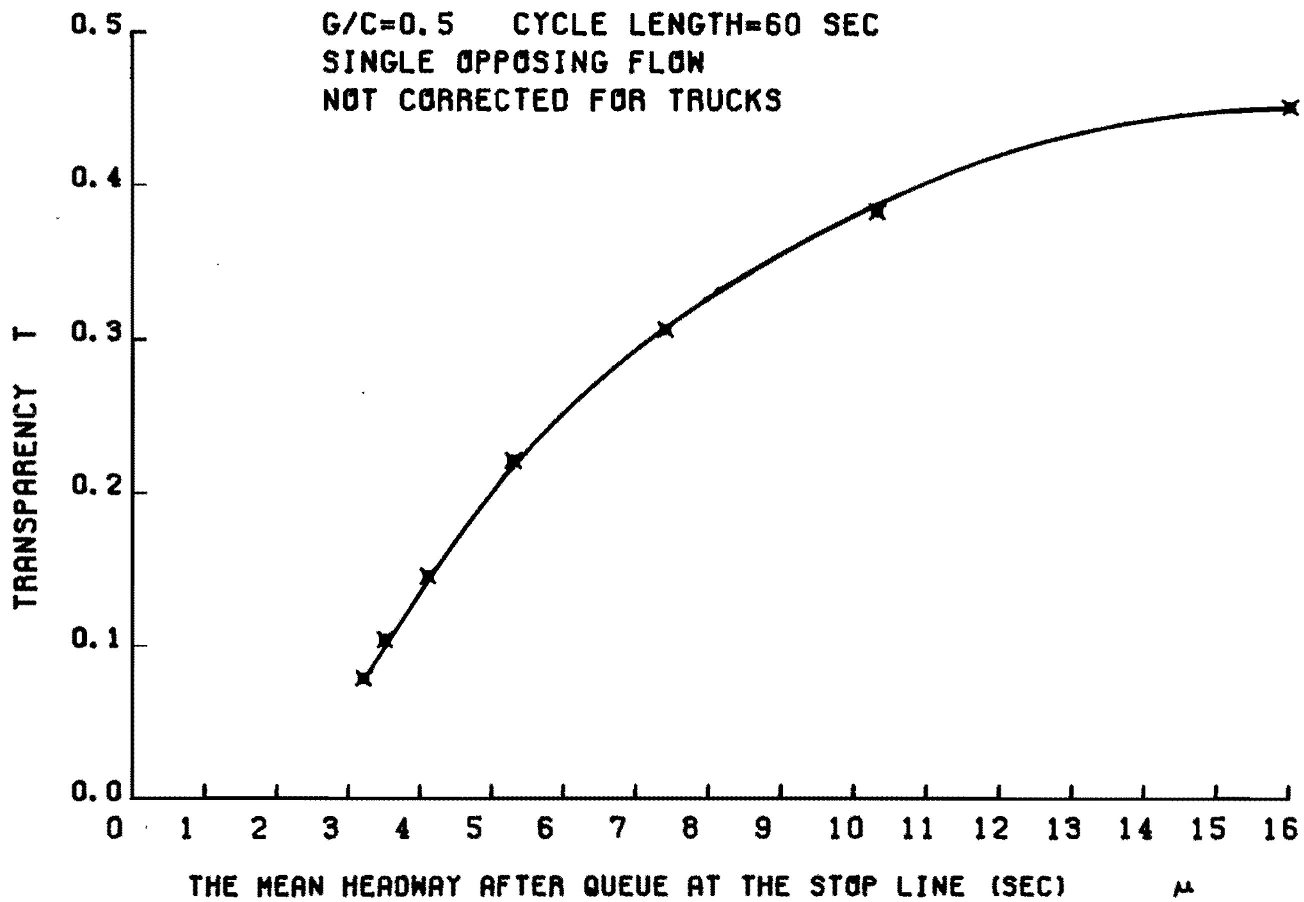


Fig 4-5. The relation between the transparency and the average opposing headway at the stop line.

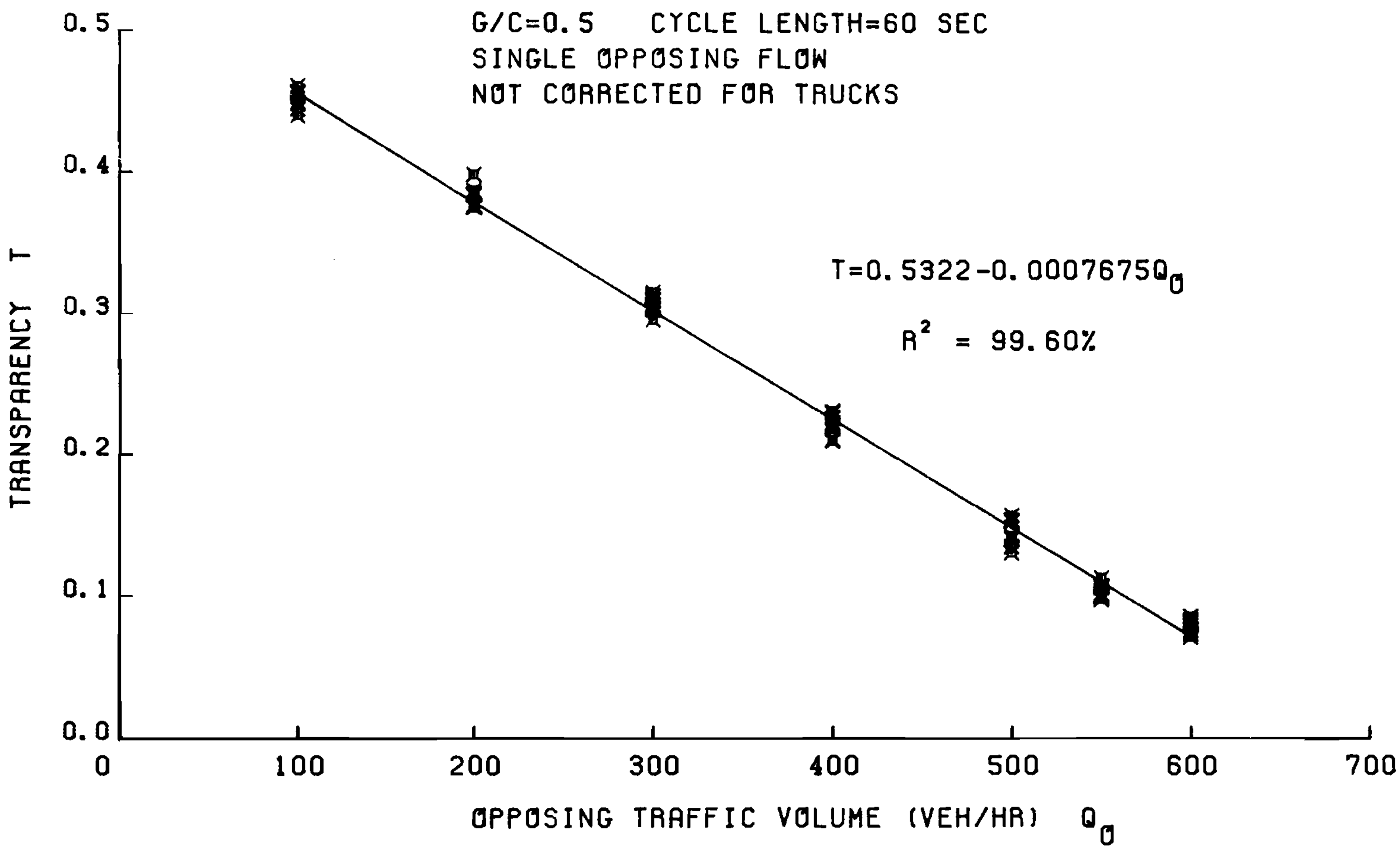


Fig 4-6. The relation between the transparency and opposing volume.

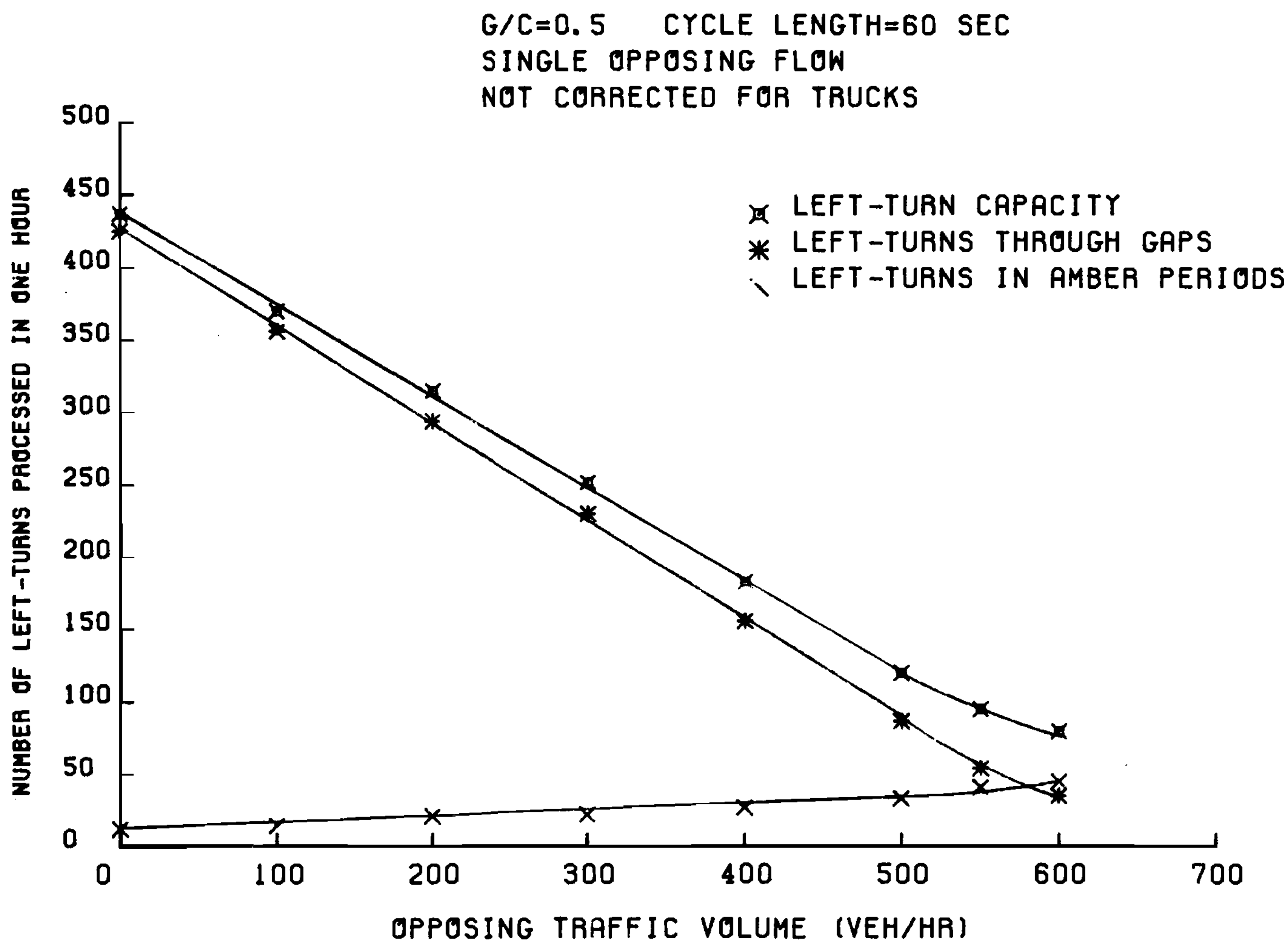


Fig 4-7. The maximum number of left turns through opposing gaps and yellow intervals, respectively, during one hour of observation.

approaches saturation, the number of left turns made through opposing gaps is reduced to almost zero, while those made through yellow intervals is increased up to one left turn per cycle. Figure 4-8 shows the average time required for these two types of left-turn movements. The average time for left turns through opposing gaps varies from 4.14 to 4.75 seconds. This left-turn time is larger than the turning headway of 3.6 seconds noted in Chapter 3 due to the discontinuity of left-turn flow. If these two types of left turns are taken together, the average left-turn processing time, $\bar{t}$, is the total time available for left turns in one hour including yellow intervals divided by the left-turn capacity:

$$\bar{t} = (3600T)/Q_L \tag{4-33}$$

The average left-turn processing time was found to be approximately constant at 4.36 seconds for opposing volumes from 100 to 500 vph (see Fig 4-8). However, as the opposing traffic approaches saturation, the average left-turn processing time converges to 3.0 seconds which is the yellow interval. Since the average left-turn processing time, $\bar{t}$, is approximately constant, the left-turn capacity can be determined by the following equation once the transparency, $\bar{t}$, is known.

$$Q_L = 36000T/\bar{t} \tag{4-34}$$

If the opposing volume is between 100 vph and 500 vph, then the left-turn capacity can be approximated by:

$$Q_L = 3600T/4.36 = 825T \tag{4-35}$$

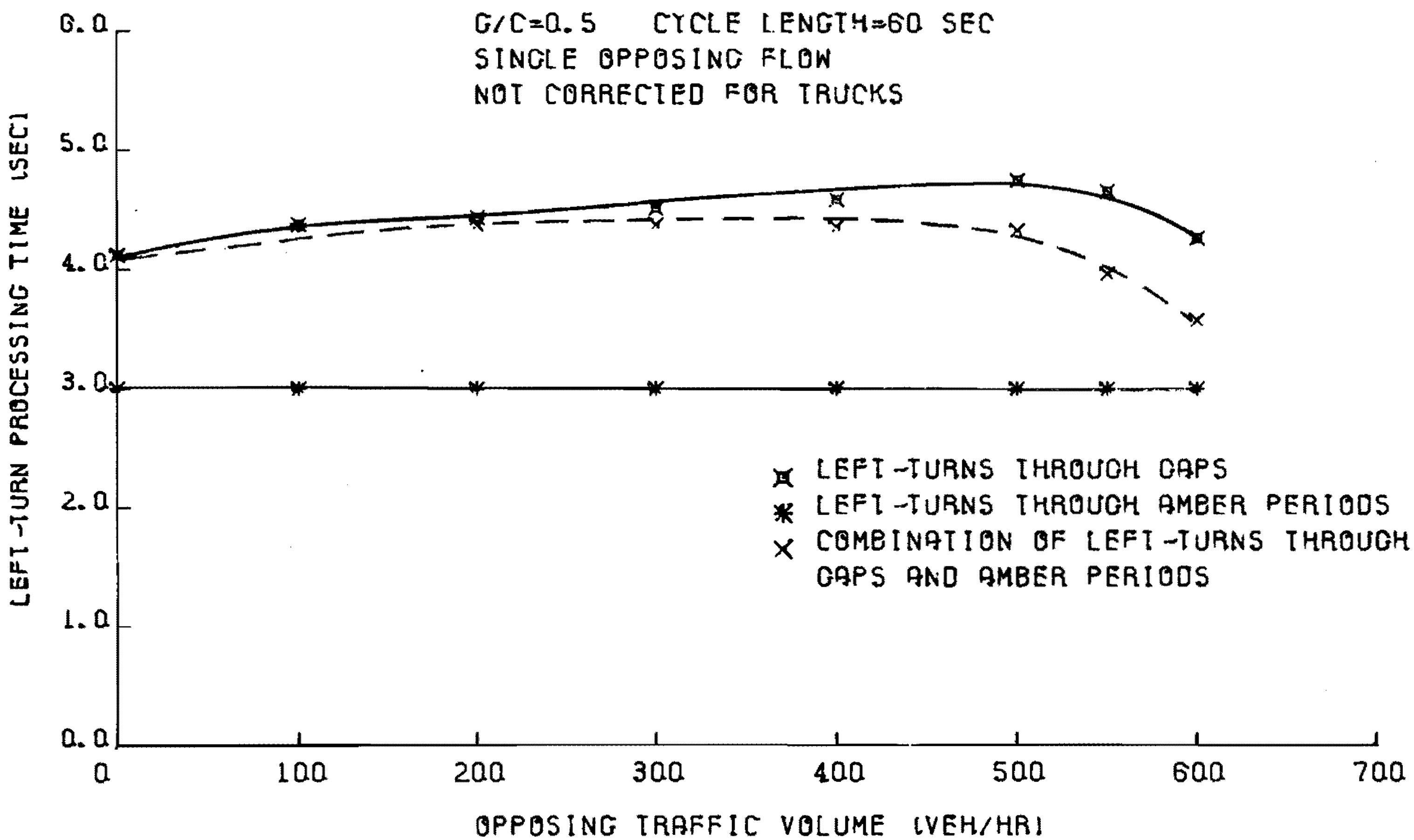


Fig 4-8. The required time for each left turn through opposing gaps and amber periods, respectively.

Left-turn operations may sometimes be more easily analyzed and understood using the notion of transparency, although the desired result is the left-turn capacity. The analysis of left-turn operations is simplified by the fact that the average left-turn processing time is approximately constant.

Single Opposing Flow. Estimation of average left-turn capacity under conditions of a single opposing flow, cycle length of 60 seconds, and 50 percent cycle split is shown in Fig 4-9. Due to randomness in traffic conditions and driver behavior, left-turn capacities may vary from observation to observation. The ranges into which left-turn capacities may fall in 95 out of 100 times are shown in Table 4-4. The average values of left-turn capacity can be approximated by a piecewise linear function of the opposing volume as follows:

$$\begin{aligned} Q_L &= 439 - 0.634 Q_o && \text{if} \quad 0 < Q_o < 500 \text{ vph} \\ Q_L &= 295 - 0.348 Q_o && \text{if} \quad 500 < Q_o < 675 \text{ vph} \end{aligned} \tag{4-36}$$

The slope of a piecewise linear function in Eq 4-36 is analogous to a sensitivity factor of left-turn capacity to the opposing traffic. The slope is 0.634 for opposing volume from 0 to 500 vph. This means that one opposing vehicle is equivalent to 0.634 left-turn vehicles, or one left-turn vehicle is equivalent to 1.6 opposing vehicles. This equivalency factor happens to be exactly the one adopted by Pignataro [Ref 21]. When the opposing volume is greater than 500 vph, most left turns are made during the yellow intervals instead of through opposing gaps, so the sensitivity factor is reduced from 0.634 to 0.348. Furthermore, Fig 4-9, which is based on results from the TEXAS Model, reveals that the left-turn capacity is approximately one left turn during each yellow interval when the opposing traffic is saturated.

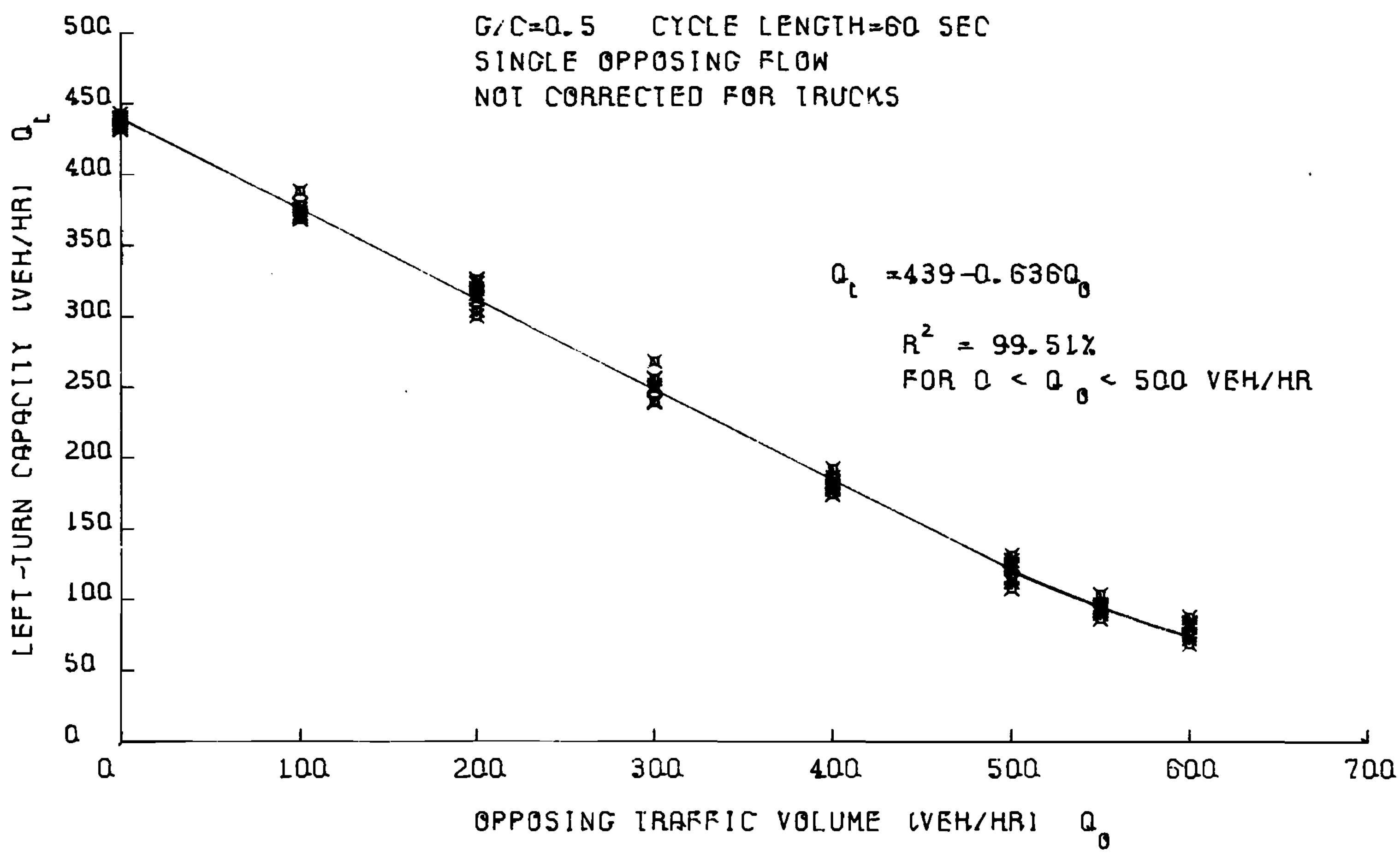


Fig 4-9. The left turn capacity under different levels of opposing volume for a simple case of left-turn movements.

TABLE 4-4. NINETY-FIVE PERCENT CONFIDENCE INVERVALS OF LEFT-TURN CAPACITY UNDER DIFFERENT LEVELS OF OPPOSING VOLUMES

Opposing Volume, vph	Left-Turn Capacity, vph		
	Average Value	Standard Deviation	95% Confidence Interval
0	439	3.7	428 ~ 446
100	376	6.7	360 ~ 392
200	317	9.8	294 ~ 340
300	252	12.1	221 ~ 283
400	183	5.6	170 ~ 196
500	121	8.1	102 ~ 140
550	95	5.0	83 ~ 107
600	80	6.4	65 ~ 95

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks or buses
Number of replications = 8

However, the Highway Capacity Manual [Ref 13] allows two left turns per cycle when the opposing flow is saturated. Australians [Ref 14 and 22] indicate that at least 1.5 vehicles per cycle can turn left after the end of the green phase. The State Department of Highways and Public Transportation in Texas [Ref 23] has sometimes used a value of 1.6 left turns per cycle as a minimum in capacity analyses. In view of this, the left-turn capacity based on the TEXAS Model is conservative by about 0.6 vehicle per cycle when the opposing volume is high. On the other hand, if there is no opposing traffic, the TEXAS Model shows the left-turn capacity in this case is 439 vph. This implies that the turning headway is 3.6 seconds if the effective green time is assumed to be 26 seconds per cycle. Pignataro [Ref 21] contends that a turning headway is 1.3 seconds more than the minimum discharging headway of 2.1 seconds. Hence, the turning headway is 3.4 seconds which is nearly that of the TEXAS Model results. Fambro et al [Ref 11] observed 2.5 seconds of turning headway in the field at signalized intersections having a left-turn bay. Notice that the turning headway 3.6 seconds is obtained by regression analysis on gap size and number of left turns accommodated. This technique is different from that used by Fambro et al in observing the turning headway.

Effect of Cycle Length. The left-turn capacity estimates produced thus far are based on a cycle length of 60 seconds. Although 60 seconds of cycle time is quite typical for many signalized intersections, it is interesting to know whether the cycle length affects left-turn capacity. A transparency study similar to that conducted for 60-second cycle length was conducted for a 90-second cycle (see Table 4-5). Transparency and left-turn capacities exhibited a maximum change of ten percent and average change of less than five percent for opposing volume of 0 to 600 vph. Thus, changing the cycle

TABLE 4-5. TRANSPARENCY STUDY FOR CYCLE SPLIT 0.5 AND AND CYCLE LENGTH NINETY SECONDS

Statistics	Opposing Traffic Volume, vph					
	0	200	300	400	500	600
Total observed time, seconds	2700	2700	2700	2700	2700	2700
Total time after the opposing queue is cleared, seconds	1350	1033	984	939	814	704
Total acceptable gap time, seconds	1350	920	781	581	393	212
Percent of time after the opposing queue is cleared, percent	50.0	38.9	37.1	35.4	30.7	26.5
Transparency, percent	50.0	34.6	29.4	21.8	14.8	8.0
Total number of left-turns through gaps during simulation time	318	226	175	128	82	42
Total number of left-turns made in amber periods during simulation	9	11	11	14	15	20
The left-turn capacity, vph	437	315	250	190	130	84
The average left-turn processing time, seconds	4.13	4.0	4.23	4.13	4.10	3.40

G/C = 0.5 C = 90 sec
Single opposing flow
Not corrected for trucks

length from 60 seconds to 90 seconds does not have a pronounced effect on left-turn capacity.

Effect of Cycle Split. As Fig 4-4 shows, cycle split not only determines the percentage of cycle time that left turns are blocked by the red phase but also affects left-turn operations during the green phase. For a 50 percent cycle split, Eq 4-31 illustrates the relationship between transparency and the opposing volume. The corresponding left-turn capacity can be obtained by Eq 4-35 or directly from Eq 4-36. In order to determine what the left-turn capacity would be if the cycle split differs from 0.5, it is necessary to assess how the transparency is affected by the cycle split.

For any G/C ratio, it is clear that left turns will be blocked by the red phase for 100(1-G/C) percent of the time. During the remaining [100(G/C) percent] green time, left turns may be further blocked for some periods by the opposing queue and by unacceptable gaps. The total unblocked time which exists only during the green phase primarily depends on the opposing volume and the cycle split. However, at signalized intersections the opposing volume in terms of vehicles per hour of green is the actual traffic volume which left turners face during the green phase. If two opposing flows with different cycle splits have the same volume in terms of vehicles per hour of green, then the left turners will be impeded to the same "degree" during the green phase in either case despite different lengths of the red phase. This "degree of impedance" can be characterized by a quantity F which is the ratio of the total unblocked time to the total green time. For example, an opposing flow of 300 vph with cycle split 0.6 is equivalent to an opposing flow of 250 vph with cycle split 0.5. Since both opposing flows are equivalent to 500 vehicles per hour of green, the F ratios in both conditions will be the same. Since the transparency and F ratio for the cycle split 0.5

have been presented in previous sections, the transparency for any cycle split can be determined by considering the following two cases.

Case I: Cycle split=0.5
Cycle length=60 seconds
Opposing volume= Q_o vph
Transparency= T

$$F = \frac{\text{total unblocked time in one hour, sec}}{\text{total green time in one hour, sec}}$$

$$= \frac{3600T}{3600(0.5)}$$

$$= 2T \tag{4-37}$$

Case II: Cycle split= G/C
Cycle length= 60 seconds
Opposing volume= Q'_o vph
Transparency= T'

$$F' = \frac{\text{total unblocked time in one hour, sec}}{\text{total green time in one hour, sec}}$$

$$= \frac{3600T'}{3600(G/C)}$$

$$= T'/(G/C) \tag{4-38}$$

If Q_o and Q'_o have the same traffic volume in terms of vehicles per hour of green, then

$$\frac{Q'_o}{(G/C)} = \frac{Q_o}{0.5}$$

$$Q_o = \frac{Q'_o}{2(G/C)} \tag{4-39}$$

From Eq 4-39, an opposing volume Q'_o under any G/C ratio can be converted to an opposing volume Q_o under a G/C ratio of 0.5. If the opposing volume is less than 1200 vehicles per hour of green, the transparency for an opposing volume Q_o with G/C = 0.5 can be computed from Eq 4-31 as follows:

$$\begin{aligned} T &= 0.5332 - 0.0007675Q_o \\ &= 0.5322 - 0.0007675Q'_o/[2(G/C)] \end{aligned} \tag{4-40}$$

Since Q_o and Q'_o have the same volume in terms of vehicles per hour of green, F must be equal to F'. By setting Eq 4-37 equal to Eq 4-38, the following equation can be obtained.

$$2T = T'/(G/C)$$

or

$$\begin{aligned} T' &= 2(G/C)T \\ &= 2(G/C)\{0.5322 - 0.0007675Q'_o/[2(G/C)]\} \\ &= 1.064(G/C) - 0.0007675Q'_o \end{aligned} \tag{4-41}$$

From Eq 4-41, the transparency under any cycle split can be determined. It can be seen that, if G/C=0.5, then Eq 4-41 is reduced to Eq 4-31. Obviously, Eq 4-41 is more general. The left-turn capacity for any G/C ratio can be calculated as follows:

$$\begin{aligned} Q'_L &= 3600T'/\overline{t} \\ &= [3830(G/C)-2.763Q'_o]/\overline{t} \end{aligned}$$

If Q'_o is less than 1000 vehicles per hour of green, then $\bar{t}$ can be approximated by a constant 4.36 seconds. The left-turn capacity in this case is:

$$Q'_L = 879(G/C) - 0.634Q'_o \tag{4-42}$$

By comparing Eq 4-42 and the results from running the TEXAS Model, it was found that Eq 4-42 predicts left-turn capacity within five percent. It not surprising to find that for G/C equal to 0.5, Eq 4-42 is reduced to Eq 4-36.

Effect of Multiple Opposing Lanes. In the field, it is more common for left-turn traffic to face two or more opposing flows than just a single opposing flow. The immediate question is how to extend the results for one opposing flow to multiple opposing flows. The HCM method and the ARCG method do not differentiate between the number of opposing lanes. Webster modifies values of parameters to account for this effect. Fambro et al, in addition, take the lane distribution of opposing volume into consideration. These methods tackle this problem basically by regarding the multiple flows as a single stream. In this section, a rational method is used to estimate the left-turn capacity for multiple opposing flows based on the information about the single opposing flow. The estimation is then compared with that from the TEXAS Model.

The major differences between one opposing flow and multiple opposing flows are the stagger of opposing gaps, the multiple check for acceptable gaps on each lane, and the longer travel distance for left turners. The stagger of opposing gaps means that vehicles on different lanes cross the stop line at various times. Webster adds one second to the critical gap to account for the effect on the gap acceptance decision. However, the stagger of gaps in multiple opposing flows is the most critical consideration. There are two extreme cases representing the best and the worst way that opposing gaps can be staggered. Figure 4-10 shows the best case in which the opposing volume is evenly distributed over the lanes and the vehicles on each lane cross the stop line simultaneously. In this special case, left turners are in effect facing only one half or one third the opposing volume depending on whether there are two or three lanes. The left-turn capacity under this condition is the upper limit. On the other hand, the worst case occurs when the opposing gaps are staggered in such a way that no more than one vehicle is crossing the stop line at any time (Fig 4-11). In this case, left turners in effect observe a single traffic stream. The left-turn capacity in the worst case, therefore, serves as a lower limit. If the average case lies somewhere middle way between the best and worst cases, then the average value of the upper limit and the lower limit might represent the left-turn capacity for multiple opposing flows under average conditions. In general, the left-turn capacity for two and three opposing flows can be derived as follows.

Let $Q_L = f(Q_o)$, a function of the opposing volume Q_o, be the left-turn capacity for single opposing flow. Let Q_{L2} and Q_{L3} be the left-turn capacities for two and three opposing flows, respectively. Then, in terms of best and worst cases, Q_{L2} and Q_{L3} can be estimated as follows:

$$Q_{L2} = [f(Q_o)+f(Q_o/2)]/2 \tag{4-43}$$

$$Q_{L3} = [f(Q_o)+f(Q_o/3)]/2 \tag{4-44}$$

Table 4-6 compares the estimated left-turn capacity calculated by Eq 4-43 and 4-44 with that from the Texas Model. The estimated left-turn capacities on the average are only about six percent less than the simulation

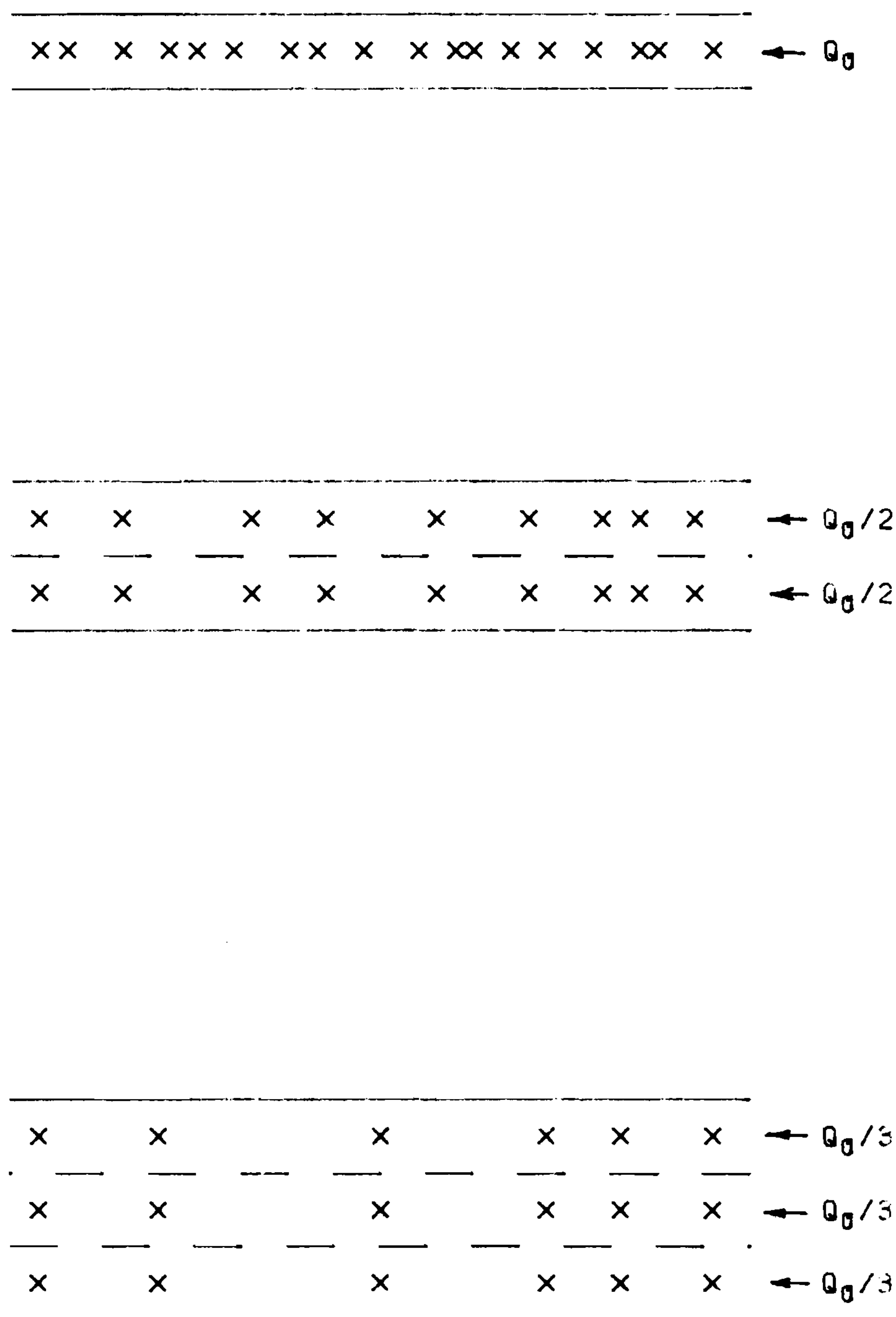


Fig 4-10. The best case for the stagger of opposing gaps.

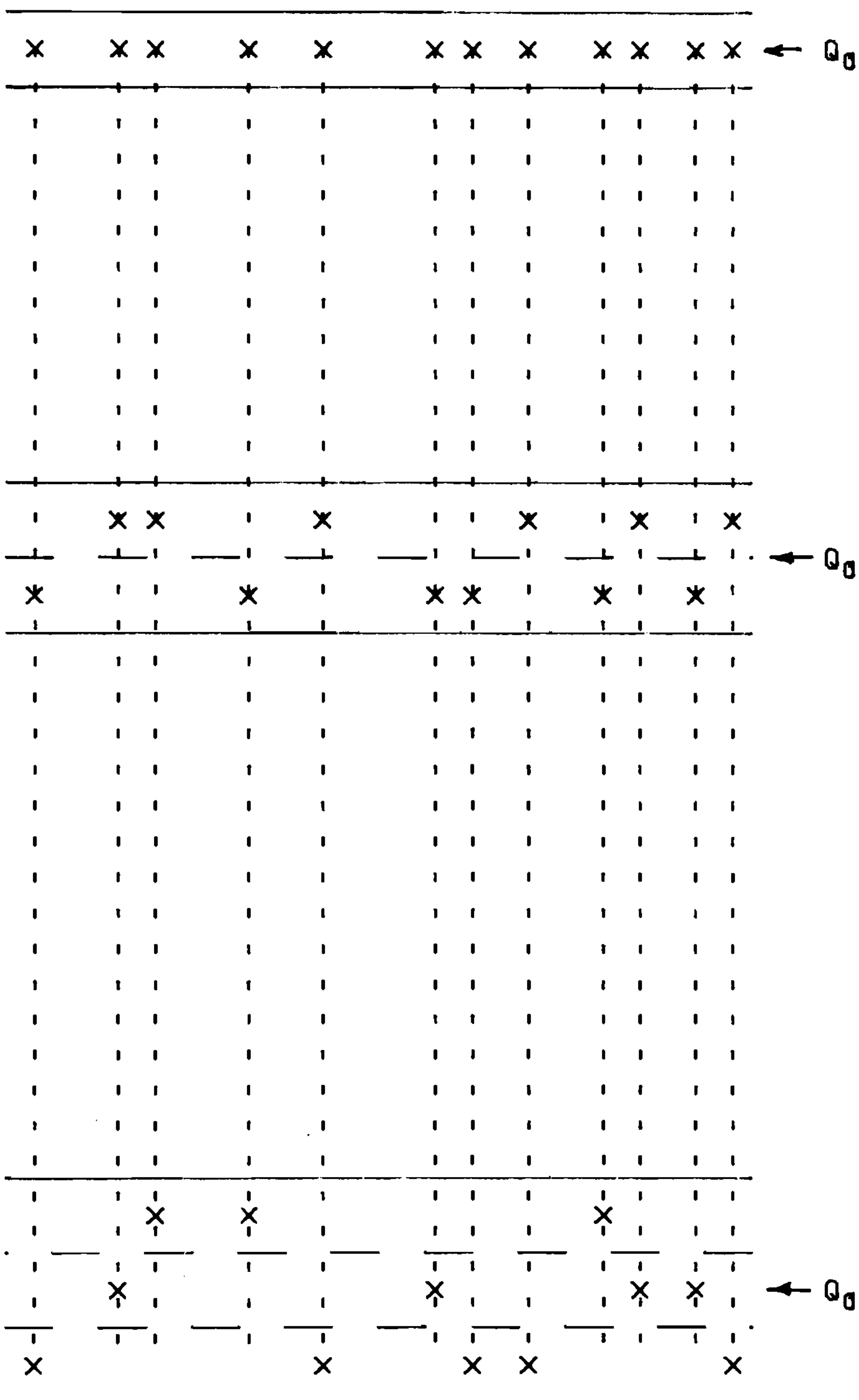


Fig 4-11. The worst case for the stagger of opposing gaps.

TABLE 4-6. COMPARISONS BETWEEN LEFT-TURN CAPACITIES FROM THE TEXAS MODEL AND THAT ESTIMATED BY EQUATION (4-43) or (4-44)

Number of Opposing Flows	Opposing Volume (vph)	Left-Turn Capacity, vph		Ratio
		Estimated	TEXAS Model	
2	200	334	353	1.06
2	300	297	310	1.04
2	400	250	256	1.02
2	500	200	218	1.09
2	600	166	175	1.05
3	200	356	375	1.05
3	300	302	322	1.07
3	400	268	278	1.04
3	500	227	246	1.08
3	600	199	212	1.06
Average				1.06

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks

results. This six percent deviation may result from geometry effects and difference between the actual and hypothetical staggers. This study shows that treating the multiple flows as a single stream tends to far underestimate the left-turn capacity.

Effect of Left-Turn Bays. Thus far, a left-turn bay of adequate length has been assumed in all cases. If there is no bay, left-turn capacity will likely be reduced due to interactions between the left-turn and through traffic in the median lane. Fambro et al [Ref 11] approached this problem by assuming that 50 percent of the traffic in the median lane, from which left turns are made, were left-turn vehicles. Furthermore, he assumed that in the average case, a through vehicle will be followed by a left-turn vehicle. So, he defined an effective turning headway which is the average turning headway 2.6 seconds plus a through headway 2.06 seconds and used it in Drew's equation to compute the left-turn saturation flow for the no bay case. This simplified approach is not reasonable. First, a through vehicle in the median lane may not affect left-turn operations at all if it is discharged while the left turns are blocked by the opposing queue or unacceptable gaps. Second, in a real situation, through vehicles tend to use the curb lane, if a left-turn queue is impeding through movement in the median lane. Consequently, in the median lane, the through traffic volume will not be the same as the left-turn volume. Actually, the number of through vehicles in the median lane depends not only on the approach volume but also on the left-turn volume. The left-turn capacity in turn is affected by the through traffic in the median lane. The manner in which the through traffic in the median lane influences left-turn operations is the key issue for determining left-turn capacity at signalized intersections without a bay. Theoretically, if the median lane is regarded as a bay, then the no-bay problem is equivalent to that of a left-turn bay but with through traffic in it. The effect of through vehicles on the left-turn capacity will be discussed in the following paragraphs.

Case I. No Left-Turn Vehicles In Opposing Flows. Once again, the effect of through traffic on left-turn operations can be more easily understood by studying the transparency. The total time available for left turns in one hour when there is a bay is 3600T seconds, where T is the transparency defined in Eq 4-41. If a left-turn bay is not provided, through vehicles in the median lane may or may not consume the left-turn time, depending on whether at the time they are crossing the stop line there exists an acceptable gap in the opposing flow. Let p be the probability that a through vehicle in the median lane is crossing the stop line while an acceptable gap is available in the opposing flow. If the through volume processed in the median lane is V_T vph, then there will be pV_T through vehicles that might use some left-turn time (V_T will include right-turn vehicles if there is a one lane approach, N=1). Suppose each through vehicle requires h seconds for discharging, then the total time in one hour remaining for left turns when there are V_T through vehicles processed in the median lane can be computed as follows:

$$\Gamma' = 3600T - hpV_T \tag{4-45}$$

where

$'$ = total seconds available for left turns in one hour when there is no bay;

T = the transparency when there is a bay, computed by Eq 4-41;

V_T = through volume processed in median lane, vph;

h = discharging headway of through vehicles, sec; and

p = probability that a through vehicle in median lane will consume left-turn time.

In Eq 4-45, the only unknown is the probability p which can be estimated in the following way:

Let

Event A = A through vehicle in the median lane is crossing the stop line at time, t, during green time;

Event B = There is an acceptable gap among opposing vehicles at the time, t, during green time;

V_L = left-turn volume processed in median lane, vph; and

V_T = through volume processed in median lane, vph.

From above definitions, the following relations can be written

$$Pr\{A\} = V_T/(V_T+V_L)$$

$$Pr\{B\} = \frac{\text{total acceptance gap time, sec}}{\text{total green time in one hour, sec}}$$

$$= 3600T/[3600(G/C)] = T/(G/C)$$

By the definition of probability p and independence of Events A and B, the probability p can be obtained as follows:

$$p = Pr\{A,B\} = Pr\{A\}Pr\{B\}$$

$$= \left(\frac{V_T}{V_T+V_L}\right)\left(\frac{T}{G/C}\right) \tag{4-46}$$

Placing Eq 4-46 in Eq 4-45, the total time available for left turns when there are V_T vph of the through traffic in the median lane becomes

$$\Gamma' = 3600T - h\left(\frac{V_T}{V_T+V_L}\right)\left(\frac{T}{G/C}\right)V_T \tag{4-47}$$

From Eq 4-34, the left-turn capacity under the no-bay condition can be computed by the following equation:

$$\tilde{Q}_L = \Gamma'/\bar{t}$$

$$= \frac{3600T}{\bar{t}} - h\left(\frac{V_T^2}{V_T+V_L}\right)\left(\frac{T}{(G/C)\bar{t}}\right)$$

$$= \frac{3600T}{\bar{t}} - \left(\frac{h}{3600(G/C)}\right)\left(\frac{V_T^2}{V_T+V_L}\right)\left(\frac{3600T}{\bar{t}}\right)$$

$$= Q_L\left\{1-\left(\frac{h}{3600(G/C)}\right)\left(\frac{V_T^2}{V_T+V_L}\right)\right\} \qquad (4\text{-}48)$$

where

Q_L = left-turn capacity when there is an adequate length of bay

$= 3600T/\bar{t}$

It should be noted that the left-turn capacity can be attained only when the left-turn traffic is saturated. In this case, the left-turn volume processed in the median lane V_L must also be the left-turn capacity so that $V_L=\tilde{Q}_L$. By rearranging Eq 4-48, it becomes a quadratic equation of left-turn capacity $\tilde{Q}_L$.

$$\tilde{Q}_L^2 + (V_T - Q_L)\tilde{Q}_L + V_T Q_L \left(\frac{hV_T}{3600(G/C)} - 1\right) = 0 \tag{4-49}$$

Solving the quadratic equation, the left-turn capacity, $\tilde{Q}_L$, is:

$$\tilde{Q}_L = 0.5(-b+\sqrt{b^2-4c}) \tag{4-50}$$

where

$$b = V_T - Q_L$$

$$c = V_T Q_L \left(\frac{hV_T}{3600(G/C)} - 1\right)$$

The discharging headway, h, in the TEXAS Model is found to be 2.6 seconds. Thus, the left-turn capacity under the no-bay condition can be computed from Eq 4-50 if the through traffic in the median lane and the left-turn capacity for a bay are known. Figure 4-12 shows the left-turn capacity under different levels of opposing volume and through traffic. Equation 4-50 has been tested against the simulation results of the TEXAS Model and found to be within five percent.

Case II. Left-Turn Vehicles In Opposing Flows. If there are left-turn vehicles in opposing flows, through vehicles in opposing traffic will not be evenly distributed over the lanes. Let P_c and N be the percentage of the total opposing traffic (excluding left turns) that is carried on the lane with the heaviest opposing volume and the number of opposing lanes, respectively. Using the same argument as presented in the discussion of multiple opposing lanes, the best case for the stagger of opposing gaps will be equivalent to a single traffic stream of $p_c Q_o$ instead of Q_o/N.

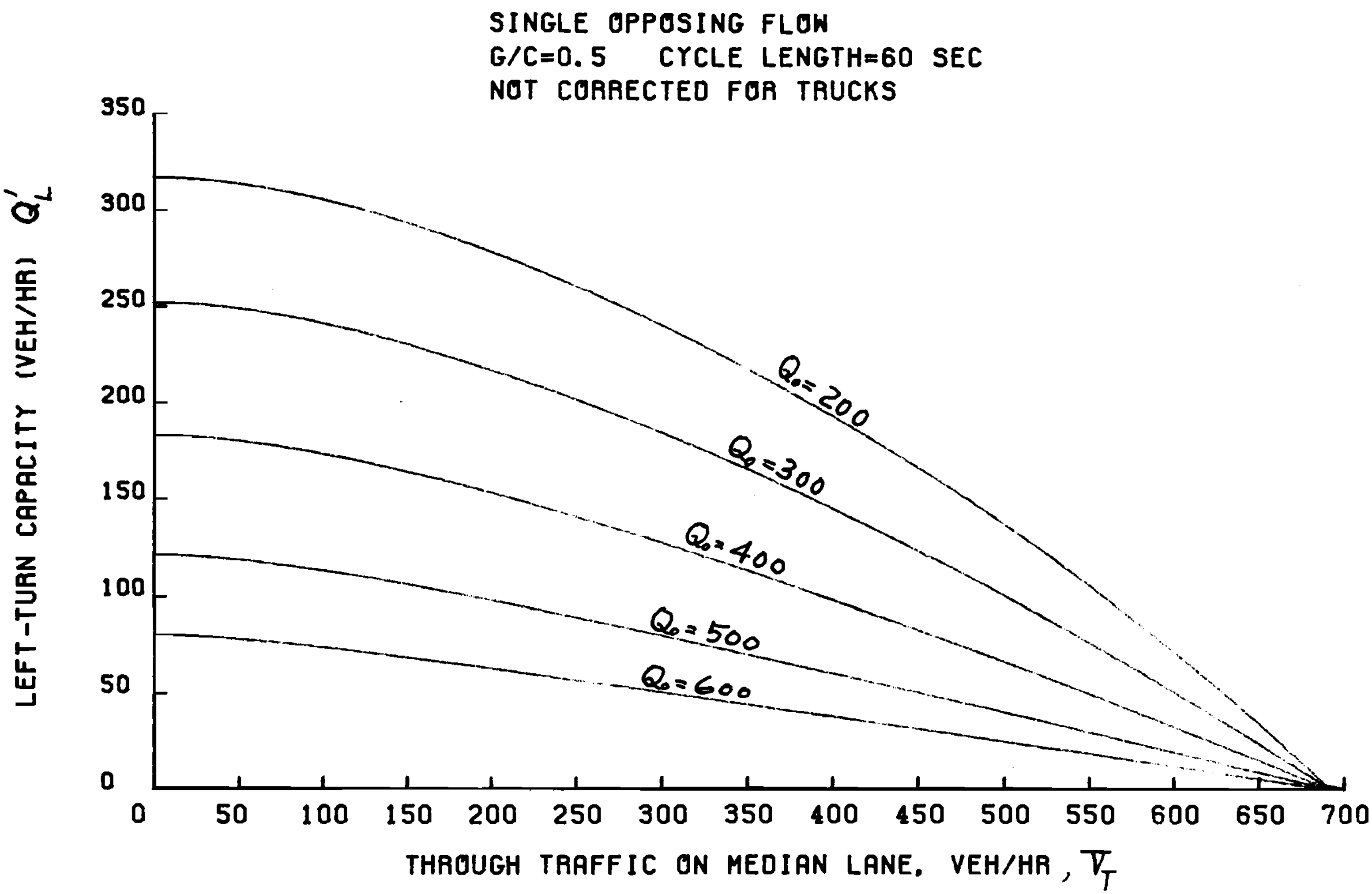


Fig 4-12. The left-turn capacity under different levels of opposing volume at two-by-two signalized intersection without a bay.

From Eq 4-42 and Eq 4-43, the corresponding left-turn capacity can be obtained as follows:

$$\hat{Q}_L = 0.5[f(Q_o)+f(p_c Q_o)]$$

$$= 0.5[f(Q_o)+f(Q_o/N)] + 0.5[f(p_c Q_o) - f(Q_o/N)]$$

$$= \tilde{Q}_L + 0.5[-0.634 p_c Q_o + 0.634(Q_o/N)]$$

$$= \tilde{Q}_L - 0.317(p_c - 1/N)Q_o \qquad (4-52)$$

Let

$$a = 0.317(p_c - 1/N) \qquad (4-53)$$

where

N = number of opposing lanes

Then

$$\hat{Q}_L = \tilde{Q}_L - aQ_o \qquad (4-54)$$

From Eq 4-50 and 4-54, it can be seen that the left-turn capacity under the no-bay condition will be affected by the interactions of left-turn and through traffic flows on the approach of concern and on the opposing approach.

Effect of Headway Distributions. The present simulation studies are based on the assumption that the headway of traffic injected into the system 1000 feet from the stop line has a shifted negative-exponential distribution. The headways at the stop line, as discussed in Chapter 3, are different from those injected into the system as a result of car-following and signalization. However, the headways upstream of an intersection may not reflect a shifted negative-exponential distribution. In fact, many other types of statistical distributions have been reported as fitting field data very well. Thus, it is necessary to study the sensitivity of the simulation output to the injected headway distributions in order to generalize results obtained in this study. Research on simulation [Ref 24] has found that simulation results are insensitive to various types of statistical distributions as long as they have similar shapes. The coefficient of variation, defined as the ratio of the standard deviation and the mean, is a relative measure of dispersion. The shifted negative exponential distribution has a coefficient of variation equal to 1.0. The coefficient of variation of the Erlang distribution is less than 1.0. In this sense, the Erlang distribution is said to be more "regular" than the shifted negative-exponential distribution. In this section, the effect of headway distributions on the left-turn capacity is studied by changing the input distribution from a shifted negative-exponential to an Erlang distribution with coefficient of variation 0.5. The left-turn capacity is not changed considerably as the coefficient of variation is reduced from 1.0 to 0.5 (see Fig 4-13). This does not imply that the actual headway distribution can be modeled by the shifted negative-exponential distribution satisfactorily. However, this study shows that the headway distribution is not very critical in affecting the left-turn capacity. There are two main reasons for this. First, the opposing gaps at the stop line are different from those at the upstream of an approach due to car-following and signalization. Second, a long gap which permits N left turns has essentially the same effect as N short gaps, each for one left turn only. Therefore, as long as the mean gap sizes are approximately the same, the left-turn capacities will be approximately equal.

Effects of Trucks and Buses. Trucks and buses have been ignored in the previous discussions. The presence of trucks and buses will make gap-acceptance decisions more difficult and, therefore, will reduce the left-turn capacity. To account for the effects of trucks and buses, the

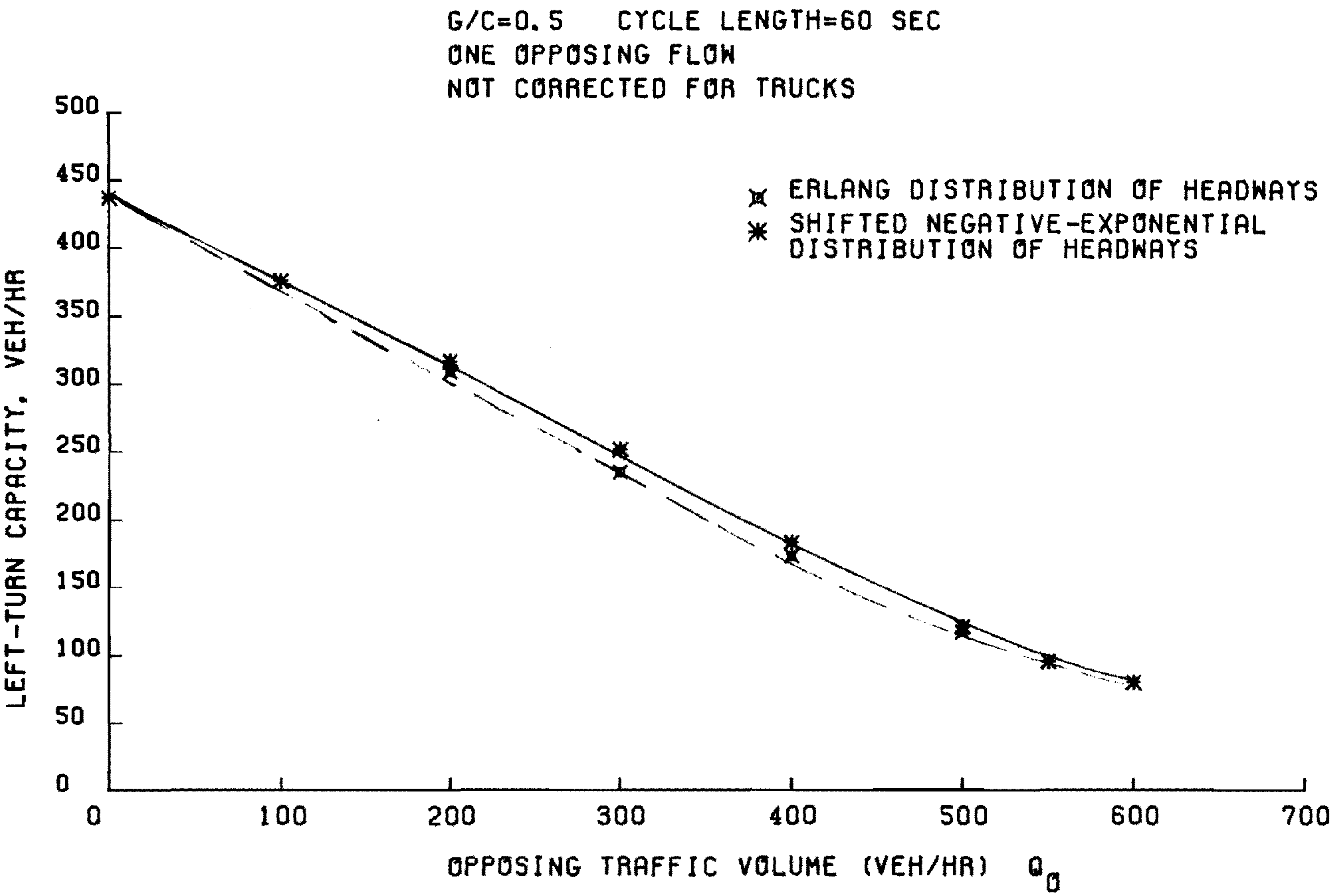


Fig 4-13. The effect of the type of headway distribution on left-turn capacity.

concepts of equivalence factor and correction factor are usually used. Greenshields et al [Ref 10] found trucks and buses equal to 1.5 passenger cars in a study of discharging headway. However, for left-turn operations this equivalence factor may not be pertinent. Moreover, it is misleading in general to use an equivalence factor for left-turn trucks and buses. Left-turning trucks and buses require larger gaps than passenger cars. Therefore, the presence of left-turning trucks and buses effectively increases the left-turn demand relative to a homogeneous stream of passenger cars. In a recent report [Ref 25], TTI suggested adjusting left-turn capacity for trucks and buses in the left-turn stream according to the following equation:

$$Q^*_L = Q_L (1-p_T) \qquad (4\text{-}55)$$

where

Q^*_L = left-turn capacity for mixed left-turn traffic, vph;

Q_L = left-turn capacity assuming no trucks and buses, vph; and

p_T = percent left-turn trucks and buses as a decimal.

Equation 4-55 is simple but not convincing. It predicts zero left-turn capacity when the left-turn vehicles are all trucks and buses. This, in general, is not true. Moreover, TTI did not suggest how to correct left-turn capacity for the effects of opposing trucks and buses. In view of this, left-turn capacity for different combinations of opposing and left-turn trucks and buses was studied by means of the TEXAS Model. Figure 4-14 shows that for a given percentage of opposing trucks and buses, the left-turn capacity is approximately linearly reduced as the percentage of left-turn trucks and buses increases. The left-turn capacity when there are trucks and

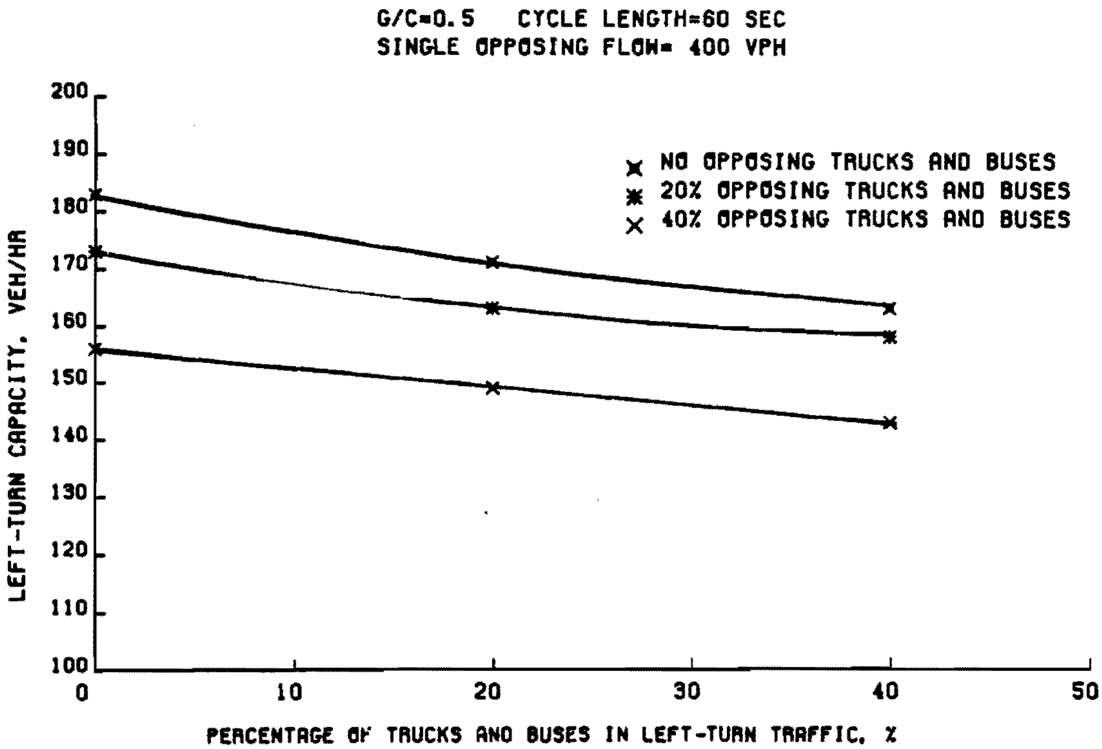


Fig 4-14. The left-turn capacity for different combinations of opposing and left-turn truck percentages.

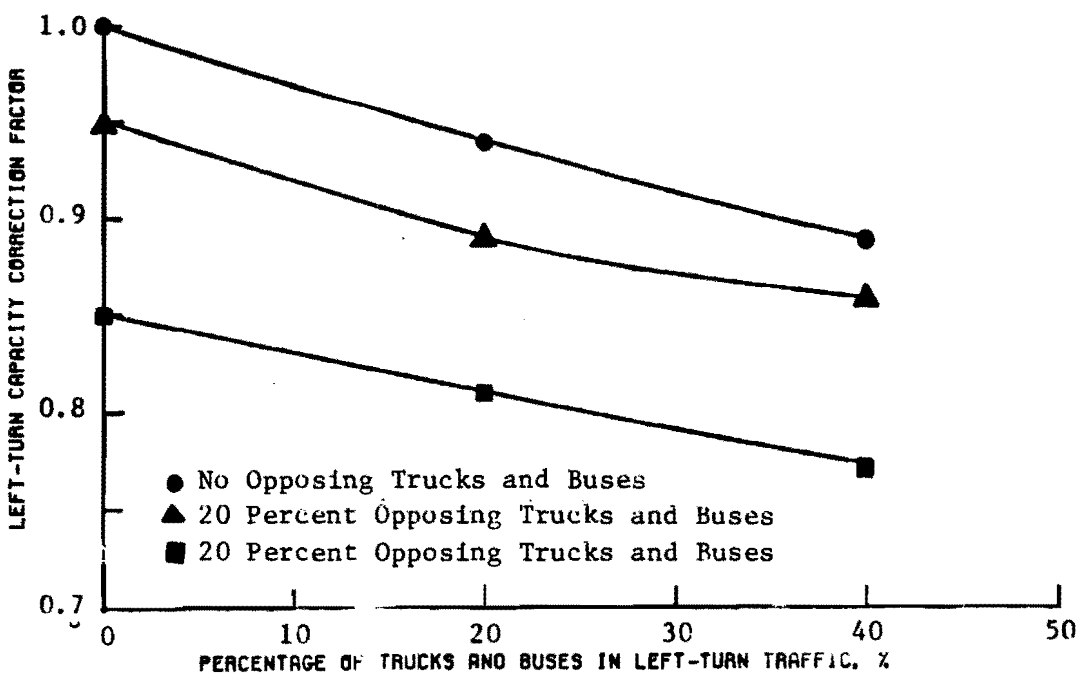


Fig 4-15. Factors for adjusting left-turn capacity for different combinations of opposing and left-turn truck percentages.

buses in the traffic population can be corrected using the following equation:

$$Q^*_L = f_T Q_L \tag{4-56}$$

where

Q^*_L = left-turn capacity for mixed traffic flows, vph;

Q_L = left-turn capacity for traffic without trucks and buses, vph; and

f_T = trucks and buses correction factor obtained from Fig 4-15.

<u>Summarized Results</u>. From the above discussions, it can be seen that the unprotected left-turn capacity at signalized intersections under various traffic conditions and geometric configurations can be obtained on the basis of the information about a basic case of cycle split 0.5, cycle length 60 seconds, and a single opposing lane. Since the left-turn capacity for the basic case was found to be approximately a piecewise linear function of opposing volume, the left-turn capacity in general can be computed in the following cases.

<u>Case I: Adequate Length of Bay, No Trucks or Buses</u>

$$Q_L = Q_c (G/C) - e_o Q_o \tag{4-57}$$

Values of Q_c and e_o under various levels of opposing volumes and number of opposing lanes are summarized in Table 4-7. The left-turn capacities under different levels of opposing volumes at signalized intersections with cycle length 60 seconds and various cycle splits are shown in Table 4-8 and Fig 4-16 through Fig 4-20.

TABLE 4-7. VALUES OF Q_c AND e_o UNDER DIFFERENT LEVELS OF OPPOSING VOLUMES AND NUMBER OF OPPOSING LANES

Number of Opposing Lanes	Opposing Volume Q_o, vph	Q_c	e_o
Single	$0 < Q_oC/G < 1000$	879	0.634
	$1000 < Q_oC/G < 1350$	590	0.348
Two	$0 < Q_oC/G < 1000$	930	0.500
	$1000 < Q_oC/G < 1350$	780	0.353
	$1350 < Q_oC/G < 2000$	465	0.167
Three	$0 < Q_oC/G < 1000$	930	0.448
	$1000 < Q_oC/G < 1350$	780	0.297
	$1350 < Q_oC/G < 3000$	465	0.112

TABLE 4-8. THE UNPROTECTED LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS HAVING ADEQUATE LENGTH OF BAY WITHOUT A SEPARATE LEFT-TURN PHASE (CYCLE LENGTH = 60 SEC)

	Opposing Approach Volume, vph						
	200	300	400	500	600	800	1000
G/C = 0.3							
N = 1	135	71	60	-	-	-	-
N = 2	177	126	92	60	60	60	-
N = 3	189	143	114	83	72	60	60
G/C = 0.4							
N = 1	223	159	94	62	-	-	-
N = 2	270	219	168	134	84	60	60
N = 3	282	236	191	162	118	95	73
G/C = 0.5							
N = 1	317	252	183	121	80	-	-
N = 2	353	316	256	218	175	97	63
N = 3	375	330	284	239	210	142	119
G/C = 0.6							
N = 1	400	335	270	206	142	76	-
N = 2	457	406	355	303	252	183	109
N = 3	468	423	377	332	286	229	166
G/C = 0.7							
N = 1	487	422	358	294	229	135	-
N = 2	550	499	448	397	346	261	156
N = 3	561	516	470	425	380	307	213

N = number of opposing lanes

Not corrected for trucks or buses

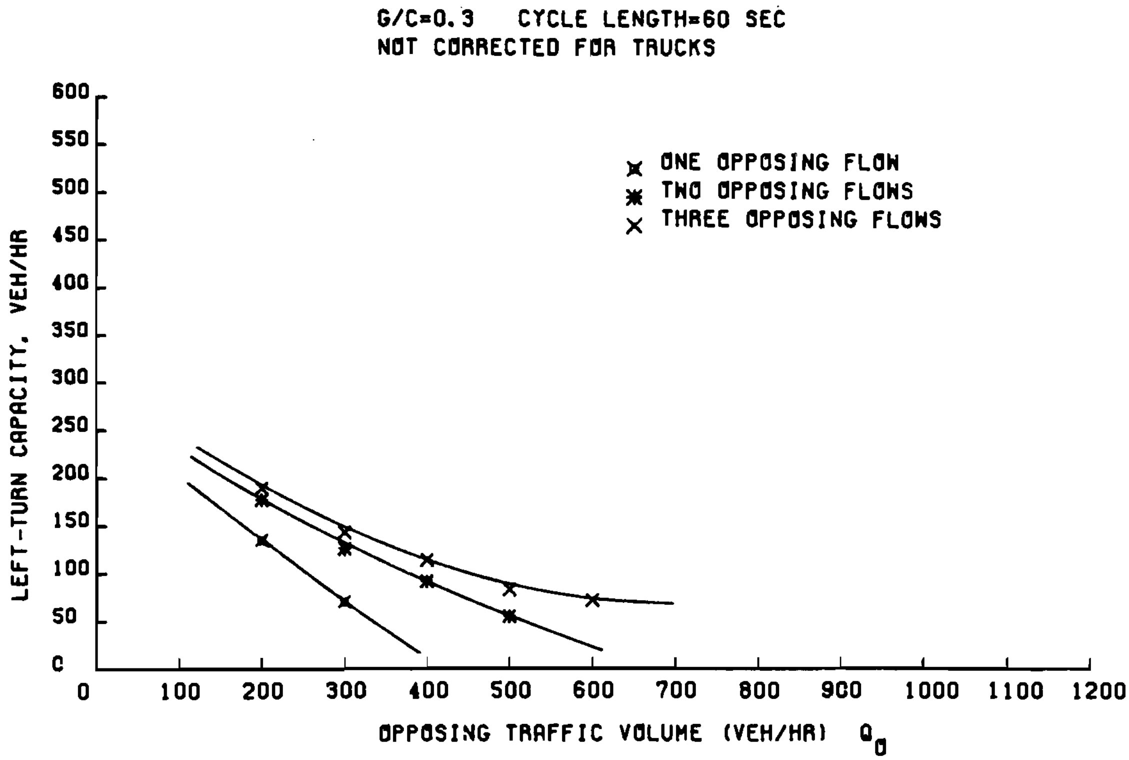


Fig 4-16. Left-turn capacity at signalized intersections having adequate length of bay for G/C = 0.3 and C = 60 sec.

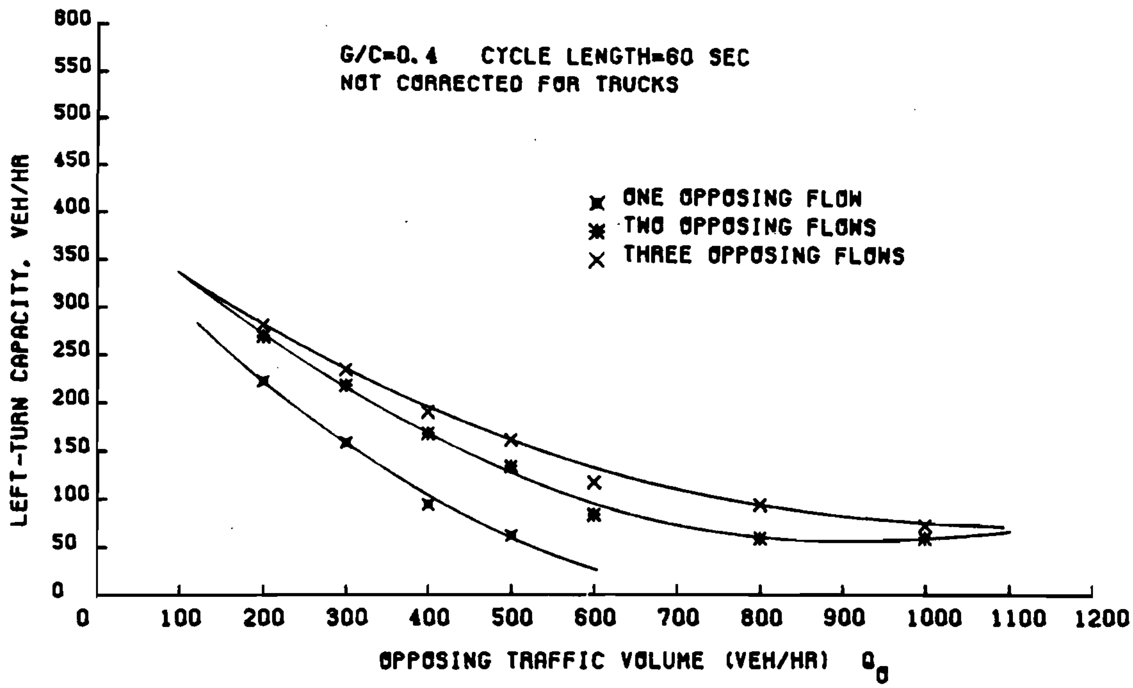


Fig 4-17. Left-turn capacity at signalized intersections having adequate length of bay for G/C = 0.4 and C = 60 sec.

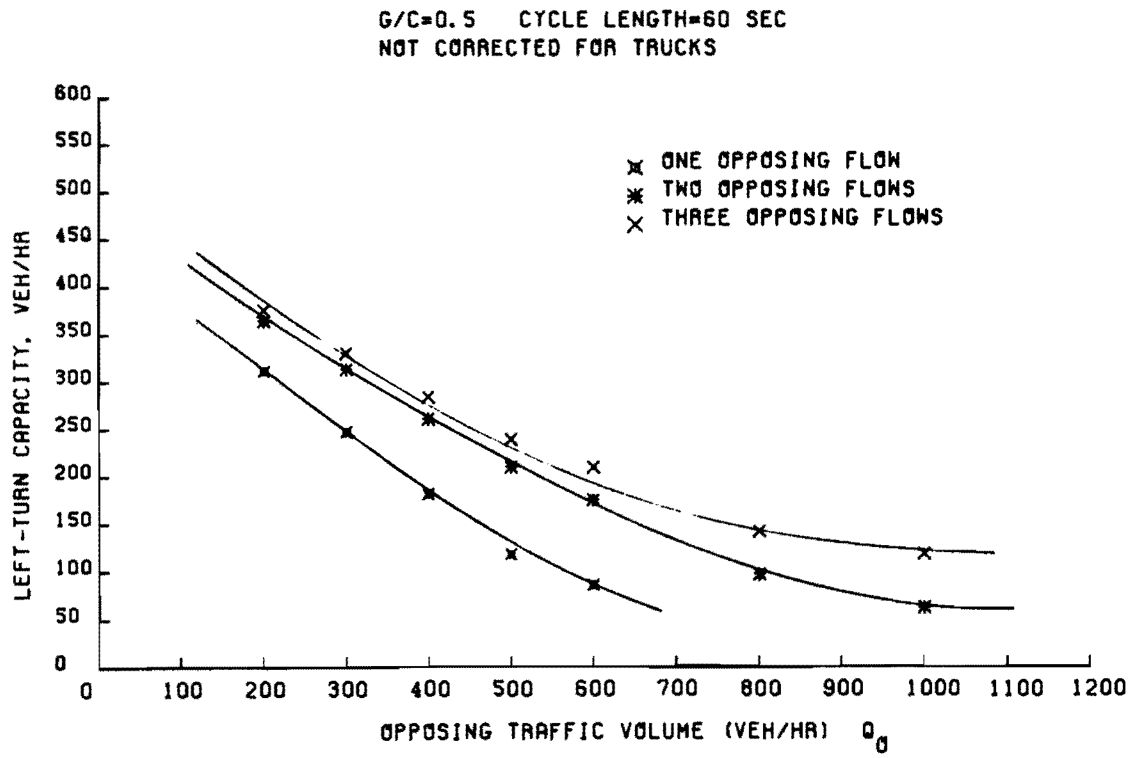


Fig 4-18. Left-turn capacity at signalized intersections having adequate length of bay for G/C = 0.5 and C = 60 sec.

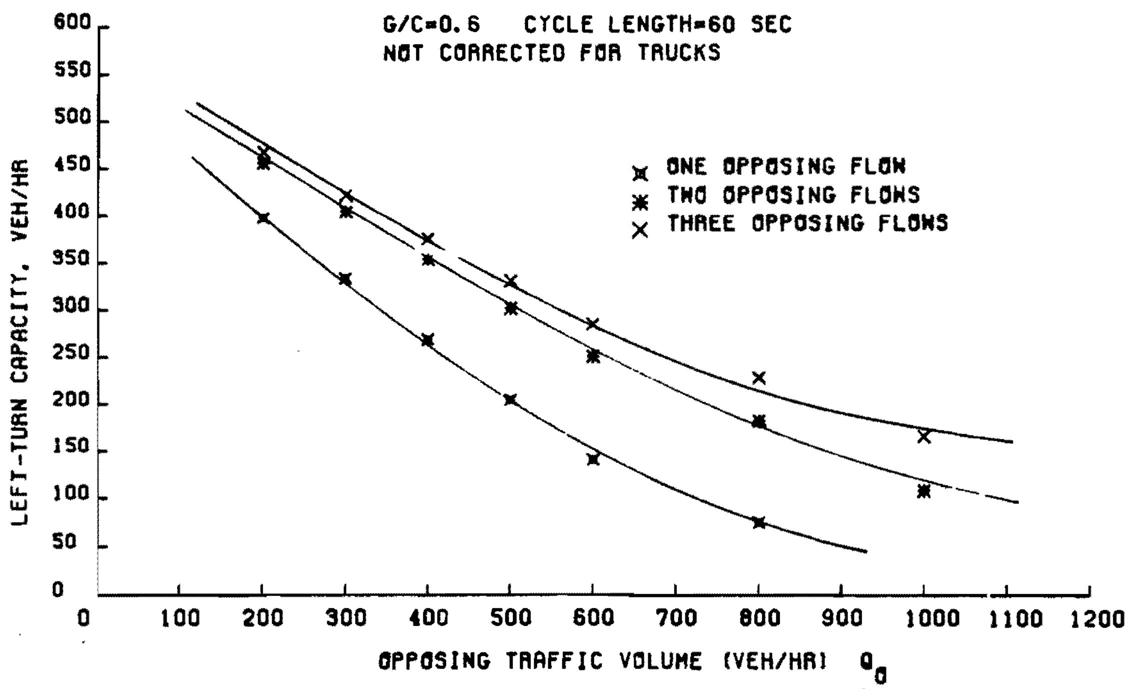


Fig 4-19. Left-turn capacity at signalized intersections having adequate length of bay for G/C = 0.6 and C = 60 sec.

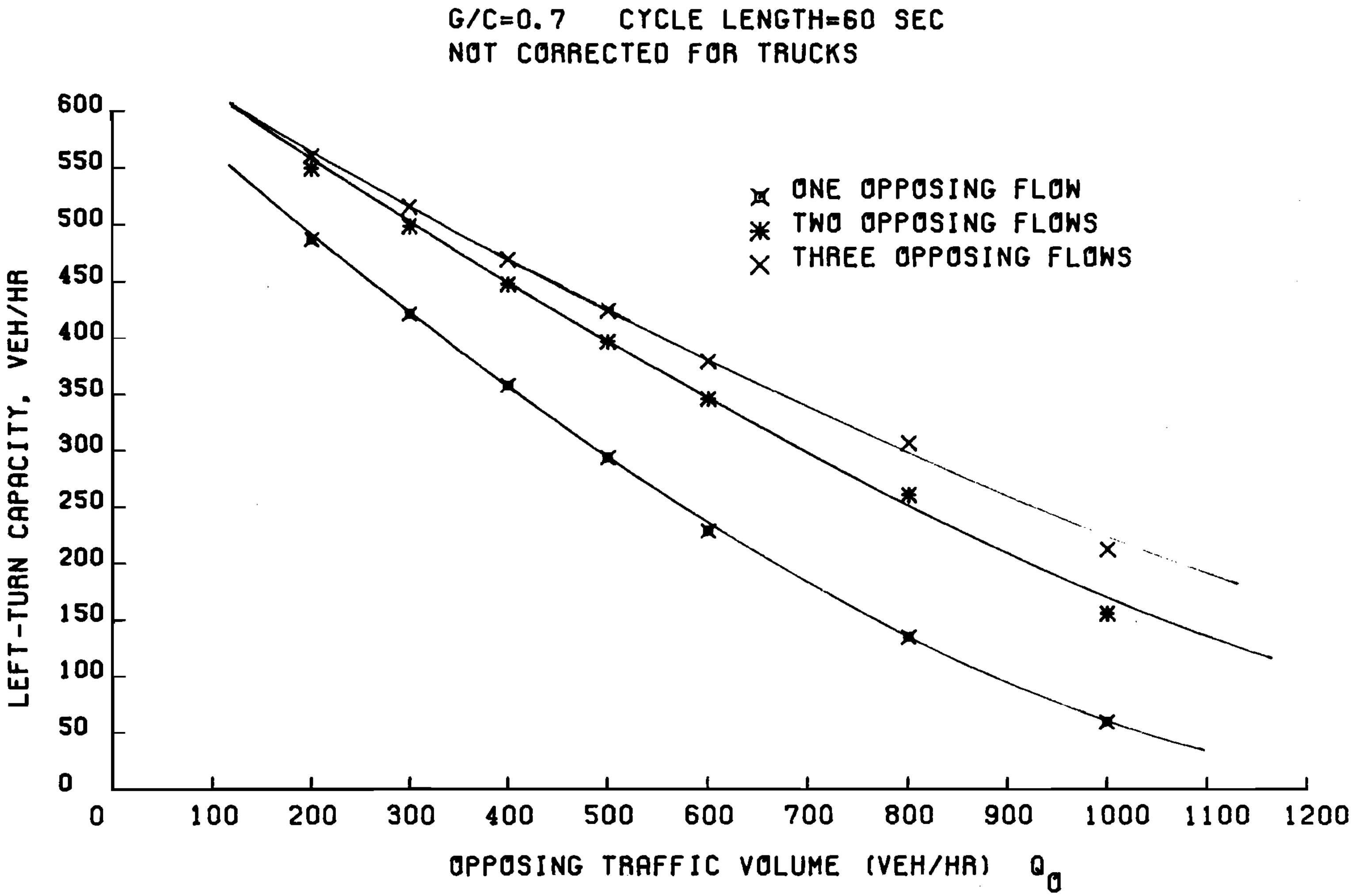


Fig 4-20. Left-turn capacity at signalized intersections having adequate length of bay for G/C = 0.7 and C = 60 sec.

Case II: No Left-turn Bay, No Trucks or Buses

When there are no left-turn vehicles in the opposing flows, left-turn capacity can be computed from Eq 4-50. It can be approximated more simply by a piecewise linear equation as follows:

$$\tilde{Q}_L = \tilde{Q}_c\,(G/C) - \tilde{e}_o Q_o \qquad (4\text{-}58)$$

where

$\tilde{Q}_L$ = left-turn capacity when there are no left-turn vehicles in opposing flows, vph

Values of Q_c and e_o are summarized in Table 4-9. Left-turn capacities computed from Eq 4-58 are summarized in Tables 4-10 through 4-14. If there are left-turn vehicles in the opposing flows, the left-turn capacity will be modified as follows:

$$\hat{Q}_L = \tilde{Q}_L - aQ_o \qquad (4\text{-}59)$$

where

$\hat{Q}_L$ = left-turn capacity when there are left-turn vehicles in opposing flows, vph; and

a = correction factor for left-turn vehicles in opposing traffic as defined in Eq 4-53.

Case III: With Trucks and Buses

$$Q^*_L = f_T Q_L \qquad (4\text{-}60)$$

$$\tilde{Q}^*_L = f_T \tilde{Q}_L \qquad (4\text{-}61)$$

$$\hat{Q}^*_L = f_T \hat{Q}_L \qquad (4\text{-}62)$$

TABLE 4-9. VALUES OF $\tilde{Q}_c$ AND $\tilde{e}_o$ UNDER DIFFERENT LEVELS OF OPPOSING VOLUMES AND NUMBER OF OPPOSING LANES

Number of Opposing Lanes	Opposing Volume Q_o, vph	Through Volume in Median Lane, vph									
		100		200		300		400		500	
		Q_c	e_c	Q_c	e_c	Q_c	e_c	Q_c	e_c	Q_c	e_c
One	$0 < Q_oC/G < 800$	855	0.634	820	0.593	680	0.526	560	0.455	415	0.340
	$800 < Q_oC/G < 1000$	855	0.634	820	0.539	680	0.526	560	0.455	295	0.250
	$1000 < Q_oC/G < 1350$	530	0.310	460	0.270	375	0.220	300	0.180	295	0.250
Two	$0 < Q_oC/G < 800$	910	0.507	840	0.483	740	0.443	615	0.380	455	0.305
	$800 < Q_oC/G < 1000$	910	0.507	840	0.483	740	0.443	615	0.380	365	0.188
	$1000 < Q_oC/G < 1600$	770	0.370	695	0.340	590	0.290	465	0.230	365	0.188
	$1600 < Q_oC/G < 2000$	435	0.160	375	0.140	310	0.115	240	0.090	160	0.060
Three	$0 < Q_oC/G < 800$	910	0.450	840	0.430	745	0.400	615	0.343	460	0.280
	$800 < Q_oC/G < 1000$	910	0.450	840	0.430	745	0.400	615	0.343	375	0.173
	$1000 < Q_oC/G < 1600$	775	0.317	705	0.297	605	0.260	485	0.210	375	0.173
	$1600 < Q_oC/G < 2000$	445	0.110	395	0.100	335	0.090	260	0.070	105	0.050

TABLE 4-10. LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS WITHOUT A SEPARATE LEFT-TURN PHASE OR BAY

Through Traffic On Median Lane, vph	Number of Opposing Lanes (N)	Opposing Approach Volume, vph						
		200	300	400	500	600	800	1000
100	N = 1	120	60	31	-	-	-	-
	N = 2	161	112	79	46	31	16	-
	N = 3	172	128	101	71	61	41	26
200	N = 1	90	43	21	-	-	-	-
	N = 2	125	83	57	32	21	11	-
	N = 3	134	97	74	51	43	28	18
300	N = 1	52	23	12	-	-	-	-
	N = 2	75	48	32	17	11	6	-
	N = 3	82	56	42	28	24	15	10

G/C = 0.3 C = 60 sec
Not corrected for trucks or buses

TABLE 4-11. LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS WITHOUT A SEPARATE LEFT-TURN PHASE OR BAY

Through Traffic On Median Lane, vph	Number of Opposing Lanes (N)	Opposing Approach Volume, vph						
		200	300	400	500	600	800	1000
100	N = 1	210	147	85	55	-	-	-
	N = 2	257	206	156	123	75	44	28
	N = 3	268	223	179	151	108	86	65
200	N = 1	181	123	69	44	-	-	-
	N = 2	224	177	132	102	61	35	22
	N = 3	235	193	152	126	89	70	52
300	N = 1	141	93	51	32	-	-	-
	N = 2	179	138	100	76	44	25	16
	N = 3	188	151	116	96	66	51	38
400	N = 1	92	59	31	19	-	-	-
	N = 2	120	90	63	47	27	15	10
	N = 3	127	100	75	61	41	32	23

G/C = 0.4 C = 60 sec
Not corrected for trucks or buses

TABLE 4-12. LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS WITHOUT A SEPARATE LEFT-TURN PHASE OR BAY

Through Traffic On Median Lane, vph	Number of Opposing Lanes (N)	Opposing Approach Volume, vph						
		200	300	400	500	600	800	1000
100	N = 1	300	236	173	110	79	-	-
	N = 2	352	301	251	200	167	89	57
	N = 3	363	318	273	228	200	133	111
200	N = 1	273	212	152	95	68	-	-
	N = 2	323	274	226	178	147	76	48
	N = 3	334	291	247	205	178	116	96
300	N = 1	235	180	127	77	55	-	-
	N = 2	282	237	192	149	122	62	39
	N = 3	293	252	212	173	150	95	78
400	N = 1	189	141	98	59	41	-	-
	N = 2	230	190	152	116	94	47	29
	N = 3	239	203	169	136	116	73	59
500	N = 1	134	98	66	39	27	-	-
	N = 2	167	135	106	79	63	31	19
	N = 3	174	145	118	93	79	49	39

G/C = 0.5 C = 60 sec
Not corrected for trucks or buses

TABLE 4-13. LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS WITHOUT A SEPARATE LEFT-TURN PHASE OR BAY

Through Traffic On Median Lane, vph	Number of Opposing Lanes (N)	Opposing Approach Volume, vph 200	300	400	500	600	800	1000
100	N = 1	389	325	261	198	134	70	-
	N = 2	447	396	345	294	243	175	103
	N = 3	458	413	368	322	277	220	158
200	N = 1	365	302	241	180	120	62	-
	N = 2	421	372	322	273	224	159	91
	N = 3	433	388	344	300	256	201	143
300	N = 1	330	271	213	157	104	52	-
	N = 2	385	337	289	243	197	138	78
	N = 3	395	353	310	269	228	177	123
400	N = 1	287	233	181	131	85	43	-
	N = 2	338	293	249	207	167	115	64
	N = 3	348	308	269	230	193	148	102
500	N = 1	236	188	144	103	66	33	-
	N = 2	281	241	203	166	132	90	49
	N = 3	290	254	219	186	155	117	80
600	N = 1	176	138	104	73	47	23	-
	N = 2	213	180	150	121	95	64	35
	N = 3	221	191	163	137	112	84	56

G/C = 0.6 C = 60 sec Not corrected for trucks or buses

TABLE 4-14. LEFT-TURN CAPACITY FOR SIGNALIZED INTERSECTIONS WITHOUT A SEPARATE LEFT-TURN PHASE OR BAY

Through Traffic On Median Lane, vph	Number of Opposing Lanes (N)	Opposing Approach Volume, vph						
		200	300	400	500	600	800	1000
100	N = 1	478	414	350	286	222	129	-
	N = 2	541	490	439	388	337	254	149
	N = 3	552	507	462	417	371	299	205
200	N = 1	456	393	330	268	206	117	-
	N = 2	518	468	418	368	318	236	137
	N = 3	530	485	440	396	351	280	190
300	N = 1	424	363	303	244	186	104	-
	N = 2	485	436	387	339	291	214	121
	N = 3	496	452	409	366	323	255	171
400	N = 1	384	326	270	215	162	89	-
	N = 2	442	395	349	304	259	188	105
	N = 3	453	411	370	329	289	226	149
500	N = 1	337	283	232	183	136	74	-
	N = 2	391	347	304	262	222	159	87
	N = 3	401	361	323	285	249	192	125
600	N = 1	282	234	190	148	109	59	-
	N = 2	330	291	253	216	181	128	69
	N = 3	339	304	269	236	204	156	100

G/C = 0.7 C = 60 sec Not corrected for trucks or buses

DISCUSSION

The left-turn capacity based on different methods under conditions of a single opposing flow, cycle length of 60 seconds, and equal cycle split are compared in Table 4-15 and Table 4-16. Table 4-16 shows that the left-turn capacity obtained from the TEXAS Model is the smallest while the Australian Road Capacity Guide method predicts the largest capacity among the methods examined. Webster and Fambro's method are not reasonable as far as the physical meaning of models is concerned. If corrections are made on Fambro's method, the results will be close to that of TEXAS Model. The Highway Capacity Manual method on the average predicts left-turn capacity about 20 percent larger than that of the TEXAS Model. This is probably because the HCM method recognizes the linear relation between the left-turn capacity and the opposing volume, but unfortunately assumes the equivalency factor between opposing and left-turning vehicles is to be 1.0. It can be seen that the recommended method based on the TEXAS Model, though conservative, is a comprehensive approach for estimating left-turn capacity.

TABLE 4-15. COMPARISONS AMONG LEFT-TURN CAPACITIES BASED ON VARIOUS METHODS

Opposing Volume, vph	T_A/C	Method of Computing Left-Turn Capacity, vph							
		HCM	ARGC	Webster	Fambro	Michalopoulos	Modified I	Modified II	TEXAS Model
200	0.419	400	486	481	503	383	419	274	317
300	0.379	300	438	383	415	310	315	250	252
400	0.333	200	390	293	333	241	230	196	183
500	0.285	120	357	215	260	177	163	142	120
600	0.218	120	324	137	181	120	103	88	80

* K values are the same as the TEXAS Model

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks

TABLE 4-16. RATIOS OF LEFT-TURN CAPACITY FROM THE TEXAS MODEL AND THAT ESTIMATED FROM OTHER METHODS

Opposing Volume (vph)	Ratio of Left-Turn Capacities						
	HCM	ARGC	Webster	Fambro	Michalopoulos	Modified I	Modified II
200	1.26	1.53	1.52	1.59	1.21	1.32	0.86
300	1.19	1.74	1.52	1.65	1.23	1.25	1.00
400	1.09	2.13	1.60	1.82	1.32	1.26	1.07
500	1.00	2.95	1.78	2.15	1.46	1.35	1.18
600	1.50	4.05	1.71	2.26	1.50	1.29	1.10
Average	1.21	2.48	1.63	1.89	1.34	1.29	1.04

G/C = 0.5 C = 60 sec
Single opposing flow
Not corrected for trucks

CHAPTER 5. LEFT-TURN PERFORMANCE MEASURES

In order to develop warrants for the use of left-turn bays or separate left-turn phases, appropriate quantitative performance measures must be selected. Guidelines for use of chosen control measures would state that bays or phases should be used when performance deteriorates to a specified level. Therefore, left-turn performance must be quantified in some measurable, meaningful fashion.

Theoretically, left-turn operations at a signalized intersection can be treated as a priority queue problem [Ref 26]. The left-turn demand and the left-turn capacity correspond to the arrival rate and service rate, respectively. Knowing the left-turn demand and capacity is not sufficient to determine the left-turn performance in this queueing system. In addition to the arrival and service processes, the left-turn performance strongly depends on how the queue was created. For such a complicated queueing situation, defining how the queue builds up is too difficult a problem to tackle analytically. Hence, simulation models are deemed appropriate for this study. Webster [Ref 16] has deduced a formula for computing the average through delay at signalized intersections from results of a simulation model while leaving the left-turn delay unmentioned. Agent and Deen [Ref 27] explored the relation between the average left-turn delay observed in field studies in Kentucky and the product of the opposing and the left-turn volumes. This relation should probably be used cautiously because of limited data upon which it is based. Furthermore, how the cycle split and cycle length affect the left-turn delay were not addressed.

In this chapter, left-turn performance under various traffic conditions and geometric configurations is described by using the TEXAS Model. Left-turn performance at signalized intersections having an adequate length of turn bay will be evaluated using selected performance measures. The effects of the cycle length, cycle split, and absence of a bay on vehicular delay will be addressed. All these results provide the basis for developing left-turn warrants in the next chapter. In all cases, a 50 percent cycle split is assumed and eight independent replications are employed to find the average value of each performance measure. These average values represent the average condition in the long run if the system is stable. Variations in each performance measure will be also assessed in order to reflect the fluctuations that occur in the real world. In addition to the simulation results, explicit relations among performance measures and traffic variables will be derived.

PERFORMANCE MEASURES

Saaty [Ref 28] and Feller [Ref 29] indicate that there are eight measures of effectiveness usually applied to queue problems, and most of these are some form of delay. In traffic engineering there are various definitions of delay such as total delay, stopped delay, and queueing delay. As far as signalized intersections are concerned, it is perhaps most appropriate to use queueing delay. Queueing delay is defined as the time duration from when a vehicle joins a queue until it crosses the stop line, and includes stop time and move-up time while in the queue. Hereafter, unless otherwise specified, the term delay refers to queueing delay. Among the eight performance measures mentioned previously, those pertinent to this study may be as follows:

(1) Average delay. The average delay is the sum of each driver's delay divided by the total number of drivers. The average value of delay is usually used in both practice and theory for evaluating a queueing system. The average delay represents the delay for an average driver under an average condition.

(2) Ninety-percentile delay. The delay that ninety percent of drivers will incur less than is called ninety-percentile delay. The ninety-percentile delay will reveal dispersion of delay among drivers.

(3) Percentage of drivers incurring excessive delay. At signalized intersections, drivers waiting more than two cycles are likely to become impatient and may attempt to make hazardous maneuvers. Hence, the percentage of drivers delayed more than two cycles can be a concern for setting warrants.

(4) Average queue length. The average value of the queue length during selected time interval is defined as average queue length. In contrast to average delay in the time domain, average queue length is a performance measure in the space domain.

(5) Degree of saturation. The degree of saturation, equivalent to the terminology of traffic intensity in queueing theory, is defined as the ratio of demand to capacity. It can be used as an indicator of the level of service.

In the following sections, the left-turn performance evaluated by the above measures using the TEXAS Model will be individually presented and discussed.

AVERAGE LEFT-TURN DELAY

The delay that an individual left turner will incur at a signalized intersection may vary from zero to a large value depending on the traffic conditions encountered. Typical histograms of left-turn delay are shown in Fig 5-1 and Fig 5-2. The coefficient of variation, defined as the ratio of the standard deviation and the average value, is usually used to compare the dispersions of different distributions. Figure 5-3 shows that the coefficient of variation slightly decreases as the average left-turn delay increases. Overall, the coefficients of variation in most cases fall between 65 percent and 90 percent with an average of 75 percent. This average value

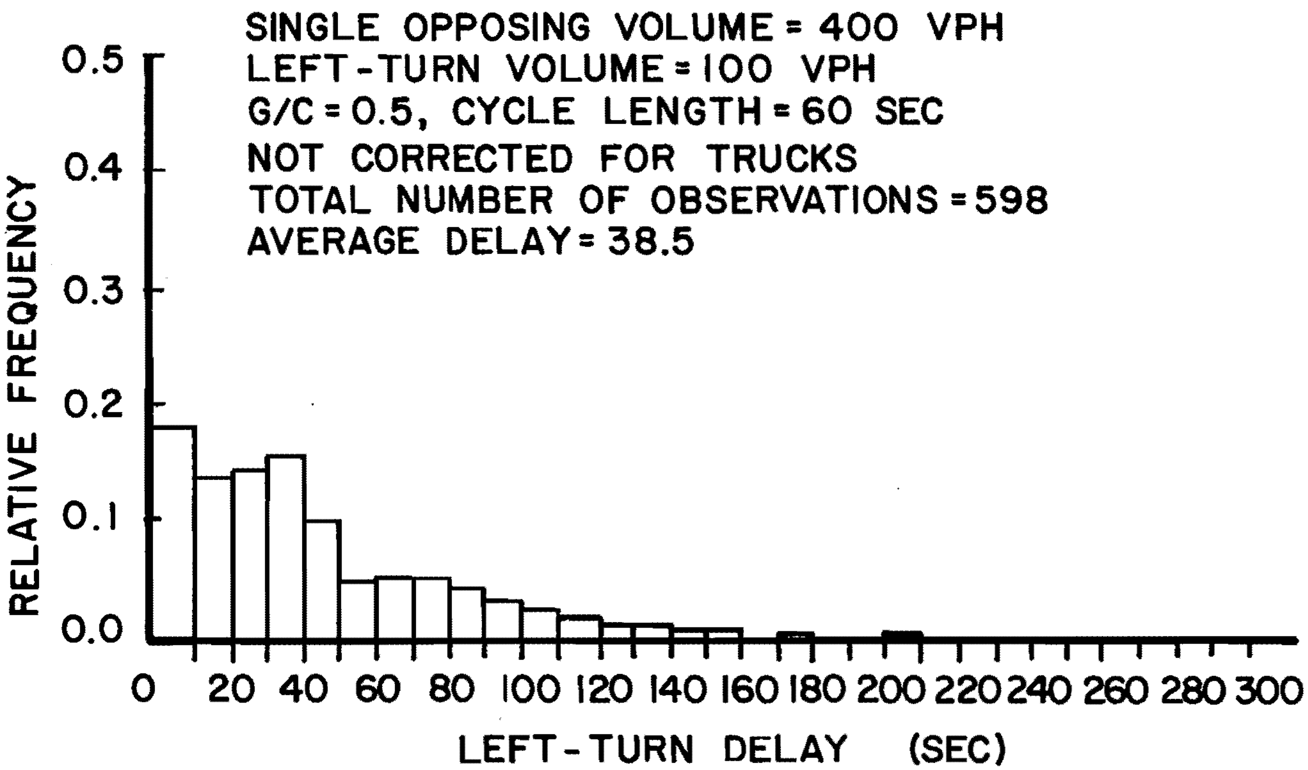


Fig 5-1. Histogram of the left-turn delay for low left-turn demand.

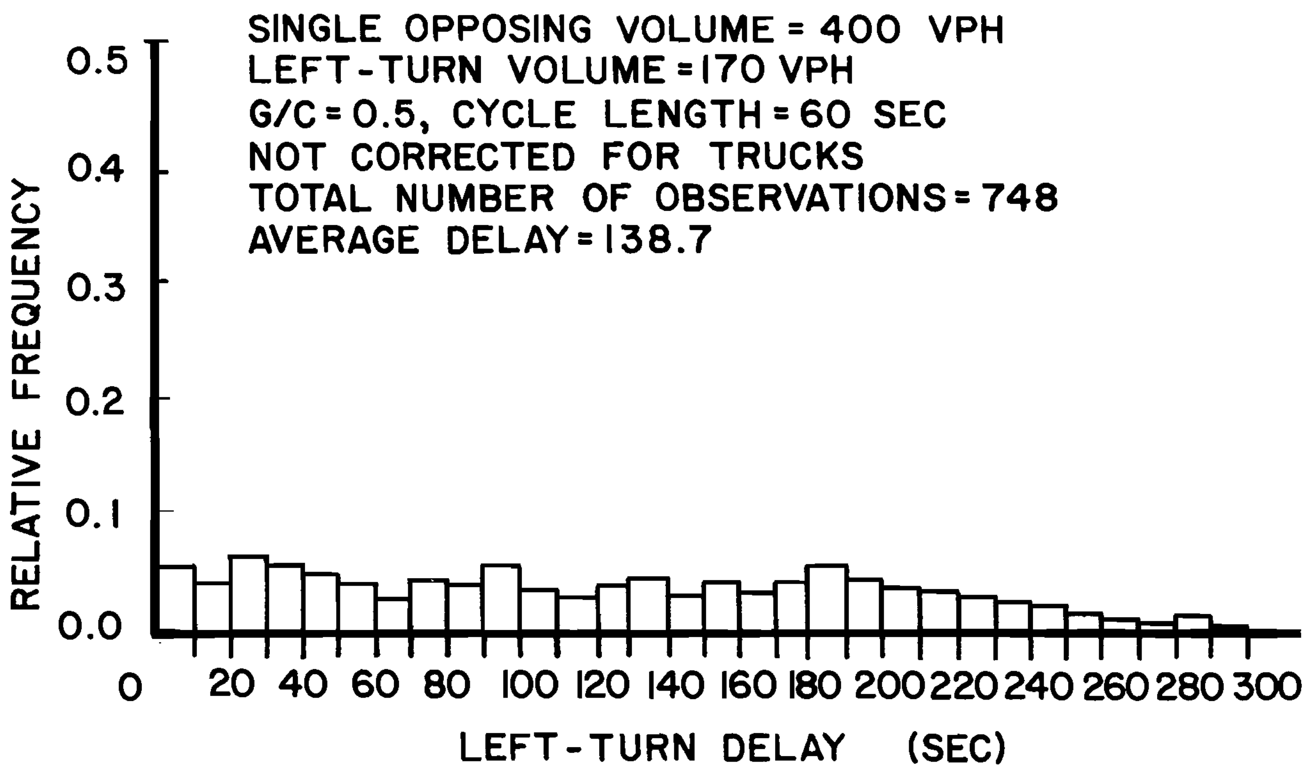


Fig 5-2. Histogram of left-turn delay for high left-turn demand.

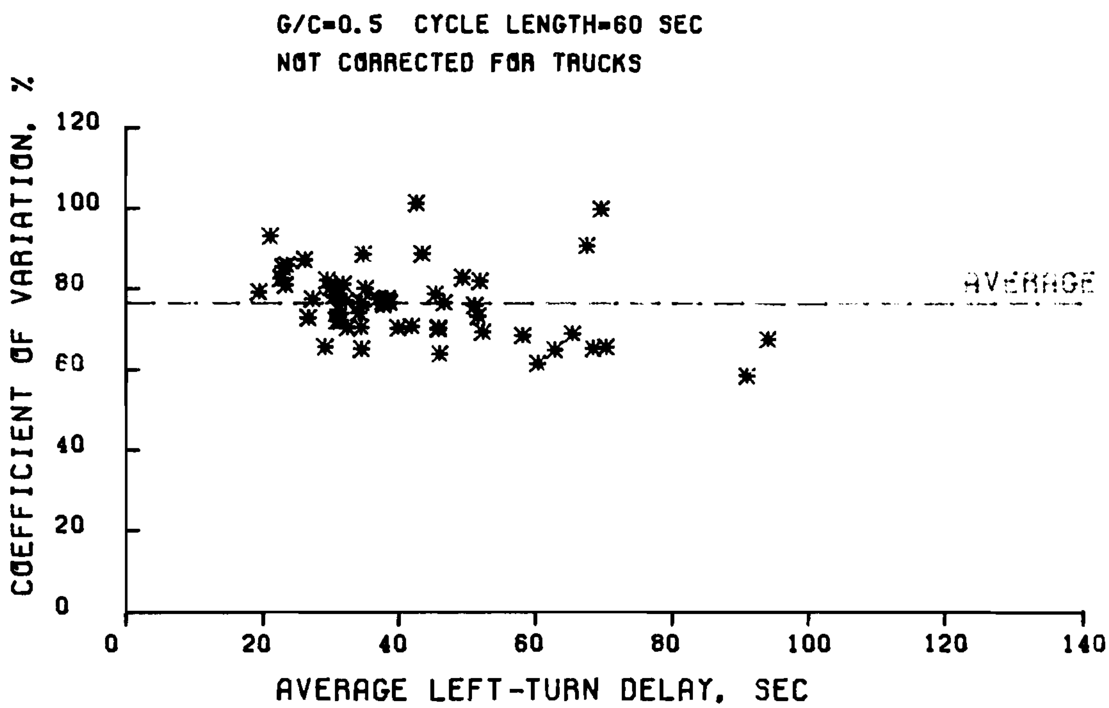


Fig 5-3. Relation between the average left-turn delay and the coefficient of variation.

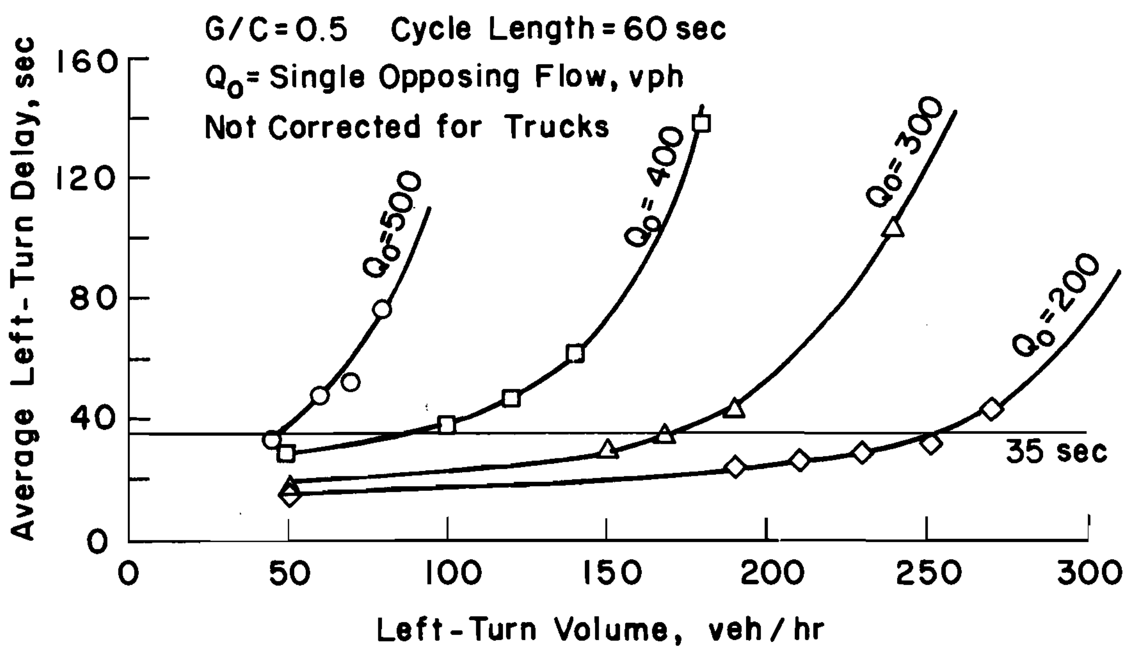


Fig 5-4. The average left-turn delay under various traffic conditions at two-by-two signalized intersections with adequate bay length.

coincides with that observed by Webster [Ref 16] at several sites in London. Agent and Deen [Ref 27] found only a slightly different value of coefficient of variation from field data. Hence, the dispersion of left-turn delay can be estimated if the average left-turn delay is known, so it becomes desirable to know the average left-turn delay under a given traffic condition.

For the same traffic conditions, the average left-turn delay may be different from observation to observation because of randomness in traffic conditions and driver behavior. With such variable data, it seems difficult to find any relations among the average left-turn delay, the left-turn demand, and the opposing volume. However, according to the Central Limit theorem, the average left-turn delays of independent replications will scatter around a mean value and follow a sample mean distribution which is approximately normal. The mean of this sampling distribution will represent the average conditions in the long run and may exhibit some relations among the traffic variables. The average left-turn delays at signalized intersections having adequate length of bay under various traffic conditions and geometric configurations are shown in Figs 5-4 through 5-6 for eight replications. It can bė seen that the average left-turn delay increases sharply as the left-turn demand reaches some critical point.

A major disadvantage inherent to the use of a simulation model is that it does not report explicit relations among variables. In order to have a clear idea about how the average left-turn delay is related to the traffic variables, a simple mathematical model for left-turn operations at signalized intersection with adequate length of bay will be developed. For clarity, the following definitions are given:

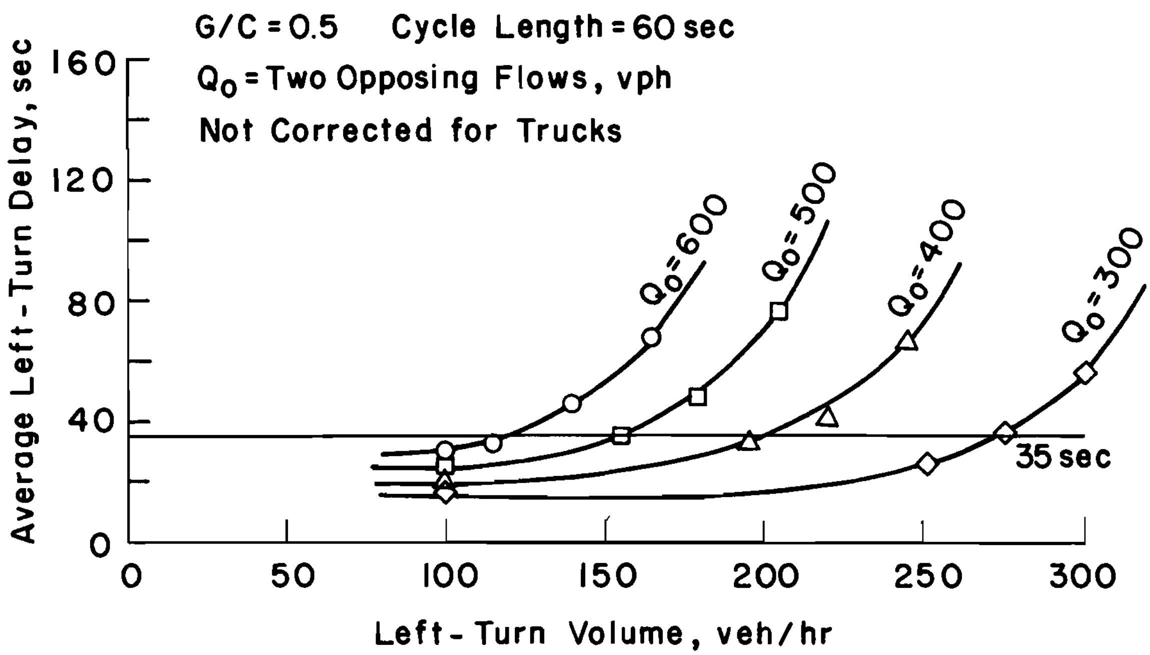


Fig 5-5. The average left-turn delay under various traffic conditions at four-by-four signalized intersections with adequate bay length.

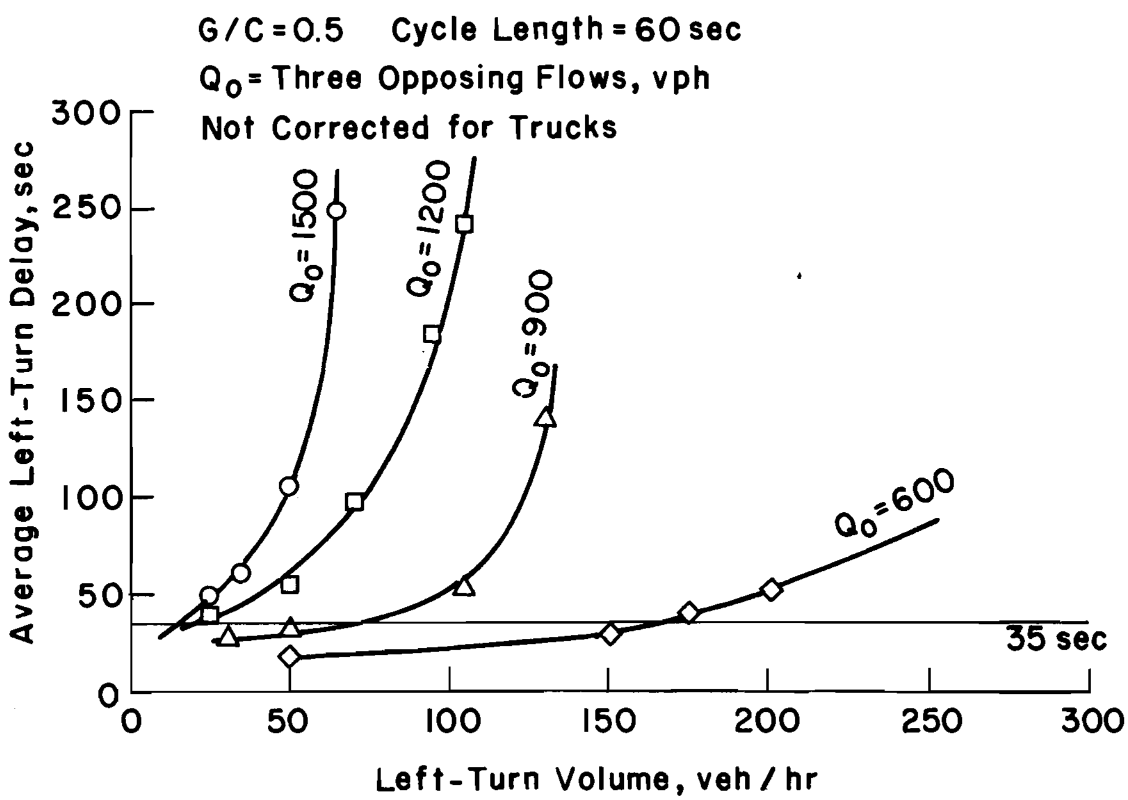


Fig 5-6. The average left-turn delay under various traffic conditions at six-by-six signalized intersections with adequate bay length.

V_L = left-turn demand, vph;

Q_L = left-turn capacity, vph;

λ = arrival rate of left-turn traffic, $V_L/3600$ = veh/sec;

ρ = degree of left-turn saturation = V_L/Q_L;

T = transparency as computed from Eq 4-41;

μ = average left-turn processing rate during unblocked period = $Q_L/(3600T) = \lambda/(\ T)$;

C = cycle length, sec;

$\bar{z}$ = average residual left-turn queue at the end of green phase, veh; and

$\bar{D}$ = average left-turn delay, sec.

At signalized intersections with adequate length of bay, in an average cycle left-turn vehicles will queue up at the rate, λ, during the blocked period of length (1-T)C. This queue will be discharged at the rate, $\mu - \lambda$, during the unblocked period, TC. Unless the left-turn demand is far less than the left-turn capacity, due to randomness there will be an average residual queue at the end of the green phase no matter how small it is (see Fig 5-7). Since the total area under the queue diagram is the total left-turn delay, the average left-turn delay is the total area divided by the number of left turners in each cycle. The average left-turn delay can be deduced as follows.

Referring to Fig 5-7, let the time for discharging the left-turn queue be t_Q:

$$t_Q = \frac{\lambda(1-T)C}{\mu - \lambda} \qquad (5\text{-}1)$$

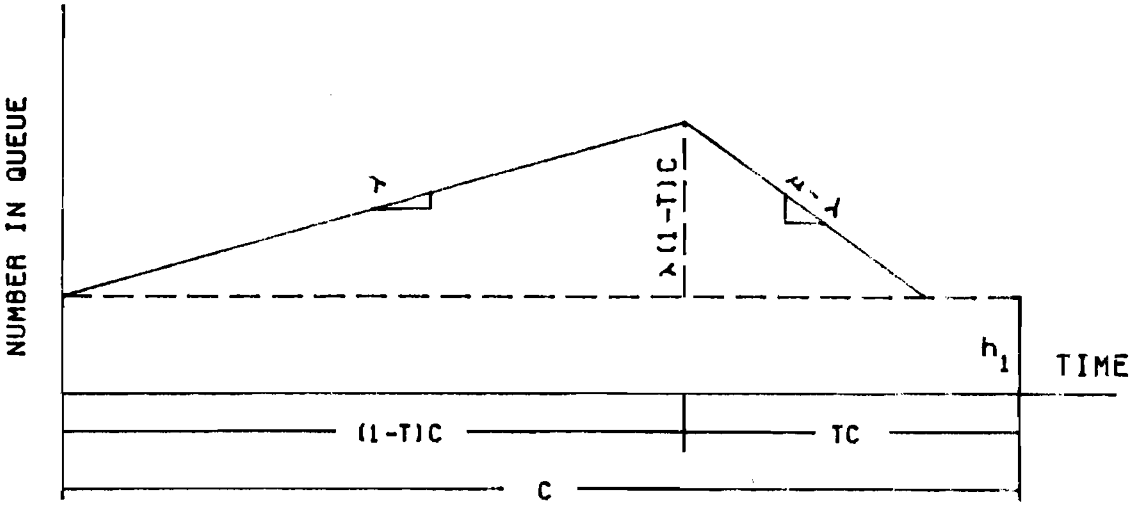


Fig 5-7. The queue diagram of left-turn operations.

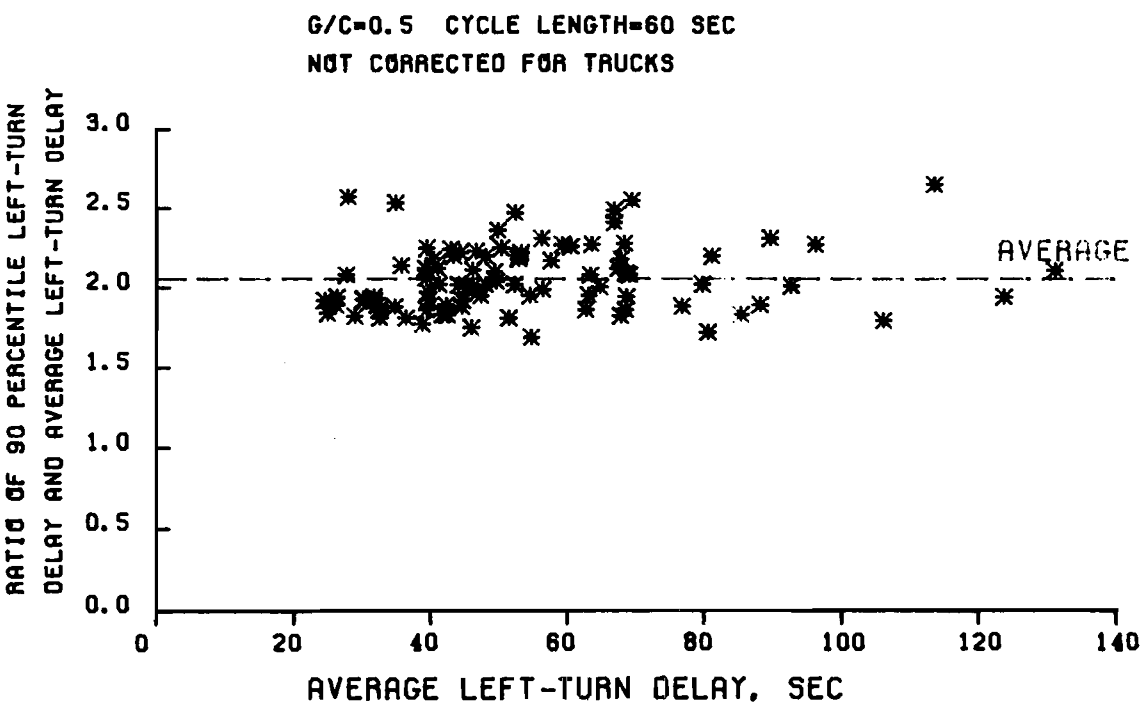


Fig 5-8. The relation between the average and 90 percentile values of left-turn delay.

Let D_c be the total left-turn delay per cycle; it follows that

$$D_c = \text{triangular area} + \text{rectangular area}$$

$$= 0.5\lambda(1-T)C[(1-T)C+t_Q] + \bar{z}C$$

$$= 0.5\lambda(1-T)C[(1-T)C+\frac{\lambda(1-T)C}{\mu-\lambda}] + \bar{z}C$$

$$= 0.5\lambda(1-T)^2C^2(1-\rho T)^{-1} + \bar{z}C$$

$$\bar{D} = \frac{\text{total left-turn delay per cycle}}{\text{number of left-turns per cycle}}$$

$$\bar{D} = \frac{0.5\lambda(1-T)^2C^2(1-\rho T)^{-1} + \bar{z}C}{\lambda C}$$

$$= \frac{0.5(1-T)^2C}{1-\rho T} + \frac{\bar{z}}{\lambda} \tag{5-2}$$

Equation 5-2 was tested against results from the TEXAS Model and was found to hold satisfactorily for a wide range of traffic conditions (see Table 5-1). The residual queue length $\bar{z}$ depends on the stochastic behavior of the left-turn and opposing traffic. Although, this value perhaps could be obtained from a mathematical model, it is beyond the scope of this chapter. Nevertheless, Eq 5-2 provides a functional form of left-turn operations.

TABLE 5-1. COMPARISONS BETWEEN THE AVERAGE LEFT-TURN DELAY FROM THE TEXAS MODEL AND THAT PREDICTED BY EQUATION 5-2

Opposing Volume, vph	Left-Turn Volume, vph	Degree of Left-Turn Saturation	Transparency T	Cycle Length, sec	Average Residual Queue (z)	Average Left-Turn Delay		Absolute Deviation	
						Equation 5-2, sec	TEXAS Model, sec	Sec	%
200	50	0.1577	0.383	60	0.0367	14.7	15.7	1.0	6.4
200	190	0.5990	0.383	60	0.5375	25.0	24.5	0.5	2.0
200	230	0.7255	0.383	60	0.7800	28.0	29.0	1.0	3.4
200	250	0.7886	0.383	60	0.9525	30.1	32.0	1.9	5.9
200	250	0.7886	0.383	90	0.6780	34.3	35.0	0.7	2.0
200	270	0.852	0.383	60	2.5000	50.3	52.7	2.4	4.6
200	300	0.9460	0.383	60	4.3730	70.4	72.7	2.3	3.2
300	50	0.1980	0.307	60	0.0575	19.0	17.8	1.2	6.7
300	150	0.5950	0.307	60	0.5100	29.3	30.0	0.7	2.3
300	170	0.674	0.307	60	0.7730	34.5	34.0	0.5	1.5
300	190	0.7540	0.307	60	1.2600	42.7	43.0	0.3	0.7
300	190	0.7540	0.307	90	0.9090	45.4	45.0	0.4	0.9
300	240	0.9520	0.307	60	5.4250	101.2	103.0	1.8	1.7
400	50	0.273	0.221	60	0.1600	30.9	28.6	1.4	4.9
400	100	0.5460	0.221	60	0.5600	40.9	38.6	2.3	6.0
400	120	0.6560	0.221	60	0.9430	49.6	47.0	2.6	5.5
400	120	0.6560	0.221	90	0.6680	52.0	47.6	4.4	9.2
400	140	0.7650	0.221	60	1.5600	62.0	61.0	1.0	1.6
400	140	0.7650	0.221	90	1.0500	59.8	60.5	0.7	1.2
400	140	0.7650	0.221	120	0.7000	61.9	60.2	1.7	2.8
400	170	0.9289	0.221	60	5.2380	134.0	137.8	3.8	2.8
500	40	0.3300	0.152	60	0.1625	37.3	33.5	3.8	11.3
500	60	0.4960	0.152	60	0.4400	49.7	47.5	2.2	4.6
500	70	0.5790	0.152	60	0.4900	48.8	50.0	1.2	2.4
500	80	0.6610	0.152	60	1.1580	76.1	76.5	0.4	0.5
500	80	0.6610	0.152	90	0.9280	77.8	74.0	4.8	6.5
Average								1.7	3.9

G/C = 0.5 Single opposing flow Not corrected for trucks

NINETY PERCENTILE LEFT-TURN DELAY

It has been noticed that histograms of left-turn delay are not bell-shaped. Thus, the percentage of left turners incurring delay within a specific range can not be easily determined from the average and the standard deviation of left-turn delay. In order to reveal the skewness of the delay histogram, the ninety percentile delay is needed. Figure 5-8 shows comparisons between the ninety percentile and the average values of left-turn delay. It was found that the ninety percentile left-turn delay is about 1.5 to 2.5 times the average. Generally, the ninety percentile left-turn delay is approximately twice as large as the average. Using field measurements, Agent and Deen [Ref 27] also reached the same conclusion. Ninety percentile left-turn delays under various traffic conditions and geometric configurations are shown in Figs 5-9 through 5-11.

PERCENTAGE OF LEFT-TURNERS BEING DELAYED MORE THAN TWO CYCLES

Because of wide variations in left-turn delay, some left turners may incur delay much larger than the average value. At signalized intersections drivers waiting more than two cycles will become impatient, and some may try to make hazardous maneuvers. The average percentages of left turners being delayed more than two cycles under various traffic conditions are shown in Figs 5-12 through 5-14.

AVERAGE LEFT-TURN QUEUE LENGTH

In queueing theory, the two types of average queue lengths which are usually discussed include the average queue length observed by an outsider and that observed by customers arriving in the system. In general, these two types of average queue length are different. However, for a queueing system having a single server and a negative-exponential distribution of arrival and

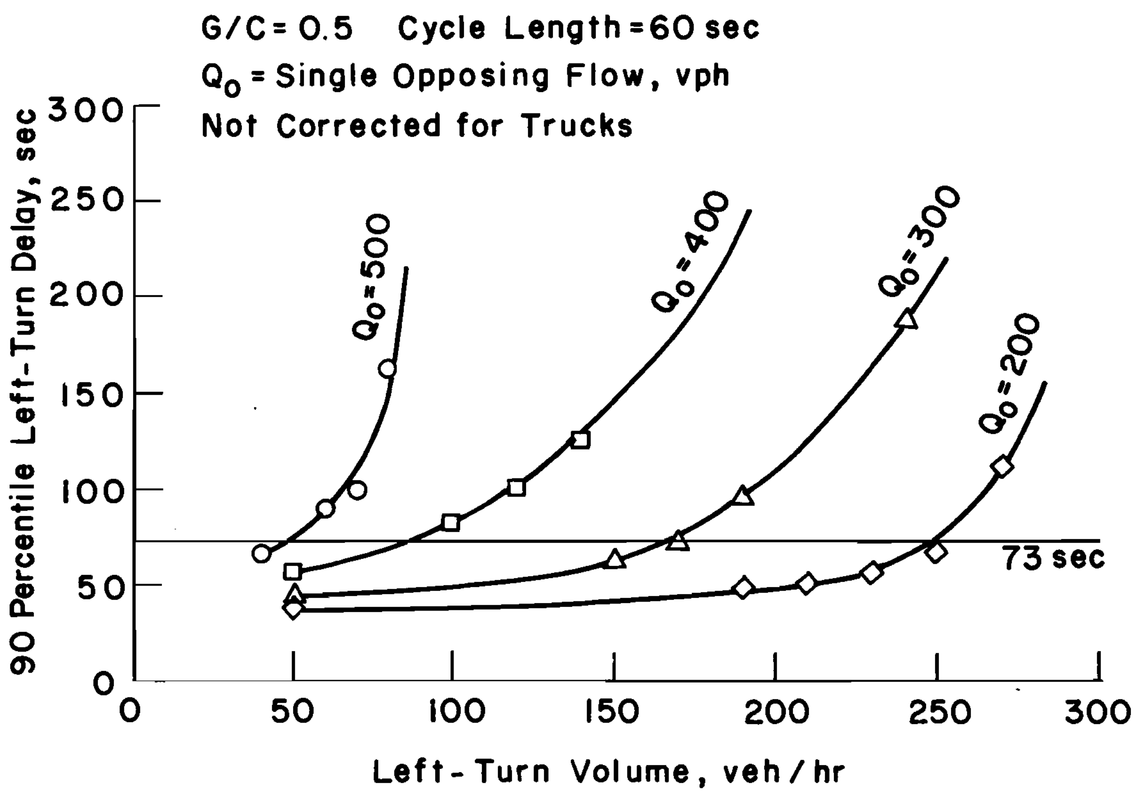


Fig 5-9. The ninety percentile left-turn delay under various traffic conditions at two-by-two signalized intersections with adequate length of bay.

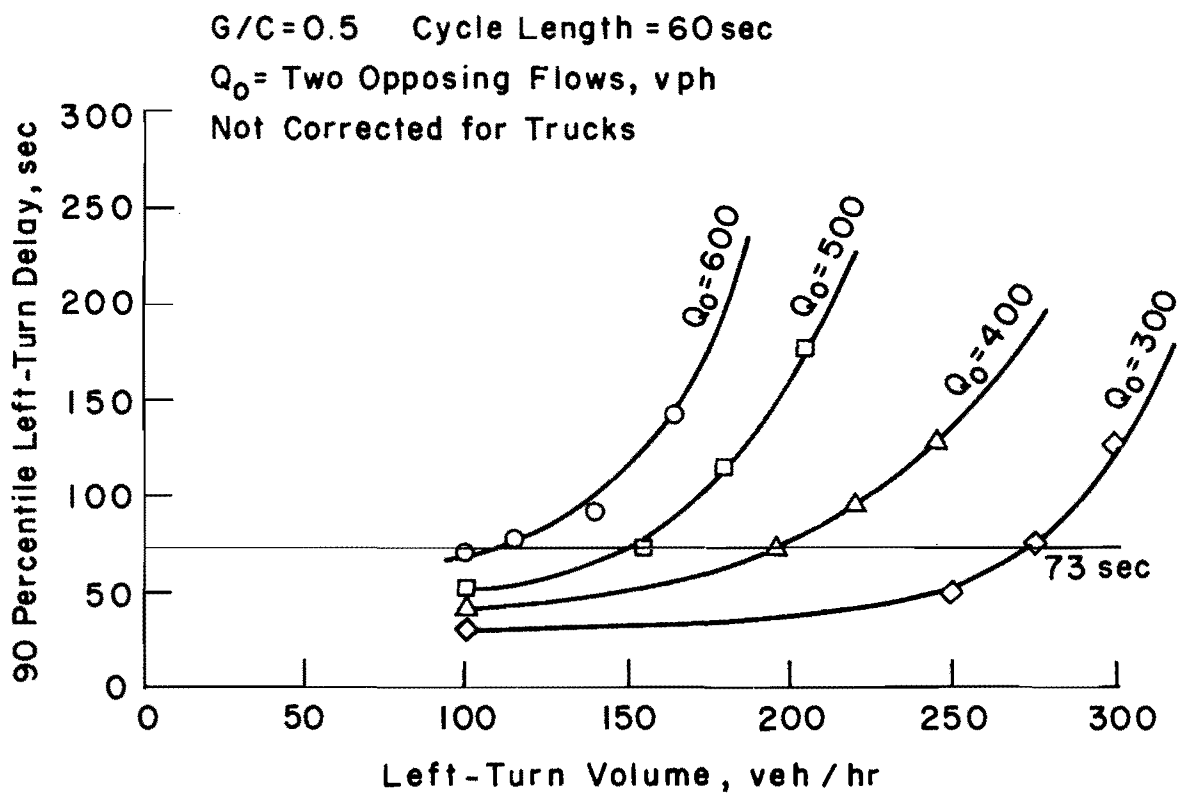


Fig 5-10. The ninety percentile left-turn delay under various traffic conditions at four-by-four signalized intersections with adequate length of bay.

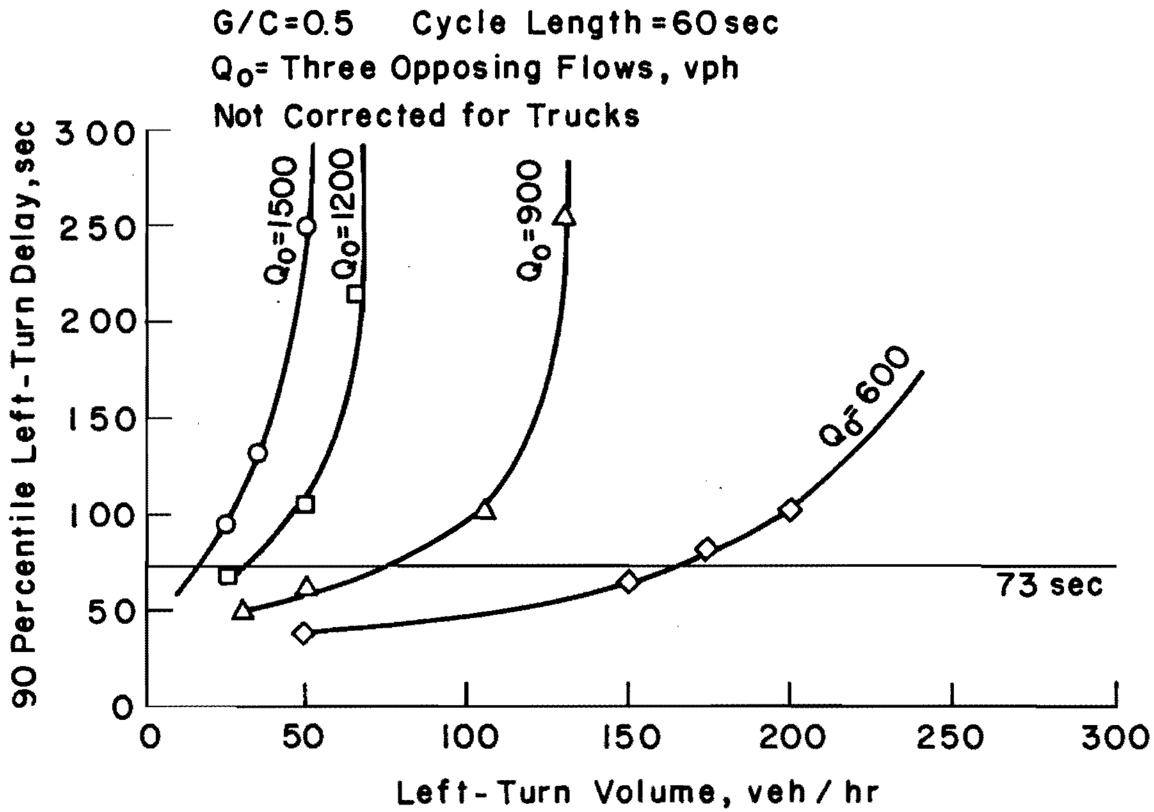


Fig 5-11. The ninety percentile left-turn delay under various traffic conditions at six-by-six signalized intersections with adequate length of bay.

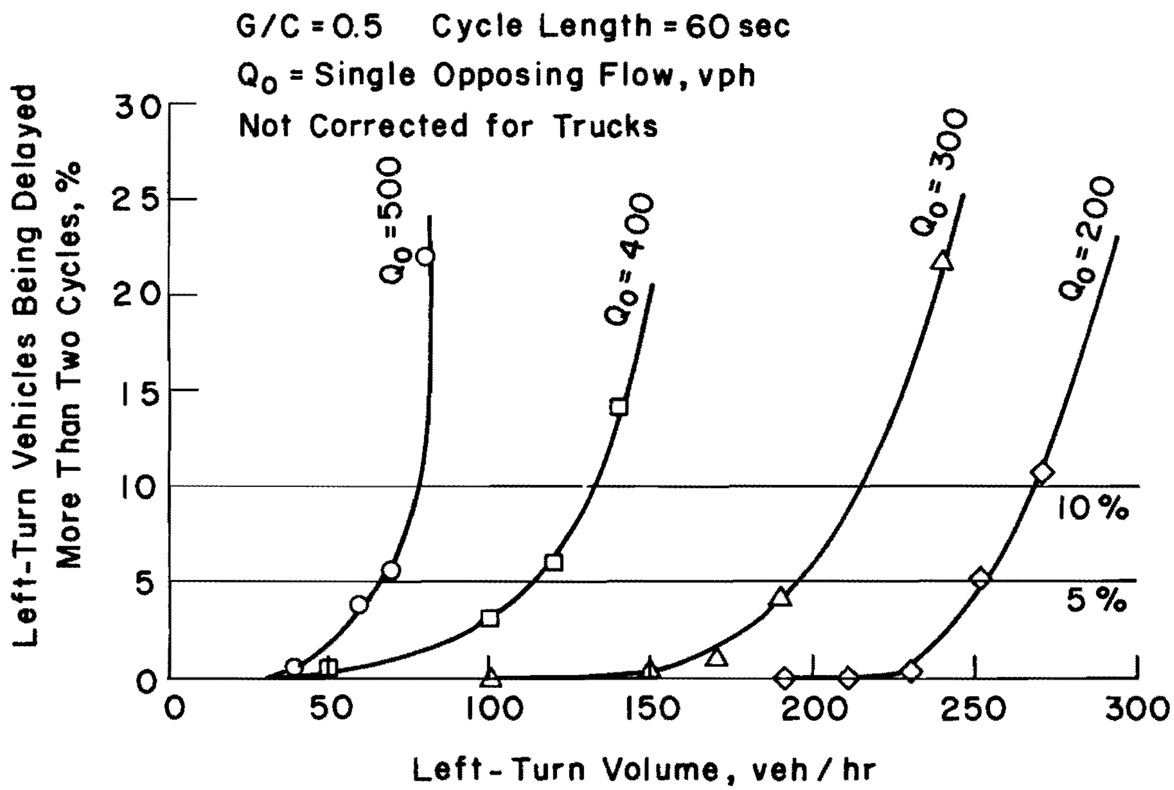


Fig 5-12. The percentage of left turners being delayed more than two cycles under various traffic conditions at two-by-two signalized intersections with adequate length of bay.

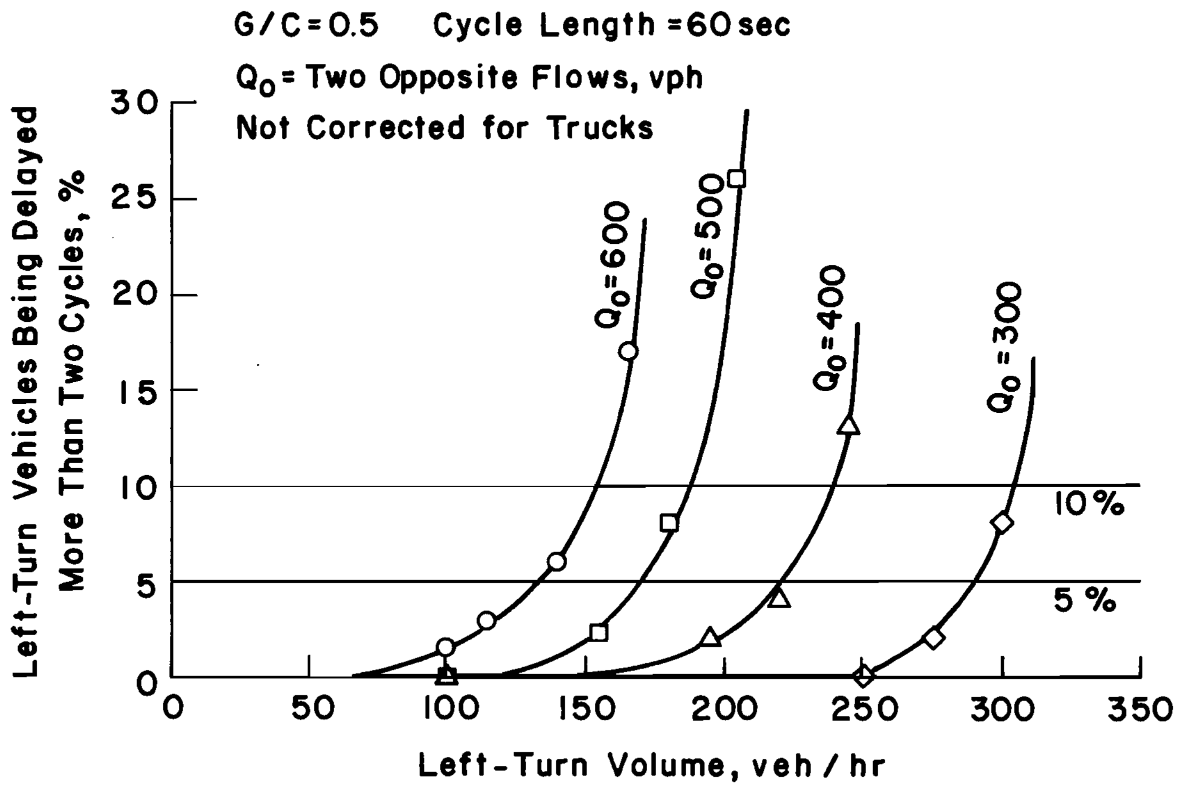


Fig 5-13. The percentage of left turners being delayed more than two cycles under various traffic conditions at four-by-four signalized intersections with adequate length of bay.

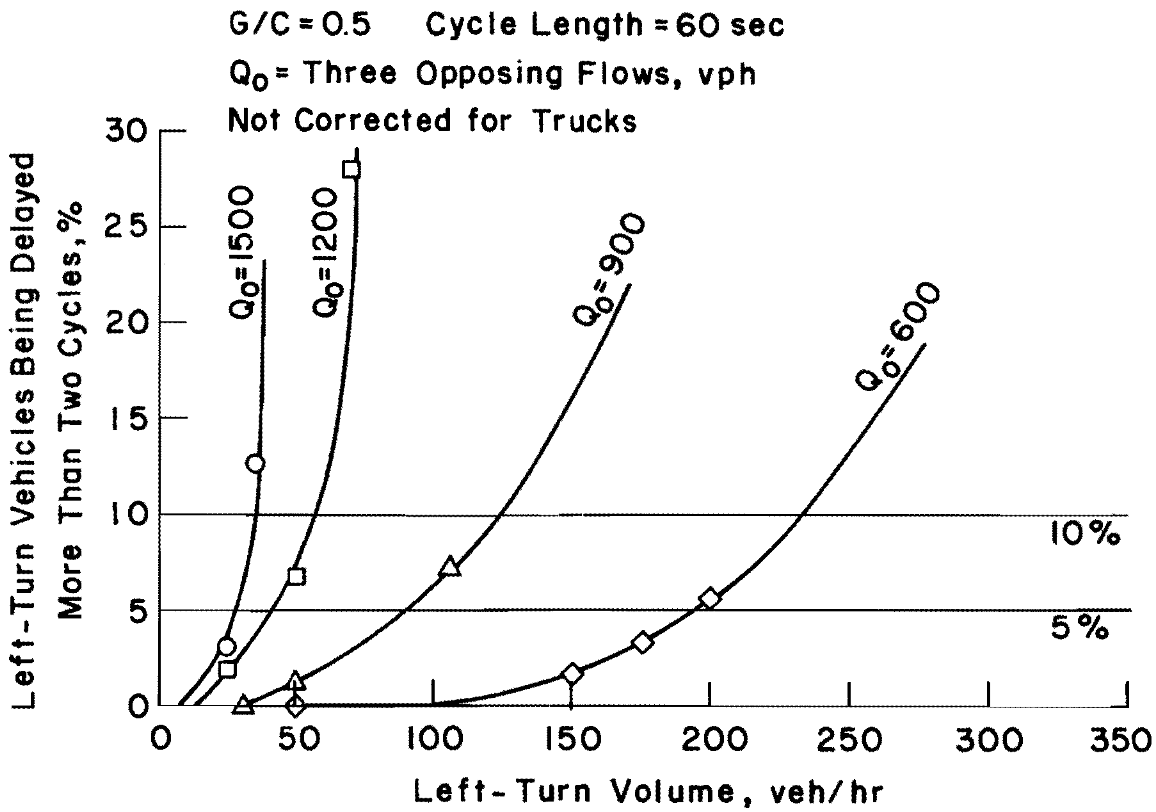


Fig 5-14. The percentage of left turners being delayed more than two cycles under various traffic conditions at six-by-six signalized intersections with adequate length of bay.

service times (referred to as an M/M/1 model), these two types of queue length happen to be equal due to the memoriless property inherent in the negative-exponential distribution. The memoriless property in this particular case means the "remaining" waiting time for a driver is independent of how long he has already been waiting in the system. Ordinarily, unless otherwise specified, the average queue length refers to that observed by an outsider. There are two reasons for choosing the average queue length as a performance measure. First, it relates the physical size of the queue to the space available. Second, the average queue length is germane to the average delay, as will be shown later. Once the average queue length is known, the average delay can be determined, and vice versa. Sometimes it is easier in the field to observe the queue length than the delay. In this section, relations between the average left-turn delay and the average left-turn queue length at signalized intersections with adequate length of bay will be derived and discussed.

J. D. C. Little [Ref 30] has derived a well known formula: $L=\lambda W$, where L, λ, and W are the average queue length, the average arrival rate, and the average waiting time, respectively. It has been shown [Ref 31] that this formula holds under very general conditions. As mentioned earlier, the left-turn operation at signalized intersections is basically a general type of queueing problem. If $\bar{L}$, $\bar{D}$, and V_L are defined as the average left-turn queue length, the average left-turn delay, and the left-turn volume, respectively, then the following equivalencies are conceptually justified:

$$\bar{L} = L \qquad (5\text{-}3)$$

$$\bar{D} = W \qquad (5\text{-}4)$$

$$V_L/3600 = \lambda \qquad (5\text{-}5)$$

According to Little's formula:

$$\overline{L} = \lambda\overline{D} = V_L\overline{D}/3600 \qquad (5\text{-}6)$$

or

$$\overline{D} = 3600\overline{L}/V_L \qquad (5\text{-}7)$$

Equations 5-6 and 5-7 show how the average left-turn delay is related to the average left-turn queue length and the left-turn demand. It is not surprising to find that simulation results from the TEXAS Model verify Little's formula. As a matter of fact, it is very difficult to find a counter example for Little's formula. The relation between the average queue length and the maximum queue length is shown in Fig 5-15. With this relation the bay length required for a critical traffic condition can be determined.

The average left-turn queue length is obtained by recording the variations in queue length every second. This is very time consuming. An easier way to obtain the average queue length is to measure the left-turn queue length as each left turner joins the queue. In other words, this is the average queue length observed by left turners when joining the queue. The definition of when queue joining occurs may affect the queueing delay and queue length. One reasonable definition might be that a vehicle becomes part of a queue when it is travelling at a speed less than 3 ft/sec and is located 30 feet from the end of the queue or the stop line. A simple mathematical model for relating the average left-turn queueing delay to the average queue length observed by left turners will be explored. First, the following variable definitions will be used:

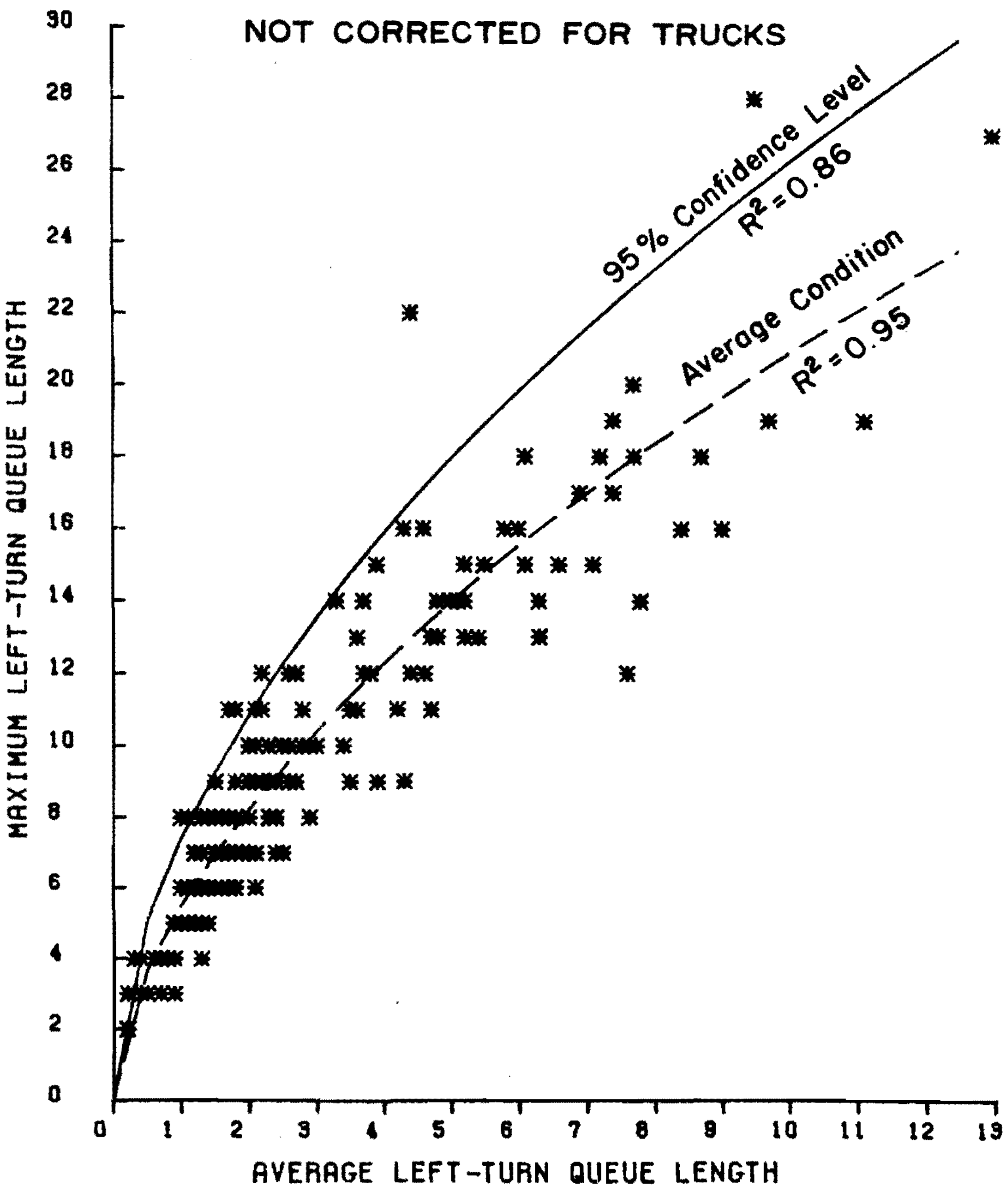


Fig 5-15. The relation between the maximum and average values of left-turn queue length.

D_i = delay of the i-th left turner, sec;

$\overline{D}$ = average left-turn delay, sec;

n_i = queue length observed by the i-th left turner when joining the queue;

$\overline{n}$ = the average queue length observed by left turners when joining the queue;

Q_L = left-turn capacity, veh/hr;

$\overline{d}$ = average left-turn time including blocked period, sec/veh = $3600/Q_L$;

N = total number of left turners recorded;

u = the average spacing among vehicles in the queue, ft;

w = average car length, ft; and

x = distance from the end of queue that a vehicle is defined as joining the queue, ft.

Suppose that there are N left turns being made over some period of time. Referring to Fig 5-16, a vehicle is said to be joining a queue when it is x feet from the end of the queue or the stop line. The average spacing among vehicles in the queue is u feet. Since the left-turn capacity is Q_L vph, on the average, the time between two successive left turns processed will be $\overline{d}$ seconds. The time $\overline{d}$ includes move-up time over the distance u+w and the waiting time at the stop line. If the left-turn time has memoriless property, then at the instant that the i-th vehicle joins the queue, the remaining waiting time for the first vehicle in the queue will still be $\overline{d}$ seconds. Let the time for the i-th vehicle to travel the distance x-u be $y\overline{d}$. By this argument, on the average, the i-th left turner who observes a queue length n_i when joining the queue will have to wait $(1+y+n_i)\overline{d}$ seconds before going through the intersection. The average left-turn delay by definition is:

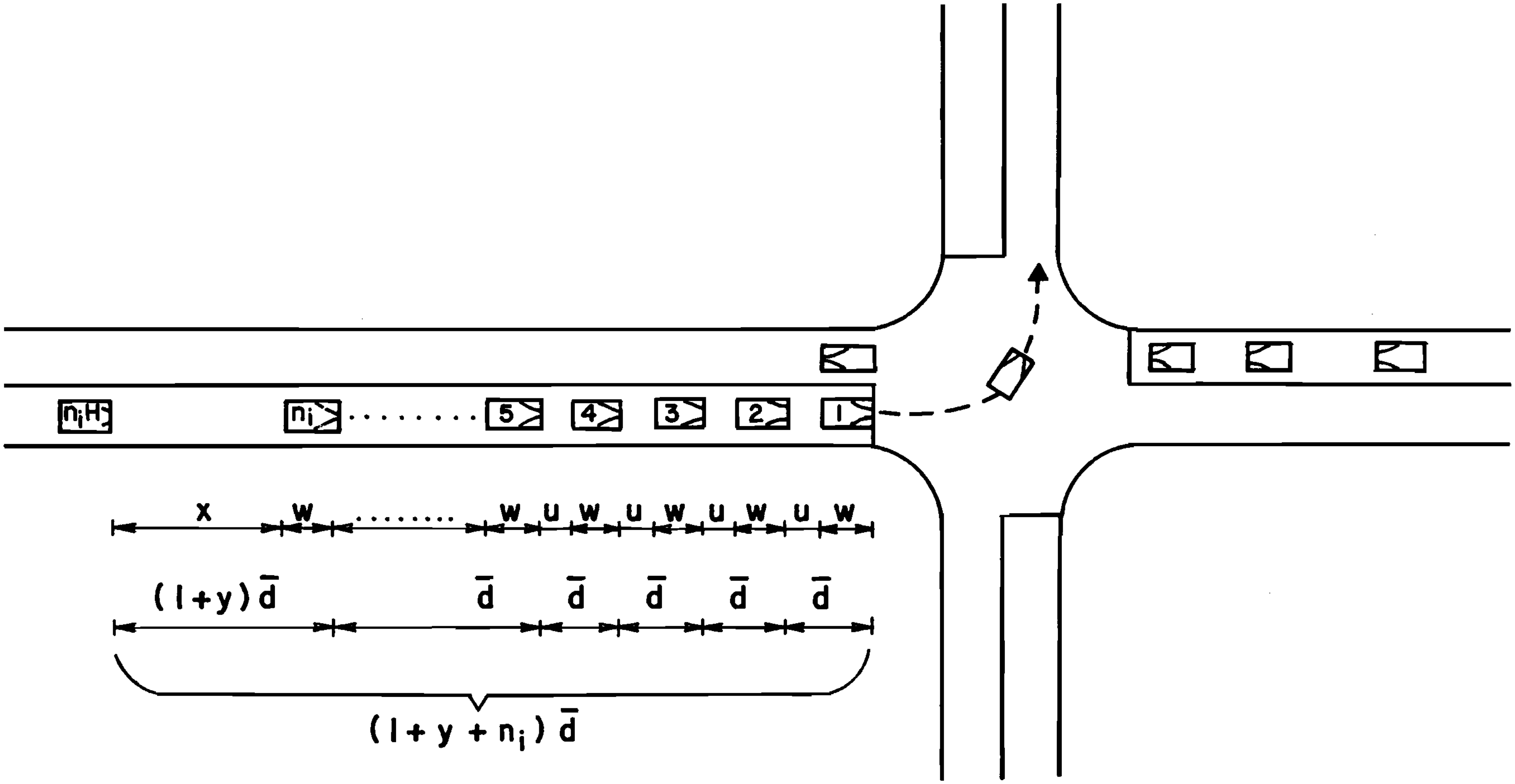


Fig 5-16. The queuing delay for the left-turner joining a queue.

$$\overline{D} = \lim_{N\to\infty} E\left[\frac{\sum_{i=1}^{N} D_i}{N}\right]$$

$$= \lim_{N\to\infty} E\left[\frac{\sum_{i=1}^{N} (1+y+n_i)\overline{d}}{N}\right]$$

$$= \overline{d} \lim_{N\to\infty}\left\{1+y+E\left[\frac{\sum_{i=1}^{N} n_i}{N}\right]\right\}$$

$$= (1+y+\overline{n})\overline{d} \tag{5-8}$$

$$= 3600(1+y+\overline{n})/Q_L \tag{5-9}$$

Equations 5-8 and 5-9 are stated appropriately for analysis of field data, since there is always a human judgment error. Nevertheless, in practice it is very unlikely that a vehicle would be assumed to have joined the queue when it is much too far from the end of the queue. The difference between x and u would likely be less than 20 feet. The travel time over the distance x-u thus is only a few seconds. This implies that y is only slightly larger than zero and can be neglected. In a simulation model, the error in judgement can be totally excluded, which means that x can be made equal to u. In this case, Eqs 5-8 and 5-9 can be simplified as follows without losing much accuracy:

$$\overline{D} = (1+\overline{n})/\overline{d} \tag{5-10}$$

$$\bar{D} = 3600(1+\bar{n})/Q_L \qquad (5\text{-}11)$$

Equation 5-10 is verified by the simulation results from the TEXAS Model as Table 5-2 shows. Equation 5-11 reveals that the left-turn capacity plays an important role in the average left-turn delay. The variable $\bar{n}$, like $\bar{L}$, depends on how the queue built up. An examination of Eqs 5-7 and 5-11 indicates clearly why the left-turn demand and the left-turn capacity are not sufficient for determining the average left-turn delay. The physical meaning and the useful application of Eq 5-11 can be demonstrated by the following example. A traffic engineer makes a left turn at some signalized intersection with adequate length of bay on the way to his office every morning during the peak hour. Assume that traffic conditions at that peak period are very much the same every day. He determines queue length in front of him when he joins the queue. Also, he records the time he has spent in the queue before crossing the stop line. These data are denoted as n_i and D_i, respectively. After N days (N must be large enough), he averages n_i and D_i and thus obtains $\bar{n}$ and $\bar{D}$. Knowing $\bar{n}$ and $\bar{D}$, he will be able to find, from Eq 5-11, the left-turn capacity associated with the traffic conditions during that period. On the other hand, if the traffic engineer has a good estimate of the left-turn capacity and adopts a criterion for the maximum average delay, say 35 seconds, he may compute the tolerable average queue length from Eq 5-11. If the average left-turn queue length observed is greater than this tolerable value, he may judge that some type of left-turn treatment is needed without having to conduct a laborious delay study or volume count.

TABLE 5-2. COMPARISONS BETWEEN THE AVERAGE LEFT-TURN DELAY FROM THE TEXAS MODEL AND THAT PREDICTED BY EQUATION (5-10)

Opposing Volume, vph	Left-Turn Volume, vph	Left-Turn Capacity, vph	Cycle Length, sec	Sample Size	$\bar{n}$, veh	$\bar{d}$, sec/veh	Average Left-Turn Delay, sec		Absolute Deviation	
							TEXAS Model	Eq 5-10	Sec	%
200	210	317	60	1265	1.211	11.36	26.0	25.1	0.9	3.5
200	230	317	60	1383	1.61	11.36	29.0	29.6	0.6	2.1
200	250	317	90	1510	2.10	11.36	35.0	35.2	0.2	0.6
200	270	317	60	1644	3.70	11.36	52.7	53.4	0.7	1.3
300	150	252	60	899	2.02	14.29	30.0	28.9	1.1	3.7
300	170	252	60	999	1.42	14.29	34.0	34.6	0.6	1.8
300	190	252	60	1135	2.04	14.29	43.0	43.5	0.5	1.2
300	190	252	90	1131	2.02	14.29	45.0	43.1	1.9	4.2
400	100	183	60	605	1.05	19.70	38.6	40.4	1.8	4.7
400	120	183	60	707	1.46	19.70	47.0	48.5	1.5	3.2
400	140	183	60	817	2.18	19.70	61.0	62.5	1.5	2.5
500	60	121	60	375	0.78	29.75	47.5	53.5	6.0	12.6
500	70	121	60	423	0.90	29.75	50.0	56.5	6.5	13.0
500	80	121	60	459	1.64	29.75	76.5	78.5	2.0	2.6
Average									1.8	4.1

G/C = 0.5
Single opposing flow
Adequate length of bay
Not corrected for trucks

Finally, the relation between $\bar{L}$ and $\bar{n}$ can be explored. Comparing Eqs 5-7 and 5-11, the following equality can be obtained:

$$\bar{D} = 3600\bar{L}/V_L = 3600(1+\bar{n})/Q_L$$

It follows that

$$\bar{L} = (1+\bar{n})V_L/Q_L$$

$$= \rho(1+\bar{n}) \qquad (5\text{-}12)$$

where ρ is the degree of left-turn saturation or equivalently the traffic intensity. For the M/M/1 model, it can be proved that $\bar{L}$ is equal to $\bar{n}$ such that Eq 5-12 will become

$$\bar{L} = \rho(1+\bar{L})$$

It follows that

$$\bar{L} = \frac{\rho}{1 - \rho} \qquad (5\text{-}13)$$

Equation 5-13 is the basic equation derived from elementary queueing theory. This demonstrates that Eqs 5-10 and 5-11 are theoretically sound.

DEGREE OF LEFT-TURN SATURATION

It has been recognized that the left-turn delay will be excessive as the left-turn demand approaches capacity. The degree of left-turn saturation is defined as the ratio of the left-turn demand to left-turn capacity and may be used as an indicator of the level of service. Figures 5-17 through 5-19 show the relations between the average left-turn delay and degree of left-turn saturation. It was found that for different opposing volumes or geometric

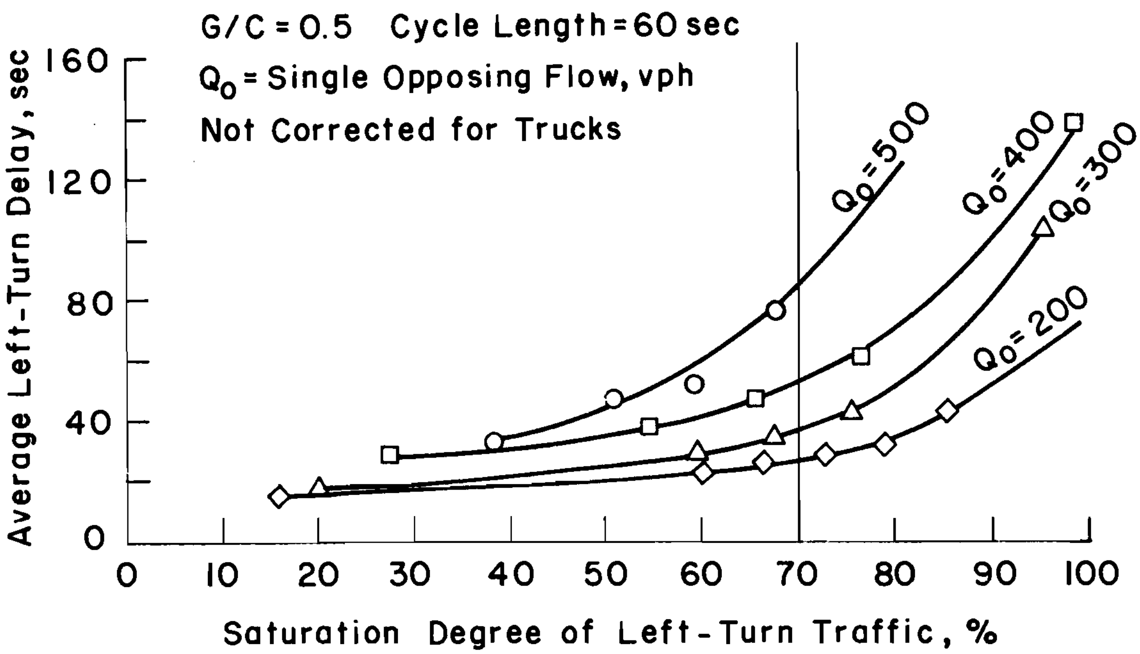


Fig 5-17. The relation between the average left-turn delay and degree of left-turn saturation at two-by-two signalized intersections.

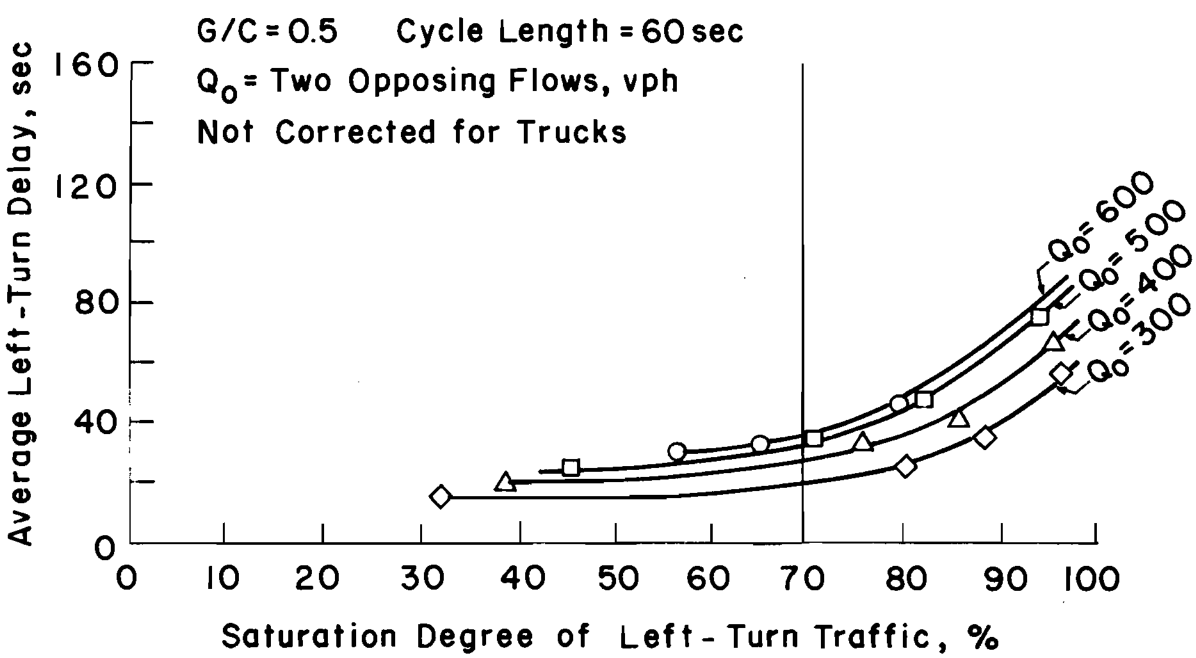


Fig 5-18. The relation between the average left-turn delay and degree of left-turn saturation at four-by-four signalized intersections.

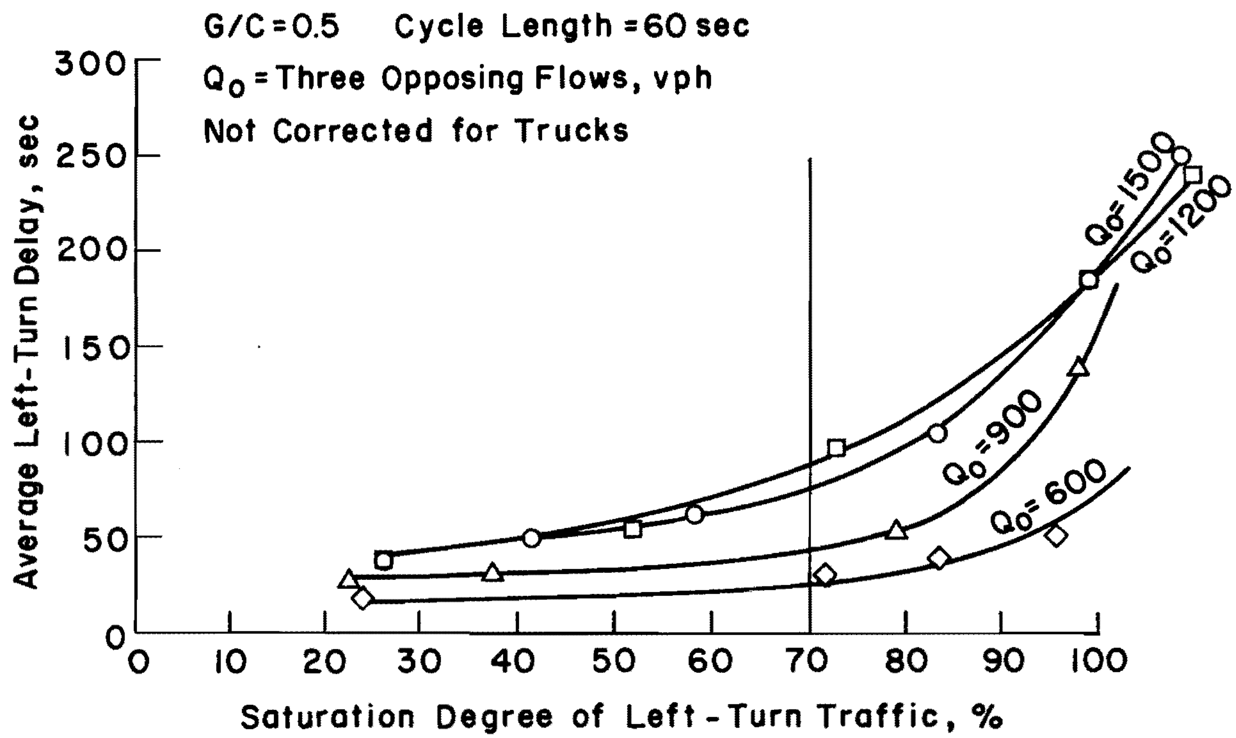


Fig 5-19. The relation between the average left-turn delay and degree of left-turn saturation at six-by-six signalized intersections.

configurations, even if the degree of left-turn saturations are the same, left turners may not be delayed the same amount. This implies that degree of left-turn saturation alone is not enough for characterizing the average left-turn delay. This also become clear after checking with Eq 5-2.

THE EFFECT OF CYCLE LENGTH

In Chapter 4, it was found that increasing the cycle length from 60 to 90 seconds does not cause a significant change in the left-turn capacity. Accordingly, after examining the relation between the average left-turn delay and left-turn capacity in Eq 5-11, one would surmise that the average left-turn delay will not be affected considerably by the change of cycle length. However, from Eq 5-11, it can be seen that the average left-turn delay also depends on the average queue length. When the cycle length is increased, more left turners will queue up during the blocked period, and more left turners will be processed during the unblocked period. This means that the queue length will be larger at the beginning of the unblocked period but will be smaller at the end of the unblocked period if the cycle length is increased. The queue diagram is changed, and in this sense, the average left-turn delay will be different (see Fig 5-20). Referring to Eq 5-2, if the cycle length is increased from C to C', the average left-turn delay will become

$$\overline{D}' = \frac{0.5(1-T)^2 C'}{1 - \rho T} + \frac{\overline{z}'}{\lambda} \qquad (5\text{-}14)$$

The first term in Eq 5-14 will increase with cycle length. The second term, a function of $\overline{z}'$, however, will be reduced if the cycle length is increased. As a result, the average left-turn delay may be larger or smaller

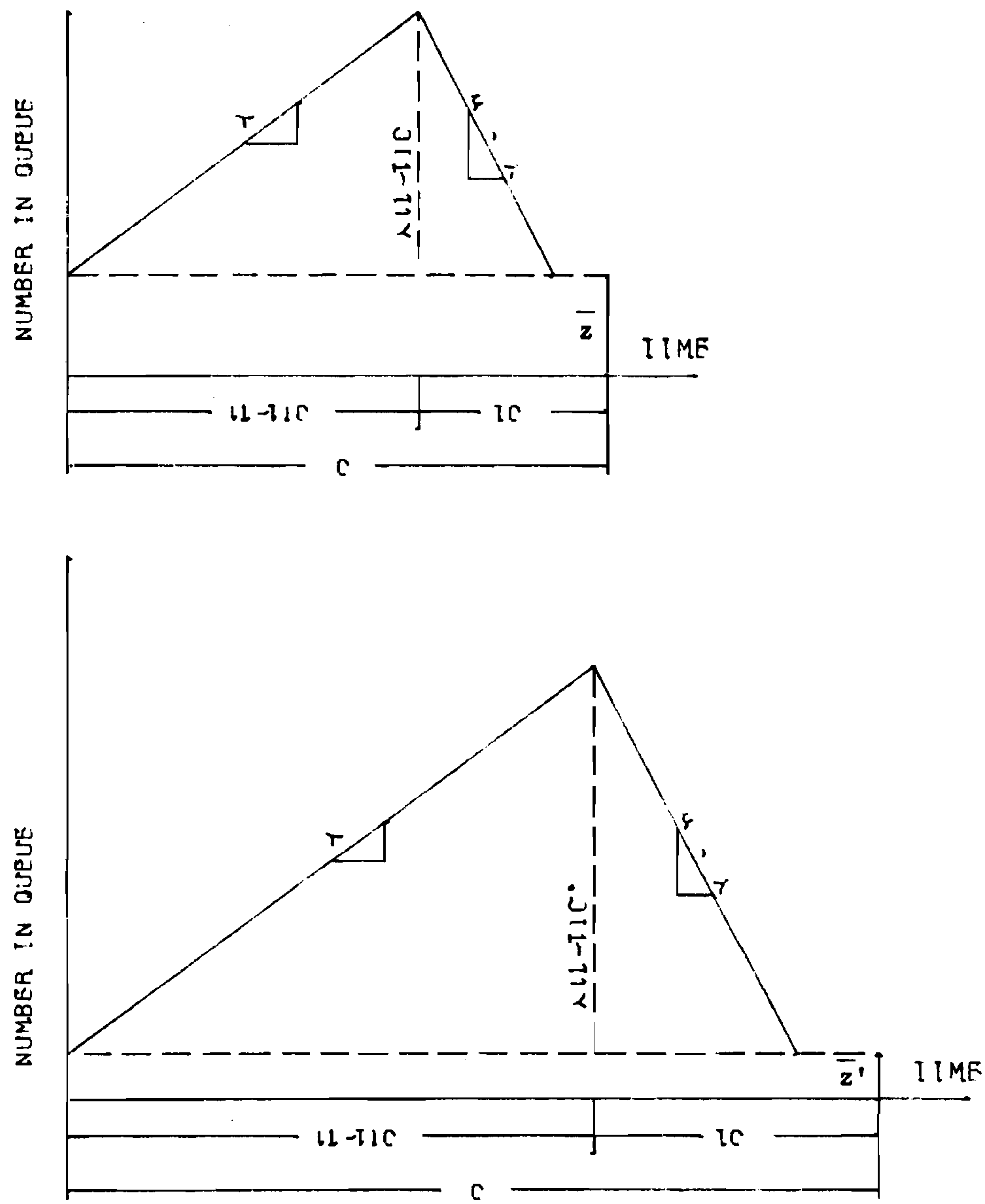


Fig 5-20. The change in queue diagram when the cycle length is increased.

depending on how the residual queue length varies with the cycle length. Simulation results from the TEXAS Model show that the average left-turn delay is not appreciably changed when the cycle length is increased from 60 to 90 seconds (see Table 5-3). This implies that the effect of longer queue in the blocked period is more or less offset by that of fewer vehicles remaining at the end of green phase.

THE EFFECT OF CYCLE SPLIT

From Eq 5-2, it can be seen that the average left-turn delay depends on the transparency. The transparency is a function of cycle split and opposing volume as shown in Eq 4-41. Hence, the average left-turn delay for any cycle split can be determined by placing Eq 4-41 in Eq 5-2. If the cycle split is increased, the blocked period will be reduced, while the unblocked period will be increased. Consequently, the queue length at the beginning and the end of unblocked period will be shorter (see Fig 5-21). The average left-turn delay is:

$$\overline{D}' = \frac{0.5(1-T')\ C}{1 - \rho T'} + \frac{\overline{z}'}{\lambda} \tag{5-15}$$

Since both the first and second terms in Eq 5-15 are reduced as the cycle split is increased, the average left-turn delay will also be reduced.

THE EFFECT OF NO BAY

At signalized intersections without a left-turn bay, there will be interaction between the through and left-turn vehicles. More specifically, if a left-turn vehicle is waiting for an acceptable gap at the stop line during the green time, the median lane is in fact being blocked for through

TABLE 5-3. COMPARISONS BETWEEN LEFT-TURN DELAY UNDER CYCLE LENGTH SIXTY SECONDS AND NINETY SECONDS

Opposing Volume, vph	Left-Turn Volume, vph	Average Left-Turn Delay, Sec		90 Percentile Delay, Sec		Percentage of Left-Turners Being Delayed More Than 120 Seconds	
		C=60 Sec	C=90 Sec	C=60 Sec	C=90 Sec	C=60 Sec	C=90 Sec
200	250	32.0	35.0	68.0	67.2	0.27	0.20
200	270	52.7	46.8	110.7	93.5	10.4	2.74
300	190	43.0	45.0	94.1	91.2	4.3	3.5
400	120	47.0	47.6	101.4	100.6	6.0	5.7
400	140	61.0	60.5	126.8	126.0	14.0	12.5
500	60	47.5	52.3	95.8	119.0	3.8	9.1
500	80	76.5	74.0	163.0	152.7	22.0	22.3

G/C = 0.5
Single opposing flow
Adequate length of bay
Not corrected for trucks

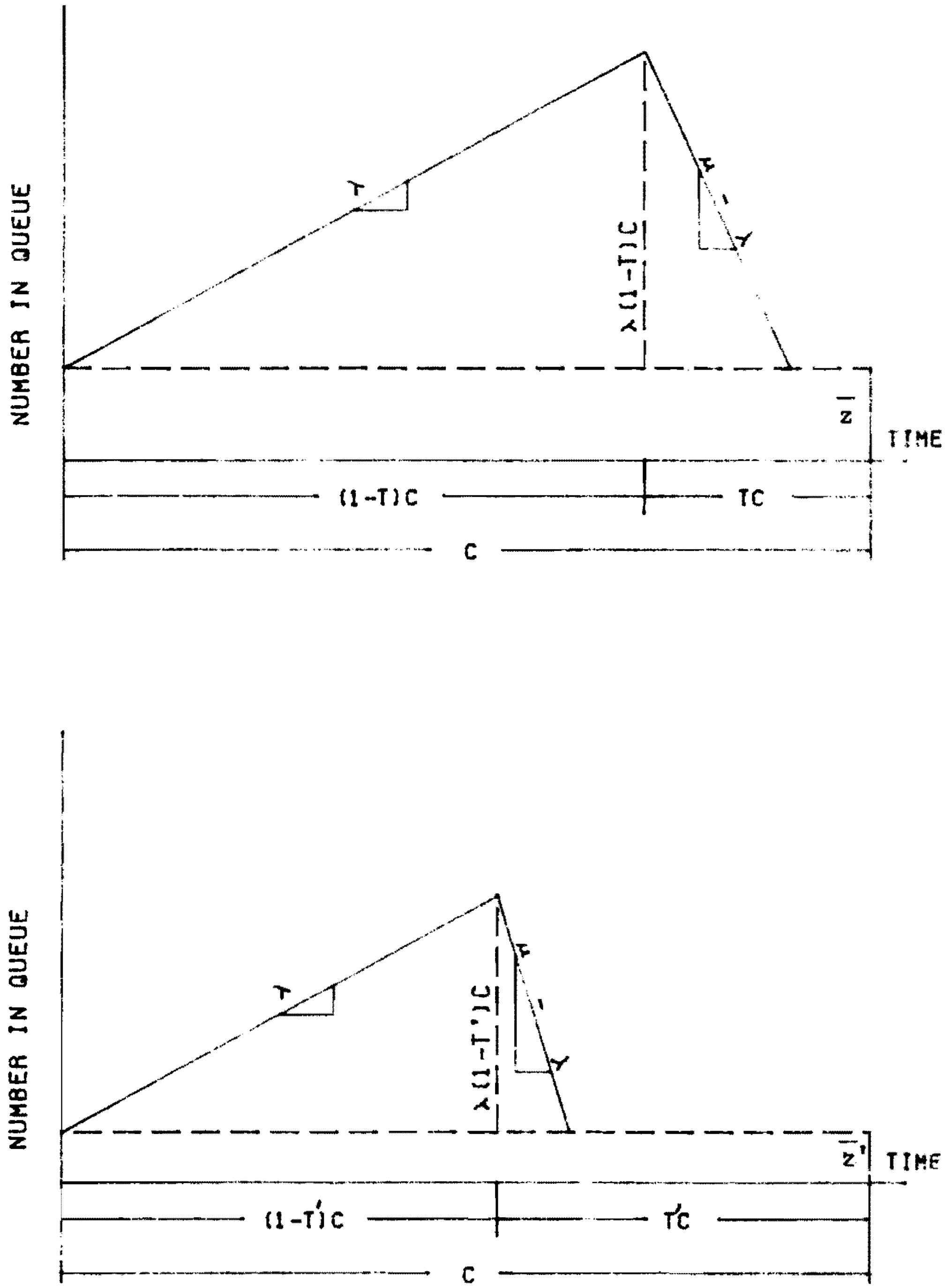


Fig 5-21. The change in queue diagram when the cycle split is increased.

movements. Through vehicles behind left turners will ordinarily suffer some delay. Furthermore, if the median lane is blocked by left turners very often, more through vehicles will choose to use the curb lane. As a result, the through traffic in the curb lane will be much heavier. When this is also the case for the opposing traffic, the left turners will face smaller gaps in the opposing traffic. The average left-turn delay and, thus, the through delay will be increased. In this section, through delay and left-turn delay under no bay condition will be studied. The through delay in the median and curb lanes will be compared.

Through Delay. Figure 5-22 shows that the average delay for through vehicles in the curb and median lanes are increased with the left-turn volume. When the left-turn volume exceeds some level, the average through delays increase sharply. It was found that the average through delay in the median lane will not be considerably greater than that in the curb lane if no more than 5 percent of left turners incur delay greater than two cycles. In other words, the effect of the absence of a bay on through vehicles is correlated to some degree with left-turn performance. If the left-turn performance is good, absence of bay will not cause too much problem to through movements.

Left-turn Delay. Table 5-4 compares the average left-turn delay when left-turn traffic interacts with 0, 400, and 600 through vehicles per hour. It was found that the average left-turn delay is increased by 3 to 10 seconds if the number of through vehicles interacting with left turners is increased from 0 to 600 vph. The percentage of left turners being delayed more than two cycles is also slightly increased.

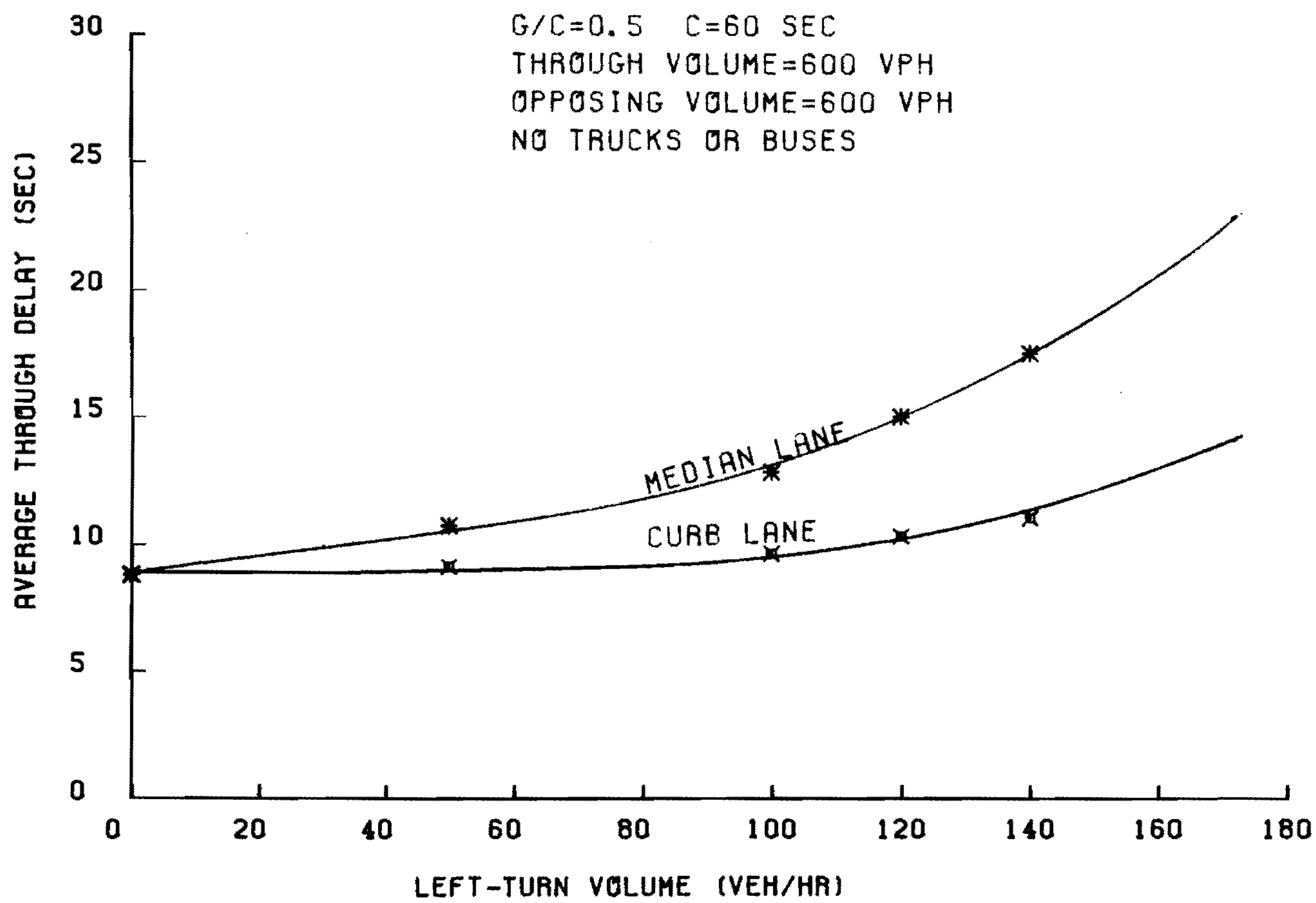


Fig 5-22. Comparison between the through delay in the median lane and that in the curb lane.

TABLE 5-4. THE AVERAGE LEFT-TURN DELAY FOR DIFFERENT MIX OF LEFT TURN AND THROUGH TRAFFIC, SEC

Left-Turn Volume, vph	Through Traffic Volume, vph		
	0	400	600
50	22.0	23.5	25.0
100	29.0	32.2	34.5
120	33.5	34.8	44.3
140	45.7	48.4	56.3

G/C = 0.5
Cycle length = 60 sec
Two opposing flows = 600 vph
Not corrected for trucks

DISCUSSION

The average left-turn delay is the key element required to provide an understanding of left-turn performance. Once the average left-turn delay is known, the variations of delay among left turners and the ninety-percentile delay can be estimated according to their relations with the average left-turn delay found in this Chapter. Equation 5-2 gives the functional form of left-turn operations at signalized intersections with adequate bay length. Little's formula relates the average left-turn delay to the average left-turn queue length. The average queue length $\bar{L}$ in Eq 5-7, is an implicit function of the left-turn demand, the opposing volume, the gap acceptance criteria, traffic control, and geometric configurations. In order to obtain $\bar{L}$, one must stand beside the intersection approach and record the timewise variations of the queue length. However, individuals who are part of the queue will observe the queue length $\bar{n}$ instead of $\bar{L}$. In other words, left turners themselves perceive the severity of traffic conditions by the performance measure $\bar{n}$. Equation 5-11 reveals the role that the left-turn capacity plays in the average left-turn delay. More important, Eq 5-11 permits determination of the left-turn capacity by observing unsaturated left-turn traffic. Strictly speaking, Eq 5-11 holds only when the left-turn time has the memoriless property (i.e., G/M/1 model). When the opposing volume is high or the sample size is small, Eq 5-11 becomes somewhat inaccurate. However, the deviation in the worst case is only about 13 percent as Table 5-2 shows.

Finally, it should be emphasized that Eqs 5-2, 5-7, and 5-11 are independent of simulation methods being used. Although values of variables in these equations might be different if different simulation methods are adopted, these functional forms still govern the relations among variables.

CHAPTER 6. LEFT-TURN WARRANT DEVELOPMENT

At signalized intersections the common treatment for improving left-turn performance is to increase left-turn capacity by adding a bay or a separate left-turn phase. However, given a traffic condition and a geometric configuration, there have been no universal guidelines for traffic engineers to determine whether a bay or a separate left-turn phase is justified. The variations in existing guidelines stem from different methodologies and criteria adopted for evaluating left-turn performance. The methodologies could be either analytical models, simulation models, or field observations, while the criteria may be a certain level of delay, conflict, or accident. The resulting guidelines usually will fall into five categories: (1) delay warrants, (2) volume warrants, (3) capacity warrants, (4) conflict warrants, and (5) accident warrants. Although conflict and accident warrants are useful for the trade-off analysis of a left-turn treatment, study of them by analytical or simulation analysis is very difficult. Thus, only the first three types of warrants will be discussed here. In this Chapter, existing left-turn warrants will be reviewed. By applying a set of delay criteria to left-turn performance curves in Chapter 5, critical conditions of left-turn operations can be defined. Efforts will be devoted to developing a general form of left-turn warrant which can identify the need for a left-turn treatment under various traffic conditions and geometric configurations.

WHEN A SEPARATE LEFT-TURN PHASE IS JUSTIFIED

Agent and Deen [Ref 27] conducted a survey of warrants currently being used by state highway agencies for installing a separate left-turn phase and found that numerous discrepancies exist (see Table 6-1). It has also been observed [Refs 1 and 2] that a left-turn phase, when not required, will cause more delay to drivers during other phases and even to left turners. Therefore, it is very important to have clear and effective guidelines for implementing a separate left-turn phase.

In order to develop warrants, a set of criteria must be chosen. If criteria on delay are employed, warrants for a separate left-turn phase can be stated in terms of delay, volume, and capacity. A volume warrant may be a minimum left-turn volume level or a product of the left-turn and opposing volumes. The latter is also called the volume-product warrant. From Table 6-1, it can be seen that a minimum left-turn volume level is the most popular type of left-turn warrant. However, this type of warrant is undesirable because it is not related to the opposing volume and the number of opposing lanes. In view of this, Agent and Deen [Ref 27], Southern Section of ITE [Ref 32], and Texas Transportation Institute [Ref 33] proposed various volume-product warrants (see Table 6-2). Unfortunately, these volume-product warrants do not seem completely robust, since they make no distinction between the left-turn and opposing volumes. For example, if a left-turn phase is justified when the product of the left-turn and opposing volumes is greater than 50,000, it does not matter whether there are 500 vph and 100 vph of opposing and left-turn volumes, respectively, or the other way around. Moreover, for a single opposing flow of 100 vph, according to the volume-product warrants in Table 6-2, the warranted left-turn volumes would

TABLE 6-1. SUMMARY OF WARRANTS FOR A SEPARATE LEFT-TURN PHASE CURRENTLY BEING USED BY STATE HIGHWAY AGENCIES

Type of Warrant	Left-Turn Warrant
Delay	Left-turn delay in excess of two cycles One left turner in one hour being delayed more than one cycle
Volume	Product of left-turn and opposing volumes equal to 50,000 or greater Product of left-turn and opposing volumes greater than 100,000 More than two vehicles per approach per cycle during a peak hour 50 or more left-turn vehicles in one hour on one approach and average speed of through traffic exceeding 45 mph More than 100 left-turn vehicles during a peak hour Left-turn volume greater than 90 vph Left-turn ADT above 500 for two-lane roadway 100 to 150 left-turn vehicles during peak hour (small cities) 150 to 200 left-turn vehicles during peak hour (large cities) 120 left-turn vehicles in the design hour 90 to 120 left-turn vehicles in the design hour More than 100 turns per hour
Accident	5 or more left-turn accidents within a 12-month period

be higher than the left-turn capacity estimated by any of the method discussed in Chapter 4.

TABLE 6-2. VOLUME-PRODUCT WARRANTS FOR INSTALLING A SEPARATE LEFT-TURN PHASE AT SIGNALIZED INTERSECTIONS WITH ADEQUATE LENGTH OF BAY

Number of Opposing Lanes	Product of Opposing and Left-turn Peak Hour Volumes		
	Agent and Deen	SSITE	TTI
1	50,000	45,000	50,000
2	100,000	90,000	90,000
3	-	135,000	110,000

In a recent report [Ref 25], Texas Transportation Institute (TTI) presented a capacity warrant in which a separate left-turn phase is recommended if the ratio of left-turn demand to capacity is greater than 0.7. This capacity warrant can be misleading, as pointed out in Chapter 5, because two traffic conditions with the same degree of left-turn saturation may not be equally severe for left-turn operations. Since no existing warrant appears to be satisfactory, a different type of left-turn warrant will be explored in this section.

Left-turn operations evaluated with different performance measures using the TEXAS Model have been studied in Chapter 5. For the purpose of developing warrants, the following left-turn delay criteria are used to define critical conditions for left-turn operations:

(1) the average left-turn delay reaches 35 seconds,

(2) the ninety-percentile left-turn delay reaches 73 seconds,

(3) five percent of left turners are delayed more than two cycles, and

(4) four left turners in one hour are delayed more than two cycles.

By applying each of these criteria to its corresponding left-turn performance curve in Chapter 5, critical left-turn volumes can be determined as shown in Tables 6-3 through 6-5. It can be seen that the criteria of 35 seconds for the average left-turn delay and 73 seconds for the ninety-percentile left-turn delay will usually generate the lowest critical left-turn volumes. On the other hand, the criteria of 5 percent of left turners delayed more than two cycles and 4 left turners in one hour delayed more than two cycles generally will lead to the highest critical left-turn volumes. Traffic engineers may choose any level between the highest and lowest critical left-turn volumes as the warranted left-turn volume depending on which criterion they regard more important. The decision regarding a separate left-turn phase can be made as follows: a separate left-turn phase is required if all the four delay criteria are met; no separate left-turn phase is needed if none of the four criteria are satisfied. When some but not all of the four delay criteria are satisfied, a judgement is required by the traffic engineer. A typical decision chart is illustrated in Fig 6-1.

Tables 6-6 and 6-7 show that neither volume-products nor volume-to-capacity ratios remain constant over opposing volumes. This tends to confirm the findings of the previous section that warrants based upon constant volume-products or volume-capacity ratios are inadequate. A warrant with a different conceptual basis is presented in the following section.

TABLE 6-3. CRITICAL LEFT-TURN VOLUMES BASED ON DIFFERENT CRITERIA FOR TWO-BY-TWO SIGNALIZED INTERSECTIONS WITH ADEQUATE LENGTH OF BAY

Criteria	Opposing Traffic Volume, vph			
	200	300	400	500
Average left-turn delay = 35 sec	255	170	90	50
90 percentile left-turn delay = 73 sec	255	170	90	50
5 percent of left turners being delayed more than two cycles	255	195	120	70
4 left turners in one hour being delayed more than two cycles	260	180	110	70
Ratio of left-turn demand to capacity = 0.7	222	176	128	85
Product of left-turn and opposing volume = 50,000	250	167	125	100

G/C = 0.5 C = 60 sec
Not corrected for trucks and buses

TABLE 6-4. CRITICAL LEFT-TURN VOLUMES BASED ON DIFFERENT CRITERIA FOR FOUR-BY-FOUR SIGNALIZED INTERSECTIONS WITH ADEQUATE LENGTH OF BAY

Criteria	Opposing Traffic Volume, vph			
	300	400	500	600
Average left-turn delay = 35 sec	275	200	155	110
90 percentile left-turn delay = 73 sec	275	195	155	110
5 percent of left turners being delayed more than two cycles	290	220	170	130
4 left turners in one hour being delayed more than two cycles	275	195	160	120
Ratio of left-turn demand to capacity = 0.7	217	179	153	122
Product of left-turn and opposing volumes = 90,000	300	225	180	150

G/C = 0.5 C = 60 sec
Not corrected for trucks and buses

TABLE 6-5. CRITICAL LEFT-TURN VOLUMES BASED ON DIFFERENT CRITERIA FOR SIX-BY-SIX SIGNALIZED INTERSECTIONS WITH ADEQUATE LENGTH OF BAY

Criteria	Opposing Traffic Volume, vph			
	600	900	1200	1500
Average left-turn delay = 35 sec	165	65	25	15
90 percentile left-turn delay = 73 sec	165	75	30	15
5 percent of left turners being delayed more than two cycles	195	90	40	30
4 left turners in one hour being delayed more than two cycles	175	75	55	35
Ratio of left-turn demand to capacity = 0.7	147	93	68	45
Product of left-turn and opposing volumes = 110,000	183	122	92	73

G/C = 0.5 C = 60 sec

Not corrected for trucks and buses

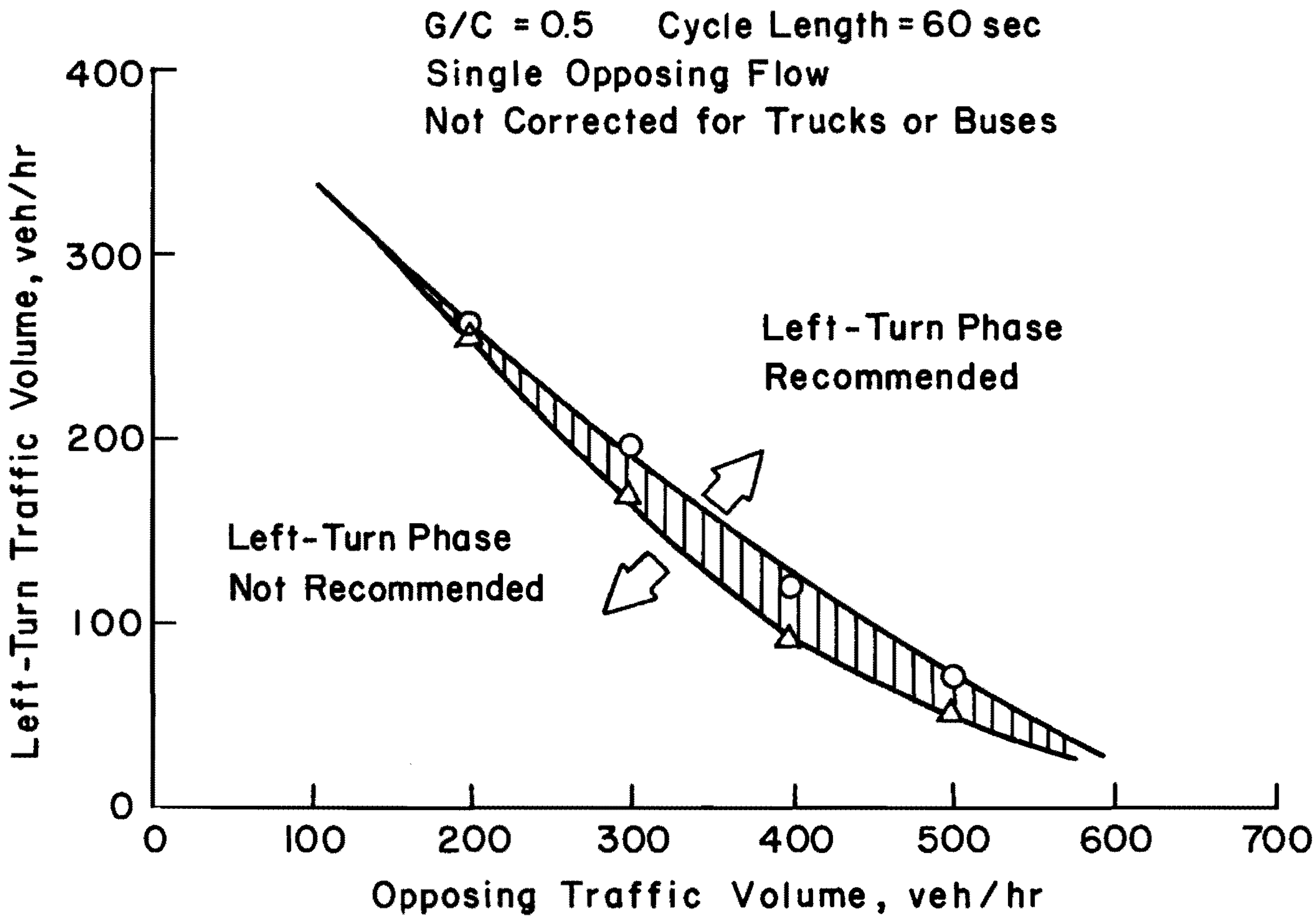


Fig 6-1. A typical decision chart for **implementing** a left-turn treatment.

TABLE 6-6. RATIOS OF CRITICAL LEFT-TURN VOLUMES TO LEFT-TURN CAPACITIES UNDER DIFFERENT LEVELS OF OPPOSING VOLUME AND NUMBER OF OPPOSING LANES

Number of Opposing Lanes	Opposing Volume, vph	Criteria for Determining Critical Left-Turn Volumes			
		Average Left-Turn Delay = 35 sec	90 Percentile Left-Turn Delay = 73 sec	5% of Left Turners Being Delayed More Than Two Cycles	4 Left Turners In One Hour Being Delayed More Than Two Cycles
Single	200	0.80	0.80	0.80	0.82
	300	0.67	0.67	0.77	0.71
	400	0.49	0.49	0.65	0.60
	500	0.41	0.41	0.58	0.58
Two	300	0.87	0.87	0.92	0.87
	400	0.78	0.76	0.86	0.76
	500	0.71	0.71	0.78	0.74
	600	0.69	0.63	0.74	0.69
Three	600	0.79	0.79	0.93	0.83
	900	0.49	0.56	0.68	0.56
	1200	0.26	0.31	0.41	0.57
	1500	0.23	0.23	0.47	0.54

G/C = 0.5 C = 60 sec
Not corrected for trucks and buses

TABLE 6-7. CROSS PRODUCTS OF CRITICAL LEFT-TURN VOLUMES AND OPPOSING VOLUMES UNDER DIFFERENT LEVELS OF OPPOSING VOLUME AND NUMBER OF OPPOSING LANES

Number of Opposing Lanes	Opposing Volume, vph	Criteria for Determining Critical Left-Turn Volumes			
		Average Left-Turn Delay = 35 sec	90 Percentile Left-Turn Delay = 73 sec	5% of Left Turners Being Delayed More Than Two Cycles	4 Left Turners In One Hour Being Delayed More Than Two Cycles
One	200	51,000	51,000	51,000	52,000
	300	51,000	51,000	58,500	54,000
	400	36,000	36,000	48,000	44,000
	500	25,000	25,000	35,000	35,000
Two	300	82,500	82,500	87,000	82,500
	400	80,000	78,000	88,000	78,000
	500	77,500	77,500	85,000	80,000
	600	72,000	66,000	78,000	72,000
Three	600	99,000	99,000	117,000	105,000
	900	58,500	67,500	81,000	67,000
	1200	30,000	36,000	48,000	66,000
	1500	22,500	22,500	45,000	52,500

G/C = 0.5 C = 60 sec
Not corrected for trucks and buses

Referring to Eq 4-42, for a single opposing flow with volume Q_o, the left-turn capacity Q_L at signalized intersections with adequate length of bay can be obtained as follows:

If

$$Q_o < Q_o C/G < 1000 \text{ vpgh}$$

then

$$Q_L = 879(G/C) - 0.634Q_o$$

In general, the left-turn capacity Q_L can be obtained from a linear equation as follows:

$$Q_L = Q_c(G/C) - e_o Q_o \tag{6-1}$$

where Q_c and e_o assume different values over different ranges of opposing volume. Equation 6-1 can also be written as

$$Q_L + e_o Q_o = Q_c(G/C) \tag{6-2}$$

The physical meaning of Eq 6-2 can be explained as follows. The coefficient e_o, as discussed in Chapter 4, is the equivalence factor of opposing to left-turn vehicles. Thus, the left-hand side of Eq 6-2 is the sum of total conflicting flows in terms of left-turn vehicles. This is produced by converting the opposing traffic to left-turn vehicles using the equivalence factor e_o. In this sense, the right-hand side of Eq 6-2 is the maximum volume of total conflicting flows that can be processed through the signalized intersection and can be regarded as "the capacity of the conflict area". It follows that Q_c will be the maximum volume of conflicting flows that can be processed in one hour of green time and can be called the effective capacity of the conflict area. When utilizing the capacity of the conflict area, opposing vehicles not only have priority over left-turn vehicles but are also weighted less than left-turn vehicles.

Notice that if Eq 6-2 is divided by e_o, then it will become

$$Q_L/e_o + Q_o = (Q_c/e_o)(G/C) \tag{6-3}$$

Let

$$e_L = 1/e_o$$

$$Q'_c = Q_c/e_o$$

then Eq 6-3 will become

$$e_L Q_L + Q_o = Q'_c(G/C) \tag{6-4}$$

Equation 6-4 has a similar physical meaning to that of Eq 6-2, except that the total conflicting flows are represented in terms of opposing vehicles instead of left-turn vehicles. Left-turn vehicles are converted to equivalent opposing vehicles using the left-turn equivalence factor e_L. A left-turn equivalence factor of 1.6 has been used in the literature and found suitable for single opposing flow less than 1000 vpgh in the TEXAS Model. However, the left-turn equivalence factor e_L, as will be shown later, is not a constant value for all opposing volumes and geometric configurations.

In order to preclude critical conditions of left-turn operations, left-turn demand or the total conflicting flows should not be near capacity. Let Q_w be a critical left-turn volume at signalized intersections having adequate length of bay without a separate left-turn phase. Let f_c be the allowable utilization factor of the conflict area and defined as follows:

$$f_c = \frac{Q_w + e_o Q_o}{Q_c (G/C)} \tag{6-5}$$

Hence, for any critical left-turn volume $Q_w < Q_L$, there exists an allowable utilization factor of the conflict area $f_c \leq 1.0$ such that the following equation holds:

$$Q_w + e_o Q_o = f_c Q_c (G/C) \qquad (6\text{-}6)$$

or

$$Q_w = f_c Q_c (G/C) - e_o Q_o \qquad (6\text{-}7)$$

As Q_w approaches Q_L, f_c will approach 1.0. In this case, Eq 6-6 is reduced to Eq 6-2. If values of e_o, f_c, and Q_c under various traffic conditions and geometric configurations are known, then the critical left-turn volume Q_w can be determined from Eq 6-7. Therefore, Eq 6-7 can serve as a left-turn warrant. Typical values of e_L, e_o, Q_c, and f_c are shown in Table 6-8. To assist traffic engineers in using their judgement, f_c values for predicting the lowest and highest critical left-turn volumes are provided. From Table 6-8, the following conclusions can be drawn:

(1) For a given intersection geometry and cycle split, the left-turn equivalence factor e_L (= $1/e_o$), the effective capacity of the conflict area Q_c, and the allowable utilization factor of the conflict area f_c have different values for different ranges of opposing volume.

(2) The left-turn equivalence factor varies from 1.6 for low volume of single opposing flow to 8.9 for high volumes of three opposing flows. Generally, the fewer the number of acceptable gaps, the larger the left-turn equivalence factor will be.

(3) The effective capacity of the conflict area varies from 465 to 930 vehicles per hour of green. For the same opposing volume, the effective capacity increases with the number of opposing lanes.

(4) The allowable utilization factor of the conflict area varies from 0.79 to 0.96. For a given intersection geometry, the allowable utilization factor of the conflict area decreases as the opposing volume increases.

TABLE 6-8. VALUES OF e_L, e_o, f_c, AND Q_c FOR DIFFERENT OPPOSING VOLUMES AND NUMBER OF OPPOSING LANES

Number of Opposing Lanes	Opposing Volume Q_o, vph	Equivalence Factor e_L	Equivalence Factor e_o	Effective Capacity of the conflict Area Q_c, vpgh	Allowable Utilization Factor, f_c
One	$0<Q_o C/G<1000$	1.6	0.634	879	0.84-0.87
	$1000<Q_o C/G<1350$	2.9	0.348	590	0.79-0.82
Two	$0<Q_o C/G<1000$	2.0	0.500	930	0.86-0.92
	$1000<Q_o C/G<1350$	2.8	0.353	780	0.82-0.87
	$1350<Q_o C/G<2000$	6.0	0.167	465	0.79-0.84
Three	$0<Q_o C/G<1000$	2.2	0.448	930	0.91-0.96
	$1000<Q_o C/G<1350$	3.4	0.297	780	0.88-0.94
	$1350<Q_o C/G<2400$	8.9	0.112	465	0.72-0.84

From Eq 6-7, the relation between the critical left-turn volume and left-turn capacity can be obtained as follows:

$$\begin{aligned} Q_w &= f_c Q_c (G/C) - e_o Q_o \\ &= [Q_c (G/C) - e_o Q_o] - [Q_c (G/C) - f_c Q_c (G/C)] \\ &= Q_L - (1-f_c) Q_c (G/C) \end{aligned} \tag{6-8}$$

Let

$$M = (1-f_c)Q_c(G/C) \tag{6-9}$$

Then

$$Q_w = Q_L - M \tag{6-10}$$

Equation 6-10 reveals that the critical left-turn volume is M vehicles less than the left-turn capacity. This implies that there exists a threshold located at M vehicles lower than the left-turn capacity, and once the left-turn demand reaches this threshold, the left-turn operations will become critical. The value of M depends on the geometric configuration, signal-timing scheme, and the level of the opposing volume. Left-turn warrants for separate left-turn phase under various traffic conditions and geometric configurations can be obtained from Table 6-9. Decision charts for a separate left-turn phase are provided in Figs 6-2 through 6-4. If a left-turn demand is greater than the warranted left-turn volume obtained from Table 6-9 or Figs 6-2 through 6-4, then the four left-turn delay criteria are all satisfied. Thus, a separate left-turn phase is required.

Compared with simulation results from the TEXAS Model, the recommended left-turn warrants in Table 6-9 predict the highest critical left-turn volume within about 10 vehicles for the case of 0.5 cycle split and a 60 second cycle length. The volume-product warrant, the volume-capacity-ratio warrant, and the recommended warrant are compared in Figs 6-5 through 6-7.

WHEN A LEFT-TURN BAY IS REQUIRED

An adequate length of bay has been assumed in studying warrants for a separate left-turn phase. Should a left-turn bay not be adequately long or not provided at all, left-turn and through vehicles will incur more delay due to interactions among them. Moreover, through vehicles impeded by the left-turn queue may attempt hazardous lane changes. Although a left-turn bay

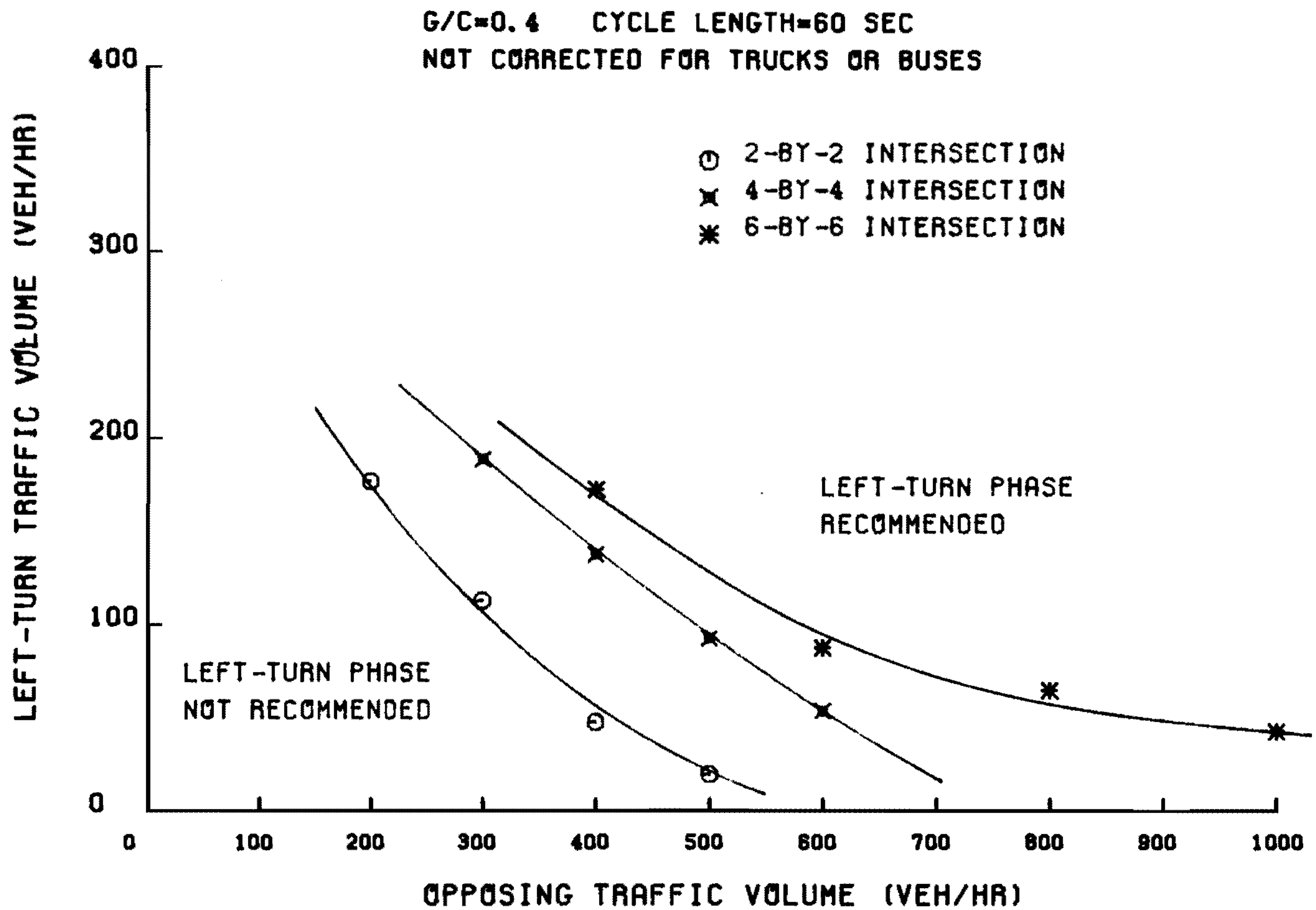


Fig 6-2. Decision chart for implementing a separate left-turn phase at signalized intersections with G/C = 0.4 and C = 60 sec.

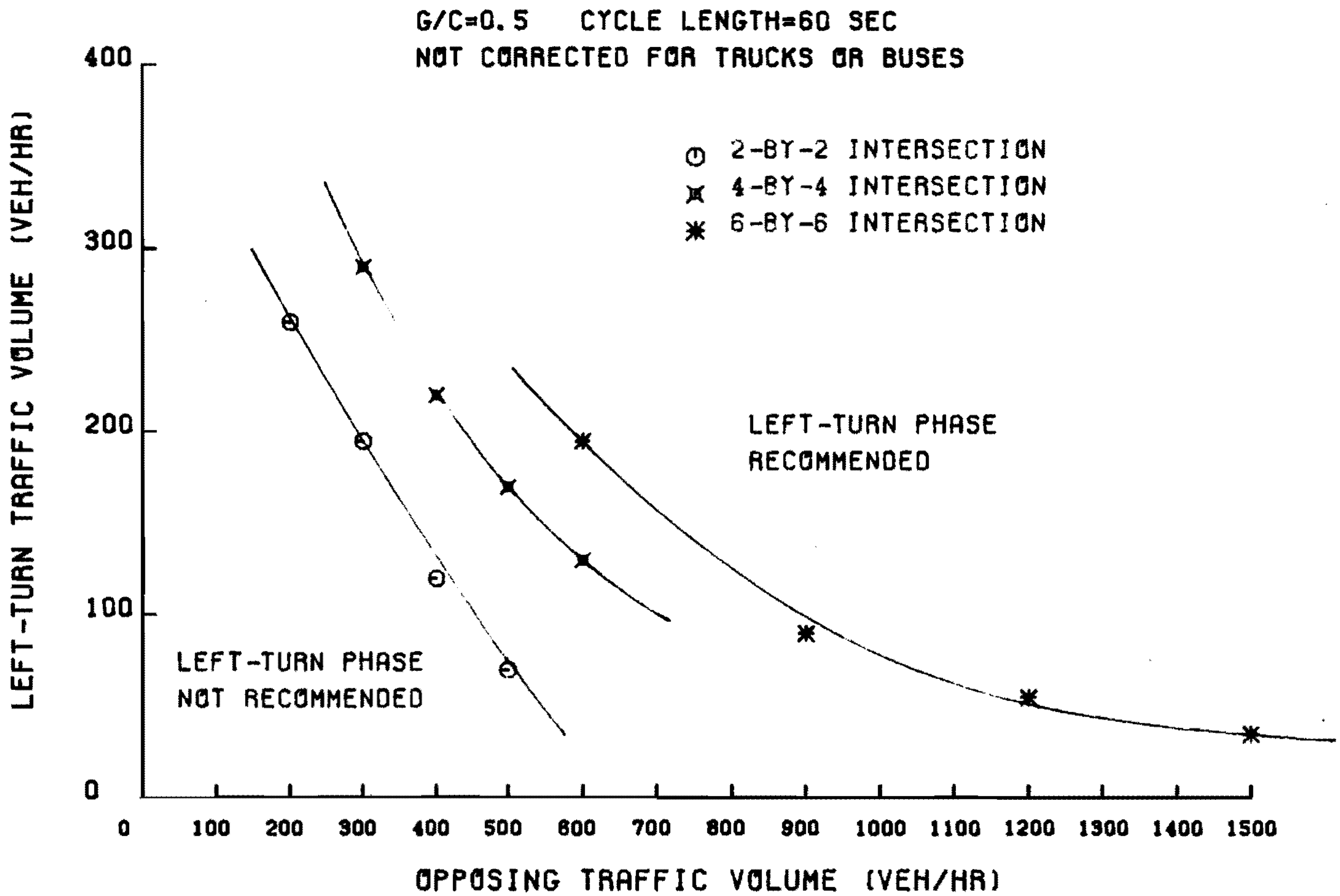


Fig 6-3. Decision chart for implementing a separate left-turn phase at signalized intersections with G/C = 0.5 and C = 60 sec.

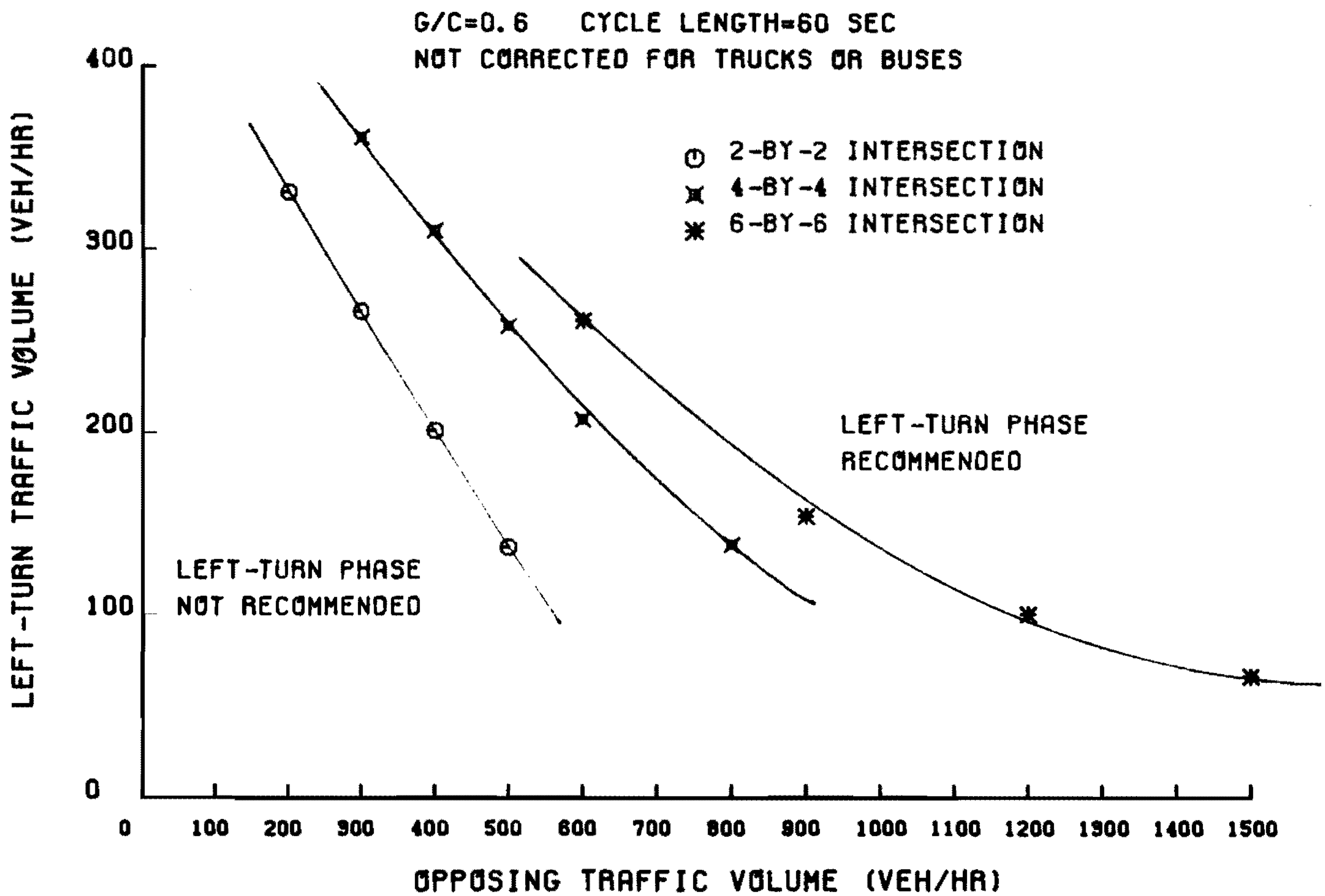


Fig 6-4. Decision chart for implementing a separate left-turn phase at signalized intersections with G/C = 0.6 and C = 60 sec.

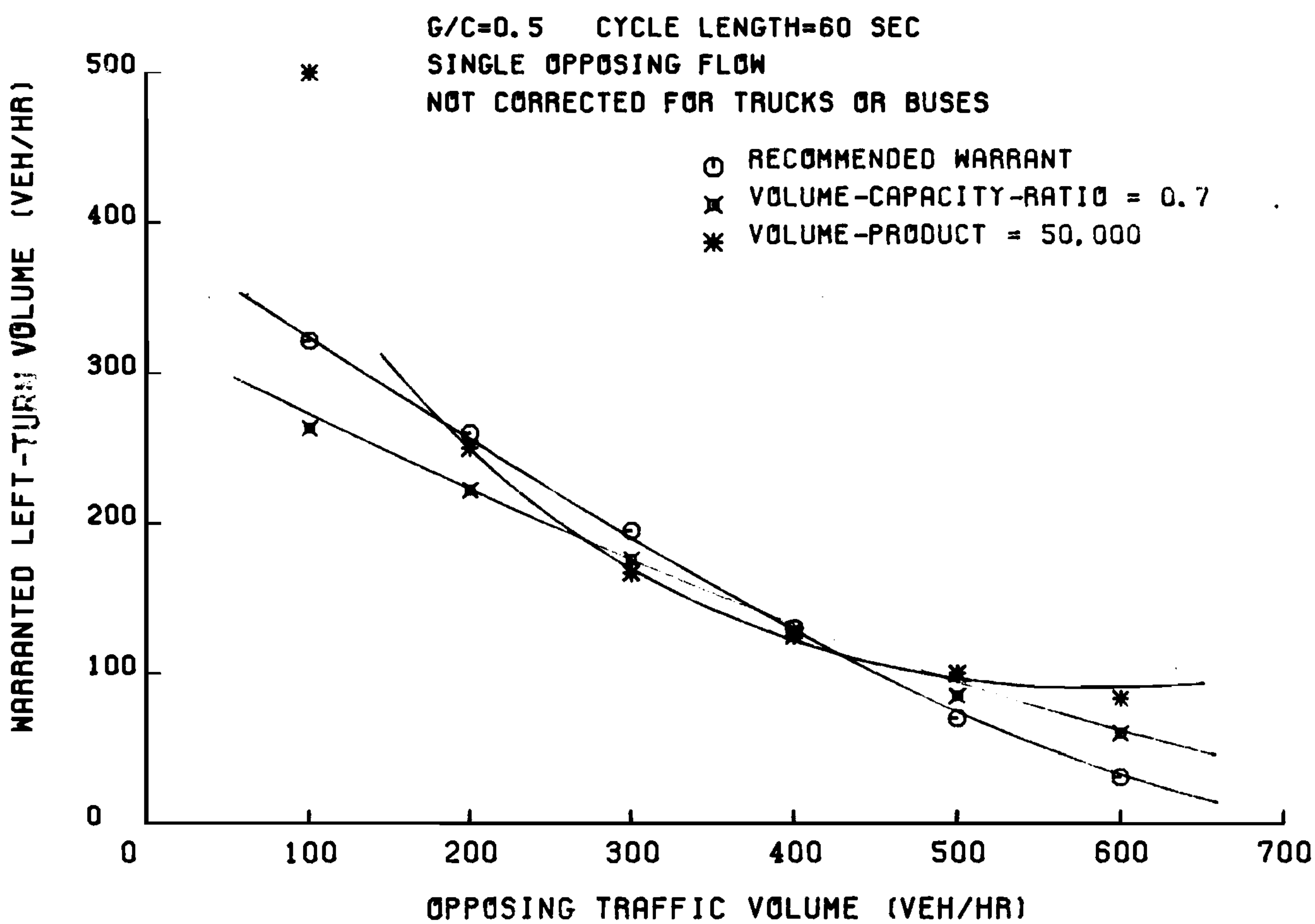


Fig 6-5. Comparisons among different warrants for a separate left-turn phase at two-by-two signalized intersections.

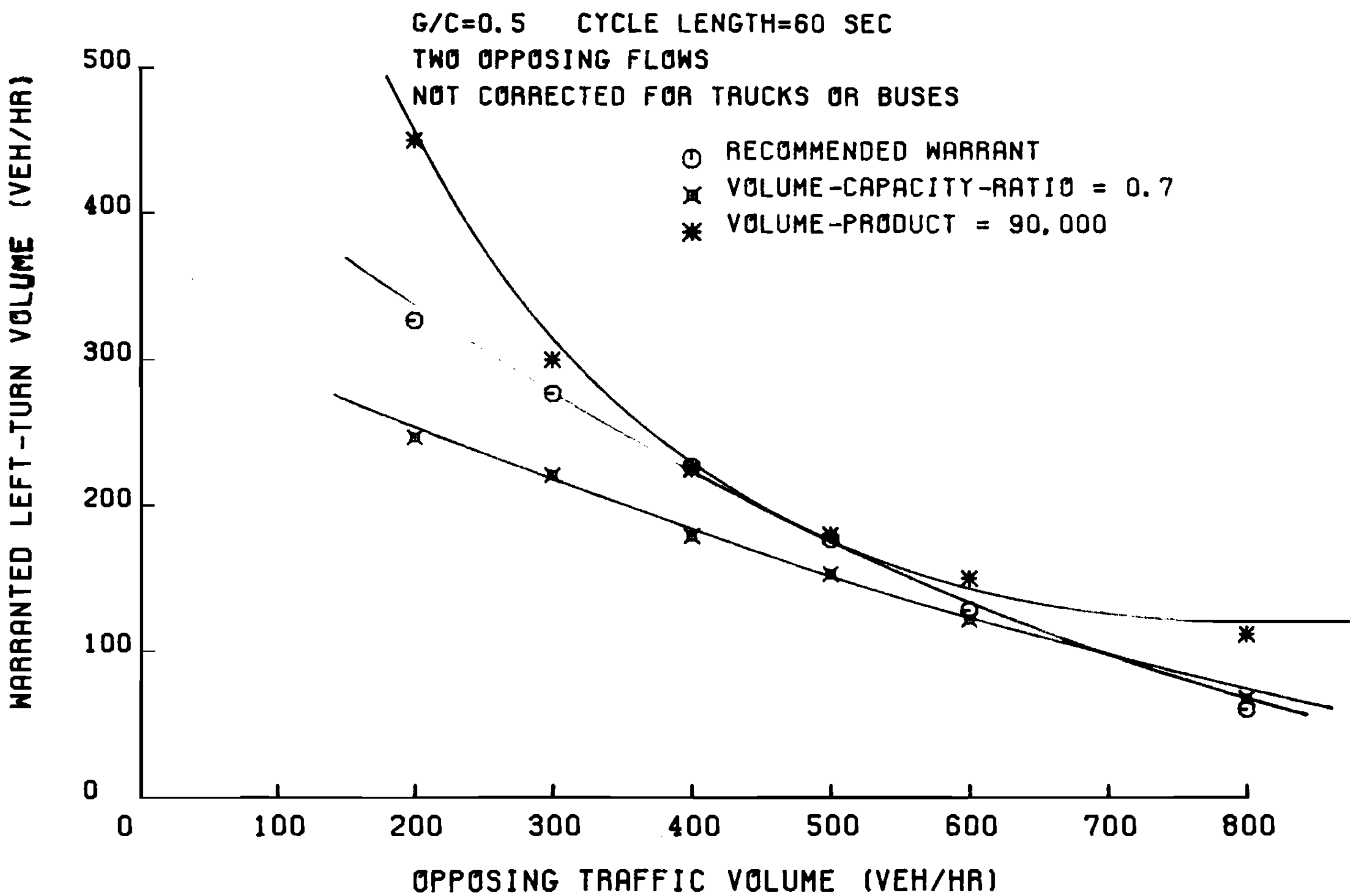


Fig 6-6. Comparisons among different warrants for a separate left-turn phase at four-by-four signalized intersections.

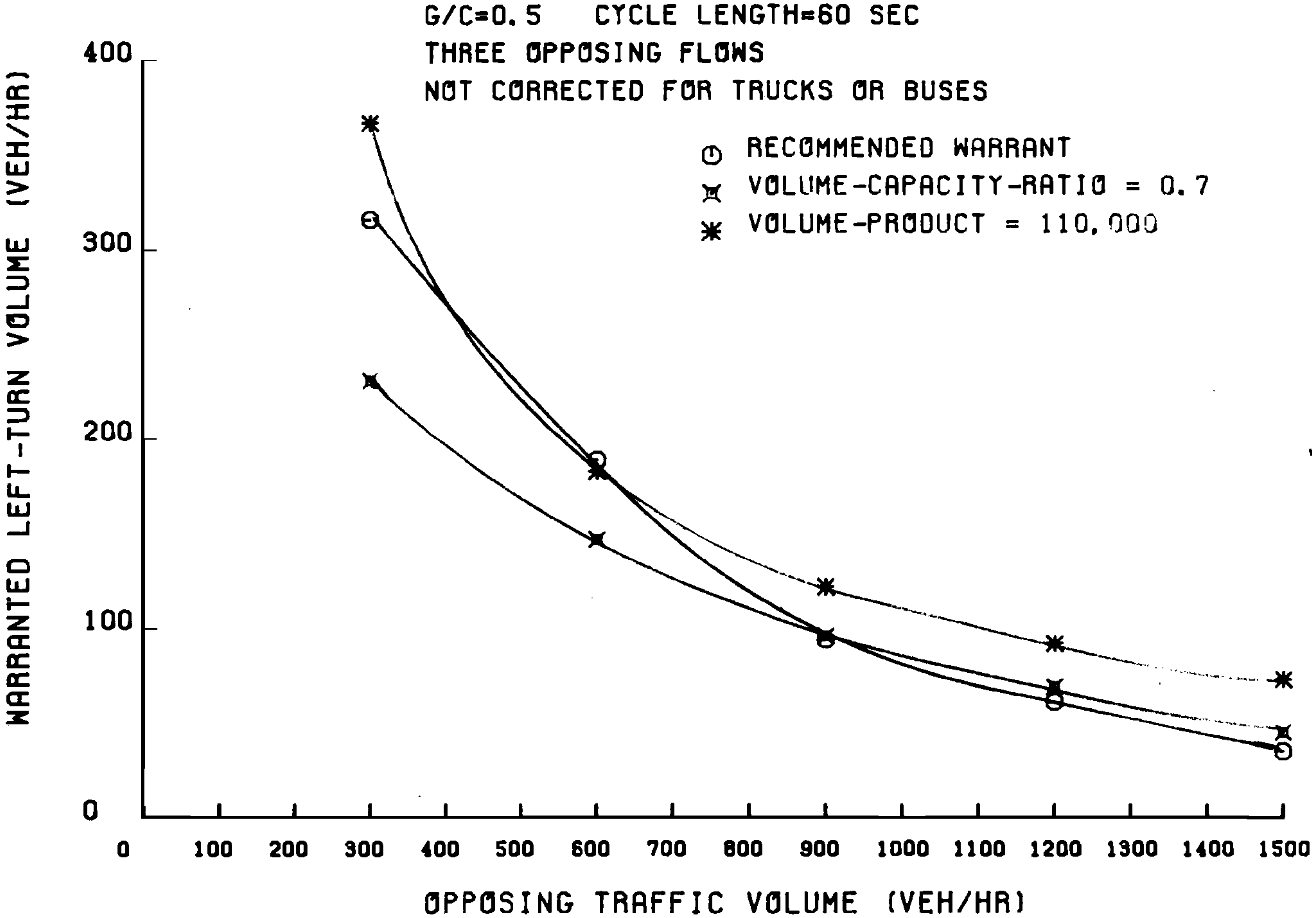


Fig 6-7. Comparisons among different warrants for a separate left-turn phase at six-by-six signalized intersections.

TABLE 6-9. RECOMMENDED LEFT-TURN WARRANTS FOR A SEPARATE LEFT-TURN PHASE UNDER DIFFERENT LEVELS OF OPPOSING VOLUMES AND NUMBER OF OPPOSING LANES

Number Of Opposing Lanes	Opposing Volume Q_o, vph	Critical Left-turn Volume Q_w, vph
One	$0 < Q_o C/G < 1000$	$(G/C) - 0.634Q_o$
	$1000 < Q_o C/G < 1350$	$(G/C) - 0.348Q_o$
Two	$0 < Q_o C/G < 1000$	$855(G/C) - 0.500Q_o$
	$1000 < Q_o C/G < 1350$	$680(G/C) - 0.353Q_o$
	$1350 < Q_o C/G < 2000$	$390(G/C) - 0.167Q_o$
Three	$0 < Q_o C/G < 1000$	$(G/C) - 0.448Q_o$
	$1000 < Q_o C/G < 1350$	$735(G/C) - 0.297Q_o$
	$1350 < Q_o C/G < 2400$	$390(G/C) - 0.112Q_o$

is always desired, the construction of a bay usually involves redesigning the intersection and is costly. Therefore, it is important to know when a left-turn bay is required and how long the bay should be. This section will concentrate on developing warrants for a left-turn bay, while the bay length will be discussed in the next section.

For unsignalized intersections, Failmezger [Ref 34] and Harmelink [Ref 35] proposed a relative warrant and volume warrants, respectively, for the construction of a left-turn bay. The relative warrant is based on an index of hazards, construction costs, and past traffic accident data. If the numerical value of the indicator parameters of relative warrant is greater than one, then a left-turn bay is recommended. The volume warrants developed by Harmelink are based on queueing theory analysis and field studies of traffic behavior. If the opposing and left-turn volumes are known, the bay length required can be determined from charts provided. As to signalized intersections, Dart [Ref 36] performed a computer simulation to develop warrants for a left-turn bay. If delay is used as a design criterion, the need for a bay can be ascertained. In this section, warrants for a left-turn bay will be related to left-turn capacity.

Before developing warrants for a left-turn bay, criteria for defining critical conditions when there is no bay have to be chosen. The four left-turn delay criteria used in developing warrants for a separate left-turn phase remain relevant in this case. The through delay in the median lane should also be considered, since through vehicles in the median lane will be impeded by left-turn vehicles if there is no bay. It has been found in Chapter 5, however, that the average through delay in the median lane is not considerably greater than that in the curb lane as long as no more than 5 percent of left turners are delayed greater than two cycles. In view of this, the four left-turn delay criteria alone would be appropriate for developing warrants for a left-turn bay.

Since the same criteria are used, warrants for a left-turn bay can be derived through an approach similar to that for a separate left-turn phase. For the convenience of discussion, left-turn vehicles in the opposing flows are ignored first and then taken back into consideration later.

Case I. No Left-turn Vehicles In Opposing Flows. Referring to Equation 4-58, the left-turn capacity for no bay when there are no left-turn vehicles in opposing flows in general can be obtained as follows:

$$\tilde{Q}_L = \tilde{Q}_c (G/C) - \tilde{e}_o Q_o \tag{6-11}$$

By the same argument as in the previous section, warrants for a left-turn bay can be expressed as follows:

$$\tilde{Q}_w = \tilde{Q}_L - (1-\tilde{f}_c)\tilde{Q}_c\,(G/C) \tag{6-12}$$

Typical values of $\tilde{e}_L$, $\tilde{e}_o$, $\tilde{Q}_c$, and $\tilde{f}_c$ are summarized in Tables 6-10 through 6-12.

Case II. Left-turn Vehicles In Opposing Flows. As discussed in Chapter 4, the left-turn capacity when there is no bay and there are V_{oL} and Q_o vph left-turn and through vehicles, respectively, in opposing flows will be as follows:

$$\hat{Q}_L = \tilde{Q}_L - aQ_o \tag{6-13}$$

where

$\hat{Q}_L$ = left-turn capacity with no bay when there are V_{oL} vph, left-turn vehicles in opposing flows;

$\tilde{Q}_L$ = left-turn capacity with no bay when there are no left-turn, vehicles in opposing flows as defined in Eq 4-58, vph; and

a = correction factor as defined in Eq 4-53.

Thus, the warrant for a left-turn bay when there are left-turn vehicles in opposing flows can be obtained as follows:

$$\hat{Q}_w = \tilde{Q}_w - aQ_o \tag{6-14}$$

Since the warranted left-turn volume $\tilde{Q}_w$ can be obtained from Eq 6-12, the left-turn volume $\hat{Q}_w$ required for construction of a bay when there are left-turn vehicles in opposing flows can be determined from Eq 6-14.

TABLE 6-10. VALUES OF $\tilde{e}_L$, $\tilde{e}_o$, $\tilde{Q}_c$, AND $\tilde{f}_c$ FOR SINGLE OPPOSING FLOW

Opposing Volume Q_o, vph	Through Volume In Median Lane, vph	$\tilde{e}_L$	$\tilde{e}_o$	$\tilde{Q}_c$	$\tilde{f}_c$
$0 < Q_oC/G < 1000$	100	1.6	0.634	855	0.84 - 0.87
	200	1.7	0.593	820	0.84 - 0.87
	300	1.9	0.526	680	0.84 - 0.87
	400	2.2	0.455	560	0.84 - 0.87
$0 < Q_oC/G < 800$	500	2.9	0.340	415	0.84 - 0.87
$1000 < Q_oC/G < 1350$	100	3.2	0.310	530	0.79 - 0.82
	200	3.7	0.270	460	0.79 - 0.82
	300	4.5	0.220	375	0.79 - 0.82
	400	5.6	0.180	300	0.79 - 0.82
$800 < Q_oC/G < 1350$	500	4.0	0.250	295	0.79 - 0.82

TABLE 6-11. VALUES OF $\tilde{e}_L$, $\tilde{e}_o$, $\tilde{Q}_c$, AND $\tilde{f}_c$ FOR TWO OPPOSING FLOWS

Opposing Volume Q_o, vph	Through Volume In Median Lane, vph	$\tilde{e}_L$	$\tilde{e}_o$	$\tilde{Q}_c$	$\tilde{f}_c$
$0 < Q_oC/G < 1000$	100	2.0	0.507	910	0.86 - 0.92
	200	2.1	0.483	840	0.86 - 0.92
	300	2.3	0.443	740	0.86 - 0.92
	400	2.6	0.380	615	0.86 - 0.92
$0 < Q_oC/G < 800$	500	3.3	0.305	455	0.86 - 0.92
$1000 < Q_oC/G < 1600$	100	2.7	0.370	770	0.82 - 0.87
	200	2.9	0.340	695	0.82 - 0.87
	300	3.4	0.290	590	0.82 - 0.87
	400	4.4	0.230	465	0.82 - 0.87
$800 < Q_oC/G < 1600$	500	5.3	0.188	365	0.82 - 0.87
$1600 < Q_oC/G < 2000$	100	6.3	0.160	435	0.79 - 0.84
	200	7.1	0.140	375	0.79 - 0.84
	300	8.7	0.115	310	0.79 - 0.84
	400	11.1	0.090	240	0.79 - 0.84
	500	16.7	0.06	160	0.79 - 0.84

TABLE 6-12. VALUES OF $\tilde{e}_L$, $\tilde{e}_o$, $\tilde{Q}_c$, AND $\tilde{f}_c$ FOR THREE OPPOSING FLOWS

Opposing Volume Q_o, vph	Through Volume In Median Lane, vph	$\tilde{e}_L$	$\tilde{e}_o$	$\tilde{Q}_c$	$\tilde{f}_c$
$0 < Q_oC/G < 1000$	100	2.2	0.450	910	0.91 - 0.96
	200	2.3	0.430	840	0.91 - 0.96
	300	2.5	0.400	745	0.91 - 0.96
	400	2.9	0.343	615	0.91 - 0.96
$0 < Q_oC/G < 800$	500	3.6	0.280	460	0.91 - 0.96
$1000 < Q_oC/G < 1600$	100	3.2	0.317	775	0.88 - 0.94
	200	3.4	0.297	705	0.88 - 0.94
	300	3.9	0.260	605	0.88 - 0.94
	400	4.8	0.210	485	0.88 - 0.94
$800 < Q_oC/G < 1600$	500	5.8	0.173	375	0.88 - 0.94
$1600 < Q_oC/G < 2000$	100	9.1	0.110	445	0.72 - 0.84
	200	10.0	0.100	395	0.72 - 0.84
	300	11.1	0.090	335	0.72 - 0.84
	400	14.3	0.070	260	0.72 - 0.84
	500	20.0	0.050	105	0.72 - 0.84

THE REQUIRED LENGTH OF LEFT-TURN BAY

Once a decision has been made regarding the construction of a left-turn bay at at a signalized intersection, the required length of bay must be determined. AASHTO [Ref 37] states that the left-turn bay should be able to accommodate 1.5 to 2.0 times the average number of vehicles that would be stored per cycle based on design volume. Unfortunately, this guideline fails to recognize that the average number of left-turn vehicles stored per cycle will depend on the opposing volume and the signal-timing scheme. For the same left-turn demand, the number of left-turn vehicles stored in the bay for high opposing volume will be much larger than that for low opposing volume. Messer [Ref 38] used a combination of theory and traffic simulation to develop the relation between left-turn volume and left-turn bay length required for a protected left-turning movement. In this section the bay length required for an unprotected left-turn movement will be derived based on the simulation results from the TEXAS Model.

From Fig 5-15, it can be found that the relations between the average and the maximum values of left-turn queue length can be approximately represented by the following equations:

Based On The Average Condition

$$L_m = 5.5\bar{L}^{0.58} \qquad (R^2=0.95) \tag{6-15}$$

Based on 95 percent Confidence Level

$$L_m = 7.4\bar{L}^{0.55} \qquad (R^2=0.86) \tag{6-16}$$

where

L_m = the maximum left-turn queue length, vehicle; and

$\bar{L}$ = the average left-turn queue length, vehicle.

If the bay length design is based on the average condition, the bay length will be exceeded under a given traffic condition with a probability 0.5. On the other hand, if the bay length is based on the 95 percent confidence level, the bay length will be exceeded with a probability 0.05. Any bay length in between will have a probability to be exceeded greater than 0.05 but less than 0.5.

By assuming that a passenger car and a truck or bus will occupy w_c ft and w_T ft of bay length, respectively. The required bay length, L_B, can be determined from the following equation:

$$L_B = w_T p_T L_m + w_c (1-p_T) L_m \tag{6-17}$$

where

p_T = percentage of trucks in the left-turn traffic flow (decimal).

Based on the 95 percent confidence level, the maximum queue length for G/C = ratio 0.5 and cycle length 60 seconds are shown in Figs 6-8 through 6-10. For G/C ratio other than 0.5, some modifications have to be made on the opposing volume in order to use Figs 6-8 through 6-10. Basically, if two opposing volumes with different G/C ratios have the same left-turn capacity, left turners might be delayed to the same degree. Therefore, an opposing volume, Q'_o, with any G/C ratio can be converted using Eq 4-39 to an opposing volume Q_o with G/C = 0.5 such that the left-turn capacity remains the same.

CORRECTIONS FOR TRUCKS AND BUSES

So far it has been assumed that the traffic population consists of passenger cars only. For traffic flows with passenger cars mixed with trucks and buses the left-turn warrants obtained in the previous sections have to be modified. As discussed in Chapter 4, the left-turn capacity for mixed

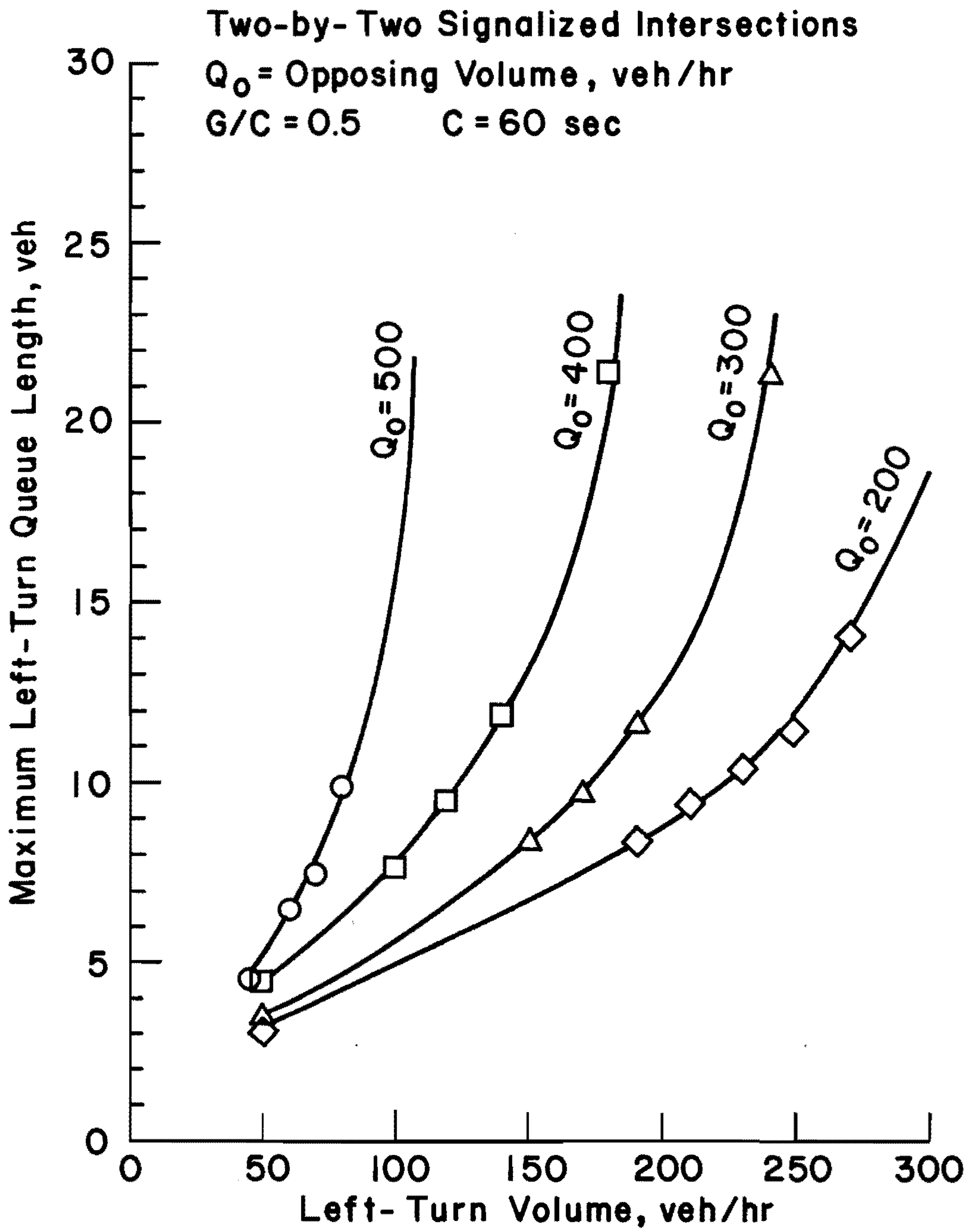


Fig 6-8. The maximum number of left-turn vehicles stored in the bay under various traffic conditions at two-by-two signalized intersections.

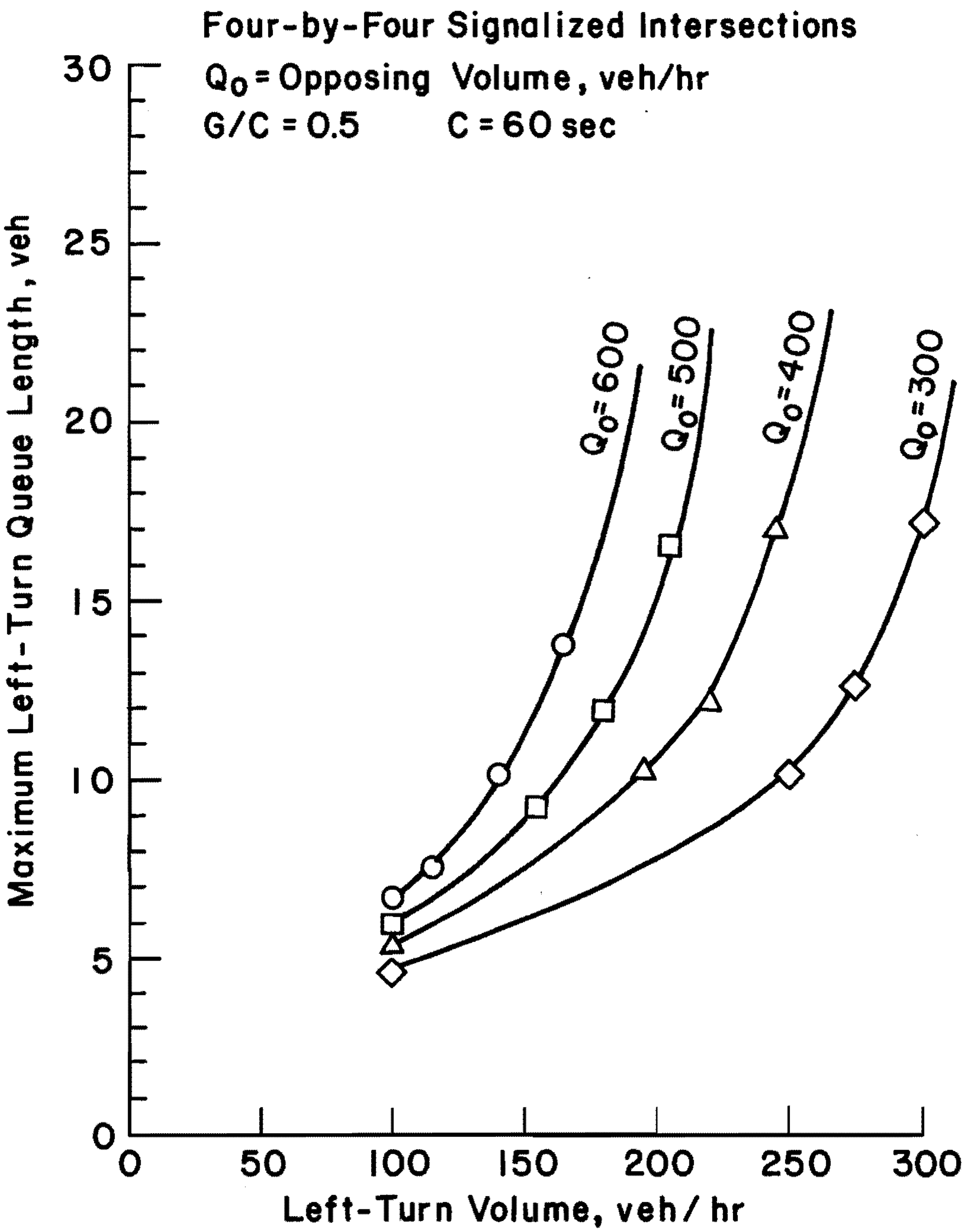


Fig 6-9. The maximum number of left-turn vehicles stored in the bay under various traffic conditions at four-by-four signalized intersections.

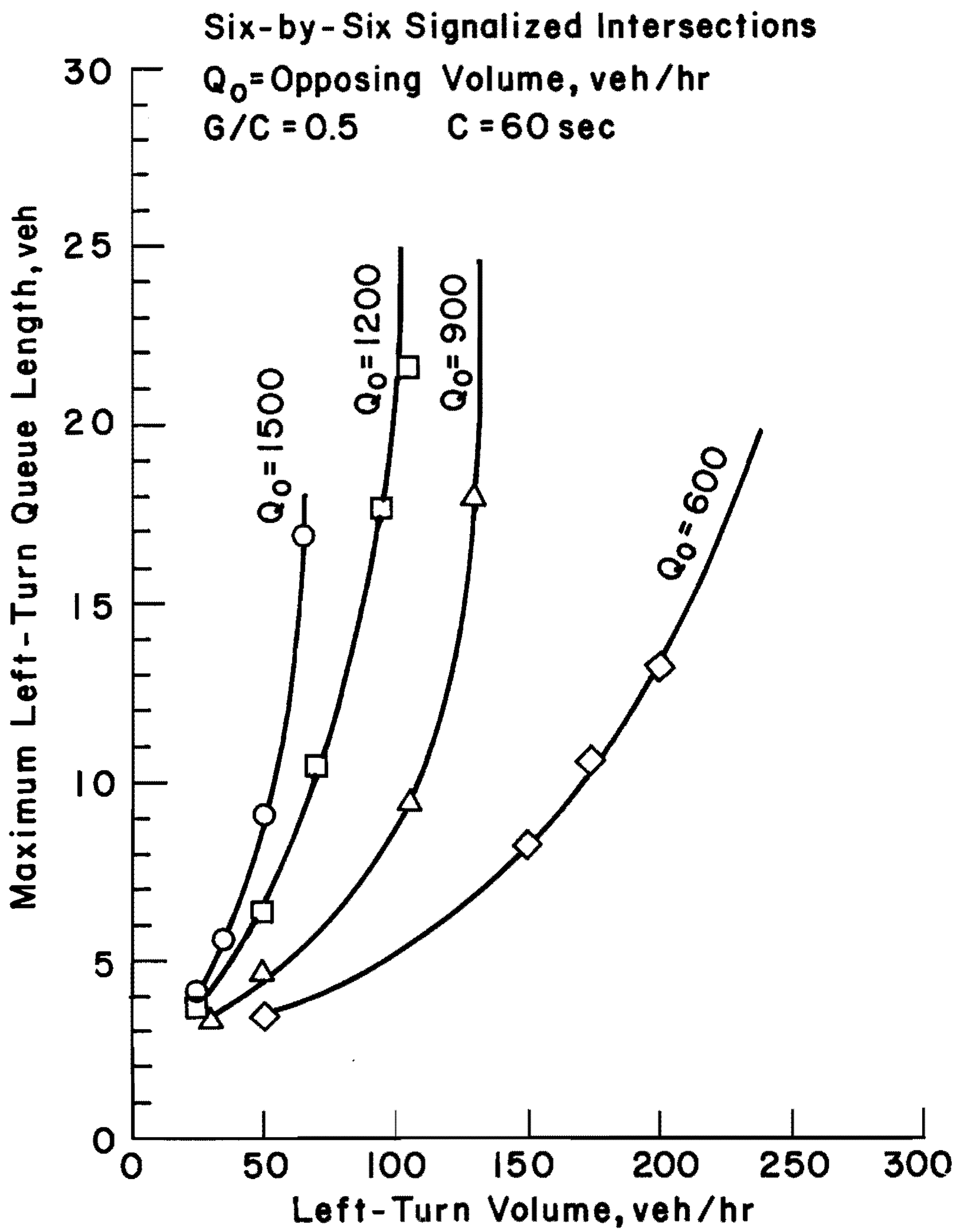


Fig 6-10. The maximum number of left-turn vehicles stored in the bay under various traffic conditions at six-by-six signalized intersections.

traffic flows can be obtained by adjusting the "truck free" capacity as follows:

$$Q^*_L = f_T Q_L$$

where

Q^*_L = left-turn capacity for mixed traffic flows, vph;

Q_L = left-turn capacity for traffic without trucks and buses, vph; and

f_T = correction factor for trucks and buses, obtained from Fig 4-15.

Therefore, the left-turn warrant for mixed traffic will be

$$Q^*_w = Q^*_L - M \tag{6-18}$$

DISCUSSION

Although the four left-turn delay criteria adopted in this study have been suggested by researchers and practicing engineers, it is recognized that different criteria and methodologies might bring out different left-turn warrants. It seems appealing to have simplified left-turn warrants such as constant volume-capacity ratios or cross-products of volumes; however, simulation results from the TEXAS Model show little evidence of such simple relations. Alternatively, this study reveals a new type of capacity warrant. The warranted left-turn volume is set at some level below the left-turn capacity. This level may have different constant values over different ranges of opposing volume. This type of left-turn warrant, though more complicated, is more reasonable.

CHAPTER 7. SUMMARY AND CONCLUSIONS

The objective of this study has been to develop warrants for left-turn treatments at signalized intersections. The effort started with discussing how to determine the capacity for left-turn movements. Then different measures of effectiveness were selected for evaluating left-turn performance under various traffic conditions and geometric configurations. Finally, four delay criteria were employed for identifying critical conditions of left-turn operations: (1) 35 seconds of average left-turn delay, (2) 73 seconds of ninety percentile left-turn delay, (3) five percent left turners being delayed more than two cycles, and (4) four left turners in one hour being delayed more than two cycles. After careful examination of the relations between the critical left-turn volumes and their corresponding left-turn capacities, a new capacity warrant has been proposed. The recommended capacity warrant states that a left-turn treatment may be needed if the left-turn demand reaches the threshold located at M vehicles lower than the left-turn capacity. The value of M depends on the opposing volume, cycle split, geometric configuration, and the criteria adopted for identifying critical conditions. Moreover, if a left-turn bay is warranted, the required bay length was also suggested.

The capacity discussion first dealt with the simple case of cycle split 0.5, 60 second cycle length, and single opposing flow. The results were then modified to cope with more general cases of left-turn movements such as different cycle length, different cycle split, multiple opposing lanes, no left-turn bay, and the presence of trucks or buses. The recommended method, though conservative, is comprehensive. It is pointed out that both Webster and Fambro failed to recognize that the opposing traffic after queue dissipation does not have the same character as that of an uninterrupted flow. Consequently, the Webster method and Fambro's method tend to overestimate left-turn capacity. However, if the opposing volume were in terms of vehicles per hour of green, and the maximum number of left-turns through a single gap were made finite, Fambro's method would predict left-turn capacity within 10 percent of the results from the TEXAS Model. The Highway Capacity Manual method and Michalopoulos's method, on the average, estimate left-turn capacities about 20 percent and 35 percent higher than that of the TEXAS Model, respectively. It was found that the Australian Road Capacity Guide predicts left-turn capacity higher than any other methods examined.

Average left-turn delay is the key indication of left-turn performance. It was observed that the standard deviation and the ninety percentile value of left-turn delay are about 75 percent and 200 percent of the average left-turn delay, respectively. The average left-turn queue length can also be related to the average left-turn delay by Little's formula, as Eq 5-6 shows. If the average value of left-turn queue length is specified, the maximum left-turn queue length can be estimated from Eqs 6-15 and 6-16. Thus, once the average left-turn delay is known, the distribution of left-turn delay and variations in the left-turn queue length can be approximated. Moreover, the role that the left-turn capacity plays in the left-turn performance is revealed in Eq 5-11.

It has been recognized that different criteria and methodologies could generate different left-turn warrants. Generally, a desirable warrant would have reasonable accuracy and relative ease in application. From a practical point of view, delay warrants are undesirable because measuring delay is laborious and costly. Therefore, whenever delay can be related to traffic volumes or left-turn capacity, it is preferable to have volume warrants or capacity warrants. However, a single minimum level of left-turn volume is not an effective guideline, since it fails to reflect the effects of the opposing volume, signal-timing scheme, and geometric configuration on left-turn movements. The volume-product type of warrant, though it takes the opposing volume into consideration, has been found inadequate, especially for low opposing volume. The volume-capacity ratio type of warrant seems conceptually sound. However, from Eq 5-2, the degree of left-turn saturation is not the only parameter in determining left-turn performance. For the development of left-turn warrants, it must be emphasized that reasonableness and accuracy should be the primary concern instead of simplicity alone. From this study it has been concluded that complicated left-turn operations can not be characterized by a single parameter with reasonable accuracy, especially over a wide range of traffic conditions.

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