A METHOD OF ESTIMATING TENSILE PROPERTIES OF MATERIALS TESTED IN INDIRECT TENSION

by

William O. Hadley
W. Ronald Hudson
Thomas W. Kennedy

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Evaluation of Tensile Properties of Subbases For Use in New Rigid Pavement Design

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PREFACE

This is the seventh report in the series dealing with research findings concerned with the evaluation of the properties of stabilized subbase materials. This report presents equations which were developed for estimating values of modulus of elasticity, Poisson's ratio, and failure strains for materials tested in the indirect tensile test. The report also includes the results of studies conducted on a circular aluminum specimen to verify these equations.

This report is a product of the combined efforts of many people. The assistance of the Texas Highway Department contact representative, Mr. Larry Buttler, is appreciated and the support of the U. S. Bureau of Public Roads is gratefully acknowledged. Special appreciation is due Mr. Pat Hardeman for his assistance in the test program, and thanks are also due to the Center for Highway Research staff who assisted with the manuscript.

William O. Hadley
W. Ronald Hudson
Thomas W. Kennedy

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LIST OF REPORTS

Report No. 98-1, "An Indirect Tensile Test for Stabilized Materials," by W. Ronald Hudson and Thomas W. Kennedy, summarizes current knowledge of the indirect tensile test, reports findings of limited evaluation of the test, and describes the equipment and testing techniques developed.

Report No. 98-2, "An Evaluation of Factors Affecting the Tensile Properties of Asphalt-Treated Materials," by William O. Hadley, W. Ronald Hudson, and Thomas W. Kennedy, discusses factors important in determining the tensile strength of asphalt-treated materials and reports findings of an evaluation of eight of these factors.

Report No. 98-3, "Evaluation of Factors Affecting the Tensile Properties of Cement-Treated Materials," by Humberto J. Pendola, Thomas W. Kennedy, and W. Ronald Hudson, presents factors important in determining the strength of cement-treated materials and reports findings of an evaluation by indirect tensile test of nine factors thought to affect the tensile properties of cement-treated materials.

Report No. 98-4, "Evaluation of Factors Affecting the Tensile Properties of Lime-Treated Materials," by S. Paul Miller, Thomas W. Kennedy, and W. Ronald Hudson, presents factors important in determining the strength of cement-treated materials and reports findings of an evaluation by indirect tensile test of eight factors thought to affect the tensile properties of lime-treated materials.

Report No. 98-5, "Evaluation and Prediction of the Tensile Properties of Lime-Treated Materials," by Walter S. Tulloch, II, W. Ronald Hudson, and Thomas W. Kennedy, presents a detailed investigation by indirect tensile test of five factors thought to affect the tensile properties of lime-treated materials and reports findings of an investigation of the correlation between the indirect tensile test and standard Texas Highway Department tests for lime-treated materials.

Report No. 98-6, "Correlation of Tensile Properties with Stability and Cohesiometer Values for Asphalt-Treated Materials," by William O. Hadley, W. Ronald Hudson, and Thomas W. Kennedy, presents a detailed correlation of indirect tensile test parameters, i.e., strength, modulus of elasticity, Poisson's ratio, and failure strain, with stability and cohesiometer values for asphalt-treated materials.

Report No. 98-7, "A Method of Estimating Tensile Properties of Materials Tested in Indirect Tension," by William O. Hadley, W. Ronald Hudson, and Thomas W. Kennedy, presents the development of equations for estimating material properties such as modulus of elasticity, Poisson's ratio, and tensile strain based upon the theory of the indirect tensile test and reports verification of the equations for aluminum.

ABSTRACT

Equations were developed for estimating values of modulus of elasticity, Poisson's ratio, and tensile failure strains for circular specimens based upon total horizontal and vertical deformations created in the specimen during indirect tensile testing.

A study was undertaken to verify the theoretical relationships for estimating the elastic constants of modulus of elasticity, Poisson's ratio, and tensile strains. To substantiate the equations a circular aluminum specimen, which is considered to exhibit a high degree of elasticity, was tested in indirect tension. The aluminum specimen was instrumented with rosette strain gages at the center. The results indicated that the elastic properties can be obtained from total horizontal and vertical deformations of an elastic material tested in indirect tension.

Additional tests were conducted to evaluate the effect of the dimensions of the curved loading strip used in the indirect tensile test and to evaluate the effect of loading rate (vertical strain rate). The results indicated that for best results a 1/2-inch-wide curved loading strip should be used to estimate the modulus of elasticity and Poisson's ratio from total deformation information. It was also found that the dimensions of the curved loading strip used had no significant effect on the tensile strain at the center of the circular aluminum specimen. The loading rate had a significant effect on Poisson's ratio but had no practical engineering effect on the modulus of elasticity.

KEY WORDS: indirect tensile test, modulus of elasticity, Poisson's ratio, tensile strain, loading rate, loading strip, total deformation, center strains, aluminum

SUMMARY

The increased use of stabilized subbases as a part of the rigid pavement structure has stimulated interest in their tensile properties. Since at present no standard test or procedure provides an adequate estimate of modulus of elasticity and Poisson's ratio, there is a need for a method of determining these properties so that they can be used in layered system theory for analyzing subbases.

The indirect tensile test is based on a well defined theory; thus it appears to have the greatest potential of all available tests for the evaluation of the tensile properties of stabilized materials. Based upon the theory of the test formulations for estimating the modulus of elasticity and Poisson's ratio were developed. Experiments were then conducted to verify the theoretical equations by testing aluminum specimens which exhibit a high degree of elasticity. The measured output data were the strains at the center of the specimen and the total deformations along the major axes. The values for Poisson's ratio and modulus of elasticity obtained from center strains were then compared to those calculated from total deformations and with the generally accepted values for aluminum.

Additional tests were also conducted to evaluate the effect of the width of the curved loading strip used in the indirect tensile test and to evaluate the effect of loading rate.

Based on the results of the study the following conclusions were made:

(1) The indirect tensile theory is valid when testing elastic materials with loads applied through a short curved loading strip.

(2) The estimated center strain values are essentially constant over the middle inch of a 4-inch-diameter specimen and are numerically equal to approximately one-half of the total horizontal deformation.

(3) The elastic properties, E and ν, and center strains can be obtained from total deformations in the x and y-directions of an indirect tensile test specimen.

(4) The width of curved loading strip used (1/2 inch or 1 inch) affects the modulus of elasticity and Poisson's ratio values obtained from total deformation values but does not effect center strains. For best results the 1/2-inch loading strip should be used when calculating E and ν from total deformation information.

(5) From an engineering standpoint the loading rate used in the indirect tensile test had no effect on modulus values obtained from total deformation information but did affect Poisson's ratio values.

IMPLEMENTATION STATEMENT

The ability to estimate values of elastic properties is a major step in the development of a design procedure for stabilized subbases and can lead to evaluation of the pavement structure as a layered system.

Since the results of this study indicate that center tensile strain created in a circular specimen tested in indirect tension approximately equals one-half of its total horizontal deformation, the tensile strains at failure for stabilized materials evaluated in previous studies (Research Report Nos. 98-2, 98-3, and 98-4) can be estimated. In addition, the equations developed in this report can be used in subsequent studies and in future testing of highway materials to estimate values of modulus of elasticity, Poisson's ratio, and tensile strains.

TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION

The increased use of stabilized subbases as a part of the rigid pavement structure has stimulated interest in the tensile properties of such subbase materials. Since no standard test or procedure presently appears to provide an adequate estimate of material properties of stabilized subbases, there is a need for a method of evaluating such tensile properties as modulus of elasticity and Poisson's ratio for the subbase materials in use. With these material properties, it is possible to conduct a rational evaluation of stabilized subbases, using layered system theory.

The indirect tensile test, because of a well-defined theory, appears to have the greatest potential of all presently available tests for the evaluation of tensile properties of stabilized materials. Although previous studies (Refs 1 through 14) have evaluated only the tensile strength of different materials, the indirect tensile test can also be used to estimate material properties such as modulus of elasticity and Poisson's ratio (Ref 15).

The objectives of this study were to develop and verify formulations for determining modulus of elasticity and Poisson's ratio, based upon equations derived by Hondros (Ref 15) for materials tested in the indirect tensile test.

CHAPTER 2. DISCUSSION OF INDIRECT TENSILE TEST

The indirect tensile test involves the loading of a circular element with compressive loads acting along two opposite generators (Fig 1). Hondros (Ref 15) developed equations for stresses created in a circular element subjected to short strip loading (Fig 2) assuming that body forces are negligible. These equations for the stresses along the principal diameters are presented below:

(1) Stresses along the vertical axis

(a) tangential stress:

$$\sigma_{\theta y} = + \frac{2P}{\pi a t} \left[\frac{\left(1 - \frac{r^2}{R^2} \right) \sin 2\alpha}{\left(1 - \frac{2r^2}{R^2} \cos 2\alpha + \frac{r^4}{R^4} \right)} - \tan^{-1} \left(\frac{\left(1 + \frac{r^2}{R^2} \right)}{\left(1 - \frac{r^2}{R^2} \right)} \tan \alpha \right) \right] \tag{1}$$

(b) radial stress:

$$\sigma_{ry} = - \frac{2P}{\pi a t} \left[\frac{\left(1 - \frac{r^2}{R^2} \right) \sin 2\alpha}{\left(1 - \frac{2r^2}{R^2} \cos 2\alpha + \frac{r^4}{R^4} \right)} \right.$$

(Eq Continued)

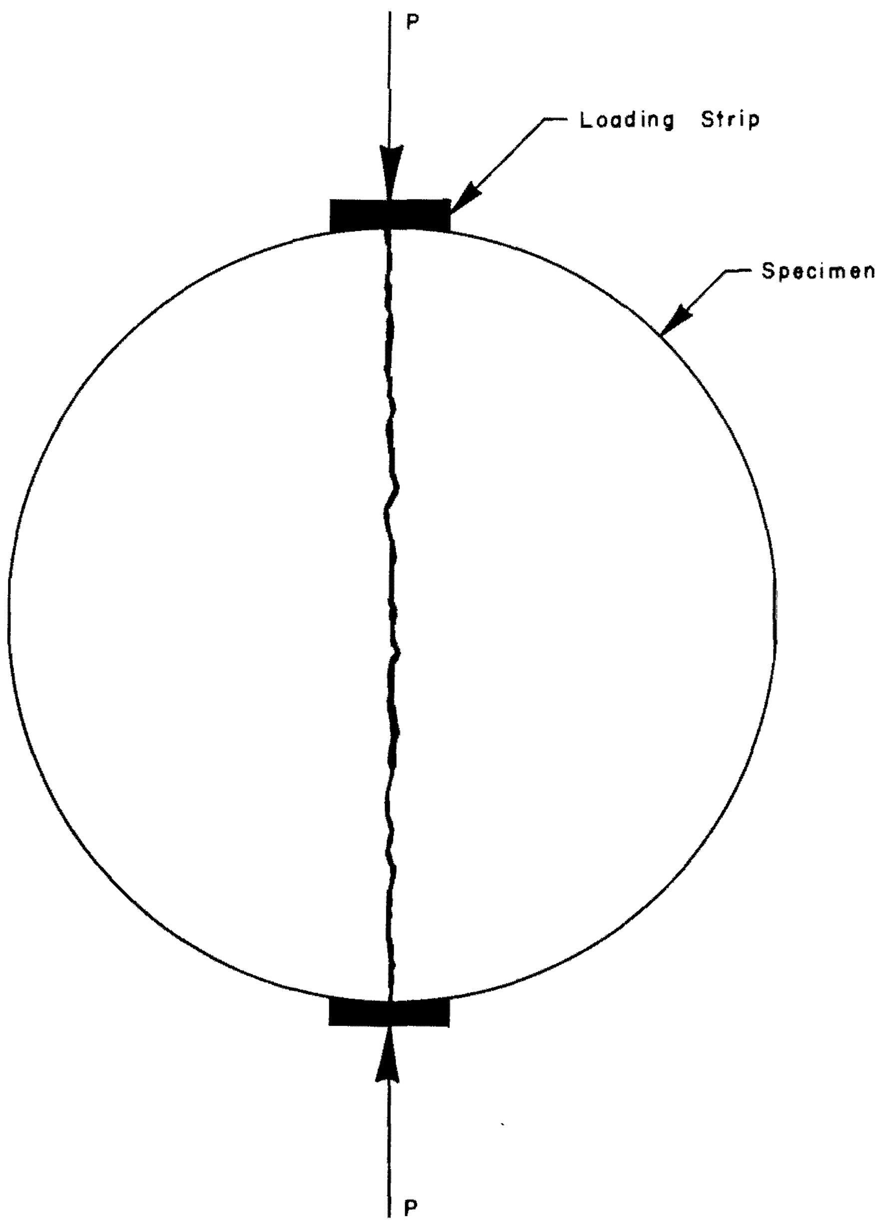

Fig 1. The indirect tensile test.

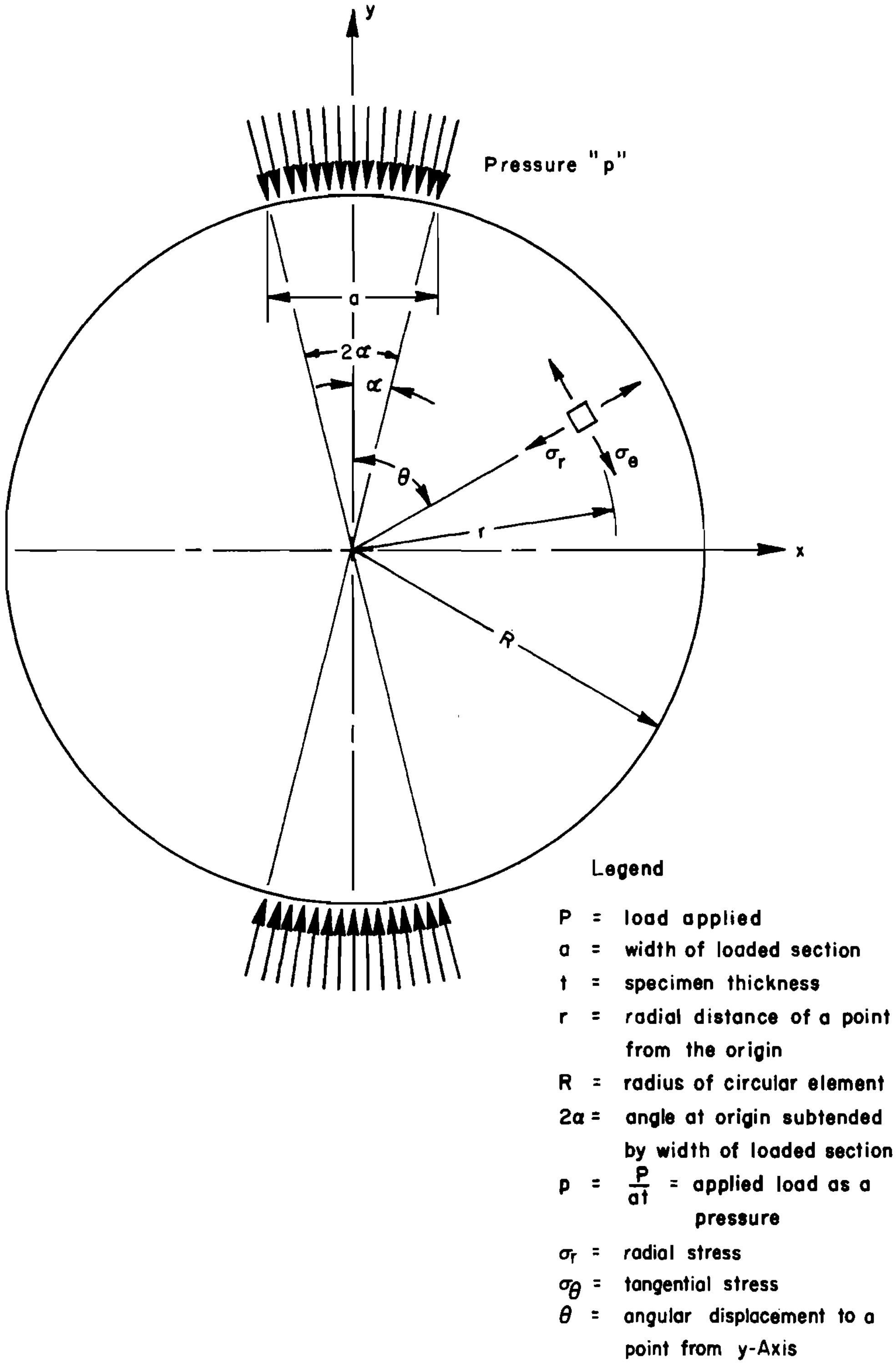

Fig 2. Notation for polar stress components in a circular element compressed by short strip loadings (from Ref 15).

$$+ \tan^{-1}\left(\frac{\left(1 + \frac{r^2}{R^2}\right)}{\left(1 - \frac{r^2}{R^2}\right)} \tan \alpha \right)\Bigg] \tag{2}$$

(c) shear stress:

$$\tau_{r\theta} = 0 \tag{3}$$

(2) Stresses along the horizontal axis

(a) tangential stress:

$$\sigma_{\theta x} = -\frac{2P}{\pi a t}\left[\frac{\left(1 - \frac{r^2}{R^2}\right) \sin 2\alpha}{\left(1 + \frac{2r^2}{R^2}\cos 2\alpha + \frac{r^4}{R^4}\right)} \right.$$

$$\left. + \tan^{-1}\left(\frac{\left(1 - \frac{r^2}{R^2}\right)}{\left(1 + \frac{r^2}{R^2}\right)} \tan \alpha \right)\right] \tag{4}$$

(b) radial stress:

$$\sigma_{rx} = +\frac{2P}{\pi a t}\left[\frac{\left(1 - \frac{r^2}{R^2}\right) \sin 2\alpha}{\left(1 + \frac{2r^2}{R^2}\cos 2\alpha + \frac{r^4}{R^4}\right)} \right.$$

$$\left. - \tan^{-1}\left(\frac{\left(1 - \frac{r^2}{R^2}\right)}{\left(1 + \frac{r^2}{R^2}\right)} \tan \alpha \right)\right] \tag{5}$$

(c) shear stress:

$$\tau_{\theta x} = 0 \tag{6}$$

The stress distributions along the principal planes through the diameters corresponding to the OX and OY-axes for a loading strip width less than D/10 are presented in Fig 3.

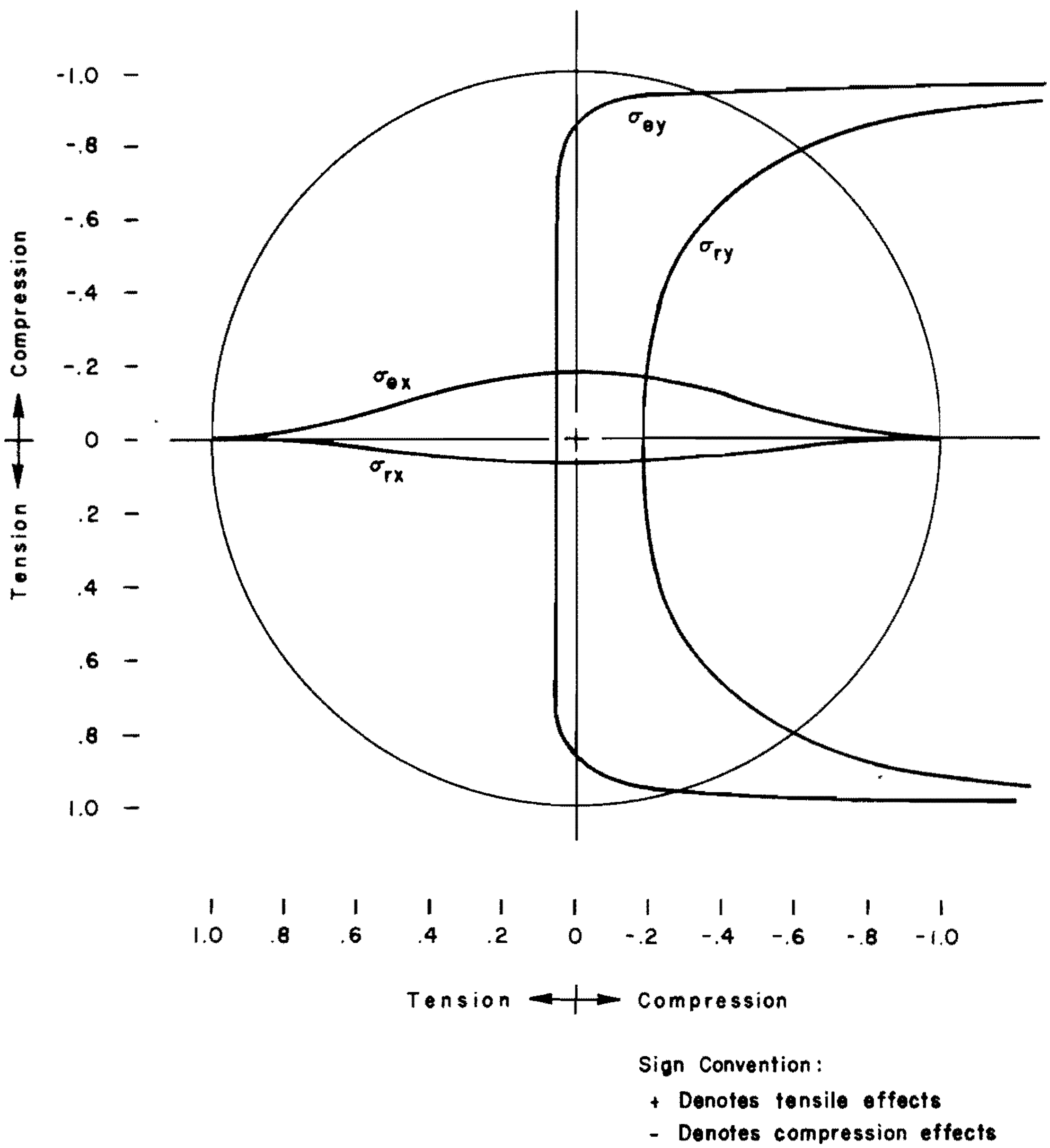

Fig 3. Stress distribution along the principal axes for loading strip width (a) less than D/10. $2p/\pi = 1$

CHAPTER 3. DEVELOPMENT OF FORMULATIONS

The equipment used at The University of Texas at Austin for testing specimens in indirect tension can be used to measure the load-deformation characteristics of a circular specimen along the principal stress planes. The deformation data, however, consist of the overall vertical and horizontal deformations rather than strain information and cannot be substituted directly into theoretical equations relating stress and strain. In order to obtain the required estimates of the material properties, it is necessary to develop relationships based upon the total horizontal and vertical deformations experienced by a circular specimen during indirect tensile testing.

The total deformation in either direction equals the integration of the strains of all individual elements along the principal axes and can be written as a function of modulus of elasticity E and Poisson's ratio ν. The equations for the integrated strains can be set equal to the total measured deformation in the two principal directions, leaving two equations and two unknowns. Equations for E and ν can be obtained by solving the two equations simultaneously.

Assumptions Made for Theoretical Development

Several assumptions should be recognized when the theoretical analysis of the indirect tensile test is utilized. The more important of these are discussed below.

The mathematical stress analysis assumes that the material is isotropic and homogeneous. This is not true for any existing structural material, but materials such as steel and aluminum more closely approximate homogeneity than portland cement concrete, asphaltic concrete, or stabilized materials.

The effect of heterogeneity on the general distribution of stress has not been determined but is probably quite small for aluminum and steel. It is thought that the random distribution of aggregate particles in portland cement concrete and asphaltic concrete tends to minimize the effect of heterogeneity.

Probably the most important assumption in the theoretical development is that Hooke's law is valid. The assumption is reasonable for aluminum and steel but would probably not hold for highway construction materials such as portland cement concrete or asphaltic concrete.

The theory also assumes that a state of plane stress exists in the specimen, but it does not occur in the practical situation. The conditions in a thin disc approximate plane stress, while those in a long cylinder approximate plane strain. The 2-inch-thick by 4-inch-diameter specimens tested at The University of Texas at Austin can be considered as thin discs; therefore, the assumption of plane stress is thought to be reasonable.

Derivation of Equations

The general stress-strain relationships for an element subjected to the action of uniformly distributed normal stresses σ_x , σ_y , and σ_z are

$$\epsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \tag{7}$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] \tag{8}$$

and

$$\epsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] \tag{9}$$

Since $\sigma_z = 0$ (plane stress assumed) and $\tau_{r\theta} = 0$ for the principal planes, the relationships can be simplified and reduced to

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu\sigma_y) \tag{10}$$

and

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu\sigma_x) \tag{11}$$

To further simplify the equations, notation for tangential and radial stresses is adopted so that the equations can be rewritten

$$\epsilon_x = \frac{1}{E}(\sigma_{rx} - \nu\sigma_{\theta x}) \tag{12}$$

and

$$\epsilon_y = \frac{1}{E}(\sigma_{ry} - \nu\sigma_{\theta y}) \tag{13}$$

where

ϵ_x, ϵ_y = strains in x and y-directions,

E = modulus of elasticity,

σ_{rx}, σ_{ry} = radial stresses along x and y-axes,

$\sigma_{\theta x}$, $\sigma_{\theta y}$ = tangential stresses along x and y-axes,

ν = Poisson's ratio.

The values for total deformation in the x and y-directions can be obtained by integrating the equations for the strains ϵ_x and ϵ_y over the total diameter from -r to +r (r = radius) :

$$X_T = \int_{-r}^{+r} \epsilon_x = \int_{-r}^{+r} \frac{1}{E}(\sigma_{rx} - \nu\sigma_{\theta x})$$

$$X_T = \frac{1}{E}\int_{-r}^{+r} (\sigma_{rx} - \nu\sigma_{\theta x})$$

$$X_T = \frac{1}{E}\left[\int_{-r}^{+r} \sigma_{rx} - \nu\int_{-r}^{+r} \sigma_{\theta x}\right] \tag{14}$$

and

$$Y_T = \int_{-r}^{+r} \epsilon_y = \int_{-r}^{+r} \frac{1}{E}(\sigma_{ry} - \nu\sigma_{\theta y})$$

$$Y_T = \frac{1}{E}\int_{-r}^{+r} (\sigma_{ry} - \nu\sigma_{\theta y})$$

$$Y_T = \frac{1}{E}\left[\int_{-r}^{+r} \sigma_{ry} - \nu\int_{-r}^{+r} \sigma_{\theta y}\right] \tag{15}$$

Solving Eqs 14 and 15 for E yields

$$E = \frac{1}{X_T}\left[\int_{-r}^{+r} \sigma_{rx} - \nu\int_{-r}^{+r} \sigma_{\theta x}\right] \tag{16}$$

and

$$E = \frac{1}{Y_T}\left[\int_{-r}^{+r} \sigma_{ry} - \nu\int_{-r}^{+r} \sigma_{\theta y}\right] \tag{17}$$

Equating Eqs 16 and 17 and solving for ν

$$\frac{1}{X_T}\left[\int_{-r}^{+r} \sigma_{rx} - \nu\int_{-r}^{+r} \sigma_{\theta x}\right] = \frac{1}{Y_T}\left[\int_{-r}^{+r} \sigma_{ry} - \nu\int_{-r}^{+r} \sigma_{\theta y}\right]$$

$$Y_T\left[\int_{-r}^{+r} \sigma_{rx} - \nu\int_{-r}^{+r} \sigma_{\theta x}\right] = X_T\left[\int_{-r}^{+r} \sigma_{ry} - \nu\int_{-r}^{+r} \sigma_{\theta y}\right]$$

$$\nu\left[-Y_T\int_{-r}^{+r} \sigma_{\theta x} + X_T\int_{-r}^{+r} \sigma_{\theta y}\right] = X_T\int_{-r}^{+r} \sigma_{ry} - Y_T\int_{-r}^{+r} \sigma_{rx}$$

$$\nu = \frac{\left[X_T\int_{-r}^{+r} \sigma_{ry} - Y_T\int_{-r}^{+r} \sigma_{rx}\right]}{\left[-Y_T\int_{-r}^{+r} \sigma_{\theta x} + X_T\int_{-r}^{+r} \sigma_{\theta y}\right]} \tag{18}$$

Since Y_T is always a negative number, the sign preceding it was changed so that only absolute values of deflections are required.

The equation then becomes

$$\nu = \frac{\left[X_T \int_{-r}^{+r} \sigma_{ry} + Y_T \int_{-r}^{+r} \sigma_{rx} \right]}{\left[Y_T \int_{-r}^{+r} \sigma_{\theta x} + X_T \int_{-r}^{+r} \sigma_{\theta y} \right]} \tag{19}$$

After solving for Poisson's ratio ν , the modulus E can be obtained by substituting the value for ν in either Eq 14 or 15.

The values for $\int_{-r}^{+r} \sigma_{ry}$, $\int_{-r}^{+r} \sigma_{rx}$, $\int_{-r}^{+r} \sigma_{\theta x}$, and $\int_{-r}^{+r} \sigma_{\theta y}$ are obtained by integrating Hondros' equations for each of the stress functions (Eqs 1 through 4). A closed-form solution of each equation involves complicated mathematical integrals; therefore, the integration process was completed numerically through the use of a computer.

Modifications to Theoretical Equations

Because there is slack in the mechanical equipment used in the indirect tensile test, minor equipment take-up occurs in the initial loading phase of the test. It is inadequate, then, to use the total deformations, i.e., X_T and Y_T , at one particular load to estimate the modulus of elasticity and Poisson's ratio. A least-squares analysis was selected to provide line-of-best-fit relationships between load-horizontal deformation and load-vertical deformation. This technique reduces the effect of equipment slack as well as observational and equipment output errors.

Corrections must be made to the equations for E and ν in order to use the least squares relationships between load and deflection values. Equation 19 is modified by defining a constant R equal to the slope of the least squares line of best fit between total vertical deformations and total horizontal deformation. Then

$$R = \frac{Y_T}{X_T}$$

$$\nu = \frac{\left[\int_{-r}^{+r} \sigma_{ry} + R \int_{-r}^{+r} \sigma_{rx} \right]}{\left[R \int_{-r}^{+r} \sigma_{\theta x} + \int_{-r}^{+r} \sigma_{\theta y} \right]} \tag{20}$$

The equations for modulus of elasticity in terms of total vertical deformations (16 and 17) must also be related to the load-deformation data obtained as output from indirect tensile test. Since the stresses σ_{rx} , $\sigma_{\theta x}$, σ_{ry} , and $\sigma_{\theta y}$ are a function of the load P , Eqs 16 and 17 can be rewritten as

$$E = \frac{P}{X_T} \left[\int_{-r}^{+r} \frac{\sigma_{rx}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta x}}{P} \right] \tag{21}$$

and

$$E = \frac{P}{Y_T} \left[\int_{-r}^{+r} \frac{\sigma_{ry}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta y}}{P} \right] \tag{22}$$

The integration of unit stresses $\frac{\sigma_{rx}}{P}$, $\frac{\sigma_{\theta x}}{P}$, $\frac{\sigma_{ry}}{P}$, and $\frac{\sigma_{\theta y}}{P}$ can be completed numerically in the computer. The values for $\frac{P}{X_T}$ and $\frac{P}{Y_T}$ can be obtained by calculating the slopes of the least squares best fit lines between load-horizontal deformation and load-vertical deformation, respectively.

It is possible also to obtain equations for modulus of elasticity E and Poisson's ratio ν in terms of center strain values. This can be accomplished by integrating the equations for strains ϵ_x and ϵ_y over the appropriate limits. The following development indicates the method. From the equations previously presented for strains (Eqs 12 and 13), the following can be obtained:

$$X_\ell = \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \epsilon_x = \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{1}{E} (\sigma_{rx} - \nu\sigma_{\theta x})$$

(Eq Continued)

$$= \frac{1}{E}\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{rx} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta x} \right] \tag{23}$$

$$\epsilon_{x\ell} = \frac{X_L}{\ell} = \frac{1}{E\ell}\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{rx} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta x} \right] \tag{24}$$

and

$$Y_\ell = \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \epsilon_y = \frac{1}{E}\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{ry} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta y} \right] \tag{25}$$

$$\epsilon_{y\ell} = \frac{Y_L}{\ell} = \frac{1}{E\ell}\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{ry} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta y} \right] \tag{26}$$

where

ℓ = length over which strain is measured;

$\epsilon_{x\ell}$, $\epsilon_{y\ell}$ = strains over length ℓ in x and y-directions, respectively.

Solving Eqs 24 and 26 simultaneously results in equations for E and ν in terms of absolute values of strain:

$$\nu = \frac{\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{ry} + SR \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{rx} \right]}{\left[SR \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta x} + \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \sigma_{\theta y} \right]} \tag{27}$$

$$E = \frac{P}{(\epsilon_{x\ell})(\ell)}\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{rx}}{P} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{\theta x}}{P} \right] \tag{28}$$

and

$$E = \frac{P}{(\epsilon_{y\ell})(\ell)} \left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{ry}}{P} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{\theta y}}{P} \right] \tag{29}$$

where

$$SR = \text{strain ratio} = \frac{\epsilon_{y\ell}}{\epsilon_{x\ell}} .$$

The values for SR , $\frac{P}{\epsilon_{x\ell}}$, and $\frac{P}{\epsilon_{y\ell}}$ can be obtained from least squares analysis of vertical strain with horizontal strain, load with horizontal strain, and load with vertical strain, respectively.

Since equations for E have been developed in terms of total deformations as well as center strains, it is possible to estimate center strains in terms of total deformations by equating the two equations for E . Using the equations for total horizontal deformation (Eq 21) and horizontal center strain (Eq 28) gives

$$\frac{P}{X_T} \left[\int_{-r}^{+r} \frac{\sigma_{rx}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta x}}{P} \right] = \frac{P}{(\epsilon_{x\ell})(\ell)} \left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{rx}}{P} \right.$$

$$\left. - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{\theta x}}{P} \right]$$

Then, solving for $\epsilon_{x\ell}$ results in

$$\epsilon_{x\ell} = \frac{X_T}{\ell} \left[\frac{\left[\int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{rx}}{P} - \nu \int_{\frac{-\ell}{2}}^{\frac{+\ell}{2}} \frac{\sigma_{\theta x}}{P} \right]}{\left[\int_{-r}^{+r} \frac{\sigma_{rx}}{P} - \nu \int_{-r}^{+r} \frac{\sigma_{\theta x}}{P} \right]} \right] \tag{30}$$

With this equation it is possible to approximate center strain values from known total horizontal deformations providing ν is also known. This allows the estimation of failure strains without the extensive use of strain gages.

CHAPTER 4. EXPERIMENTAL EVALUATION

Experimental studies were conducted to verify the theoretical equations by testing a material which behaved somewhat elastically and to determine the effects produced by changes in the loading rate and the width of the curved loading strip.

A study was undertaken to verify the theoretical relationships for the elastic constants E and ν . To substantiate the equations, the decision was made to test a specimen made from aluminum, which exhibits a high degree of elasticity. The measured output was center strains and total deformations along the major axes. The equations were verified by comparing the values for Poisson's ratio and modulus of elasticity calculated using measured center strains with those calculated from measured total deformations. The values for measured center strains were considered to be the best estimate of E and ν because the recorded strain output, which is measured in micro-units, should have been more accurate than the total deformation values. The resulting modulus of elasticity E and Poisson's ratio ν were also compared with the generally accepted values of modulus of elasticity and range of Poisson's ratio for aluminum of 10×10^6 psi and 0.33 through 0.35, respectively.

Additional tests were included in the study to evaluate the effect of the width of the curved loading strip used in the indirect tensile test and to evaluate the effect of loading rate. The strip widths under consideration were 1/2 inch and 1 inch. The 1-inch strip had been used in previous studies (Refs 5, 7, and 8); however, it was thought that the smaller, 1/2-inch, strip might provide better results since it approximates a point load more closely. In addition, it was felt that there would also be a reduction of the possible confining effect caused by the curved strip. The loading rates used in the study were .05 inch/minute, 0.5 inch/minute, and 1.0 inch/minute. Total deformations were obtained from these latter tests.

Test Equipment

The circular test specimen used for this study was an aluminum disc 3.996 inches in diameter and 1.00-inch thick with a rosette strain gage (90^{o}) attached at the center of one of the faces. The loading equipment (Fig 4) consisted of a loading table, shoe die loading head, and mechanical screw jack loading system. The horizontal and vertical center strains were obtained using strain indicators while the total vertical and horizontal deformations were measured by a LVDT and horizontal strain device, respectively. The horizontal strain device has been discussed in Reports 98-1 and 98-2 (Refs 4 and 5).

Test Procedure

The procedure for static testing consisted of loading the aluminum specimen in increments of 800 (±10) pounds beginning at 800 pounds and ending with 8,000 pounds. Separate tests were run to obtain center strain and total deformation data. At each load level either the total deformations or center strains were measured. The strains were read from the strain indicators and recorded by project personnel while deformations versus loads were plotted on X-Y plotters. The procedure for testing at a certain load rate involved only the measurement of total horizontal and vertical deformations.

Experiment Design and Analysis

The statistical experiment design for static testing (Experiment No. 1) consists of an evaluation of the four different test configurations:

(1) 1/2-inch-wide curved loading strip - center strain measurements,
(2) 1/2-inch-wide curved loading strip - total deformation measurements,
(3) 1-inch-wide curved loading strip - center strain measurements, and
(4) 1-inch-wide curved loading strip - total deformation measurements.

Ten tests were conducted on the same aluminum specimen for each of the test configurations.

A similar statistical experiment design (Experiment No. 2) was used to evaluate the effect of loading rate on the modulus of elasticity and Poisson's ratio. Ten separate tests were conducted on the same aluminum specimen for each of the four loading rates:

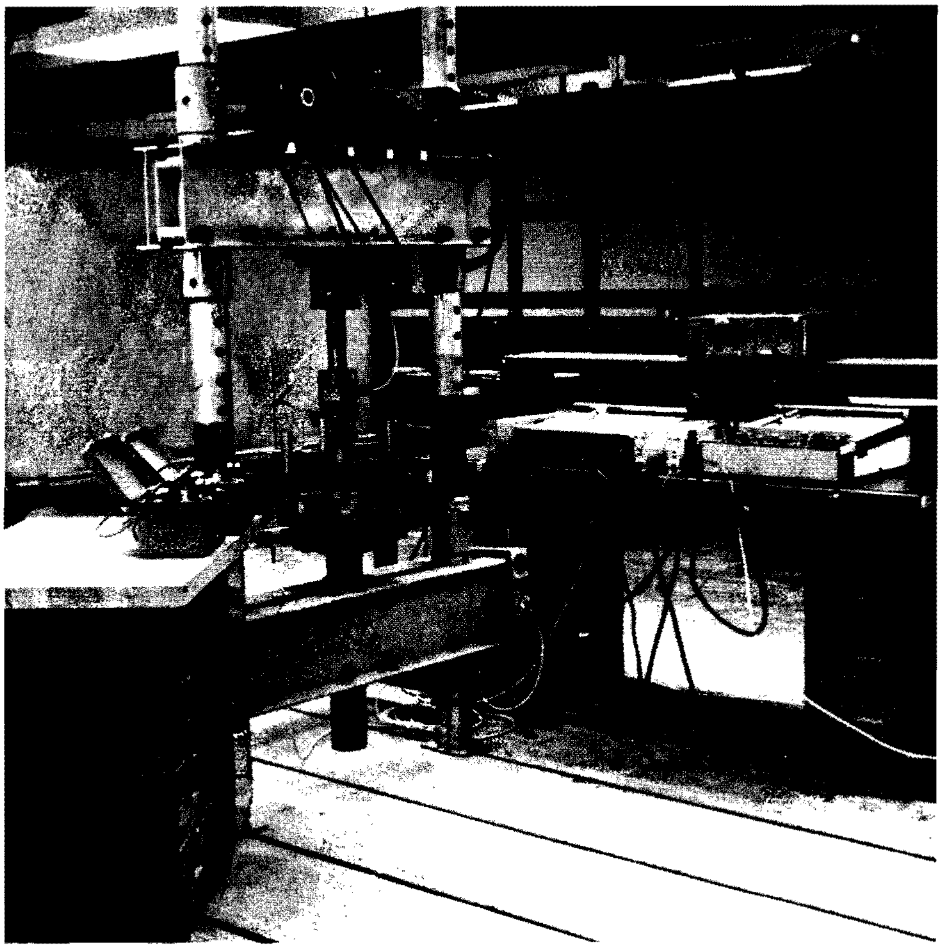

Fig 4. Loading equipment.

(1) static,

(2) .05 inch/minute,

(3) 0.5 inch/minute, and

(4) 1.0 inch/minute.

The analysis technique selected for this portion of the study is called the "Method of Orthogonal Contrasts" (Ref 15) and is used to determine at some given level of statistical significance whether treatment means are different. The means for the treatments in the first experiment were the average of the ten separate values of modulus of elasticity and Poisson's ratio for each test configuration (i.e., 1/2-inch curved loading strip - center strain measurements, 1-inch curved loading strip - total deformation measurements, etc.), while the means for the second experiment were the average of the ten separate values of modulus of elasticity and Poisson's ratio for each of the four loading rates. It should be noted that the method required that the contrast among the mean values be selected prior to the experiment and that the number of contrasts could not exceed the number of degrees of freedom between the treatment means.

The three orthogonal contrasts selected for Experiment No. 1 (static tests) were

$$
\begin{array}{lllll}
C_1 = & +T.1 & & -T.3 & \\
C_2 = & & +T.2 & & -T.4 \\
C_3 = & +T.1 & -T.2 & +T.3 & -T.4
\end{array}
$$

where T.1 , T.2 , T.3 , and T.4 were the mean values for Treatments 1 through 4, respectively. Contrast C_1 compared the mean of the first treatment with that of the third, C_2 compared the mean of the second treatment with that of the fourth, and C_3 compared the average of Treatments 1 and 3 with the average of Treatments 2 and 4. The coefficients of the treatment means for the three contrasts are given in Table 1.

TABLE 1. ORTHOGONAL COEFFICIENTS - EXPERIMENT NO. 1

	T.1	T.2	T.3	T.4
C_1	+1	0	-1	0
C_2	0	+1	0	-1
C_3	+1	-1	+1	-1

The three orthogonal contrasts selected for Experiment No. 2 (evaluation of loading rate effect) were

$$C_1 = +T.1 \quad -T.2$$

$$C_2 = \quad +T.3 \quad -T.4$$

$$C_3 = +T.1 \quad +T.2 \quad -T.3 \quad -T.4$$

with the corresponding coefficients outlined in Table 2.

TABLE 2. ORTHOGONAL COEFFICIENTS - EXPERIMENT NO. 2

	T.1	T.2	T.3	T.4
C_1	+1	-1	0	0
C_2	0	0	+1	-1
C_3	+1	+1	-1	-1

CHAPTER 5. EXPERIMENTAL RESULTS

Experiment No. 1 - Static Tests to Verify Equations

The values of the two parameters evaluated in the first part of this study are presented in Table 3.

The analysis of variance (AOV) for modulus of elasticity values obtained in the first part is presented in Table 4 while the AOV for Poisson's ratio values is shown in Table 5.

Modulus of Elasticity. The analysis of variance for modulus of elasticity indicated there was a highly significant treatment effect among the four treatments of Experiment No. 1. Comparisons C_2 and C_3 were also found to be highly significant at a probability level of 0.01. Since comparison C_1 was found to be not significant, there was no reason to expect that the means of the two treatments were different. The loading strips evaluated in this study, therefore, apparently had no effect on modulus of elasticity values obtained from center strain data. A review of the strain values for each strip type indicated the same results, and it was concluded that center strains are relatively unaffected by the type of compressive loading applied to the surface of the circular specimen.

In comparison C_2 the mean of modulus of elasticity values obtained from total deformation of the aluminum specimen loaded with a 1/2-inch curved loading strip was compared with the mean value determined from total deformation of the specimen with load applied through a 1-inch curved loading strip. The fact that this comparison was highly significant indicated there was a real difference between the means of Treatments 2 and 4. The width of loading strip used in this study therefore had a highly significant effect on the modulus of elasticity value obtained from total deformation data. This can be seen in the data: the mean of Treatment 4 was 6.25×10^6 psi while the mean of Treatment 2 was 10.43×10^6 psi. The mean of Treatment 2 compared well with the accepted value for the modulus of elasticity of aluminum of 10×10^6 psi. Based on the results of this study it was concluded that a 1/2-inch loading

TABLE 3. EXPERIMENTAL RESULTS FOR EXPERIMENT NO. 1.

	Treatment 1 1/2-Inch Strip, Center Strains		Treatment 2 1/2-Inch Strip, Total Deformations		Treatment 3 1-Inch Strip, Center Strains		Treatment 4 1-Inch Strip, Total Deformations	
	E($\times 10^6$), psi	ν	E($\times 10^6$), psi	ν	E($\times 10^6$), psi	ν	E($\times 10^6$), psi	ν
	10.40	.321	10.46	.386	9.35	.299	6.21	.082
	10.63	.330	10.09	.363	9.38	.300	6.21	.094
	10.66	.332	10.48	.348	9.37	.300	6.11	.077
	10.69	.331	10.86	.378	9.38	.301	6.20	.089
	10.76	.333	10.86	.370	9.39	.305	6.39	.092
	10.26	.314	10.48	.374	10.38	.332	6.11	.088
	10.25	.316	10.45	.353	10.42	.331	6.13	.089
	10.17	.307	10.47	.362	10.50	.337	6.21	.093
	10.18	.313	10.09	.363	10.47	.335	6.57	.123
	10.19	.314	10.08	.331	10.51	.340	6.38	.103
Means	10.419	.321	10.432	.363	9.915	.318	6.252	.093

TABLE 4. AOV FOR MODULUS OF ELASTICITY, EXPERIMENT NO. 1

Source	df	SS($\times 10^{14}$)	MS($\times 10^{14}$)	F	Significance Level, %
Total	39	1.263274			
Treatment	3	1.219383	0.406461	333.44	1
Comparison C_1	1	.000008	.000008	0.00	not significant
Comparison C_2	1	.670878	.670878	550.35	1
Comparison C_3	1	.548496	.548496	449.96	1
Residual	36	.043891	.001219		

TABLE 5. AOV FOR POISSON'S RATIO, EXPERIMENT NO. 1

Source	df	SS	MS	F	Significance Level, %
Total	39	.454937			
Treatment	3	.447530	.149177	725.00	1
Comparison C_1	1	.008644	.008694	41.96	1
Comparison C_2	1	.252945	.252945	1227.89	1
Comparison C_3	1	.185940	.185940	902.62	1
Residual	36	.007407	.000206		

strip should be used in the indirect tensile test for determining the modulus of elasticity from total deformation data.

Comparison C_3 contrasted the average of means of modulus of elasticity values obtained from center strain data with the average of means from total deformations. The analysis indicated a highly significant difference between the average of the means. The low value for Treatment 4 (6.25×10^6) appeared to cause the significant difference since the values for Treatments 1, 2, and 3 all compare favorably with the accepted value of 10×10^6 psi.

Poisson's Ratio. The analysis of variance for Poisson's ratio values from Experiment No. 1 indicated that all three comparisons were highly significant. The comparison between the means of Poisson's ratio values obtained from center strain data under the two loading configurations (1/2-inch and 1-inch curved loading strips) (C_1) was found to be highly significant. However, the difference between the mean values of Poisson's ratio (.321 - .318 = .003) was of no practical engineering significance; the effect was measurable but was not considered large enough to affect a significant difference in the application of the results.

The comparison between the mean values of Poisson's ratio obtained from total deformation data for the specimen loaded through 1/2-inch and 1-inch curved loading strips (C_2) was also highly significant. The reason for the significant difference can be seen by reviewing the two corresponding mean values. The mean value for the specimen tested with the 1-inch-wide curved loading strip was .093 while the mean value for the 1/2-inch-wide curved loading strip was 0.363. The latter value compared reasonably well with the accepted value of 0.33 through 0.35.

The third comparison (C_3), involving the results obtained from center strain information and with those from total deformation information, was found to be highly significant. The cause for the significant difference was attributed to Treatment 4 ($\nu = 0.093$) since the mean values for Treatments 1, 2, and 3 compared well with the generally accepted range of 0.33 through 0.35.

Effect of Poisson's Ratio on Modulus of Elasticity. From a review of Table 3 it can be seen that the values for both the modulus of elasticity E and Poisson's ratio ν in Treatment 4 (1-inch curved loading strip, total deformation) were quite different from those for the other three treatments.

Since the Poisson's ratio value has a direct effect on the value of the modulus of elasticity (see Eqs 21 and 22), Treatment 4 was further evaluated to determine whether or not the values of modulus of elasticity were affected by the Poisson's ratio value primarily or by a combination of Poisson's ratio and errors in total deformation data. Modified values for modulus of elasticity E in Treatment 4 were calculated assuming a value for Poisson's ratio of 0.318, the mean value obtained from center strain data (Treatment 3). The modified values are summarized in Table 6 along with the original values for Treatments 3 and 4. A comparison of the modulus of elasticity values for Treatment 3 with the modified values for Treatment 4 shows that the value of Poisson's ratio obtained in Treatment 4 could have caused the low original modulus of elasticity values associated with that treatment.

The configuration of the loading strip had the greatest effect on the value obtained for Poisson's ratio. The 1-inch curved loading strip had a 1/2-inch curved section in the center and a 1/4-inch section on each side, and it is possible that a portion of the tangent sections came in contact with the circular aluminum specimen during the test. If so, the width of the actual loaded area would have been between 1/2 inch and 1 inch. If the exact value of the loaded area for each run had been known, then better estimates of both modulus of elasticity and Poisson's ratio could have been obtained. The range of estimates for Poisson's ratio, however, can be established by calculating the lower limit, assuming that the width of the loaded area was 1/2 inch. These calculations were made and are presented in Table 7 along with the original values for Treatments 3 and 4.

A comparison of the modified Poisson's ratio values with the original values in Treatments 3 and 4 showed that the values obtained from total deformation data for specimens tested with a 1-inch loading strip were much lower. The assumption of a 1/2-inch loaded area, however, did not produce Poisson's ratio values comparable to those obtained from center strain data (Treatment 3). Thus, the configuration of the loading strip could have been the primary cause for the underestimation of Poisson's ratio values in Treatment 4.

Experiment No. 2 - Evaluation of Effect of Loading Rate

The values of the two elastic parameters evaluated in the second portion of the study, concerning the effect of loading rate, are presented in Table 8.

TABLE 6. COMPARISON OF MODULUS OF ELASTICITY VALUES, 1-INCH-WIDE STRIP AND TOTAL DEFORMATION

	Treatment 3, psi ($\times 10^6$)	Treatment 4, psi ($\times 10^6$)	Treatment 4, Modified, psi ($\times 10^6$)
	9.35	6.21	9.81
	9.38	6.21	9.52
	9.37	6.11	9.76
	9.38	6.20	9.61
	9.39	6.39	9.65
	10.38	6.11	9.50
	10.42	6.13	9.50
	10.50	6.21	9.53
	10.47	6.57	9.42
	10.51	6.38	9.57
Means	9.92	6.25	9.61

TABLE 7. COMPARISON OF POISSON'S RATIO VALUES

	Treatment 3	Treatment 4	Treatment 4 Modified to 1/2-Inch Strip
	.299	.082	.175
	.300	.094	.189
	.300	.077	.168
	.301	.089	.184
	.305	.092	.187
	.332	.088	.182
	.331	.089	.183
	.337	.093	.189
	.335	.123	.227
	.340	.103	.202
Means	.318	.093	.189

TABLE 8. EXPERIMENTAL RESULTS FOR EXPERIMENT NO. 2

Loading Rate								
	Static		.05 in./min		.5 in./min		1.0 in./min	
	$E(\times 10^6)$, psi	ν	$E(\times 10^6)$, psi	ν	$E(\times 10^6)$, psi	ν	$E(\times 10^6)$, psi	ν
	10.46	.385	8.91	.211	10.22	.296	9.58	.295
	10.09	.363	9.51	.276	9.28	.278	9.29	.269
	10.48	.348	9.21	.259	8.58	.239	9.75	.299
	10.86	.378	8.93	.240	10.27	.329	8.86	.219
	10.86	.370	9.13	.260	9.32	.304	9.68	.292
	10.48	.374	8.99	.243	10.49	.286	9.60	.266
	10.45	.353	9.00	.252	9.92	.315	9.60	.287
	10.47	.362	8.93	.229	8.97	.248	9.91	.303
	10.09	.363	9.14	.258	9.54	.302	9.62	.294
	10.08	.331	8.85	.266	9.40	.279	9.67	.282
Means	10.432	.363	9.060	.249	9.599	.288	9.556	.281

The analyses of variance for the two parameters are presented in Tables 9 and 10, respectively.

Modulus of Elasticity. As indicated in the analysis of variance (Table 9) the test loading rate had a significant effect on the value of the modulus of elasticity obtained from results of the indirect tensile test. The only contrast found to be significant was C_1, which compared the mean value for static loading with that for a loading rate of .05 inch per minute. The mean modulus of elasticity value for .05 inch per minute loading rate was 9.06×10^6 psi while the mean for static loading was 10.432×10^6 psi. Both values compared well with the generally accepted value for modulus of elasticity of aluminum of 10×10^6 psi. Although the analysis indicated a highly significant difference in the means, it was felt that the difference was not of practical engineering significance. The remaining two contrasts were found to be not significant. The results of C_2 indicated no difference in the means of modulus of elasticity obtained at loading rates of 0.5 and 1.0 inch per minute. The last contrast, the comparison between the average of static testing and the .05-inch per minute loading rate with the average of .5 and 1.0-inch per minute loading rates, indicated no significant difference between slow and fast loading rates.

Poisson's Ratio. Two of the three comparisons were found to be highly significant (see Table 10). There was also a significant treatment effect because of loading rate. The contrast between static testing and loading rate of .05 inch per minute was found to be highly significant. The mean value for static testing was 0.363 while the mean for a loading rate of .05 inch per minute was 0.249. The results of static testing compare better with the accepted range of values of Poisson's ratio for aluminum of 0.33 through 0.35.

There was no significant difference between mean values of Poisson's ratio for the two higher loading rates. The mean values of 0.288 and 0.281 for loading rates of 0.5 and 1.0 inch per minute, respectively, closely approximated the accepted range and are both greater than the mean value obtained for the slower loading rate of .05 inch per minute. The third contrast, involving comparisons of the mean values of Poisson's ratio for slow loading rates with those for faster loading rates, was found to be highly significant; however, it is considered to have no practical engineering significance.

TABLE 9. AOV FOR MODULUS OF ELASTICITY, EXPERIMENT NO. 2

Source	df	SS($\times 10^{14}$)	MS($\times 10^{14}$)	F	Significance Level, %
Total	39	.149262			
Treatment	3	.097051	.032350	22.31	1
Comparison C_1	1	.094119	.094119	64.91	1
Comparison C_2	1	.000092	.000092	0.06	not significant
Comparison C_3	1	.002339	.002839	1.96	not significant
Residual	36	.052211	.001450		

TABLE 10. AOV FOR POISSON'S RATIO, EXPERIMENT NO. 2

Source	df	SS	MS	F	Significance Level, %
Total	39	.087324			
Treatment	3	.069114	.023038	45.71	1
Comparison C_1	1	.064116	.064116	127.21	1
Comparison C_2	1	.000240	.000240	0.48	not significant
Comparison C_3	1	.004807	.004807	9.54	1
Residual	36	.018160	.000504		

Verification of Theory

The development of an equation relating center strains to total deformation values allows one to verify the indirect tensile theory when using curved loading strips. Center strains over a length of 1/2 inch were measured by strain indicators and provided a good estimate of the strains produced in the specimen. The center strains were also estimated by Eqs 24 and 26 from total deformation data. The comparisons were made for the tests conducted with the 1/2-inch loading strip. The mean values for actual strains and estimated strains for loads from 800 to 8,000 pounds are presented in Table 11.

The values for the estimated and measured strains were quite close, as can be seen in Figs 5 and 6 and Table 12. The values of estimated strains were obtained from least squares analysis and were portrayed by the line of best fit. The closeness of the results verified that the theory holds when testing elastic materials in indirect tension with loads applied by a short curved loading strip.

An Evaluation of the Equation Relating Center Strain and Total Horizontal Deformation

Although the equation relating center strain to total horizontal deformation (Eq 30) provides a method of estimating strain over some length ℓ , it does not provide an estimate of strain for a specimen which fails by splitting along its vertical axis, unless the length over which the strain is estimated is extremely small and corresponds to the strain at the exact center of the circular specimen.

It was thought necessary to evaluate the relationship between center strain and total horizontal deformation from Eq 30 for a number of lengths ℓ ranging from 0.001 to 1.0 inch. At the same time the effect of Poisson's ratio on the strain-horizontal deformation relationship was also evaluated by substituting values of 0.0, 0.25, and 0.5 inch into Eq 30 for each length ℓ . The calculations were completed for a 4-inch-diameter specimen with load applied through a 1/2-inch curved loading strip.

The results of the evaluation are presented in Table 13 and Fig 7 and indicated that the strains near the center of the specimen approached a constant ratio of the total horizontal deformation for all three Poisson's ratio values. The coefficients approached the values of 0.579, 0.529, and 0.512 for

TABLE 11. AVERAGE STRAINS (μ IN./IN.)

Load in Pounds	Horizontal Strains		Vertical Strains	
	Measured	Estimated	Measured	Estimated
0				
800	26.3	24.75	49.0	41.32
1600	52.1	49.5	95.9	82.65
2400	76.3	74.25	137.9	123.97
3200	99.4	99.0	178.7	165.30
4000	121.2	123.75	217.6	206.62
4800	143.9	148.5	257.1	247.95
5200	167.7	173.25	295.7	289.27
6400	189.5	198.0	334.1	330.60
7200	212.2	222.75	372.5	371.92
8000	234.6	247.50	409.4	413.24

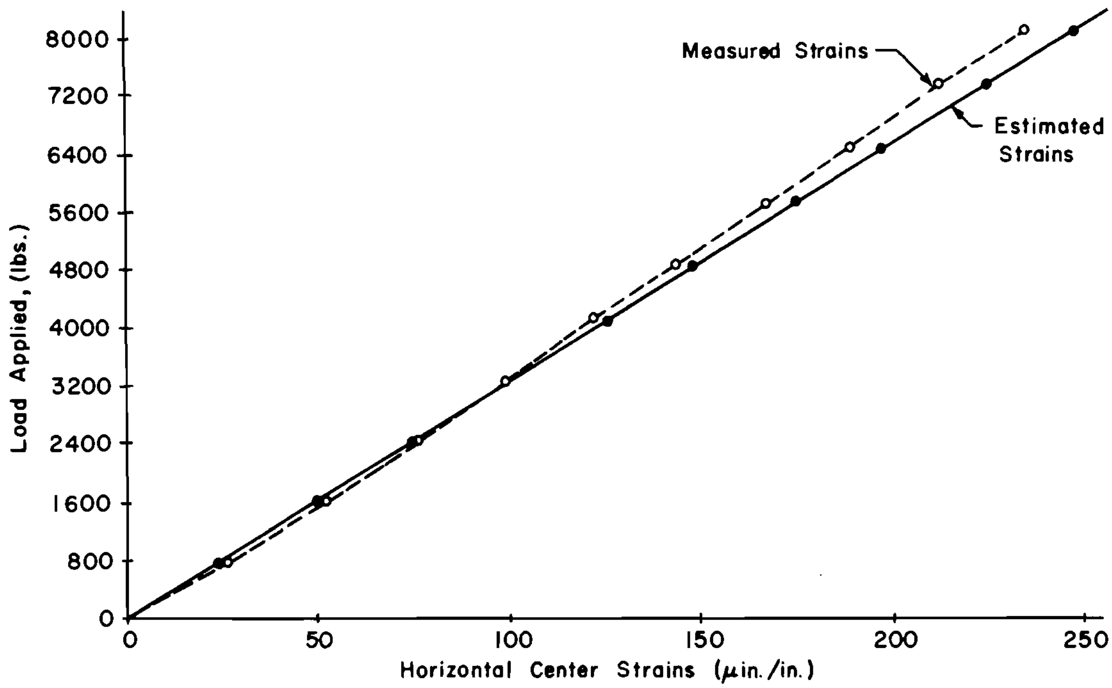

Fig 5. Comparison of measured and estimated horizontal strains.

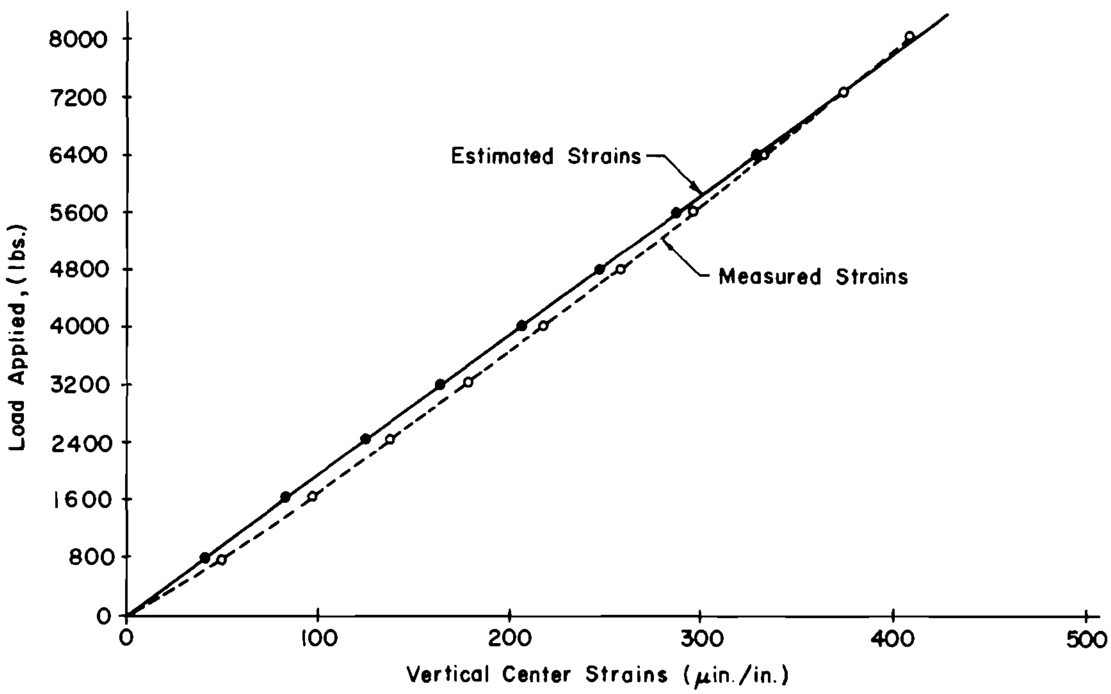

Fig 6. Comparison of measured and estimated vertical strains.

TABLE 12. PERCENT ERROR* FOR ESTIMATED STRAINS

Load, Pounds	Percent Error	
	Horizontal Strains	Vertical Strains
0		
800	5.9	15.7
1600	5.0	13.8
2400	2.7	10.1
3200	0.4	7.5
4000	-2.1	5.1
4800	-3.2	3.6
5600	-3.3	2.2
6400	-4.5	1.0
7200	-5.0	.2
8000	-5.5	- .9

$$* \text{ Percent Error} = \left(\frac{\epsilon_m - \epsilon_e}{\epsilon_m} \right) 100$$

where

ϵ_m = measured strain and

ϵ_e = estimated strain.

TABLE 13. COEFFICIENTS FOR THE FORMULA $\epsilon_{x\ell} = C_o X_T$ FOR DIFFERENT ℓ AND POISSON'S RATIO VALUES

ℓ , in.	Poisson's Ratio Values		
	0.0	0.25	0.50
1.0	.535	.495	.480
.8	.550	.506	.491
.6	.562	.516	.500
.4	.572	.523	.506
.2	.577	.528	.510
.1	.578	.529	.511
.08	.579	.529	.512
.06	.579	.529	.512
.04	.579	.529	.512
.02	.579	.529	.512
.016	.579	.529	.512
.012	.579	.529	.512
.008	.579	.529	.512
.001	.579	.529	.512

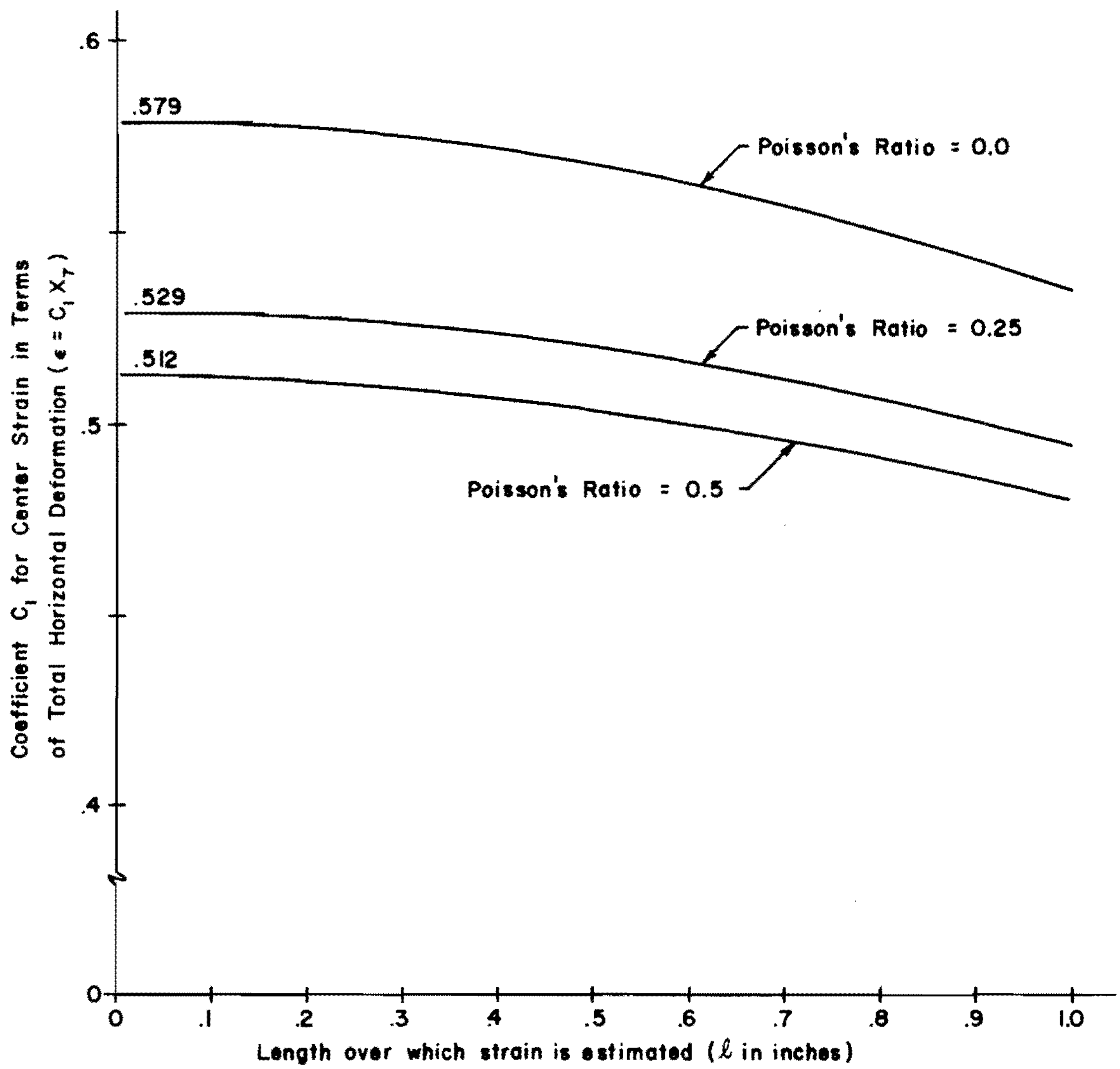

Fig 7. Relationship between center strain and total horizontal deformation.

Poisson's ratio values of 0.0, 0.25, and 0.5 inch, respectively. It should also be noted from Fig 7 that the estimated strains remained fairly constant up to a length of 1 inch (1/2 inch each side of the center of the circular specimen) and were approximately equal to one-half of the total horizontal deformation. The failure strains can best be approximated by using a value of ℓ less than 0.1 in Eq 27.

CHAPTER 6. CONCLUSIONS AND APPLICATION OF RESULTS

Conclusions

Based on the results of this study the following conclusions were made:

(1) The elastic properties E and ν can be obtained from total overall deformations in x and y-directions of a specimen tested in indirect tension through theoretical development.

(2) Center strains created in a specimen during the indirect tensile test can be estimated from known total horizontal and vertical deformation by theoretical development based upon Hondros' equations.

(3) The width of curved loading strip used (1/2 inch or 1 inch) had no significant effect on center strains created in the specimen.

(4) The width of curved loading strip used (1/2 inch or 1 inch) had a highly significant effect on modulus of elasticity and Poisson's ratio values obtained from total deformation values. For best results the 1/2-inch loading strip should be used when calculating E and ν from total deformation information.

(5) From an engineering standpoint the loading rate used in the indirect tensile test had no practical significant effect on modulus values obtained from total deformation information.

(6) The loading rate had a significant effect upon the Poisson's ratio of the material. The mean value obtained at static testing was 0.363 while the mean values for loading rates of 0.05, 0.5, and 1.0 in./min are 0.249, 0.288, and 0.281, respectively. The two higher rates compare better with the static test results. At the higher loading rates the ν may be underestimated which would result in lower values of E.

(7) The indirect tensile theory is valid when testing elastic materials with loads applied through a short curved loading strip.

(8) The center strain values estimated from Eq 30 remain fairly constant over the middle inch of a 4-inch-diameter specimen and are approximately equal to one-half of the total horizontal deformation.

Application of Results

The equations developed in this report cannot be directly applied in the field but can be used in subsequent studies of stabilized materials to estimate values of modulus of elasticity, Poisson's ratio, and tensile strain for a variety of highway materials.

The ability to estimate values of elastic properties is a major step in the development of a design procedure for stabilized subbase and can lead to evaluation of the pavement structure as a layered system.

Since the results of this study indicate that center tensile strain created in a circular specimen tested in indirect tension approximately equals one-half of its total horizontal deformation, the tensile strains at failure for stabilized materials evaluated in previous studies (Research Reports 98-2, 98-3, and 98-4) can be estimated.

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THE AUTHORS

William O. Hadley is a Research Associate with the Center for Highway Research at The University of Texas at Austin. He has had experience in the areas of highway design, power plant construction, and missile complex construction at Cape Kennedy, Florida. He is the author of several technical papers and reports and is currently involved in researching and evaluating the use of asphalt-treated subbases in rigid pavement construction.

W. Ronald Hudson is an Associate Professor of Civil Engineering and Associate Dean of the College of Engineering at The University of Texas at Austin. He has had a wide variety of experience as a research engineer with the Texas Highway Department and the Center for Highway Research at The University of Texas at Austin and was Assistant Chief of the Rigid Pavement Research Branch of the AASHO Road Test. He is the author of numerous publications and was the recipient of the 1967 ASCE J. James R. Croes Medal. He is presently concerned with research in the areas of (1) analysis and design of pavement management systems, (2) measurement of pavement roughness performance, (3) slab analysis and design, and (4) tensile strength of stabilized subbase materials.

Thomas W. Kennedy is an Associate Professor of Civil Engineering at The University of Texas at Austin. His experience includes work with the Illinois Division of Highways and research at the University of Illinois and the Center for Highway Research at The University of Texas at Austin, where he has conducted extensive investigations in the areas of (1) highway geometrics, (2) concrete durability, (3) tensile strength of stabilized subbase materials, and (4) time-dependent deformation of concrete and has contributed numerous publications in the field of transportation engineering. He is a member of several professional societies and has participated in committee work for the Highway Research Board and the American Concrete Institute.