

1. Report No. FHWA/TX-98/1435-1	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle CHARTS FOR REMEDIATION OF HIGHLY PLASTIC CLAY EMBANKMENTS IN TEXAS		5. Report Date October 1997	
		6. Performing Organization Code	
7. Author(s) Jeffrey Alan Fippin and Stephen G. Wright		8. Performing Organization Report No. 1435-1	
9. Performing Organization Name and Address Center for Transportation Research The University of Texas at Austin 3208 Red River, Suite 200 Austin, TX 78705-2650		10. Work Unit No. (TRAIS)	
		11. Contract or Grant No. 0-1435	
12. Sponsoring Agency Name and Address Texas Department of Transportation Research and Technology Transfer Section/Construction Division P.O. Box 5080 Austin, TX 78763-5080		13. Type of Report and Period Covered Research Report (9/96 — 8/97)	
		14. Sponsoring Agency Code	
15. Supplementary Notes Project conducted in cooperation with the Federal Highway Administration.			
16. Abstract  Clay embankments along Texas highways have been prone to shallow slope failures that occur a number of years after construction. Typical embankments that have failed are constructed of highly plastic clays, with heights ranging from 3 to 9 m (10 to 30 ft) and their side slopes at 2:1 or flatter.  This report presents the development of design charts for shallow slides. Strengthening of the soil with additives and reinforcement with geosynthetics are the two methods considered. Numerous limit equilibrium slope stability analyses were performed to define the effects of soil and reinforcement strength properties and the geometry of the remediated zone. Results of these analyses were plotted in dimensionless form to produce design charts to assist engineers in the remediation of shallow slope failures.			
17. Key Words Geotechnical engineering, slope stability, slope maintenance, landslides		18. Distribution Statement No restrictions. This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161.	
19. Security Classif. (of report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 114	22. Price

**CHARTS FOR REMEDIATION OF HIGHLY PLASTIC  
CLAY EMBANKMENTS IN TEXAS**

by  
Jeffrey Alan Fippin  
and  
S. G. Wright

Research Report Number 1435-1

Research Project 0-1435  
*Development of Simplified Procedures for Design of Slope Reinforcement  
for Slide Repair*

Conducted for the

**TEXAS DEPARTMENT OF TRANSPORTATION**

in cooperation with the

**U.S. DEPARTMENT OF TRANSPORTATION  
FEDERAL HIGHWAY ADMINISTRATION**

by the

**CENTER FOR TRANSPORTATION RESEARCH**  
Bureau of Engineering Research  
**THE UNIVERSITY OF TEXAS AT AUSTIN**

October 1997



This report was prepared in cooperation with the Texas Department of Transportation (TxDOT) and the U.S. Department of Transportation, Federal Highway Administration.

### **DISCLAIMERS**

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration or the Texas Department of Transportation. This report does not constitute a standard, specification, or regulation.

There was no invention or discovery conceived or first actually reduced to practice in the course of or under this contract, including art, method, process, machine, manufacture, design, or composition of matter, or any new and useful improvement thereof, or any variety of plant, which is or may be patentable under the patent laws of the United States of America or any foreign country.

### **ACKNOWLEDGEMENTS**

The authors acknowledge the expert assistance provided by the research project director, C. Fu, of TxDOT's Materials and Tests Division, and by C. Utley of the Lubbock District, who served as project coordinator. The authors also thank the other TxDOT members of the research team, which included J. Cravens of the FHWA, S. Ekstrom of the Paris District, L. Kelley of the Dallas District, M. McFarland of the Design Division, and S. Yin of the Houston District.

**NOT INTENDED FOR CONSTRUCTION,  
BIDDING, OR PERMIT PURPOSES**

Stephen G. Wright, P.E. (Texas No. 49007)  
*Research Supervisor*



## TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION .....	1
1.1 BACKGROUND.....	1
1.2 REPORT ORGANIZATION .....	1
CHAPTER 2. DEVELOPMENT OF DESIGN CHARTS FOR SLOPES STRENGTHENED WITH ADDITIVES.....	3
2.1 INTRODUCTION.....	3
2.2 PARAMETRIC STUDIES.....	4
2.3 EFFECT OF PORE WATER PRESSURE COEFFICIENT ( $R_u$ ).....	6
2.4 EFFECT OF VARYING DEPTH, D, OF STRENGTHENED ZONE.....	7
2.5 SLOPES ON SAME FOUNDATION MATERIALS .....	9
2.5.1 Minimum Widths and Dimensionless Charts .....	11
2.5.2 Threshold Strengths and Dimensionless Charts.....	12
2.5.3 Example Problem.....	15
2.6 SLOPES ON MUCH STRONGER FOUNDATIONS.....	16
2.6.1 Dimensionless Charts.....	17
2.6.2 Example Problems.....	28
2.7 CONCLUSIONS.....	29
CHAPTER 3. GEOSYNTHETIC SLOPE REINFORCEMENT .....	31
3.1 INTRODUCTION.....	31
3.2 SCHMERTMANN ET AL. (1987).....	31
3.2.1 Minimum Reinforcement Force Required .....	32
3.2.2 Minimum Required Reinforcement Length.....	34
3.3 JEWELL (1990).....	38
3.3.1 Minimum Reinforcement Force Required .....	39
3.3.2 Minimum Required Reinforcement Lengths.....	42
3.3.3 Vertical Spacing of Reinforcement Layers .....	45
3.4 LESHCHINSKY AND REINSCHMIDT .....	47
3.4.1 Required Reinforcement Strength.....	47
3.5 RESLOPE .....	50
3.5.1 Tieback Analysis (Part I).....	50
3.5.2 Compound Stability (Part II).....	51
3.5.3 Direct Sliding (Part III) .....	53
3.5.4 Length of Reinforcement Layers.....	54
3.5.5 Deep-Seated Failure (Part IV).....	55
3.5.6 Final Design .....	55
3.6 COMPARISON OF DESIGN METHODS.....	55
3.7 SUMMARY .....	61
3.8 APPLICABILITY OF EXISTING METHODS .....	61

CHAPTER 4. NEW CHARTS FOR GEOSYNTHETIC-REINFORCED SLOPE REMEDIATION.....	63
4.1 INTRODUCTION.....	63
4.2 TOTAL REQUIRED REINFORCEMENT FORCE AND REINFORCEMENT SPACING.....	63
4.2.1 Calculation Procedure .....	65
4.2.2 Chart Presentation .....	65
4.3 LENGTH OF REINFORCEMENT .....	72
4.3.1 Length for Global Stability .....	72
4.3.2 Length for Direct Sliding .....	80
4.3.3 Determination of Reinforcement Length .....	84
4.4 USE OF CHARTS FOR FACTORS OF SAFETY GREATER THAN 1.0 .....	84
4.5 EFFECT OF SURCHARGE .....	84
4.6 COMPARISON OF RESULTS WITH EXISTING METHODS .....	87
4.7 SUMMARY AND CONCLUSIONS.....	90
CHAPTER 5. SUMMARY .....	93
5.1 CHARTS FOR STRENGTHENED SOIL.....	93
5.2 CHARTS FOR GEOSYNTHETIC-REINFORCED SLOPES.....	93
5.3 RECOMMENDATIONS FOR FURTHER STUDY .....	94
BIBLIOGRAPHY .....	95
APPENDIX .....	97

## **CHAPTER 1. INTRODUCTION**

### **1.1 BACKGROUND**

Along Texas highways a large number of slope failures have occurred in embankments composed of highly plastic clays. These embankment failures are for the most part shallow slope failures that occur several years after construction. The embankments typically have side slopes less than 2:1 (horizontal:vertical) and have heights between 3 and 9 m (10 and 30 feet). Numerous slope failures of this type are described by Stauffer and Wright (1984). Kayyal and Wright (1991) also showed that after cycles of wetting and drying, clays similar to those found in the failed embankments would have shear strengths significantly lower than those of the as-compacted clay. They showed that, using the reduced shear strengths caused by wetting and drying, relatively high pore water pressures must exist for slopes flatter than 3:1 to fail. The Texas Department of Transportation (TxDOT) is looking for a simple means for designing repair measures for these slopes.

The current study was undertaken to produce charts that can be used for the design of selected remedial measures for slopes that have failed. Repair of slopes both by strengthening with lime and cement and by the use of geosynthetic reinforcement have been considered. Limited equilibrium slope stability analyses have been performed and the results used to develop appropriate charts for both types of remediation.

### **1.2 REPORT ORGANIZATION**

In Chapter 2, studies are presented that show the effect of geometry and strength for a zone of soil strengthened by adding lime or cement. Appropriate charts employing dimensionless coefficients for design are also presented. In Chapter 3, current procedures for design of slopes with geosynthetic reinforcement are reviewed. The applicability of the existing methods to the slopes of interest is discussed. In Chapter 4, new charts for design of geosynthetic-reinforced slopes are presented. These new charts cover the range of slope conditions that are applicable to the embankments of interest in Texas. Chapter 5 summarizes the work of this study and presents recommendations for further work.



## CHAPTER 2. DEVELOPMENT OF DESIGN CHARTS FOR SLOPES STRENGTHENED WITH ADDITIVES

### 2.1 INTRODUCTION

*Strengthening* soil by mixing it with such additives as lime and cement has frequently been used to repair slides in slopes. Previous studies have shown that highly plastic, expansive clays that are treated by adding lime and cement exhibit significant increases in unconfined compressive strength (Ingles and Metcalf 1972; Kennedy and Smith 1986). Table 2.1 represents strengths reported by Kennedy and Smith (1986) for saturated specimens of the highly plastic Beaumont clay after treatment with cement and with lime. Beaumont clay is commonly used in the Houston, Texas, area to construct embankments, some of which have experienced slides. The strengths shown in Table 2.1 are believed to be typical of those that could be obtained by using either portland cement or lime to strengthen the soil.

*Table 2.1 Average unconfined compressive strengths for compacted specimens of Beaumont clay treated with additives of portland cement and hydrated lime (Kennedy and Smith 1986)*

Total Cure Time, Days	Untreated Soil	4% Portland Cement Added	4% Hydrated Lime Added
7	0.6	14.7	6.8
14	0	50.5	36
35	0	66.7	58
126	1.3	75.5	58

Note: All values in psi, all specimens pulverized by 100 percent passing number 4 sieve before adding additives and compacting, additives expressed as percent of dry weight of soil.

For the present study, a number of slope stability calculations were performed using the slope stability software UTEXAS3 (Wright 1990) to investigate the effect of strengthening the soil with additives on slope stability. Results of the calculations were then used to develop design charts. Because the embankment failures that have been observed in Texas involving highly plastic clays have occurred several years after construction, it is assumed that long-term drained shear strengths prevailed. Accordingly, all calculations and charts are based on drained (effective stress) shear strength parameters. Based on the work of Kayyal and Wright, the appropriate drained shear strength is the fully softened strength representing the effects of repeated wetting and drying cycles. Figure 2.1 shows strength envelopes for saturated specimens of Paris and Beaumont clays that have been weakened by wetting and drying cycles (Kayyal 1991). The effective stress shear strength envelopes for

these materials, which are similar to materials found in the failed embankments in this study, can be modeled using an effective-stress shear strength envelope with a minimal (zero) effective stress cohesion intercept, i.e.,  $\bar{c} = 0$ .

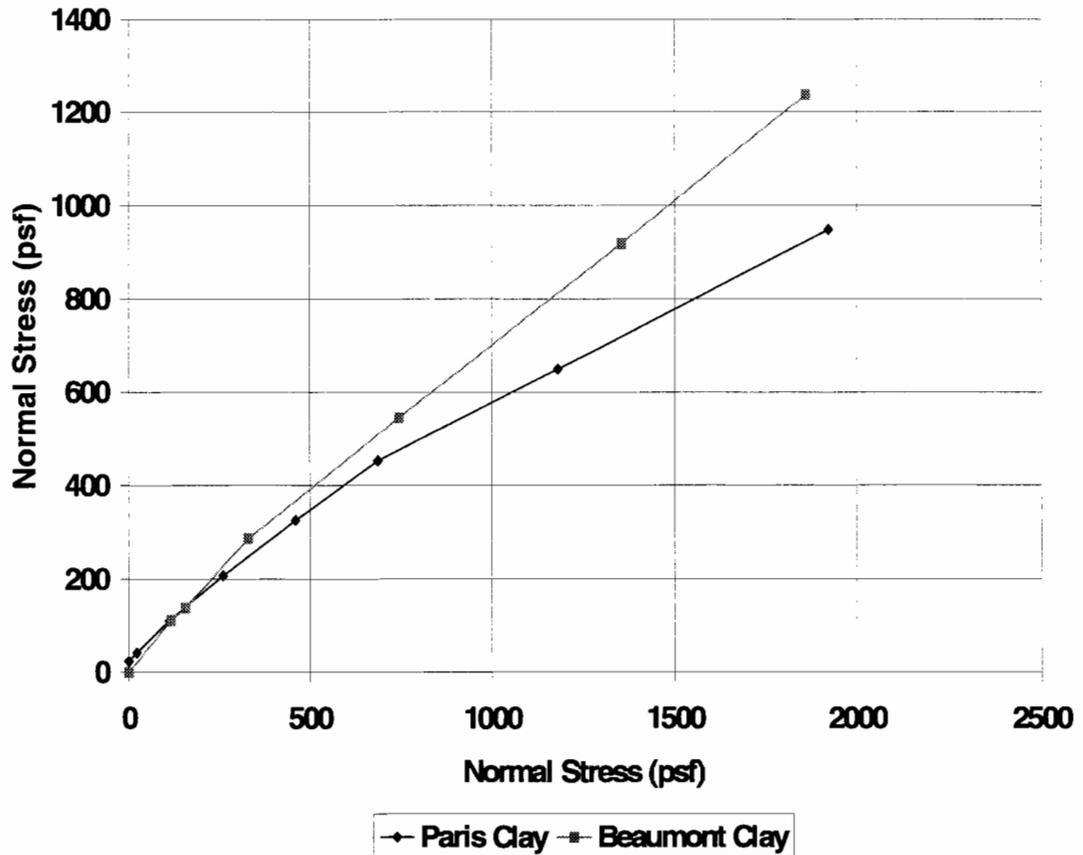


Figure 2.1 Effective stress Mohr-Coulomb shear strength envelopes for compacted Paris and Beaumont clays after wetting and drying (Kayyal 1990)

## 2.2 PARAMETRIC STUDIES

To define the strength of the slope prior to stabilization, the slope was assumed to have failed and the shear strength was then determined by back-analysis. Since the soils of interest appear to have minimal effective stress cohesion ( $\bar{c}$ ), the value of  $\bar{c}$  was assumed to

be zero. The following equation, based on an infinite slope, was then used to calculate the effective-stress friction angle ( $\bar{\phi}$ ).

$$F = [\cot \beta - r_u (\cot \beta + \tan \beta)] \tan \bar{\phi} \quad (2.1)$$

where  $F$  is the factor of safety,  $r_u$  is the pore water pressure coefficient, and  $\beta$  is the slope angle measured from the horizontal. The pore water coefficient is defined as

$$r_u = \frac{u}{\gamma z} \quad (2.2)$$

where  $u$  is the pore water pressure at a depth,  $z$ , in the slope, and  $\gamma$  is the total unit weight of the soil. Since the slope was assumed to have failed, the factor of safety was assumed to be unity. For each slope geometry (slope angle),  $\bar{\phi}$  was calculated for assumed values of  $r_u$ .

Once the shear strength parameters were found for the unstabilized slope, stabilization was modeled by *replacing* a section of the slope with a stronger material, with the geometry shown in Figure 2.2. The strength of the stabilized soil was assumed to be controlled primarily by the cementing agent, rather than by effective stress. Thus, the strength was represented by cohesion,  $c$ , and  $\phi$  was assumed to be zero. The strengthened zone extended the full height ( $H$ ) of the slope. The effects of varying the width ( $W$ ) of the strengthened zone as well as the effect of the depth ( $D$ ) of the strengthened zone below the toe were examined.

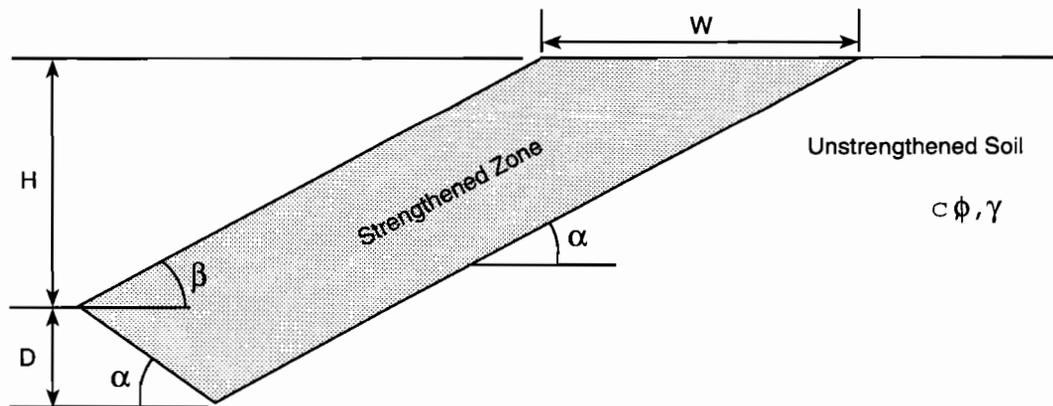


Figure 2.2 Slope parameters for strengthened earth slopes

The unit weights of the unstrengthened and strengthened materials were assumed to be the same. Two sets of calculations were performed based on two different assumptions for the foundation material. The first set of calculations was performed assuming that the foundation material had the same strength as the unstrengthened slope material. This case is referred to as a slope on the *same* foundation material. The second set of calculations was performed assuming that the foundation was much stronger than the slope material, such that failure surfaces would not pass into the foundation.

### 2.3 EFFECT OF PORE WATER PRESSURE COEFFICIENT ( $r_u$ )

The effect of pore water pressures on slope stability and the requirements for strengthening unstable slopes were investigated by comparing factors of safety for two different slope and strengthened soil geometries using several different pore water pressures. In each case, the strength of the unstrengthened slope ( $\bar{\phi}$ ) was calculated from Equation 2.1 assuming  $F = 1$  and the appropriate values of pore water pressure coefficient ( $r_u$ ) and slope angle ( $\beta$ ). For these studies, the foundation was assumed to have the same shear strength as the slope. Pore water pressures of 0.0, 0.25, and 0.5 were assumed for  $r_u$ . The following values were assumed:

- 1) Slope height ( $H$ ) = 9.14 m (30 feet)
- 2) Width of strengthened zone ( $W$ ) = 4.57 m (15 feet)
- 3) Unit weight of soils = 2,002 kg/m<sup>3</sup> (125 pcf)
- 4) Slope angles of 2.5:1 and 3:1

As shown in Table 2.2, the cases in which the pore pressure was lowest ( $r_u = 0$ ) produced the smallest increase in factor of safety when the strengthened zone was added. This is because the lowest pore water pressure coefficients produced the lowest (back-calculated) shear strengths. Based on these observations, all further calculations of strengthening requirements were based on zero pore water pressures ( $r_u = 0$ ) because this represents the most conservative assumption for pore water pressures. If zero pore water pressures are assumed, Equation 2.1 reduces to:

$$F = \frac{\tan \bar{\phi}}{\tan \beta} \quad (2.3)$$

and the friction angle  $\bar{\phi}$  is given by:

$$\bar{\phi} = \tan^{-1} \left( \frac{\tan \beta}{F} \right) \quad (2.4)$$

The value of  $\bar{\phi}$  derived from Equation 2.4 for a factor of safety of 1.0 is equal to the slope angle  $\beta$ . Equation 2.3 yields lower values for the friction angle,  $\bar{\phi}$ , than would be obtained if pore water pressures were greater than zero.

*Table 2.2 Effect of pore water pressure coefficient,  $r_u$ , on stability of strengthened slopes (factor of safety = 1.0 before stabilization,  $H = 30$  feet,  $W = 15$  feet,  $\gamma = 125$  pcf)*

Slope Angle	$r_u$	Friction Angle (degrees)	Strengthened Zone Shear Strength (psf)	Factor of Safety
3:1	.5	36.8	9375	1.405
3:1	.25	24.8	9375	1.372
3:1	0.0	18.4	9375	1.340
2.5:1	.5	43.5	9375	2.351
2.5:1	.25	29.4	9375	1.432
2.5:1	0.0	21.8	9375	1.390

## 2.4 EFFECT OF VARYING DEPTH, D, OF STRENGTHENED ZONE

A series of parametric studies was conducted to examine the effect that extending the depth (D) of the strengthened zone to below the toe of the slope would have on the factor of safety (Figure 2.2). For this series of calculations, the foundation soil and the unstrengthened soil in the slope were assumed to be the same. The slope was assumed to have a factor of safety of unity and pore water pressures were zero. Thus, the friction angle is equal to the slope angle ( $\bar{\phi} = \beta$ ). The shear strength of the strengthened zone was modeled by a cohesion value,  $c$  ( $\phi = 0$ ).

Extending the strengthened zone to a depth below the toe produced a significant increase in the factor of safety. Adding a given volume of strengthened soil with a portion extending below the toe of the slope (Figure 2.3b) results in a significantly greater increase in factor of safety compared with strengthening the same volume of soil without extending below the slope (Figure 2.3a). This is illustrated by the example of a 25-foot high, 3:1 slope shown in Table 2.3. In this example, two sets of calculations were performed for a given volume of strengthened soil. In the first case, the strengthened zone has a width of 15 feet and shear strength of 3125 psf, but does not extend below the toe of the slope. In this case, the factor of safety is increased to 1.18 from the prestrengthened value of 1.0. In the second case shown in Table 2.3, the volume of the strengthened zone is the same, but the strengthened zone is extended 5 feet into the foundation. In this case, the factor of safety is increased to 1.28 (from 1.0), representing an over 50 percent greater increase in stability ( $F = 1.18$  vs. 1.28).

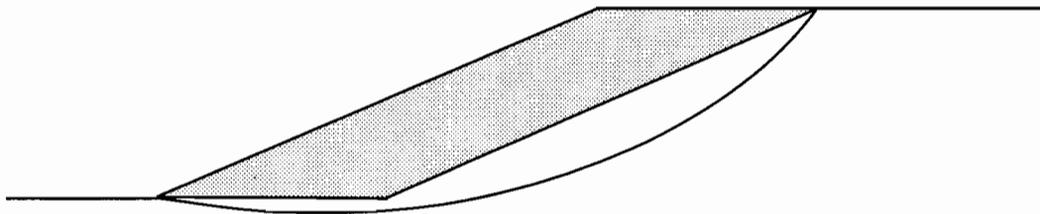
*Table 2.3 Effect of depth of strengthened zone on factor of safety for identical volumes of strengthened soil (Strengthened volume = 375 cu. ft/ft; 3:1 Slope,  $\phi = 18.4^\circ$ ,  $H = 25$  feet,  $c = 3125$  psf)*

Width of Strengthened Zone (feet)	Depth of Strengthened Zone (feet)	Factor of Safety*	Percent Increase in Factor of Safety*
12.5	5	1.28	28
15.0	0	1.18	18

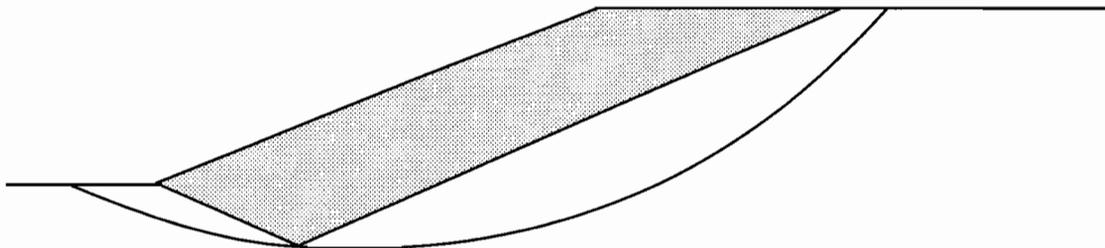
\* Based on factor of safety of 1.0 for unstrengthened slope

When the strengthened soil is much stronger than the unstrengthened soil the failure surfaces are forced to pass outside the strengthened zone. Adding additional material below the toe of the slope significantly increases the stability as the additional strengthened material forces the critical circle to pass deeper into the foundation material. The differences between the critical circles for strengthened zones that extend to depths ( $D$ ) below the toe and for strengthened zones that do not extend below the toe can be seen in Figure 2.3.

(a) Depth below Toe,  $D = 0$



(b) Depth below Toe,  $D > 0$



*Figure 2.3 Effect of depth,  $D$ , of strengthened zone below toe on failure surface (on same foundation material)*

Although strengthening soil below the toe of the slope appears to have a relatively large effect on improving the factor of safety when compared with strengthening the soil only above the toe of the slope, uncertainties exist regarding the stresses in the area of the slope below the toe. For example, there is the possibility of significant flexural stresses developing in the strengthened soil near the toe of the slope, which can lead to failure of the soil in flexure. A much more detailed study, perhaps one employing finite element analyses, would be required to examine the stresses in strengthened soil beneath the toe of the slope to determine what strengths can be relied on. No further consideration was given in this study to strengthening soil below the toe of the slope; only strengthening of soil above the toe, in the slope itself, was considered for the balance of this work.

## 2.5 SLOPES ON SAME FOUNDATION MATERIALS

Several series of slope stability calculations were performed to develop charts that accounted for the effects of various geometries and strengths of strengthened soil in improving slope stability. The first series of calculations and charts were for foundations having the same strengths as the overlying slopes. For each slope geometry, the strength and width of the strengthened zone were varied and the factor of safety was calculated. Slopes inclined at 1.5:1, 2:1, 2.5:1, 3:1 and 3.5:1 (horizontal:vertical) were studied to cover the typical range of slopes of interest.

If the shear strength of the strengthened soil is significantly greater than the unstrengthened soil, the failure surface passes outside the strengthened zone. However, if the soil is only moderately strengthened, the failure surface can fall largely within the strengthened zone and the stability will decrease as the zone is widened. This is illustrated by the following example: A 3:1 slope with no pore water pressures was assumed to rest on a foundation having the same shear strength as the overlying slope ( $\bar{\phi} = \beta$ ,  $\bar{c} = 0$ ). Two separate values of shear strength ( $c = 187.5$  psf and  $c = 375$  psf) were assumed for the strengthened zone. The width of the strengthened zone was varied from 0 to 30 feet and the factor of safety was calculated. The results are plotted in Figure 2.4. As shown in this figure, the factor of safety for both cases was assumed to be 1.0 for no strengthened zone ( $W = 0$ ). As the width of the strengthened zone was increased, the factor of safety first increased, and then decreased. The critical circular failure surfaces for both these cases ( $c = 187.5$  psf and  $c = 375$  psf) and a strengthened zone 15 feet wide are plotted in Figure 2.5. For the higher strength ( $c = 375$  psf) the critical circle passes almost completely outside the strengthened zone, but for the lower strength (187.5 psf) a noticeable portion of the critical circle passes through the strengthened zone.

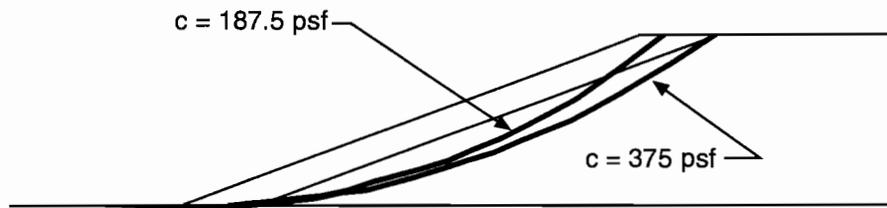


Figure 2.4 Variation in factor of safety with width of strengthened zone with for two different strengths of strengthened zone (3:1 slope,  $H = 30$  feet,  $\gamma = 125$  pcf,  $\beta = \phi$ )

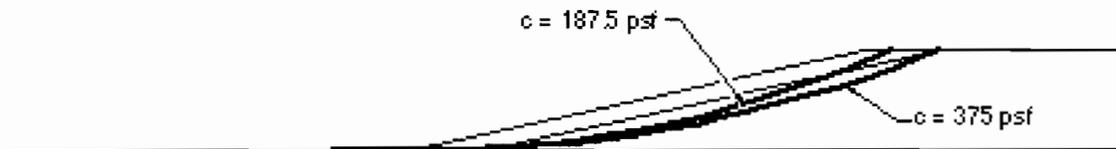


Figure 2.5 Critical circular failure surfaces for different shear strengths for strengthened zone (3:1 slope,  $W = 15$  feet,  $H = 30$  feet,  $\gamma = 120$  pcf)

As the shear strength of the strengthened zone is increased, for a given width of reinforcement, the critical circle will cut through less and less of the strengthened material, and the factor of safety will increase. This increase in factor of safety will continue until the shear strength of the strengthened zone is greater than the strength of the unstrengthened zone. Once the shear strength of the strengthened zone reaches this point, further strength increases will not raise the factor of safety beyond the *terminal* value because the critical shear surfaces all pass completely outside the strengthened zone. Table 2.4 shows that the factor of safety reaches a maximum value for a 3:1 slope, 30 feet high, with a unit weight of soil of 125 pcf and a width of strengthened zone of 7.5 feet. Various strengths (75 psf, 190 psf, 1500 psf, 2000, psf, and 7500 psf). Strengths exceeding about 1500 psf show that the factor of safety reaches a terminal value of approximately 1.09. For design charts, strengths less than the strengths required to force the failure surface outside the strengthened zone and develop the terminal factor of safety were not considered. It is assumed that the strength of the strengthened zone is sufficient to develop the terminal value of the factor of safety. The strengths required to achieve this condition are discussed later in Section 2.5.2.

*Table 2.4 "Terminal" factor of safety for strengthened slopes on foundation with same shear strength as unstrengthened slope (3:1 Slope, H = 30 ft,  $\gamma = 125$  pcf, W = 7.5 ft,  $\beta = \phi$ )*

Shear Strength of Strengthened Zone (psf)	Factor of Safety
75	1.06
188	1.08
1500	1.09
2000	1.09
7500	1.09

### **2.5.1 Minimum Widths and Dimensionless Charts**

The variables of significance in developing design charts are the slope angle ( $\beta$ ), the width of the strengthened zone (W), the height of the slope (H), the unit weight of the soils ( $\gamma$ ), and the shear strength of the strengthened zone (c). As previously mentioned, the shear strength of the strengthened zone was assumed to be sufficient to develop the terminal factor of safety. Analysis showed that for a given slope angle and strength of unstrengthened soil ( $\phi = \beta$ ), the terminal factor of safety was uniquely related to the dimensionless width ratio, W/H. For given values of the ratio W/H, and a given slope angle ( $\beta$ ), the terminal factor of safety is unique. For illustration purposes, two slope inclinations (2:1 and 3:1) were selected and the values for the variables W, H, and  $\gamma$  were varied, while keeping the ratio W/H constant. The strength of the strengthened zone was high enough to force failure surfaces outside the strengthened zone, and the factors of safety were calculated. The factors of safety are nearly the same for all the cases with a given slope angle (Table 2.5).

*Table 2.5 Dimensionless parameter W/H for slope strengthening: Slope on same strength foundation with shear strength of strengthened soil large enough to force failure surface outside strengthened zone (W/H = 0.4,  $\phi = \beta$ )*

Case	W (feet)	H (feet)	$\gamma$ (pcf)	F.S.
2:1 Slope	4	10	125	1.18
2:1 Slope	4	10	120	1.18
2:1 Slope	20	50	125	1.17
3:1 Slope	4	10	125	1.11
3:1 Slope	4	10	120	1.11
3:1 Slope	20	50	125	1.11

The range of width ratios,  $W/H$ , chosen for developing charts was 0 to 1. The upper limit ( $W/H = 1$ ) was chosen because excavating widths greater than the slope height seemed unrealistic for most highway slopes. Figure 2.6 shows the increase in factor of safety with increasing width to height ratio of the strengthened zone for slopes on a foundation with the same strength properties as the overlying slope. In this figure, the strength of the strengthened zone equals or exceeds the threshold value.

*Table 2.6 Threshold shear strengths ( $c$ ) for slope strengthening expressed as dimensionless parameter  $c/\gamma H$ : ( $W/H = 0.4$ ,  $\phi = \beta$ )*

Case	W (feet)	H (feet)	$\gamma$ (pcf)	$c$ (psf)	$c/\gamma H$
2:1 Slope	4	10	125	85	0.07
2:1 Slope	4	10	120	82	0.07
2:1 Slope	20	50	125	420	0.07
3:1 Slope	4	10	125	50	0.04
3:1 Slope	4	10	120	48	0.04
3:1 Slope	20	50	125	250	0.04

### **2.5.2 Threshold Strengths and Dimensionless Charts**

The minimum strength of the strengthened zone that is required to force the failure surface outside the strengthened zone and develop the terminal factor of safety can be presented as a dimensionless ratio. Analysis showed that for a given slope angle, width ratio ( $W/H$ ) and strength of unstrengthened soil ( $\phi=\beta$ ), the minimum (threshold) strength required to obtain the terminal factor of safety could be described by the dimensionless strength ratio  $c/\gamma H$ . To illustrate this, the threshold strength for the examples shown in Table 2.5 was determined through trial and error. The threshold strengths are shown in Table 2.6. As shown in this table, the value for the threshold strength ratio ( $c/\gamma H$ ) is constant for a given slope angle and width ratio ( $W/H$ ). Figure 2.7 shows the required threshold strength ratio as a function of the width ratio ( $W/H$ ) for various slope angles.

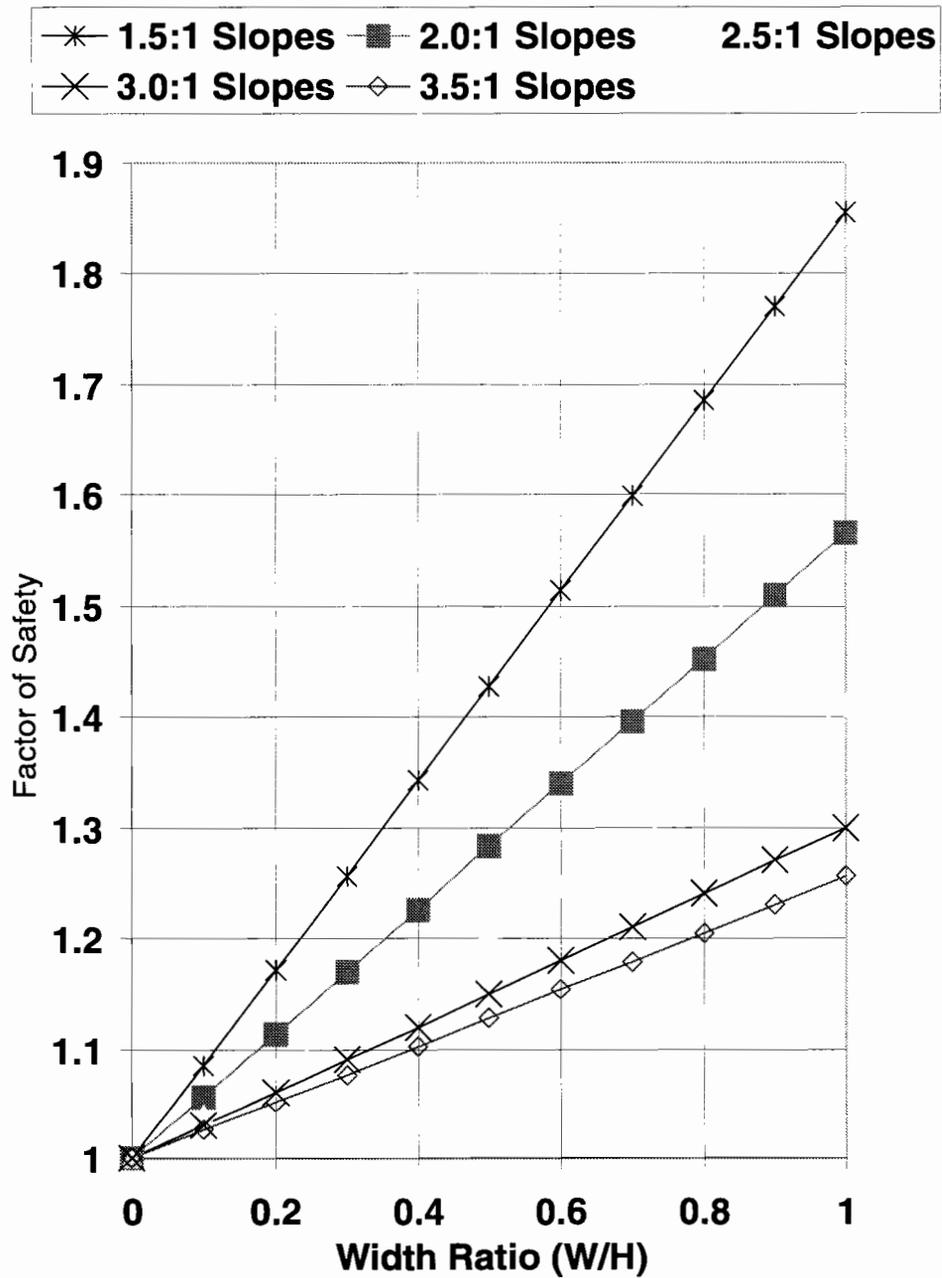


Figure 2.6 Variation in factor of safety with dimensionless width ratio for slopes of 1.5, 2, 2.5, 3, and 3.5:1 (same foundation; strength ratio  $(c/\gamma H) >$  "threshold" value)

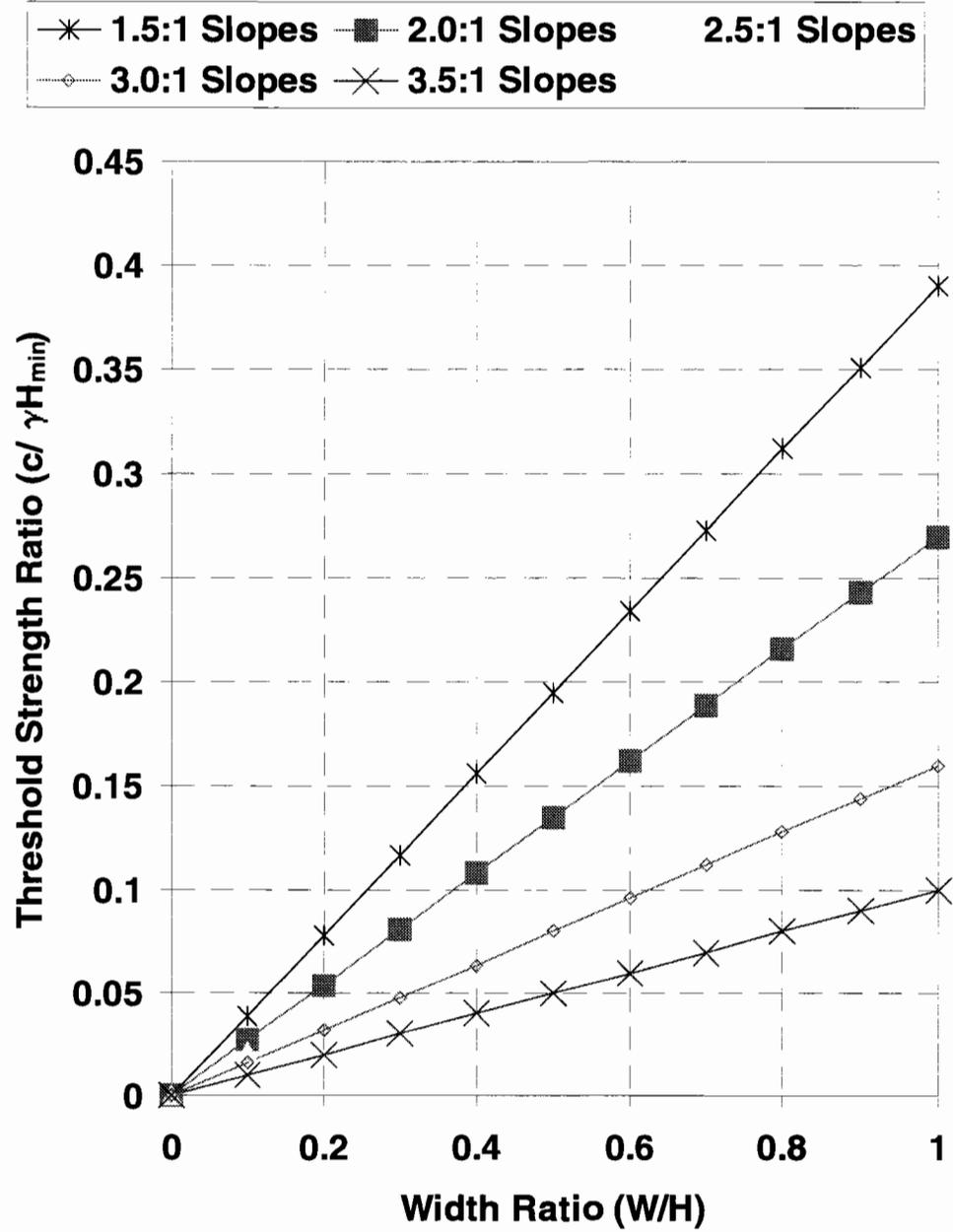


Figure 2.7 Threshold values of dimensionless strength required for obtaining maximum factor of safety for given width ratio (slopes on same foundation)

To determine likely maximum values for the strength ratio,  $c/\gamma H$ , the strength values reported by Kennedy and Smith (1986) were examined. They report a maximum unconfined compressive strength ( $q_u$ ) of 76 psi (11,000 psf) as shown previously in Table 2.1. The corresponding shear strength, assuming  $\phi = 0$ , is one-half the unconfined compressive strength or 38 psi (5,500 psf). A typical slope of interest might have a height of 20 feet, and the unit weight might be approximately 120 pcf. Based on these values ( $c = 5,500$  psf,  $H = 20$  feet, and  $\gamma = 120$  pcf), the value of  $c/\gamma H$  is 2.3. This value easily exceeds the threshold values shown on Figure 2.7.

### 2.5.3 Example Problem

The following example illustrates the use of the charts for the design of remediation of a slope resting on the same foundation soil as the slope. The slope is inclined at 1.5:1 (Figure 2.8), is 30 feet high and has a unit weight of soil of 120 pcf. Suppose the desired factor of safety for the repaired slope is 1.5. Using a factor of safety of 1.5, the necessary width ratio,  $W/H$ , is read from Figure 2.6. The value of  $W/H$  determined from this figure is approximately 0.58. The minimum required width ( $W$ ) is obtained by multiplying the slope height ( $H$ ) by the width ratio

$$W = H \left( \frac{W}{H} \right) = (30 \text{ feet})(0.58) \approx 18 \text{ feet} \quad (2.5)$$

Thus, the width of the strengthened zone must be at least approximately 18 feet. Next, the required minimum strength ratio ( $c/\gamma H$ ) is obtained from Figure 2.7 for a width ratio ( $W/H$ ) of 0.58 and 1.5:1 slope. The value of the strength ratio determined from Figure 2.7 is approximately 0.215. The required shear strength,  $c$ , of the strengthened zone is obtained by multiplying the strength ratio by the slope height ( $H$ ) and unit weight of the soil ( $\gamma$ ):

$$c = \frac{c}{\gamma H} (\gamma)(H) = 0.215(120 \text{ pcf})(30 \text{ feet}) = 775 \text{ psf} \quad (2.6)$$

Thus, the minimum required strength for the strengthened zone is approximately 775 psf. Whatever method of additive is chosen should provide at least this minimum value of long-term strength.

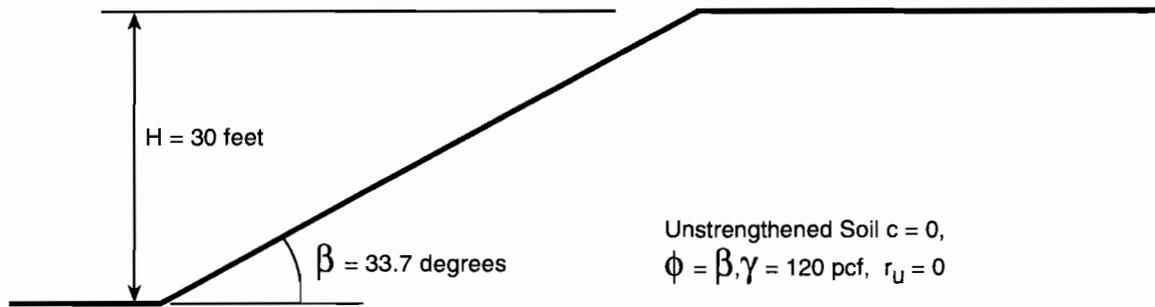


Figure 2.8 Slope parameters for example problem—Slope and foundation soils same

## 2.6 SLOPES ON MUCH STRONGER FOUNDATIONS

The second series of charts was developed for slopes on much stronger foundations where the failure surface cannot pass into the foundation. For this case, the failure surfaces were restricted from passing into the foundation as shown in Figure 2.9. Computations were performed for the same range in slope geometry as the previous charts. Pore water pressures were assumed to be zero and the factor of safety of the unstrengthened slope was assumed to be unity, i.e.  $\bar{\phi} = \beta$  ( $\bar{c} = 0$ ). The shear strength and width of the strengthened zone were again varied, and the factor of safety was calculated.

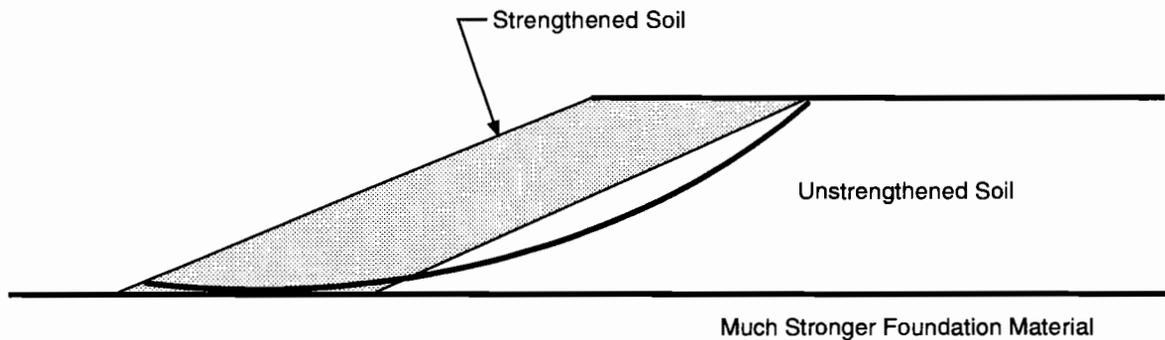


Figure 2.9 Critical circle when slope rests on much stronger foundation

In all cases where the slope rests on a much stronger foundation, the factor of safety continues to increase, as the shear strength increases, for a given width of strengthened zone.

Figure 2.10 shows the continuous increase in factor of safety for a 3:1, 30-foot high slope with a width of strengthened soil of 3 feet. No *terminal* factor of safety is reached for this, or any other width of strengthened zone. However, as shown for the case of slopes on foundations with the same strength properties, for some strengths of the strengthened zone, factors of safety could decrease and even be less than 1.0 as the width of the strengthened zone is increased. Cases where the factor of safety began to decrease were excluded from the charts.

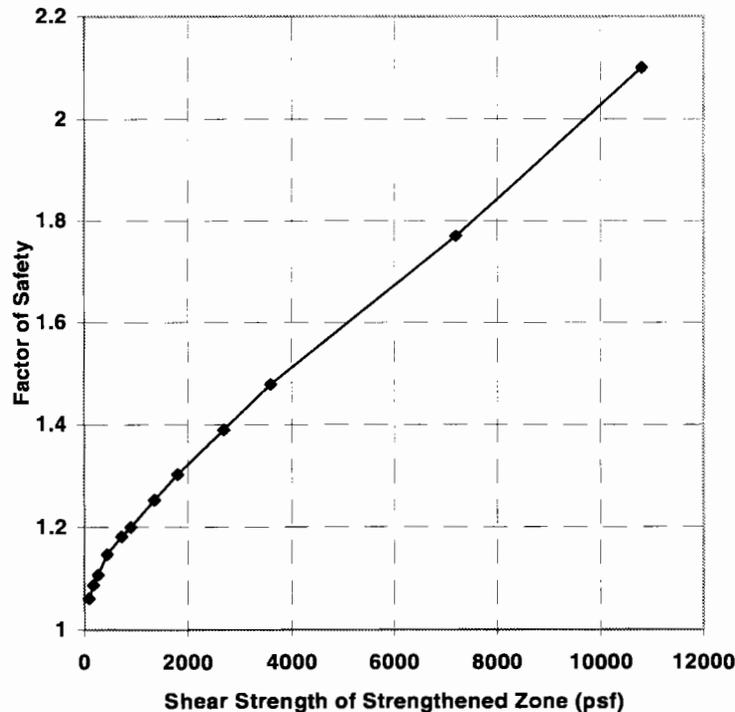


Figure 2.10 Variation in factor of safety with shear strength of strengthened zone for slope on strong foundation material (3:1 slope,  $H = 30$  feet,  $W = 3$  feet,  $\gamma = 120$  pcf,  $\phi = \beta$ )

### 2.6.1 Dimensionless Charts

A large number of stability calculations was again performed and used to develop plots of factor of safety vs. width ratio ( $W/H$ ) for various values of the strength ratio  $c/\gamma H$ . Separate charts were plotted for each slope angle. In addition, for each slope angle charts were plotted to two different scales corresponding to height ratios from 0.0 to 0.25 and from

0.25 to 1.0. These charts are shown in Figures 2.11 to 2.20. However, caution should be used in relying on the strength of relatively thin zones, and values of width ratio less than 0.25 should probably not be used.

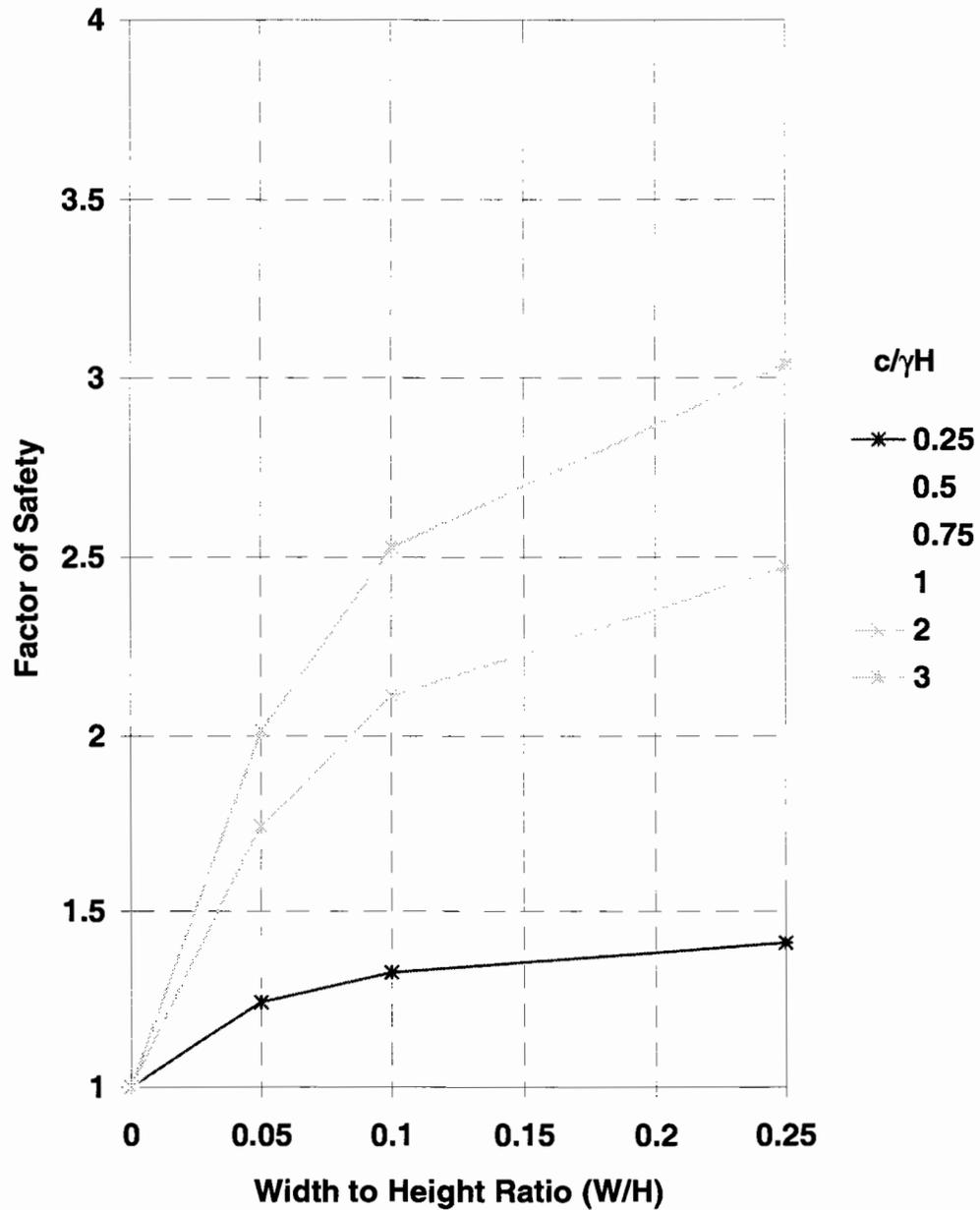


Figure 2.11 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 1.5:1 slope;  $W/H = 0 - 0.25$

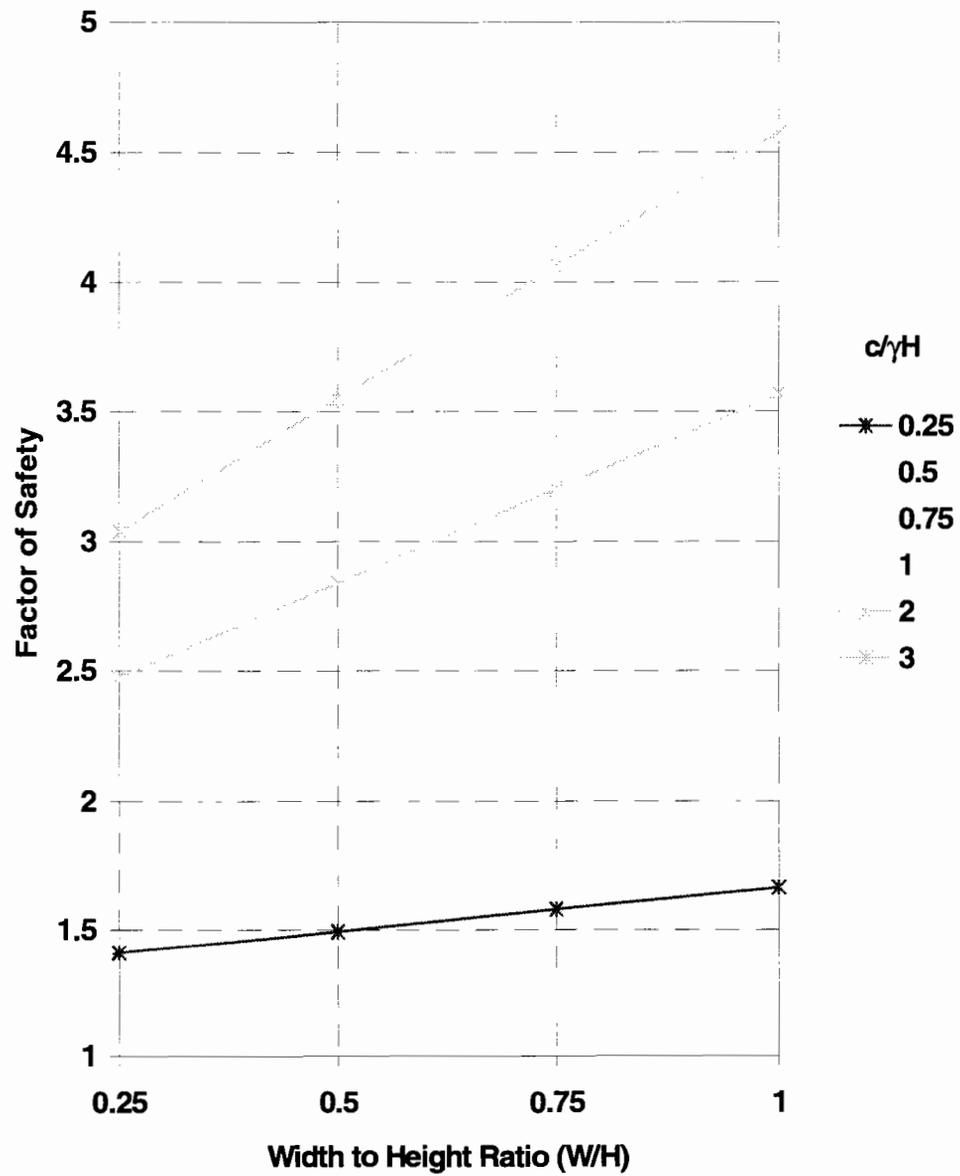


Figure 2.12 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 1.5:1 slope;  $W/H = 0.25 - 1.0$

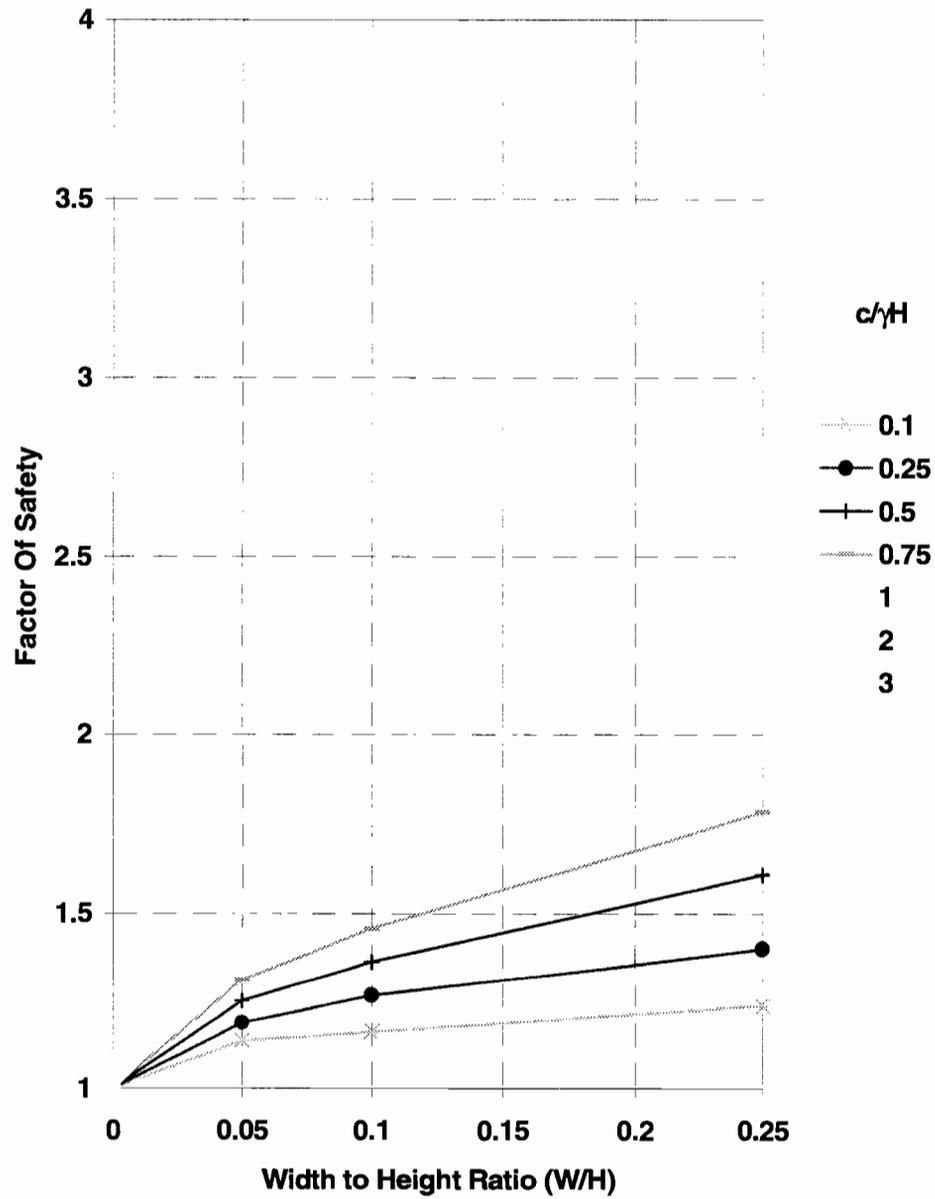


Figure 2.13 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 2:1 slope;  $W/H = 0 - 0.25$

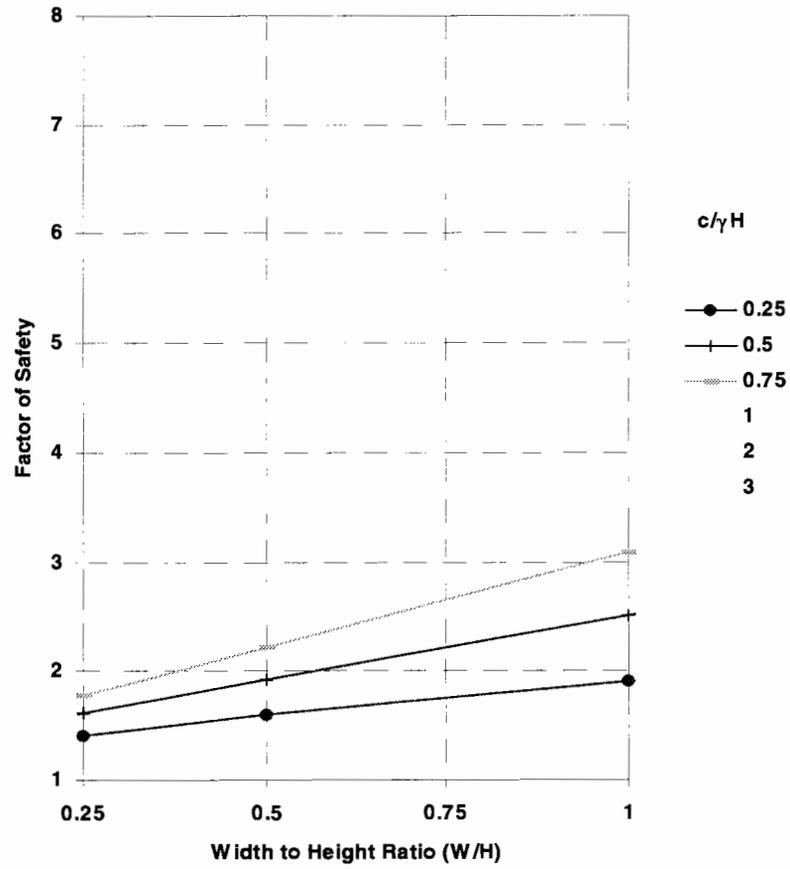


Figure 2.14 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 2:1 slope;  $W/H = 0.25 - 1.0$

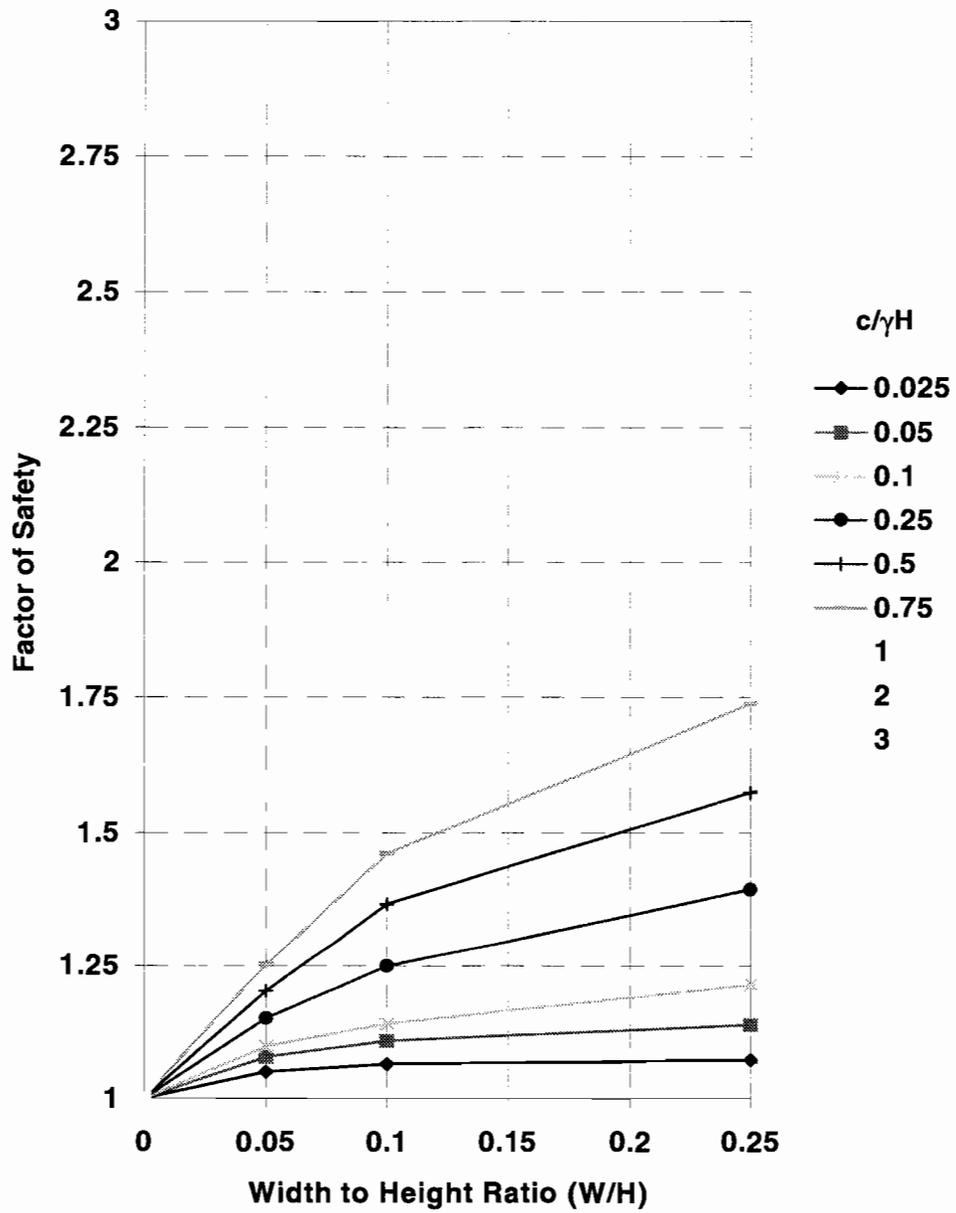


Figure 2.15 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 2.5:1 slope;  $W/H = 0 - 0.25$

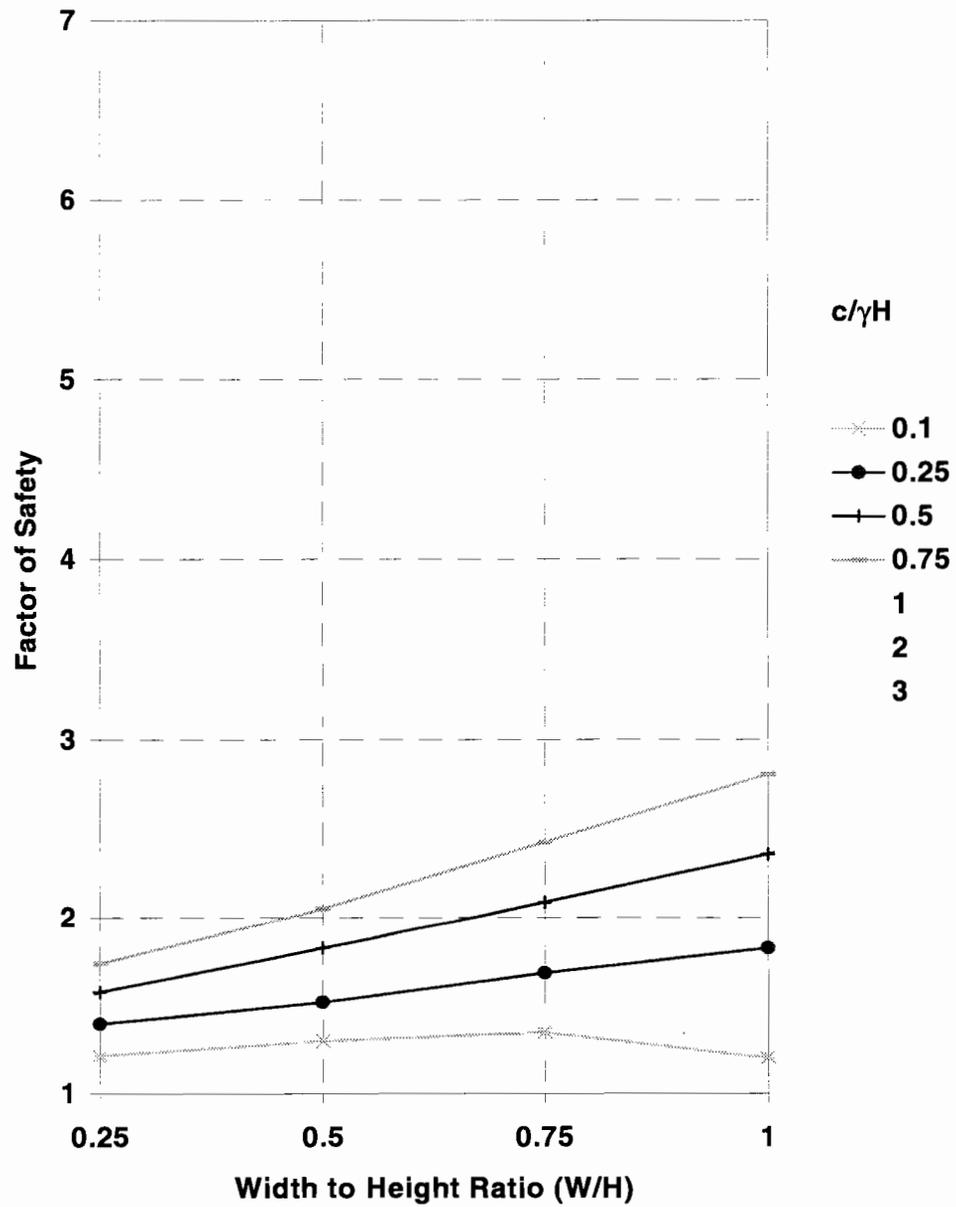


Figure 2.16 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 2.5:1 slope;  $W/H = 0.25 - 1.0$

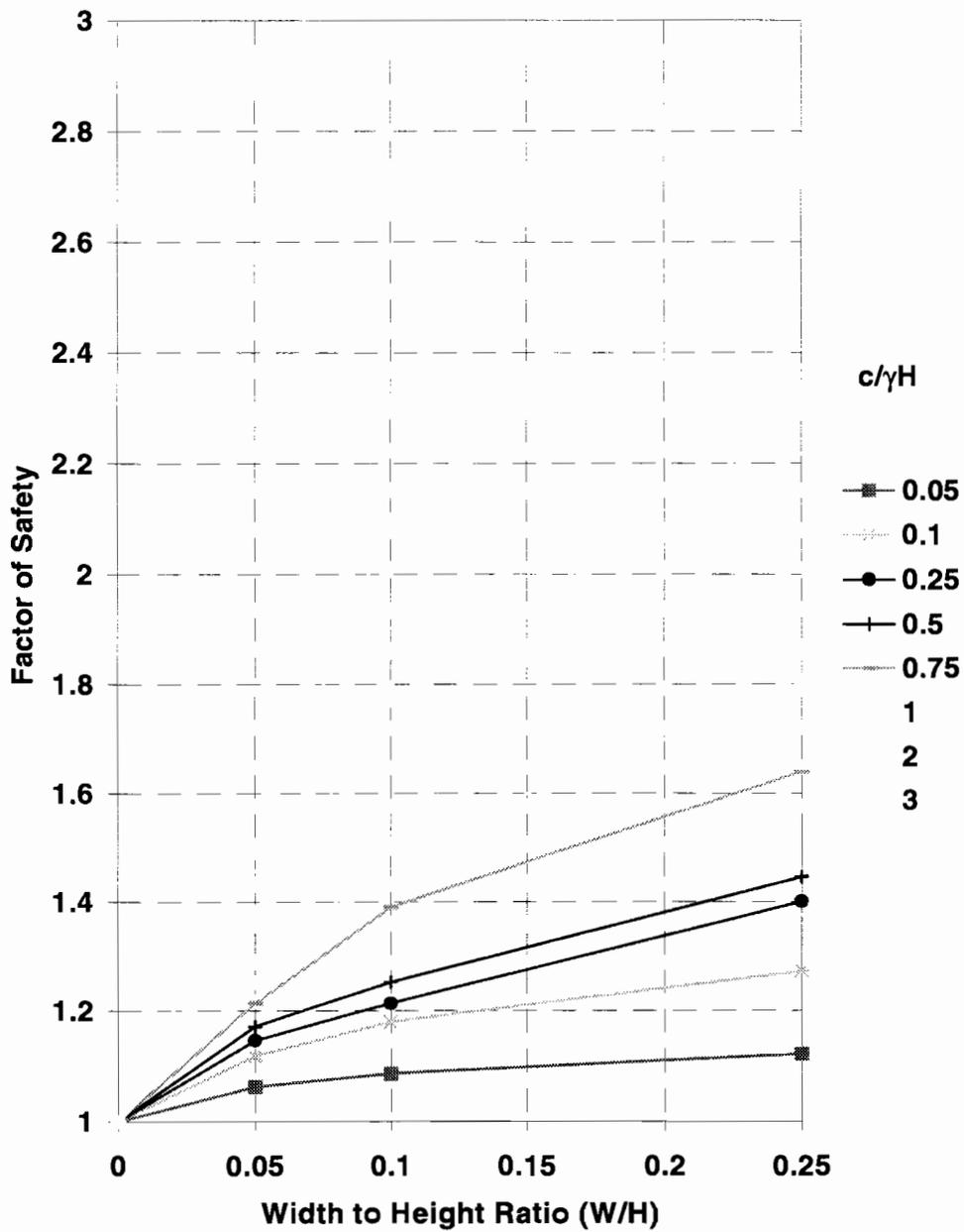


Figure 2.17 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 3:1 slope;  $W/H = 0 - 0.25$

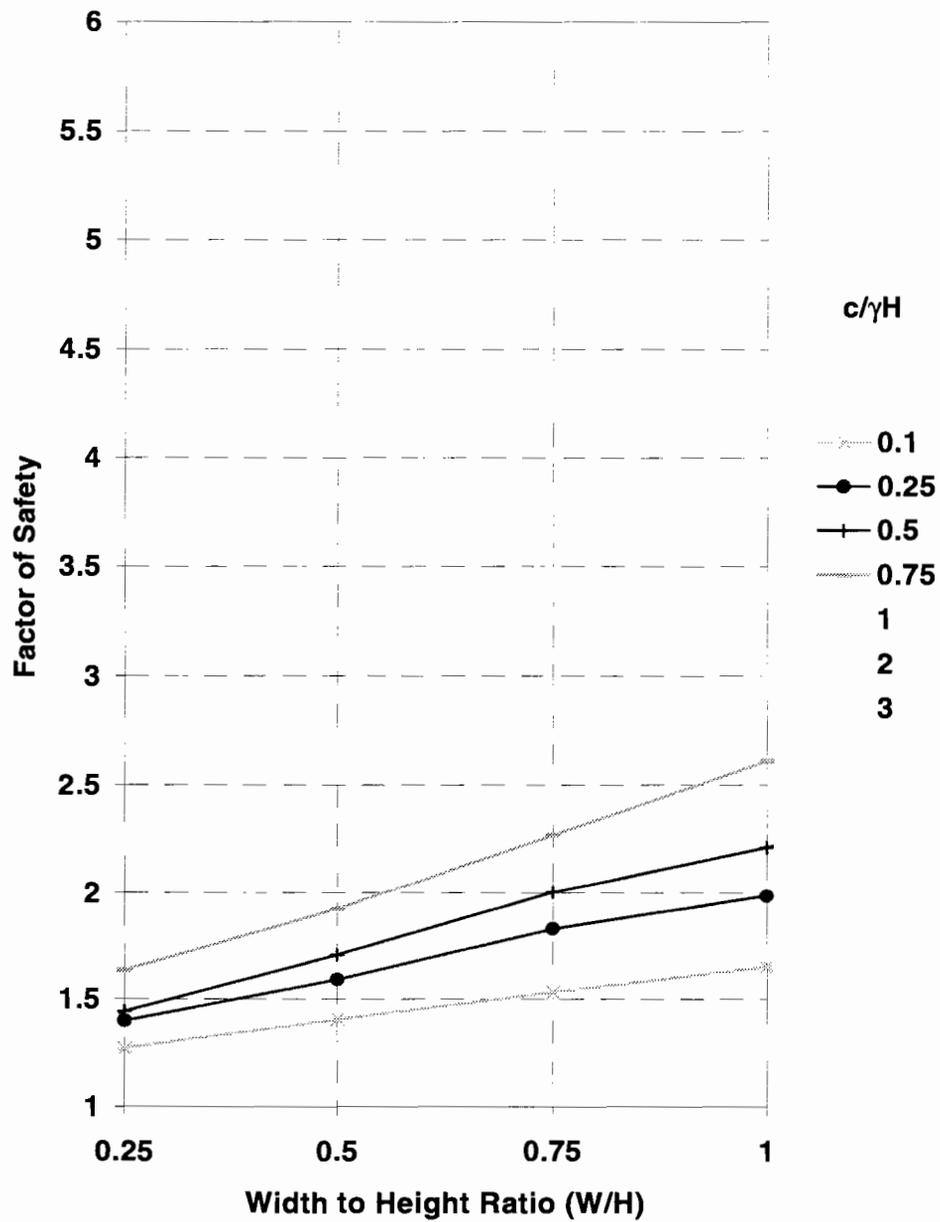


Figure 2.18 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 3:1 slope;  $W/H = 0.25 - 1.0$

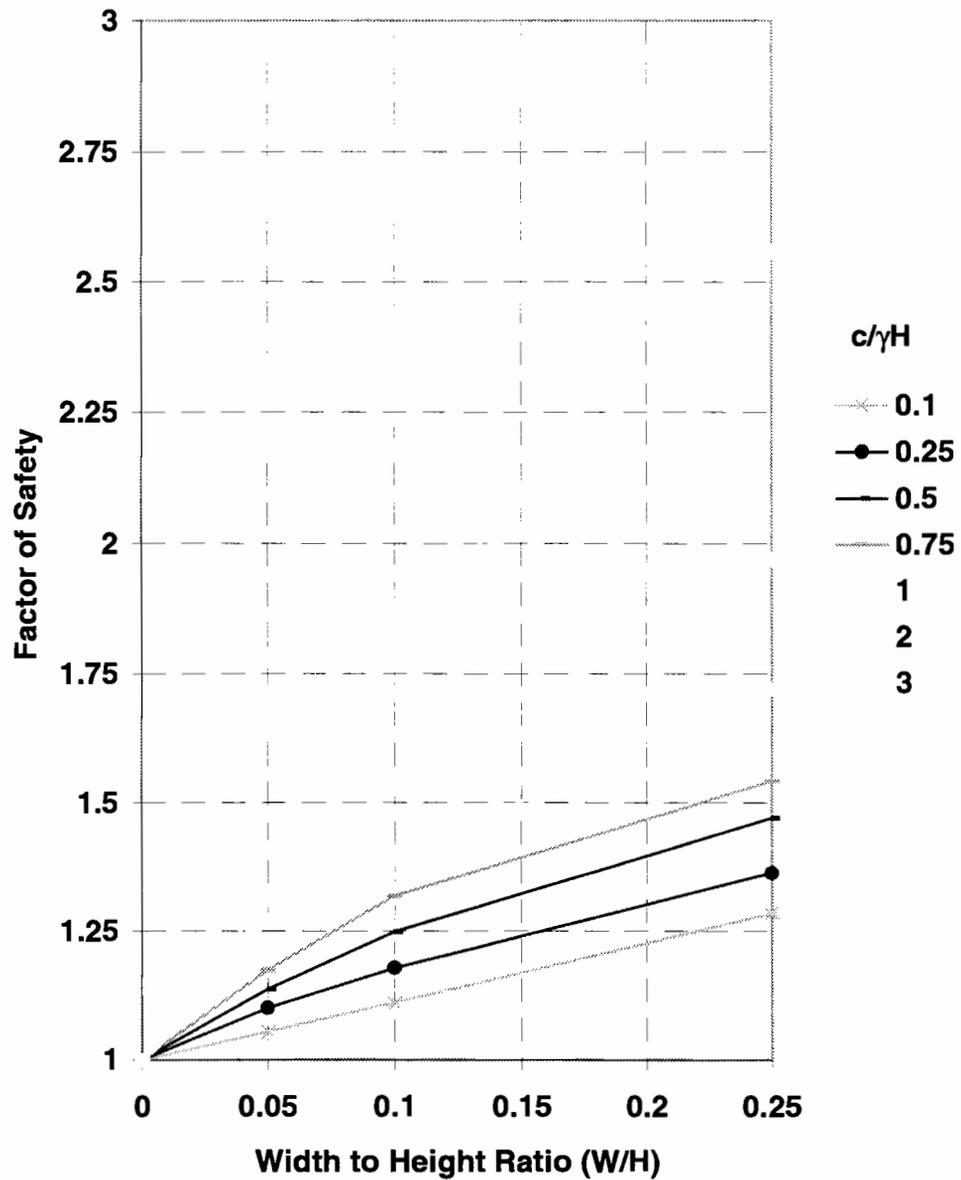


Figure 2.19 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 3.5:1 slope;  $W/H = 0 - 0.25$

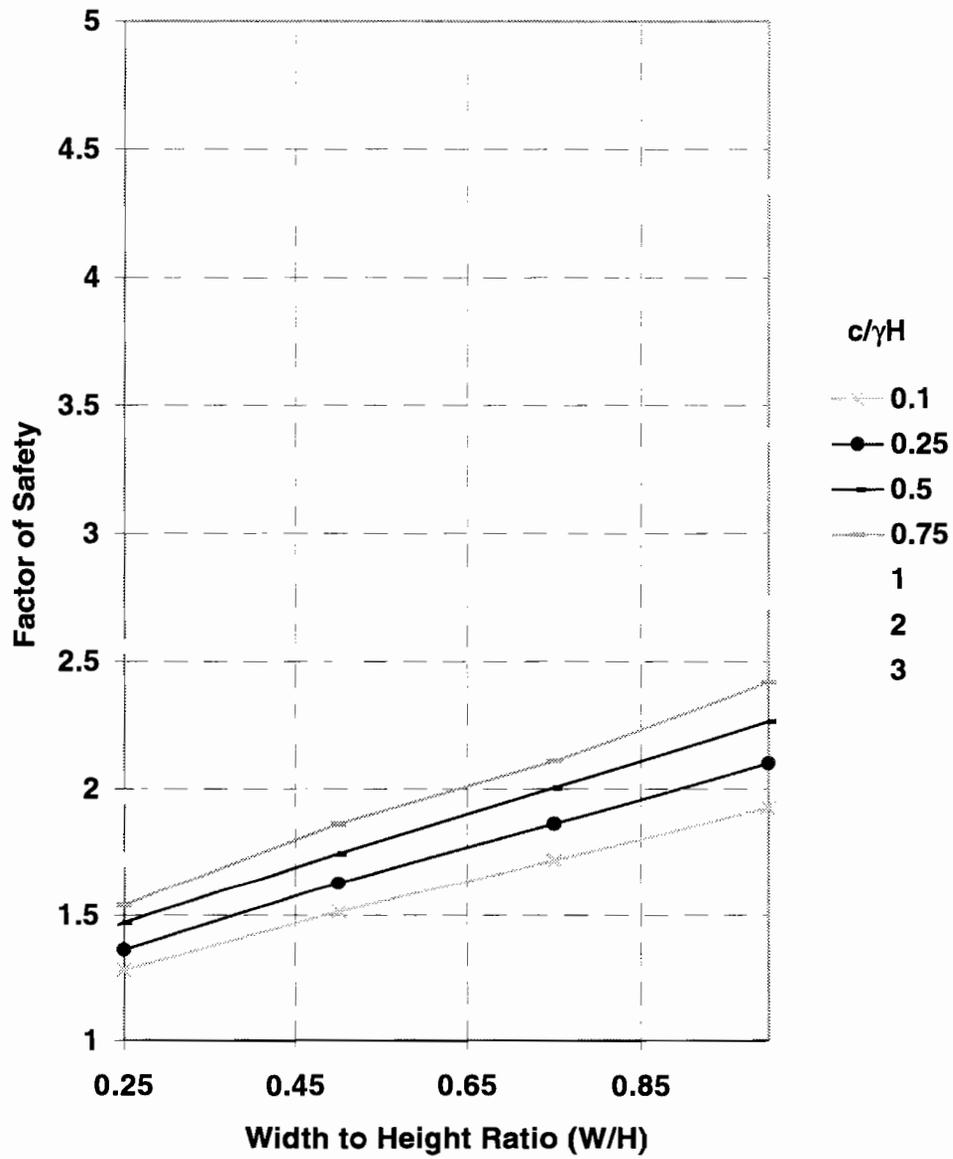


Figure 2.20 Variation in factor of safety with dimensionless width ratio for various dimensionless strength ratios for slopes on strong foundations – 3.5:1 slope;  $W/H = 0.25 - 1.0$

### 2.6.2 Example Problems

The following example illustrates the use of the charts for a slope on a stronger foundation. Again, a slope inclined at 1.5:1 (Figure 2.21), 30 feet high, and with a unit weight of soil of 120 pcf is assumed. A factor of safety of 1.5 is also assumed for the repaired slope. Two sets of calculations are performed for this case. In the first set of calculations, the width is assumed, and the strength of the strengthened soil is found. In the second, the shear strength of the strengthened soil is assumed, and the width is found.

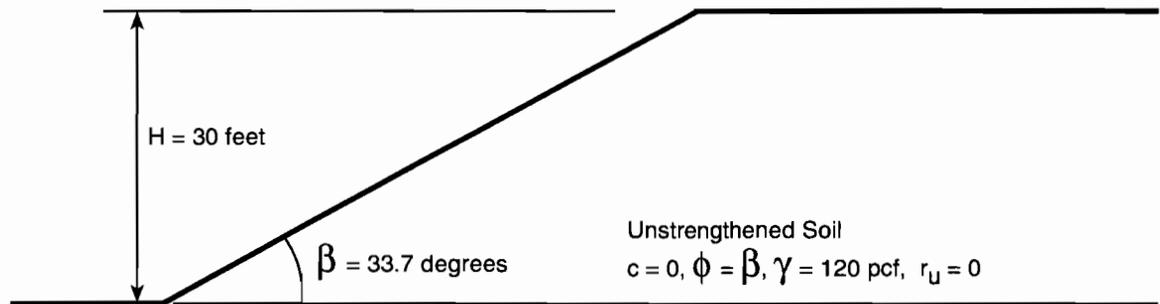


Figure 2.21 Slope parameters for example problem—Slope on much stronger foundation

For the first set of calculations, a width of 15 feet is chosen for the strengthened zone. This width corresponds to a width ratio ( $W/H$ ) of 0.5:

$$\frac{W}{H} = 15 \text{ feet} / 30 \text{ feet} = 0.5 \quad (2.7)$$

From Figure 2.12 the required minimum strength ratio ( $c/\gamma H$ ) is determined using the intersection of the design factor of safety (1.5) and the width ratio (0.5). The minimum strength ratio is found to be 0.25. The required shear strength,  $c$ , of the strengthened zone is obtained by multiplying the strength ratio by the slope height ( $H$ ) and the unit weight of the soil ( $\gamma$ ):

$$c = \frac{c}{\gamma H} (\gamma)(H) = 0.25(120 \text{ pcf})(30 \text{ feet}) = 900 \text{ psf} \quad (2.8)$$

Thus, the strengthened zone must have a shear strength of 900 psf. Whatever additive and method of strengthening are chosen should provide at least this strength (900 psf).

The second set of calculations also assumes a factor of safety of 1.5, but assumes the strength of the strengthened soil is 2,500 psf. Thus, the strength ratio ( $c/\gamma H$ ) is calculated as follows:

$$\frac{c}{\gamma H} = 2500 \text{ psf} / (120 \text{ pcf} \times 30 \text{ feet}) = 0.7 \quad (2.9)$$

The minimum required width ratio ( $W/H$ ) is estimated from Figure 2.11 for a strength ratio of 0.7 and factor of safety of 1.5. Linear interpolation between the lines for strength ratios of 0.75 and 0.5 is required to estimate the required width for a strength ratio of 0.7. The value of the width ratio from Figure 2.11 is approximately 0.14. Thus, the minimum width of the strengthened zone is determined by multiplying the height times the width ratio as,

$$W = H \left( \frac{W}{H} \right) = 30 \text{ feet} (0.14) = 4.2 \text{ feet} \quad (2.10)$$

The minimum width of the strengthened zone should be at least 4.2 feet<sup>1</sup>. However, since this width is so small relative to the slope height, a minimum width of 7.5 feet ( $W/H = 0.25$ ) is recommended.

## 2.7 CONCLUSIONS

Charts have been developed for determining the width and strength requirements for strengthened zones near the face of the slope. Production of the charts was greatly simplified by the dimensionless quantities,  $c/\gamma H$  and  $W/H$ . The effect of strengthening a portion of soil near the face of a slope is less if the foundation is the same rather than much stronger. When the foundation is the same strength as the slope, failure surfaces tend to pass into the foundation, avoiding the strengthened zone and resulting in lower factors of safety than if the foundation was much stronger. When the foundation is much stronger than the slope, failure surfaces must pass through the strengthened zone and much higher factors of safety are possible than if the foundation had the same strength as the overlying slope. However, caution must be used in relying on the strength of relatively thin zones. Strengthened zones with widths resulting in width ratios ( $W/H$ ) of less than 0.25 are not recommended.

Theoretically, for slopes on a foundation having the same strength as the slope, extending the strengthened zone below the toe, into the foundation, is more effective in increasing the stability than increasing the width of the strengthened zone. Extending the strengthened area deeper forces failure surfaces deeper into the foundation, through stronger material. However, questions then arise regarding how much strength can be mobilized in

---

<sup>1</sup> Precautionary note: caution should be taken when using width ratios < 0.25, and assuming a strong foundation.

the strengthened soil beneath the toe of the slope. Even if such questions were easily answered, strengthened zones below the toe of the slope would require charts more complex than the ones presented. This topic might be addressed in further research.

## **CHAPTER 3. REVIEW OF DESIGN METHODS FOR GEOSYNTHETIC SLOPE REINFORCEMENT**

### **3.1 INTRODUCTION**

Slopes are commonly repaired with geosynthetics by removing part of the soil in the failed slope, and then recompacting it with horizontal layers of geosynthetic-reinforcement. Tensile forces are developed in the reinforcement layers as a result of construction and postconstruction-induced extension of the reinforcement. These tensile forces produce corresponding increases in the compressive stresses in the soil. The increase in compressive stresses in the soil increases the soil's strength. The increase of the soil strength, along with the direct resistance provided by the tension in the reinforcement, increases the stability of the slope. Thus, it is possible to construct slopes with geosynthetic reinforcement that are steeper and higher than the soil would otherwise allow.

The number of layers and length of reinforcement required to produce a stable slope depend on the height of the slope, the weight of the soil, the shear strength of the soil, the required factors of safety, and the amount of strain (elongation) allowed. Several researchers, including Jewell et al. (1990), Leshchinsky and Reinschmidt (1985), and Schmertmann et al. (1987), have developed charts for designing geosynthetic-reinforced soil slopes. Recently, Leshchinsky developed a computer program, ReSlope (1995), to facilitate reinforced slope design. Each of the methods assumes that the factor of safety of the unreinforced slope is less than 1.0. The first step in the methods is to determine the total force required for equilibrium with an acceptable factor of safety on shear strength. This force can be expressed in the form of a force coefficient,  $K$ , which is multiplied by the square of the height of the slope and the unit weight of the soil to determine the total force. Once the force is determined, the minimum number of layers of reinforcement is determined based on the total force required and the type of reinforcement selected. Finally, the required length of the reinforcement is determined.

The various methods for designing reinforced slopes are reviewed in this chapter. The applicability of each method to the repair of slopes similar to those that are encountered in Texas is also examined.

### **3.2 SCHMERTMANN ET AL. (1987)**

Schmertmann et al. (1987) developed design charts for geogrid-reinforced slopes for the Tensar Corporation. Several assumptions were made in developing these charts. The assumptions are:

- 1) Homogeneous slope.
- 2) The foundation is sufficiently strong to prevent failure surfaces from passing into it.

- 3) The soil is cohesionless ( $\bar{c} = 0$ ) – this should be appropriate for granular materials or long-term (*drained*) stability of many clays.
- 4) Pore water pressures are zero.
- 5) No seismic loads.
- 6) Simple slope geometry (slope face is planar, and the ground surface extends horizontally beyond the crest and toe of the slope).
- 7) Reinforcement layers are horizontal.
- 8) The shear strength at the interface between the reinforcement and soil is reduced. This strength reduction is characterized by a coefficient of interaction,  $\mu$ , i.e., the interface friction angle is expressed as  $\tan^{-1}(\mu \tan \bar{\phi}_{soil})$ . Schmertmann et al. assumed a value of 0.9 for the coefficient of interaction.
- 9) A uniformly distributed surcharge load exists on the top of the slope.
- 10) The failure surface is bilinear. The total reinforcement force acts at the mid-height of the two wedges above the failure surface.

Schmertmann's charts are not applicable to short-term stability where undrained strengths apply, to slopes on weak foundations, or to slopes with nonzero pore water pressures.

### 3.2.1 *Minimum Reinforcement Force Required*

Schmertmann et al. (1987) used a bilinear sliding surface and two-part wedge, like the one shown in Figure 3.1, to determine the total force,  $T_b$ , required for equilibrium of the slope (the factor of safety of the unreinforced slope was always less than 1.0). Slopes with inclinations,  $\beta$ , between 30 and 80 degrees were considered. Soil shear strengths were characterized by effective stress friction angles,  $\bar{\phi}$ , ranging from 15–35 degrees ( $\bar{c}=0$ ). The inclination of the forces between the two wedges,  $\delta$ , was assumed to be equal to the friction angle of the soil.

Schmertmann et al. chose the method they used because it was simple and they expected it to give results similar to those obtained from more rigorous methods. The method satisfies only force equilibrium, not moment equilibrium; nor do the forces obtained satisfy complete static equilibrium. Schmertmann et al. checked the forces by comparing the values for several cases with those obtained using Bishop's simplified method of analysis. Bishop's simplified method employs circular failure surfaces and satisfies overall moment equilibrium and force equilibrium in the vertical direction, but not force equilibrium in the horizontal direction. Schmertmann et al. reported that results from analyses using Bishop's simplified method were within 10 percent of the results obtained from the bilinear wedge model. Accordingly, they chose to use the bilinear wedge model to compute initial values for forces and then adjusted the values in some cases to be consistent with those calculated using Bishop's simplified method. Details of how values were adjusted were not presented by Schmertmann et al.

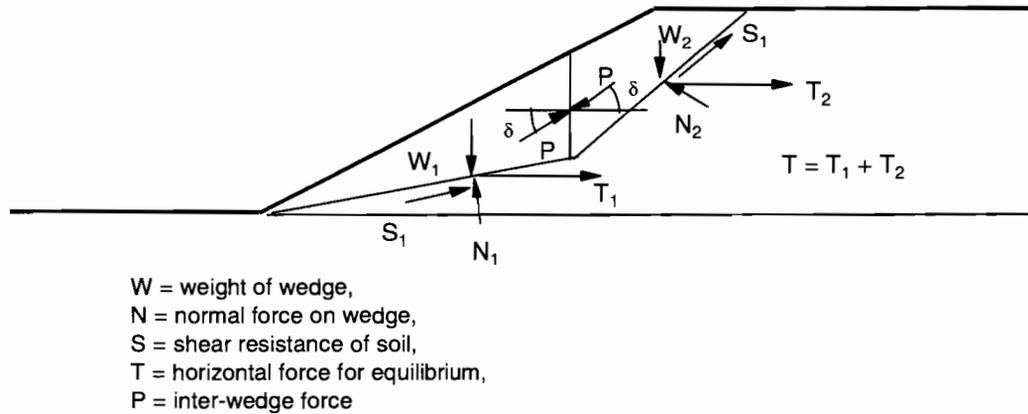


Figure 3.1 Bilinear failure surface and two-part wedge used in force equilibrium model by Schmertmann et al.

Schmertmann et al. presented the results of their analysis in dimensionless form. The force for equilibrium,  $T_t$ , is expressed in terms of a reinforcement coefficient  $K$  defined by

$$K = \frac{2T_t}{\gamma H'^2} \quad (3.1)$$

where  $\gamma$  is the unit weight of the soil, and  $H'$  is a *modified* slope height. The modified slope height is used to account for surcharge at the top of the slope. The modified height  $H'$  is computed from the following equation:

$$H' = H + \frac{q}{\gamma} \quad (3.2)$$

where  $q$  is the surcharge pressure.

Schmertmann et al. developed their chart for calculating the required force assuming a factor of safety on shear strength of 1.0, i.e., the shear strength ( $\bar{\phi}$ ) was assumed to be fully mobilized. Their chart is shown in Figure 3.2. The chart is used to determine values of  $K$  for various slope angles,  $\beta$ , and friction angles,  $\bar{\phi}$ . For factors of safety on shear strength greater than 1.0, a mobilized friction angle,  $\bar{\phi}_m^1$ , is used. The mobilized friction angle is expressed as

$$\bar{\phi}_m = \tan^{-1} \left( \tan \frac{\bar{\phi}}{F} \right) \quad (3.3)$$

<sup>1</sup> Schmertmann et al. called this a *factored* friction angle and designated it by the symbol  $\bar{\phi}_f$ .

where  $F$  is the required factor of safety on shear strength.

Schmertmann recommends using the peak soil friction angle,  $\bar{\phi}_p$ , in cases where allowable strains in the slope are small. In other cases, Schmertmann recommends using the “critical state” friction angle,  $\bar{\phi}_{cv}$ . The critical state friction angle corresponds to large strains during shear where volume changes cease.

Once the total required force,  $T_t$ , is determined using the chart in Figure 3.2, the minimum number of reinforcement layers,  $N_{min}$  is calculated by dividing the total required force by the long-term, allowable design tensile strength of the geosynthetic,  $T_a$ . The allowable tensile strength,  $T_a$ , is determined using the following equation:

$$T_a = \frac{T_{ult}}{F_{CR} F_{ID} F_{CD} F_{BD} F_{JNT}} \quad (3.4)$$

where  $F_{CR}$  is a partial reduction factor for creep deformation,  $F_{ID}$  is a partial reduction factor for installation damage,  $F_{CD}$  is a partial reduction factor for chemical deterioration,  $F_{BD}$  is a partial reduction factor for biological deterioration, and  $F_{JNT}$  is a partial reduction factor for joints, seams, and connections. Typical values of these partial reduction factors are shown in Table 3.1 (Koerner 1994).

Reinforcement forces are assumed to be distributed in a triangular pattern, increasing linearly with depth. Accordingly, the spacing of layers should be inversely proportional to depth below the crest of the slope. In order to conform to this triangular distribution of forces, Schmertmann recommends the maximum allowable spacing,  $s_v$ , of the reinforcement at a depth  $z$  be determined from the following equation:

$$s_v = \frac{T_a}{K\gamma z} \quad (3.5)$$

### 3.2.2 Minimum Required Reinforcement Length

Schmertmann et al. (1987) also considered the minimum length required for the reinforcement. The length of the reinforcement at the top and bottom of the slope,  $L_T$  and  $L_B$ , respectively, was calculated using two different methods. Both methods are based on finding the minimum depth a failure surface must extend into the slope to produce a factor of safety of 1.0.

The required length of reinforcement at the top of the slope was determined using a simple sliding wedge like the one shown in Figure 3.3. The angle  $\theta$  was varied to find the maximum reinforcement force,  $T$ . The intersection of the most critical wedge and the slope crest was used to define the minimum required reinforcement length at the top. Schmertmann et al. checked some of the lengths,  $L_T$ , they obtained using the simple wedge against lengths they would have obtained if they had used Bishop’s simplified method and circular shear surfaces. They found that in some cases Bishop’s simplified method gave lengths that were 40 percent larger. Consequently, they stated that they modified some of the

results from the wedge analysis to match results from the analyses with Bishop's simplified method. Details of the modification are not given in Schmertmann's paper.

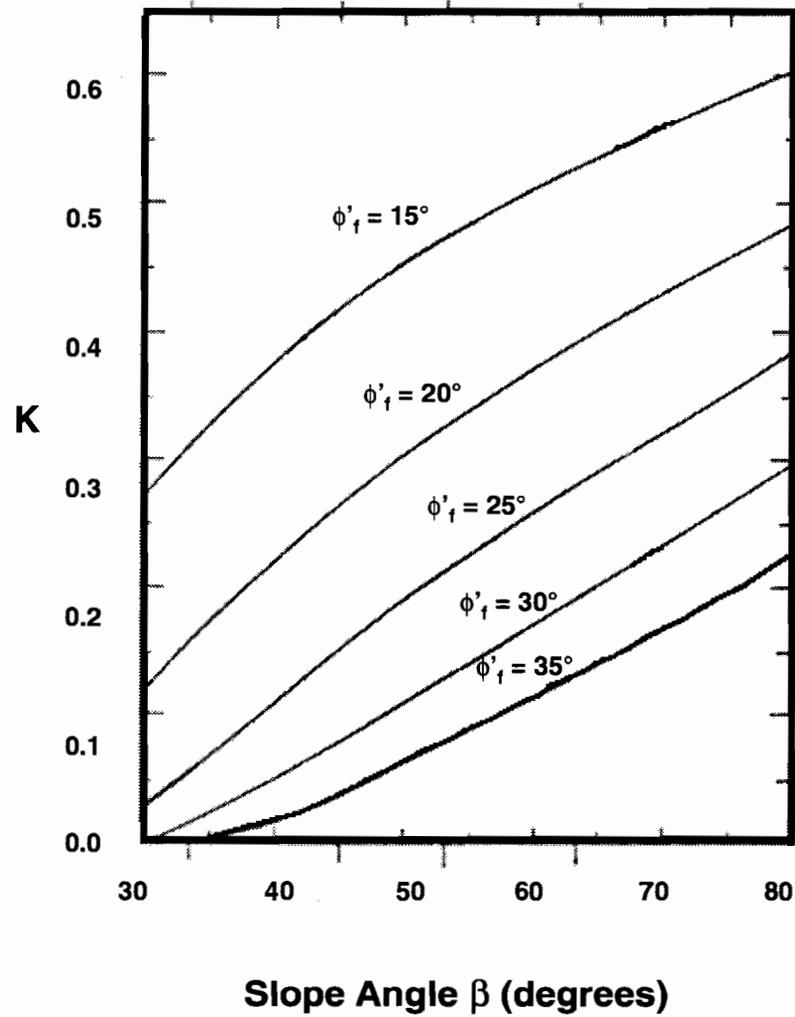


Figure 3.2 Required reinforcement force coefficient  $K$ —from Schmertmann et al. (1987)

Table 3.1 Recommended values for partial reduction factors in geosynthetic reinforcement—Values from Koerner (1994)

	Geogrids	Geotextiles
$FS_{ID}$	1.1 to 1.4	1.1 to 1.5
$FS_{CR}$	2.0 to 3.0	2.0 to 3.0
$FS_{BD}$	1.0 to 1.3	1.0 to 1.3
$FS_{CD}$	1.0 to 1.4	1.0 to 1.5

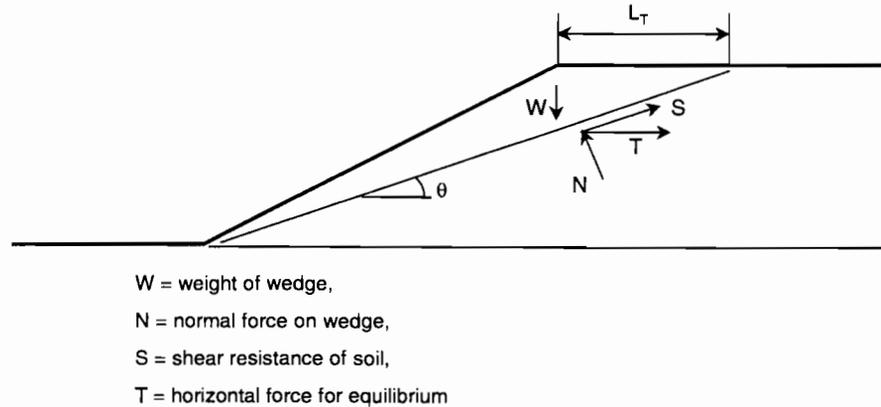


Figure 3.3 Sliding wedge force equilibrium model used by Schmertmann et al. for calculating required reinforcement length at top of slope

Schmertmann et al. calculated the required reinforcement lengths at the bottom of the slope using a two-part wedge like the one shown in Figure 3.4. The wedge was used to model sliding along a reinforcement layer at the bottom of the slope. The resistance to sliding along the reinforcement layer was computed using a coefficient of interaction,  $\mu$ , equal to 0.9, i.e.,  $\phi = \tan^{-1}(\mu \tan \phi) = \tan^{-1}(0.9 \tan \phi)$ .

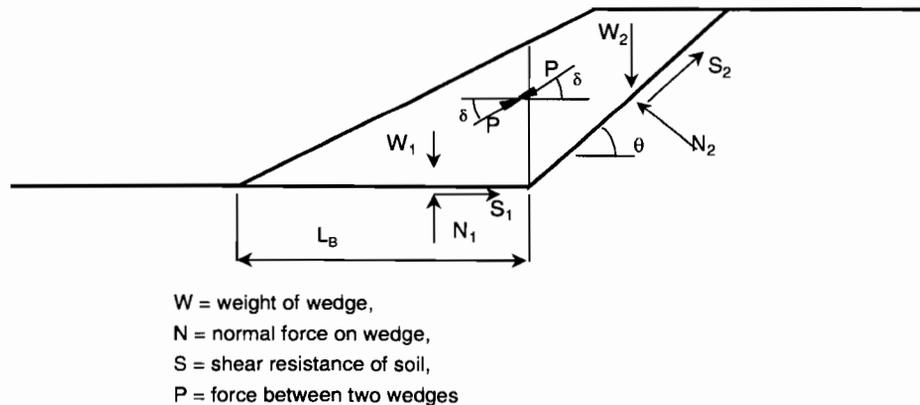


Figure 3.4 Two-part wedge used in force equilibrium model by Schmertmann et al. for calculating required reinforcement length at bottom of slope

The inclination of the force between the two wedges was represented by an interwedge friction angle,  $\delta$ . The value of the interwedge friction angle,  $\delta$ , was assumed to be equal to the friction angle of the soil. The inclination of the active wedge ( $\theta$ ) and the length

( $L_B$ ) was varied to find the largest length,  $L_B$ , required to achieve a factor of safety of 1.0. The lengths from the two-part wedge analyses were then checked using the same-shaped wedge but employing Spencer's limit equilibrium procedure of slices to compute the factor of safety. Spencer's procedure satisfies complete static equilibrium, while the two-part wedge analysis does not. The results from the wedge analyses were adjusted to match the results from analyses with Spencer's procedure. Schmertmann et al., do not mention the details of the adjustments. They also further adjusted the values of  $L_T$  so that  $L_T$  did not exceed  $L_B$ .

Schmertmann et al. presented the required reinforcement lengths ( $L_B$  and  $L_T$ ) in dimensionless form. The lengths were normalized by dividing them by the slope height and presented as values of the dimensionless ratios,  $L_B/H'$  and  $L_T/H'$ , where  $H'$  is the modified height described previously<sup>2</sup>. The normalized lengths,  $L_B/H'$  and  $L_T/H'$ , were presented on a single chart as shown in Figure 3.5. The required lengths depend on the slope angle,  $\beta$ , and the friction angle,  $\bar{\phi}$ .

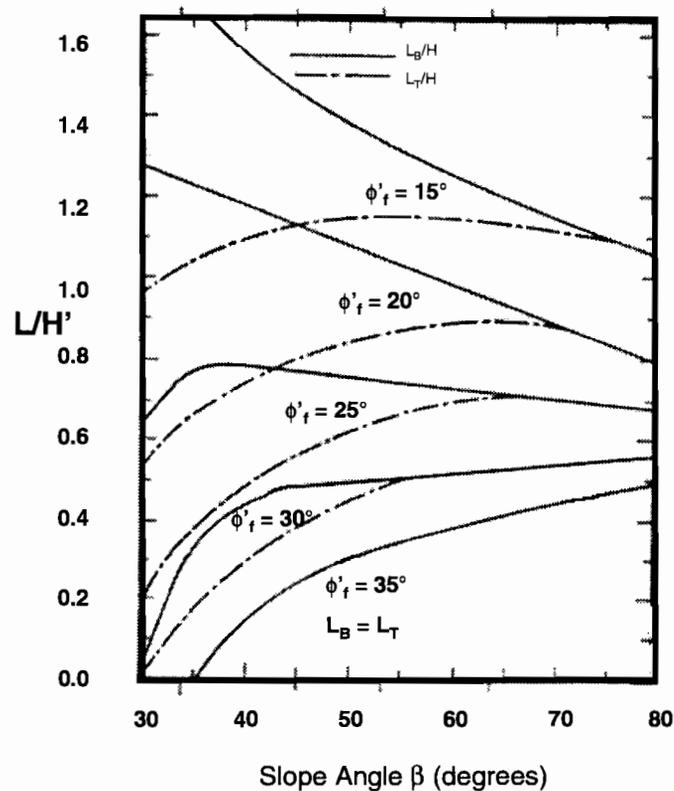


Figure 3.5 Chart for determining required reinforcement lengths at top and bottom of slope—from Schmertmann et al. (1987)

<sup>2</sup> Schmertmann et al. apparently never considered surcharges directly in their analyses, but assumed an equivalent height,  $H'$ , could be used to represent surcharge.

### 3.3 JEWELL (1990)

Jewell et al. (1984) published design charts for slope angles ranging from 30–80 degrees. Jewell (1984) used a bilinear wedge similar to what Schmertmann used. Jewell's (1984) model differed, however, in that the shear force between the wedges was assumed to be zero. Upon reviewing slopes designed with his charts presented in 1984, Jewell concluded that the charts led to overly conservative designs. Jewell (1990) then revised the charts based on log spiral and a different bilinear failure surface (two-part wedge) and extended the range of his charts to cover slopes inclined at up to 90 degrees. The assumptions used in Jewell's (1990) analysis are:

- 1) Homogeneous slopes.
- 2) Foundation is sufficiently strong to prevent failure surfaces from passing into it.
- 3) Soil is cohesionless ( $\bar{c} = 0$ ) – this should be appropriate for granular materials or long-term stability of many clays.
- 4) Pore water pressures are expressed by the pore water pressure coefficient ( $r_u$ ) defined by

$$r_u = \frac{u}{\gamma z} \quad (3.6)$$

where  $z$  is the depth below the ground surface,  $u$  is the pore pressure at the depth  $z$ , and  $\gamma$  is the unit weight of the soil.

- 5) No seismic loads.
- 6) Simple slope geometry (slope face is planar, and the ground surface extends horizontally beyond the crest and toe of the slope).
- 7) Reinforcement layers are horizontal.
- 8) The shear strength at the interface between the reinforcement and soil is reduced. The strength is reduced using a direct sliding coefficient,  $f_{ds}$ , equivalent to Schmertmann's coefficient of interaction,  $\mu$ . The shear strength at the reinforcement-soil interface is expressed by a reduced friction angle,  $\phi_{ds} = \tan^{-1}(f_{ds} \tan \bar{\phi})$ . Jewell assumed a value of 0.8 for the direct sliding coefficient.
- 9) A uniformly distributed surcharge load is applied to the top of the slope.
- 10) The resultant reinforcement force acts at one-third the height of the slope.
- 11) The length of reinforcement required to develop a given force is governed by a bond coefficient  $f_b$ . The bond coefficient governs the load transfer between the reinforcement and the soil. The bond coefficient is assumed to be between 0 and 1.

### 3.3.1 Minimum Reinforcement Force Required

Jewell (1990) used both bilinear wedges and log spiral failure surfaces to determine the minimum reinforcement force required for stability. The effects of pore water pressures were represented by the pore water pressure coefficient  $r_u$ . First introduced by Bishop and Morganstern (1960), the pore water pressure coefficient ( $r_u$ ) has been shown to be a useful parameter for characterizing pore water pressures in slope stability charts.

The log spiral failure surface, as shown in Figure 3.6, provides a solution that fully satisfies static equilibrium. A log spiral can be defined by its center point coordinates ( $x_c$  and  $y_c$ ) and an initial radius,  $r_o$ . The radius varies with the angle  $\theta$  as follows:

$$r_\theta = r_o e^{\theta \tan \bar{\phi}} \quad (3.7)$$

Jewell assumed that all log spirals passed through the toe of the slope; spirals extending into the foundation were not considered.

The force required for equilibrium is determined by summing moments due to forces acting on the soil mass above the log spiral failure surface (Figure 3.6). Because the reaction due to stresses in the soil along the shear surface acts through the center of the log spiral, these stresses do not need to be known to determine the required force for equilibrium  $T$ .

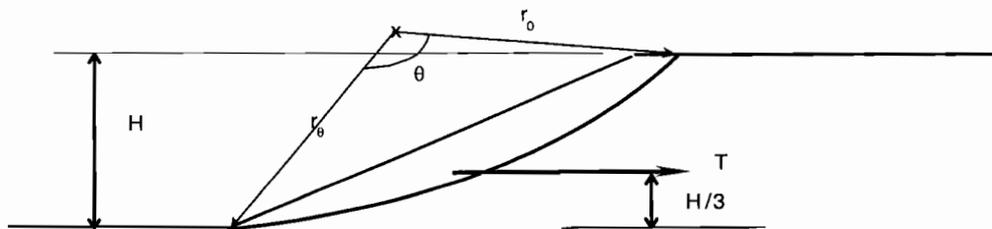


Figure 3.6 Log spiral limit equilibrium model used by Jewell for calculating required reinforcement coefficient,  $K_{Req}$

Jewell also used bilinear failure surfaces and a two-part wedge to compute the forces required for equilibrium. He compared the values to those obtained from the log spiral analyses. The two-part wedge model is illustrated in Figure 3.7. The model differs from the model used by Schmertmann et al. because the angle between the two wedges,  $\theta_3$ , is also varied in the search for the critical wedge. The three angles,  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ , are systematically varied to find the largest required reinforcement force,  $T_r$ . The force  $T_r$  is the sum of the two forces  $T_1$  and  $T_2$  acting at the lower one-third height of their respective wedges (Figure 3.7).

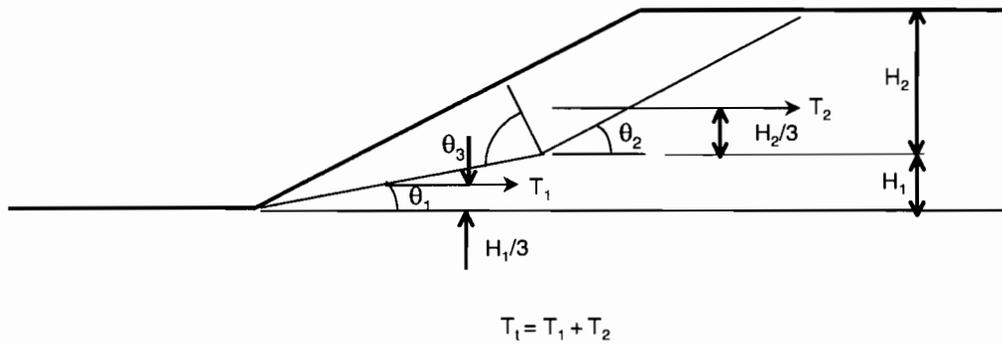


Figure 3.7 Two-part wedge used by Jewell in force equilibrium model for calculating required reinforcement coefficient,  $K_{Req}$

Jewell presented results in dimensionless form similar to the form used by Schmertmann et al. Jewell used a reinforcement coefficient,  $K_{Req}$ , which is identical in form to the coefficient used by Schmertmann et al. The reinforcement coefficient  $K_{Req}$  was calculated as follows:

$$K_{Req} = \frac{2T_1}{\gamma H^2} \quad (3.8)$$

Like Schmertmann's charts, Jewell's charts can be used to find the force required for any factor of safety by using the mobilized friction angle,  $\bar{\phi}_m$ .

Jewell examined several cases (slope angles and friction angles) and reported the values of  $K_{Req}$  computed using both log spiral and wedge models. Table 3.2 shows a comparison of some of the values reported by Jewell for the bilinear wedge and log spiral analyses. The difference between the values of  $K_{Req}$  for the two approaches ranges from approximately 10–22 percent. In all reported cases, the values obtained using the log spiral procedure are greater than or equal to those obtained using the bilinear wedge model. The differences are greatest for materials with small friction angles.

Table 3.2 Comparison of required reinforcement force coefficient,  $K_{Req}$  from bilinear wedge and log spiral failure surfaces—Values from Jewell (1990)

$\bar{\phi}_m$ (degrees)	$\beta$ (degrees)	Jewell et al. Bilinear Wedge	Log Spiral	Change (percent)
40	70	0.100	0.103	16
40	60	0.059	0.064	17
40	50	0.023	0.028	13
20	60	0.275	0.284	22
20	50	0.223	0.239	21
20	40	0.162	0.187	17

Note:  $r_u = 0$

Jewell used the results from the log spiral analysis to create his charts. Values of  $K_{Req}$  were plotted versus slope angle for various friction angles, as shown in Figure 3.8. In Jewell's procedure, the *critical state* friction angle,  $\bar{\phi}_{cv}$ <sup>3</sup>, is used to determine the value of  $K_{Req}$ . According to Jewell, this is equivalent to applying a factor of safety to the shear strength of the slope and no additional factor of safety is used.

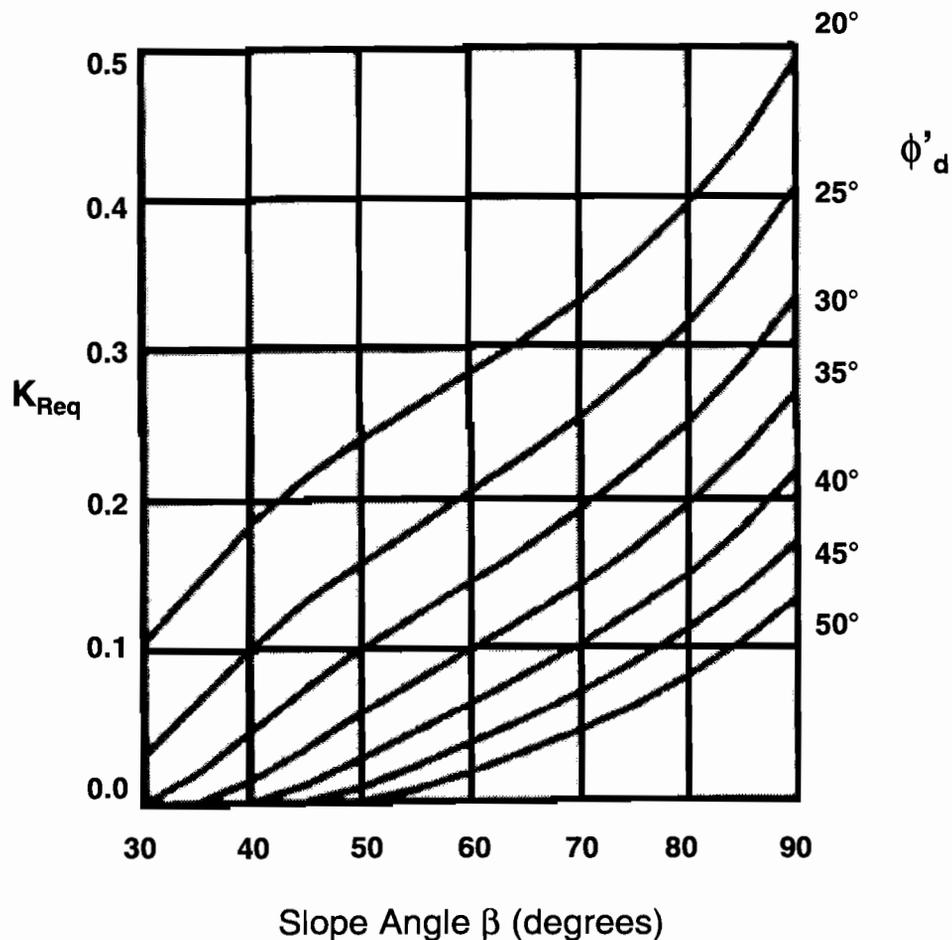


Figure 3.8 Chart for required reinforcement coefficient ( $K_{Req}$ ) for  $r_u = 0$ —from Jewell (1990)

<sup>3</sup> Jewell uses the symbol  $\bar{\phi}_{cs}$  to represent the critical state friction angle.

Jewell developed separate charts for values of  $r_u$  of 0, 0.25, and 0.5. He suggested that linear interpolation can be used to obtain values of  $K_{Req}$  for intermediate values of  $r_u$ . As illustrated by typical values of  $K_{Req}$  in Table 3.3, the required force for slopes with pore water pressures greater than zero ( $r_u > 0$ ) is higher than that for slopes with no pore water pressures.

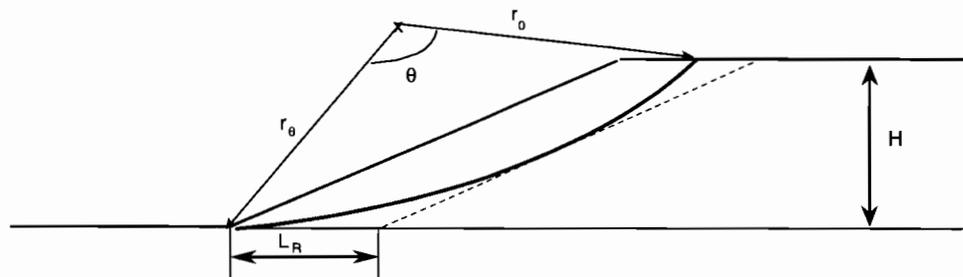
*Table 3.3 Effect of pore water coefficient on values of required reinforcement coefficient ( $K_{Req}$ )—Values from Jewell (1990)*

$\bar{\phi}_m$ (degrees)	$\beta$ (degrees)	$r_u$	$K_{Req}$
20	30	0.0	0.11
20	30	0.25	0.27
25	40	0.0	0.10
25	40	0.25	0.23

### 3.3.2 Minimum Required Reinforcement Lengths

Jewell investigated the minimum length required for reinforcement assuming a constant length of reinforcement for all layers. Two models were used to determine minimum the length. Jewell considered both overall stability and direct sliding along the soil-reinforcement interface. The pore water pressure coefficient,  $r_u$ , was again used to characterize pore water pressures.

Overall stability was considered using a log spiral failure surface. For a given slope ( $\bar{\phi}$ ,  $\beta$ , and  $r_u$ ) the log spiral that resulted in the largest required reinforcement force was located. The reinforcement length ( $L_R$ ) required for overall stability was determined using the location of the most critical log spiral, as shown in Figure 3.9. The broken line shown in this figure is parallel to the slope and tangent to the log spiral surface that produced the largest required force.



*Figure 3.9 Log spiral limit equilibrium model used by Jewell for calculating required reinforcement length,  $L_R$*

Direct sliding along the soil-reinforcement interface was considered using a bilinear failure surface and two-part wedge (Figure 3.10). The two-part wedge model was used because log spirals cannot conform to a planar failure surface along the reinforcement. The direct sliding coefficient ( $f_{ds}$ ) was assumed to be 0.8.

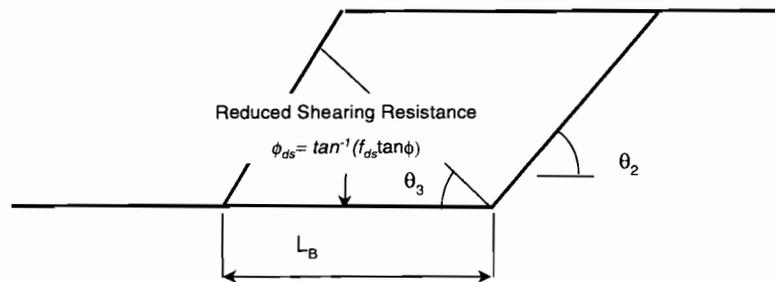


Figure 3.10 Direct sliding force equilibrium model used by Jewell for determining  $L_R$

Jewell expressed the length required for equilibrium,  $L_R$ , in dimensionless form by dividing the length by the slope height ( $H$ ). Separate charts were developed for overall stability (Figure 3.11) and direct sliding along a soil-reinforcement interface (Figure 3.12). Separate charts were also provided for values of  $r_u$  equal to 0, 0.25, and 0.5. The normalized lengths depend on the value of  $r_u$ , the slope inclination ( $\beta$ ), and the critical state friction angle,  $\bar{\phi}_{cv}$ . The larger of the two lengths determined for overall stability and for direct sliding are used for design.

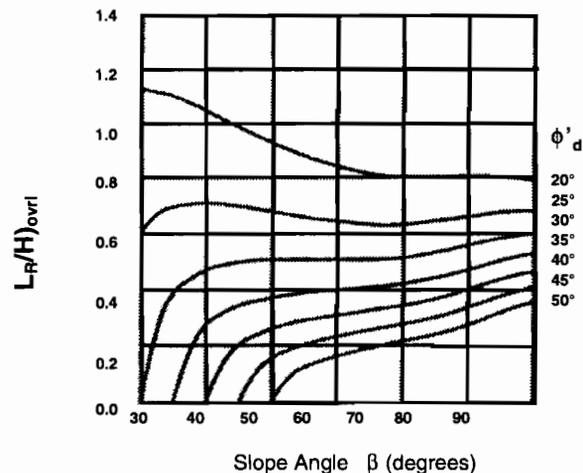


Figure 3.11 Chart for  $L_R$  for overall stability,  $r_u = 0$ —from Jewell (1990)

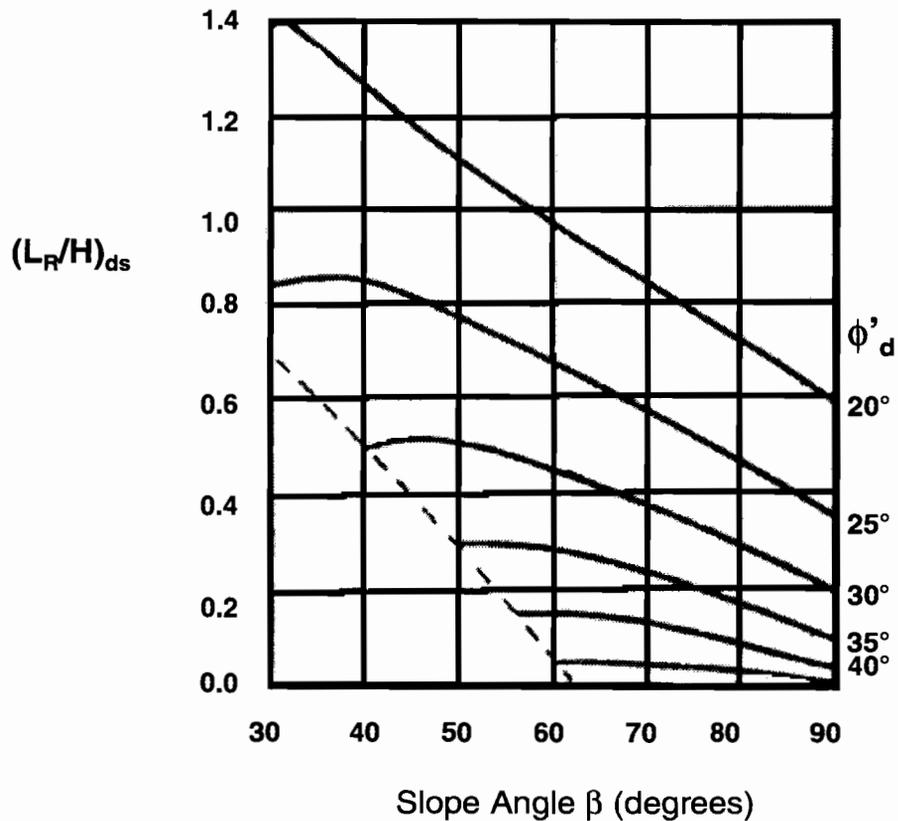


Figure 3.12 Chart for  $L_R$  for direct sliding,  $r_u = 0$ —from Jewell (1990)

Values for normalized lengths,  $L_R/H$ , for overall stability and direct sliding are compared in Table 3.4 for two slopes ( $\bar{\phi} = 20$  degrees,  $\beta = 30$  degrees and  $\bar{\phi} = 25$  degrees,  $\beta = 45$  degrees). As shown in this table, direct sliding tends to govern the length for flatter slopes, while overall stability tends to govern the length of reinforcement required for steeper slopes.

Table 3.4 Comparison of minimum required reinforcement lengths determined for direct sliding and overall stability using charts by Jewell (1990)

	$\bar{\phi}_m$ (degrees)	$\beta$ (degrees)	$L_R/H$
Direct Sliding	20	30	1.15
Overall Stability	20	30	1.41
Direct Sliding	35	60	0.40
Overall Stability	35	60	0.30

Note:  $r_u = 0$

### 3.3.3 Vertical Spacing of Reinforcement Layers

The reinforcement coefficient and the minimum length of reinforcement are used to determine the vertical spacing of the reinforcement. The spacing layout is selected such that it provides the minimum required stress according to a design *earth pressure* distribution. This earth pressure distribution is constructed using the required length ( $L_R$ ), the reinforcement coefficient ( $K_{Req}$ ), and the bond coefficient ( $f_b$ ). The procedure for constructing the earth pressure distribution is discussed below.

First, the bond coefficient is used to determine the length of reinforcement required to develop resistance to pullout forces. The length of reinforcement required to fully develop the allowable tensile force in a reinforcement layer is called the bond length ( $L_{Bond}$ ) and is calculated as follows:

$$L_{Bond} = \frac{T_a}{2\gamma H W_r f_b \tan \phi} \quad (3.9)$$

where  $W_r$  is the width of the reinforcement layer,  $\gamma$  is the unit weight of the soil,  $H$  is the slope height, and  $T_a$  is the allowable tensile force of the reinforcement. If the required length of reinforcement is less than the bond length, the reinforcement layer will not be able to develop its allowable tensile force. This shortfall can be accounted for either by increasing the length of the reinforcement or by adding layers of reinforcement. Jewell recommends adding layers of reinforcement.

Once the bond length is determined, an earth pressure distribution is calculated starting with the following equation:

$$\sigma_{Req} = \gamma K_d z \quad (3.10)$$

where  $K_d$  is a design earth pressure coefficient calculated from

$$K_d = \frac{K_{Req}}{\left(1 - \frac{L_B}{L_R}\right)} \quad (3.11)$$

The term  $1 - L_B/L_R$  is called the *bond allowance* of the reinforcement. Equation 3.9 defines a linear distribution of earth pressure with depth. This is modified near the top of the slope, such that the earth pressure has a minimum value given by

$$\sigma_{min} = \gamma H \left(\frac{L_B}{L_R}\right) K_{Req} \quad (3.12)$$

The resulting earth pressure distribution is illustrated in Figure 3.13 (shown by the bold line). The reinforcement must be spaced to provide at least the force indicated by this earth pressure distribution.

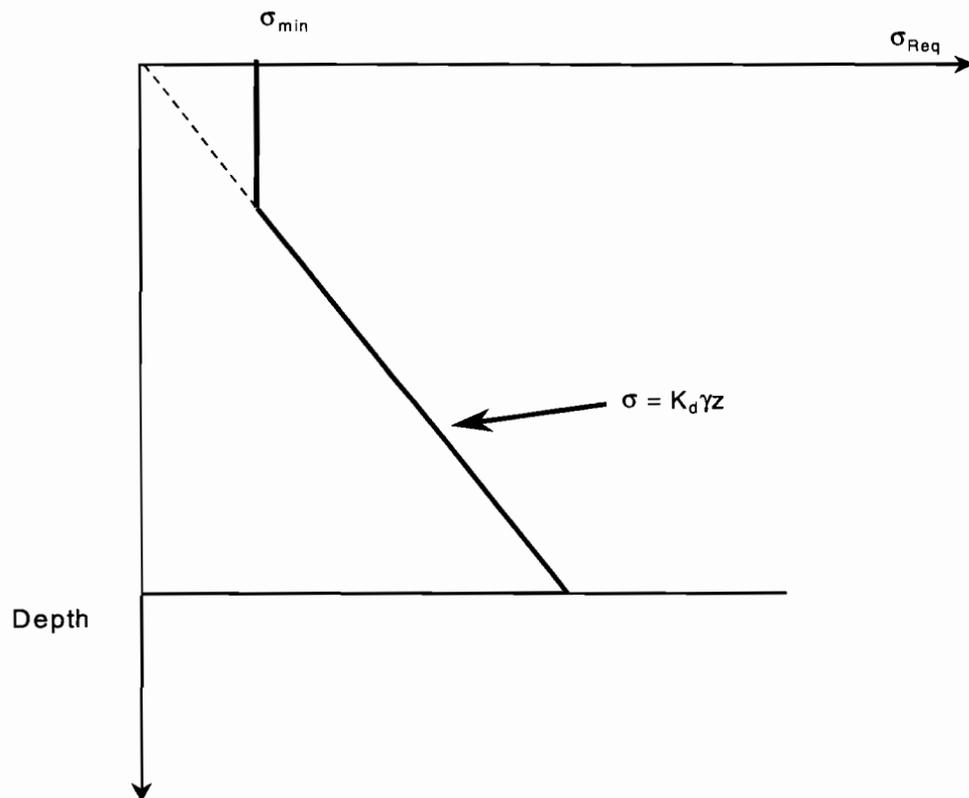


Figure 3.13 Earth pressure distribution used to compute vertical reinforcement spacing using method by Jewell (1990)

### 3.4 LESHCHINSKY AND REINSCHMIDT

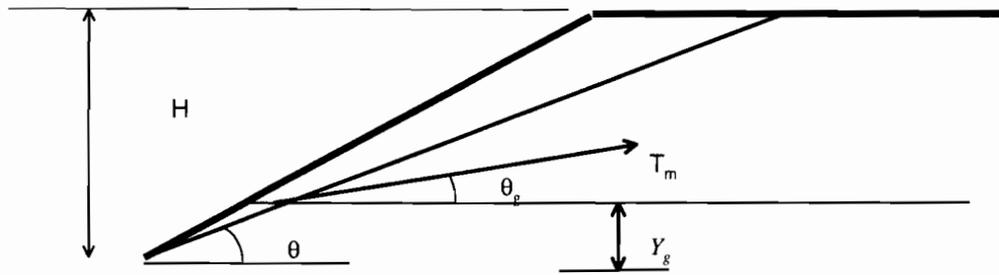
Leshchinsky and Reinschmidt (1986) also developed charts for use in design of geosynthetic-reinforced earth structures. Their charts provide only the required total force for equilibrium, but cover a range of slope angles and shear strengths not covered by other charts. Their charts cover slope angles of 15, 30, 45, 60, and 90 degrees. Their charts also allow for soils with both cohesion ( $\bar{c}$ ) and friction ( $\bar{\phi}$ ). Finally, failure surfaces are allowed to enter the foundation, which permits slopes having foundations with the same strength as the slope to be considered. Leshchinsky and Reinschmidt's charts are somewhat more complicated than the charts used by Schmertmann et al. and Jewell because of the larger number of variables. The charts were developed by evaluating both a critical *rotational* mode of failure, assuming a log-spiral failure surface and a critical *translational* mode of failure, assuming a linear (planar) failure surface. Leshchinsky and Reinschmidt made the following assumptions:

- 1) Homogeneous slope and foundation.
- 2) Soil strengths expressed by linear Mohr-Coulomb envelope with cohesion ( $c$ ) and friction ( $\phi$ ). The cohesion and friction may be either total stress or effective stress values, depending on the condition of interest.
- 3) Pore water pressures are zero.
- 4) No seismic loads.
- 5) Simple slope geometry (slope face is planar, and the ground surface extends horizontally beyond the crest and toe of the slope).
- 6) Reinforcement is horizontal.
- 7) A uniformly distributed surcharge load is applied at the crest of the slope.

#### 3.4.1 Required Reinforcement Strength

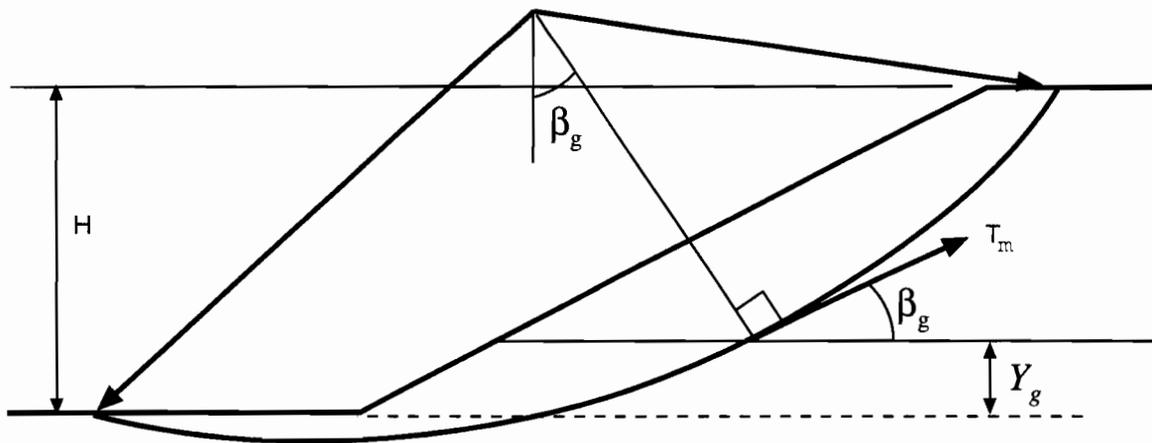
Leshchinsky and Reinschmidt (1984) used both log spiral and planar failure surfaces to determine the force required for equilibrium. They also investigated the effect that different elevations for the line of action of the reinforcement force had on factors of safety computed using the log spiral procedure.

The planar failure surface considered by Leshchinsky and Reinschmidt is shown in Figure 3.14. Leshchinsky and Reinschmidt assumed the reinforcement rotated where the planar surface intersected the reinforcement. The inclination of the reinforcement force was assumed to be defined by the angle of the failure plane and the friction angle of the soil. They assumed that if the angle between the failure plane and the horizontal was  $\theta$ , then the tensile force provided by the reinforcement acted at an angle  $\theta_g = \theta - \phi_m$  from the horizontal, where  $\phi_m$  is the mobilized friction angle of the soil. This maximizes the contribution of the reinforcement force on the factor of safety.



*Figure 3.14 Planar shear surface model used in force equilibrium procedure by Leshchinsky and Reinschmidt to model translational failure*

The log spiral failure surface considered by Leshchinsky and Reinschmidt is illustrated in Figure 3.15. As with the planar surface, the reinforcement was assumed to rotate at the failure surface. The reinforcing force was assumed to act in a direction perpendicular to the radius of the failure surface and opposing the direction of failure. The angle between the reinforcing force and the horizontal was represented as  $\beta_g$ . Unlike Jewell, Leshchinsky and Reinschmidt allowed the log spiral failure surfaces to enter the foundation. This allows the reinforcement force to be determined for cases in which the foundation is composed of the same material as the slope. Jewell's charts are restricted to slopes where the foundations are much stronger than the slope.



*Figure 3.15 Log spiral limit equilibrium model used by Leshchinsky and Reinschmidt for modeling rotational failure*

Leshchinsky and Reinschmidt expressed the force required for equilibrium by the dimensionless parameter  $T_m$ , defined by

$$T_m = \frac{nt_g}{\gamma H^2 F} \quad (3.13)$$

where  $t_g$  is the allowable tensile force in a single reinforcement layer,  $n$  is the number of layers of reinforcement, and  $F$  is a factor of safety for equilibrium. The factor of safety is applied equally to both the soil shear strength and the reinforcement force.

The elevation,  $y_g$ , of the line of action of the reinforcement force above the toe of the slope is expressed by the dimensionless parameter  $Y_g$ , defined as

$$Y_g = \frac{y_g}{H} \quad (3.14)$$

The cohesion of the soil,  $c$ , is represented by a dimensionless parameter  $N_m$  defined as

$$N_m = \frac{c}{\gamma H F} \quad (3.15)$$

Leshchinsky and Reinschmidt plotted values of  $T_m$  as a function of  $\phi_m$  for various values of  $N_m$  as shown in Figure 3.16. Separate charts were developed for each combination of slope angle ( $i$ ) and dimensionless elevation of the reinforcement force ( $Y_g$ ). Leshchinsky and Reinschmidt's charts can be used for total stress analyses as well as for effective stress analyses. However, for effective stress analyses no method is provided for accounting for pore water pressures.

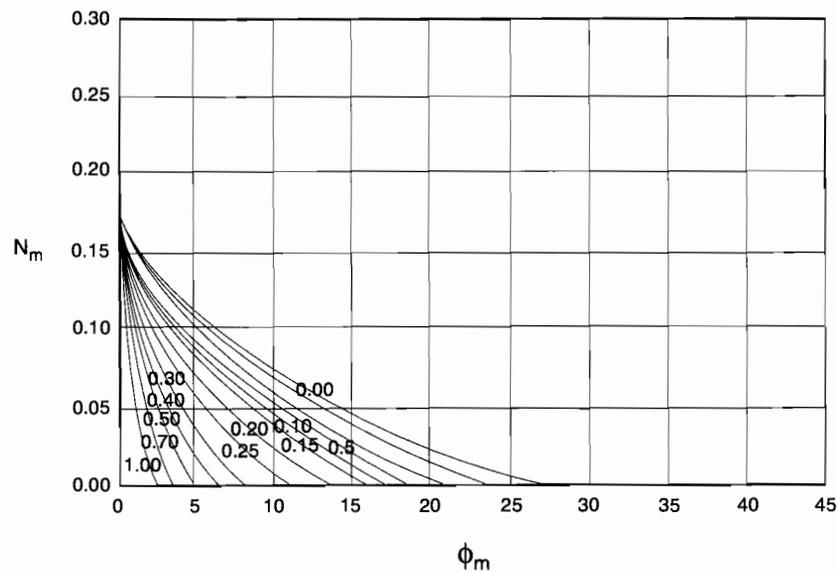


Figure 3.16 Chart developed by Leshchinsky and Reinschmidt (1985) for determining reinforcement force:  $i = 30^\circ$ ,  $Y_g = 0.5$

Thus, the charts are limited to either completely submerged slopes with no flow or slopes with zero pore water pressures. For effective stress analyses of completely submerged slopes, pore water pressures are accounted for indirectly by using the submerged unit weight,  $\gamma' (= \gamma - \gamma_{water})$  in place of the total unit weight.

Leshchinsky and Reinschmidt indicated on their charts which of the two failure modes, rotational (log spiral) or translational (planar), is more critical. For rotational surfaces, they also indicated when the log spiral entered the foundation. They found that the translational mode of failure was critical only for steep slopes where the reinforcement acted at a point near the toe of the slope. They also showed that, the stronger the reinforcement, the deeper the failure surface extends into the foundation.

### 3.5 RESLOPE

Leshchinsky (1995 and 1997) developed a computer software program, ReSlope, to determine the optimal number, length, and spacing of primary geosynthetic reinforcement layers for design. ReSlope uses limit equilibrium analyses with both log-spiral and bilinear failure surfaces to determine the pattern of reinforcement required to provide a given factor of safety. The software allows the user to specify factors of safety for sliding, pullout, and for soil strength. ReSlope also utilizes several *partial* safety factors to define the allowable tensile force,  $T_a$ , in the reinforcement in terms of its ultimate tensile strength.

ReSlope is versatile in that it can be used for soils with both cohesion and friction; the strengths of the foundation and overlying slope may also differ. Pore water pressures can be included and are defined by either a value of a pore water pressure coefficient,  $r_w$ , or a piezometric surface. ReSlope allows for pseudostatic earthquake analysis through the use of both horizontal and vertical seismic coefficients,  $k_h$  and  $k_v$ , respectively. Surcharge loads can also be included in the analysis. Also, the crest of the slope does not need to be horizontal.

ReSlope uses a four-part design procedure. The first part determines the required reinforcement forces and reinforcement spacing. ReSlope uses log spirals and a *tieback* analysis to do this. The second part of the analysis determines minimum required lengths of reinforcement using a *compound stability* analysis. Log spiral failure surfaces are also used for this part. In the third part of the analysis, direct sliding along the soil-reinforcement interface for each layer is checked to determine if the length determined in the second part of the analysis is adequate. A two-part wedge and force equilibrium procedure is used for the direct sliding analysis. The fourth part of the analysis uses Bishop's simplified method with circular failure surfaces to check the factor of safety against global failure, including the foundation.

#### 3.5.1 Tieback Analysis (Part I)

The first part of an analysis by ReSlope consists of determining the required reinforcement forces using a tieback analysis. Log spiral failure surfaces are used and

multiple sets of computations are performed to compute the required reinforcement force and vertical spacing. This multistep scheme is represented in Figure 3.17.

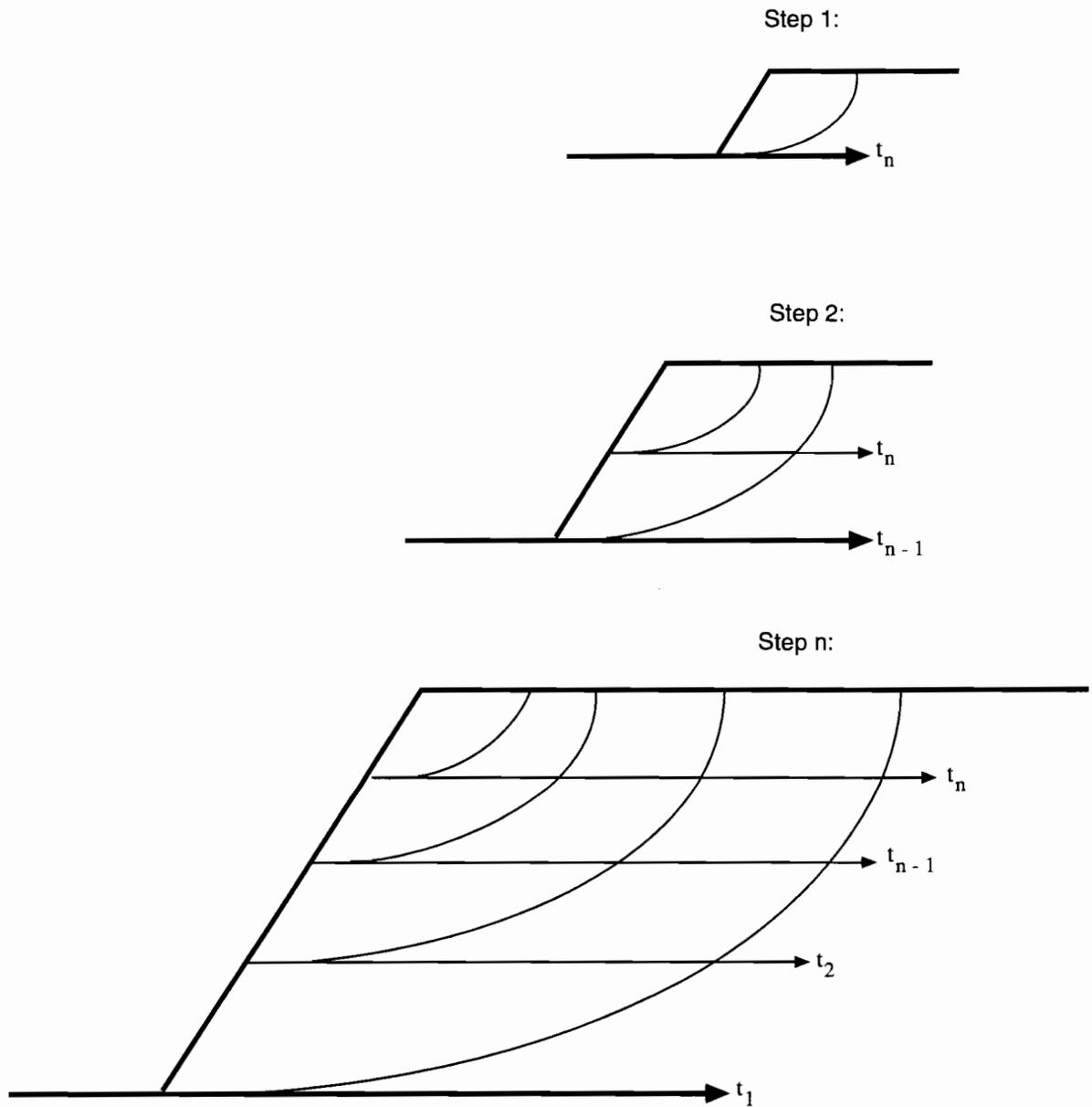
In this procedure, ReSlope first locates the critical log spiral, producing the largest reinforcement force,  $t_n$ , for a top layer of reinforcement. The location of this layer is restricted to being at a distance below the crest no more than the maximum spacing specified. The value of  $t_n$  must be less than the allowable force,  $t_a$ , in the reinforcement. The allowable force of the reinforcement is determined in the same way as explained previously for Schmertmann's (1987) method. The allowable load,  $t_a$ , is based on the ultimate strength of the reinforcement and the partial reduction factors for (1) installation damage, (2) chemical degradation, (3) biological degradation, and (4) creep. An additional factor of safety against uncertainties in the geosynthetic material is also applied to the strength of the reinforcement. If the calculated force  $t_n$  is greater than the allowable force  $t_a$ , ReSlope tries a shallower depth for the reinforcement until  $t_n$  does not exceed  $t_a$ . Once the location of the top layer of reinforcement is set, the next tieback analysis is performed to determine the location of the second layer of reinforcement. This step is similar to the first step except that the force from the first layer of reinforcement is applied at the elevation determined for the first layer. The location of the second layer is found in much the same way as is the top layer. That is, the location is chosen such that the second layer of reinforcement combined with the first layer can provide the required force to produce the desired factor of safety in the overlying slope. The procedure of calculating forces and spacing for each layer is repeated from the top down, until the bottom layer of reinforcement is reached. In all analyses, the reinforcement forces are assumed to be tensile and to act horizontally.

### 3.5.2 Compound Stability (Part II)

The second part of the design procedure implemented in ReSlope is also based on log spiral failure surfaces. The second part of the analysis is used to determine the lengths of the reinforcement layers. For this part, ReSlope first sums the individual values of  $t_n$  calculated from the tieback analysis for each layer to determine the total required reinforcement force. ReSlope then determines the minimum number of reinforcement layers that will just provide this force, assuming each layer develops its full allowable strength,  $T_a$ . If  $m$  is the number of layers of reinforcement necessary to provide the required total force, then the bottom  $m$  layers of reinforcement are assumed to develop their maximum allowable strength and any upper layers are ignored. The upper layers are assumed to have lengths determined by the outermost log spiral in the tieback analysis.

Beginning with the  $m$  lowest layers, the uppermost of these layers is omitted and the lateral distance that the log spiral must pass (at the elevation of the omitted layer) to produce the desired factor of safety is determined. This distance is indicated as the distance  $L_m$  in Figure 3.18. Next, the next-to-topmost of the  $m$  layers is omitted and the distance the log spiral must extend into the slope at the elevation of this  $(m-1)$  layer is found ( $L_{m-1}$  in Figure

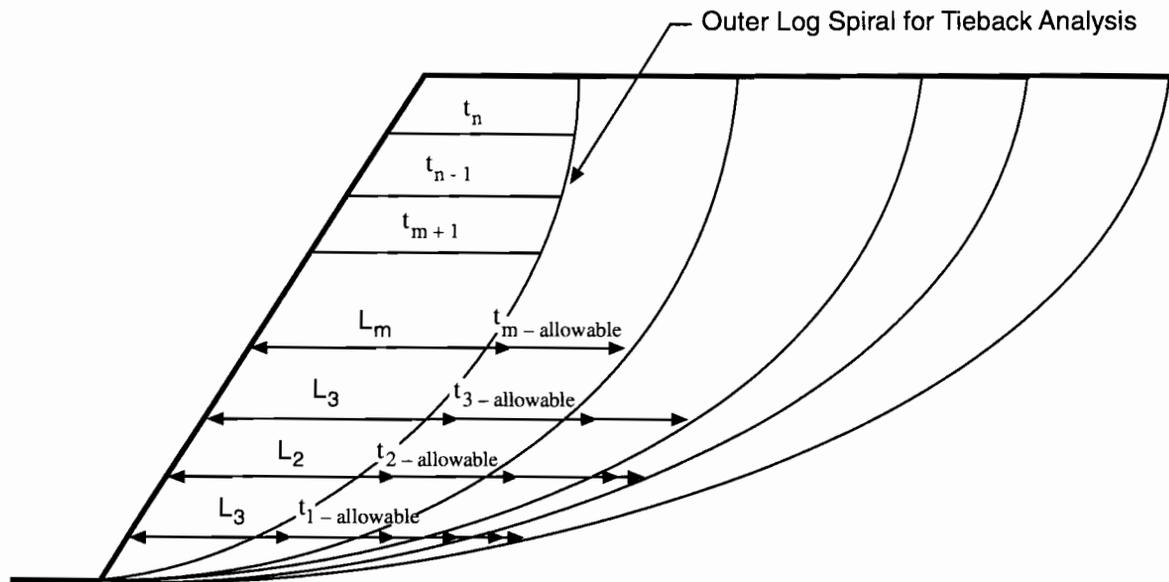
3.18). This process is repeated, removing one layer of reinforcement at a time from the top down to determine the lateral distance the log spiral must extend into the



*Figure 3.17 Tieback analysis scheme used in ReSlope to determine required reinforcement forces and spacing of reinforcement layers*

slope to produce the required factor of safety without the benefit of the upper reinforcement.

After all the lengths ( $L$ ) have been determined, ReSlope checks using log spiral failure surfaces that emerge above the toe of the slope to determine if the lengths need to be increased. The length of any reinforcement is increased as necessary to provide the required factor of safety.



*Figure 3.18 Compound stability analysis used by ReSlope to determine required reinforcement lengths*

### **3.5.3 Direct Sliding (Part III)**

The third part of the analysis performed by ReSlope involves direct sliding along the interface between the reinforced soil and reinforcement. ReSlope uses a two-part wedge and the force-equilibrium method illustrated in Figure 3.19 to perform the direct sliding analysis. The analysis is used to determine the length of each layer,  $L_{ds}$ , required to provide a specified factor of safety for direct sliding,  $F_{s-ds}$ . The factor of safety for direct sliding is defined as:

$$F_{s-ds} = T_B / P \cos \delta \quad (3.16)$$

where  $T_B$  is the shear strength at the soil-reinforcement interface,  $P$  is the force between the two wedges, and  $\delta$  is the inclination of the force from the horizontal. The required length of the reinforcement is related to the shear strength,  $T_B$ , by the following equation:

$$T_B = (N_B \tan \phi_d + c_d L_{ds}) C_{ds} \quad (3.17)$$

where  $C_{ds}$  is a *direct sliding coefficient*,  $N_B$  is the normal force acting on the reinforcement-soil interface, and  $c_d$  and  $\phi_d$  are the developed shear strength parameters. When the bottom layer of reinforcement rests directly on the foundation, the value of  $T_B$  is calculated using values of  $c_d$  and  $\phi_d$  for both the slope and the foundation. The smaller of the two values of  $T_B$  is then used. For Equation 3.15, Leshchinsky suggests using a value of  $\delta$  between  $(2/3)\phi$  and  $\phi$ .

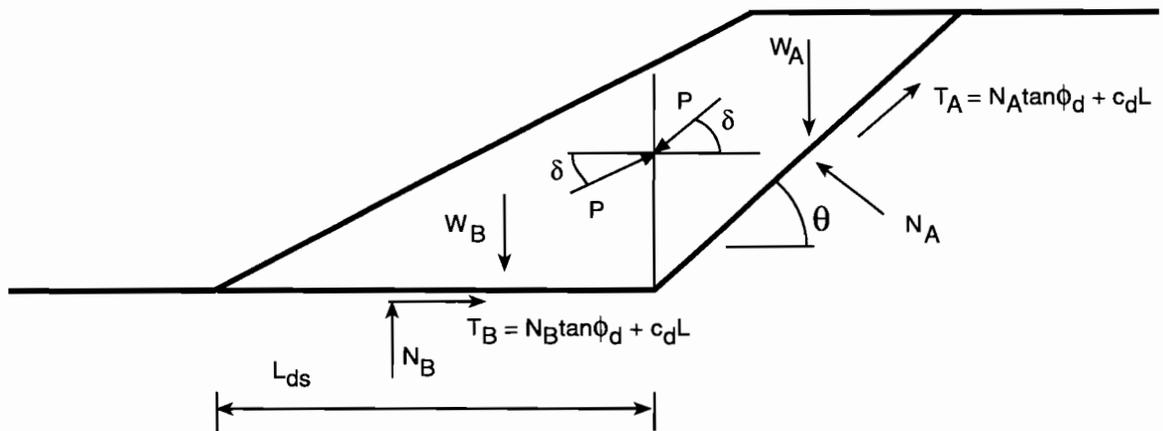


Figure 3.19 Direct sliding model used in ReSlope to determine required reinforcement lengths

### 3.5.4 Length of Reinforcement Layers

ReSlope compares the length from the compound analysis,  $L_c$ , with the length from the direct sliding analysis,  $L_{ds}$ , for each layer of reinforcement. The greater of these two lengths is used for design. Once the lengths of the layers are determined, they are increased by the amount required to develop full capacity against pullout.

The ability of each layer to develop full capacity against pullout is determined using a pullout interaction coefficient,  $C_i$ .  $C_i$  is similar to the bond coefficient used by Jewell. The length of reinforcement,  $L_{a,j}$ , required to develop resistance to pullout forces in layer  $j$  is calculated using

$$L_{a,j} = \frac{t_j}{[\sigma_j C_i (\tan \phi_d + c_d)]} \quad (3.18)$$

where  $\sigma_j$  is the normal stress acting on layer  $j$ . The length calculated from Equation 3.18 is added to the length calculated from the compound stability and/or direct sliding analyses for each layer to determine the final length of each reinforcement layer.

### 3.5.5 Deep-Seated Failure (Part IV)

The final step in the analyses performed by ReSlope is a check for deep-seated failure. ReSlope uses Bishop's simplified method and circular failure surfaces that pass through the unreinforced soil to determine the minimum factor of safety. Failure surfaces are permitted to pass below the toe into the foundation. The foundation soil can be different from the slope's (i.e., different strengths or unit weights). Thus, the adequacy of the foundation for supporting the reinforced slope is evaluated. However, ReSlope does not adjust the design if the required factor of safety is not met; it simply reports the lowest factor of safety in the output and it is left to the user to modify the design as necessary.

### 3.5.6 Final Design

ReSlope offers three choices for determining final lengths for the reinforcement. The first choice is for lengths to be reported as the individual maximum lengths calculated for each layer of reinforcement from the first three parts of the analysis. These lengths may not be practical for construction since they will likely be nonuniform. As an alternative, the second choice is for the final lengths to be constant. In this case, the maximum length calculated for any layer is used for all layers. The third choice for defining the reinforcement lengths is for the lengths to vary linearly between the top layer of reinforcement and the bottom layer. The length at the top,  $L_t$ , is the greatest length calculated from the compound and tieback analyses. The length at the bottom,  $L_b$ , is the greatest length calculated for the compound, tieback, and direct sliding analyses.

Neither ReSlope nor any other chart-based procedure addresses local stability against sloughing at the exposed face of the slope. It is assumed that the designer will address this by adding secondary reinforcement between primary layers. Secondary reinforcement requirements can be established by performing a stability analysis for the portion of the slope face between primary layers. If the factor of safety computed from such analyses is at least equal to the required factor of safety for the design, secondary reinforcement is not necessary.

## 3.6 COMPARISON OF DESIGN METHODS

Each of the design methods reviewed in this chapter was used to determine reinforcement requirements for two example slopes. The example slopes were selected so that all of the four design methods would apply. Both examples consist of simple slopes with

no pore water pressures. Both slopes are 30-feet high and have unit weights of 120 pounds per cubic foot. A factor of safety of 1.5 is used for both overall stability and direct sliding. The foundations were assumed strong enough to keep failure surfaces from entering the foundation. The two slopes and soil properties are shown in Figure 3.20. The allowable strength of the reinforcement used in the design is 1525 pounds per foot; this strength corresponds to the reported allowable strength for Geogrid type UX1400HT as reported in Tensar's *Sierra Slope Retention System Design Manual* (1994). The strength presumably accounts for all the partial reduction factors typically used in determining a geosynthetic material's long-term strength. Thus, no additional factors of safety were applied to the reinforcement force. Tables 3.5 and 3.6 summarize the required forces and total lengths of reinforcement force for Examples 1 and 2, respectively, for each of the design methods. Although it is not called for in Jewell's (1990) paper, a factor of safety was applied to the shear strength for the values from Jewell's charts for consistency with other methods. The value of required force from Leshchinsky and Reinschmidt's chart was calculated using two methods. In the first method, a factor of safety was applied to both the shear strength and to the total force. In the second method, the factor of safety was applied only to the shear strength.

The design layout (number, vertical spacing, and horizontal length) for the reinforcement layers was determined for each slope using the charts by Schmertmann et al. (1987) and Jewell (1990). ReSlope was also used to calculate the design layout for both examples. In order to compare designs using similar design criteria, several of the factors of safety in ReSlope (geosynthetic uncertainties and geosynthetic pullout, as well as all the partial reduction factors) were set equal to 1.0. Leshchinsky and Reinschmidt (1985) did not provide charts for determining the required length of reinforcement or recommendations about vertical spacing; thus, their charts could be used only to determine the total required force. Layouts for Leshchinsky and Reinschmidt's method are not presented. The height of each layer above the foundation and its length for each design method are presented in Tables 3.7 to 3.12. Once the reinforcement layouts were determined, each slope was analyzed using the computer program UTEXAS3. The factor of safety for global stability of each slope and reinforcement layout was calculated using circular failure surfaces and Spencer's procedure. The factor of safety for direct sliding of each slope was calculated using direct sliding coefficients of 0.8 and 0.9 with noncircular failure surfaces and Spencer's procedure. For the analyses with UTEXAS3, the reinforcement was modeled as horizontal tensile forces applied at the intersection of the failure surface with each layer of reinforcement. Any flexural strength of the reinforcement was ignored. The factors of safety calculated using UTEXAS3 are presented in Tables 3.13 and 3.14. The minimum factor of safety from the two failure modes, *global* and *direct sliding*, is underlined.

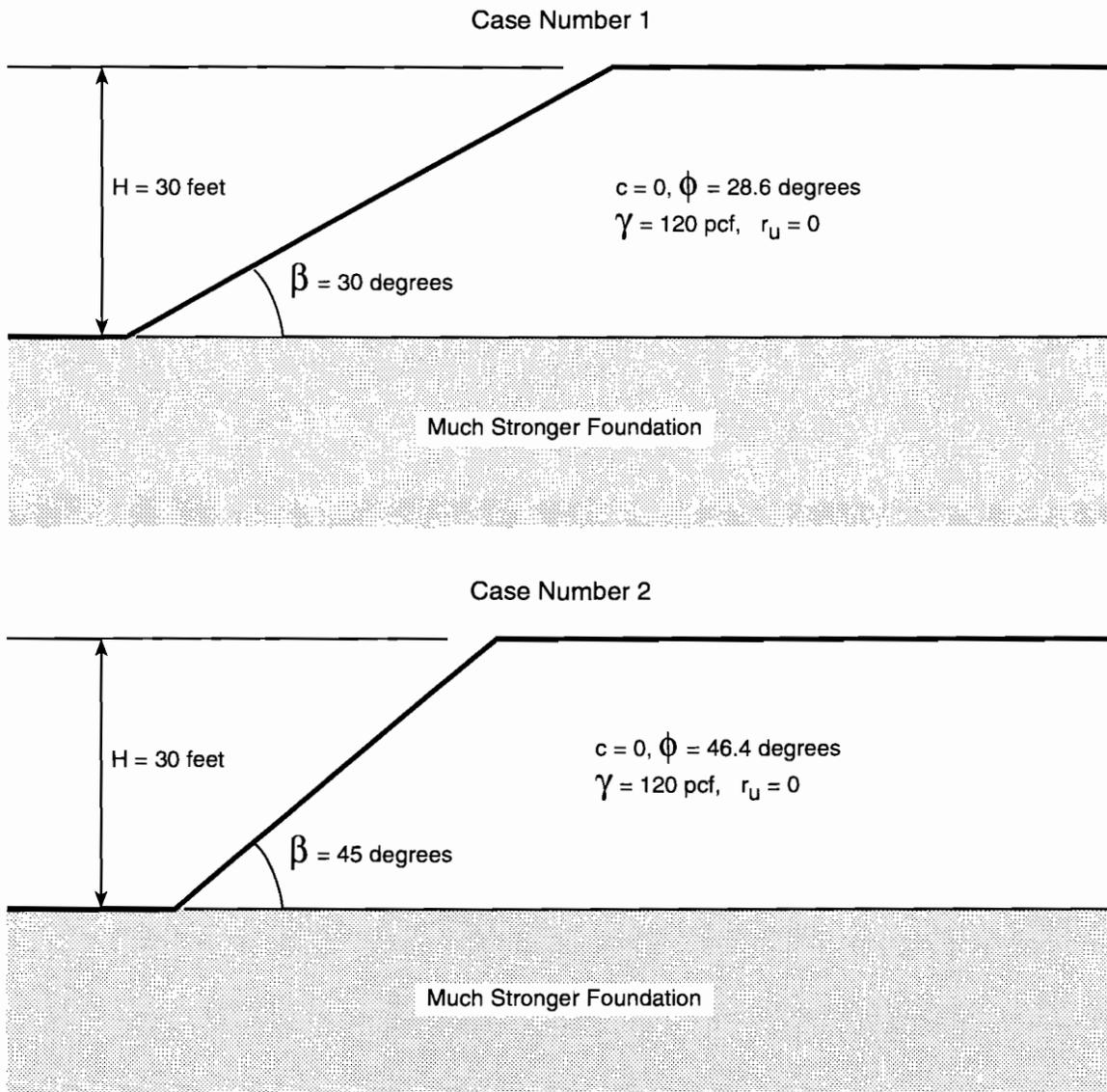


Figure 3.20 Two example slopes used to compare design methods

*Table 3.5 Design quantities for example reinforced slope number 1 using various design methods (allowable reinforcement strength = 1525 lb/ft)*

Method	Total Force (lb/ft)	L <sub>B</sub> (feet)	L <sub>T</sub> (feet)
Leshchinsky and Reinschmidt (1984) with F.S. applied to both shear strength and total force	9720	N/A	N/A
Leshchinsky and Reinschmidt (1984) With F.S. applied only to shear strength	6480	N/A	N/A
Jewell (1990)	5670	43.5	43.5
Schmertmann et al. (1987)	6750	38.4	16.4
ReSlope	6653	55.6	35.1

*Table 3.6 Design quantities for example reinforced slope number 2 using various design methods (allowable reinforcement strength = 1525 lb/ft)*

Method	Total Force (lb/ft)	L <sub>B</sub> (feet)	L <sub>T</sub> (feet)
Leshchinsky and Reinschmidt (1984) with F.S. applied to both shear strength and total force	3240	N/A	N/A
Leshchinsky and Reinschmidt (1984) With F.S. applied only to shear strength	2160	N/A	N/A
Jewell (1990)	2700	11.25	11.25
Schmertmann et al. (1987)	2160	6.3	6.3
ReSlope	2261	22.1	12.6

*Table 3.7 Height above foundation and lengths of reinforcement layers for example slope number 1 using Schmertmann's method*

Layer	Height (feet)	Length (feet)
1	0	38.4
2	4	35.5
3	9	31.8
4	14	28.1
5	20	23.7

*Table 3.8 Height above foundation and lengths of reinforcement layers for example slope number 2 using Schmertmann's method*

Layer	Height (feet)	Length (feet)
1	0	6.5
2	9	6.5
3	24	6.5

*Table 3.9 Height above foundation and lengths of reinforcement layers for example slope number 1 using Jewell's method*

Layer	Height (feet)	Length (feet)
1	0	43.5
2	4	43.5
3	8	43.5
4	16	43.5
5	24	43.5

*Table 3.10 Height above foundation and lengths of reinforcement layers for example slope number 2 using Jewell's method*

Layer	Height (feet)	Length (feet)
1	0	11.25
2	7	11.25
3	14	11.25
4	22	11.25

*Table 3.11 Height above foundation and lengths of reinforcement layers for example slope number 1 using ReSlope*

Layer	Height (feet)	Length (feet)
1	0	55.6
2	1.5	54.1
3	6	49.4
4	11	43.8
5	20	35.1

*Table 3.12 Height above foundation and lengths of reinforcement layers for example slope number 2 using ReSlope*

Layer	Height (feet)	Length (feet)
1	0	22.2
2	10	17.4
3	20	12.6

*Table 3.13 Factors of safety calculated for example slope number 1 using UTEXAS3*

Case	Factor of Safety Global <sup>(1)</sup>	Factor of Safety Direct Sliding <sup>(2)</sup> $\mu = 0.8$	Factor of Safety Direct Sliding <sup>(2)</sup> $\mu = 0.9$
Schmertmann et al.	1.64	<u>1.44</u>	<u>1.54</u>
Jewell	<u>1.61</u>	1.86	1.99
ReSlope	<u>1.53</u>	1.95	2.12

(1) Based on Spencer's procedure and circular shear surfaces

(2) Based on Spencer's procedure and bilinear shear surfaces

*Table 3.14 Factors of safety calculated for example slope number 2 using UTEXAS3*

Case	Factor of Safety Global <sup>(1)</sup>	Factor of Safety Direct Sliding <sup>(2)</sup> $\mu = 0.8$	Factor of Safety Direct Sliding <sup>(2)</sup> $\mu = 0.9$
Schmertmann et al.	1.46	<u>1.44</u>	<u>1.51</u>
Jewell	1.71	<u>1.62</u>	<u>1.71</u>
ReSlope	<u>1.71</u>	2.18	2.33

(1) Based on Spencer's procedure and circular shear surfaces

(2) Based on Spencer's procedure and bilinear shear surfaces

As shown in Tables 3.5 and 3.6, the forces from Leshchinsky and Reinschmidt's charts, using their recommended method of applying the factor of safety to both the shear strength and the total force, are much higher than those of any other method. This difference is due to three assumptions made for their charts: The other methods assume that the reinforcement forces act horizontally, while Leshchinsky and Reinschmidt assumed the reinforcement rotated. The angle of rotation selected allows for the maximum contribution of the total force to resist sliding of the failure zone. This should cause the required force to be less than the other methods. However, the elevation of the line of action of the total reinforcement force is assumed to be at half the height of the slope in the charts used. The elevation of the total force in the model by Jewell (which also uses log spirals) is at the lower one-third point of the slope. The higher elevation assumed by Leshchinsky and Reinschmidt means a larger force is necessary to get the same moment as that obtained in the model by

Jewell, which has a larger moment arm for the reinforcement force. The most important reason Leshchinsky and Reinschmidt's procedure gives the largest force is that the same factor of safety is applied to both the shear strength and the reinforcement force. As shown in Tables 3.5 and 3.6, when the factor of safety is applied only to the shear strength, the total force is similar to the values obtained by the other methods. In the other methods based on charts, a factor of safety is only applied to the shear strength. The values of total force required are similar for all the other methods. The factors of safety shown in Tables 3.13 and 3.14 for a direct sliding mechanism with a coefficient of interaction of 0.8 for the two cases using Schmertmann's charts are less than the design value of 1.5. This is expected because Schmertmann et al. assumed the coefficient of interaction was 0.9. When 0.9 is used for the coefficient of interaction, the factor of safety against direct sliding is greater than 1.5. The other two methods (Jewell and ReSlope) result in factors of safety that are 1.5 or higher. The reason these factors of safety are higher than the design value of 1.5 is, in part, because the methods are conservative. Also, it is generally impossible to obtain a layout of reinforcement that produces exactly the required force. Because reinforcement is available only in specific types and capacities (allowable forces), the force provided by the actual layout will usually be greater than the required force. The largest factors of safety in Table 3.13 and 3.14 are from the direct sliding analyses of the ReSlope designs. This is because ReSlope produced the longest lengths. ReSlope uses a factor of safety against direct sliding as well as a factor of safety against the shear strength of the soil. This additional factor of safety makes final design lengths greater than those associated with the other methods.

### **3.7 SUMMARY**

Numerous charts exist for the design of reinforced slopes. They differ in the limit equilibrium models employed to develop the charts, pore water pressure assumptions, assumptions about the foundation, use of factors of safety, and shear strength parameters. Other than the differences caused by the various factors of safety used, the methods give comparable results when used for cases where the methods (charts) are applicable. Therefore, the choice of which method to use should be based on the applicability of the available charts to the problem of interest.

### **3.8 APPLICABILITY OF EXISTING METHODS**

The primary objective of reviewing the various design methods and charts for reinforced slopes was to determine their applicability to slopes like the ones typically encountered by the Texas Department of Transportation. For charts to be applicable to the failed slopes of interest, the charts must cover the following conditions:

- 1) Effective stress (*drained*) friction angles in the range of 20–25 degrees or less with minimal to no cohesion. For a factor of safety of 2.0, the mobilized friction angle would need to be as low as 10 degrees.

- 2) Foundations composed of both stronger materials and materials of similar strength as the overlying slope.
- 3) Slopes inclined from 1.5:1 (horizontal:vertical) to 3.5:1, i.e., approximately 16–34 degrees.
- 4) Various values of pore water pressures.

The charts by Leshchinsky and Reinschmidt (1985) are the least useful because they do not provide a means for determining the length of the reinforcement. Additionally, the charts are based on the assumption that the slope is founded on a material having the same strength as the slope. Thus, the charts may not be applicable to slopes on much stronger foundations. Neither the charts by Schmertmann et al. nor the charts by Jewell provide for slope angles of less than 30 degrees or mobilized friction angles of less than 20 degrees. Additionally, Schmertmann's charts do not address pore water pressures at all. The computer program ReSlope is versatile and can be utilized for virtually any slope angle and shear strength of interest. However, charts are preferred for use by Texas Department of Transportation's maintenance engineers. None of the available charts is applicable to the range of design characteristics needed, especially the flatter slopes and lower friction angles with significant pore water pressures.

## CHAPTER 4. NEW CHARTS FOR GEOSYNTHETIC-REINFORCED SLOPE REMEDIATION

### 4.1 INTRODUCTION

In order for design charts to be applicable to the Texas highway slopes of interest, the charts must cover the following conditions:

- 1) Effective stress friction angles ( $\bar{\phi}$ ) in the range of 20–25 degrees with minimal to no cohesion (with a factor of safety of 2.0, the mobilized friction angle would need to be as low as 10 degrees).
- 2) Foundations composed of both much stronger materials and materials with the same strength as the overlying slope.
- 3) Slopes inclined between 1.5:1 (horizontal: vertical) and 3.5:1, i.e., approximately 16–34 degrees.
- 4) Various values of pore water pressure.

None of the existing charts meets all these requirements. Therefore, a new set of charts was developed that can be used for the design of slopes like those of interest in Texas. Shear strengths were assumed to be expressed in terms of effective stress because experience shows the long-term, drained condition is most critical for the slopes of interest. Cohesion ( $\bar{c}$ ) was neglected because the high plasticity index clays of interest have been found to have minimal cohesion in the long-term condition. Charts were developed for slopes on foundations that are much stronger than the slope material and foundations that have the same soil properties as the overlying slope.

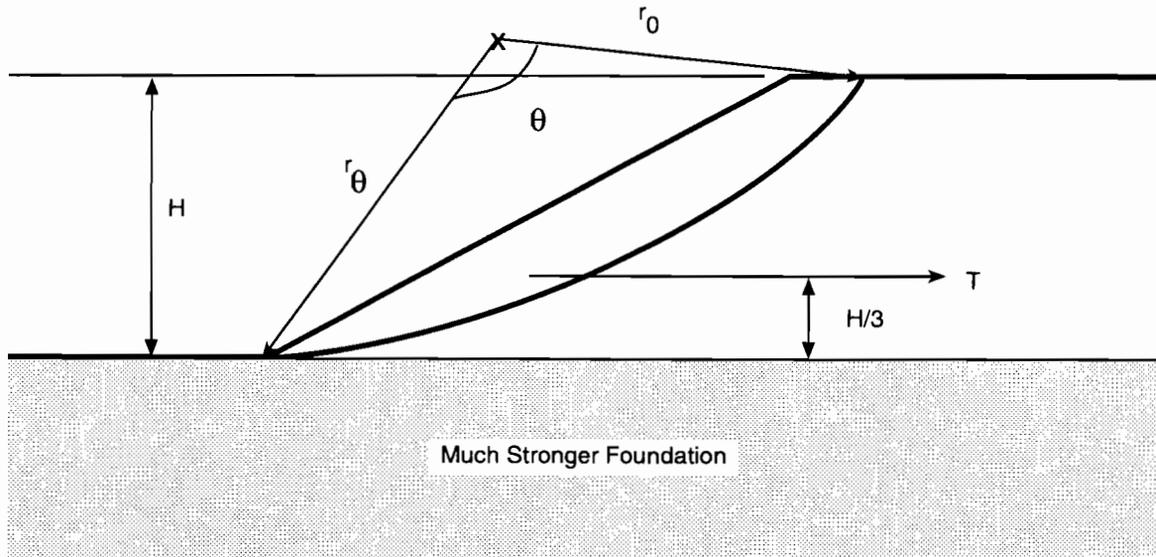
The form of the charts is similar to that of the existing charts. That is, the designer first determines the mobilized friction angle, based on the desired factor of safety on soil shear strength. The mobilized friction angle, the pore water pressure conditions and the slope angle are then used with the charts to obtain a value for the total force and minimum length of reinforcement required to produce the desired factor of safety.

### 4.2 TOTAL REQUIRED REINFORCEMENT FORCE AND REINFORCEMENT SPACING

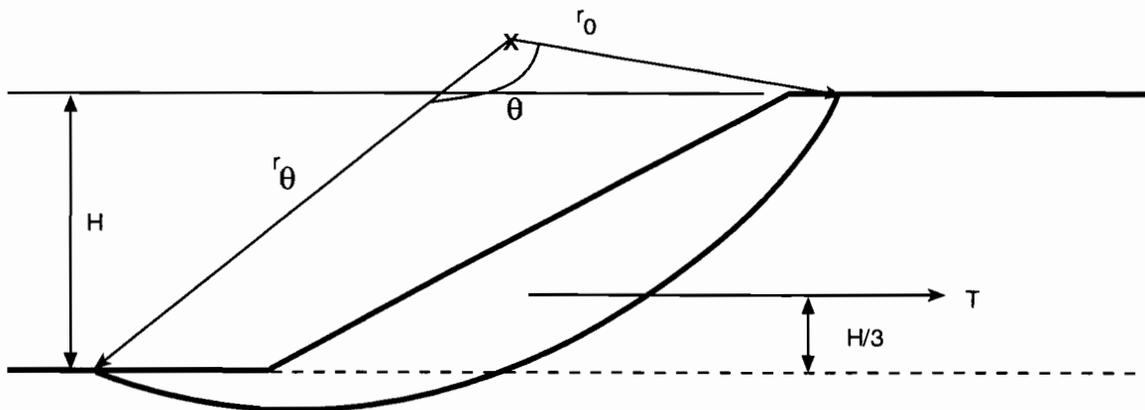
Charts for determining the required force were developed using a log spiral failure surface and limit equilibrium procedure. Figures 4.1 and 4.2 illustrate the models used to determine the force. The log spiral procedure was chosen because it satisfies complete static equilibrium. In this model, the force was assumed to act at the one-third height of the slope. Pore water pressures were modeled using the pore water pressure coefficient,  $r_u$ , defined as

$$r_u = \frac{u}{\gamma H} \quad (4.1)$$

where  $u$  is the pore water pressure at a depth  $z$  in the slope, and  $\gamma$  is the unit weight of the soil. Values of  $r_u$  of 0.0, 0.25, and 0.5 were used. The shear strength of the soil was expressed by an effective stress friction angle ( $\bar{\phi}$ ). Values of  $\bar{\phi}$  ranged from 10–40 degrees. Slope angles ( $\beta$ ) ranging from 10–80 degrees were used.



*Figure 4.1 Log spiral shear surface and parameters used to determine the required reinforcement force for slopes on much stronger foundations*



*Figure 4.2 Log spiral shear surface and parameters used to determine the required reinforcement force for slopes on foundations with the same strength*

#### 4.2.1 Calculation Procedure

The first step in searching for the required force was to choose a center and radius for a *trial* log spiral. The force required for static equilibrium of the log spiral was then calculated assuming the shear strength was fully developed along the shear surface (i.e., factor of safety = 1.0). The center of the log spiral was then systematically moved and the force required for equilibrium was again calculated. This process of trying various log spiral shear surfaces and calculating the required force was repeated until the spiral producing the maximum required force was found.

Very strong foundations were modeled by considering only log spirals that did not enter the foundation. For foundations having the same strength as the slope, the log spirals were allowed to enter the foundation.

#### 4.2.2 Chart Presentation

The required forces calculated for each slope and set of soil properties were expressed in the same dimensionless form as used by Jewell (1990) and Schmertmann et al. (1987). The force required for equilibrium ( $T_t$ ) was expressed by a reinforcement force coefficient  $K_{Req}$ . The value of  $K_{Req}$  was determined using the following equation:

$$K_{Req} = \frac{2T_t}{\gamma H^2} \quad (4.2)$$

Separate charts were plotted for slopes on both strong foundations and foundations that have the same properties as the slope for each value of  $r_u$  (0.0, 0.25, and 0.5). These charts are presented as Figures 4.3 through 4.8.

Because the reinforcement force is assumed to act at the bottom one-third point of the slope, the distribution of reinforcement forces over the height of the slope is assumed triangular. The vertical spacing of the reinforcement should be smallest near the toe of the slope, and largest near the crest. The following equation suggested by Schmertmann et al. (1987) can be used for estimating an initial vertical spacing of reinforcement layers:

$$s_v = \frac{T_a}{K\gamma z} \quad (4.3)$$

where  $z$  is the depth below the crest of the slope and  $T_a$  is the allowable strength of the reinforcement. Spacings will then need to be adjusted to conform to boundaries between lifts. The allowable strength ( $T_a$ ), as described in Chapter 3, can be calculated from Equation 3.4.

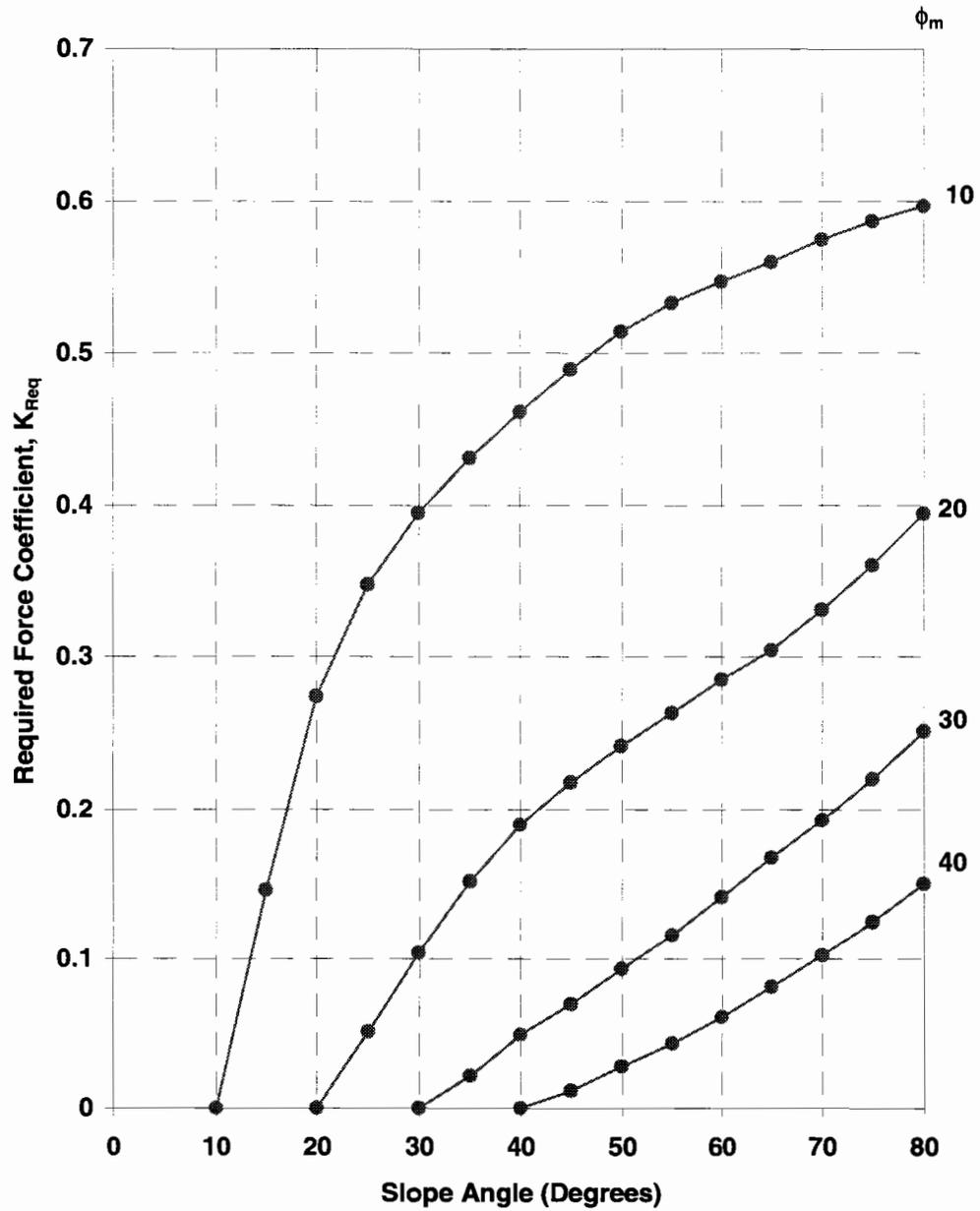


Figure 4.3 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on foundations with the same strength -  $r_u = 0$

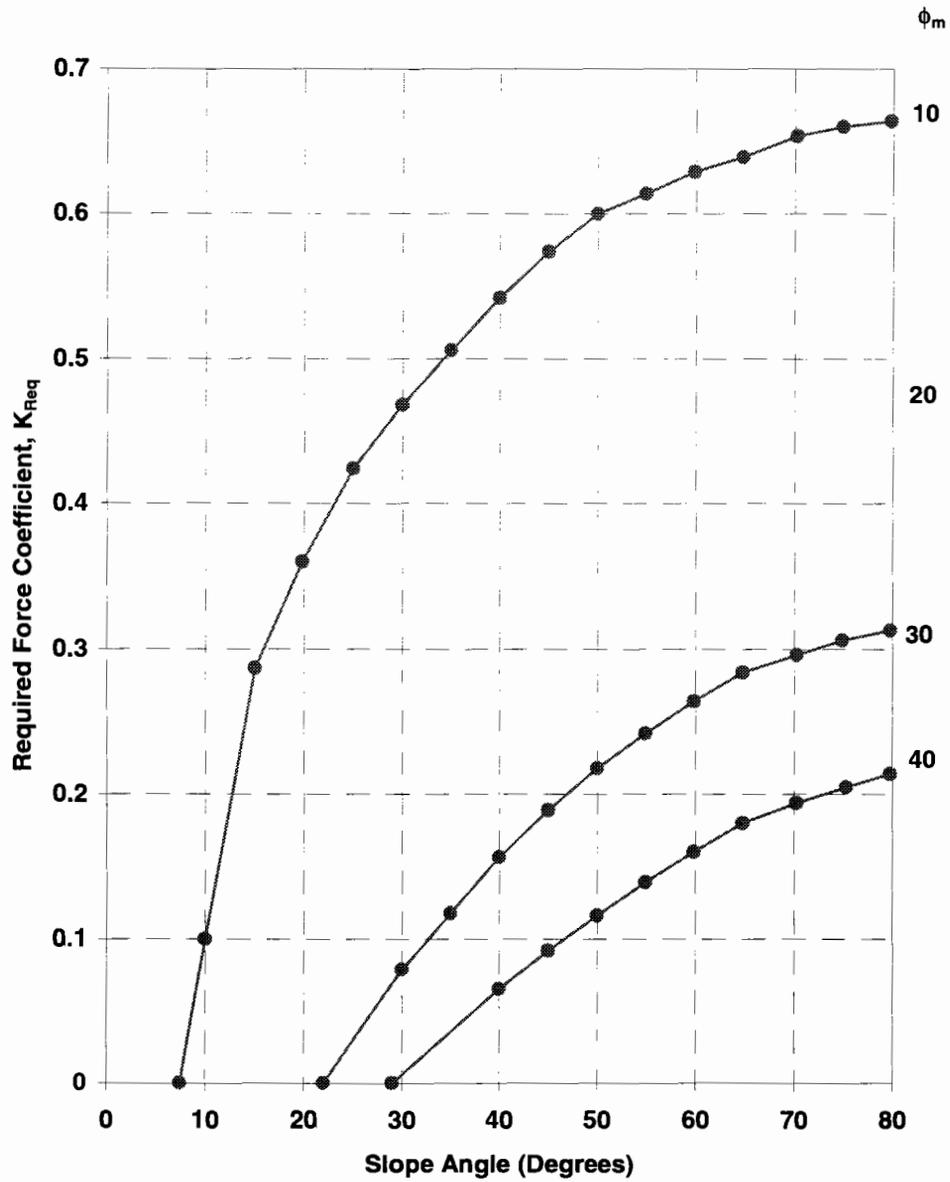


Figure 4.4 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on foundations with the same strength -  $r_u = 0.25$

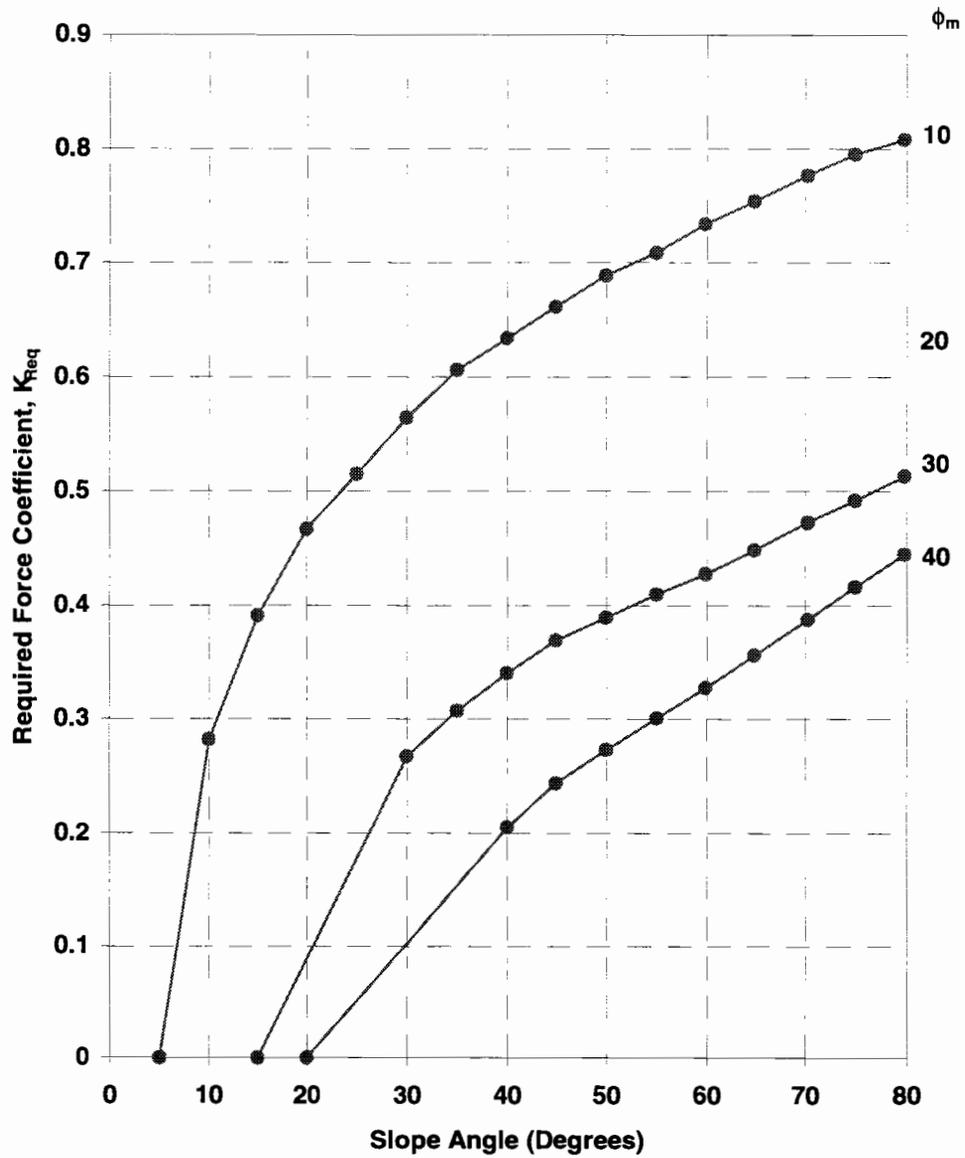


Figure 4.5 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on foundations with the same strength -  $r_u = 0.5$

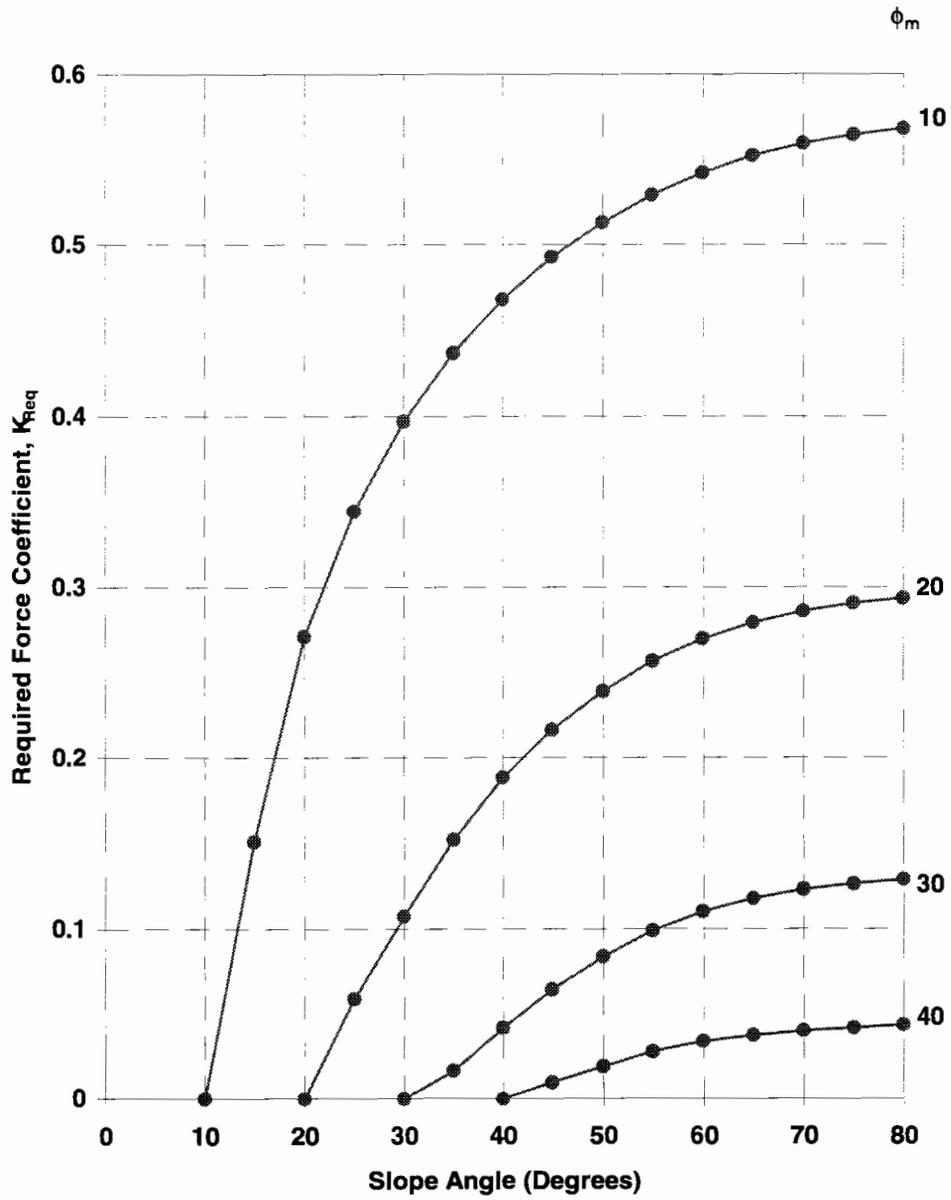


Figure 4.6 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on much stronger foundations -  $r_u = 0.0$

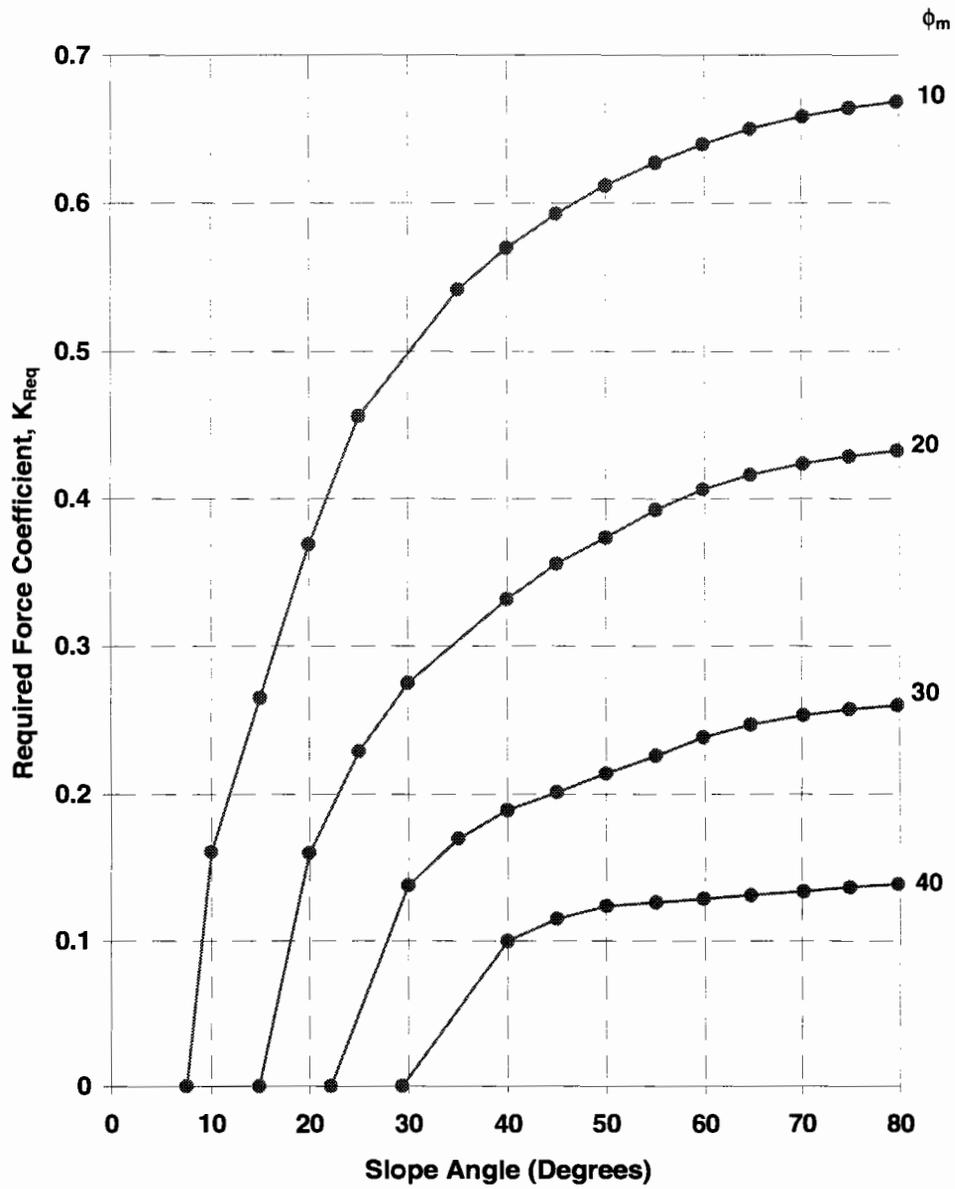


Figure 4.7 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on much stronger foundations -  $r_u = 0.25$

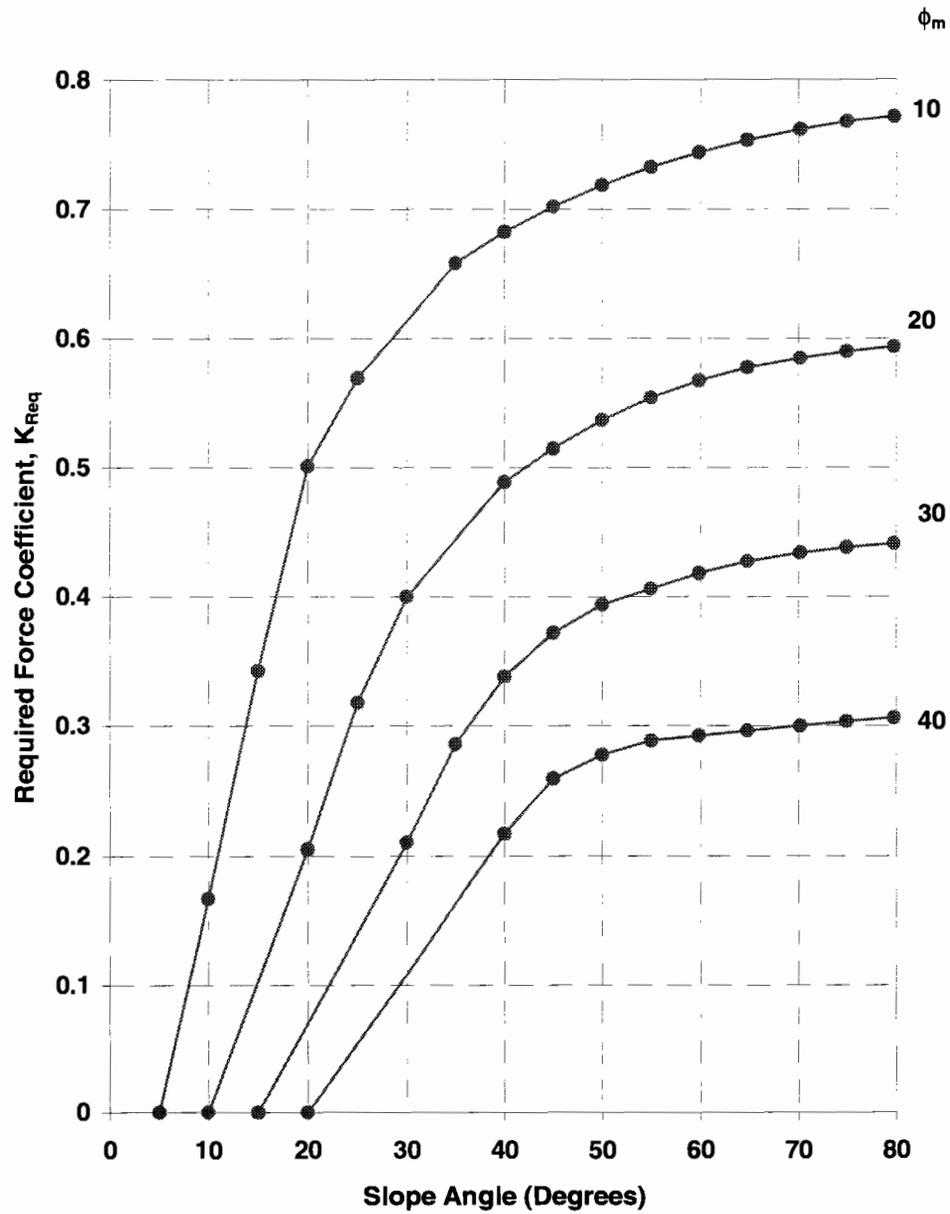


Figure 4.8 Chart for required reinforcement force coefficient ( $K_{Req}$ ) for slopes on much stronger foundations -  $r_u = 0.5$

### 4.3 LENGTH OF REINFORCEMENT

Additional charts were developed for determining the length of the reinforcement. Separate charts were developed for the length required to prevent global failure and to prevent direct sliding along the soil-reinforcement interface. The charts for global stability consider both conditions in which the foundation is much stronger than the slope and conditions in which the foundation and overlying slope have the same strength. The charts for direct sliding were created assuming the soil-reinforcement interaction coefficient,  $\mu$ , was 0.8. The interaction coefficient,  $\mu$ , as previously defined in Chapter 3, is used to determine the shear strength of the interface between the reinforced soil and the reinforcement. The shear strength at the soil-reinforcement interfaces is computed as  $\bar{\sigma}(\mu \tan \bar{\phi})$ . The same range of slope angles and friction angles used in the analyses to calculate required forces was used to compute required lengths. Separate charts were again developed for pore water pressure coefficients ( $r_u$ ) of 0.0, 0.25, and 0.5.

#### 4.3.1 Length for Global Stability

The required length for *global stability* was also calculated using log spirals. For very strong foundations, the log spirals were restricted from entering the foundation. For foundations with the same properties as the slope, the log spirals were allowed to pass into the foundation. The shear strength of the soil was assumed to be fully mobilized (factor of safety = 1.0). A search was conducted to find the required length. The search was controlled by first assuming a *control* point. Rays were drawn from the control point toward the slope at various angles to *sweep* the probable area of the center points of log spirals of interest (Figure 4.9). Center points of log spirals were located along each ray. The factor of safety was assumed to be 1.0, and the force required was calculated for each center, starting at the control point and moving outward along the ray. The search along a given ray was halted when a log spiral requiring no additional force (to obtain a factor of safety of 1.0) was found. The spiral at this point defined a region where no reinforcement was required. The search then proceeded to the next ray, and the process of calculating forces for log spirals with center points along the ray was repeated until a log spiral requiring no reinforcement was found. For each of the log spirals that required no reinforcement, the horizontal distance ( $L_{gs}$ ) into the slope was calculated at each of ten equally spaced elevations, as shown in Figure 4.10. The length used in creating the charts was the maximum length found at any level.

The required lengths were normalized by dividing them by the slope height,  $H$ , to create the nondimensional ratio  $L_{gs}/H$ . Values of  $L_{gs}/H$  were then plotted versus slope angle for various friction angles. These charts are presented as Figures 4.11–4.16.

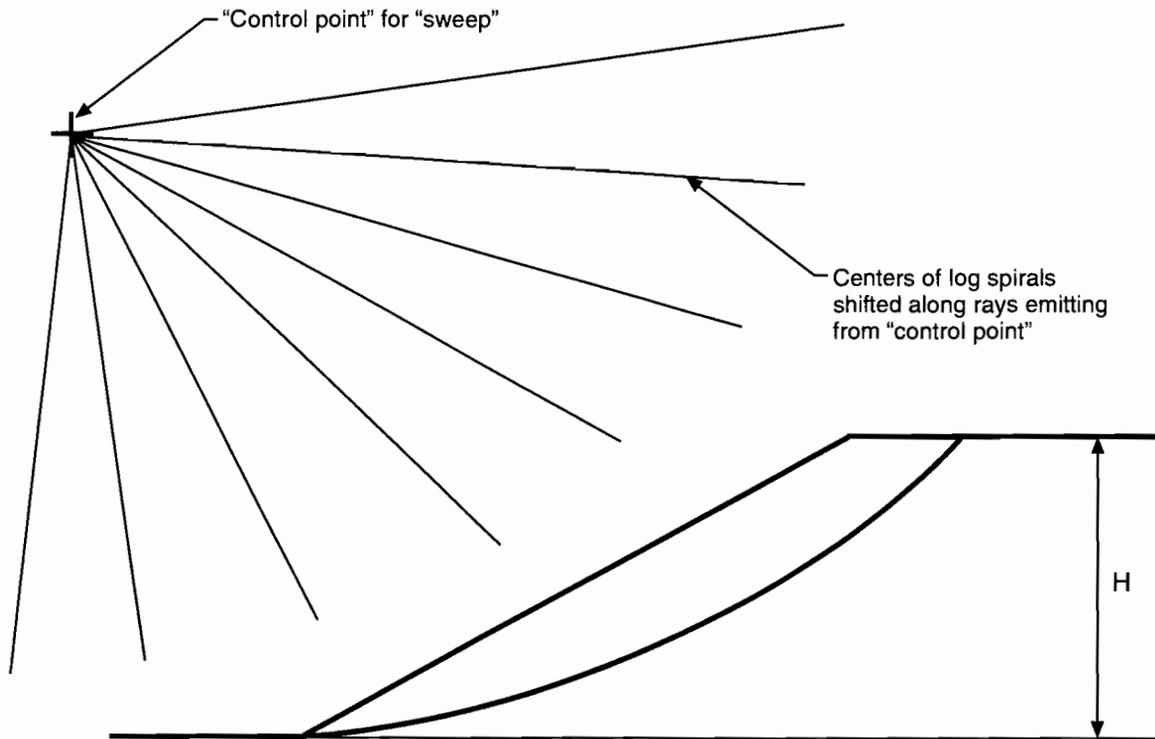


Figure 4.9 Search for critical log spiral in global stability analysis

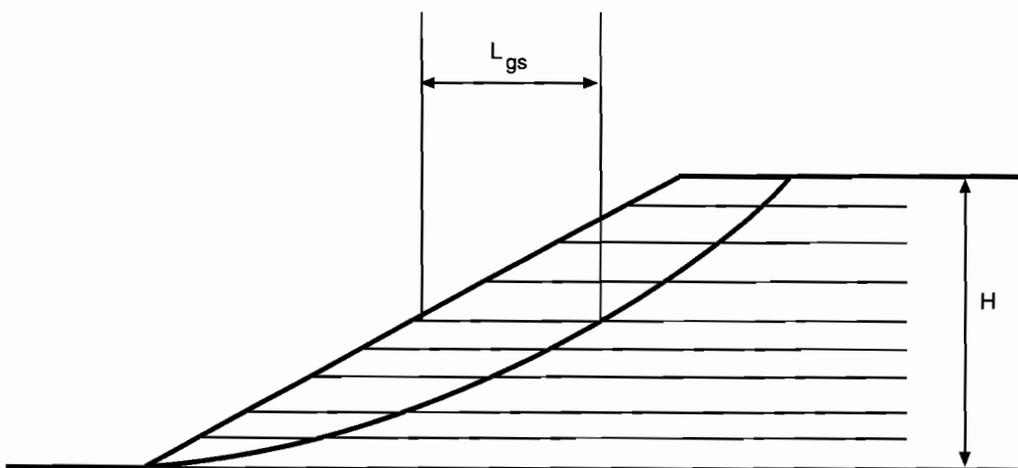


Figure 4.10 Scheme used determine required reinforcement length for global stability with log spiral shear surfaces

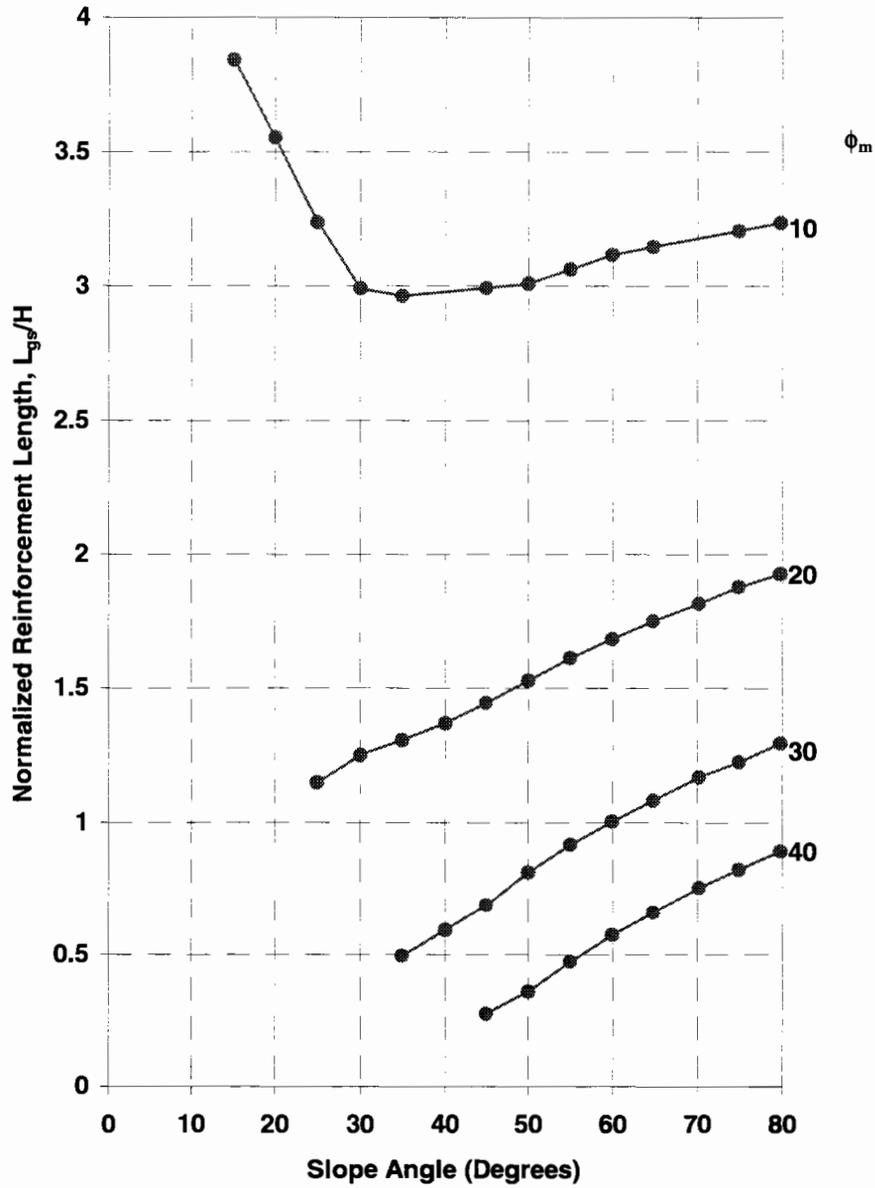


Figure 4.11 Minimum length of reinforcement for global stability of slopes on foundations with the same strength –  $r_u = 0.0$

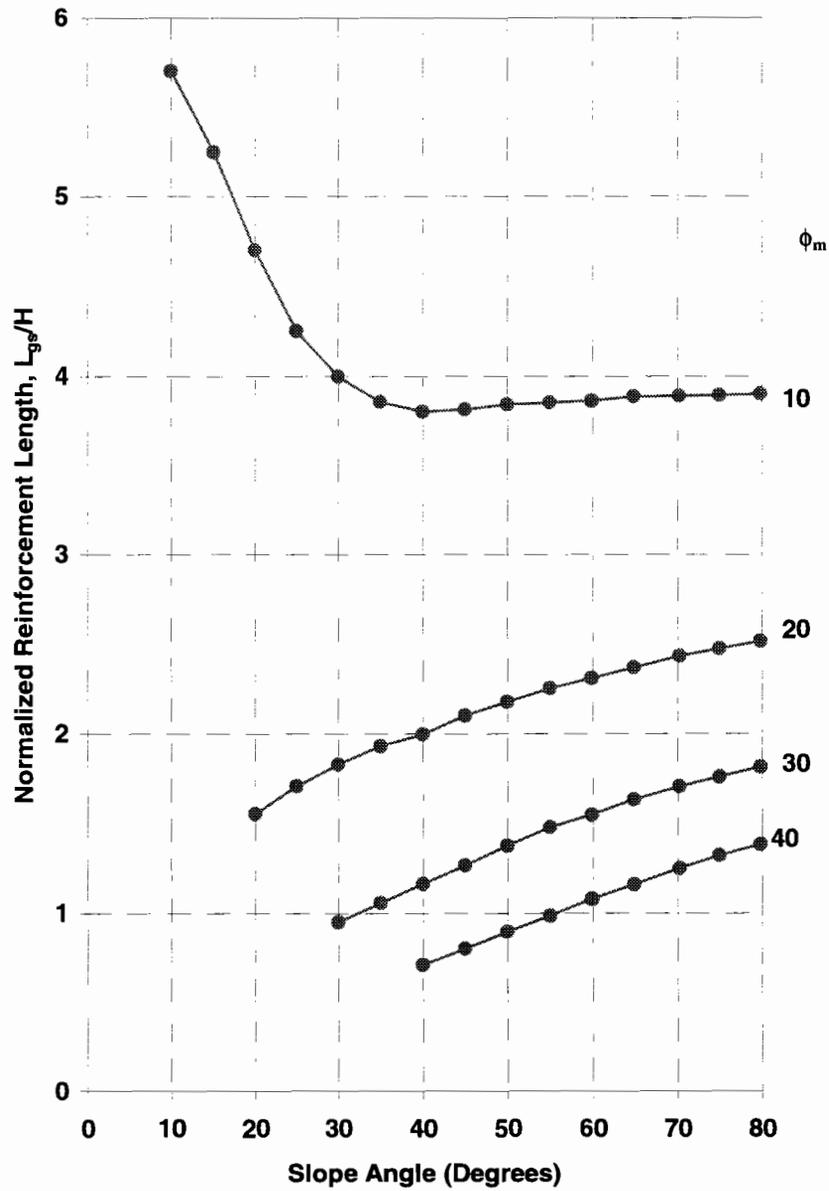


Figure 4.12 Minimum length of reinforcement for global stability of slopes on foundations with the same strength –  $r_u = 0.25$

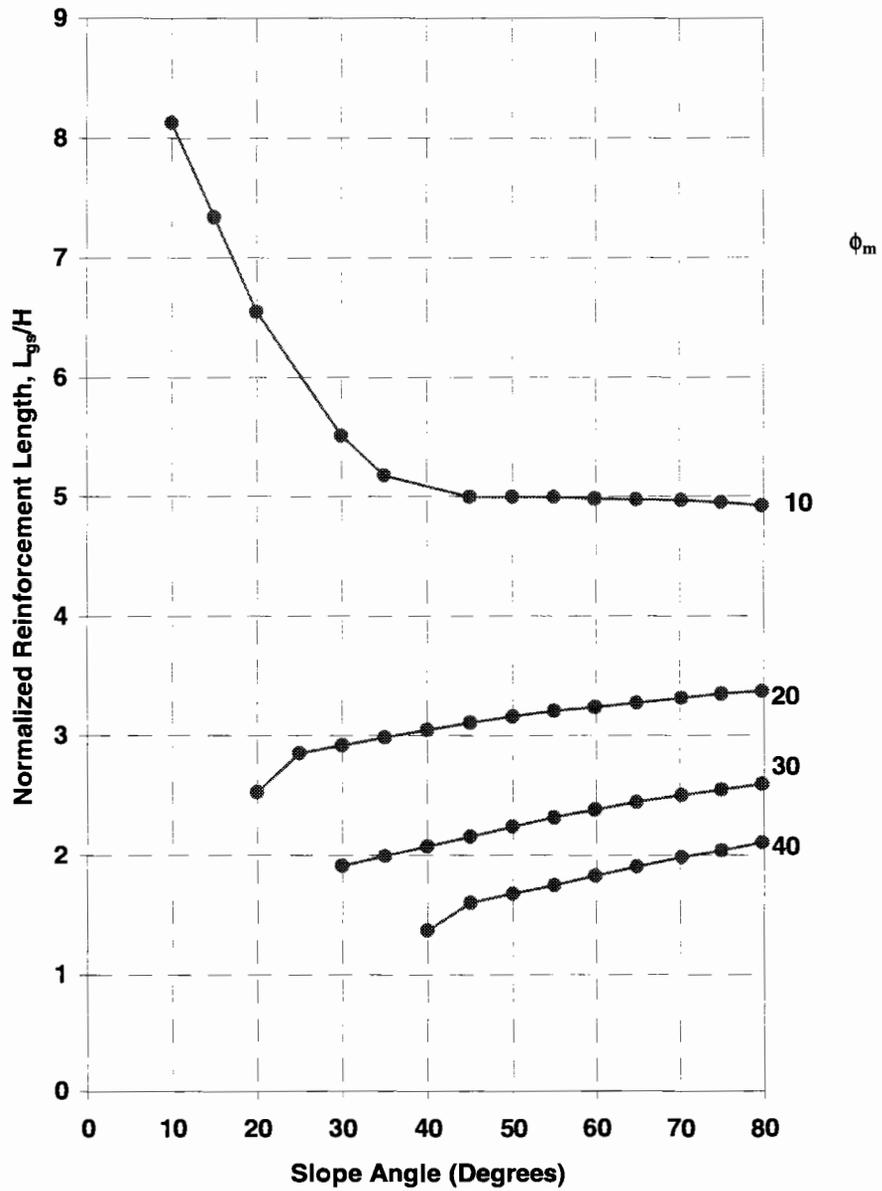


Figure 4.13 Minimum length of reinforcement for global stability of slopes on foundations with the same strength –  $r_u = 0.5$

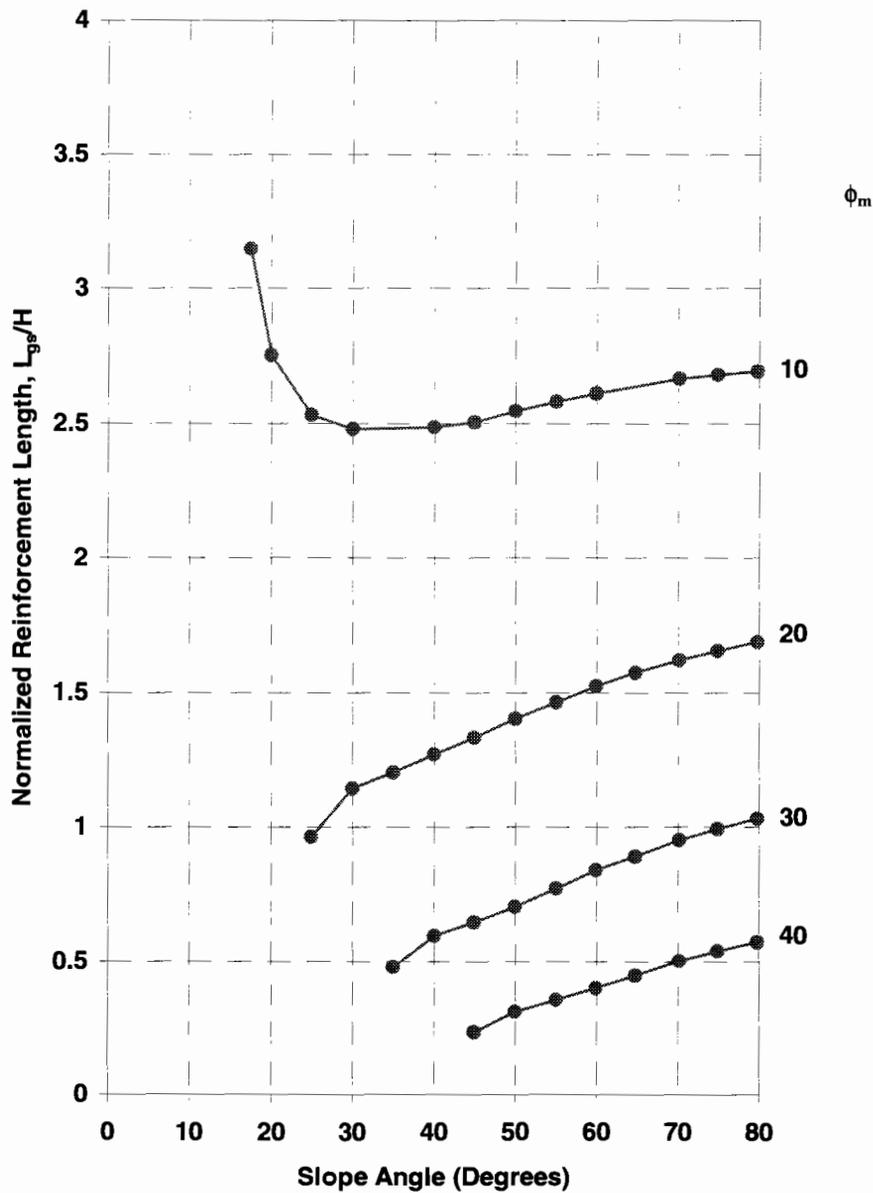


Figure 4.14 Minimum length of reinforcement for global stability of slopes on much stronger foundations –  $r_u = 0.0$

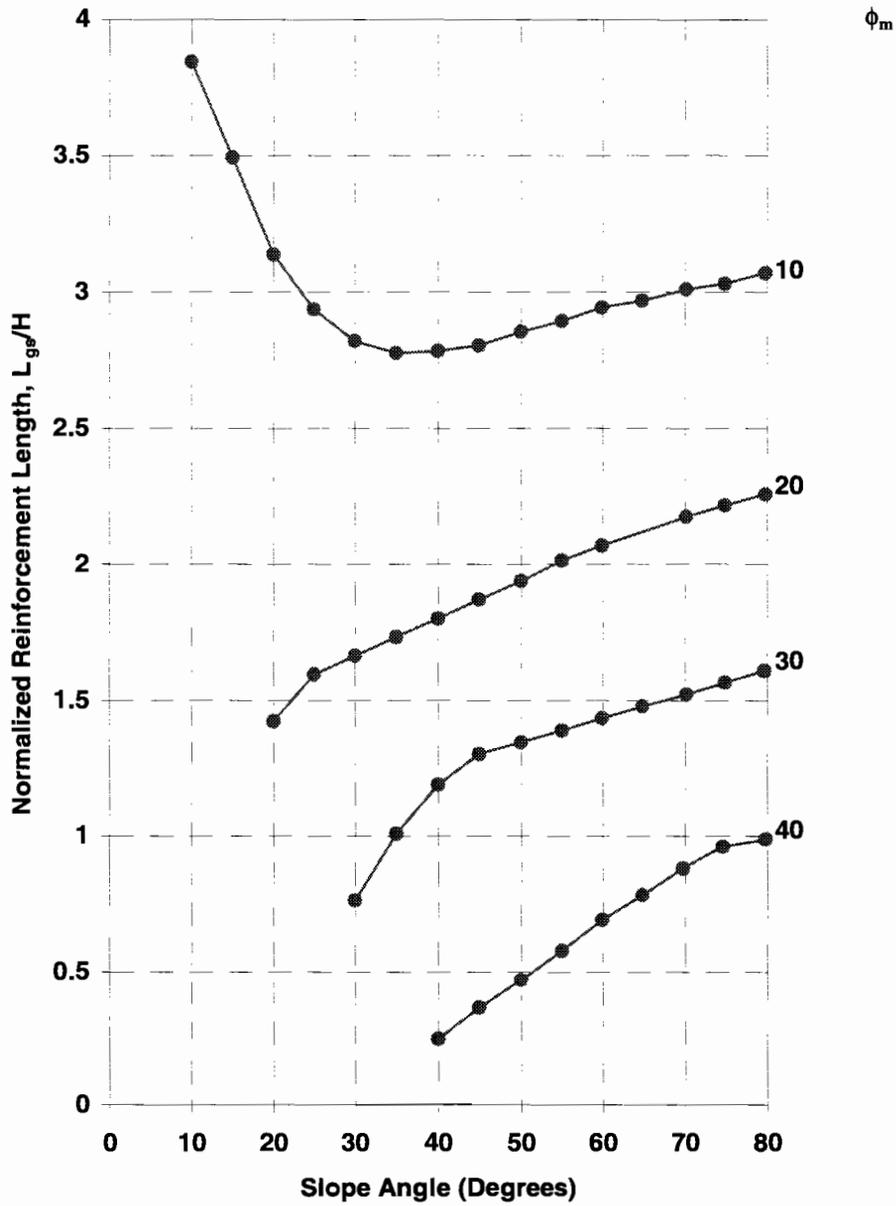


Figure 4.15 Minimum length of reinforcement for global stability of slopes on much stronger foundations –  $r_u = 0.25$

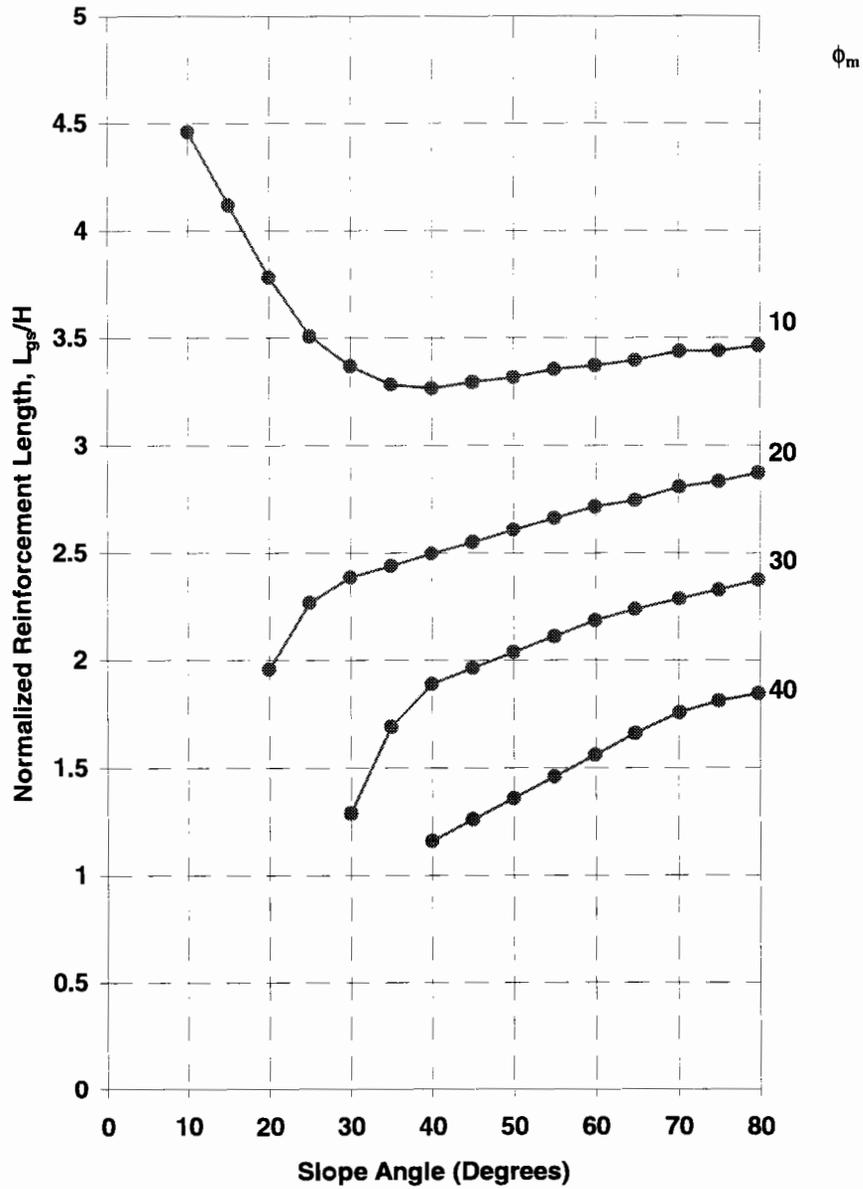


Figure 4.16 Minimum length of reinforcement for global stability of slopes on much stronger foundations  $-r_u = 0.5$

### 4.3.2 Length for Direct Sliding

Reinforcement lengths required to prevent direct sliding along a reinforcement layer were calculated using the bilinear failure surface illustrated in Figure 4.17. Spencer's limit equilibrium procedure of slices was used to calculate the factor of safety. For each combination of slope angle and friction angle, a horizontal length of sliding along the reinforcement was assumed. For each length,  $L_{ds}$ , the inclination,  $\theta$ , of the *active* wedge was varied to find the minimum factor of safety. The length was then changed and the angle  $\theta$  was varied again until the minimum factor of safety for the new length was found. This process was repeated until the maximum length was located that yielded a factor of safety of unity. The strength of the foundation was ignored for these calculations because the failure surface was assumed to be confined to the soil and reinforcement in the slope.

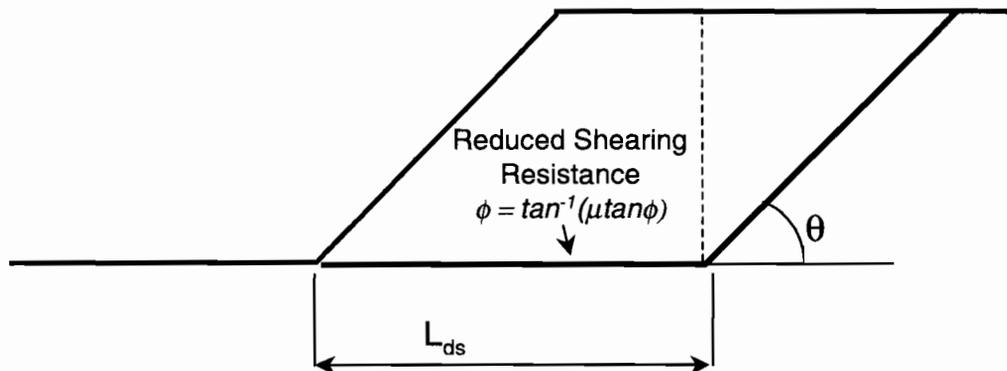


Figure 4.17 Wedge model used with Spencer's procedure to determine minimum length for direct sliding

Lengths were normalized by dividing them by the slope height and expressed as the dimensionless ratio  $L_{ds}/H$ . Values of  $L_{ds}/H$  were then plotted against slope angles for various friction angles. The charts for direct sliding are presented as Figures 4.18–4.20.

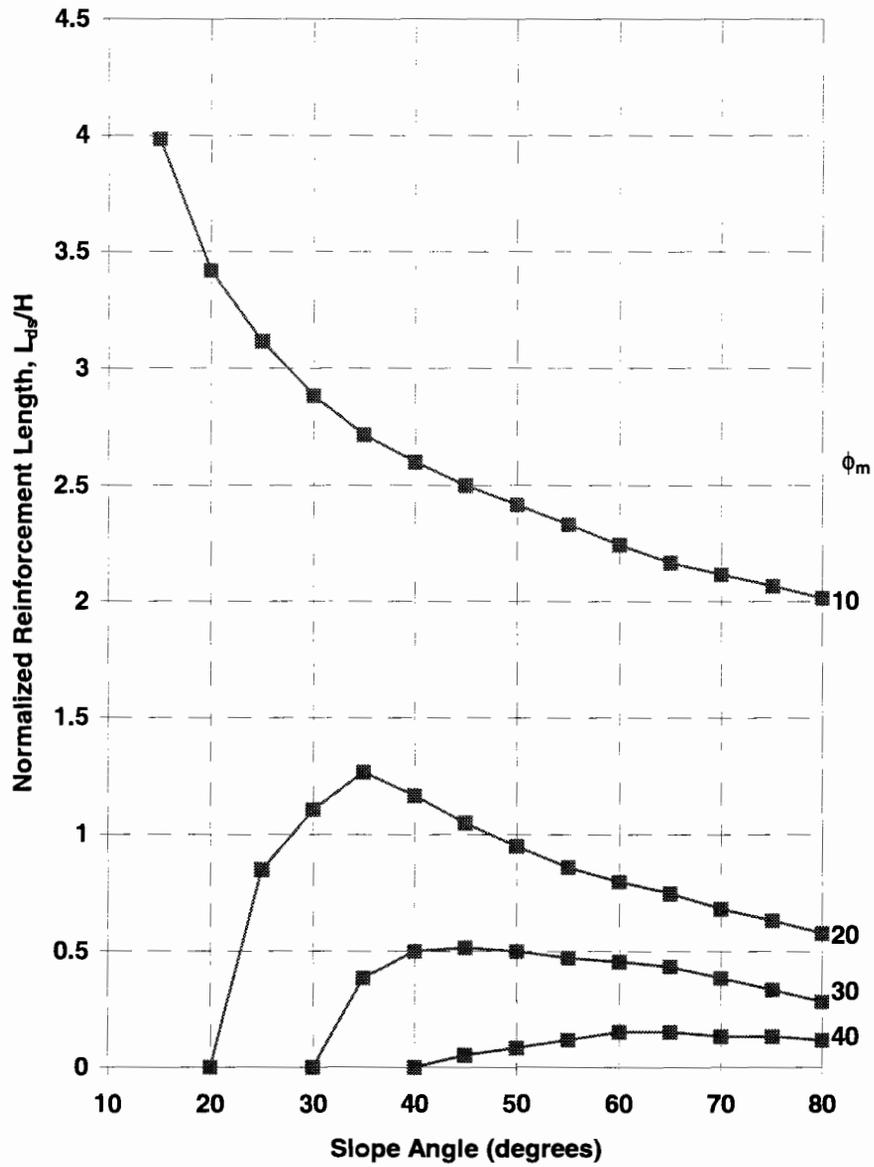


Figure 4.18 Minimum length of reinforcement for direct sliding –  $r_u = 0.0$

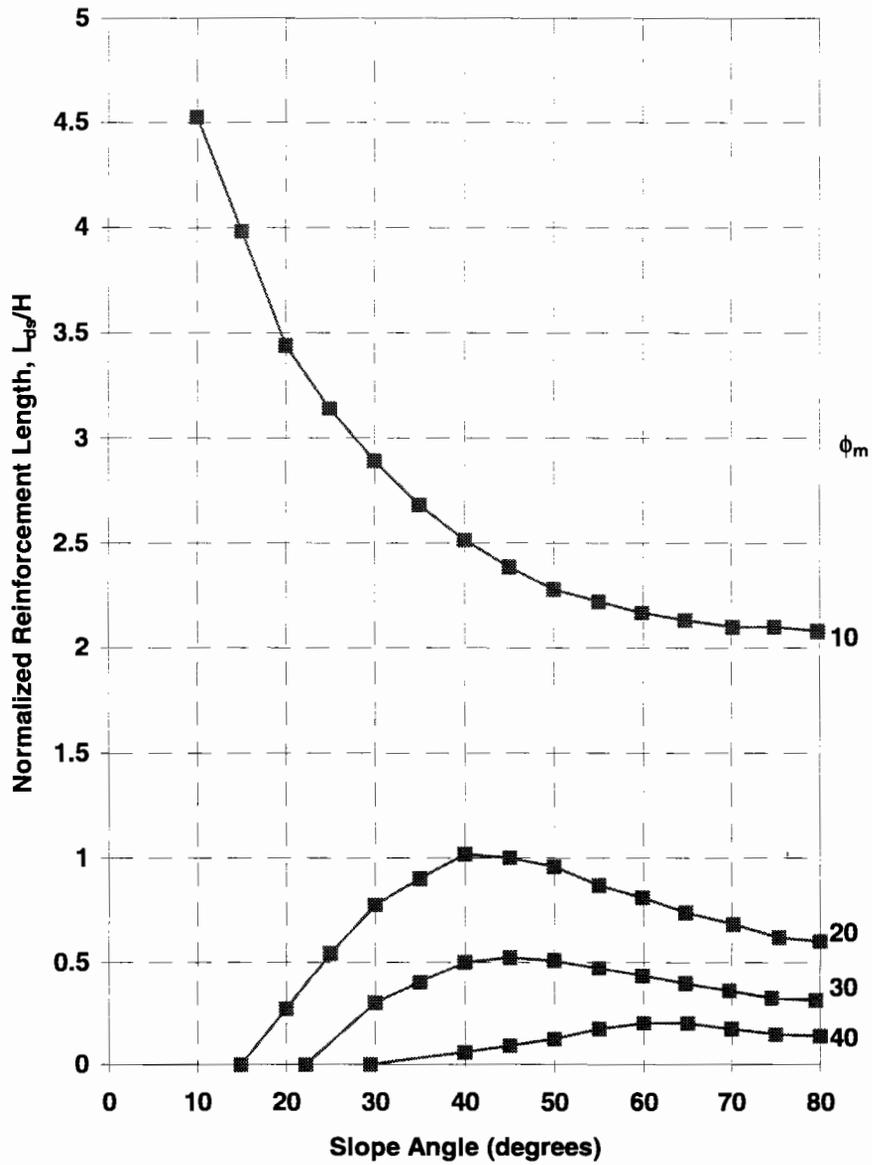


Figure 4.19 Minimum length of reinforcement for direct sliding –  $r_u = 0.25$

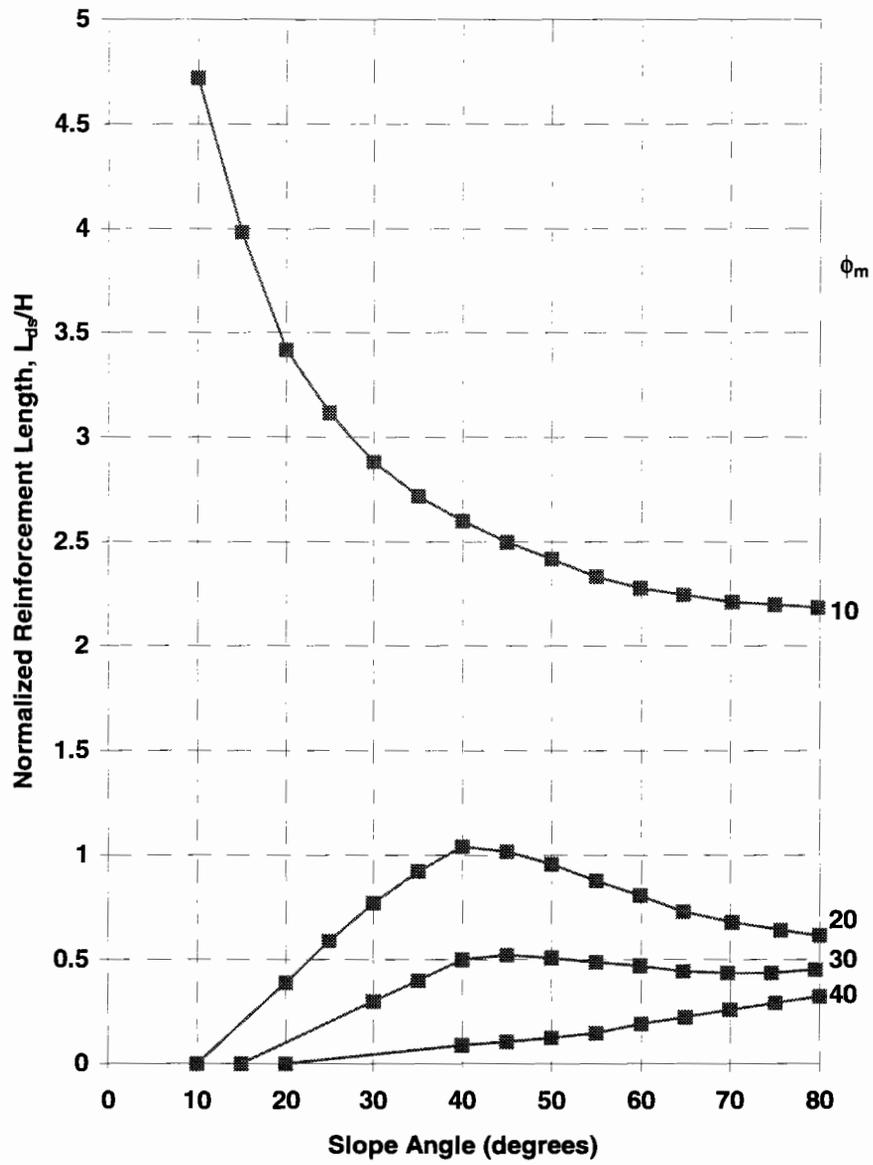


Figure 4.20 Minimum length of reinforcement for direct sliding –  $r_u = 0.5$

### 4.3.3 Determination of Reinforcement Length

The required lengths are obtained by multiplying the slope height by the maximum length ratio ( $L/H$ ) from the charts for global stability and direct sliding. The charts are used to determine the appropriate values of  $L_{gs}/H$  and  $L_{ds}/H$ . The larger of the two values is multiplied by the slope height,  $H$ , to determine the length of reinforcement required. For some cases, global stability will govern the required length; in other cases, direct sliding will govern.

## 4.4 USE OF CHARTS FOR FACTORS OF SAFETY GREATER THAN 1.0

The charts presented in Figures 4.3–4.19 were created assuming that the friction angle,  $\bar{\phi}$ , is fully developed, i.e., the factor of safety is unity. To use the charts for a factor of safety greater than unity, the mobilized friction angle,  $\bar{\phi}_m$ , should be used. The mobilized friction angle is calculated using the following equation:

$$\bar{\phi}_m = \tan^{-1} \left( \tan \frac{\bar{\phi}}{F} \right) \quad (4.4)$$

where  $F$  is the factor of safety on shear strength.

## 4.5 EFFECT OF SURCHARGE

All of the charts presented in this chapter were developed assuming no surcharge (external loads) on the slope. However, Schmertmann et al. (1987) suggested that a uniform surcharge,  $q$ , along the crest of the slope could be accounted for by using a modified height,  $H'$ . The value of  $H'$  is determined from the following equation:

$$H' = H + \frac{q}{\gamma} \quad (4.5)$$

To determine the validity of such an approach, an additional series of calculations was performed. Two example slopes with surcharges of 240 psf (in Figure 4.21) were selected. The charts were then used with a modified height to determine the reinforcement layout. The modified height,  $H'$ , for both slopes is twenty feet.

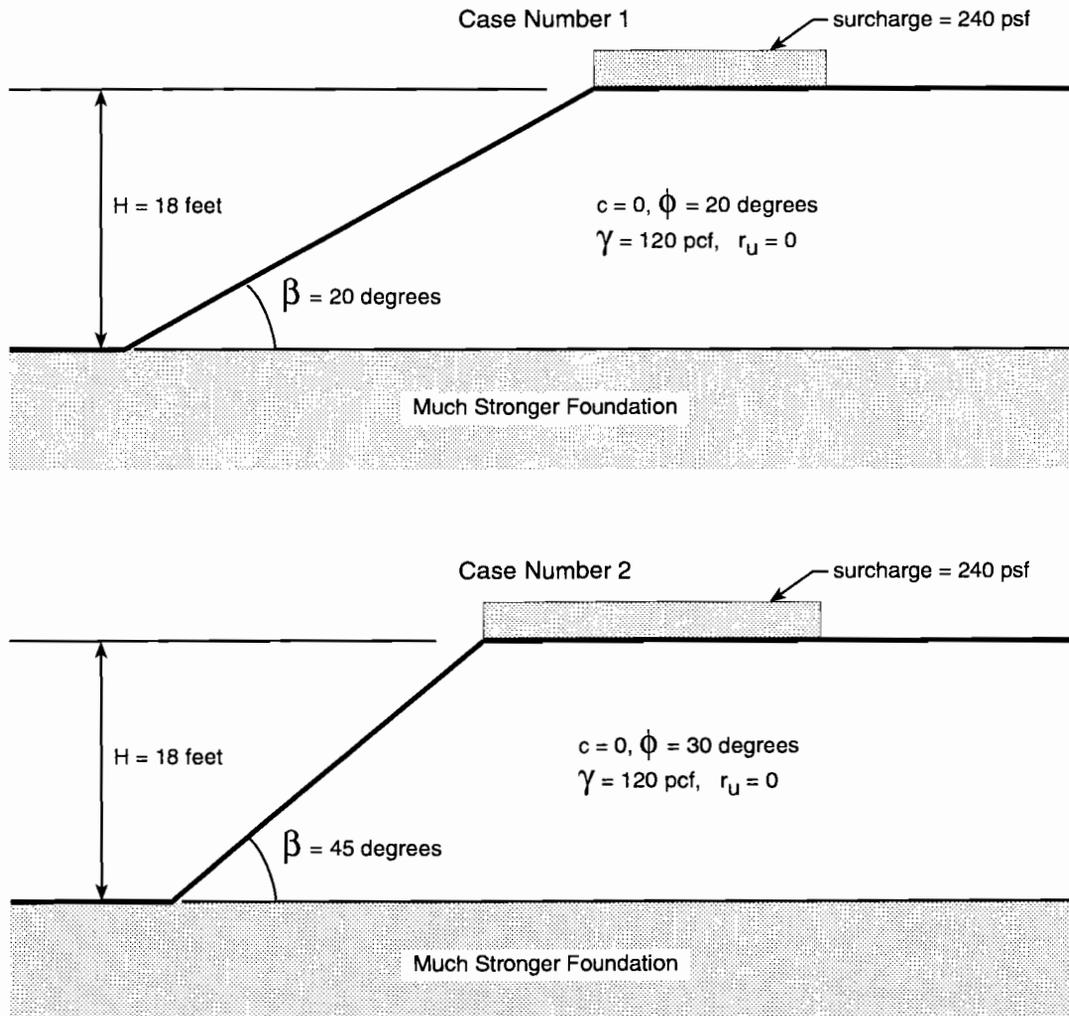


Figure 4.21 Examples used in surcharge study

A design factor of safety of 1.5 was used for both examples. The allowable strength of the reinforcement used in the design was 1,525 pounds per foot; this strength corresponds with the reported allowable strength for Geogrid type UX1400HT, as reported in Tensar's *Sierra Slope Retention System Design Manual* (1994). The strength accounts for the various partial reduction factors for construction damage, biological degradation, creep, and joints that are typically used in determining a geosynthetic material's long-term strength. The values for the design variables (reinforcement coefficient, length for direct sliding, and length for global stability) were obtained from the charts presented earlier in this chapter. The required force and lengths determined in this manner for the example slopes are summarized in Table 4.1.

*Table 4.1 Design quantities used to study effects of surcharge (shown in Figure 4.20)*

Case	Total Force (lb/ft)	$L_{ds}$ (feet)	$L_{gs}$ (feet)
1	4,080	44.1	40
2	4,800	20	25

Once the required force and lengths were determined, the vertical spacing was determined using Equation 4.3. The height above the toe and length of each layer of reinforcement for each example are shown in Tables 4.2 and 4.3. Once the vertical spacing of the reinforcement was determined, the factors of safety for direct sliding and global failures were calculated using UTEXAS3. The factor of safety for global stability was calculated using circular failure surfaces; the factor of safety for direct sliding was calculated using noncircular failure surfaces. Spencer's procedure was used for all the calculations. For the analyses with UTEXAS3, the reinforcement was modeled as horizontal layers with tensile forces. The tensile forces were applied by UTEXAS3 at the intersection of the failure surface and each layer of reinforcement. Any flexural strength of reinforcement was ignored. Two different slope configurations were used for each example slope: an 18-foot high slope with a 240 psf surcharge and a 20-foot high *equivalent* slope with no surcharge. The 20-foot high slope was assumed to have the same reinforcement layout as the 18-foot high slope (number, length, and vertical location of reinforcement layers). The factors of safety calculated using UTEXAS3 are presented in Table 4.4. For both examples, the factor of safety calculated for the actual 18-foot slope and the equivalent 20-foot slope are essentially the same (maximum difference = 2.4 percent). This indicates that a modified height,  $H'$ , can be used to model slopes with moderate levels of uniform surcharge loads.

*Table 4.2 Height above foundation and lengths of reinforcement layers for surcharge example slope Number 1*

Layer	Height (feet)	Length (feet)
1	0	44.1
2	3	44.1
3	7	44.1
4	12	44.1

*Table 4.3 Height above foundation and lengths of reinforcement layers for surcharge example slope Number 2*

Layer	Height (feet)	Length (feet)
1	0	25
2	3	25
3	7	25
4	12	25

*Table 4.4 Factors of safety for example slopes used in surcharge study*

Case	Factor of Safety Global	Factor of Safety Direct Sliding
Example 1 H = 28 feet q = 240 psf	1.69	<u>1.53</u>
Example 1 H = 30 feet Q = 0 psf	1.65	<u>1.54</u>
Example 2 H = 28 feet q = 240 psf	<u>1.58</u>	1.64
Example 2 H = 30 feet Q = 0 psf	<u>1.57</u>	1.64

#### 4.6 COMPARISON OF RESULTS WITH EXISTING METHODS

To verify the design charts presented in this chapter, designs were performed using the new charts, and the factors of safety for direct sliding and global stability were calculated. The same two example slopes considered in Chapter 3 (shown in Figure 4.22) were used in this comparison. Both slopes are simple slopes with zero pore water pressures. Both slopes are 30-feet high and have unit weights of 120 pounds per cubic foot. The coefficient of interaction ( $\mu$ ) for direct sliding of the interface of the soil and reinforcement is assumed to be 0.8. A factor of safety of 1.5 is used for both overall stability and direct sliding. In order to make the designs comparable, in the ReSlope design the other factors of safety (geosynthetic uncertainty and pullout) are assumed to be 1.0. The allowable strength of the reinforcement used in the design is 1525 pounds per foot; this strength corresponds with the reported allowable strength for Geogrid type UX1400HT, as reported in Tensar's *Sierra Slope Retention System Design Manual* (1994). The strength accounts for all the partial reduction

factors typically used in determining a geosynthetic material's long-term strength. The pore water pressures were assumed to be zero.

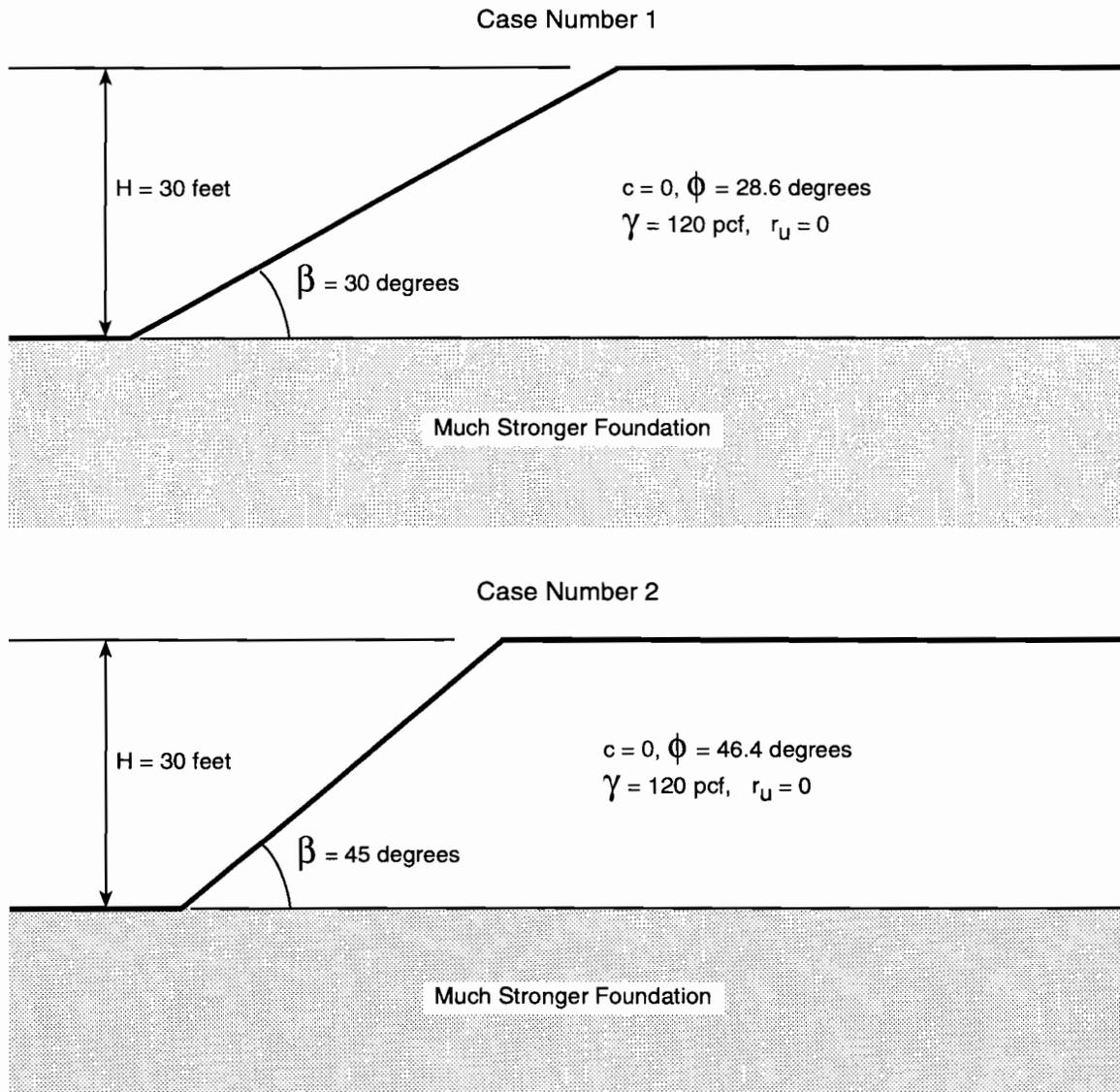


Figure 4.22 Examples used in comparison of new charts with existing methods

The design requirements for these two example slopes were determined using the charts presented in this chapter as well as the charts by Jewell (1990), Schmertmann et al. (1985), and the computer software ReSlope. The design requirements using the charts presented in this chapter are summarized in Tables 4.5 and 4.6<sup>1</sup>. The vertical spacing was

<sup>1</sup> Tables presenting the design requirements for the other methods are presented in Chapter 3.

determined using Equation 4.3. The height of each reinforcement layer above the foundation and the length of each layer are shown in Tables 4.7 and 4.8.

*Table 4.5 Design quantities for reinforced slope example 1 using various methods (allowable reinforcement strength = 1525 lb/ft)*

Method	Total Force (lb/ft)	L <sub>B</sub> (feet)	L <sub>T</sub> (feet)
New Charts	5792	34.3	34.3
Jewell (1990)	5670	43.5	43.5
Schmertmann et al. (1987)	6750	38.4	16.4
ReSlope	6653	55.6	35.1

*Table 4.6 Design quantities for reinforced slope example 2 using various methods (allowable reinforcement strength = 1525 lb/ft)*

Method	Total Force (lb/ft)	L <sub>B</sub> (feet)	L <sub>T</sub> (feet)
New Charts	2160	12.75	12.75
Jewell (1990)	2700	11.25	11.25
Schmertmann et al. (1987)	2160	6.3	6.3
ReSlope	2261	22.2	12.6

*Table 4.7 Height above foundation and lengths of reinforcement layers for example slope Number 1*

Layer	Height (feet)	Length (feet)
1	0	34.3
2	4	34.3
3	8	34.3
4	13	34.3
5	20	34.3

*Table 4.8 Height above foundation and lengths of reinforcement layers for example slope Number 2*

Layer	Height (feet)	Length (feet)
1	0	12.75
2	10	12.75
3	25	12.75

Once the reinforcement layout was established, the factor of safety for global stability was calculated for each slope and reinforcement layout using the computer program UTEXAS3 with circular failure surfaces and Spencer's procedure. The factor of safety for direct sliding of each slope was calculated using UTEXAS3 with noncircular failure surfaces and Spencer's procedure. For the analyses with UTEXAS3, the reinforcement was modeled as horizontal tensile forces applied at the intersection of the failure surface with each layer of reinforcement. Any flexural strength of the reinforcement was ignored. The factors of safety calculated using UTEXAS3 are presented in Tables 4.9 and 4.10. The minimum factor of safety from the two failure modes, *global* versus *direct sliding*, is underlined.

*Table 4.9 Factors of safety calculated for example slope Number 1*

Case	Factor of Safety Global <sup>(1)</sup>	Factor of Safety Direct Sliding <sup>(2)</sup>
Schmertmann et al.	1.64	<u>1.44</u>
Jewell	<u>1.61</u>	1.86
ReSlope	<u>1.43</u>	1.95
New Charts - This Report	1.51	<u>1.50</u>

(1) Based on Spencer's procedure and circular shear surfaces

(2) Based on Spencer's procedure and bilinear shear surfaces ( $\mu = 0.8$ )

*Table 4.10 Factors of safety calculated for example slope Number 2*

Case	Factor of Safety Global <sup>(1)</sup>	Factor of Safety Direct Sliding <sup>(2)</sup>
Schmertmann et al.	1.46	<u>1.44</u>
Jewell	1.71	<u>1.62</u>
ReSlope	<u>1.71</u>	2.18
New Charts - This Report	<u>1.66</u>	1.71

(1) Based on Spencer's procedure and circular shear surfaces

(2) Based on Spencer's procedure and bilinear shear surfaces ( $\mu = 0.8$ )

The factors of safety for the two cases designed with the new charts presented in this chapter are all at least equal to the design value of 1.5 and are never more than 10 percent greater. The factors of safety for the design based on the new charts are as close to the specified value of 1.5 as any of the other methods.

#### 4.7 SUMMARY AND CONCLUSIONS

New charts have been presented for the design of reinforced slopes. These new charts are the only charts that cover slopes on both foundations with the same strength as the overlying slope and foundations that are much stronger. The current charts that allow for complete reinforcement design (required force and length of reinforcement) allow only for

the design of slopes with very strong foundations. The new charts also cover slope angles as flat as 10 degrees and mobilized friction angles as low as 10 degrees. Current charts allow only for slope angles and friction angles as low as 20 degrees. The charts also cover pore water pressures given by pore water pressure coefficients ( $r_u$ ) of 0, 0.25, and 0.5. The equilibrium models used to create the new charts are fundamentally more precise than those used by Jewell (1990) and Schmertmann et al. (1987). The slope stability analysis methods used satisfy complete static equilibrium, while those used by Jewell and Schmertmann et al. do not.

The values of the factors of safety from the various methods examined in Chapters 3 and 4 (as shown in Tables 4.9 and 4.10) are all close. Thus, the approximations employed in earlier methods appear reasonable. The primary advantage of the new charts is the fact that they extend the range of conditions that can be considered. They also provide a simple means for designing that avoids the need for running such complex software as ReSlope or UTEXAS3.

The charts presented in this chapter provide only the design variables for the primary reinforcement design, and they assume the full development of the reinforcement forces. Additional length should be added to each reinforcement layer to provide resistance for pullout forces acting on the layer. The pullout length can be determined as described earlier in the discussion of ReSlope in Chapter 3. Additional reinforcement layers will be necessary to provide stability of the slope near the face. The *secondary* reinforcement requirements can be determined by using the method outlined in the discussion of ReSlope in Chapter 3 or the method presented by Collin (1996).



## CHAPTER 5. SUMMARY

Two new series of charts have been developed for use in the design of remedial measures for failed slopes. The first series of charts is for use in the design of *strengthened* soil slopes, where soil is mixed with additives such as lime and cement. The second series of charts pertains to slopes that are stabilized using geosynthetic (geogrids, geotextiles) reinforcement.

Emphasis has been placed on remediation of shallow slides in slopes consisting of highly plastic clays. The long-term, *drained* shear strengths of these clays are characterized by effective stress shear strength envelopes with no cohesion intercept ( $\bar{C} = 0$ ). The charts can be used in cases where either the foundation is much stronger than the overlying slope (such that failure surfaces cannot pass into the foundation) or the foundation has the same strength as the overlying slope and the failure surface may pass into the foundation.

### 5.1 CHARTS FOR STRENGTHENED SOIL

The first series of charts applies to slopes where a portion of soil near the face of the slope is strengthened with cementing additives, typically lime or cement. The charts are based on the assumption that the original slope has failed and the soil strength is backcalculated assuming a factor of safety of unity. Using the appropriate backcalculated shear strength, charts were then developed for various slope angles, widths, and strengths of the strengthened zone. The dimensionless quantities  $c/\gamma H$  and  $W/H$  were used to simplify the charts. It was found that when the foundation is much stronger than either the unstrengthened or the strengthened soils, such that the failure surface must pass through the strengthened zone, much higher factors of safety are possible than if the foundation had the same strength as the overlying slope. However, reliance on a much stronger foundation and development of the strength for shear through the strengthened soil should be approached cautiously.

### 5.2 CHARTS FOR GEOSYNTHETIC-REINFORCED SLOPES

Existing methods for the design of geosynthetic-reinforced slopes were reviewed to determine their applicability to slopes like those typically encountered by the Texas Department of Transportation. Charts by Schmertmann et al. (1987), Jewell (1990), and Leshchinsky and Reinschmidt (1984), as well as a computer program, ReSlope, developed by Leshchinsky (1994), were examined and evaluated for their suitability. In cases where the assumptions used in creating the charts apply, the methods all give similar results. However, all of the available charts were found inadequate to cover the slopes of interest. The charts either did not cover flat enough slope angles, did not include pore water pressures, or did not consider foundations having the same strength as the slope. The computer software ReSlope offers a powerful tool for engineering design; however, it is more complex than methods

employing charts. The software was judged to be of limited use to highway maintenance engineers, to whom much of this work was directed.

A series of charts was developed for the design of geosynthetic-reinforced slopes. These charts enable the number of reinforcement layers and the minimum lengths of those layers to be determined. The charts can be used in cases in which pore water pressures are either zero or positive. Also, the foundation can be either much stronger than the overlying slope or have the same strength. The range of slope angles, as well as strength ( $\bar{\phi}$ ) of the soil, is much larger than previous charts have covered.

### **5.3 RECOMENDATIONS FOR FURTHER STUDY**

Theoretically, for slopes on foundations having the same soil properties as the slope, extending the strengthened zone below the toe is more efficient for raising the stability of the slope than increasing the width of the strengthened zone. Extending the strengthened area deeper makes failure surfaces cut deeper through stronger material. However, the amount of strength that can be mobilized beneath the toe of the slope is questionable. Also, for slopes on much stronger foundations, strengthening the soil to shallow depths near the face of the slope appears to greatly improve stability but leads to questions regarding the integrity of the strengthened material. Further studies might address the feasibility of strengthening soil beneath the toe of the slope and at shallow depths near the face when the slope is on a much stronger foundation.

## BIBLIOGRAPHY

- Abrams, T. G., and S. G. Wright, (1972), "A Survey of Earth Slope Failures and Remedial Measures in Texas," *Research Report No. 161-1*, Center for Highway Research, The University of Texas at Austin.
- Bishop, A. W., and Norbert Morganstern, (1960), "Stability Coefficients for Earth Slopes," *Geotechnique*, Vol. 10, No. 4, pp. 129–150.
- Collin, J. G. (1996), "Controlling Surficial Stability Problems on Reinforced Steeped Slopes," *Geotechnical Fabrics Report*, April 1996, pp. 26–29.
- Cuenca, F., and S. G. Wright, (1986), "Stability Computation Procedures for Earthen Slopes Containing Internal Equilibrium," *Research Report No. 435-1*, Center for Transportation Research, The University of Texas at Austin.
- Gourlay, A. W., and S. G. Wright, (1984), "Initial Laboratory Study of the Shear Strength Properties of Compacted, Highly Plastic Clays Used for Highway Embankment Construction in the Area of Houston, Texas," *A Report on Laboratory Testing Performed Under Interagency Contract Nos. (82-83) 2187 and (84-85) 1026*, Center for Transportation Research, The University of Texas at Austin.
- Green, Roger, and S. G. Wright, (1986), "Factors Effecting the Long-Term Strength of Compacted Beaumont Clay," *Research Report No. 436-1*, Center for Transportation Research, The University of Texas at Austin.
- Ingles, O. G., and J. B. Metcalf, (1972), *Soil Stabilization*, Halsted Press, New York, pp. 103–169.
- Jewell, R. A., N. Paine, and R. I. Woods, (1984), "Design Methods for Steep Reinforced Embankments," *Polymer Grid Reinforcement*, Thomas Telford, London, pp. 70–81.
- Jewell, R. A., (1990), "Revised Design Charts for Steep Reinforced Slopes," *Proceedings of the Symposium on Reinforced Embankments: Theory and Practice in the British Isles*, Cambridge, Thomas Telford, London, pp. 1–31.
- Jewell, R. A., (1991), "Application of Revised Design Charts for Steep Reinforced Slopes," *Geotextiles and Geomembranes*, Vol. 10, pp. 203–233.
- Kayyal, M. K., and S. G. Wright, (1991), "Investigation of Long-Term Strength Properties of Paris and Beaumont Clays in Earthen Embankments," *Research Report No. 1195-2F*, Center for Transportation Research, The University of Texas at Austin.
- Kennedy, T. W., and R. D. Smith, (1986), "Lime and Cement Treatment of Soils for Repair of Earthen Slopes," *Research Report No. 435-2F*, Center for Transportation Research, The University of Texas at Austin.

- Koerner, R. M., (1994), *Designing with Geosynthetics*, 3rd edition, Prentice-Hall, Englewood Cliff, NJ., p. 783.
- Leshchinsky, D., and A. J. Reinschmidt, (1985), "Stability of Membrane Reinforced Slopes," *Journal of Geotechnical Engineering*, ASCE, Vol. 111, No. 11, pp. 1285–1300.
- Leshchinsky, D., H. Ling, and G. Hanks, (1995), "Unified Design Approach to Geosynthetic Reinforced Slopes and Segmented Walls," *Geosynthetics International*, Industrial Fabrics Association International, Vol. 2, No. 5, pp. 845–881.
- Leshchinsky, D., (1995), *ReSlope: Supplementary Notes*, Newark, Delaware, February, 1995.
- Leshchinsky, D., (1997), "Software to Facilitate Design of Geosynthetic-Reinforced Steep Slopes," *Geotechnical Fabrics Report*, Vol. 15, No. 1, pp. 40–46.
- Rogers, L. E., and S. G. Wright, (1986), "The Effects of Wetting and Drying on the Long-Term Shear Strength Parameters for Compacted Beaumont Clay," *Research Report No. 436-2F*, Center for Transportation Research, The University of Texas at Austin.
- Schmertmann, G. R., V. E. Chouery-Curtis, R. D. Johnson, and R. Bonaparte, (1987), "Design Charts for Geogrid-Reinforced Soil Slopes," *Proceedings of Geosynthetic '87 Conference*, New Orleans, LA, pp. 108–120.
- Stauffer, P.A., and S. G. Wright, (1984), "An Examination of Earth Slope Failures in Texas," *Research Report No. 353-3F*, Center for Transportation Research, The University of Texas at Austin.
- The Tensar Corporation, "Slope Reinforcement with Tensar Geogrids: Design and Construction Guideline," *Tensar Technical Note SR1*, The Tensar Corporation, February 1988.
- The Tensar Corporation, "Sierra Slope Retention System: Design Manual and Overview For Mechanically Stabilized Earth Slope Retention System," The Tensar Corporation, 1994.
- Wright, S. G., (1990), "UTEXAS3: A Computer Program for Slope Stability Calculations," Austin, Texas, May 1990.

## **APPENDIX**



Table A.1

Factors of Safety for "Strengthened" Slopes on Strong Foundations

1.5 to 1 Slope on Strong Foundation

W/H	c/H = 0.025	c/H = 0.05	c/H = 0.075	c/H = 0.1	c/H = 0.2	c/H = 0.25	c/H = 0.375	c/H = 0.5	c/H = 0.75	c/H = 1	c/H = 2	c/H = 3
0	1	1	1	1	1	1	1	1	1	1	1	1.000
0.05	1.096	1.112	1.154	1.135	1.210	1.196	1.259	1.260	1.333	1.367	1.407	1.510
0.1	0.995	1.126	1.143	1.185	1.296	1.285	1.314	1.361	1.454	1.532	1.680	1.996
0.25	0.525	0.561	1.235	1.262	1.394	1.408	1.549	1.643	1.832	2.014	2.728	3.184
0.5	0.390	0.541	1.166	1.163	1.572	1.667	1.842	2.038	2.396	2.716	3.864	5.143
1	0.185	0.416	0.564	0.648	1.868	1.977	2.310	2.652	3.386	3.882	6.286	7.767
1.25	0.156	0.314	0.470	0.628	1.430	1.910	2.491	3.003	3.782	4.485	7.882	9.558
1.5	0.173	0.344	0.517	0.688	1.376	1.936	2.685	3.369	4.324	5.088	9.926	11.349
1.75	0.179	0.355	0.534	0.705	1.380	1.777	2.924	3.882	4.910	5.691	11.970	13.140
2	0.179	0.355	0.534	0.705	1.376	1.630	3.156	4.250	5.602	6.294	14.014	14.931

2 to 1 Slope on Strong Foundation

W/H	c/H = .025	c/H = .05	c/H = .075	c/H = .1	c/H = .2	c/H = .25	c/H = .375	c/H = .5	c/H = .75	c/H = 1	c/H = 2	c/H = 3
0	1	1	1	1	1	1	1	1	1	1	1	1
0.05	1.073	1.095	1.122	1.137	1.175	1.188	1.217	1.252	1.313	1.366	1.41	1.52
0.1	1.03	1.117	1.135	1.163	1.249	1.268	1.311	1.364	1.458	1.541	1.69	1.89
0.25	0.799	1.05	1.207	1.238	1.359	1.401	1.519	1.609	1.785	1.946	2.509	3.013
0.5	0.53	0.837	1.205	1.23	1.513	1.592	1.774	1.931	2.223	2.502	3.517	4.568
1	0.25	0.52	0.75	0.925	1.79	1.902	2.209	2.503	3.096	3.563	5.63	7.143
1.25	0.207	0.415	0.622	0.83	1.651	1.94	2.41	2.82	3.488	4.107	6.816	8.67
1.5	0.203	0.405	0.608	0.81	1.62	2.025	2.609	3.13	3.953	4.651	8.226	10.197
1.75	0.203	0.405	0.608	0.81	1.62	2.025	2.82	3.54	4.44	5.195	9.636	11.724
2	0.203	0.405	0.608	0.81	1.62	2.025	3.038	3.9	4.98	5.739	11.046	13.251

2.5 to 1 Slope on Strong Foundation

W/H	c/H = .025	c/H = .05	c/H = .075	c/H = .1	c/H = .2	c/H = .25	c/H = .375	c/H = .5	c/H = .75	c/H = 1	c/H = 2	c/H = 3
0	1	1	1	1	1	1	1	1	1	1	1	1
0.05	1.05	1.078	1.09	1.098	1.14	1.151	1.175	1.203	1.254	1.298	1.446	1.571
0.1	1.065	1.108	1.127	1.141	1.202	1.25	1.308	1.367	1.462	1.55	1.7	2.127
0.25	1.073	1.139	1.179	1.214	1.324	1.394	1.489	1.575	1.738	1.878	2.29	2.842
0.5	0.67	1.133	1.244	1.297	1.454	1.517	1.706	1.824	2.05	2.288	3.17	3.993
1	0.315	0.624	0.936	1.202	1.712	1.827	2.108	2.354	2.806	3.244	4.974	6.519
1.25	0.258	0.516	0.774	1.032	1.872	1.97	2.329	2.637	3.194	3.729	5.75	7.782
1.5	0.233	0.466	0.699	0.932	1.864	2.114	2.533	2.891	3.582	4.214	6.526	9.045
1.75	0.227	0.455	0.682	0.915	1.86	2.273	2.716	3.198	3.97	4.699	7.302	10.308
2	0.227	0.455	0.682	0.915	1.864	2.42	2.92	3.55	4.358	5.184	8.078	11.571

3 to 1 Slope on Strong Foundation

W/H	c/H = .025	c/H = .05	c/H = .075	c/H = .1	c/H = .2	c/H = .25	c/H = .375	c/H = .5	c/H = .75	c/H = 1	c/H = 2	c/H = 3
0	1	1	1	1	1	1	1	1	1	1	1	1
0.05	1.046	1.062	1.081	1.118	1.118	1.146	1.171	1.171	1.214	1.253	1.382	1.492
0.1	1.061	1.087	1.107	1.18	1.19	1.214	1.253	1.253	1.39	1.5	1.77	2.1
0.25	0.99	1.122	1.156	1.273	1.339	1.401	1.445	1.445	1.64	1.8	2.29	2.77
0.5	0.794	1.159	1.224	1.405	1.463	1.593	1.709	1.709	1.922	2.133	2.92	3.659
0.75	0.541	1.081	1.256	1.532	1.62	1.83	2.001	2.001	2.266	2.56	3.55	4.548
1	0.414	0.828	1.176	1.656	1.766	1.986	2.211	2.211	2.612	2.987	4.18	5.437
1.25	0.338	0.676	1.014	1.779	1.917	2.204	2.468	2.468	2.958	3.414	4.81	6.326
1.5	0.288	0.57	0.856	1.938	2.061	2.422	2.767	2.767	3.304	3.841	5.44	7.215
1.75	0.266	0.543	0.784	2.002	2.27	2.645	3.05	3.05	3.65	4.268	6.07	8.104
2	0.253	0.505	0.758	2.019	2.426	2.838	3.38	3.33	3.996	4.695	6.7	8.993

3.5 to 1 slopes on strong foundations

W/H	c/H = .025	c/H = .05	c/H = .075	c/H = .1	c/H = .2	c/H = .25	c/H = .375	c/H = .5	c/H = .75	c/H = 1	c/H = 2	c/H = 3
0	1	1	1	1	1	1	1	1	1	1	1	1
0.05	1.042	1.046	1.072	1.0541	1.096	1.1	1.167	1.137	1.174	1.208	1.318	1.413
0.1	1.057	1.066	1.087	1.111	1.178	1.178	1.198	1.249	1.318	1.3918	1.5278	1.839
0.25	0.907	1.105	1.133	1.284	1.354	1.363	1.401	1.471	1.542	1.722	1.969	2.558
0.5	0.918	1.185	1.204	1.513	1.472	1.629	1.712	1.746	1.863	2.058	2.484	3.257
0.75	0.679	1.356	1.357	1.719	1.645	1.863	2.091	2.004	2.108	2.361	2.98	3.84
1	0.513	1.032	1.146	1.926	1.82	2.1	2.314	2.263	2.418	2.73	3.386	4.356
1.25	0.418	0.836	1.254	2.526	1.962	2.438	2.607	2.299	2.722	3.099	3.87	4.87
1.5	0.343	0.674	1.013	2.944	2.258	2.73	3.001	2.643	3.026	3.466	4.354	5.385
1.75	0.305	0.631	0.886	3.089	2.68	3.017	3.384	2.902	3.33	3.837	4.838	5.9
2	0.279	0.555	0.834	3.123	2.988	3.256	3.84	3.11	3.634	4.206	5.322	6.415

**Table A.2**

**Values for Charts for “Strengthened” Slopes on Foundations with Same Strengths**

Factors of Safety for various width ratios of strengthened zone assuming strength greater than threshold

W/H	1.5:1 Slopes	2.0:1 Slopes	2.5:1 Slopes	3.0:1 Slopes	3.5:1 Slopes
0	1	1	1	1	1
0.1	1.09	1.06	1.04	1.03	1.03
0.2	1.17	1.11	1.08	1.06	1.05
0.3	1.26	1.17	1.12	1.09	1.08
0.4	1.34	1.23	1.16	1.12	1.10
0.5	1.43	1.28	1.21	1.15	1.13
0.6	1.51	1.34	1.25	1.18	1.15
0.7	1.60	1.40	1.29	1.21	1.18
0.8	1.68	1.45	1.33	1.24	1.20
0.9	1.77	1.51	1.37	1.27	1.23
1	1.86	1.57	1.41	1.30	1.26

Threshold strength ratios for strengthened slopes on same strength foundations

W/H	1.5:1 Slopes	2.0:1 Slopes	2.5:1 Slopes	3.0:1 Slopes	3.5:1 Slopes
0	0	0	0	0	0
0.1	0.04	0.03	0.02	0.02	0.01
0.2	0.08	0.05	0.04	0.03	0.02
0.3	0.12	0.08	0.06	0.05	0.03
0.4	0.16	0.11	0.08	0.06	0.04
0.5	0.20	0.14	0.11	0.08	0.05
0.6	0.23	0.16	0.13	0.10	0.06
0.7	0.27	0.19	0.15	0.11	0.07
0.8	0.31	0.22	0.17	0.13	0.08
0.9	0.35	0.24	0.19	0.14	0.09
1	0.39	0.27	0.21	0.16	0.10

**Table A.3**

**Required Force for Equilibrium for Reinforced Slopes on Strong Foundations**

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0$ , strong foundation

$\phi$	$\beta$	T (lbs)	K
10	10	0	0.00
10	15	8154	0.15
10	20	14638	0.27
10	25	18598	0.34
10	30	21447	0.40
10	35	23596	0.44
10	40	25273	0.47
10	45	26611	0.49
10	50	27691	0.51
10	55	28570	0.53
10	60	29278	0.54
10	65	29811	0.55
10	70	30202	0.56
10	75	30481	0.56
10	80	30665	0.57
20	20	0	0.00
20	25	3186	0.06
20	30	5792	0.11
20	35	8230	0.15
20	40	10157	0.19
20	45	11683	0.22
20	50	12896	0.24
20	55	13852	0.26
20	60	14557	0.27
20	65	15065	0.28
20	70	15427	0.29
20	75	15676	0.29
20	80	15838	0.29
30	30	0	0.00
30	35	900	0.02
30	40	2252	0.04
30	45	3487	0.06
30	50	4524	0.08
30	55	5354	0.10
30	60	5947	0.11
30	65	6359	0.12
30	70	6641	0.12
30	75	6830	0.13
30	80	6948	0.13
40	40	0	0.00
40	45	520	0.01
40	50	1039	0.02
40	55	1505	0.03
40	60	1820	0.03
40	65	2029	0.04
40	70	2164	0.04
40	75	2259	0.04
40	80	2354	0.04

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0.25$ , strong foundation

$\beta$	$\phi$	T (lbs)	K
8	10	0	0.00
10	10	8694	0.16
15	10	14310	0.27
20	10	19926	0.37
25	10	24630	0.46
35	10	29247	0.54
40	10	30777	0.57
45	10	32010	0.59
50	10	33020	0.61
55	10	33852	0.63
60	10	34539	0.64
65	10	35099	0.65
70	10	35545	0.66
75	10	35874	0.66
80	10	36096	0.67
15	20	0	0.00
20	20	8640	0.16
25	20	12366	0.23
30	20	14850	0.28
40	20	17928	0.33
45	20	19224	0.36
50	20	20180	0.37
55	20	21180	0.39
60	20	21930	0.41
65	20	22478	0.42
70	20	22876	0.42
75	20	23153	0.43
80	20	23337	0.43
22	30	0	0.00
30	30	7452	0.14
35	30	9180	0.17
40	30	10219	0.19
45	30	10883	0.20
50	30	11547	0.21
55	30	12211	0.23
60	30	12876	0.24
65	30	13348	0.25
70	30	13681	0.25
75	30	13909	0.26
80	30	14055	0.26
29	40	0	0.00
40	40	5400	0.10
45	40	6210	0.12
50	40	6683	0.12
55	40	6819	0.13
60	40	6955	0.13
65	40	7091	0.13
70	40	7217	0.13
75	40	7385	0.14
80	40	7489	0.14

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0.5$ , strong foundation

$\beta$	$\phi$	T (lbs)	K
5	10	0	0.00
10	10	9018	0.17
15	10	18522	0.34
20	10	27054	0.50
25	10	30746	0.57
35	10	35550	0.66
40	10	36852	0.68
45	10	37916	0.70
50	10	38803	0.72
55	10	39547	0.73
60	10	40174	0.74
65	10	40695	0.75
70	10	41122	0.76
75	10	41448	0.77
80	10	41671	0.77
10	20	0	0.00
20	20	11070	0.21
25	20	17172	0.32
30	20	21600	0.40
40	20	26371	0.49
45	20	27799	0.51
50	20	28962	0.54
55	20	29911	0.55
60	20	30640	0.57
65	20	31180	0.58
70	20	31577	0.58
75	20	31857	0.59
80	20	32043	0.59
15	30	0	0.00
30	30	11340	0.21
35	30	15444	0.29
40	30	18252	0.34
45	30	20088	0.37
50	30	21266	0.39
55	30	21935	0.41
60	30	22604	0.42
65	30	23091	0.43
70	30	23442	0.43
75	30	23684	0.44
80	30	23844	0.44
20	40	0	0.00
40	40	11718	0.22
45	40	13986	0.26
50	40	15012	0.28
55	40	15587	0.29
60	40	15790	0.29
65	40	15992	0.30
70	40	16194	0.30
75	40	16396	0.30
80	40	16530	0.31

**Table A.4**

**Required Force for Equilibrium for Reinforced Slopes on Foundations with the Same Strength**

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0$ , same foundation

$\phi$	$\beta$	T (lbs)	K
10	10	0	0.00
10	15	7912	0.15
10	20	14799	0.27
10	25	18763	0.35
10	30	21318	0.39
10	35	23278	0.43
10	40	24905	0.46
10	45	26419	0.49
10	50	27756	0.51
10	55	28782	0.53
10	60	29538	0.55
10	65	30240	0.56
10	70	31050	0.58
10	75	31698	0.59
10	80	32238	0.60
20	20	0	0.00
20	25	2771	0.05
20	30	5632	0.10
20	35	8208	0.15
20	40	10260	0.19
20	45	11772	0.22
20	50	13035	0.24
20	55	14202	0.26
20	60	15390	0.29
20	65	16431	0.30
20	70	17863	0.33
20	75	19453	0.36
20	80	21295	0.39
30	30	0	0.00
30	35	1161	0.02
30	40	2647	0.05
30	45	3780	0.07
30	50	5043	0.09
30	55	6274	0.12
30	60	7656	0.14
30	65	9068	0.17
30	70	10411	0.19
30	75	11875	0.22
30	80	13557	0.25
40	40	0	0.00
40	45	623	0.01
40	50	1505	0.03
40	55	2339	0.04
40	60	3299	0.06
40	65	4396	0.08
40	70	5545	0.10
40	75	6752	0.13
40	80	8130	0.15

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0.25$ , same foundation

$\beta$	$\phi$	T (lbs)	K
7.5	10	0	0.00
10	10	5400	0.10
15	10	15498	0.29
20	10	19440	0.36
25	10	22896	0.42
30	10	25272	0.47
35	10	27324	0.51
40	10	29268	0.54
45	10	30996	0.57
50	10	32400	0.60
55	10	33156	0.61
60	10	33966	0.63
65	10	34513	0.64
70	10	35297	0.65
75	10	35640	0.66
80	10	35856	0.66
15	20	0	0.00
20	20	4698	0.09
25	20	8154	0.15
30	20	11286	0.21
35	20	13608	0.25
40	20	15984	0.30
45	20	17820	0.33
50	20	19440	0.36
55	20	21180	0.39
60	20	22132	0.41
65	20	23037	0.43
70	20	24190	0.45
75	20	24899	0.46
80	20	25272	0.47
22	30	0	0.00
30	30	4266	0.08
35	30	6341	0.12
40	30	8424	0.16
45	30	10206	0.19
50	30	11772	0.22
55	30	13068	0.24
60	30	14256	0.26
65	30	15336	0.28
70	30	15984	0.30
75	30	16524	0.31
80	30	16902	0.31
29	40	0	0.00
40	40	3510	0.07
45	40	4968	0.09
50	40	6264	0.12
55	40	7506	0.14
60	40	8640	0.16
65	40	9720	0.18
70	40	10476	0.19
75	40	11070	0.21
80	40	11556	0.21

Raw Data for  $K_{req}$  charts - H = 30,  
 $\gamma = 120$  pcf,  $r_c = 0.5$ , same foundation

$\phi$	$\beta$	T (lbs)	K
10	5	0	0.00
10	10	15228	0.28
10	15	21114	0.39
10	20	25218	0.47
10	25	27810	0.52
10	30	30456	0.56
10	35	32724	0.61
10	40	34236	0.63
10	45	35724	0.66
10	50	37206	0.69
10	55	38286	0.71
10	60	39636	0.73
10	65	40716	0.75
10	70	41958	0.78
10	75	42930	0.80
10	80	43632	0.81
20	10	0	0.00
20	20	16200	0.30
20	25	19872	0.37
20	30	21963	0.41
20	35	23760	0.44
20	40	25650	0.48
20	45	27324	0.51
20	50	28404	0.53
20	55	29538	0.55
20	60	30348	0.56
20	65	31496	0.58
20	70	32094	0.59
20	75	32761	0.61
20	80	33480	0.62
30	15	0	0.00
30	30	14438	0.27
30	35	16578	0.31
30	40	18407	0.34
30	45	19937	0.37
30	50	21016	0.39
30	55	22116	0.41
30	60	23092	0.43
30	65	24236	0.45
30	70	25538	0.47
30	75	26568	0.49
30	80	27702	0.51
40	20	0	0.00
40	40	11070	0.21
40	45	13187	0.24
40	50	14744	0.27
40	55	16235	0.30
40	60	17707	0.33
40	65	19245	0.36
40	70	20938	0.39
40	75	22464	0.42
40	80	24030	0.45

**Table A.5**

**Lengths of Reinforcement Required for Global Stability on Foundations with Same Strength**

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.5, Same strength

β	α	L (ft)	L/H
10	10	0.06	0.002
15	10	115.27	3.842333
20	10	106.59	3.553
25	10	97.1	3.236667
30	10	89.69	2.989667
35	10	88.9002	2.96334
45	10	89.79	2.993
50	10	90.24	3.008
55	10	91.86	3.062
60	10	93.45	3.115
65	10	94.31	3.143667
75	10	96.08	3.202667
80	10	96.99	3.233
20	20	0	0
25	20	34.49	1.149667
30	20	37.56	1.252
35	20	39.27	1.309
40	20	41.1	1.37
45	20	43.38	1.446
50	20	45.86	1.528667
55	20	48.42	1.614
60	20	50.52	1.684
65	20	52.5	1.75
70	20	54.49	1.816333
75	20	56.33	1.877667
80	20	57.74	1.924667
30	30	0	0
35	30	14.84	0.494667
40	30	17.83	0.594333
45	30	20.65	0.688333
50	30	24.36	0.812
55	30	27.55	0.918333
60	30	30.16	1.005333
65	30	32.47	1.082333
70	30	35.12	1.170667
75	30	36.86	1.228667
80	30	38.92	1.297333
40	40	0	0
45	40	8.24	0.274667
50	40	10.84	0.361333
55	40	14.21	0.473667
60	40	17.28	0.576
65	40	19.82	0.660667
70	40	22.62	0.754
75	40	24.75	0.825
80	40	26.79	0.893

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.25, Same strength

β	α	L (ft)	L/H
10	7.5	0	0
10	10	171	5.7
15	10	157.45	5.248333
20	10	141.03	4.701
25	10	127.58	4.252667
30	10	120	4
35	10	115.61	3.853667
40	10	114.01	3.800333
45	10	114.43	3.814333
50	10	115.26	3.842
55	10	115.52	3.850667
60	10	115.87	3.862333
65	10	116.58	3.886
70	10	116.67	3.889
75	10	116.82	3.894
80	10	116.99	3.899667
15	20	0	0
20	20	46.62	1.554
25	20	51.24	1.708
30	20	54.84	1.828
35	20	58	1.933333
40	20	59.92	1.997333
45	20	63.1	2.103333
50	20	65.4	2.18
55	20	67.63	2.254333
60	20	69.27	2.309
65	20	71.12	2.370667
70	20	72.98	2.432667
75	20	74.25	2.475
80	20	75.53	2.517667
22	30	0	0
30	30	28.53333	0.951111
35	30	31.685	1.056167
40	30	34.86	1.162
45	30	38	1.266667
50	30	41.28	1.376
55	30	44.35	1.478333
60	30	46.45	1.548333
65	30	49.01	1.633667
70	30	51.21	1.707
75	30	52.82	1.760667
80	30	54.43	1.814333
29	40	0	0
40	40	21.36	0.712
45	40	24.1	0.803333
50	40	26.84	0.894667
55	40	29.58	0.986
60	40	32.32	1.077333
65	40	34.77	1.159
70	40	37.5	1.25
75	40	39.64	1.321333
80	40	41.5	1.383333

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.5, Same strength

β	α	L (ft)	L/H
5	10	0	0
10	10	243.91	8.130333
15	10	220.19	7.339667
20	10	196.47	6.549
30	10	165.22	5.507333
35	10	155.22	5.174
45	10	149.87	4.995667
50	10	149.8	4.993333
55	10	149.85	4.995
60	10	149.32	4.977333
65	10	149.2	4.973333
70	10	149.09	4.969667
75	10	148.4	4.946667
80	10	147.72	4.924
10	20	0	0
20	20	75.9	2.53
25	20	85.57	2.852333
30	20	87.55	2.918333
35	20	89.53	2.984333
40	20	91.51	3.050333
45	20	93.31	3.110333
50	20	94.8	3.16
55	20	96.26	3.208667
60	20	97.14	3.238
65	20	98.28	3.276
70	20	99.43	3.314333
75	20	100.53	3.351
80	20	101.11	3.370333
15	30	0	0
30	30	57.32	1.910667
35	30	59.77	1.992333
40	30	62.22	2.074
45	30	64.67	2.155667
50	30	67.12	2.237333
55	30	69.57	2.319
60	30	71.43	2.381
65	30	73.44	2.448
70	30	75.03	2.501
75	30	76.46	2.548667
80	30	77.89	2.596333
20	40	0	0
40	40	41.1	1.37
45	40	47.99	1.599667
50	40	50.28	1.676
55	40	52.57	1.752333
60	40	54.86	1.828667
65	40	57.15	1.905
70	40	59.44	1.981333
75	40	61.14	2.038
80	40	63.17	2.105667
75	40	39.64	1.321333
80	40	41.5	1.383333

**Table A.6**

**Lengths of Reinforcement Required for Global Stability on Foundations with Same Strength**

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.0, Strong foundations

$\beta$	$\phi$	L (ft)	L/H
10	10	0	0
18	10	94.38	3.146
20	10	82.62	2.754
25	10	75.93	2.531
30	10	74.38	2.479333
40	10	74.53	2.484333
45	10	75.1	2.503333
50	10	76.33	2.544333
55	10	77.41	2.580333
60	10	78.33	2.611
70	10	79.98	2.666
75	10	80.41	2.680333
80	10	80.83	2.694333
20	20	0	0
25	20	28.89	0.963
30	20	34.32	1.144
35	20	36.11	1.203667
40	20	38.09	1.269667
45	20	39.95	1.331667
50	20	42.06	1.402
55	20	43.92	1.464
60	20	45.69	1.523
65	20	47.25	1.575
70	20	48.59	1.619667
75	20	49.64	1.654667
80	20	50.67	1.689
30	30	0	0
35	30	14.33	0.477667
40	30	17.84	0.594667
45	30	19.38	0.646
50	30	21.09	0.703
55	30	23.15	0.771667
60	30	25.22	0.840667
65	30	26.71	0.890333
70	30	28.54	0.951333
75	30	29.77	0.992333
80	30	30.98	1.032667
40	40	0	0
45	40	7.09	0.226333
50	40	9.36	0.312
55	40	10.72	0.357333
60	40	12.02	0.400667
65	40	13.38	0.446
70	40	15.01	0.500333
75	40	16.15	0.538333
80	40	17.15	0.571667

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.25, strong foundations

$\beta$	$\phi$	L (ft)	L/H
7.5	10	0	0
10	10	115.33	3.844333
15	10	104.74	3.491333
20	10	94.15	3.138333
25	10	88.09	2.936333
30	10	84.58	2.819333
35	10	83.26	2.775333
40	10	83.47	2.782333
45	10	84.09	2.803
50	10	85.64	2.854667
55	10	86.75	2.891667
60	10	88.29	2.943
65	10	89.01	2.967
70	10	90.25	3.008333
75	10	90.88	3.029333
80	10	92.03	3.067667
15	20	0	0
20	20	42.69	1.423
25	20	47.82	1.594
30	20	49.89	1.663
35	20	51.96	1.732
40	20	54.03	1.801
45	20	56.1	1.87
50	20	58.16	1.938667
55	20	60.41	2.013667
60	20	62.05	2.068333
70	20	65.23	2.174333
75	20	66.46	2.215333
80	20	67.69	2.256333
22	30	0	0
30	30	22.87	0.762333
35	30	30.3	1.01
40	30	35.67	1.189
45	30	39.05	1.301667
50	30	40.37	1.345667
55	30	41.69	1.389667
60	30	43.01	1.433667
65	30	44.33	1.477667
70	30	45.65	1.521667
75	30	46.97	1.565667
80	30	48.29	1.609667
30	40	0	0
40	40	7.38	0.246
45	40	10.89	0.363
50	40	14.07	0.469
55	40	17.31	0.577
60	40	20.7	0.690343
65	40	23.49	0.783
70	40	26.43	0.881
75	40	28.83	0.961
80	40	29.67	0.989

Global Stability Lengths for Reinforcement, H = 30 feet, r = 0.5, strong foundations

$\beta$	$\phi$	L (ft)	L/H
5	10	0	0
10	10	133.74	4.458
15	10	123.58	4.119333
20	10	113.42	3.780667
25	10	105.3	3.51
30	10	101.07	3.369
35	10	98.49	3.283
40	10	98.04	3.268
45	10	98.82	3.294
50	10	99.51	3.317
55	10	100.63	3.354333
60	10	101.15	3.371667
65	10	101.88	3.396
70	10	103.13	3.437667
75	10	103.26	3.442
80	10	103.9	3.463333
10	20	0	0
20	20	58.68	1.956
25	20	68.07	2.269
30	20	71.58	2.386
35	20	73.24	2.441333
40	20	74.9	2.496667
45	20	76.56	2.552
50	20	78.22	2.607333
55	20	79.88	2.662667
60	20	81.54	2.718
65	20	82.41	2.747
70	20	84.25	2.808333
75	20	85.01	2.833667
80	20	86.25	2.875
15	30	0	0
30	30	38.7	1.29
35	30	50.7	1.69
40	30	56.64	1.888
45	30	58.88	1.962667
50	30	61.12	2.037333
55	30	63.36	2.112
60	30	65.6	2.186667
65	30	67.08	2.236
70	30	68.55	2.285
75	30	69.89	2.329667
80	30	71.22	2.374
20	40	0	0
40	40	34.84	1.161333
45	40	37.82	1.260667
50	40	40.8	1.36
55	40	43.78	1.459333
60	40	46.76	1.558667
65	40	49.74	1.658
70	40	52.72	1.757333
75	40	54.31	1.810333
80	40	55.32	1.844

Table A.7

**Lengths of Reinforcement Required for Direct Sliding**

Lengths for Direct Sliding Study - H = 30  
feet, r = 0

β	L	LH
10	0	0
10	15	3.983333
10	20	3.416667
10	25	3.116667
10	30	2.883333
10	35	2.716667
10	40	2.6
10	45	2.5
10	50	2.416667
10	55	2.333333
10	60	2.244
10	65	2.166667
10	70	2.116667
10	75	2.066667
10	80	2.016667
20	20	0
20	25	0.85
20	30	1.106
20	35	1.289
20	40	1.166667
20	45	1.05
20	50	0.95
20	55	0.859
20	60	0.8
20	65	0.75
20	70	0.683333
20	75	0.633333
20	80	0.58
30	30	0
30	35	0.383333
30	40	0.5
30	45	0.516667
30	50	0.5
30	55	0.472
30	60	0.456
30	65	0.433333
30	70	0.383333
30	75	0.333333
30	80	0.283333
40	40	0
40	45	0.05
40	50	0.083333
40	55	0.116667
40	60	0.15
40	65	0.15
40	70	0.133333
40	75	0.133333
40	80	0.116667

Lengths for Direct Sliding Study - H = 30  
feet, r = 0.25

β	L	LH
10	0	0
10	10	4.525
10	15	3.983333
10	20	3.441667
10	25	3.142
10	30	2.891667
10	35	2.681
10	40	2.515
10	45	2.385
10	50	2.28
10	55	2.22
10	60	2.167
10	65	2.132
10	70	2.1
10	75	2.1
10	80	2.08
20	15	0
20	20	0.271
20	25	0.542
20	30	0.775
20	35	0.9
20	40	1.018
20	45	1
20	50	0.958333
20	55	0.869
20	60	0.808333
20	65	0.738
20	70	0.683333
20	75	0.617833
20	80	0.6
30	22	0
30	30	0.3
30	35	0.4
30	40	0.5
30	45	0.525
30	50	0.508333
30	55	0.47
30	60	0.433333
30	65	0.392833
30	70	0.3589
30	75	0.324967
30	80	0.313
40	29	0
40	40	0.058333
40	45	0.091667
40	50	0.125
40	55	0.174
40	60	0.2
40	65	0.2
40	70	0.174
40	75	0.148
40	80	0.139

Lengths for Direct Sliding Study - H = 30  
feet, r = 0.5

β	L	LH
10	0	0
10	10	4.72
10	15	3.983333
10	20	3.416667
10	25	3.116667
10	30	2.883333
10	35	2.716667
10	40	2.6
10	45	2.5
10	50	2.416667
10	55	2.333333
10	60	2.28
10	65	2.246
10	70	2.211
10	75	2.2
10	80	2.185
20	10	0
20	20	0.387
20	25	0.59
20	30	0.775
20	35	0.923
20	40	1.041667
20	45	1.018
20	50	0.958333
20	55	0.879
20	60	0.808333
20	65	0.731
20	70	0.683333
20	76	0.644
20	80	0.618
30	15	0
30	30	0.3
30	35	0.4
30	40	0.5
30	45	0.525
30	50	0.508333
30	55	0.457
30	60	0.47
30	65	0.444
30	70	0.435
30	75	0.435
30	80	0.453
40	20	0
40	40	0.087
40	45	0.106
40	50	0.125
40	55	0.148
40	60	0.1916667
40	65	0.225
40	70	0.2583333
40	75	0.2916667
40	80	0.325