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16. Abstract

A computer program FWD-DYN has been developed to perform forward analyses simulating the Falling Weight Deflectometer test, and to perform inversion of actual field data to estimate the Young's moduli of the surface layer, the base, and the subgrade of a pavement system. In the first case the dynamic displacements at various points along the surface are computed under the effect of a transient impact load distributed over a small circular area. The analysis assumes that the layers are horizontal, that they extend to infinity in the horizontal directions, and that their properties are constant over the extent and thickness of each layer. It is further assumed that all materials are linearly elastic with a very small amount of frequency-independent, hysteretic-type damping. The analyses are performed first in the frequency domain and then converted to the time domain using the Fast Fourier Transform (FFT). Details of the formulation are presented. For the inverse analysis three options are available: to use only the peak displacements (as is done normally at present); to use a complete dynamic analysis in each cycle of iteration with the experimental data; or to use an intermediate procedure in which the dynamic motions recorded at each station are used to estimate first what would be the static displacements, and a static inversion is then performed. These three options are discussed and documented. Using the computer program, a number of parametric studies were conducted (with the forward analysis option) to assess the sensitivity of the results to variations in the layer moduli, the thickness of the layers, and the depth to bedrock. These studies complement those which had been carried out in previous projects and are intended to illustrate the capabilities and potential of the FWD, serving also as a basis for the inversion procedure adopted in the computer program. A user's manual of the program is included.

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FWD-DYN: A COMPUTER PROGRAM FOR FORWARD ANALYSIS AND INVERSION OF FALLING WEIGHT DEFLECTION DATA

by

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Research Report 1970-1F

Research Project 7-1970
Development of Dynamic Analysis Techniques
for the Falling Weight Deflectometer

conducted for

Texas Department of Transportation

by the

Center for Transportation Research
Bureau of Engineering Research
The University of Texas at Austin

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IMPLEMENTATION STATEMENT

The results of parametric studies conducted under this project have confirmed the potential importance of dynamic effects on the magnitude and shape of the deflection basins obtained with the Falling Weight Deflectometer. The use of static analyses to backfigure the elastic moduli of the subgrade, the base, and the surface layer directly from the field data can lead therefore to substantial errors in many practical cases. The computer program FWD-DYN developed as part of this project allows the user to simulate the operation of the FWD on a pavement system with known properties (forward analysis) or to backfigure the layer properties from recorded field data (inversion). Inversion can be performed statically, as normally done now, with full dynamic analyses or with static analyses applied to modified data from which the dynamic effects have been eliminated. While the full dynamic analysis is the best procedure, it is more time-consuming than the others. The program has been tested with computer generated data but needs a deeper evaluation with actual field data. A combined experimental-analytical program using the FWD as well as other alternative techniques at sites where the properties are known is highly desirable. The efficiency of the computation in the full dynamic inversion can be improved with minor modifications. It would seem that the quality of the data and the reliability of the inversion results could also be enhanced by introducing some minor modifications in the FWD, such as recording a longer duration of the station displacements. Finally, the feasibility of combining the standard FWD operation with features of other nondestructive testing techniques should be explored.

Prepared in cooperation with the Texas Department of Transportation.

DISCLAIMERS

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SUMMARY

A computer program FWD-DYN has been developed to perform forward analyses simulating the Falling Weight Deflectometer test, and to perform inversion of actual field data to estimate the Young's moduli of the surface layer, the base and the subgrade of a pavement system. In the first case the dynamic displacements at various points along the surface are computed under the effect of a transient impact load distributed over a small circular area. The analysis assumes that the layers are horizontal, that they extend to infinity in the horizontal directions and that their properties are constant over the extent and thickness of each layer. It is further assumed that all materials are linearly elastic with a very small amount of frequency independent, hysteretic type damping. The analyses are performed first in the frequency domain then converted to the time domain using the Fast Fourier Transform (FFT). Details of the formulation are presented. For the inverse analysis three options are available: to use only the peak displacements (as is done normally at present); to use a complete dynamic analysis in each cycle of iteration with the experimental data, or to use an intermediate procedure in which the dynamic motions recorded at each station are used to estimate first what would be the static displacements, and a static inversion is then performed. These three options are discussed and documented. Using the computer program, a number of parametric studies were conducted (with the forward analysis option) to assess the sensitivity of the results to variations in the layer moduli, the thickness of the layers and the depth to bedrock. These studies complement those which had been carried out in previous projects and are intended to illustrate the capabilities and potential of the FWD, serving also as a basis for the inversion procedure adopted in the computer program. A user's manual of the program is included.

CHAPTER 1. INTRODUCTION

Dynamic nondestructive testing techniques have been extensively used for some years now to evaluate the structural capacity and integrity of highway and airfield pavements. These techniques can be grouped into two general categories: (1) deflection basin tests, and (2) wave propagation tests.

Deflection basin tests are those in which the deflections are recorded along the surface of a pavement subjected to a steady state harmonic load or a transient dynamic load, while deflection basins are computed using the peak displacements at each point. Typical of this group are the Dynaflect and Road Rater tests (steady state loads) and the Falling Weight Deflectometer (transient load). For steady state tests, the peak displacements are the amplitudes of the steady state vibrations recorded at the different receivers. Using these amplitudes alone ignores the fact that, because there will be phase differences between the motions, the peak displacements will not occur at the same time at all the receivers. For transient tests, the deflection basin uses the maximum displacements recorded at any time, though again these peak deflections will not occur at the same time (it takes some time for the motion to propagate from one receiver to the next). The interpretation of the deflection basins, in order to backfigure the elastic moduli (normally Young's modulus of elasticity E) of the pavement layers, is normally performed assuming that the thicknesses of the layers (surface layer, base, and subgrade) are known, considering initial values of the moduli, performing a forward analysis to determine the theoretical deflections corresponding to the assumed pavement system, comparing the analytical deflection basin to the experimental one, introducing changes in the assumed properties, and iterating until a satisfactory agreement between theoretical predictions and field data is obtained. The direct, or forward, analyses conducted in each cycle of the iteration process are normally performed at present using a static formulation and assuming that the subgrade (bottom layer) extends to infinity. This approach neglects the dynamic nature of the tests and the fact that, in many cases, the soil will be underlain at some depth by much stiffer, rock-like material, with a consequent abrupt contrast in elastic properties.

The second general category of dynamic nondestructive tests is that of wave propagation, of which the Spectral Analysis of Surface Wave (SASW) technique is an example. In this case, the time histories of the motions generated by an applied dynamic load are again recorded at two or more receivers along the surface of the pavement. Instead of working with the amplitude of these motions, the recorded time histories are automatically converted to the frequency domain using a Fast Fourier Transform (FFT) algorithm, with the phases between the motion at two points computed as a function of frequency (spectral analysis). For each frequency one can then compute from the phase difference the travel time between adjacent receivers; from this travel time and from the receiver spacing, the apparent velocity of propagation of the waves in the horizontal direction (phase velocity) can be obtained. This provides a dispersion curve that relates phase velocities to frequencies or wavelengths. Thicknesses and stiffnesses of the pavement layers are then calculated by an inversion process based on the propagation of generalized plane surface waves of the Rayleigh type. This procedure, which is the one most commonly used in practice, works well when dealing with soil profiles where the stiffness (elastic modulus) increases gradually with depth. The results become ambiguous, however, and much harder to interpret when dealing with stiff layers overlying softer material, which is the case encountered in pavement systems. A full three-dimensional dynamic analysis is then necessary to simulate properly the actual test and to solve correctly the inverse problem.

Thus, while the two types of tests have some clear differences, the interpretation of the data they provide and the use of these data to backfigure the properties of the pavement layers require a dynamic analysis. The same programs used to perform forward modeling (accounting for dynamic effects) for one of these tests can then be easily extended to model the others. Both types of tests do in fact complement each other, such that together they provide a more complete picture of the pavement system.

Among these techniques, the Falling Weight Deflectometer (FWD) has seen the most widespread use, in large part because of its ability to impose high amplitude dynamic loads on a pavement, similar to those that may be imposed by truck traffic. The FWD consists of a drop weight mounted on a vertical shaft and housed in a trailer that can be towed by most conventional vehicles. The drop weight is hydraulically lifted to predetermined heights ranging from 2 to 20 inches (5 to 51 cm). The weight is then dropped onto an 11.8-inch-diameter (30-cm) loading plate resting on a 0.22-inch-thick (5.6 mm) rubber buffer. The resulting load is a force impulse with a duration of approximately 30 to 33 msec, and a peak magnitude ranging from about 2,000 to more than 20,000 lbs (9,000 to 90,000 N), depending on the drop height, drop weight, and pavement stiffness. The applied force and the vertical displacements at various points along the surface of the pavement are measured by a load cell and a set of vertical velocity transducers. A typical arrangement of the load and the recording stations is shown in Figure 1.1.

As pointed out earlier, the inversion process to estimate the elastic modulus of the surface layer, the base, and the subgrade is normally performed using only the peak displacements to construct a deflection basin (ignoring the fact that they occur at different times), and assuming that this is the deflection basin that would be obtained if the maximum load were applied statically. As a result, in some older FWD systems, only the peak values of the applied force and the displacements are stored. This should not, however, pose a limitation that cannot be overcome in FWD testing. In newer FWD systems, the complete time histories of the applied force and the station displacements are recorded and saved for a duration of at least 60 mseconds. A duration of 120 msec — and preferably 240 msec or more — would provide much more complete information and would increase the reliability of the computations. Records are typically digitized at time intervals of 0.2 msec.

A number of studies have been conducted over the last few years to better understand the effect of various parameters on the deflection basins obtained with the FWD. The studies conducted at The University of Texas at Austin for TxDOT have been primarily of an analytical nature. A computer program to perform forward analyses simulating the FWD tests was developed by Shao (1985), who conducted preliminary studies to investigate the effect of the depth to bedrock on the amplitude and shape of the deflection basins, after calibrating the numerical model. Shao discussed the importance of dynamic effects when there is bedrock at a shallow depth showing the ratio of dynamic to static displacements for the same pavement models, and when the subgrade is assumed to extend to infinity (as in the inversion studies normally). The effect of the finite width of a pavement and the position of the test (load and receivers) with respect to the edge of the pavement was investigated by Kang (1990), who developed a more powerful (and also more time-consuming) program to study the dynamic response of systems with two-dimensional geometries subjected to three-dimensional loads. Kang's studies showed that edge effects will be generally negligible for flexible pavements and will be important only for rigid pavements if the test is performed at distances less than about 4 ft (1.20 m) from the pavement edge. The effects are more significant when dealing with an embankment or a ramp with or without retaining walls. The possibility of nonlinear material behavior in the surface layer or the subgrade when using the largest loads (largest drop weights and heights) was investigated by Chang (1992), who developed new computer programs to perform nonlinear dynamic analyses approximately in the frequency domain or, more accurately, in the time domain. The results of Chang's studies indicated that for flexible pavements over relatively soft subgrades, nonlinear effects are likely to occur for loads of 10,000 lbs (45,000 N) or higher, whereas for stiff pavements and relatively stiff subgrades, they may not be significant until the applied load nears 20,000 lbs (90,000 N). Nonlinear effects are particularly significant under the load and decay quickly with distance. They are negligible in most cases at distances of 3 ft (1 m) or more. As a result, the material properties will change not only with depth (from layer to layer), but also in the horizontal direction; moreover, the deflection basin will change not only in amplitude, but also in shape (this is also the case when dealing with important dynamic effects versus static solutions). Additional and more extensive parametric studies for the case of linear behavior and horizontal layers of infinite extent (the normal model), extending the work of Shao, were conducted by Seng (1992).

All the above-mentioned studies were conducted using forward analyses, determining analytically for a known pavement profile the deflection basins that would result from application of FWD type loads. Except for the nonlinear studies conducted by Chang (1992), they were all concerned exclusively with the computation of the surface displacements. Inverse analyses, when performed to investigate the effect that changes in the deflection basins would produce on the estimated properties (and thus the reliability of the predictions), were conducted with an existing computer program, MODULUS, which assumes static conditions. The objectives of Project 1970 were to add to the existing computer program for forward analyses the capability to compute stresses and strains at various depths, and to implement an inversion procedure that would account, if so desired, for the dynamic nature of the test. Chapter 2 summarizes the basic formulation used for the forward analyses. It is essentially the same as that implemented by Shao (1985) in his studies, with some improvements added, including an automatic generation for the sublayers and changes to increase the efficiency. Chapter 3 summarizes the inversion procedure implemented in the new computer program, FWD-DYN. It is based on a least squares minimization of the differences between the computed and measured deflection basins. It offers three alternatives: a static solution as conducted at present (but with the least squares minimization), a full dynamic solution, and an intermediate approach in which the values of the displacements that would be obtained if a static load were applied are first obtained by eliminating dynamic effects and a static inversion is then performed on these values. Results of further parametric studies which have served as a basis for the inversion procedure are presented in Chapter 4. Chapter 5 contains a small number of conclusions and recommendations for further work. While the FWD has some limitations at present, particularly because of the simplifying assumptions and approximations used in the inversion process, it is a valuable testing procedure and it could be further improved with relatively simple modifications. There is also a need to field-verify the inversion procedure and to compare the moduli predicted from FWD data with those that would be obtained with other nondestructive testing techniques (such as the SASW, for instance). The user's manual for the FWD-DYN program is included in the Appendix.

CHAPTER 2. THEORETICAL FORMULATION

2.1 Introduction

A basic step in any inversion or back-calculation process is to have a theoretical model which predicts the response of a given system with known properties to an excitation or input acting on the system. In the case of the FWD test, the response is represented by the displacements at the different receivers, the system is the pavement structure, and the excitation is the FWD load. In this chapter, the theoretical formulation needed to compute the response of the pavement structure subject to the FWD load is presented.

The pavement is modeled as composed of horizontal layers which extend to infinity and are underlain by a rigid bedrock layer at a finite depth or by an elastic half-space. The material is assumed to be isotropic and linearly elastic with a very small amount of frequency-independent, hysteric-type damping. Also, full interface bonding is assumed at the layer interfaces. A more realistic model would take into consideration the finite width of the pavement (Kang, 1990) and the possibility of nonlinear material behavior (Chang, 1992). These more realistic models would be computationally more time-consuming than the model implemented in the FWD-DYN program. The effects of the finite width and the material nonlinear behavior have been briefly discussed in Chapter 1.

To understand how a pavement system responds to dynamic loads applied at the surface, it is helpful to review theoretical studies dealing with the dynamic response of uniform and layered systems.

2.2 Dynamic Loads on a Semi-infinite Medium

Lamb (1904) was the first to study the effect of a pulse on a uniform elastic half-space. Lamb treated four basic problems: dealing with surface line and point-load sources, and with buried line and point-load sources. He derived his solution for these problems through Fourier synthesis of the steady-state propagation solution. For the surface source problem, Lamb evaluated the surface displacements (horizontal and vertical), and pointed out that the largest disturbance in the far field is the Rayleigh surface wave. He noted the nondispersive nature of the solution, and that, for point-load excitation, it decays as $\sqrt{r}$, where r is the distance from the source. Through the years these problems have been referred to as Lamb's problem.

The first closed-form solution for Lamb's problem in three-dimensional space was provided by Pekeris (1955) for the particular case of a material with Poisson's ratio of 0.25. A generalization for arbitrary values of Poisson's ratio is due to Mooney (1974) and can also be found in Erigen and Suhubi (1975); however, the Green's functions (in the time domain) for this case are available only for a vertical point pulse with a step time-function acting on the free surface.

Miller and Pursey (1954) considered the case of a circular disk vibrating harmonically and normally on the free surface of a half space. They found explicit expressions for the displacements at points at great distances from the loaded area. These expressions for the horizontal and vertical (u,w) displacements at the surface of the medium owing to a unit disk load are of the form:

$$f(v)\cdot\frac{R^2}{G}\cdot\sqrt{\frac{\omega}{Cr\cdot r}}\cdot e^{-i\frac{\omega r}{Cr}} \qquad (2.1)$$

where R is the radius of the disk load, G is the shear modulus of the medium, w is the circular frequency of excitation, Cr is the Rayleigh wave velocity of the medium, and r is the distance to the source. The term f(u) is a constant, which is function of Poisson's ratio. For u equal 1/3 for instance, f(u) equals -0.182($\sqrt{2}$/2 + i $\sqrt{2}$/2) for the horizontal displacement and f(u) equals 0.286($\sqrt{2}$/2 - i $\sqrt{2}$/2) for the vertical displacement.

While closed-form solutions to Lamb's problem have significant theoretical interest, it is improbable that exact solutions will become available soon for somewhat more complicated material or load configurations because of the great mathematical difficulties involved. Thus, for the solution of dynamic problems in layered media like pavement systems, numerical techniques need to be used.

2.3 Dynamic Loads on Layered Media: Application to the Dynamic Analysis of Pavements

Consider a pavement system that consists of horizontal layers. The mass densities and elastic moduli change with depth, from layer to layer, but are (assumed to be) constant over each layer. For the present application, the top layer represents the pavement surface layer (assuming that it extends to infinity in both horizontal directions), the second layer is the base, and the remaining layers are a sub-base layer and/or the soil subgrade. Determination of the response of this system to dynamic loads applied on the surface (or at any point within the profile) falls mathematically into the area of wave propagation theory.

Formulation of these problems starts normally by considering steady-state harmonic forces and displacements at a given frequency. For an arbitrary transient excitation (in the case of the FWD test), the time history of the specified forces must be decomposed into different frequency components using a Fourier series or, more conveniently, a Fourier transform. Results are then obtained for each term of the series (each frequency) and combined to obtain the time history of displacements (inverse Fourier transform).

2.3.1 Steady-state Response

Considering an isolated layer with uniform properties, for a given frequency w, the stresses and displacements along the top and bottom surfaces can be expanded in a double Fourier series (or Fourier transform) in the two horizontal directions for Cartesian coordinates, or in a Fourier series in the circumferential direction and a series of modified Bessel functions in the radial direction for cylindrical coordinates. For each term of these series, corresponding to a given wave number, one can determine closed-form analytical expressions in the form of a transfer matrix relating amplitudes of stresses and displacements at the bottom surface to the corresponding quantities at the top (or vice versa). This approach (Thomson, 1950, and Haskel, 1953) has served as the basis for most studies on wave propagation through layered media for the last 35 years. An alternative is to relate the stresses at both surfaces to the displacements, obtaining a dynamic stiffness matrix for the layer (Kausel and Roesset, 1981), which can be used and understood in much the same way as is done in structural analysis. For a half space, the stiffness matrix directly relates stresses and displacements at the top surface, because the bottom surface is pushed to infinity.

For the particular case at hand (FWD testing of pavements), with an axisymmetric load, only one term of the Fourier series is needed (the 0 term), and the radial and vertical displacements u, w can be expressed as

$$U = \begin{Bmatrix} u \\ w \end{Bmatrix} = \int_0^{\infty} -k \begin{bmatrix} J_1(kr) & 0 \\ 0 & J_0(kr) \end{bmatrix} \overline{U} dk \tag{2.2}$$

where k is the wave number, r is the radial distance to the center of the loaded area, and J_1 and J_o the Bessel functions of first and zero order, respectively.

At the surface where the excitation is being applied, the load vector can be expressed in the spatial domain as

$$P = \begin{Bmatrix} P_r \\ P_z \end{Bmatrix} = q \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad 0 \le r \le R \tag{2.3}$$

where q is the amplitude of the load, and R the radius of the disk load. In the wave number domain the load can be expressed as

$$P = \frac{1}{2\pi} \int_{r=0}^{\infty} \int_{\theta=0}^{2\pi} -r \begin{bmatrix} J_1(kr) & 0 \\ 0 & J_0(kr) \end{bmatrix} P dr d\theta \tag{2.4}$$

Performing the integration the only nonzero term of the vector $\overline{P}$ is the second term, which is equal to $\frac{q \cdot R}{k} \cdot J_1(kR)$.

The displacements $\overline{U}$ and forces $\overline{P}$, in the wave number domain are then related by

$$K\,\overline{U} = \overline{P}$$

where K is the dynamic stiffness matrix of the profile obtained by assembling the stiffness matrices of the layers and the underlying half space.

If $\overline{u_1}$ and $\overline{w_1}$ are the first two terms of the vector $\overline{U}$, obtained by solving Eq. (2.5) for a vector $\overline{P}$ with all components 0 and a 1 as the second term (for every value of k), the surface displacements as a function of the distance r to the center of the loaded area become

$$u = q \cdot R \int_0^{\infty} \overline{u_1} \cdot J_1(kR) \cdot J_1(kr) dk \tag{2.6a}$$

$$u = q \cdot R \int_0^{\infty} \overline{u_1} \cdot J_1(kR) \cdot J_0(kr) dk \tag{2.6b}$$

The solution of the problem thus requires assembling the dynamic stiffness matrix K of the layered medium, solving the system of Eq. (2.5) for many different values of k, and evaluating numerically the integrals of Eq. (2.6). The numerical integration is performed shifting the poles of the integrand by including a small attenuation in the materials (for materials with damping, all of the poles are complex, so that no singularities are encountered along the real axis of integration). However, for systems with sharp variation in material properties between layers, the integrands may exhibit considerable waviness, making it difficult to evaluate the integrals. The solution of the equations is time-consuming when there is a large number of layers. The procedure is convenient when dealing with a homogeneous half-space or with a smaller number of layers.

An alternative can be obtained by expanding the terms of the dynamic stiffness matrix of a layer in terms of k and keeping terms only up to second-degree (the terms of the transfer or stiffness matrices of each layer are transcendental functions). It can be shown that this is equivalent to assuming that the displacements have a linear variation with depth over each layer using standard finite element techniques to derive the layer matrix. The stiffness matrices of each layer, the half space, and the total profile can then be expressed in the form

$$K = Ak^2 + Bk + G - \omega^2 M \tag{2.7}$$

where the expressions for the matrices A, B, G and M can be found in Kausel (1974). By computing the in-plane modes of propagation as the solution of a quadratic eigenvalue problem (Waas, 1972, and Kausel, 1974) and keeping only the modes propagating outwards, Kausel [13] has shown that the displacements $\overline{u}_i$, $\overline{w}_i$ in Eq. (2.6) can be expressed as

$$\overline{u_1} = \sum_{i=1}^{2n+2} \overline{u}_{i1} \overline{w}_{i1} \frac{k}{k_i(k^2 - k_i^2)} \tag{2.8a}$$

$$\overline{w_1} = \sum_{i=1}^{2n+2} \overline{w}_{i1} \frac{1}{(k^2 - k_i^2)} \tag{2.8b}$$

for a system with n layers, where $\overline{u}_{i1}$, and $\overline{w}_{i1}$, denote the horizontal and vertical displacements at the surface in the ith mode and k_i is the eigenvalue or wave number in the ith mode (discrete Rayleigh wave modes and wave numbers, respectively). By substituting Eq. (2.8) with Eq. (2.6), the integrals can be evaluated analytically, obtaining the following closed form expressions for the displacements at the surface (Kausel, 1981)

$$\overline{u_1} = qR \sum_{i=1}^{2n+2} \overline{u_{i1}}\,\overline{w_{i1}}\ \frac{I_{2i}}{k_i} \tag{2.9a}$$

$$\overline{w_1} = qR \sum_{i=1}^{2n+2} \overline{w_{i1}^2} I_{1i} \tag{2.9b}$$

where the integral I_{1i} is

$$I_{1i} = \int_{k=0}^{\infty} \frac{1}{k^2 - k_i^2} J_0(kr) J_1(kR)\, dk \tag{2.10}$$

which for values of the imaginary part of k_i less than 0 (waves that propagate away from the source) is given by

$$I_{1i} = \frac{\pi}{2ik_i} J_0(k_i r) H_1^{(2)}(k_i R) - \frac{1}{Rk_i^2}, \text{ when } 0 \le r \le R \tag{2.11a}$$

$$I_{1i} = \frac{\pi}{2ik_i} J_1(k_i R) H_0^{(2)}(k_i r), \text{ when } r \ge R \tag{2.11b}$$

where $i = \sqrt{-1}$ and $H_1^{(2)}$ and $H_0^{(2)}$ are the Hankel functions of the second kind of first and zero order, respectively. The expressions for I_{2i} can be found in the work of Kausel (1981).

This formulation requires a subdivision of the layers (thin layers are needed to reproduce accurately the variation of displacements with depth with a piece-wise linear approximation). It is particularly convenient when dealing with a large number of layers, as is the case when it is desired to obtain a detailed variation of soil properties with depth. Furthermore, since the fundamental solutions (or Green's functions) are known explicitly, one can determine the displacements or strains at many locations without significant additional computational effort.

2.3.2 Transient Response

One of several methods used to study wave propagation phenomena in a linear elastic or viscoelastic medium is by superposition of the response to steady-state harmonic excitations. The method, known as Fourier superposition, provides an easy way to study complicated transient events when the solution to the steady-state problem is known. It should be noted that the use of superposition techniques is limited to linear systems.

In the case of the modeling of the FWD test, the objective of the analysis is to obtain the time history of displacements $u_i(t)$ that would be recorded at receiver i due to a transient uniform disk load p(t) applied to the pavement structure (Figure 1.1). As a first step, the excitation p(t) is decomposed into its different frequency components p(w) by means of a direct Fourier transform. This Fourier transform is evaluated numerically using the Fast Fourier transform (FFT) algorithm.

The second step is to obtain the transfer functions $H_i(w)$, defined as the response of the system to a unit disturbance. In our case, $H_i(w)$ is the displacement at receiver i due to a harmonic vertical unit load acting on the surface of the pavement.

The third step is to obtain the direct Fourier transform of the displacements, $u_i(w)$, by multiplying the Fourier transform of the force by the transfer functions. That is

$$u_i(w) = H_i(w) \times p(w) \qquad (2.12)$$

which is evaluated for all the range of frequencies. Finally, the time history of displacements $u_i(t)$ at receiver i can be recovered through an inverse Fourier Transform, which is evaluated numerically with the same Fast Fourier Transform algorithm. It should be noted that in replacing the continuous Fourier transform by a discrete one using the FFT algorithm, it is assumed that the input function is periodic with a period Tp. In using the FFT, the values of the basic parameters involved (e.g., number of sampling points N, time increment t, and period Tp) have to be properly selected so that a compromise can be reached between the accuracy of results and the cost of computation. Finally, it should also be noticed that the transfer functions do not need to be computed for all the range of frequencies, and that interpolation techniques can be used effectively to reduce computation time.

2.4 Computer Implementation

The computation of the steady-state response (displacements) of a layered system due to a harmonic vertical load acting on the surface of the medium with the continuous formulation (Eq. 2.6) and the discrete formulation (Eq. 2.9) had been implemented at The University of Texas at Austin (Roesset and Shao, 1985, Foinquinos, Roesset, and Stokoe, 1993) to simulate the FWD test. Although a large number of sublayers must be used in the discrete formulation in order to obtain satisfactory results, this formulation has been found in general to be more efficient computationally than the continuous formulation. The FWD-DYN computer program implements, therefore, the discrete formulation.

The main algorithm of the FWD-DYN program for performing forward modeling can be summarized as follows:

(1) For each frequency, w

i) Assemble the total stiffness matrix of the profile (the exact continuous matrices are used in this step) and find the smallest value of k (Rayleigh wave number) that makes the determinant of the stiffness matrix zero through a determinant search technique.

ii) Use the radius of the disk load and the Rayleigh wave length corresponding to the fundamental mode (found in step i) to discretize the system in the vertical direction.

iii) Assemble the matrices A, B , G, and M (Eq. 2.7) and find the discrete Rayleigh wave numbers k, which make zero the determinant of K by solving a quadratic eigenvalue problem (Eq. 2.7). Find also the discrete Rayleigh wave modes.

iv) For each Rayleigh mode, compute the Bessel and Hankel functions to evaluate the integral I_{1i} (Eq. 2.11) and perform the summation of equation 2.9b to find the vertical displacements. The procedure is similar when computing strains or stresses using their corresponding expressions (not shown here).

(2) Fourier analysis

i) Perform the direct FFT of the load.

ii) Multiply the frequency components of the load by the transfer functions of displacements or strains found in step (1).

iii) Perform the inverse FFT on the frequency components of the displacements to find the time history of displacements (or strains).

In order to illustrate briefly the steps involved in the analysis, a flexible pavement profile with bedrock at 20 ft (6.1 m) shown in Table 2.1 was analyzed. Poisson's ratio, specific weight, and material damping were taken for all layers as 0.35, 120 pcf (18500 N/m^3), and 0.02, respectively.

Table 2.1 Flexible pavement with bedrock at 20 ft (6.1 m).

Layer	Thickness in. (m)	Young's Modulus ksi (MPa)	Shear wave velocities fps (m/sec)
Surface	6 (0.15)	436.7 (3013)	2500 (762)
Base	12 (0.30)	70 (483)	1000 (305)
Subgrade	222 (5.64)	18 (124)	500 (152)

Figure 2.1 shows the amplitude of the transfer functions (amplitude of displacements due to a unit harmonic load as a function of frequency) at station 1 (under the center of the load) and station 7 (the farthest measurement point), respectively. For each frequency, the steady-state displacements were computed following all the steps listed in the main step (1). Figure 2.2 shows the FWD load and the amplitude of its Fourier transform. By multiplying the frequency components of the load by the transfer functions at the different stations, the direct Fourier transform of the displacements is obtained. To obtain the time history of displacements at the different stations, an inverse Fourier transform of the frequency components of displacements is performed. Figure 2.3a shows the time history of displacements at each station, and Figure 2.3b shows the dynamic peak displacements at each station (deflection basin), which are obtained from the time histories.

Finally, in order to see the range of validity of Equation 2.1, which gives the far field surface displacements due to a unit vertical disk load vibrating harmonically in the surface of a homogeneous half space, the variation with distance of the real and imaginary parts of the vertical displacements due to a unit load vibrating at a frequency of 100 Hz are compared with the ones obtained using the discrete formulation and shown in Figure 2.4. The properties of the medium are: Shear wave velocity of 1000 fps (304.8 m/s), Poisson's ratio equal to 1/3, and mass density of 120 pcf (18,500 N/m^3). It can be seen that for distances of half a wave length or more the agreement is already very good between both solutions.

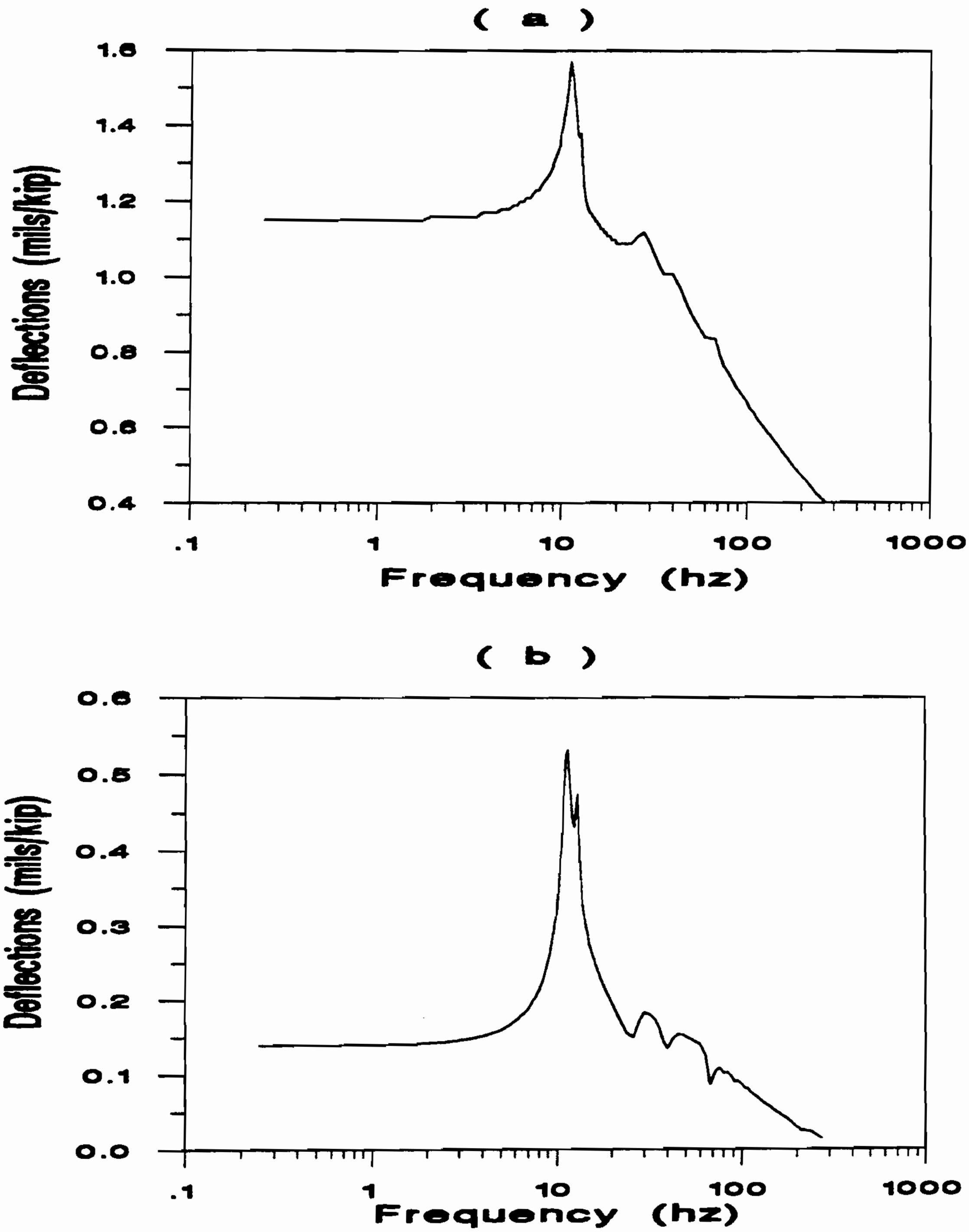


Figure 2.1. Dynamic response of a flexible pavement with bedrock at 20 ft (6.1 m) to FWD loading: (a) and (b) transfer functions at recording stations 1 and 7.

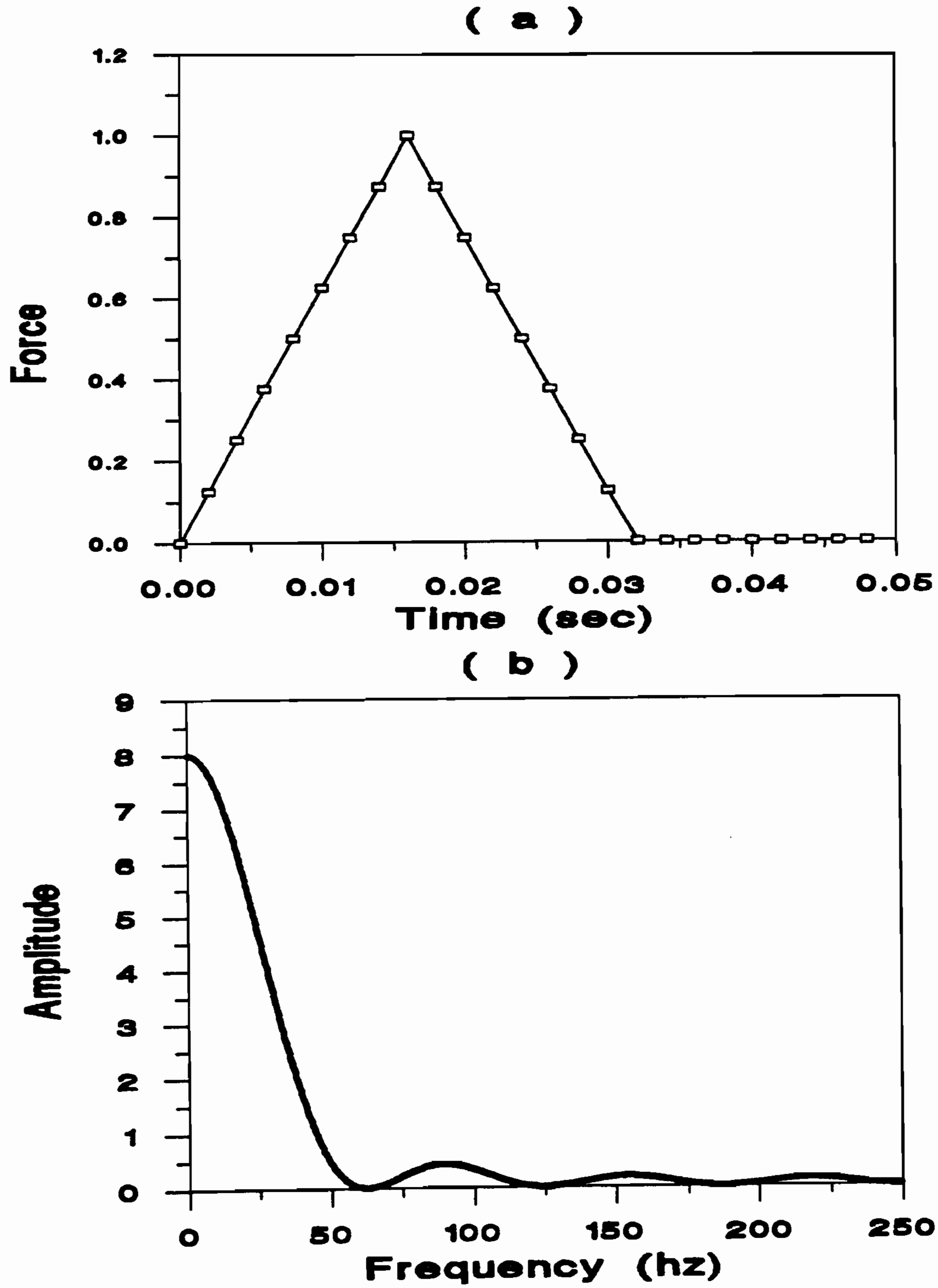


Figure 2.2. Simplified loading history of FWD test and its corresponding load spectra.

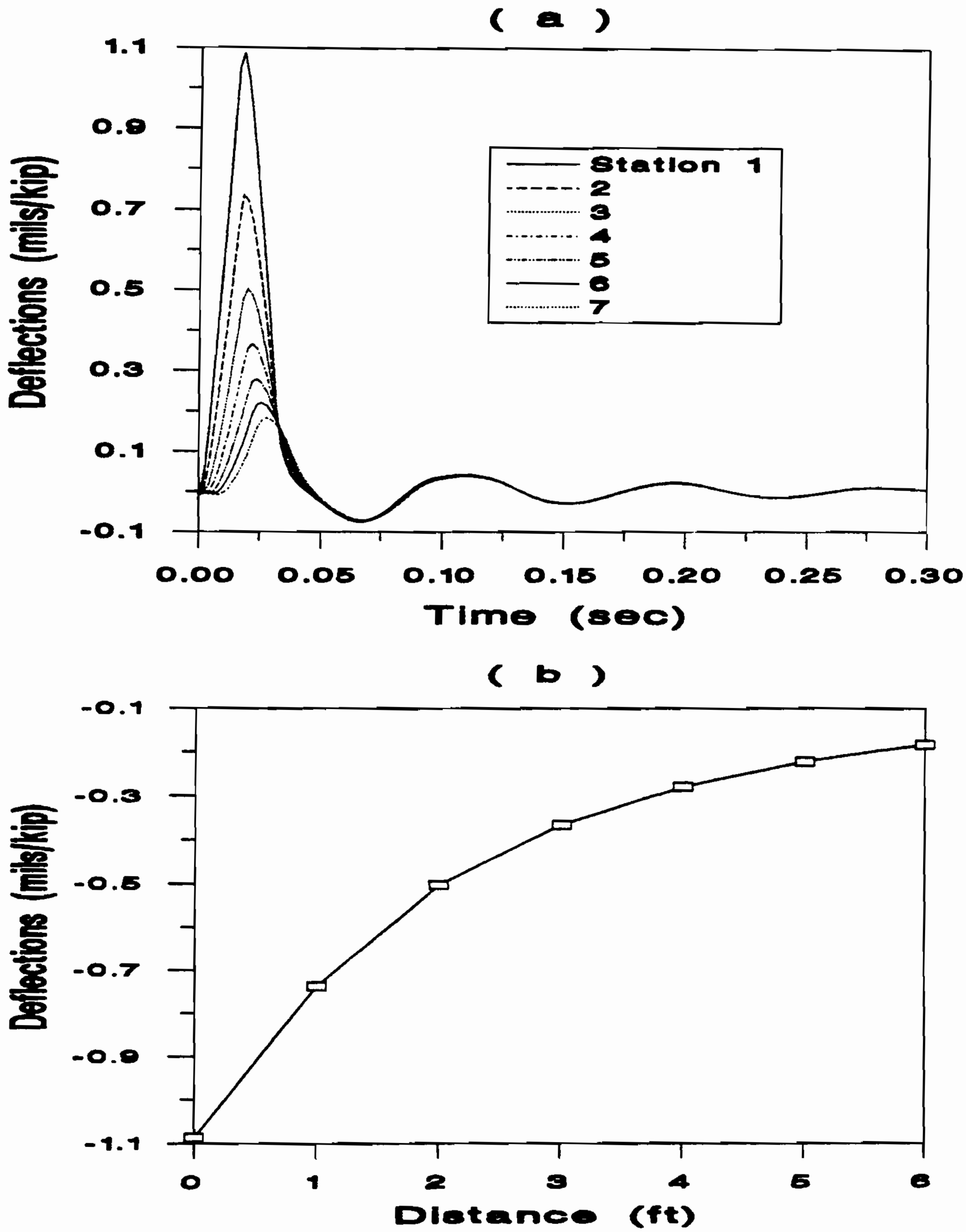


Figure 2.3. Dynamic response of a flexible pavement with bedrock at 20 ft (6.1 m) to FWD loading: (a) displacement-time histories and (b) measured deflection basin.

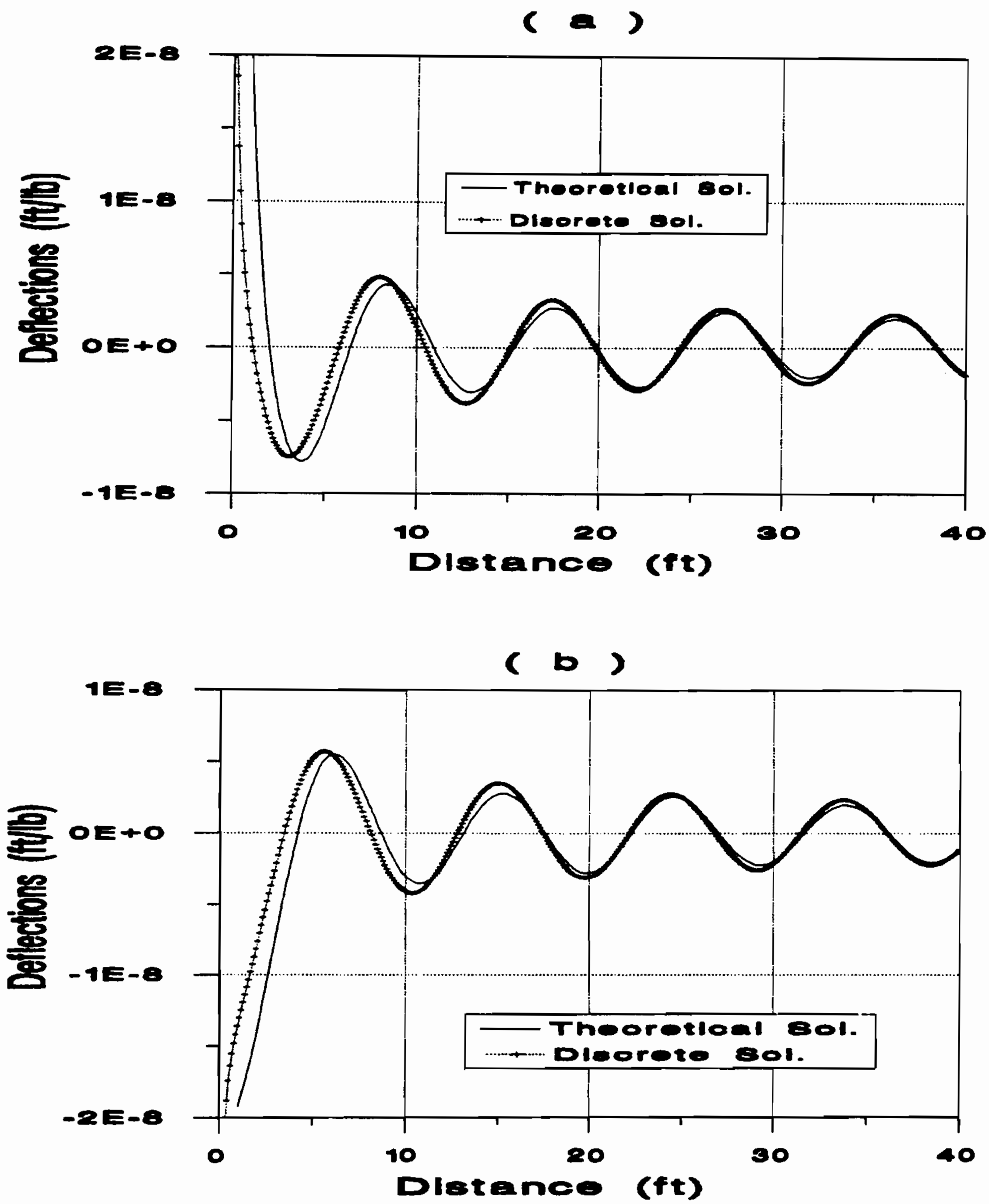


Figure 2.4. Response of elastic half space subject to a uniform vertical disk load vibrating with a frequency of 100 Hz obtained with the theoretical and the discrete solution. (a) and (b) real and imaginary part of the vertical displacements.

CHAPTER 3. INVERSION

3.1 Introduction

Inversion or back-calculation problems, also referred to as parameter identification or system identification problems, are common in many areas of science and engineering. Basically, they involve situations in which an excitation or input is applied to a system to determine from the measured response the properties of that system. An analytical or numerical model is needed to describe the response or output of a system with known properties to a given excitation or input.

In FWD test interpretation, the input is the impulse load applied to the pavement structure by the falling weight, the output is the time history of displacements at the different receivers, and the system is the pavement structure. The theoretical formulation for computing the response of the pavement structure due to the FWD load has been presented in Chapter 2. In general the dynamic response of the pavement structure depends on the elastic modulus, thickness, Poisson's ratio, and mass density of each layer.

Variations in values of the mass density and Poisson's ratio have a small effect on the dynamic response of the pavement structure for reasonable choices of these variables; accordingly, these parameters are assumed to be known. The thicknesses of the pavement can also be found from construction records, leaving as the main unknowns the elastic modulus of the layers and the depth at which bedrock may be present.

Current methods of interpretation of the results of FWD tests use the maximum displacement at each velocity transducer to define a deflection basin, which is interpreted as having resulted from a statically applied load. Further, it is generally assumed that the soil subgrade extends to infinity. This approach neglects the dynamic nature of the test, and the fact that in many cases the soil will be underlain at some depth by much stiffer, rock-like material.

When the time history of load and displacements is recorded, the additional information available provides substantial insight into the properties of the system and facilitates significantly the inversion process.

The computer program FWD-DYN offers three options for inversion: to use only the peak displacements recorded at each station and to assume that these are static displacements (as is done normally at present); to use a complete dynamic analysis in each cycle of iteration matching the deflection basins obtained from each one of these analyses with the experimental data; or to use an intermediate procedure in which the dynamic motions recorded at each station are used to estimate first what would be the static displacements if the peak load were applied statically and then a static inversion is performed.

In the three inversion options mentioned, a least square type of optimization algorithm is used to match the measured and computed deflection.

3.2 Least Squares Minimization of the Relative Difference between Measured and Computed Deflections

Let us assume first that a preliminary estimate of the properties of the pavement layers is available. Since variations in values of Poisson's ratio and mass density have a small effect on the static or dynamic response of the pavement structure, these parameters are assumed constant after the initial estimate is made. The thicknesses of the pavement layers are also assumed to be known from construction records. Therefore, the unknown parameters are the elastic modulus of the pavement layers and the thickness of the subgrade if there is a rigid bedrock layer. The thickness of the subgrade can be estimated from the dynamic response of the pavement, as will be discussed later; the only remaining unknowns are the elastic modulus of the pavement layers.

Defining:

$\mathbf{E_O} = \{ E_{O1}, E_{O2}, ..., E_{ON} \}$ the initial assumed values of the Young's moduli of the layers, where N is the number of layers of the pavement profile,

$\mathbf{E} = \{ E_1, E_2, ..., E_N \}$ the unknown moduli of the layers,

d_i^m the measured deflection at location i,

$d_i^c(\mathbf{E})$ the deflection at location i calculated for Young's modulus of the layers **E**,

$R_i = \dfrac{d_i^m - d_i^c(Eo)}{d_i^m}$ the relative error at location i at iteration 0

Changing Young's modulus of the layers by a small amount Δ **E**, the new Young's modulus are given by

$$E_1 = Eo + \Delta\ E \tag{3.1}$$

The deflection with the new values of the model parameters can be expressed by a Taylor series expansion in the vicinity of Eo as

$$\begin{aligned} d_i^c(E) = d_i^c(Eo)\ &+ \sum_{j=1}^{N} \frac{\partial d_i^c(Eo)}{\partial E_j} \Delta E_j + \\ &\sum_{j=1}^{N}\sum_{m=1}^{N} \frac{\partial^2 d_i^c(Eo)}{\partial E_j \partial E_m} \Delta E_j \Delta E_m + \ldots \end{aligned} \tag{3.2}$$

Keeping only the first variation in (3.2) and defining a prediction relative error, or misfit at location i as

$$e_i = \frac{d_i^m - d_i^c(E)}{d_i^m} \tag{3.3}$$

or

$$e_i = R_i - \sum_{j=1}^{N} d'_{ij} \Delta E_j \tag{3.4}$$

with $d'_{ij} = \dfrac{\Delta d_i^c}{\Delta d_i^m \Delta E_j}$ being an approximation to the derivative of the deflection basin at location i due to an increase in the Young's modulus of layer j by $.\Delta E_j$

Defining the overall error E as

$$E\ =\ \sum_{i=1}^{M} e_i^{\,2} = \sum_{i=1}^{M} \left[R_i - \sum_{j=1}^{N} d'_{ij} \Delta E_j \right]^2 \tag{3.5}$$

M being the number of receivers, the total prediction relative error E (the sum of the squares of the individual errors) is exactly the squared Euclidean length of the vector **e**, or $E = e^T e$

By minimizing the total prediction relative error with respect to one of the model parameters, say E_k, we get:

$$\frac{\partial E}{\partial E_k} = \sum_{i=1}^{M}\left[R_i - \sum_{j=1}^{N} d'_{ij}\Delta E_j\right] d'_{ik} = 0$$

$$= \sum_{i=1}^{M} R_i d'_{ik} - \sum_{j=1}^{N}\left[\sum_{i=1}^{M} d'_{ij} d'_{ik}\right]\Delta E_j = 0$$

which results in: $\mathbf{A}\ \Delta\ \mathbf{E} = \mathbf{P}$ (3.6)

with

$$a_{jk} = \sum_{i=1}^{M} d'_{ij} d'_{ik}$$

$$P_j = \sum_{i=1}^{M} R_i d'_{ij} \qquad (3.7)$$

$j,k = 1, 2, \ldots, N$

Solving equation (3.6) and using equation (3.1), new values of the Young's modulus can be obtained. The process can be repeated until a good agreement between the measured and computed deflections is reached, or until the root-mean-square value (RMS) of the relative error defined by:

$$RMS = \frac{1}{M}\left(\sum_{i=1}^{M}\left[\frac{d_i^m - d_i^c(E)}{d_i^m}\right]^2\right)^{1/2} \qquad (3.8)$$

is less than a given tolerance.

3.3 Description of the Inversion Procedures Implemented in the FWD-DYN Program

Three inversion procedures are implemented in the FWD-DYN program. They are all based on a least squares minimization of the relative differences between the measured and computed deflection basins, as described above. They are:

1. Static Inversion. In this option, static inversion is applied directly to the deflection basins (values of peak displacements at each receiver) recorded in the field. It is assumed that these peak displacements are equal to those that would be

obtained if the peak value of the load were applied statically. Since from the information recorded (peak force and peak displacements) it is not possible to determine the depth to bedrock directly, the assumption is made that the subgrade extends to infinity in the computer program. (This approach could be upgraded using an approach similar to that employed in MODULUS, but inversion using the pseudo-dynamic or dynamic procedures described below directly determines the depth to bedrock.) The theoretical deflection basins are obtained from static analyses. These analyses are performed using the forward modeling option for a single frequency equal to zero.

An initial estimate of the properties of the subgrade is obtained taking the deflection at the last receiver as corresponding to an elastic half-space (the parametric studies conducted and presented in the next chapter indicate that the deflections at the outer stations depend mainly on the properties of the subgrade).

The solution obtained with this option would be equivalent to those obtained at present, and thus may be in serious error in certain cases, since it neglects entirely dynamic effects. The next chapter discusses in more detail the limitations of this method.

2. Pseudo-Dynamic Inversion. In this option, the time histories of the applied force and the measured deflections are used to obtain experimental transfer functions. From these transfer functions for values of zero or very low frequencies a "static" deflection basin is obtained. These are the displacements that would have occurred if the peak load had been applied statically. Also from the transfer functions one can see if there is a rigid bedrock layer at a finite depth (in this case the transfer function presents resonant peaks). If there is bedrock, the natural frequency of the subgrade (fn) is estimated from the location of the main peak in the transfer function; this is used to obtain a first estimate of the depth of bedrock using the formulas proposed by Chang et al (1992) given by:

$$h \cong 300/fn \tag{3.9a}$$

for h in ft, or

$$h \cong 90/fn \tag{3.9b}$$

for h in meters.

An estimate of the modulus of the subgrade is obtained taking the modified "static" deflection at the last receiver as corresponding to an elastic half-space (if the subgrade extends to infinity) or a one-layer system (rigid bedrock layer at a finite depth) with the thickness obtained with equation (3.9). Once Young's modulus of elasticity of the subgrade has been determined, an improved estimate of the depth to bedrock is obtained as (Chang et al, 1992)

$$h \cong \frac{60\sqrt{E}}{fn} \tag{3.10a}$$

with E in ksi and h in ft, or

$$h \cong \frac{\sqrt{E}}{4fn} \tag{3.10b}$$

with E in kN/m^2 and h in meters.

The inversion is carried out as in the static option but using the modified "static" deflection basin and the estimated depth to bedrock. This option is particularly attractive when there is sufficient duration of the displacement time histories to obtain reasonable transfer functions. The solution converges to results that are close to the exact values when applied to computer-generated data.

3. Dynamic Inversion. In this option, the measured deflection basin is matched with the theoretical one obtained with a full dynamic analyses. This option is much more time-consuming than the other two options, since in each cycle the analysis must be conducted for the complete set of frequencies (instead of a single one), with the results then transformed from the frequency to the time domain.

When the method is applied to the case in which only the peak force and peak displacements are recorded, the dynamic analysis is performed with the typical FWD time history load (shown in Figure 2.2a) scaled to the peak force. In this case, it is assumed that the subgrade extends to infinity.

In the case in which the time histories of force and displacements are recorded, the dynamic analysis is performed with the actual time history of force; the estimation of the depth to bedrock is performed as in option 2 (Pseudo-Dynamic inversion). When applied to computer generated data, the results converge to the exact values in very few cycles of iteration.

CHAPTER 4. PARAMETRIC STUDIES

4.1 Introduction

This chapter presents analytical studies of the effect of various parameters on the dynamic response of pavement systems to forces simulating the excitations of the FWD test. The forward modeling option of the FWD-DYN program was used to carry out the analyses. The information that can be extracted from the dynamic response of the pavements is presented. The deflection basins obtained with the dynamic model and those from static analyses are compared, and the effect of the depth to bedrock is discussed. The sensitivity of the deflection basins to the properties of the different layers is also investigated.

4.2 Description of the Pavement Profiles

Two generalized pavement profiles, a flexible one and a rigid one, were selected to illustrate the dynamic response of the pavement systems to applications of the FWD. Because variations in total unit weight (γ), Poisson's ratio (υ), and damping ratio (D) have minor effects on the dynamic response (within ranges of logical values), as compared with changes in the stiffnesses of the layers, they were taken to be the same for all the layers; that is $\gamma = 120$ lb/ft^3 (18,500 N/m^3), $\upsilon = 0.35$, and D = 0.02. The elastic properties and thicknesses of the layers in both profiles are given in Table 4.1.

4.3 General Description of the Dynamic Response of a Pavement System to the FWD Load

In order to illustrate the typical behavior of a pavement system subjected to the FWD load, as well as the kind of information that can be extracted from its dynamic response, the generalized flexible pavement given in Table 4.1 with rigid rock at 20 ft (6.1 m) was analyzed. Figures 4.1a and 4.1b show the amplitude of the transfer functions of the displacements (amplitude of displacements caused by a unit harmonic load as a function of frequency) at station 1, located at the center of the load, and at station 7, which is the farthest measurement point, respectively. It can be observed that, for low frequencies, the system behaves as if the load were applied statically. A "static" deflection basin can be extracted from the response at these very low frequencies — precisely what is done in the pseudo-dynamic inversion method presented in Chapter 3. As the frequency increases, the displacements increase until they reach a peak at the same frequency at all stations. The low amplitudes of displacements at high frequencies are a result of inertial effects. These transfer functions are multiplied by the Fourier transform of the excitation, and then inverted to obtain the displacement-time histories. Figure 4.2a shows the time history of displacements at each station. The main pulse is followed by oscillations, with decaying amplitude, which represent the free vibrations of the complete pavement system and the soil subgrade layer in particular. These free oscillations have a well-defined period that lies between the natural period of the subgrade for shear and compressional waves and are essentially the same for all the recording stations. The frequency of the free vibration coincides also with the frequency of the main peak in the transfer functions. Chang et al (1992) have suggested a simple formula to estimate the depth to bedrock based on the free vibration period from the displacement-time records, which can also be obtained from the main peak in the transfer functions (this formula was presented in Chapter 3). This figure also shows that there is a time offset at the start of the motion and at the occurrence of the peak displacements at the different stations. Seng et al (1993) have suggested the use of the offset time to find the shear velocity of the subgrade. Figure 4.2b shows the dynamic peak displacement at each station (deflection basin), which is obtained from the time histories.

4.4 Effect of Depth to Bedrock

Results of the analyses for the flexible pavement with different depths to bedrock are shown in Figures 4.3, 4.4, and 4.5. Figures 4.3a and 4.3b show the transfer function at stations 1 and 7, respectively. It can be observed that, as the depth to bedrock decreases, the peak displacement and the frequency at which it occurs increase, while the static displacement decreases. It can also be seen that the dynamic effect is more important at the farthest stations. Figures 4.4a and 4.4b show the displacement-time histories when the depth to bedrock is 20 ft (6.1 m) and when it extends to infinity, respectively. In the second case, the free oscillations are no longer present because there are no reflections from bedrock. Figure 4.5a shows that the static displacements are very sensitive to the depth of bedrock. However, the dynamic deflection basins are nearly independent of the depth to bedrock for depths larger than 20 ft (6.1 m), as shown in Figure 4.5b.

The same type of analysis was performed for the rigid pavement, with the results shown in Figures 4.6, 4.7, and 4.8. It can be observed again that, while the static displacements are very sensitive to the depth to bedrock, the dynamic deflection basins are nearly independent of this depth. Also, the shape and magnitude of the static and dynamic deflection basins differ completely from those of the flexible pavement, as illustrated in Figure 4.9, which compares the deflection basins for the flexible and rigid pavement when the depth to bedrock is 20 ft (6.1 m).

The ratio of dynamic to static displacements (amplification factor) at the different stations was computed as a function of depth to bedrock. The results are shown in Figures 4.10a and 4.11a for the flexible and rigid pavements, respectively. For these profiles, the maximum amplification occurs for a depth to bedrock of about 7 to 10 ft (2.1 to 3 m). This means that, for shallow profiles, the use of back-calculation (inversion) process based on static analysis (with a known depth to bedrock) would lead to an underestimation of the stiffness of the layers. For the rigid pavement, the deflection ratio becomes less than one for depths greater than about 15 ft (4.5 m). In this range, a static inversion procedure would lead to an overestimation of the stiffness of the subgrade layer, as well as to associated complications in evaluating the other layers.

In static back-calculation procedures, it has often been assumed in the past that the subgrade is an elastic half-space. It is therefore interesting to compare the dynamic results for a given depth to bedrock with the static deflections for an infinite depth to bedrock. The ratio of these deflections are shown in Figures 4.10b and 4.11b for the flexible and rigid pavements, respectively. These results indicate that the dynamic deflections are smaller than the static deflections for a half-space (although they can be larger than the static deflections for the same profile with a finite bedrock depth). This implies that the static inversion process, as normally applied, will lead to an overestimation of the stiffness of the layers, especially for shallow profiles. It can also be observed that the dynamic peak displacements remain constant for a depth to bedrock greater than about 15 ft (4.5 m). This depth depends mainly on the properties of the subgrade (Seng, 1993).

4.5 Effects of Changes of the Stiffness and Thicknesses of the Layers in the Deflection Basins

The sensitivity of the deflection basins and the dispersion curves to variation in the stiffness of the pavement layers was investigated next. The flexible pavement with a subgrade extending to infinity was analyzed first. Figure 4.12a shows the deflection basins, for shear wave velocities of the surface layer of 1250, 2500 and 3750 fps (381, 762, 1,143 m/s), which correspond to soft, medium, and stiff pavements surface layers, respectively. Figure 4.12a indicates that, while the displacement under the load is influenced by the properties of the surface layer, this influence becomes negligible at the outer stations. Figure 4.12b shows the deflection basins when the shear wave velocity of the base is 500, 1000, and 1500 fps (152.4, 304.8 and 457.2 m/s), corresponding to soft, medium, and stiff bases. Changes in the properties of the base affect the displacements under the load and at the next two stations, but have still a negligible effect on the displacements at the last three stations. **Changes in the properties of the subgrade layer affect, on the other hand, the displacements at all the stations in the same degree, as shown in Figure 4.12c.** The same kind of sensitivity analysis was performed for the rigid pavement. Examining the results shown in Figure 4.13, it can be observed that the behavior is very similar to the flexible pavement, with the exception that in this case the variation of the stiffness of the surface layer has a more important effect on the deflection at the outer stations than on the flexible pavement, though this effect is still small.

The sensitivity of the deflection basins to changes in the thicknesses of the surface and base layer for the flexible pavement with bedrock at 20 ft (6.1m) is illustrated in Figure 4.14. The thickness of each layer was varied independently. Figure 4.14a shows the case in which the thickness of the surface layer was changed, while Figure 4.14b illustrates the case in which the thickness of the base was varied. These figures show that, while the changes in the thickness affect the deflection under the load and the near stations, they still have a negligible effect at the outer station. The case in which the depth to bedrock was varied has been presented before finding that the dynamic deflection basins are insensitive to this parameter (except for very shallow profiles).

As an extreme case, the deflections of the flexible and rigid pavement with bedrock at 20 ft (6.1 m) are compared with those of a single layer system with the properties of the subgrade and thickness equal to 20 ft (6.1 m). Figures 4.15a and 4.15b show the comparison for the flexible and rigid pavement, respectively. These figures show that the displacements at the outer stations for the uniform profile are almost the same as those of both pavement systems.

In conclusion, the parametric studies show that the deflections under the load and the near receivers are influenced by the properties (stiffnesses and thicknesses) of all the layers, though this influence disappears at the outer stations, where deflections are governed almost exclusively by the properties of the subgrade.

4.6 Dynamic vs Static Strains under the Axis of the Load

Dynamic peak strains and static strains were computed at various depths under the axis of the load for the flexible pavement profile with different depths of bedrock. Table 4.2 and 4.3 show the strains for bedrock at 20 ft (6.1 m) and infinity, respectively. The results indicate that the peak dynamic strains under the load are almost identical over the top part of the pavement system to the static strains. They are also insensitive to the depth to bedrock over this range. As the depth increases, the strains in the subgrade show more pronounced dynamic effects and a larger effect of the depth to bedrock.

The dynamic peak shear strains can be used to estimate the possibility of nonlinear behavior by comparison with typical shear stress-shear strain curves for soils and pavement layers.

4.7 Summary

The forward modeling option of the computer program FWD-DYN has been used in this chapter to perform an analytical study of the parameters affecting the response of two generalized pavement profiles (one flexible and one rigid) to the FWD load.

The results of this analytical study confirm that the dynamic response of the pavement system affects the magnitude and shape of deflection basins obtained with the FWD test, and that these basins can be substantially different from those obtained under static conditions. If the dynamic deflection basins are compared against the static ones as a function of the assumed depth to bedrock, significant dynamic amplification can be found for some range of depths to bedrock (typically less than 20 ft [6.1 m]). However, dynamic deflections can also be smaller than the static ones over a wide range of depths (typically depths greater than 50 ft [15 m]). The use of a static back-calculation procedure with the known bedrock depth will lead to underestimation of the stiffness of the subgrade layer in the first case, and to an overestimation in the second case. The evaluation of the stiffnesses for the other layers will be complicated by the errors in the subgrade layer. On the other hand, current static back-calculation practice is to consider that the subgrade extends to infinity (because the depth to bedrock is not normally known). In this case, the static back-calculation procedure will lead to a general overestimation of the stiffness of the pavement system.

The current method of interpretation of FWD test results fails to utilize the true potential of the FWD test because the dynamic nature of the pavement response is not considered. If one simply recorded a longer time history of the dynamic response of the pavement system, this would allow a simpler and faster estimation of the subgrade stiffness from the offset times. The depth to bedrock could also be estimated from the period of the free vibrations of the pavement that follow the passage of the FWD pulse. This information and the history of the force are needed if a dynamic back-calculation procedure is to be used for system identification, as was shown in Chapter 3.

Computation of strains under the axis of the FWD load reveals that the static and dynamic strains are almost the same in the upper layers, and that the effect of depth to bedrock is negligible for the strains in the top part (say upper 2 ft [0.6 m]) of the pavement. The dynamic effect on the strains increases with depth. The parametric studies indicate clearly that the dynamic deflections at the outer receivers are governed almost exclusively by the subgrade properties.

TABLE 4.1. Values of Elastic Properties and Layer Thicknesses of Generalized Pavements Profiles

Type of pavement	Layer	Thickness in. (cm)	Young's Modulus ksi (MPa)	Shear wave velocities fps (m/sec)
Flexible	Surface Base Subgrade	6 (15) 12 (30) variable	436.7 (3013) 70 (483) 18 (124)	2500 (762) 1000 (305) 500 (152)
Rigid	Surface Base Subbase Subgrade	10 (22.5) 6 (15) 12 (30) variable	5660 (39020) 436.7 (3013) 70 (483) 18 (124)	9000(2743) 2500 (762) 1000 (305) 500 (152)

			Flexible Pavement					
			(Bedrock at 20 ft)					
			Strains by Kip of load					
Layer	depth(in)	Dynamic strains			Static strains			Dyn/Stat
		Ez	Er = Eθ	Shear St.	Ez	Er = Eθ	Shear St.	
1	0	-5.28E-06	1.65E-05	2.18E-05	-5.05E-06	1.64E-05	2.15E-05	1.01621
1	3	1.08E-05	-1.31E-06	1.21E-05	1.07E-05	-1.28E-06	1.20E-05	1.005131
1	6	2.10E-05	-1.54E-05	3.64E-05	2.08E-05	-1.52E-05	3.61E-05	1.009983
2	6	4.44E-05	-1.54E-05	5.98E-05	4.44E-05	-1.52E-05	5.96E-05	1.002079
2	12	2.17E-05	-9.82E-06	3.15E-05	2.14E-05	-9.68E-06	3.11E-05	1.013042
2	18	1.78E-05	-1.21E-05	3.00E-05	1.74E-05	-1.19E-05	2.93E-05	1.021034
3	18	3.21E-05	-1.21E-05	4.42E-05	3.14E-05	-1.19E-05	4.33E-05	1.021267
3	24	2.12E-05	-8.20E-06	2.94E-05	2.07E-05	-7.73E-06	2.84E-05	1.033373
3	30	1.64E-05	-6.32E-06	2.27E-05	1.50E-05	-5.63E-06	2.06E-05	1.100325
3	36	1.30E-05	-4.92E-06	1.79E-05	1.14E-05	-4.28E-06	1.57E-05	1.136595

Ez, Er, Eθ Longitudinal Strains in the vertical, radial and azimuthal direction, respectively.

TABLE 4.2. Strains under the axis of the FWD load for a flexible pavement with bedrock at 20 ft (6.1 m).

			Flexible Pavement					
			(Bedrock at infinite)					
			Strains by Kip of load					
Layer	depth(in)	Dynamic strains			Static strains			Dyn/Stat
		Ez	Er = E●	Shear St.	Ez	Er = E●	Shear St.	
1	0	-5.28E-06	1.65E-05	2.18E-05	-5.10E-06	1.64E-05	2.15E-05	1.012132
1	3	1.08E-05	-1.31E-06	1.21E-05	1.07E-05	-1.24E-06	1.19E-05	1.012109
1	6	2.10E-05	-1.54E-05	3.64E-05	2.08E-05	-1.52E-05	3.60E-05	1.01206
2	6	4.44E-05	-1.54E-05	5.98E-05	4.41E-05	-1.52E-05	5.93E-05	1.007654
2	12	2.17E-05	-9.82E-06	3.15E-05	2.14E-05	-9.65E-06	3.10E-05	1.014785
2	18	1.78E-05	-1.21E-05	2.99E-05	1.74E-05	-1.19E-05	2.93E-05	1.022199
3	18	3.20E-05	-1.21E-05	4.42E-05	3.13E-05	-1.19E-05	4.32E-05	1.021943
3	24	2.12E-05	-8.22E-06	2.95E-05	2.06E-05	-7.70E-06	2.83E-05	1.039304
3	30	1.64E-05	-6.33E-06	2.27E-05	1.50E-05	-5.60E-06	2.06E-05	1.103725
3	36	1.29E-05	-4.92E-06	1.79E-05	1.14E-05	-4.25E-06	1.57E-05	1.140588
3	48	4.74E-06	-3.34E-06	8.07E-06	4.25E-06	-2.72E-06	6.97E-06	1.158748

Ez, Er, Eθ Longitudinal Strains in the vertical, radial and azimuthal direction, respectively.

TABLE 4.3. Strains under the axis of the FWD load for a flexible pavement with bedrock at infinity.

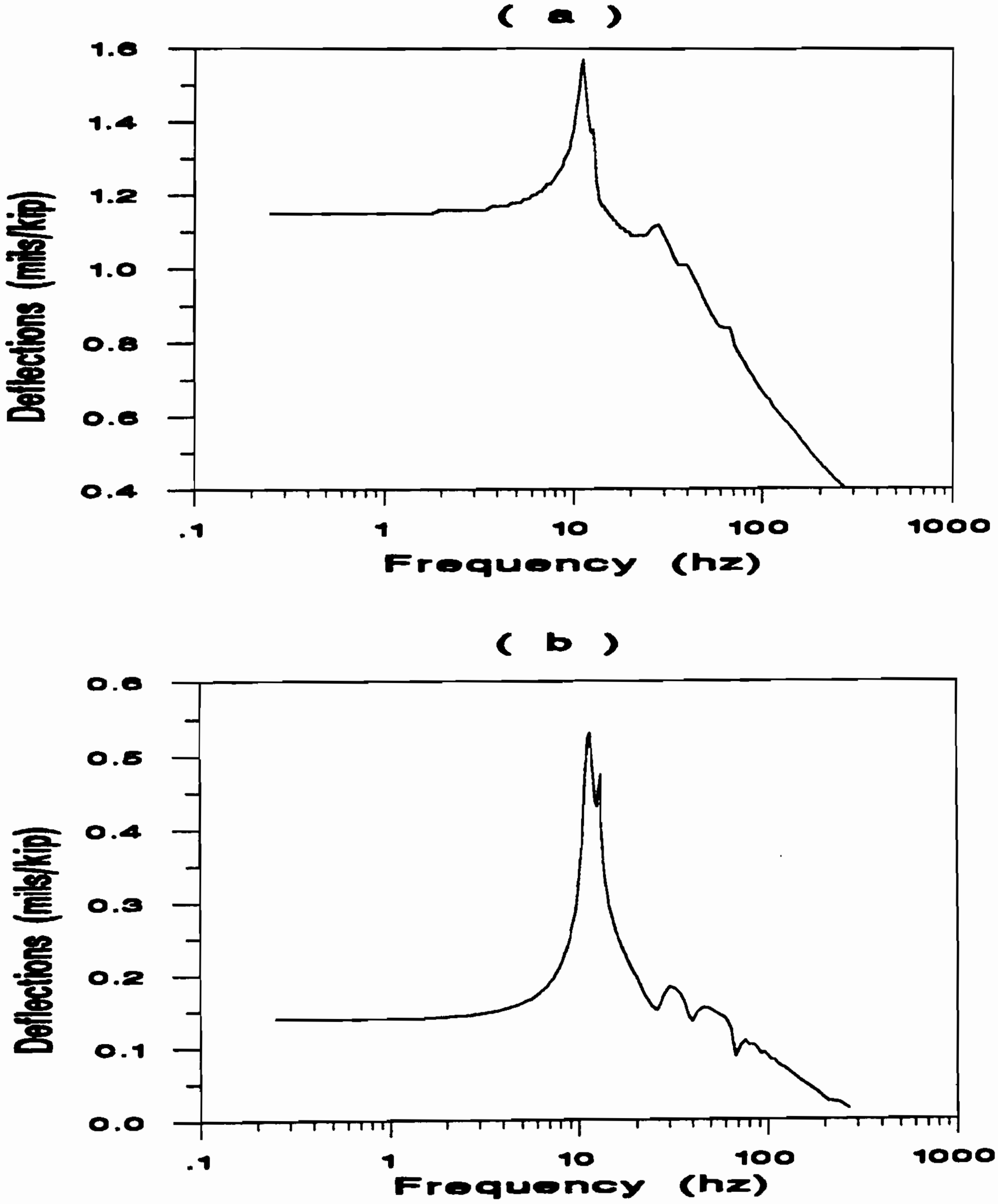


Figure 4.1. Dynamic response of a flexible pavement with bedrock at 20 ft (6.1 m) to FWD loading: (a) and (b) transfer functions at recording stations 1 and 7.

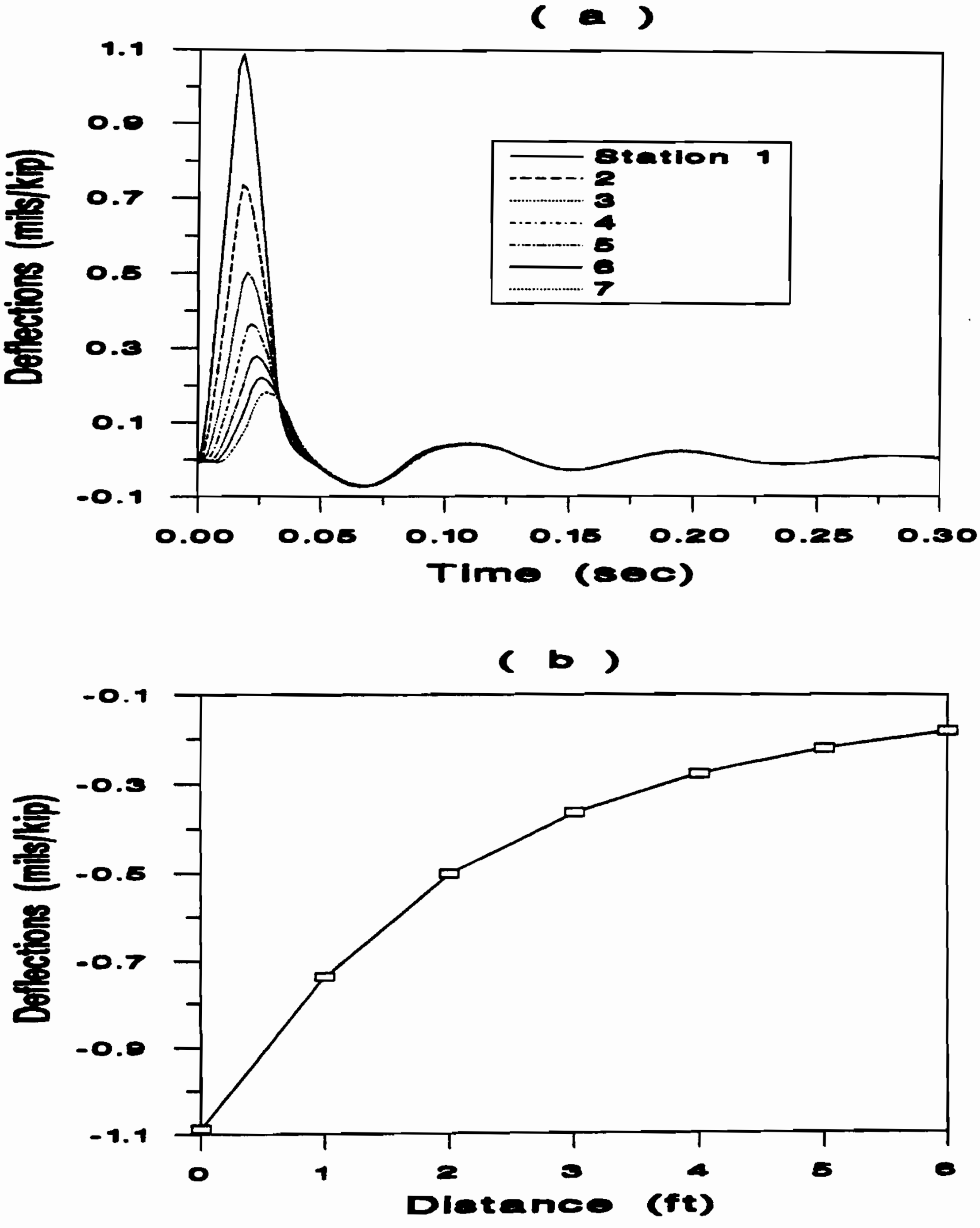


Figure 4.2. Dynamic response of a flexible pavement with bedrock at 20 ft (6.1 m) to FWD loading: (a) displacement-time histories and (b) measured deflection basin.

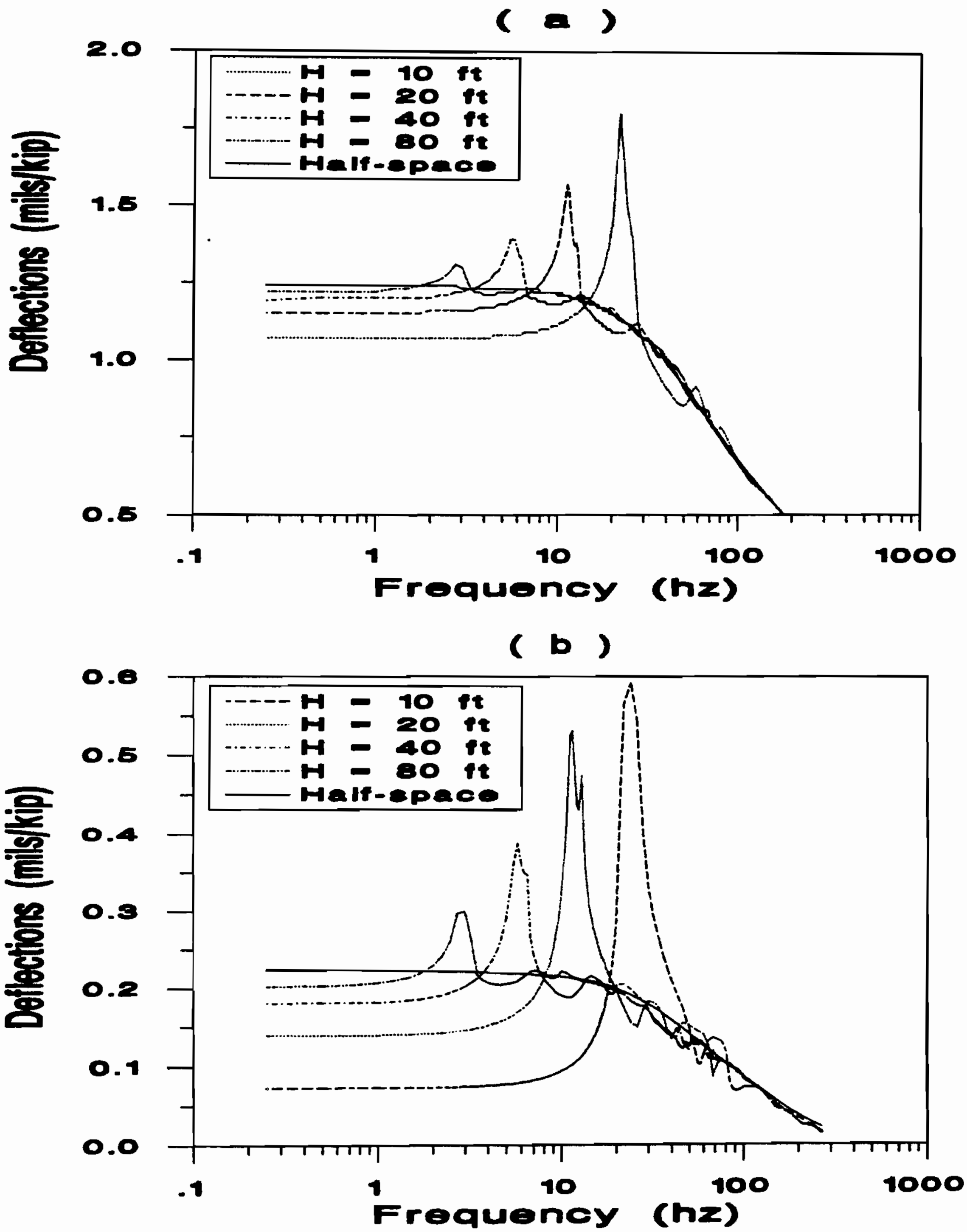


Figure 4.3. Dynamic response of a flexible pavement with different depths to bedrock to FWD loading: (a) and (b) transfer functions at recording stations 1 and 7.

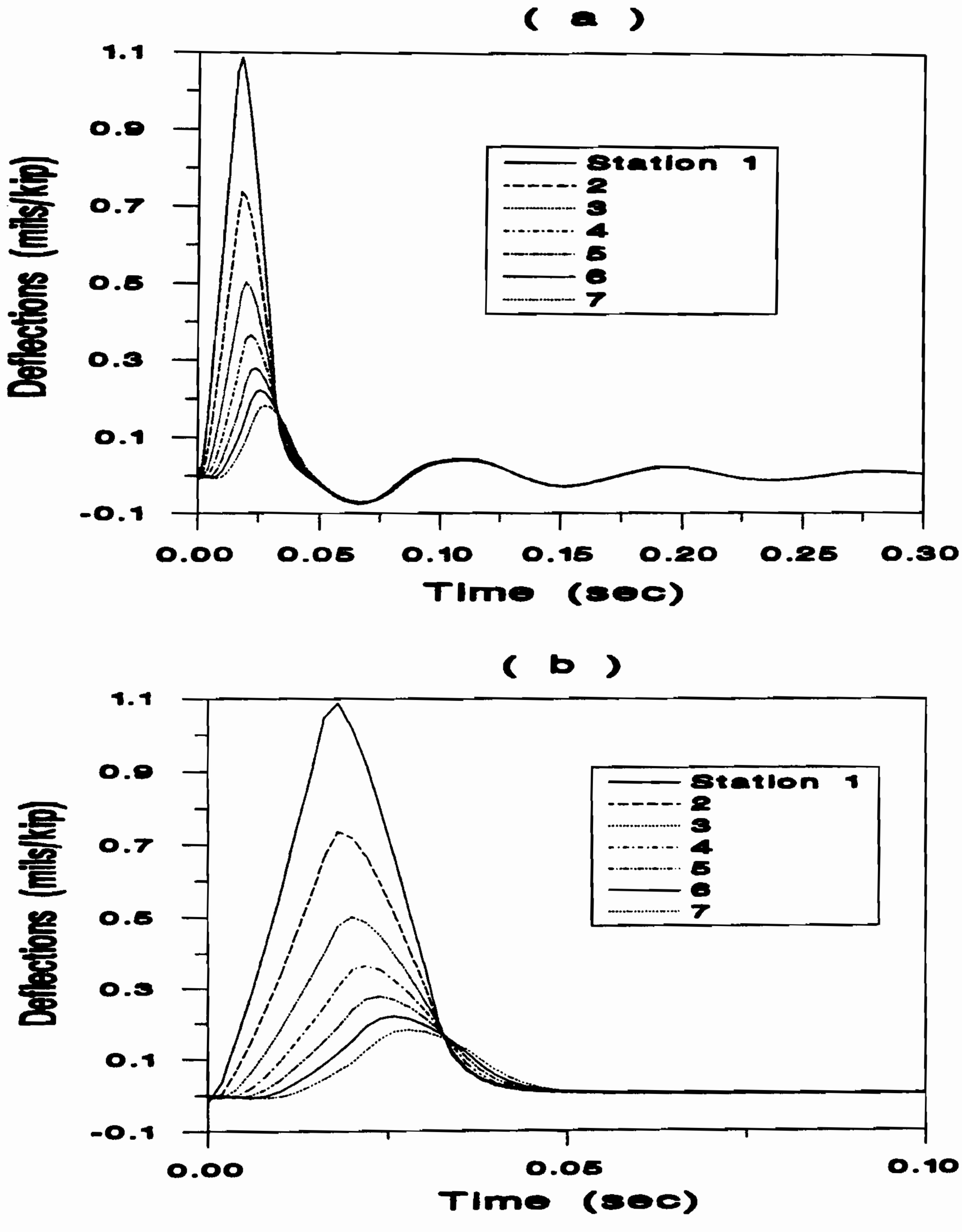


Figure 4.4. Dynamic response of a flexible pavement with different depths to bedrock to FWD loading: (a) and (b) displacement-time histories for depth to bedrock of 20 ft (6.1 m) and infinity.

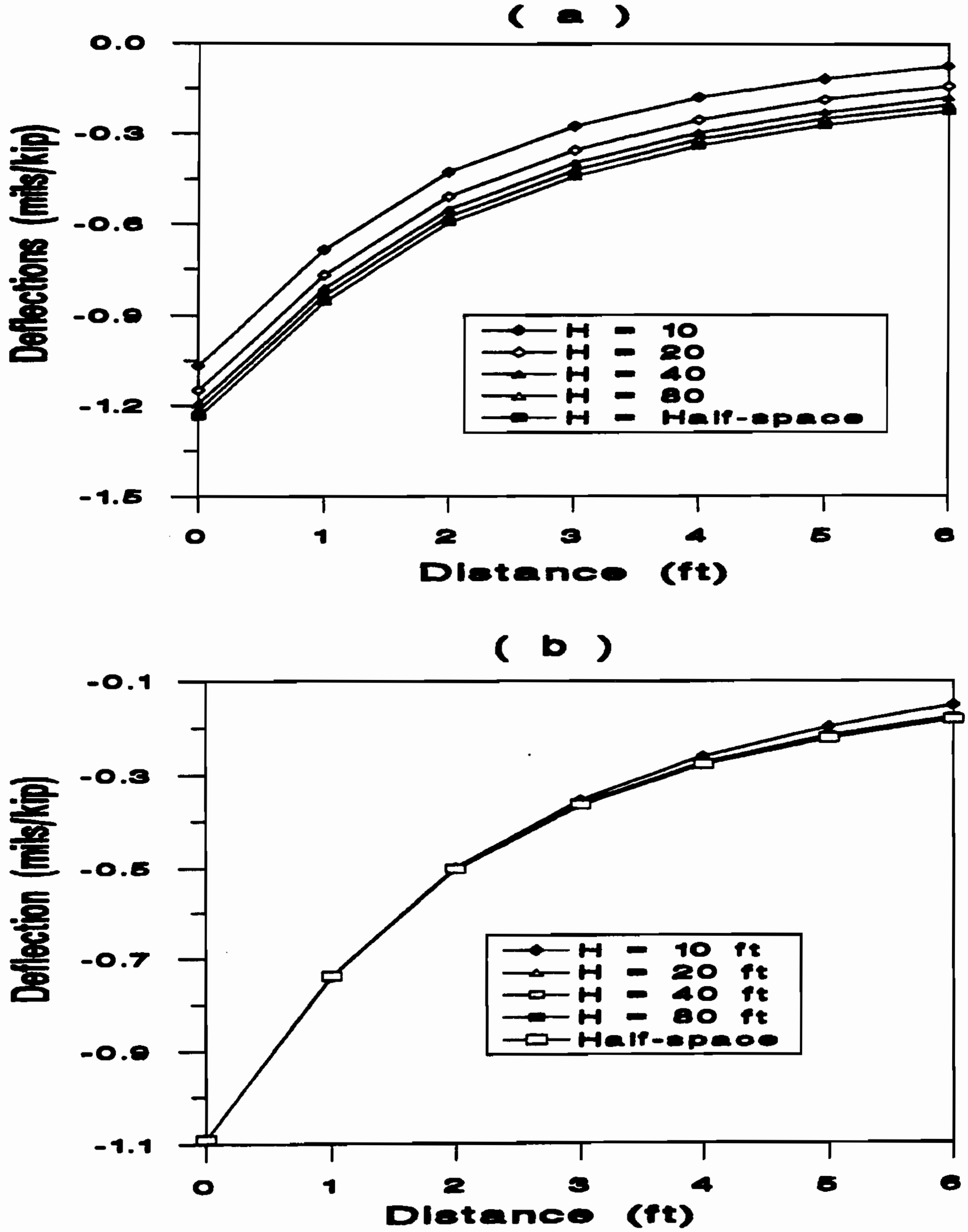


Figure 4.5. Dynamic response of a flexible pavement with different depths to bedrock to FWD loading: (a) static displacements and (b) measured deflection basins.

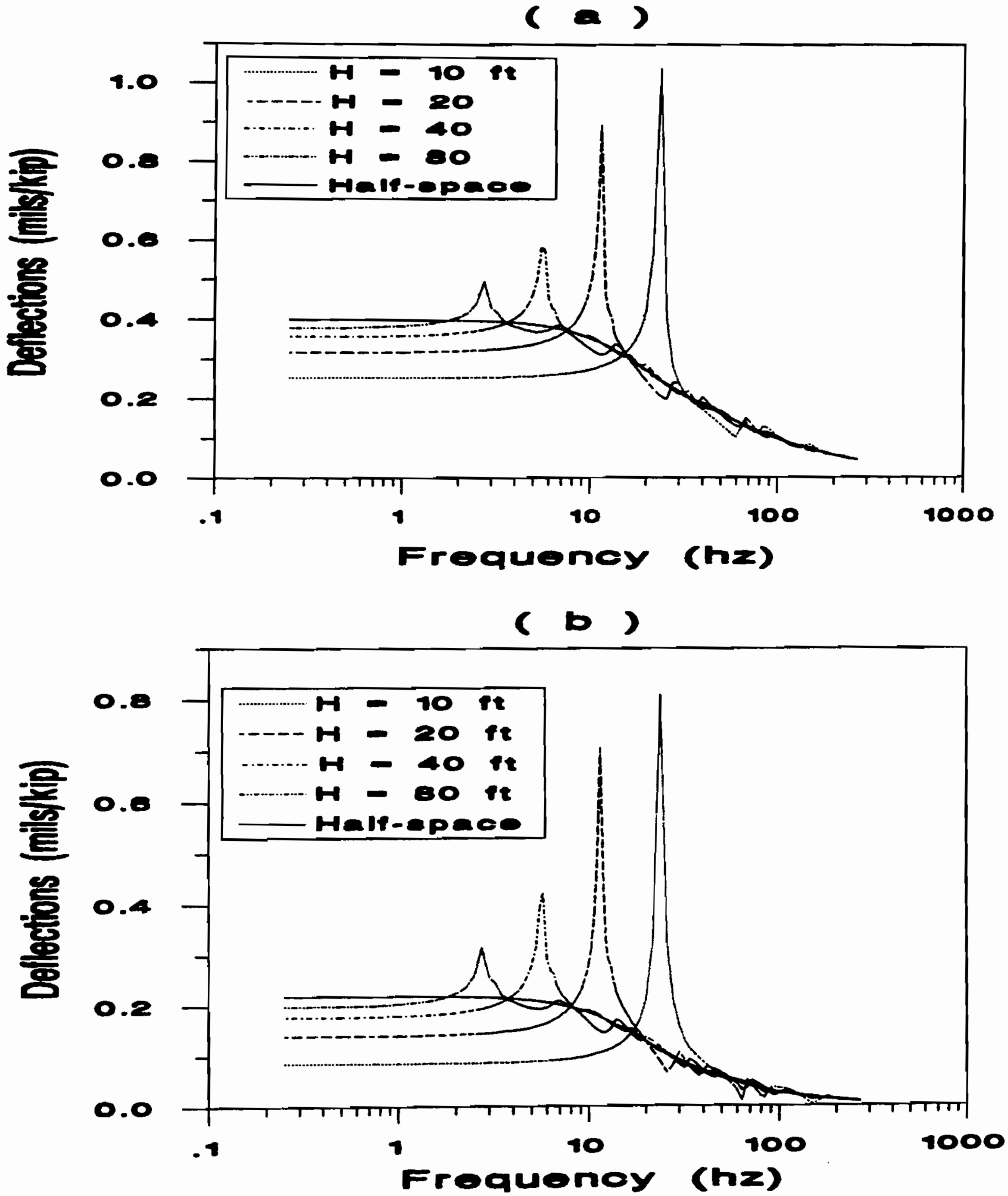


Figure 4.6. Dynamic response of a rigid pavement with different depths to bedrock to FWD loading: (a) and (b) transfer functions at recording stations 1 and 7.

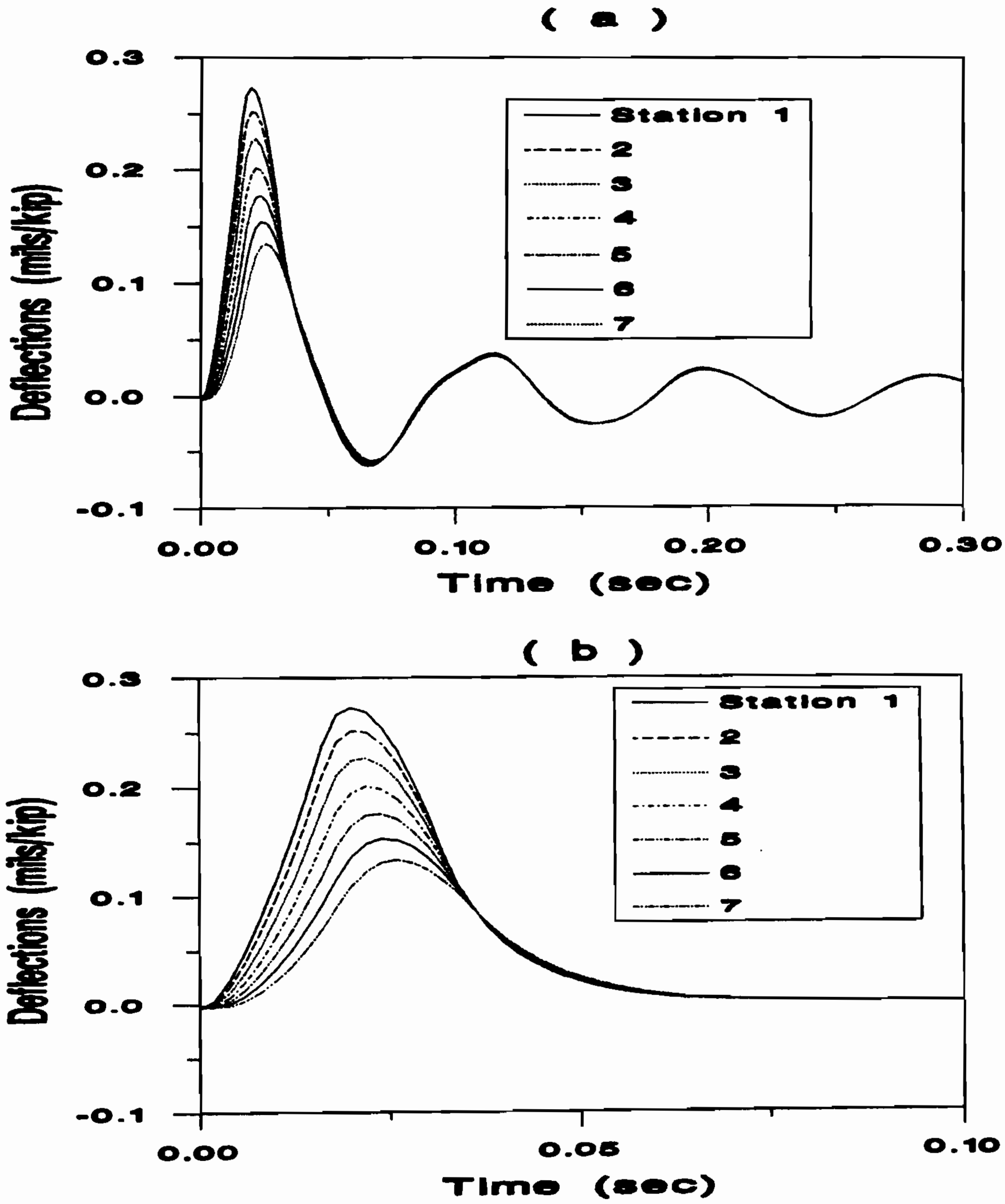


Figure 4.7. Dynamic response of a rigid pavement with different depths to bedrock to FWD loading: (a) and (b) displacement-time histories for depth to bedrock of 20 ft (6.1 m) and infinity.

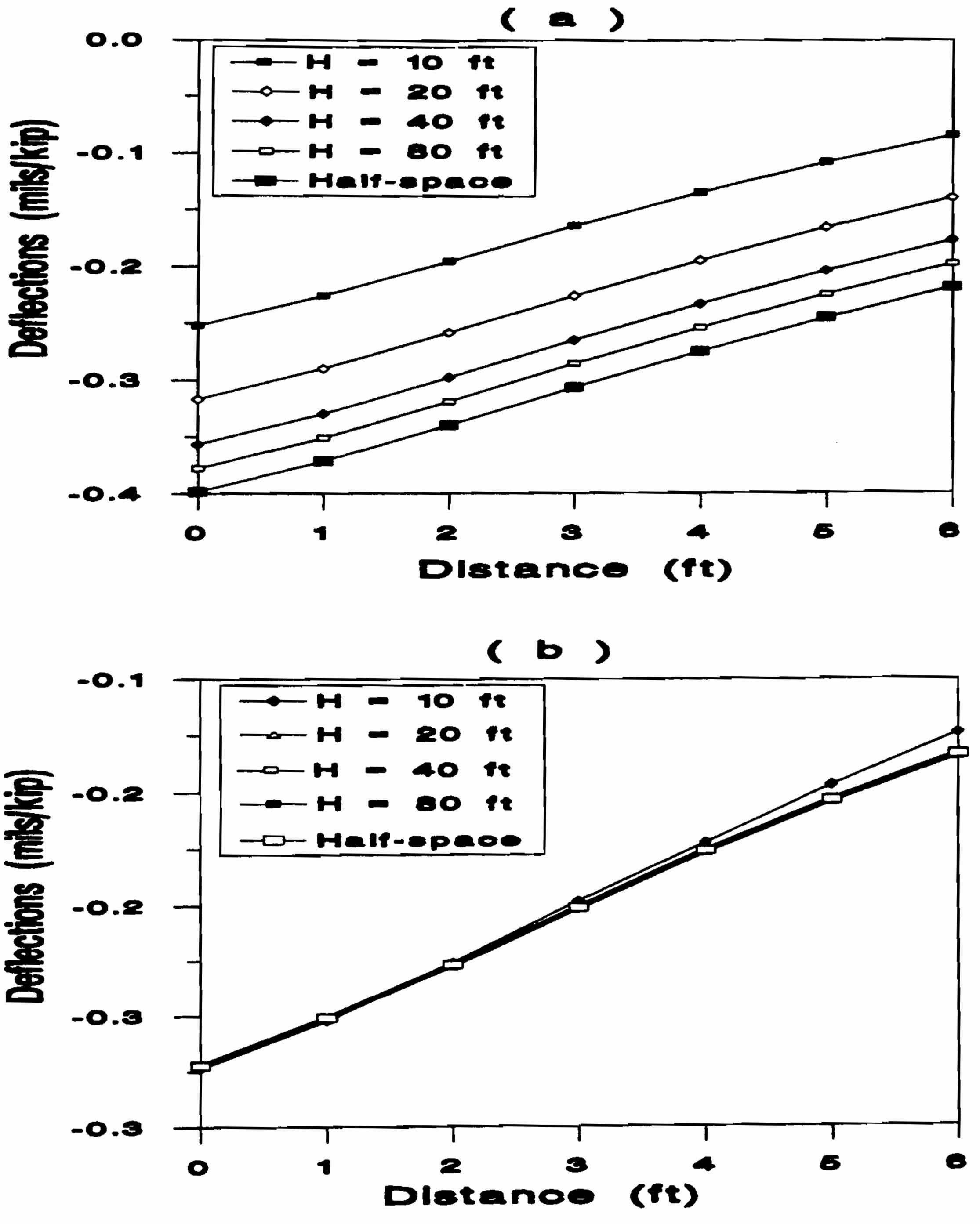


Figure 4.8. Dynamic response of a rigid pavement with different depths to bedrock to FWD loading: (a) static displacements and (b) measured deflection basins.

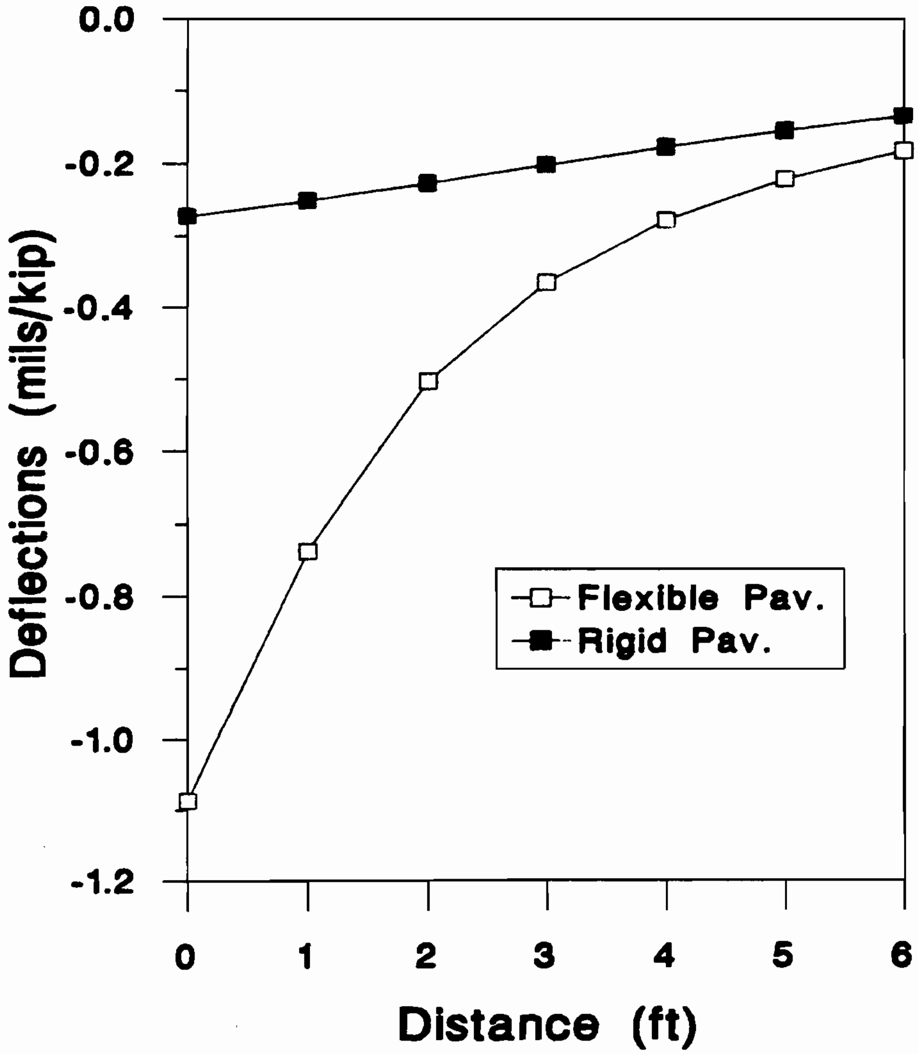
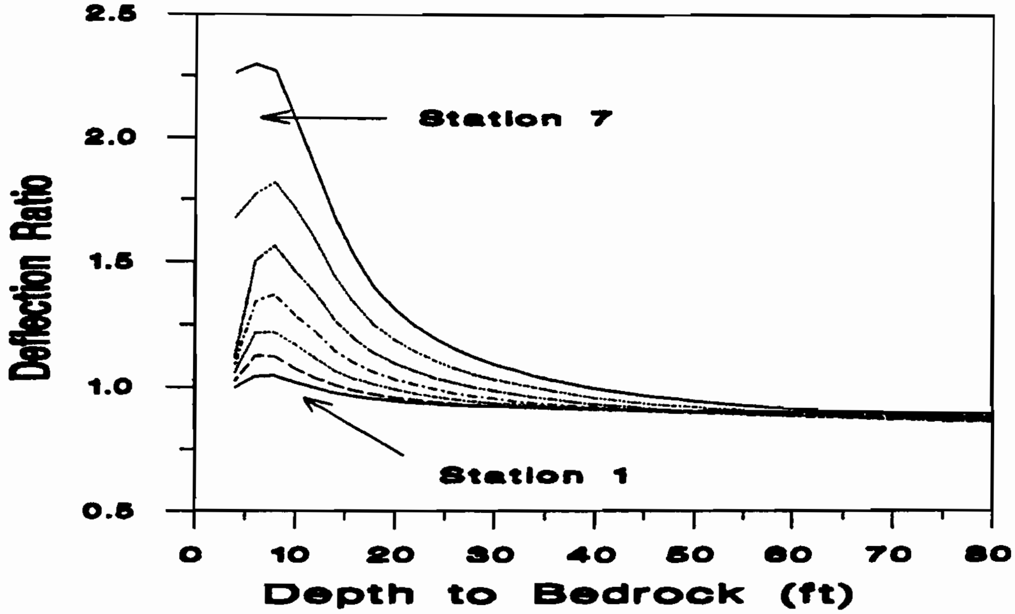


Figure 4.9. Comparison of deflection basins for a flexible and a rigid pavement with bedrock at 20 ft (6.1 m).

(a)



(b)

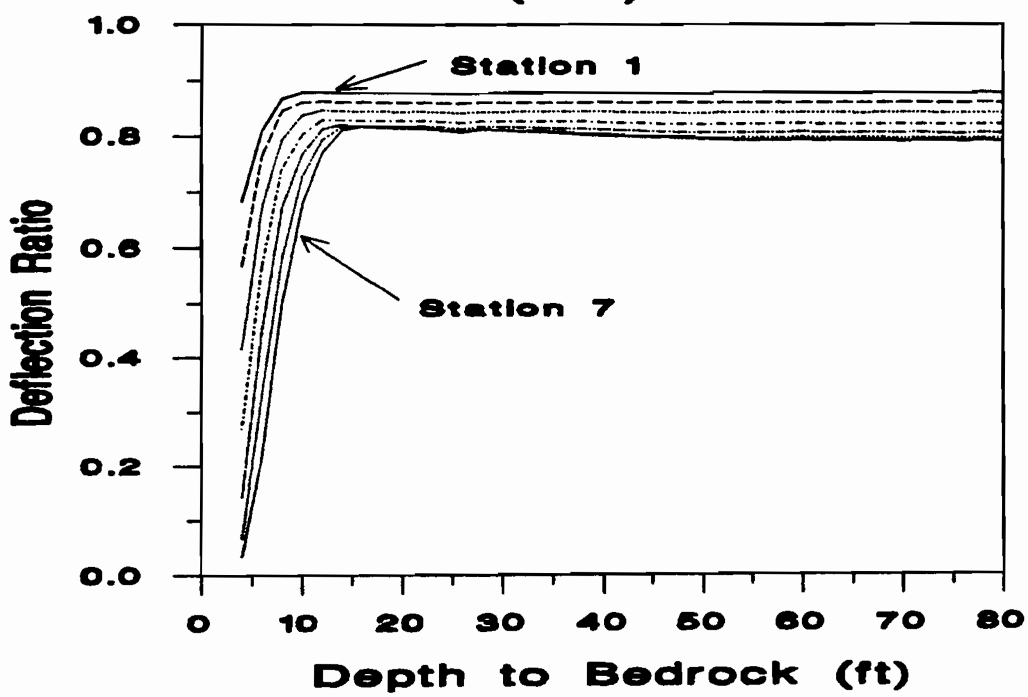


Figure 4.10. Ratio of dynamic to static displacement for a flexible pavement: (a) static displacements with a finite depth to bedrock and (b) static displacements with a infinite subgrade depth.

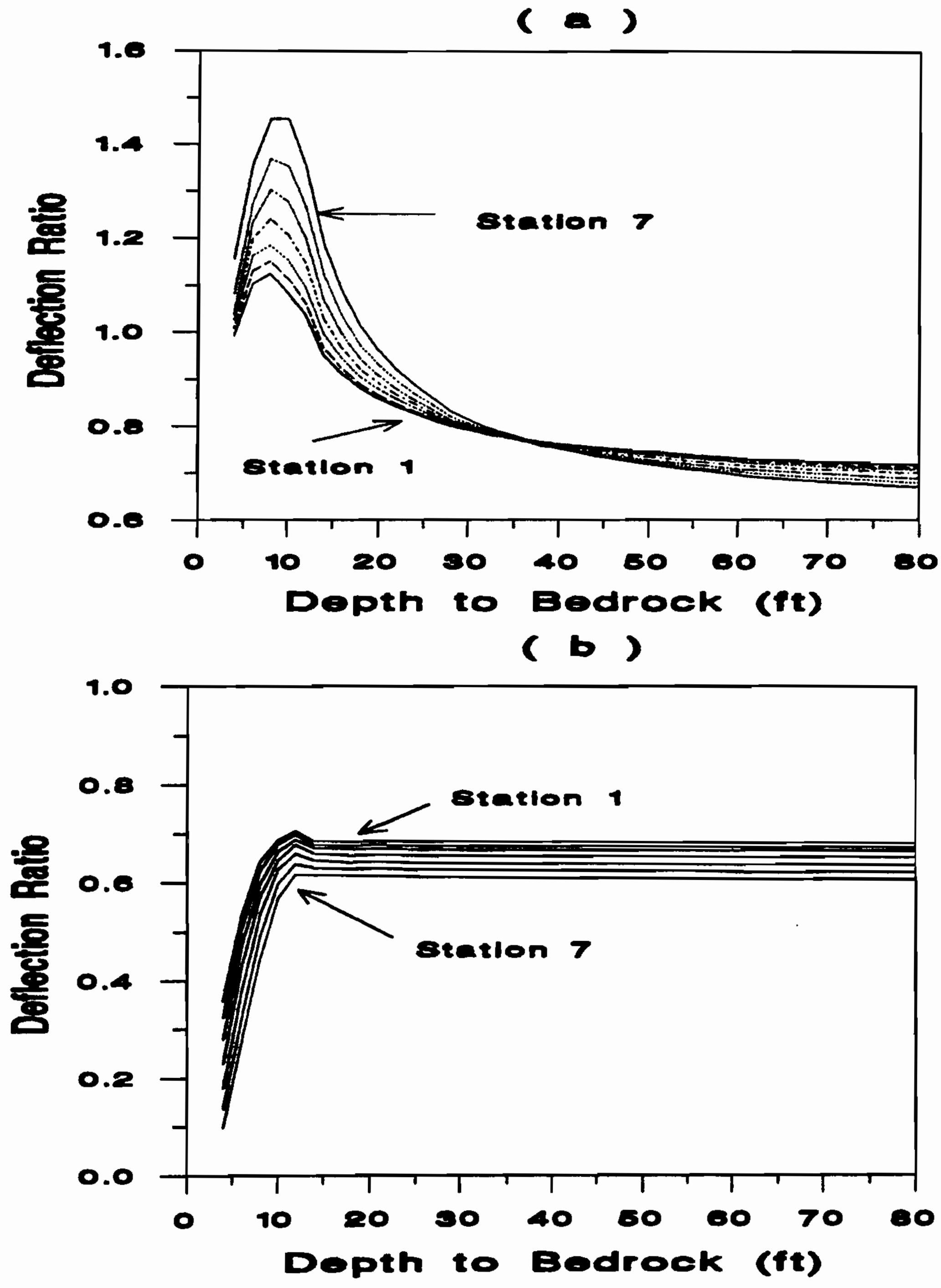


Figure 4.11. Ratio of dynamic to static displacement for a rigid pavement: (a) static displacements with a finite depth to bedrock and (b) static displacements with a infinite subgrade depth.

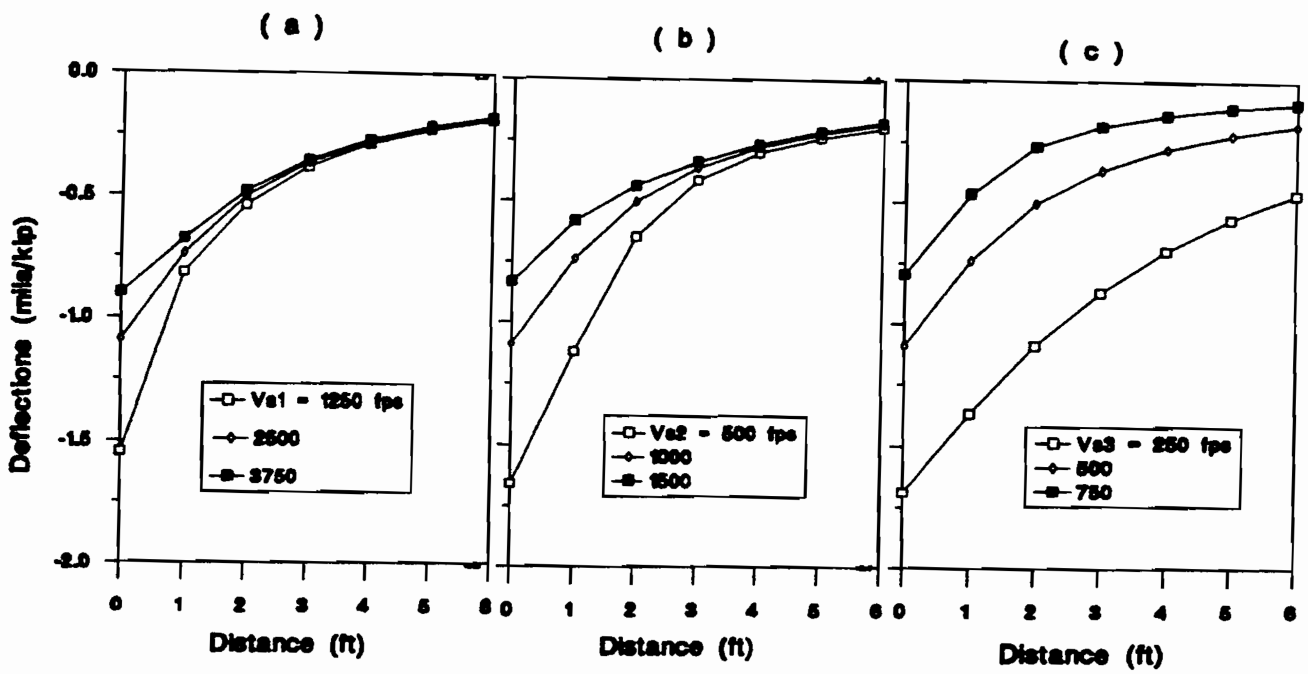


Figure 4.12. Sensitivity of deflection basins to changes in the stiffness of the layers for a flexible pavement with subgrade extending to infinity: (a) surface layer, (b) base layer, (c) subgrade layer.

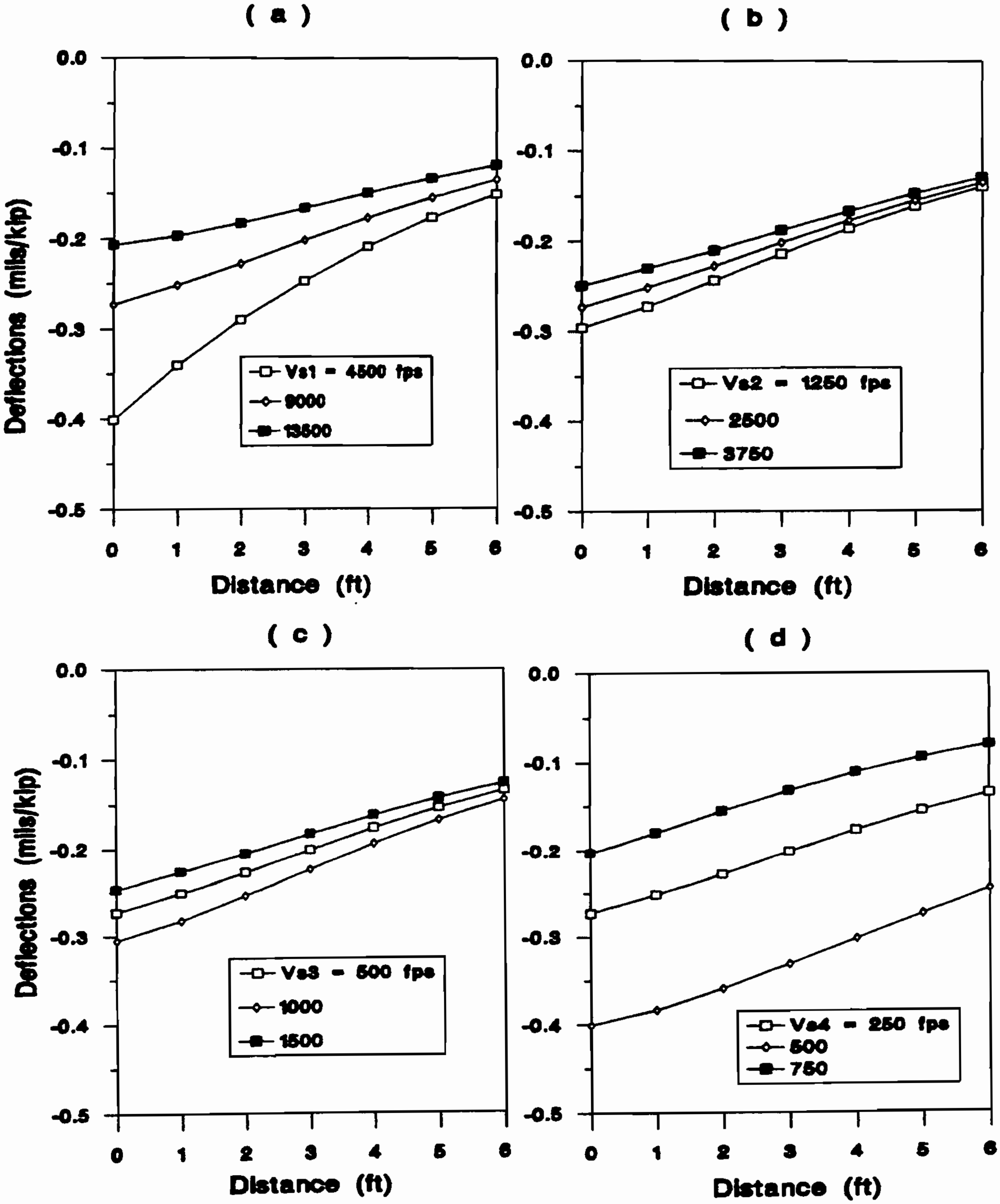


Figure 4.13. Sensitivity of deflection basins to changes in the stiffness of the layers for a rigid pavement with subgrade extending to infinity: (a) surface layer, (b) base layer, (c) subbase layer and (d) subgrade layer.

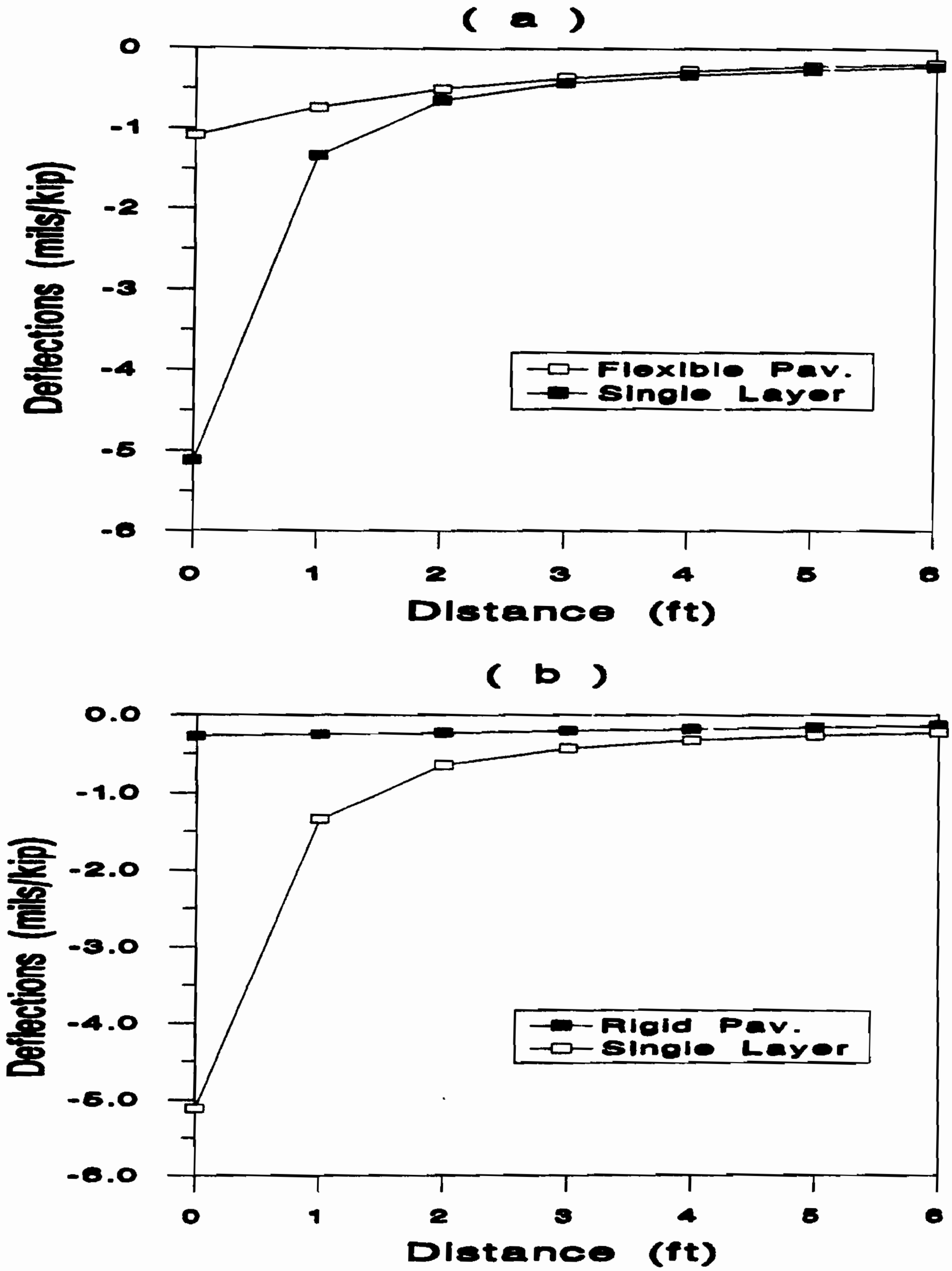


Figure 4.14. Sensitivity of deflection basins to changes in the thickness of the layers for a flexible pavement with bedrock at 20 ft (6.1 m): (a) surface layer, (b) base layer.

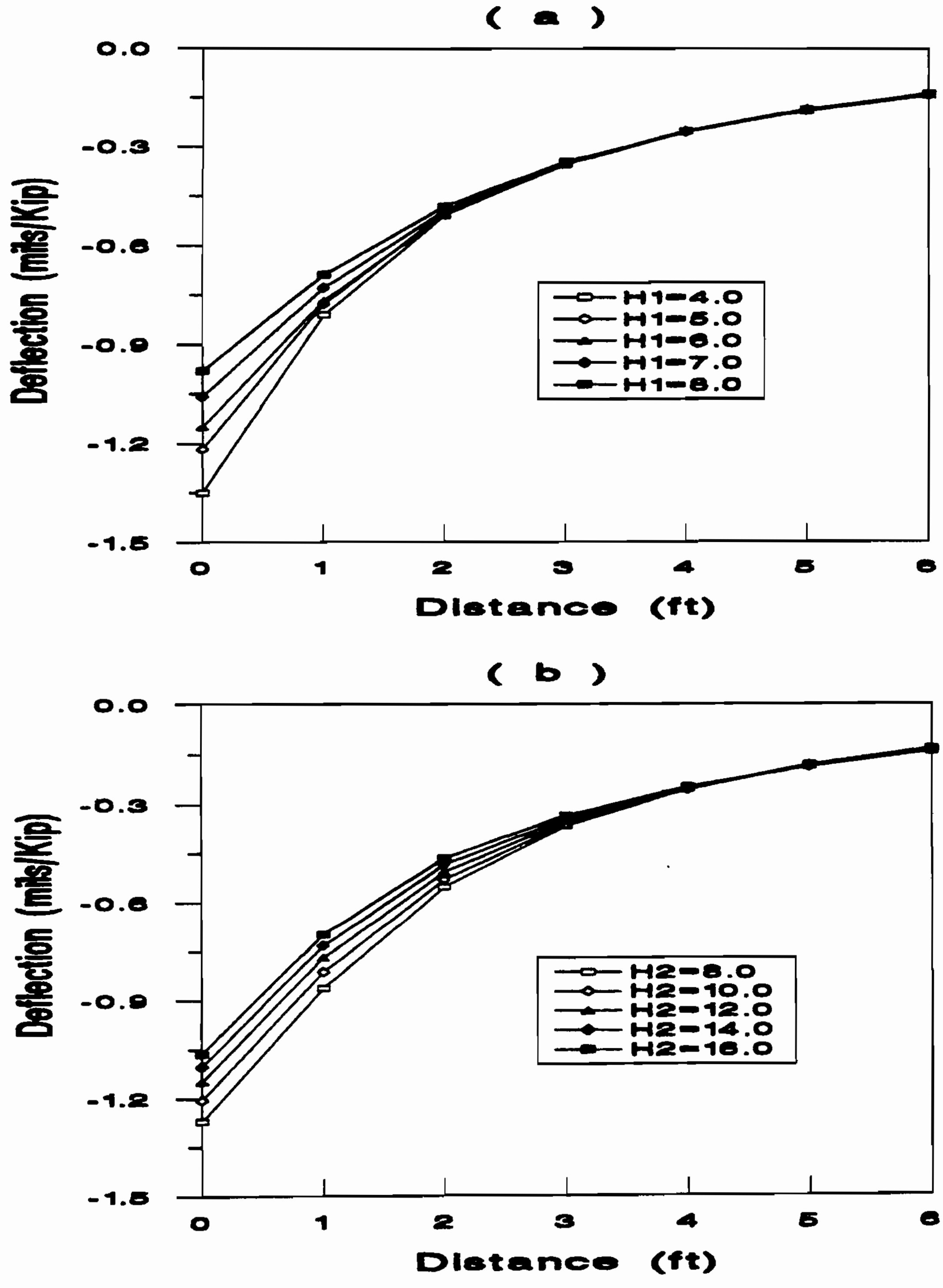


Figure 4.15. Comparison of deflection basins for (a) a flexible and (b) a rigid pavement with that of a single-layer subgrade when the depth to bedrock is 20 ft (6.1 m).

CHAPTER 5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

While the results of the parametric studies conducted as a part of this study confirm those of previous projects, they also shed some new light on the importance and nature of dynamic effects.

1. As already pointed out by previous studies, the dynamic nature of the applied loads and the resulting dynamic effects on the response of a pavement system influence both the magnitude and the shape of the deflection basins obtained with the FWD test, such that the basins can be substantially different from those obtained under static conditions. If the dynamic deflection basins are compared against the static ones as a function of the assumed depth to bedrock, significant dynamic amplifications can be found for some range of depths to bedrock (typically less than 20 ft [6m]). However, dynamic deflections can also be smaller than the static ones over a wide range of depths (typically depths greater than 50 ft [16 m]). This is particularly so for stiff pavements. The ratio of the dynamic to the static displacements for a fixed value of the depth to bedrock will not be the same for all the stations, leading thus to different dynamic and static shapes of the deflection basins.

2. The use of a static back-calculation procedure with a known depth to bedrock will lead to underestimation of the stiffness of the subgrade layer for shallow rock cases and an overestimation for deep subgrade layers. The stiffness obtained for the other layers will be affected by the errors committed in the value of the modulus of the subgrade, resulting in their overestimation or underestimation, depending on the case. In the case of previous static back-calculation practice, when it was often assumed that the subgrade extended to infinity (mostly because the depth to bedrock was neither known nor estimated), the static inversion algorithm led to a general overestimation of the stiffness of the pavement system.

3. While the static displacements are strongly affected by the depth to bedrock, the dynamic displacements are relatively insensitive to this parameter — except for the outmost receiver and very shallow bedrocks (of the order of 10 ft [3 m] or less). The

dependence of the dynamic effects, measured by the ratio of dynamic to static displacements, on the depth to bedrock is thus due mostly to the variations in the denominator (static displacements), rather than in the numerator (dynamic displacements). This indicates that in most practical cases the results of a static inversion procedure, as commonly used now, will be sensitive to the assumed depth to bedrock, whereas those of a true dynamic inversion would not be affected by this parameter.

4. The peak displacements recorded at the farthest receivers (stations 5, 6, and 7 typically) are affected only by the elastic properties of the subgrade within the normal range of values of the elastic modulus of the base and the surface layer. The deflections at the intermediate receivers (stations 3 and 4) are affected also by the properties of the base, though they are still insensitive to variations in the properties of the surface layer. Deviations in the expected values of Young's modulus of elasticity of the surface layer will only have a noticeable effect on the deflections measured directly under the load or at the next receiver. This implies that the FWD data can provide, when properly interpreted, very reliable estimates of the modulus of the subgrade and even of the base, but that it is much harder to obtain accurate estimates of the modulus of the surface layer. Other methods, such as the SASW or the impact echo technique, can provide, on the other hand, a much easier and reliable value of the stiffness of the top layer. A combination of these different techniques would provide an ideal setup for fast, reliable, and economical determination of the elastic moduli of pavement systems.

5. The above observation implies also that it is convenient to start an inversion process determining first the properties of the subgrade from the values of the displacements at the last receivers, and proceeding then up the pavement profile computing the values of the modulus of the base from the displacements at the intermediate receivers, and finally that of the surface layer from the value of the deflection under the load. The procedure implemented at present in FWD-DYN starts by determining the thickness and modulus of the subgrade when the time histories of the displacements are available. The separate consideration of the base and the surface layer has not been implemented yet, however.

6. For a static inversion, directly with the measured deflection basin or with a calculated static basin obtained by eliminating dynamic effects, it is necessary to estimate first the depth to bedrock. This is not possible if one has only the values of the peak displacements. When the time histories are stored and they are sufficiently long to include the free vibrations of the system, one can estimate the thickness and modulus of the subgrade considering the amplitude of the displacement at the farthest receiver and measuring the period of vibration of the last part of the records. Alternatively, one can obtain this period or frequency from the location of the peak of the transfer functions (ratio of the Fourier transform of the displacement record to the Fourier transform of the applied force).

7. Computations of strains under the axis of the FWD load indicates that the static and dynamic strains are very similar (almost the same) in the upper layers (the region of main interest). The effect of the depth to bedrock is essentially negligible for the strains in the upper 2 or 3 ft (0.5 to lm) of the pavement. The dynamic effect on the strains increases with depth. The option to calculate strains and stresses is now available within the forward modeling option of FWD-DYN. The accuracy of the computed strains decreases somewhat as the distance to the axis of the load increases, but seems to be very good directly under the load at any depth.

5.2 Recommendations

The computer program FWD-DYN offers three options for inversion of the field data. The static options, either applying static analyses to assumed pavement systems attempting to match the recorded deflection basin, or computing first what would be static deflection basins from the dynamic displacements (referred to as pseudo-dynamic inversion in the program), are particularly efficient and can produce very fast results in a 386 or 486 PC. (Although the program can be implemented in a 286, 386, or 486, clearly the use of a 486 is recommended.) The full dynamic analysis, on the other hand, may take on the order of half an hour per cycle of iteration on these machines (a 486/33). The following summarizes recommendations and observations, based on the present version of the program:

1. The static option should be used when only the peak displacements are available or when the recorded time histories are very short (60 msec) and do

not include therefore a sufficient duration of free vibration to estimate the depth to bedrock or to obtain a reliable transfer function. The solution obtained with this option would be equivalent to that obtained at present, and may be thus in serious error in certain cases, since it neglects entirely dynamic effects and assumes that the subgrade extends to infinity.

2. The pseudo-dynamic option is particularly attractive when there is a sufficient duration of the displacement time histories to obtain reasonable transfer functions. The solutions converges to results which are close to the exact values when applied to computer generated data. It should be noticed, however, that it still involves some approximations in the estimation of the depth to bedrock. Most of the evaluation of the inversion process has been based on the analysis of theoretical data; the forward option issued to generate the time histories of displacements at the seven receivers and these computer generated data are then used as input to the inversion process. It is necessary to gain more experience with actual field data at sites where the true properties are known in order to judge its accuracy. It is believed, in any case, that it is a better solution than the standard static analysis if one has an adequate set of time histories.

3. The dynamic inversion is clearly the best procedure. The results are insensitive to the depth to bedrock and therefore the effect of this variable is eliminated. When applied to computer generated data, the results converge to the exact values in very few cycles of iteration. The amount of time needed to perform an iteration is, however, still great. It is also necessary to test the accuracy of this procedure with actual field data. In addition, some modifications could be introduced in the formulation and in the steps of the inversion algorithm to reduce the time of computation. It is believed that significant reductions can be obtained, but this would require an additional research effort.

4. It is felt that the reliability of the inversion procedure and the estimated elastic moduli could be improved if the duration of the recorded time histories were increased. Some of the equipment available at present at TxDOT can record up to 120 msec, an improvement over the usual 60 msec. If

it is difficult or expensive to increase the number of data points stored (at present 300 points per record with a duration of 60 msec), it would be possible to increase the duration of the records with the same number of points by increasing the time step (digitization interval). The time increment between points is now typically 0.2 msec. Values of 0.4 msec or even 1 msec might be sufficient to provide a good estimate of the peak values. Some verification would be necessary before implementing this type of changes.

5. As stated earlier, it is necessary to conduct actual field tests to evaluate the accuracy and usefulness of the program FWD-DYN, and also to get a better feeling for the quality of the data and the reliability of the results. This requires conducting the tests at locations where the properties of the pavement system are well known. It would be particularly important to conduct measurements simultaneously with alternative methods (such as the SASW) and to study means by which they can be combined.

APPENDIX

FWD-DYN USERS'S MANUAL

A.1. General

FWD-DYN is a computer program which can perform:

a) Forward modeling of the FWD test. A linear dynamic analysis of a specified pavement system subjected to an impulse load distributed over a circular area is performed. The analysis is carried out in the frequency domain obtaining the transfer function of the displacement at the different receivers. The time histories of these displacement are computed multiplying the transfer functions by the Fourier transform of the load history and obtaining the inverse Fourier transform. The Fourier transformations are computed using a Fast Fourier Transform (FFT) algorithm. The transfer functions are calculated at each frequency using the discrete Green's functions for a layered system derived by Kausel (1981).

b) Inversion of field data or back-calculation of the elastic properties (Young's modulus) of the subgrade, the base, and the surface layer from the time histories of the displacements recorded at the different stations. A least squares type of optimization algorithm is used to match the measured and computed deflections (calculated using the forward modeling option). The program allows three options for the inversion:

1. Static inversion applied directly to the deflection basins (values of the peak displacements at each receiver) recorded in the field. It is assumed that these peak displacements are equal to those that would be obtained if the peak value of the load were applied statically. The theoretical deflection basins are obtained from static analyses. These analyses are performed using the forward modeling option for a single frequency equal to zero. The subgrade is assumed to extend to infinity.

2. Pseudo-Dynamic inversion, in which the time histories of the applied force and the measured displacements are used to obtain experimental transfer functions. The natural frequency of the subgrade is then estimated from the location of the main peak in the transfer functions and used to estimate the thickness and modulus of the subgrade in combination with the displacement at the farthest receiver. From the transfer functions, one obtains also the values of the displacements that would have occurred if the load had been applied statically. The inversion is then carried out as in the static option but using the modified "static" deflection basin and the estimated depth to bedrock.

3. Dynamic inversion, in which full dynamic analyses are performed in each iteration. This is clearly the optimum solution at least from a theoretical point of view. It is, however, much more time-consuming than the other two options, since in each cycle analyses must be conducted for the complete set of frequencies (instead of a single one), and then the results must be transformed from the frequency to the time domain.

The program is designed to accept directly the field data obtained from the Dynatest FWD.

A.2. Files

The computer program FWD-DYN consists of two programs written in BASIC (input and output interfaces), which act as a preprocessor and a postprocessor, and a main program, written in FORTRAN, which performs the computations. The HELVB.FON file is needed to use the graphics options of the main program. The deflection basins corresponding to the assumed properties of the layers are displayed on the screen at each iteration together with the experimental basin. Finally, a batch file called DYN.BAT is used to load the different executable files. In order to run FWD.DYN, one must have therefore in the same directory the following files:

-FWDDAT.EXE input interface (preprocessor) files (BASIC)

-FWD.EXE main program (FORTRAN)

-FWDOUT.EXE	output interface (postprocessor) file (BASIC)
-DYN.BAT	batch file to run the program
-HELVB.FON	font file used by the graphics of the program

A.3. System Requirements

-A 286, 386 or 486 based microcomputer

-640 Kb of RAM

DOS (version 5.0 or later) operating system

Math coprocessor chip (80287 or similar)

VGA monitor

A.4 Execution

To run the program, the directory in which the five above-mentioned files are contained must be activated. One must then type DYN and press <enter>. The introductory screen shown in Figure A1 will appear. Press then any key to obtain the following screen shown in Figure A2. In this screen the user can select the desired type of analysis (forward modeling or inversion) and the system of units to be used (English or International System). The default values are Inversion and English System of units at the present time. One can move between boxes using the arrow keys or pressing <enter>. The box which is active at any time appears highlighted. To modify the content of a box, one simply types the desired option when the box is activated.

To proceed with the program once all the options in a screen have been selected and the needed information has been input, one must press the <Page Down key>. If one desires to return to a previous screen the <Page Up>key must be pressed. To abort, the <Esc> key is used. These instructions are valid for all the screens and appear at the bottom of each of them. If there is information still missing and one tries to quit, a screen a message will appear, as illustrated in Figure A3.

A.4.1 Forward Modeling Option

If the forward modeling option is selected in the second screen (Figure A2), the following screen will look as shown in Figure A4. The user has then the option of entering the data interactively or using a batch file. Once an option has been selected, a message requesting a name for the input data file and a box where this name must be typed will appear in the same screen, as illustrated in Figure A4. With the N option, the printer message is "Enter Input File Name," instead of "Give a Name to this Data File."

If the user chooses to enter the data interactively, the forward modeling information shown in Figure A5 will appear next. Default values will appear for the radius of the disk load, the bottom boundary condition, and the computation of stresses and strains. The default options in the present version of the program are a radius of 5.9 inches (0.15 m), existence of rigid rock at a finite depth (0), and no computation of stresses. The following screen (Figure A6) contains the information on the material properties of the various layers from top to bottom. Default values are used for Poisson's ratio (0.35), the unit weight (120 lb/cu ft or 18,500 N/m^3), and damping (0.02 or 2 %). Default values can always be changed by activating he corresponding box and typing the desired ones. The additional information requested consists of the thickness and the Young's modulus of each layer. If the last layer extends to infinity (an elastic half space), it is recommended that one use values of 60 ft (20 m). The program will automatically place an approximate half-space boundary condition at this depth. Use of a larger value would increase the time of computation, while use of a smaller one might affect the accuracy of the results.

If one has specified that computation of stresses is desired, the next screen will ask at how many points the stresses are desired (strains will also be computed at the same points). The coordinates of these points must be entered next (Figure A7). A maximum number of 4 points can be requested.

If one has chosen the option to enter the data through a batch file instead of through the interactive mode, the format for the input is illustrated in Figures A8 and A9.

After all the data have been input (either interactively or through an existing batch file), the Fortran program will be automatically created and execution will start. Upon completion of the analysis, the results will be displayed graphically on the screen, if the user requests it. A message will appear on the screen asking the user to choose among the following options:

Exit the program (type 0)

Display plots of the transfer functions at the different receivers (type 1)

Display plot of the peak displacements (deflection basin) (type 2)

Display plots of the time histories of the displacements (type 3)

Display plots of the steady state response (type 4)

For option 1 and 3, the program will ask at what receivers the information is desired. For option 4, the program will ask for what value of the frequency the results are desired.

After viewing the requested plots on the screen, one will press <Enter> to quit and return to the menu where the options are provided. One can press the <Print Screen> key to obtain a copy of any of the plots. To quit the processing of the results in graphical form, one would type 0 (Exit the program option). At this time the program will show on the screen automatically the contents of a file FWDOUT2, which contains the values of the peak displacements at the different receivers and the maximum stresses and strains if these had been requested.

Three output files are created by the program and stored in the current directory:

FWDOUT1 which contains the transfer functions at the different receivers

FWDOUT2 which contains the peak displacements and stresses

FWDOUT3 which contains the time histories of the displacements at the different receivers

Figures A10, A11 and A12 show typical contents of these files.

A.4.2 Inversion Option

If the inversion option is selected, the next screen to appear will be as shown in Figure A13. The first question asked is whether the data are in the standard format provided by the Dynatest FWD (as obtained in the field) or in a specially created file. The default option corresponds to the standard Dynatest FWD field data because this is the equipment available at the TxDOT.

In the second box of the screen, one must provide the name of the file that will contain the field data, which can have up to 12 characters. For option 1 (Dynatest field data) it will be the name of the Dynatest file. For option 2 this file may contain only the peak displacements or the complete time histories as discussed in more detail later. The name of the desired output file must be entered in the third box. It must have a maximum of eight characters and should not have an extension. The program will create two files with this name and the extensions .RES and .ITE.

The fourth box is used to specify the number of layers with different properties in the pavement system (maximum 10). The last piece of information on this screen is the radius of the load, with a default value of 5.9 inches (0.15 m).

Whatever the option selected for the input format, the next screen will contain the information on estimated properties for the layers. Default values of 0.35, 120 lbs/cu ft (18,500 N/m^3) and 0.02 (2%) are used for Poisson's ratio, the unit weight, and the material damping. These values may be changed by activating the corresponding box and typing the desired figures. The required information is the thickness and estimated modulus of elasticity of the pavement layers, except the bottom one. The thickness and modulus of this one will be back-calculated by the program. This screen is illustrated in Figure A14.

The next screen for input option 1 (Dynatest field data) is shown in the upper part of Figure A15. It asks whether one desires a static or a pseudo-dynamic inversion as explained earlier, and whether a full dynamic inversion is to be performed, starting with the values computed in the static or pseudo-dynamic procedures. Finally, the program asks if one wants to follow the inversion process step by step, in an interactive mode, or wants just to let it go automatically.

The lower part of Figure A15 shows the screen for input option 2 (data other than the Dynatest field data). It asks first whether the input data are the time histories of force and displacement or the peak force and displacements. The remaining questions are exactly the same as the ones in the upper part of Figure A15. For this option, the format for the input is illustrated in Figures A16, A17, and A18.

After all the data have been input, the Fortran program will be automatically loaded and execution will start. If the user has chosen to perform the inversion process step by step, the program will display a plot of the computed vs the measured deflections, and the user must decide whether to continue or stop the iterations. If the inversion process is performed automatically, then the program would stop when the root-mean-square of the relative errors is less than 0.01, the relative difference between iterations is less than 0.001, or if the number of iterations is greater than 10. In both cases (step by step or automatic (inversion), the program takes as the final solution the profile that has the lowest root-mean-square of relative errors, which is not necessarily the one of the last iteration.

If the user has chosen the option of dynamic inversion, this is performed after the static or pseudo-dynamic inversion process has been completed and the initial approximation is the solution obtained with the static or pseudo-dynamic inversion procedure.

Two output files are created with the name specified by the user. One of the files has the extension .RES and contains the final results. The other file has the extension .ITE and contains the profiles for all the iterations performed. Figures A19 and A20 show typical contents of these files.

After finishing the inversion process, the file with the extension .RES is displayed on the screen automatically.

FWD-DYN

Falling Weigth Deflectometer Linear Dynamic
Analysis Program
Forward Modeling and Inversion

Civil Engineering Department
University of Texas at Austin
August, 1993

Press any key to continue

Figure A1

Do you want to perform Forward modeling
or Inversion ?
(type F or I)

I

Do you want to use the English or International
system of units ?
(type ES or SI)

ES

Use the arrow keys to move between boxes
(Up)
↑
← (Left)   (Right) →
↓
(Down)

Press <Page Down> for the next screen
Press <Esc> to exit the program

Figure A2

Inversion Information

Is the Input data:
1. Dynatest FWD field data
2. Data other than Dynatest ?
(type 1 or 2)

	1

Name of the ... D

You have not finished entering the data in this screen
Please complete it

< Press any key to leave this box >

Name of the ... UT
(Please don' ...

Number of layers of the profile

Radius of disk load. (inches) 5.9

<Page Down> Next screen. <Page Up> Previous screen
<Esc> exit the program

Figure A3

Do you want to enter the input data
from the screen ?
(type Y or N)

Y

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Do you want to enter the input data
from the screen ?
(type Y or N)

Y

Give a name to this Data File
(maximum of 12 characters)

FLEX20.DAT

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Figure A4

Forward Modeling Information

1. Title for the job

2. Radius of disk load, (inches) 5.9

3. Number of Layer (from 1 to 10)

4. Botton Boundary Condition 0
   0 for rigid base
   1 for elastic halfspace

5. Do you want to compute stresses ? (Y or N) N

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Forward Modeling Information

1. Title for the job

FLEXIBLE PAVEMENT, BEDROCK AT 20 FT

2. Radius of disk load, (inches) 5.9

3. Number of Layer (from 1 to 10) 3

4. Botton Boundary Condition 0
   0 for rigid base
   1 for elastic halfspace

5. Do you want to compute stresses ? (Y or N) Y

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Figure A5

Material Properties

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson's Ratio	Unit Weight (pcf)	Damping
1			0.35	120.0	0.02
2			0.35	120.0	0.02
3			0.35	120.0	0.02

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Material Properties

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson's Ratio	Unit Weight (pcf)	Damping
1	6	440	0.35	120.0	0.02
2	12	70	0.35	120.0	0.02
3	222	18	0.35	120.0	0.02

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Figure A6

Number of points at which stresses are desired
(from 1 to 4)

Press <Page Down> to continue, <Page Up> Previous screen
<Esc> exit the program

Number of points at which stresses are desired 4
(from 1 to 4)

Point coordinates (inches)

Point	Radial Dist.	Depth
1	0.0	0.0
2	0.0	6.0
3	0.0	18.0
4	0.0	24.0

Press <Page Down> to continue, <Page Up> Previous screen
<Esc> exit the program

Figure A7

FORWARD MODELING INPUT DATA

LINE-BY-LINE GUIDE TO INPUT

The input data can be given interactively or as a batch file

```
C----------------------------------------------------------------------
C
C               Reads input data for forward analysis ( Free format):
C   TITLE ................> Any title
C
C   RADIUS................> Radius of the disk load ( inches or meters )
C
C   NL, IROCK.............> Where:
C          NL: No. of physical layers for the profile and
C       IROCK: Botton boundary condition, specified 0 for rigid rock (fixed,
C              displacements are zero) and 1 for and elastic halfspace.
C
C   Provides now with NL lines with the following:
C                 Enter for each layer
C                 I    H     E    NU   UWE   DAM
C
C   Where:
C          I:  Layer number
C          H:  Thickness of each physical layer. (inches or meters)
C          E:  Young's modulus of each layer. (Ksi or MPa)
C        UWE:  Unit weigth of each layer. (pcf or N/m3)
C         NU:  Poisson's ratio of each layer.
C        DAM:  Material damping ratio of each layer.
C
C   ISTRESS...............> specify 1 if you want to compute stresses,
C                                   0 if you don't
C   If ISTRESS = 1 then specify:
C   NSTRESS...............> No of points at which stresses are desired
C                           NSTRESS <= 4
C   I, XS(I), ZS(I) .........> Radial and vertical coordinates of point I,
C                              (inches or meters) repeat this line NSTRESS
C                              times
C
C----------------------------------------------------------------------
```

Figure A8

```
FLEXIBLE PAVEMENT, BEDROCK AT 20 FT
  5.90
          3              0
       1      6.00    .43672E+03        .35       120.00      .02
       2     12.00    .69876E+02        .35       120.00      .02
       3    222.00    .17469E+02        .35       120.00      .02
          1
          4
1      .00       .00
2      .00      6.00
3      .00     18.00
4      .00     24.00
```

Figure A9 Example of input data for the forward modeling option.

FLEXIBLE PAVEMENT, BEDROCK AT 20 FT

The date is 11, 9,1993 The time is 11:39:56. 8
Radius of disk load = 5.90 inches

Boundary conditions:
Botton = rigid rock (displacements are zero)

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson r.	Unit Weight (pcf)	Damping
1	6.00	.43672E+03	.35	120.00	.02
2	12.00	.69876E+02	.35	120.00	.02
3	222.00	.17469E+02	.35	120.00	.02

TRANSFER FUNCTIONS

**** Solution at station No. 1 ****

Frequency(hz)	Amplitude(mils/kip)	Phase(deg)
.250	.1051E+01	2.2910
.500	.1051E+01	2.2914
.750	.1051E+01	2.2921
1.000	.1052E+01	2.2931

240.000	.3142E-01	112.5174
270.000	.2194E-01	153.2584

Resonant frequency = 11.75

**** Solution at station No. 7 ****

Frequency(hz)	Amplitude(mils/kip)	Phase(deg)
.250	.1401E+00	2.2899
.500	.1402E+00	2.2919

Figure A10 FWDOUT1

```
--------------------------------------------------------------------------
FLEXIBLE PAVEMENT, BEDROCK AT 20 FT
--------------------------------------------------------------------------

The date is  11, 9,1993                            The time is 11:39:56. 8
Radius of disk load =   5.90 inches

Boundary conditions:
Botton    = rigid rock (displacements are zero)

     Layer  Thickness  Young Modulus  Poisson r.  Unit Weight  Damping
             (inches)      (ksi)                      (pcf)

       1      6.00     .43672E+03        .35        120.00       .02
       2     12.00     .69876E+02        .35        120.00       .02
       3    222.00     .17469E+02        .35        120.00       .02

                    PEAK DISPLACEMENTS
                    ------------------

 Station No.   Distance (ft)   Arr. Time (sec)    Peak Disp. (mils/kip)

    1               .00            .0180            .10314E+01
    2              1.00            .0180            .69333E+00
    3              2.00            .0200            .47489E+00
    4              3.00            .0220            .35107E+00
    5              4.00            .0240            .27318E+00
    6              5.00            .0260            .22078E+00
    7              6.00            .0280            .18355E+00

                    PEAK STRESSES AND PEAK STRAINS
                    -----------------------------

              ( stresses (ksi) and strains by kip of load)


     Position No.1
          Radial distance ==>     .00
          Depth  ==========>      .00

     Layer =  1
                 Shear RZ     Vertical      Radial      Azimuthal
Stresses        .0000E+00    .9150E-02    .1541E-01    .1541E-01
Strains         .0000E+00   -.3746E-05    .1560E-04    .1560E-04
```

Figure A11 FWDOUT2

```
     Position No.2
          Radial distance ==>      .00
          Depth  ===========>     6.00

     Layer =  1
                        Shear RZ     Vertical      Radial       Azimuthal
Stresses               .0000E+00    .3178E-02   -.7457E-02   -.7457E-02
Strains                .0000E+00    .1923E-04   -.1365E-04   -.1365E-04
     Layer =  2
                        Shear RZ     Vertical      Radial       Azimuthal
Stresses               .0000E+00    .3178E-02    .2442E-03    .2442E-03
Strains                .0000E+00    .4303E-04   -.1365E-04   -.1365E-04

     Position No.3
          Radial distance ==>      .00
          Depth  ===========>    18.00

     Layer =  2
                        Shear RZ     Vertical      Radial       Azimuthal
Stresses               .0000E+00    .1413E-03   -.1160E-02   -.1160E-02
Strains                .0000E+00    .1364E-04   -.1149E-04   -.1149E-04
     Layer =  3
                        Shear RZ     Vertical      Radial       Azimuthal
Stresses               .0000E+00    .1413E-03   -.2328E-03   -.2328E-03
Strains                .0000E+00    .1742E-04   -.1149E-04   -.1149E-04

     Position No.4
          Radial distance ==>      .00
          Depth  ===========>    24.00

     Layer =  3
                        Shear RZ     Vertical      Radial       Azimuthal
Stresses               .0000E+00    .3457E-03    .7862E-04    .7862E-04
Strains                .0000E+00    .1664E-04   -.4000E-05   -.4000E-05
```

Figure A11 (cont.)

FLEXIBLE PAVEMENT, BEDROCK AT 20 FT

The date is 11, 9,1993 The time is 11:39:56. 8
Radius of disk load = 5.90 inches

Boundary conditions:
Botton = rigid rock (displacements are zero)

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson r.	Unit Weight (pcf)	Damping
1	6.00	.43672E+03	.35	120.00	.02
2	12.00	.69876E+02	.35	120.00	.02
3	222.00	.17469E+02	.35	120.00	.02

TIME DOMAIN RESPONSE

Time (sec)	***** Displacements (mils/kip) ***** Stat.1	Stat.2	Stat.3	Stat.4	Stat.5	Stat.6	Stat.7	Force
.000	-.4025E-01	-.3001E-01	-.1926E-01	-.1202E-01	-.7403E-02	-.4242E-02	-.2235E-02	.0000
.002	.1108E-01	-.1865E-01	-.2358E-01	-.1555E-01	-.7836E-02	-.3786E-02	-.1983E-02	.1250
.004	.1202E+00	.4032E-01	-.3465E-02	-.1541E-01	-.1249E-01	-.6034E-02	-.1886E-02	.2500
.006	.2399E+00	.1147E+00	.3982E-01	.5791E-02	-.8022E-02	-.9775E-02	-.5698E-02	.3750
.008	.3798E+00	.2053E+00	.9408E-01	.3838E-01	.9859E-02	-.3835E-02	-.8008E-02	.5000
.010	.5184E+00	.2985E+00	.1569E+00	.8065E-01	.3587E-01	.1012E-01	-.3011E-02	.6250
.012	.6721E+00	.4016E+00	.2236E+00	.1268E+00	.6883E-01	.3177E-01	.8678E-02	.7500
.014	.8169E+00	.5018E+00	.2946E+00	.1779E+00	.1045E+00	.5681E-01	.2562E-01	.8750
.016	.9896E+00	.6173E+00	.3675E+00	.2293E+00	.1439E+00	.8602E-01	.4606E-01	1.0000
.018	.1031E+01	.6933E+00	.4453E+00	.2879E+00	.1839E+00	.1155E+00	.6913E-01	.8750
.020	.9643E+00	.6783E+00	.4749E+00	.3394E+00	.2339E+00	.1522E+00	.9415E-01	.7500
.022	.8727E+00	.6310E+00	.4603E+00	.3511E+00	.2665E+00	.1928E+00	.1287E+00	.6250
.024	.7489E+00	.5564E+00	.4239E+00	.3394E+00	.2732E+00	.2150E+00	.1608E+00	.5000
.026	.6219E+00	.4741E+00	.3726E+00	.3106E+00	.2636E+00	.2208E+00	.1787E+00	.3750
.028	.4790E+00	.3801E+00	.3142E+00	.2735E+00	.2416E+00	.2127E+00	.1836E+00	.2500
.030	.3412E+00	.2866E+00	.2502E+00	.2294E+00	.2134E+00	.1968E+00	.1781E+00	.1250

Figure A12 FWDOUT3

```
                         Inversion Information

  Is the Input data:
  1. Dynatest FWD field data                                   [ 1 ]
  2. Data other than Dynatest ?
        (type 1 or 2)

  Name of the Input File                               [            ]


  Name of the Output File                                 [        ]
  (Please don't give an extension to this name)

  Number of layers of the profile                            [    ]

  Radius of disk load. (inches)                            [ 5.9   ]
```

<Page Down> Next screen. <Page Up> Previous screen
<Esc> exit the program

```
                         Inversion Information

  Is the Input data:
  1. Dynatest FWD field data                                   [ 1 ]
  2. Data other than Dynatest ?
        (type 1 or 2)

  Name of the Input File                               [ ROBC.FWD   ]


  Name of the Output File                                 [ ROBC   ]
  (Please don't give an extension to this name)

  Number of layers of the profile                            [ 3  ]

  Radius of disk load. (inches)                            [ 5.9   ]
```

<Page Down> Next screen. <Page Up> Previous screen
<Esc> exit the program

Figure A13

Initial Estimate of Material Properties

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson's Ratio	Unit Weight (pcf)	Damping
1			0.35	120.0	0.02
2			0.35	120.0	0.02
3	will be	computed	0.35	120.0	0.02

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Initial Estimate of Material Properties

Layer	Thickness (inches)	Young Modulus (ksi)	Poisson's Ratio	Unit Weight (pcf)	Damping
1	2	500	0.35	120.0	0.02
2	24	50	0.35	120.0	0.02
3	will be	computed	0.35	120.0	0.02

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Figure A14

What type of inversion proccedure do you want to perform

A. 1.Static inversion or 2.Pseudo-Dynamic inversion 2
(type 1 or 2)

B. Dynamic inversion (type Y or N) ? N

Do you want to see the inversion process step by step ? Y
(type Y or N)

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Is the input data
1. The time history of force and displacements or 1
2. The peak force and displacements ?
(type 1 or 2)

What type of inversion proccedure do you want to perform

A. 1.Static inversion or 2.Pseudo-Dynamic inversion 2
(type 1 or 2)

B. Dynamic inversion (type Y or N) ? N

Do you want to see the inversion process step by step ? Y
(type Y or N)

<Page Down> Next screen, <Page Up> Previous screen
<Esc> exit the program

Figure A15

INVERSION INPUT DATA

LINE-BY-LINE GUIDE TO INPUT

There are two options:
1.- Specify the peak displacements
2.- Specify the time history of displacements and force

```
C-----------------------------------------------------------------------
C
C              Reads input data for inversion analysis ( Free format):
C   TITLE ................> Any title
C
C   If the input data are the peak displacements,then read
C   FORCE.................> peak load
C   STAT(I)...............> peak displacements (7 lines)
C
C   In other case read the time history of displacements and load.
C   Read as many lines as data you have, each line containing the
C   displacements at the 7 receivers and the load.
C   for example for line I, we have:
C   T(I), V(I,1),........,V(I,7), FOR(I), where:
C   The sampling rate is supposed to be 0.002 sec. (the time is read
C   in miliseconds)
C   The force and displacements must be specified in kips and mils
C   for the English system of units and in KN and microns for the
C   international system of units
C-----------------------------------------------------------------------
```

Figure A16

```
FLEXIBLE PAVEMENT, BEDROCK AT 20 FT
1.0
.10314E+01
.69333E+00
.47489E+00
.35107E+00
.27318E+00
.22078E+00
.18355E+00
```

Figure A17 Inversion input data, peak force and peak displacements.

FLEXIBLE PAVEMENT, BEDROCK AT 20 FT

.000	-.4025E-01	-.3001E-01	-.1926E-01	-.1202E-01	-.7403E-02	-.4242E-02	-.2235E-02	.0000
.002	.1108E-01	-.1865E-01	-.2358E-01	-.1555E-01	-.7836E-02	-.3786E-02	-.1983E-02	.1250
.004	.1202E+00	.4032E-01	-.3465E-02	-.1541E-01	-.1249E-01	-.6034E-02	-.1886E-02	.2500
.006	.2399E+00	.1147E+00	.3982E-01	.5791E-02	-.8022E-02	-.9775E-02	-.5698E-02	.3750
.008	.3798E+00	.2053E+00	.9408E-01	.3838E-01	.9859E-02	-.3835E-02	-.8008E-02	.5000
.010	.5184E+00	.2985E+00	.1569E+00	.8065E-01	.3587E-01	.1012E-01	-.3011E-02	.6250
.012	.6721E+00	.4016E+00	.2236E+00	.1268E+00	.6883E-01	.3177E-01	.8678E-02	.7500
.014	.8169E+00	.5018E+00	.2946E+00	.1779E+00	.1045E+00	.5681E-01	.2562E-01	.8750
.016	.9896E+00	.6173E+00	.3675E+00	.2293E+00	.1439E+00	.8602E-01	.4606E-01	1.0000
.018	.1031E+01	.6933E+00	.4453E+00	.2879E+00	.1839E+00	.1155E+00	.6913E-01	.8750
.020	.9643E+00	.6783E+00	.4749E+00	.3394E+00	.2339E+00	.1522E+00	.9415E-01	.7500
.022	.8727E+00	.6310E+00	.4603E+00	.3511E+00	.2665E+00	.1928E+00	.1287E+00	.6250
.024	.7489E+00	.5564E+00	.4239E+00	.3394E+00	.2732E+00	.2150E+00	.1608E+00	.5000
.026	.6219E+00	.4741E+00	.3726E+00	.3106E+00	.2636E+00	.2208E+00	.1787E+00	.3750
.028	.4790E+00	.3801E+00	.3142E+00	.2735E+00	.2416E+00	.2127E+00	.1836E+00	.2500
.030	.3412E+00	.2866E+00	.2502E+00	.2294E+00	.2134E+00	.1968E+00	.1781E+00	.1250
.032	.1850E+00	.1823E+00	.1835E+00	.1830E+00	.1796E+00	.1741E+00	.1657E+00	.0000
.034	.9518E-01	.9743E-01	.1123E+00	.1304E+00	.1431E+00	.1481E+00	.1476E+00	.0000
.036	.5607E-01	.5564E-01	.6411E-01	.8016E-01	.9942E-01	.1160E+00	.1256E+00	.0000
.038	.3019E-01	.2978E-01	.3634E-01	.4785E-01	.6273E-01	.7978E-01	.9615E-01	.0000
.040	.1661E-01	.1508E-01	.1849E-01	.2639E-01	.3753E-01	.5100E-01	.6598E-01	.0000
.042	.5445E-02	.3988E-02	.6316E-02	.1158E-01	.1917E-01	.2913E-01	.4121E-01	.0000
.044	-.2662E-02	-.4557E-02	-.3642E-02	-.1701E-03	.5229E-02	.1239E-01	.2142E-01	.0000
.046	-.1159E-01	-.1338E-01	-.1284E-01	-.1050E-01	-.6820E-02	-.1640E-02	.5157E-02	.0000
.048	-.2016E-01	-.2209E-01	-.2217E-01	-.2058E-01	-.1781E-01	-.1396E-01	-.8891E-02	.0000
.050	-.2955E-01	-.3133E-01	-.3150E-01	-.3036E-01	-.2829E-01	-.2525E-01	-.2123E-01	.0000

Figure A18 Inversion input data, time history of displacement and force.

RIGID PAVEMENT, BEDROCK AT 20 FT

The date is 11,10,1993 The time is 8:11:43.72
Radius of disk load = 5.90 inches

Pseudo-Dynamic Inversion

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8881.72	5516.60	.35	120.00
2	6.00	2220.45	344.79	.35	120.00
3	12.00	1096.66	84.11	.35	120.00
4	224.69	481.91	16.24	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.33	.30	.27	.24	.21	.18	.15
Computed Deflection:	.33	.30	.27	.24	.21	.18	.15

Root-mean-square of relative error = .000045

Dynamic Inversion

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8991.70	5654.06	.35	120.00
2	6.00	2491.71	434.18	.35	120.00
3	12.00	986.03	67.99	.35	120.00
4	224.69	498.65	17.39	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.27	.25	.23	.20	.18	.15	.13
Computed Deflection:	.27	.25	.23	.20	.18	.16	.14

Root-mean-square of relative error = .001376

Figure A19 name. RES

RIGID PAVEMENT, BEDROCK AT 20 FT

The date is 11,10,1993 The time is 8:11:43.72
Radius of disk load = 5.90 inches

Pseudo-Dynamic Inversion

THE PRELIMINARY PROFILE IS

NUMBER OF LAYERS = 4

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	7562.95	4000.00	.35	120.00
2	6.00	2673.91	500.00	.35	120.00
3	12.00	845.56	50.00	.35	120.00
4	224.69	493.12	17.00	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.33	.30	.27	.24	.21	.18	.15
Computed Deflection:	.36	.33	.29	.25	.21	.18	.15

Root-mean-square of relative error = .019176

ITERATION NUMBER 1

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8561.25	5125.68	.35	120.00
2	6.00	2368.32	392.25	.35	120.00
3	12.00	1053.35	77.59	.35	120.00
4	224.69	482.14	16.26	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.33	.30	.27	.24	.21	.18	.15
Computed Deflection:	.34	.31	.28	.24	.21	.18	.15

Root-mean-square of relative error = .004946

Figure A20 name.ITE

ITERATION NUMBER 2

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8860.75	5490.58	.35	120.00
2	6.00	2233.81	348.95	.35	120.00
3	12.00	1094.07	83.71	.35	120.00
4	224.69	481.96	16.24	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.33	.30	.27	.24	.21	.18	.15
Computed Deflection:	.33	.30	.27	.24	.21	.18	.15

Root-mean-square of relative error = .000224

ITERATION NUMBER 3

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8881.72	5516.60	.35	120.00
2	6.00	2220.45	344.79	.35	120.00
3	12.00	1096.66	84.11	.35	120.00
4	224.69	481.91	16.24	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.33	.30	.27	.24	.21	.18	.15
Computed Deflection:	.33	.30	.27	.24	.21	.18	.15

Root-mean-square of relative error = .000045

Dynamic Inversion

THE PRELIMINARY PROFILE IS

NUMBER OF LAYERS = 4

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8881.72	5516.60	.35	120.00
2	6.00	2220.45	344.79	.35	120.00
3	12.00	1096.66	84.11	.35	120.00
4	224.69	481.91	16.24	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.27	.25	.23	.20	.18	.15	.13
Computed Deflection:	.28	.26	.23	.21	.18	.16	.14

Root-mean-square of relative error = .012502

Figure A20 name.ITE (cont.)

ITERATION NUMBER 1

Layer	Thickness	Shear Vel.	Young Mod.	Poisson r.	Unit W.
1	10.00	8991.70	5654.06	.35	120.00
2	6.00	2491.71	434.18	.35	120.00
3	12.00	986.03	67.99	.35	120.00
4	224.69	498.65	17.39	.35	120.00

RECEIVERS	R1	R2	R3	R4	R5	R6	R7
Measured Deflection:	.27	.25	.23	.20	.18	.15	.13
Computed Deflection:	.27	.25	.23	.20	.18	.16	.14

Root-mean-square of relative error = .001376

Figure A20 name.ITE (cont.)

BIBLIOGRAPHY

Chang, D. W., Roesset, J. M., and Stokoe, K. H., II, (1992), "Nonlinear Effects in Falling Weight Deflectometer Tests," Transportation Research Record 1355, pp 1-7.

Chang, D. W., Kang, V. Y., Roesset, J. M., and Stokoe, K. H., II (1992), "Effect of Depth to Bedrock on Deflection Basins Obtained with Dynaflect and FWD Tests," Transportation Research Record 1355, pp 8-16.

Eringen, A. C., and Suhubi, S., (1975), *Elasto Dynamics*, Vol 2, Academic Press, New York.

Foinquinos, R., Roesset, J. M., and Stokoe, K. H., II, (1993), "Dynamic Interpretation of FWD Deflection Basins," XII World Congress of IRF, Madrid.

Haskell, N. A., (1953), "The Dispersion of Surface Waves in Multi-Layered Media," Bulletin of the Seismological Society of America, Vol 43, No. 1.

Kang, V. K., Roesset, J. M., and Stokoe, K. H., II, (1990), "Effect of the Finite Width of Pavements on Deflection Basins Obtained with Dynaflect and FWD Tests," Transportation Research Board.

Kausel, E., (1974), "Forced Vibrations of Circular Foundations on Layered Media," Research Report R74-11, Department of Civil Engineers, M.I.T.

Kausel, E., and Roesset, J. M., (1981), "Stiffness Matrices for Layered Soils," Bulletin of the Seismology Society of American, Vol 71, No. 6.

Lamb, H., (1904), "On the Propagation of Tremors Over the Surface of an Elastic Solid," Phil. Transactions Royal Society of London., Series A.

Miller, G. F., and Pursey, H., (1954), "The Field and Radiation Impedance of Mechanical Radiator on the Free Surface of a Semi-Infinite Isotropic Solid," *Proceeding*, Royal Society of London Series A.

Mooney, H. M., (1974), "Some Numerical Solutions of Lamb's Problem," Bulletin of the Seismology Society of America, Vol 64, No. 2.

Pekeris, C., (1955), "The Seismic Surface Pulse, " *Proceedings*, National Academy of Science.

Seng, C. R., Stokoe, K. H., II, and Roesset, J. M., (1993), "Effect of Depth of Bedrock on the Accuracy of Backcalculated Moduli Obtained with Dynaflect and FWD Tests," Research Report 1175-5, Center of Transportation Research, The University of Texas at Austin.

Thomson, W. T., (1950), "Transmission of Elastic Waves Through a Stratified Soil Medium," Journal of Applied Physics, Vol 21.

Uddin, W., (1984), "A Structural Evaluation Methodology for Pavements Based on Dynamic Deflections," doctoral dissertation, The University of Texas at Austin.

Waas, G., (1972), "Linear Two-Dimensional Analysis of Soil Dynamics Problems of Semi-Infinite Layered Media," doctoral dissertation, University of California at Berkeley.