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16. Abstract

The object of this study is to proposed and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams, with the aim of clarifying and simplifying current design provisions and AASHTO standard specifications.

This report summarizes an extensive literature review which documents the development of present regulations and procedures. In addition, the report outlines the general background and derivation of a powerful three-dimensional space truss model with variable angle of inclination of the diagonal elements. This conceptual model was developed by European and Canadian engineers over the past 15 years. The model is shown to be a plasticity lower bound solution which matches the upper bound solution. Thus the model is a mathematically valid solution which represents the failure load.

Extension of the use of this model into a design procedure is outlined. Experimental verification, detailed design procedures and specifications, and example applications are given in later reports in this series.

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REVIEW OF DESIGN PROCEDURES FOR SHEAR AND TORSION IN REINFORCED AND PRESTRESSED CONCRETE

by

J. A. Ramirez and J. E. Breen

Research Report No. 248-2

Research Project 3-5-80-248

"Reevaluation of AASHTO Shear and Torsion Provisions for
Reinforced and Prestressed Concrete"

Conducted for

Texas

State Department of Highways and Public Transportation

In Cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by

CENTER FOR TRANSPORTATION RESEARCH
BUREAU OF ENGINEERING RESEARCH
THE UNIVERSITY OF TEXAS AT AUSTIN

November 1983

The contents of this report reflect the views of the authors who are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

P R E F A C E

This report is the second in a series which summarizes a detailed evaluation of AASHTO design procedures for shear and torsion in reinforced and prestressed concrete beams. The first report summarized an exploratory investigation of the shear transfer between joints using details commonly found in segmental box girder construction. This report reviews the historical development of design procedures for shear and torsion in concrete members as found in American practice. Both the AASHTO Specifications and the ACI Building Code are examined, since they have been closely related. In addition, this report presents the background and equilibrium relationships for use of a space truss with variable inclination diagonals as a design model. The third report in this series summarizes special considerations required for the practical usage of the variable inclination truss model. It also compares the theoretical capacity as computed by the truss model to experimental results for a great variety of previously reported tests as well as the results of tests run in this program to investigate several variables. The fourth and final report in this series draws on the analytical and experimental results presented in the earlier reports. It uses these results to develop design procedures and suggested AASHTO Specification procedures for girder shear and torsion. The final report also contains several examples to illustrate the application of the design criteria and procedures.

This work is part of Research Project 3-5-80-248, entitled "Reevaluation of AASHTO Shear and Torsion Provisions for Reinforced and Prestressed Concrete." The studies described were conducted at the Phil M. Ferguson Structural Engineering Laboratory as part of the overall research program of the Center for Transportation Research of The University of Texas at Austin. The work was sponsored jointly by the Texas Department of Highways and Public Transportation and the Federal Highway Administration under an agreement with The University of Texas at Austin and the Texas Department of Highways and Public Transportation.

SUMMARY

The object of this study is to propose and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams, with the aim of clarifying and simplifying current design provisions and AASHTO standard specifications.

This report summarizes an extensive literature review which documents the development of present regulations and procedures. In addition, the report outlines the general background and derivation of a powerful three-dimensional space truss model with variable angle of inclination of the diagonal elements. This conceptual model was developed by European and Canadian engineers over the past 15 years. The model is shown to be a plasticity lower bound solution which matches the upper bound solution. Thus the model is a mathematically valid solution which represents the failure load.

Extension of the use of this model into a design procedure is outlined. Experimental verification, detailed design procedures and specifications, and example applications are given in later reports in this series.

IMPLEMENTATION

This report is the second in a series which summarizes a major experimental and analytical project aimed directly at suggesting new design recommendations for treating shear and torsion in reinforced and prestressed concrete girders. The detailed recommendations are included in the fourth and concluding report of this series.

This report contains background information of interest to those responsible for deciding on specifications and codes. In addition, it contains detailed derivations of the equilibrium equations for the space truss with variable angle of inclination of the diagonals. Such relationships will be of particular value to designers since they show typical applications of equilibrium relationships to relatively simple truss models. Such familiar and consistent applications of truss statics are the main tools for designers interested in specific application of the variable angle truss model to new and unfamiliar situations.

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INTRODUCTION

1.1 General

Design provisions for shear and torsion for reinforced and prestressed concrete members and structures in both the AASHTO Specifications (17) and the ACI Building Code (24) have evolved into complex procedures in recent revisions. The complexity of such procedures results from their highly empirical basis and the lack of a unified treatment of shear and torsion. Ironically, such design procedures seem better suited for analysis, since they become cumbersome and obscure when used for design.

In the case of continuous bridges, the designer must consider several different loading combinations to obtain maximum shear and flexural effects. The use of different loading combinations in the current design procedure is unclear and contradictory. This highly complicates the design of such members.

The available design procedures for shear and torsion were derived for particular cross sections such as rectangular, T, and I shapes. They become very difficult to apply to several of the other basic forms used for bridge cross sections shown in Fig. 1.1.

Outside the laboratory, there are few examples of pure torsion. Eccentrically loaded or horizontally curved beams are subjected to the

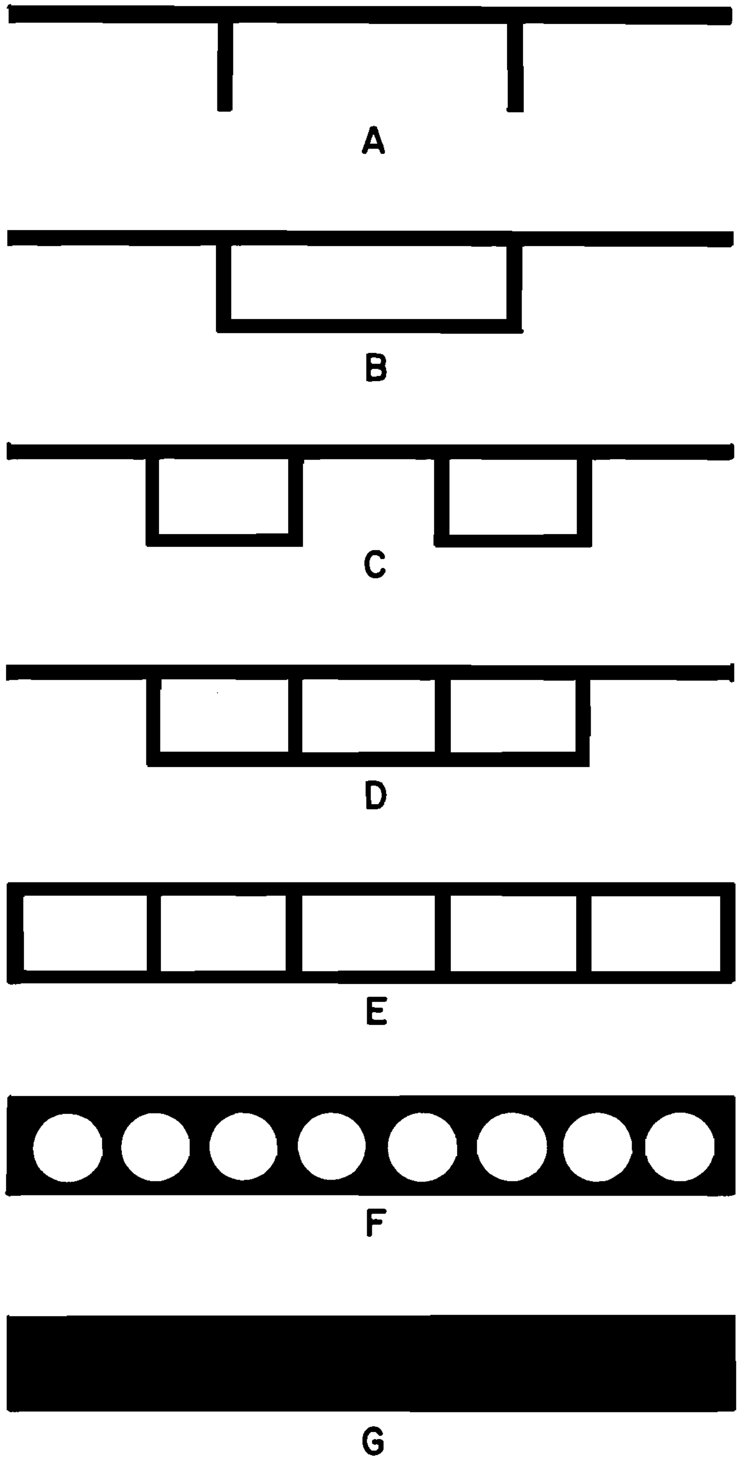


Fig. 1.1 Basic forms used for bridge cross sections

combined effect of bending moments, twisting moments and transverse shears. Staircases without intermediate supports, cantilevers with eccentric loading and edge beams of shells constitute other examples which can give rise to high twisting moments but are accompanied by bending moments and shear forces. Current ACI recommendations and AASHTO specifications follow the same approach of adding reinforcement required for torsion to that required for bending and shear. The practice of superimposing these effects is due to the lack of a unified approach to design for shear and torsion which would permit the correct evaluation of the combined actions.

Current American design practices do not emphasize enough the importance of adequate detailing for members subjected to shear and torsion. Furthermore, due to the empirical nature of such design procedures, it is not clear to the designer how to adequately detail such members.

Design provisions for shear and torsion in both the AASHTO Specifications and the ACI Building Code present a considerable void in the area of prestressed concrete members and structures. There is a total absence of design regulations for the cases of prestressed concrete members subjected to torsion or combined torsion, shear and bending.

Such deficiencies could be overcome if the design procedures in the shear and torsion areas were based on behavioral models rather than on detailed empirical equations. If the design procedures were based on a physical model, the designer would be able to envision the effects of the forces acting on the member, and then provide structural systems capable of resisting those forces. Furthermore, design provisions based on a conceptual model would become more simple and would not require as much test verification.

1.2 Problem Statement

The June 1973 report of ACI-ASCE Committee 426 "The Shear Strength of Reinforced Concrete Members" (28) indicated that for the next decade the Committee

> . . . hoped that the design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner.

The advent of computers has resulted in a quantum leap forward in the methods of analysis. There are now numerous programs based on elastic analysis techniques which can determine the sectional forces (axial loads, moments about any axis, torques, and shears) for wide variations of structures and loading cases. Now, the real difficulty starts, upon completion of the analysis process based on an idealized structure. The sectional forces must be transposed into physical arrangements of materials to provide adequate capacity to resist the applied forces. This procedure is a fundamental part of the design process and is referred to as dimensioning.

Procedures for dimensioning cross sections for reinforced and prestressed concrete members subjected to axial load, or moment, or combined axial load and moment, are generally well-established. These procedures can be explained in a few pages of text, and are based on rational, simple general design models which can be embodied in a few paragraphs of code or specification documents.

Such failure models provide the designer with means to evaluate the ultimate moment capacity of quite irregular sections in both reinforced and prestressed concrete. In addition, the same basic models can be used to study the interaction between axial load and moment, making the related design process relatively simple and straight forward. Unfortunately, design provisions in the areas of shear and torsion are not of the same level of rationality and general applicability. The absence of rational models has resulted in highly empirical design procedures characterized by large scatter when compared to test results.

In the past, those setting regulatory provisions were able to hide these deficiencies behind large factors of safety implicit in the overall design methods. Improvements in construction materials, analysis methods, and the adoption of the more refined ultimate strength design proportioning procedures, have resulted in generally smaller members. These changes have significantly reduced those hidden factors of safety. Consequently, the need for improved design procedures for shear and torsion has become increasingly important.

Due to the complexity involved in explaining the behavior of concrete members subjected to shear and torsion, and the lack of adequate knowledge in this area, most research has tended to concentrate on predicting the collapse load of such members on an almost totally empirical basis. Unfortunately, the empiricism of the analytical methods has led to design procedures which are cumbersome and obscure. With the advent of prestressed concrete, the procedures have become far more complex.

From a scientific point of view, an empirical approach is valid only when the identification and control of the main variables in the test program are assured, and sufficient tests are conducted to allow a statistical treatment of the results. In most research programs the constraints of time and money mean that the previous conditions are rarely met. Moreover, because of the large amount of work required in order to substantiate such empirical methods, the more general studies of the basic behavior of beams and the way in which the overall member carries shear and torsional forces have often been neglected.

The lack of fundamental behavioral models for concrete members subjected to shear and torsional loadings seems to be the prime reason for the unsatisfactory nature of the current highly empirical design procedures used in North American codes and standards.

In the late 60's, researchers in Europe were working with the idea of a conceptual model to properly represent the behavior of concrete members subjected to torsion and shear. The main objectives were to rationalize and at the same time simplify the design procedures in these areas. In Switzerland, Lampert and Thürlimann (93) developed a conceptual model based on theory of plasticity. The model was a Space Truss with variable angle of inclination of the diagonal compression members. This model was a refined version of the Truss Model with a constant 45 degree angle of inclination of the diagonal compression members introduced in Switzerland at the beginning of this century by Ritter (150) for the case of shear in reinforced concrete members. During the late 70's, in Canada, Mitchell and Collins (118-120) also proposed a generalized design approach based on a theoretical model. This was a major departure from the highly empirical approach followed in the American practice. Mitchell and Collins were able to treat general problems of shear and torsion in both prestressed and reinforced concrete members in a unified rational fashion. However, the authors fell short of providing the designer with a simple and easy-to-apply design method. The advantages of the procedure proposed by Mitchell and Collins were obscured because of the overtheoretical approach followed in the proposed design recommendations. The design procedure ends up being complex with long semiempirical equations. These equations, although adequate to evaluate the strength and deformations of members subjected to shear and torsion, tend to obscure the physical model on which the overall procedure is based.

It seems obvious that designers will not be too eager to adopt new complex design methods, even if these are accurate, when previously they have ignored torsion without disastrous consequences. For this reason, a rational and easy to apply approximate design approach based on a simplified model, considering only the main variables, is preferable.

1.3 Objectives and Scope of the Study

The present study attempts to answer the challenge posed by the ACI-ASCE Committee 426 (28):

> During the next decade it is hoped that design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner.

An overall review of the current AASHTO Specifications and the ACI Building Code in the Areas of shear and torsion is summarized in Chapter 2. This study shows that design procedures have become more and more complex with every revision. The highly empirical provisions are difficult to use in many design situations.

The wide array of previous tests and detailed empirical equations resulting from these tests have not provided designers with simple general procedures which could be represented as a clear model to handle special and unusual variations.

Ironically, it has been precisely the extensive amounts of detailed testing required to substantiate the empirical approaches which are the probable cause for the lack of studies focusing on the basic generalized shear and torsion behavior of beams. A clearer understanding of such mechanisms within an overall framework encompassing a wide variety of applications would have directed researchers towards more basic conceptual models. This in turn would have led to simpler design rules that would not involve as many detailed and test-dependent variables.

A clear example of the benefits of a good physical model in shear design in reinforced concrete is given by the approach followed in the design of brackets and corbels using the shear friction theory combined with the elementary truss model. It is remarkable how easily the designer is able to envision the effects of the forces acting on such elements, and then provide structural systems capable of resisting those effects. More striking yet are the relatively simple design procedures and code requirements stemming from such an approach.

Consequently, it is the nature of the empirical approach, and of its consequence, the lack of a conceptual model, which are the primary reasons for the complex and fragmented design approach to shear and torsion reflected in current codes and specifications.

The main objective of this study is to propose and evaluate a design procedure for shear and torsion in reinforced and prestressed concrete beams. The goal is to clarify and simplify current design recommendations and AASHTO requirements in such areas. The basic reevaluation of the current procedures and development of new procedures are to be carried out using a conceptual structural model rather than detailed empirical equations wherever practical.

Chapter 3 summarizes the space truss model with variable angle of inclination of the diagonal elements. This model was selected as the one which best represents the behavior of reinforced and prestressed concrete beams subjected to shear and torsion. This conceptual model was developed by a number of European engineers over the past 15 years. Principal contributions were made by the Zürich group (Thürlimann et al.), the Copenhagen group (Nielsen et al.), and more recently by the Canadian group in North America (Collins and Mitchell). Much of the European work has been based on highly complex proofs of the application of plasticity theorems in the fields of shear and torsion. The complete formulations are generally not in English and are quite complex. The more limited reports, which are in English, have not had wide American readership. The apparent complexity of the proofs of the plasticity theorems as applied to shear and torsion can cause the more design-oriented reader to lose sight of the fact that the authors use these proofs only as a theoretical basis for proving the application of a refined truss model. The truss model is shown to be a plasticity lower bound solution giving the same result as the upper bound solution. Hence, it is a mathematically valid solution which correctly represents the failure load.

The highlights of the refined truss model approach are the relatively simple design procedures that can be developed from the space truss model, and the extremely logical way the designer can envision providing and proportioning reinforcement for shear and torsion under special circumstances as in the case of box sections, concentrated loads on lower flanges, etc.

However, it was felt that before the generalized refined truss model approach could be used as the basic design procedure in American practice, a complete evaluation of the accuracy of the model using a significant body of the available test data reported in the American literature was necessary. In companion Report 248-3, thorough comparisons of the space truss model with a wide range of test data and with predicted failure loads from other design procedures are presented.

In companion Report 248-4F, the general procedures derived from the space truss model are translated into design recommendations and draft AASHTO requirements are suggested. Design applications for typical highway structures using the proposed design recommendations as well as the current AASHTO approach are presented for comparison in Report 248-4F.

CHAPTER 2

REVIEW OF AASHTO AND ACI DESIGN PROCEDURES FOR SHEAR AND TORSION IN REINFORCED AND PRESTRESSED CONCRETE BEAMS

2.1 Introduction

A comprehensive review, dealing with all of the factors influencing behavior and strength of reinforced and prestressed concrete beams failing in shear and/or torsion, and all of the ways researchers and designers have attempted to mold these factors into code or specification formats would be a monumental task. Not only are these factors numerous and complex, but the individual contributions of researchers are difficult to integrate into an orderly and comprehensive body of knowledge.

In Chapter 2, the historical development of AASHTO and ACI design procedures for shear and torsion in prestressed and reinforced concrete members is followed. An effort is made to try to illustrate the factors that previous researchers have considered to be of great influence in the overall behavior of members subjected to shear and/or torsional stresses. Following a parallel course, a presentation of the manner in which those factors have been translated into Code or Specification formats is carried out.

The driving force behind the overall review rests in the hope that such study might provide some clues which will indicate the reasons for present design approaches and make decision makers less hesitant to adopt a major shift in the basic approach.

2.2 Shear in Reinforced Concrete Beams

It is reasonable to believe that the concepts of vertical dowel action formed the basis for early designs of web reinforcement (78). Early pioneers of reinforced concrete before the year 1900 developed two schools of thought pertaining to the mechanism of shear failures in reinforced concrete members. One school of thought considered horizontal shear as the basic cause of shear failures. This seemed a reasonable approach at a time when scholars and engineers were familiar with the action of shear-keys in wooden beams, for which horizontal shearing stresses were computed using the well-known equation for the shearing stress in a homogeneous beam

$$v = VQ/Ib \tag{2.1}$$

where:

v = horizontal shear stress

V = applied shear force

Q = static moment of cross section area, above or below the level being investigated for shear

I = moment of Inertia

b = width of cross section

Reinforced concrete beams were treated as an extension of the older materials such as wood assuming that concrete without web reinforcement could only resist low horizontal shearing stresses, and that the role of vertical stirrups was to act as shear keys for higher shearing stresses (see Fig. 2.1).

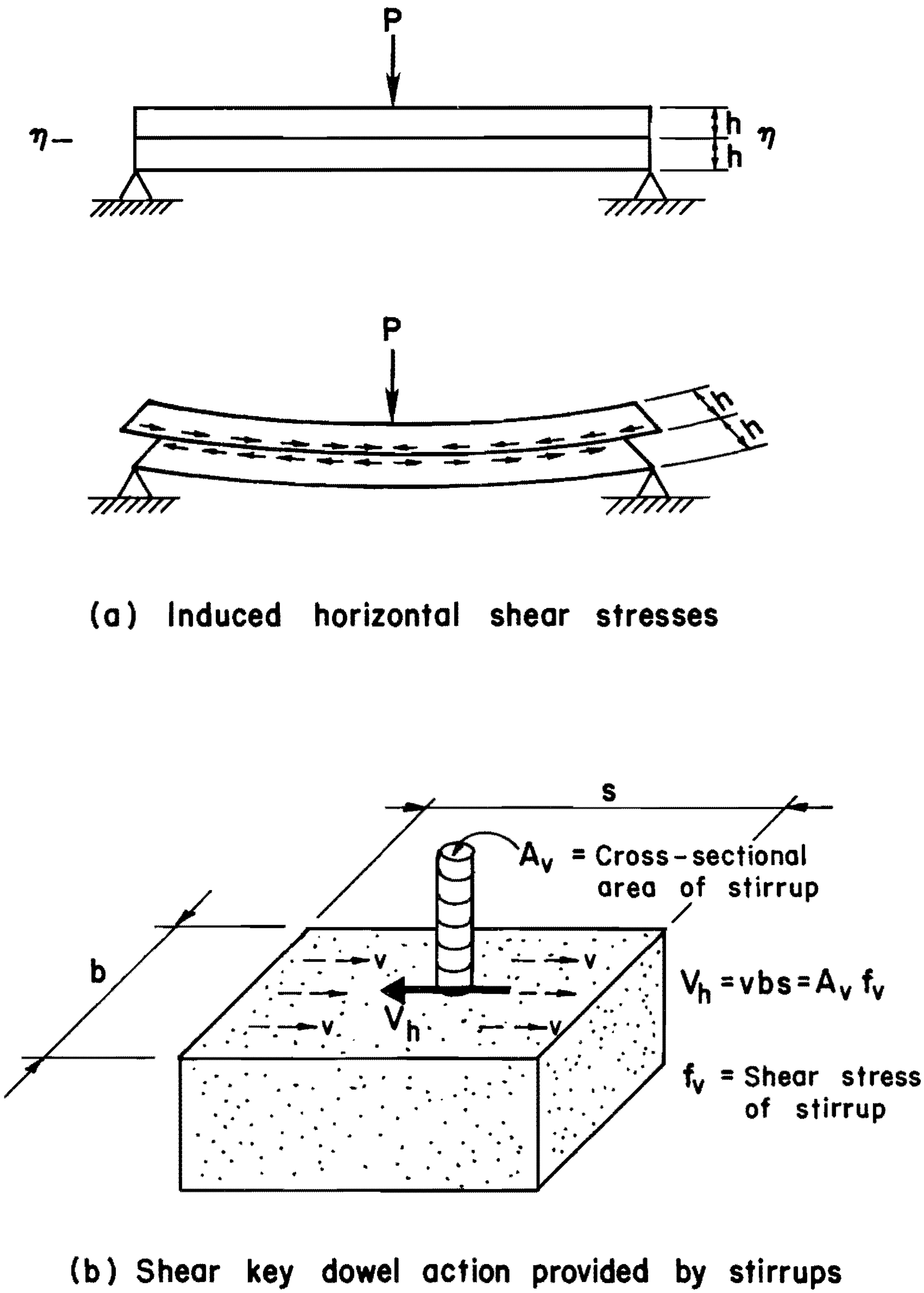


Fig. 2.1 Horizontal shear stresses and concept of dowel action as a shear key

For instance, P. Christophe (51) recommended, "This stress (horizontal shear) divided by the sectional area of the metal (in stirrups) gives the unit resistance of the reinforcement which must be below its limit of resistance to shearing." This theory concerning horizontal shear was apparently founded and developed in Europe and only a few American engineers defended these concepts (78).

The second school of thought, accepted by nearly all engineers today, considered diagonal tension the basic cause of shear failures.

Consider the case of a beam subjected to a shear force and a bending moment. Prior to cracking, the state of stresses would correspond to that shown on Figure 2.2(a). The state of stress for an element at the neutral axis and a corresponding Mohr circle for such state of stresses (see Fig. 2.2b) would show that maximum tensile stresses exist at an angle of 45 degrees with respect to the longitudinal axis of the member. Such stresses were referred to as diagonal tension stresses.

Based on the concepts of diagonal tension, W. Ritter (150) presented as early as 1899 the concept of a "Truss Analogy" for design of web reinforcement. After specifically referring to Hennebique's view that the stirrups resisted the horizontal shearing stresses, Ritter stated:

> In this connection, one ordinarily imagines that the stirrups together with the stem and the concrete form a type of truss [Ritter refers to the figure given in Fig. 2.3(a)] in which the stirrups act as the hinged end tension bars and the concrete working in the direction of the dashed lines, acts as the diagonal struts. These lines will routinely be assumed at 45° corresponding to the compressive pressure curve.

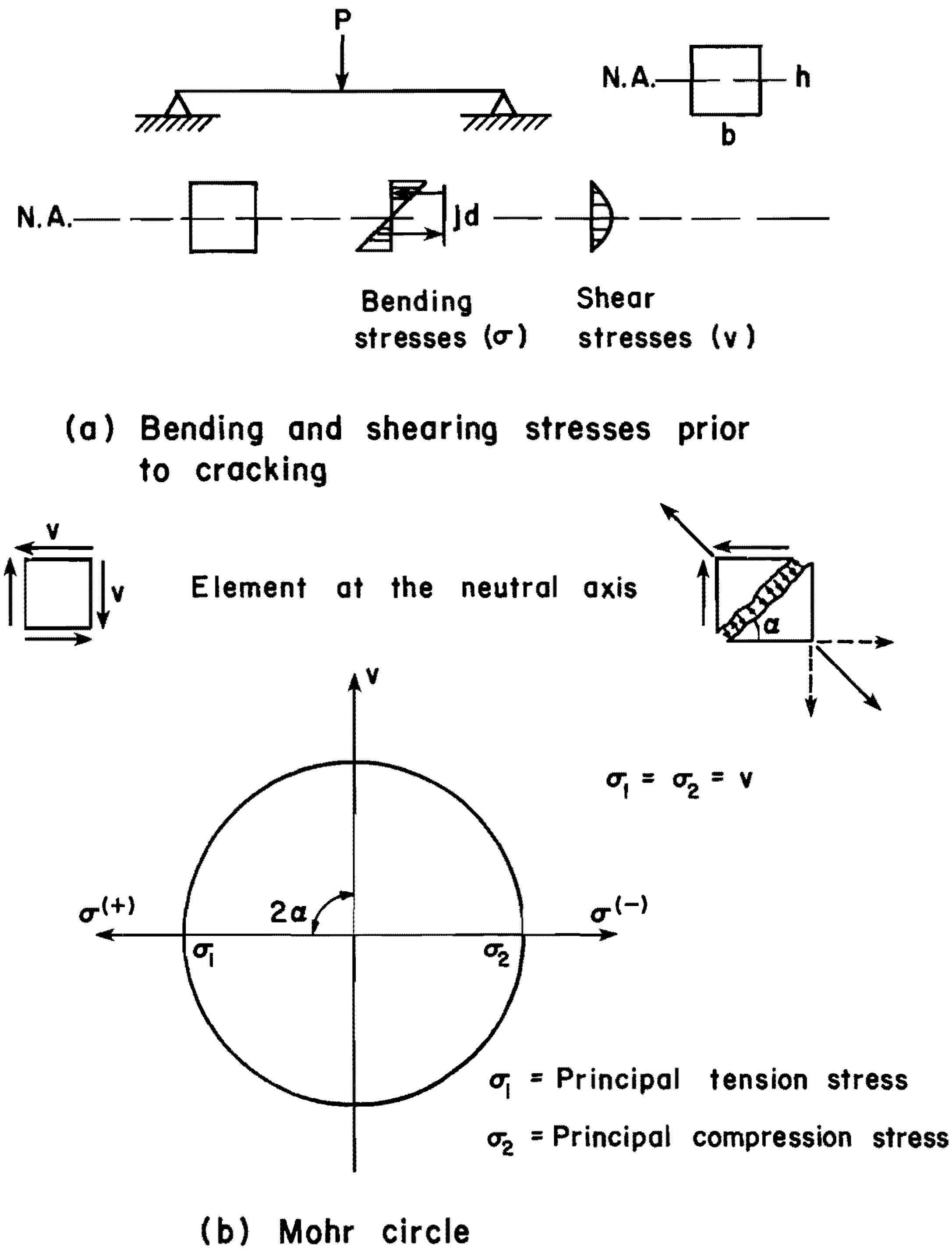
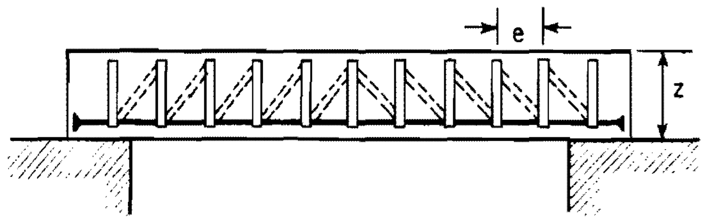
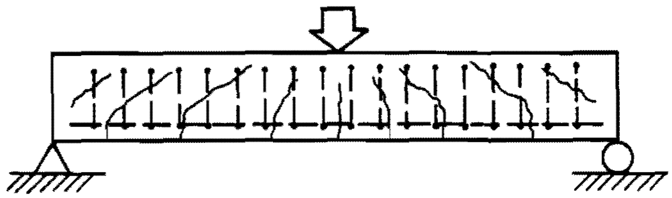


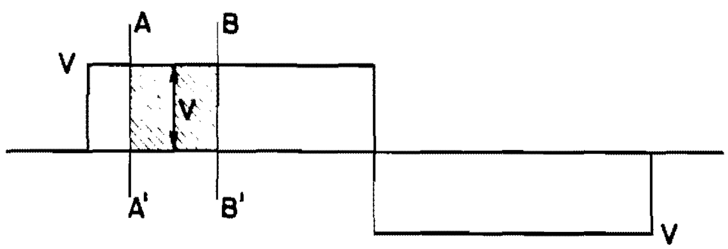
Fig. 2.2 Concept of diagonal tension stress



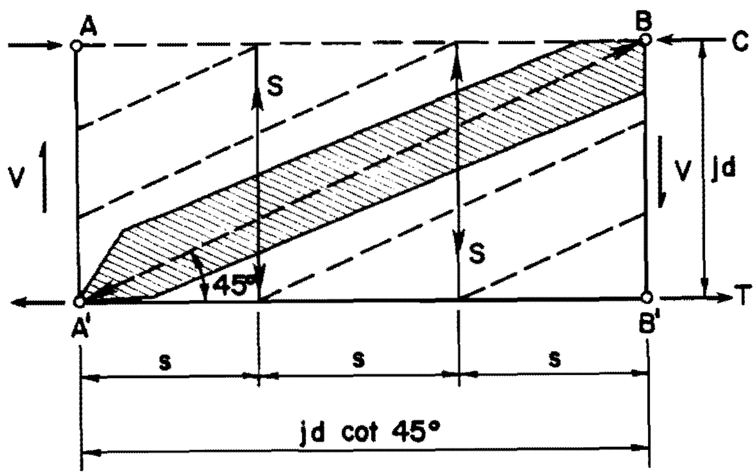
(a) Ritter's truss model



(b) Analogous model



(c) Shear zone AB



(d) Compression strut

Fig. 2.3 45° truss model

[Translator's note...the German text says "unter 45^o", although with qualification it could also mean "below or up to 45^o"]

On the basis of this view, the stirrups will be only calculated from statics and in fact according to the formula Q = 2 σbd, where Q indicates the shear force, and b and d are the width and thickness (respectively) of the flat steel bars used as stirrups. The factor 2 is included therein because each stirrup has two legs...

The formula presupposes that the stirrup spacing e is equal to the distance z between the compression and the tension centroids. If one makes e greater than z, the stresses increase proportionately. It is thus general as

$$\sigma = Qe/2bdz$$

....The mode of action of the stirrups however, exists in my opinion as expressed earlier herein, that they resist the tensile stresses acting in the direction of the (diagonal) tension curves and they prevent the premature formation of cracks. To this end they have to of course be provided approximately at 45^o; yet this arrangement would complicate the construction.

That the stirrups in a vertical position also increase the load capacity of the beam, one can hardly deny; however, in what way the formula above can make claims to reliability and corresponds to the relative relationship, cannot easily be determined on a theoretical basis. Here, comparative tests might be appropriate.

Putting Ritter's truss model theory into a more modern context leads to the analogy shown in Fig. 2.3(b) to (d).

After a reinforced concrete beam cracks due to diagonal tension stresses, it can be idealized as a truss member. In this truss model, the horizontal compression chords are provided by the concrete and the steel in compression, C. The vertical elements are provided by the web reinforcement (stirrups), S. The horizontal tension chord is provided by the longitudinal steel reinforcement acting in tension, T. Finally, between the diagonal tension cracks, concrete diagonal struts subjected to compression stresses are formed. Ritter assumed their inclination at

45 degrees and stated that those diagonal struts are the inclined members of the truss model. Based completely on vertical equilibrium, the design of vertical stirrups is given as:

$$V = A_v f_v jd/s \tag{2.2}$$

where V = shearing force acting at the section under consideration, A_v = area of stirrups crossing the crack, jd = internal moment arm, f_v = yield stress of stirrup reinforcement, and s = spacing of stirrups along the beam axis (see Fig. 2.3d). This expression is essentially the same one currently used for V_s by ACI 318-77 (24) and AASHTO Specifications (17).

Discussion and debate between the proponents of horizontal shear theories and of diagonal tension theories continued for nearly a decade until laboratory tests resolved the issue, mainly through the efforts of E. Morsch (122) in Germany. He concluded that it was diagonal tension that caused the shear failures and, like Ritter, presented the Truss Analogy for the design of web reinforcement.

In 1906 M. O. Withey (171,172) introduced Ritter's equation into the American literature. He found that this equation gave tensile stresses in the stirrups which were too high when compared with values obtained from actual test results. Withey indicated that the concrete of the compression zone may carry considerable shear even after the web below the neutral axis is cracked in diagonal tension. He also indicated a possible vertical shear transfer by dowel action of the longitudinal reinforcement (78). A large number of tests in which beams failed in diagonal tension were also carried out in the United States in these early years. One of the first laboratory studies was reported at the University of Wisconsin in 1906 (78). The author, E. A. Moritz, presented a basically sound discussion of "inclined tension failures". The first study by A. N. Talbot was also presented in 1906 (78). He developed a formula similar to that previously suggested by E. Morsch (122):

$$v = V/bjd \tag{2.3}$$

Talbot pointed out that the diagonal tension stress equals the horizontal shearing stress if no flexural tension is taken by the concrete as assumed in the standard theory.

In 1909, Talbot (160) presented a study of web stresses, including tests of 188 beams. The conclusions of this report are indeed important. In particular, the conclusion referring to beams with stirrups said:

> Stirrup stresses computed by Ritter's equation appear too high. It is therefore recommended that stirrups be dimensioned for two-thirds of the external shear, the remaining one-third being carried by the concrete in the compression zone. (78)

The National Association of Cement Users, the forerunner of the present American Concrete Institute, published its first code recommendations in 1908 (74). This report was essentially based on what has later become known as ultimate strength design. In this report the NACU specified that:

> .....when the shearing stresses developed in any part of a reinforced concrete constructed building exceeds, under the multiplied loads, the shearing strength as fixed by the section, a

> sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance of shear is overcome.

Hence, the various sections were dimensioned on an ultimate basis for a load 4 times the total working load. No formulas were presented for design of web reinforcement. The progress report of the First Joint Committee (137) was revised in 1909, and adopted as NACU Standard No. 4 in 1910. This code introduced the concept of working stresses, departing from the ultimate load concept introduced in the 1908 report. This was to be the format of later codes up to 1963. The NACU Standard No. 4 clearly indicated the principles of diagonal tension, and following Talbot's recommendation based on laboratory tests, proposed that the web reinforcement be designed to carry two-thirds of the total shear with the concrete carrying the remaining one-third (78).

Thus, by 1910, in the United States, the concepts of diagonal tension and a dual shear carrying mechanism, formed by the web reinforcement contribution obtained from a 45 degree truss analogy and a concrete compression shear contribution, were established in the treatment of shear in normal reinforced concrete beams.

The progress report of the First Joint Committee (137) established the general shear design philosophy followed by succeeding codes:

> Calculations of web resistance shall be made on the basis of maximum shearing stresses or determined by the formulas hereinafter given ($v = V/bjd$).
>
> When the maximum shearing stresses exceed the value allowed for concrete alone, web reinforcement must be provided to aid in carrying diagonal tensile stresses. The following allowable values

for the maximum shearing stresses are recommended (based on f'_c = 2000 psi; may be increased proportional to f'_c but this increase shall not exceed 25%):

a. beams with no web reinforcement, 40 psi.

b. for beams in which a part of the horizontal reinforcement is used in the form of bent-up bars, arranged with respect to the shearing stresses, a higher value may be allowed, but not exceeding 60 psi.

c. For beams thoroughly reinforced for shear, a value not exceeding 120 psi.

In the calculation of web reinforcement to provide the strength required in (c), the concrete may be counted upon as carrying 1/3 of the shear. The remainder is to be provided for by means of metal reinforcement consisting of bent-up bars or stirrups, but preferably both. (See Fig. 2.4a.)

The development of code regulations continued along these same lines. ACI reports in 1916 and 1917 (26) recommended the allowable shearing stress to be resisted by the concrete as $0.02f'_c$ with 66 psi inferred to be the maximum limit. The excess shear up to a ceiling value of $0.075f'_c$ (247 psi maximum inferred) could be resisted by web reinforcement (see Fig. 2.4b).

Another ACI report appeared in 1919 (78), which departed radically from earlier reports. The allowable nominal shearing stresses for beams without web reinforcement was maintained at $0.02f'_c$ (66 psi maximum implied), and the ceiling value for beams with web reinforcement was maintained at $0.075f'_c$ (248 psi maximum implied) provided that the longitudinal bars were anchored. In this report there was a change in the whole philosophy of design for shear. It was indicated that if the shear stress was greater than $0.02f'_c$, the shear reinforcement had to be provided for the entire shear with no allowance for v_c (see Fig. 2.4c).

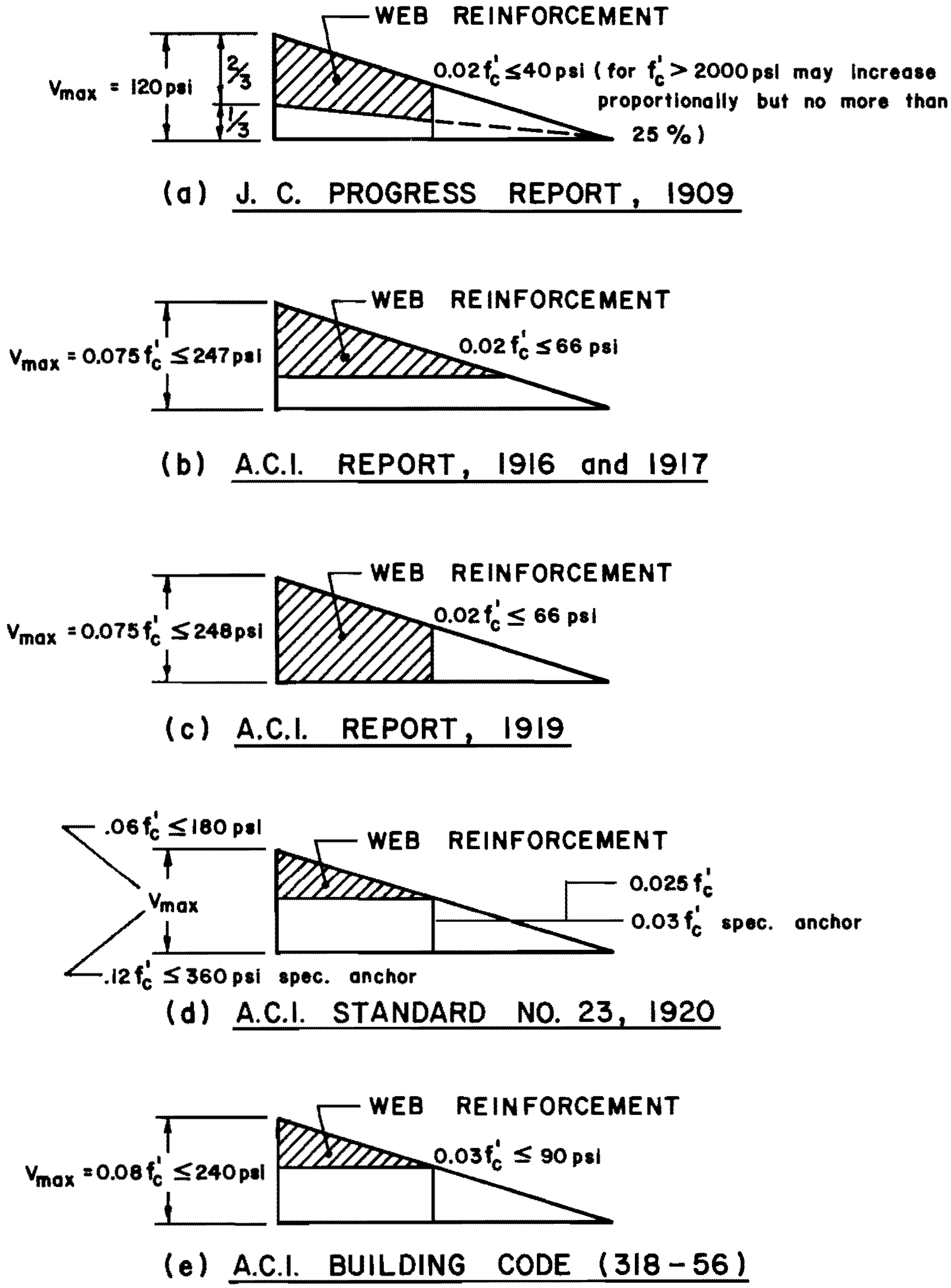


Fig. 2.4 American specifications for shear design
[from Ref. 26]

The web reinforcement was designed using Ritter's Equation 2.2, but V was the total shear force. This stipulation was perhaps inspired by contemporary German Codes (78) which follow this same philosophy.

The ACI Standard Specification No. 23 of 1920 (78) represented an almost complete development of American philosophy on design of web reinforcement. In this version the change was made back to the design philosophy existing prior to the 1919 report. The 1920 Standard Specification again recognized the concrete contribution in shear past the limiting value accepted for beams with no web reinforcement (see Fig. 2.4d). The specification allowed the following nominal shearing stresses: For beams without web reinforcement, $0.02f'_c$ (60 psi maximum); for beams without web reinforcement, with special anchorage of longitudinal reinforcement, $0.03f'_c$ (90 psi maximum). Web reinforcement was designed by the equation

$$A_v f_v = V's \sin\theta/jd \tag{2.4}$$

where:

V' = total shear minus $0.02\ f'_c bjd$ (or $0.025\ f'_c bjd$ with special anchorage).

s = spacing of shear steel measured perpendicular to its direction.

θ = angle of inclination of the web reinforcement with respect to the horizontal axis of the beam.

The limiting value for nominal shearing stresses was $0.06f'_c$ (180 psi maximum), or with anchorage of longitudinal steel $0.12f'_c$ (360 psi maximum) (78) (see Fig. 2.4d).

This basic procedure lasted from 1921 to 1956 with only minor changes in allowable web stresses and limitations on f'_c (18). However, in ACI 318-51

> the provision for beams with web reinforcement and special anchorage of longitudinal reinforcement was omitted and replaced by the specification that all plain bars must be hooked, and deformed bars must meet ASTM A305. Therefore, $0.12f'_c$ was the maximum allowable unit stress for all beams with web reinforcement. (19)

The 1956 ACI Building Code (20), based on allowable stresses, specified that if the unit shearing stress is greater that $0.03f'_c$ web reinforcement must be provided for the excess shear. For beams with longitudinal and web reinforcement, the allowable unit stress was reduced to $0.08f'_c$ with a maximum value of 240 psi (see Fig. 2.4e).

Calculation of the area of vertical stirrups continued to be based on Ritter's truss model in which it was assumed that the shear on a section less the amount assumed to be carried by the concrete, is carried by the web reinforcement in a length of beam equal to its depth.

At this time, early 1950's, important changes in the design procedures for shear in reinforced concrete members were about to take place. One of the major difficulties in relating theoretical and laboratory investigations to failures of full scale structures is that in actual structures, failures usually occur from several contributory causes. It is often difficult to gather all of the pertinent facts and to determine the degree to which they contributed to failure. An exception was the warehouse failure at Wilkins Air Force Depot in Shelby, Ohio, which occurred in 1955. This massive shear induced failure intensified doubts and questions about design procedures used to evaluate the diagonal tension strength of beams. This failure in conjunction with intensified research work brought about a clear realization that shear and diagonal tension was a complex problem involving many variables. This actually represented a return to forgotten fundamentals.

Based on a general concept that shear failure in reinforced concrete beams is a tensile phenomenon, design specifications in the United States up until 1956 considered the nominal shearing stress, $v = V/bjd$, to be a measure of diagonal tension, and related it to the cylinder compressive strength f'_c as the only principal variable.

A. N. Talbot (160) pointed out the fallacies of such procedures as early as 1909:

> It will be found that the value of v (nominal shearing stress) will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam.

He substantiated these statements with test results for 106 beams without web reinforcement, and he concluded as follows:

> In beams without web reinforcement, web resistance depends upon the quality and strength of the concrete....
>
> The stiffer the beam the larger the vertical stresses which may be developed. Short, deep beams give higher results than long slender ones, and beams with high percentage of reinforcement than beams with a small amount of metal....

Unfortunately Talbot's findings were not expressed in mathematical terms, and became lost as far as design equations were concerned. In the interval between 1920 and the late 1940's, the early experiments regarding effects on shear strength of the percentage of longitudinal reinforcement, and the length to depth ratio, were forgotten.

A return to these fundamentals began in 1945 with O. Moretto (121). In reporting a series of tests he presented an empirical equation for shear strength which included the percentage of longitudinal tensile reinforcement as a variable. The beneficial effect of the steel percentage on the shear strength of beams with no web reinforcement may be explained in two ways:

1. Dowel action of the reinforcement: in this case the longitudinal steel crossing the crack acts as a horizontal dowel resisting the shearing displacements along the crack.

2. With reducing amounts of longitudinal reinforcement the flexural cracks extend higher into the beam and are wider, reducing the amount of shear that can be transferred across the crack.

In 1951, A. P. Clark (53) introduced an expression which involved the span-to-depth ratio a/d, where a was the length of shear span and d was the effective depth of beam. He thus recognized the effect that small a/d ratios have on the shear strength of such members (see Fig. 2.5). The increasing shear strength obtained with smaller a/d ratios may be explained by the fact that in regions such as supports or under point loads local state of compression might be induced. This state of compression delays the appearance of diagonal tension cracks, thus increasing the shear strength of the beam. Therefore, the closer the point of application of the load is to the reaction producing local compression in the member, the more difficult it will be for diagonal cracking to occur. This concept of compression bulbs in zones where the support would induce such stresses led to the provision that sections

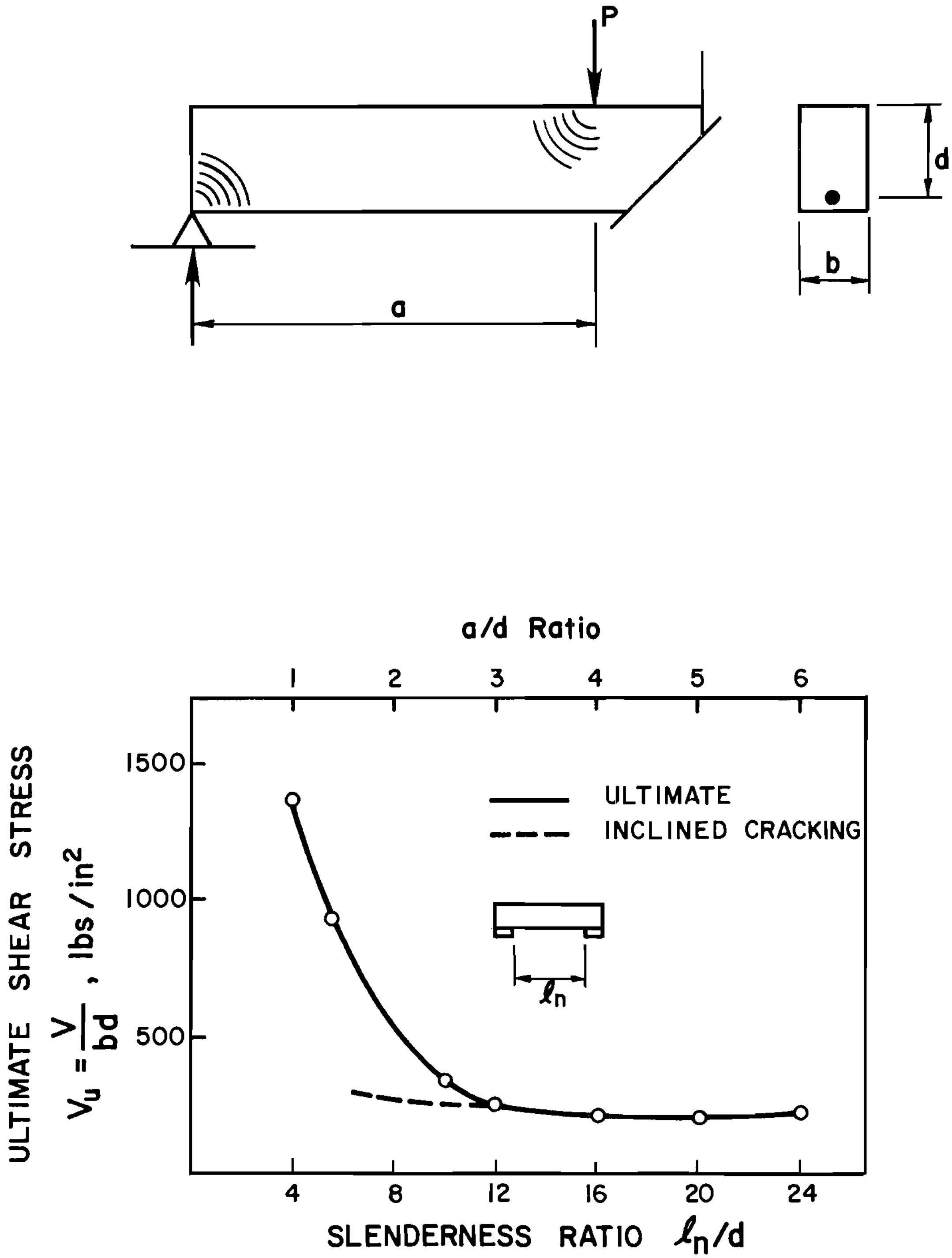


Fig. 2.5 Effect of shear span-to-depth ratio on shear strength of beams with no web reinforcement [from Ref. 26]

located less than a distance d from the face of the support might be designed for the same shear force as that computed at a distance "d".

However, use of an a/d term was handicapped since the shear span "a" could not be defined for generalized cases of loading. In simple beams with a single point load, or with two symmetrical point loads the term "a" is the distance from a load point to the nearest support. For other loading conditions such as uniformly distributed loads, the term "a" has no direct physical meaning.

The difficulty was later overcome by a slight modification of the general concepts of diagonal tension. Shear failures of beams are characterized by the occurrence of inclined cracks. The manner in which inclined cracks develop and grow and the type of failure that subsequently develops is strongly affected by the relative magnitudes of the shearing stress, v, and the flexural stress, f_x. As a first approximation, these stresses may be defined as:

$$v = k_1 \, V/bd \quad \text{and} \quad f_x = k_2 \, M/bd^2 \tag{2.5}$$

in which k_1 and k_2 are coefficients depending on several variables, including geometry of the beam, the type of loading, the amount and arrangement of reinforcement, and the type of steel. The values, V and M, are the shear and moment at a given section respectively; b = width of the web section for rectangular beams; and d = distance from the tension reinforcement to the extreme compression fiber. The ratio f_x/v is thus:

$$f_x/v = k_3 \, M/V_d \tag{2.6}$$

in which $k_3 = k_2/k_1$. The shear, V, is a measure of moment gradient; V = dM/dx. For beams subjected to concentrated loads this relation may be expressed by V = M/a where "a" is the shear span. Thus a = M/V and a/d = M/V_d. Hence, the shear span to depth ratio is in reality relating the effect of horizontal flexural tension on diagonal tension. This thought then led to the adoption of the M/V_d ratio as a substitute for the a/d term. For the case of simple beams with point loads both expressions are synonymous. For any other loading condition M/V_d still has physical significance at any cross section of the beam (see Fig. 2.6). A large percentage of the laboratory tests used to verify basic shear theories consisted of beams with no web reinforcement and subjected to one or two concentrated loads in any one span (26).

Based on the results of 194 beams from studies carried out in the late 1940's and continued through the 1950's, ACI Committee 326 (26) in 1962 proposed a design equation to evaluate the diagonal tension strength of members without web reinforcement. The equation, which is still present in current AASHTO (17) and ACI (24) ultimate strength design specifications is:

$$v_c = 1.9\sqrt{f'_c} + 2500 \quad V_d/M \leq 3.5\sqrt{f'_c} \qquad (2.7)$$

Finally, in 1962, Talbot's notions were expressed by an empirical equation involving three variables--percentage of longitudinal reinforcement, ρ, ratio of beam length to depth, M/V_d, and concrete strength $\sqrt{f'_c}$ rather than f'_c.

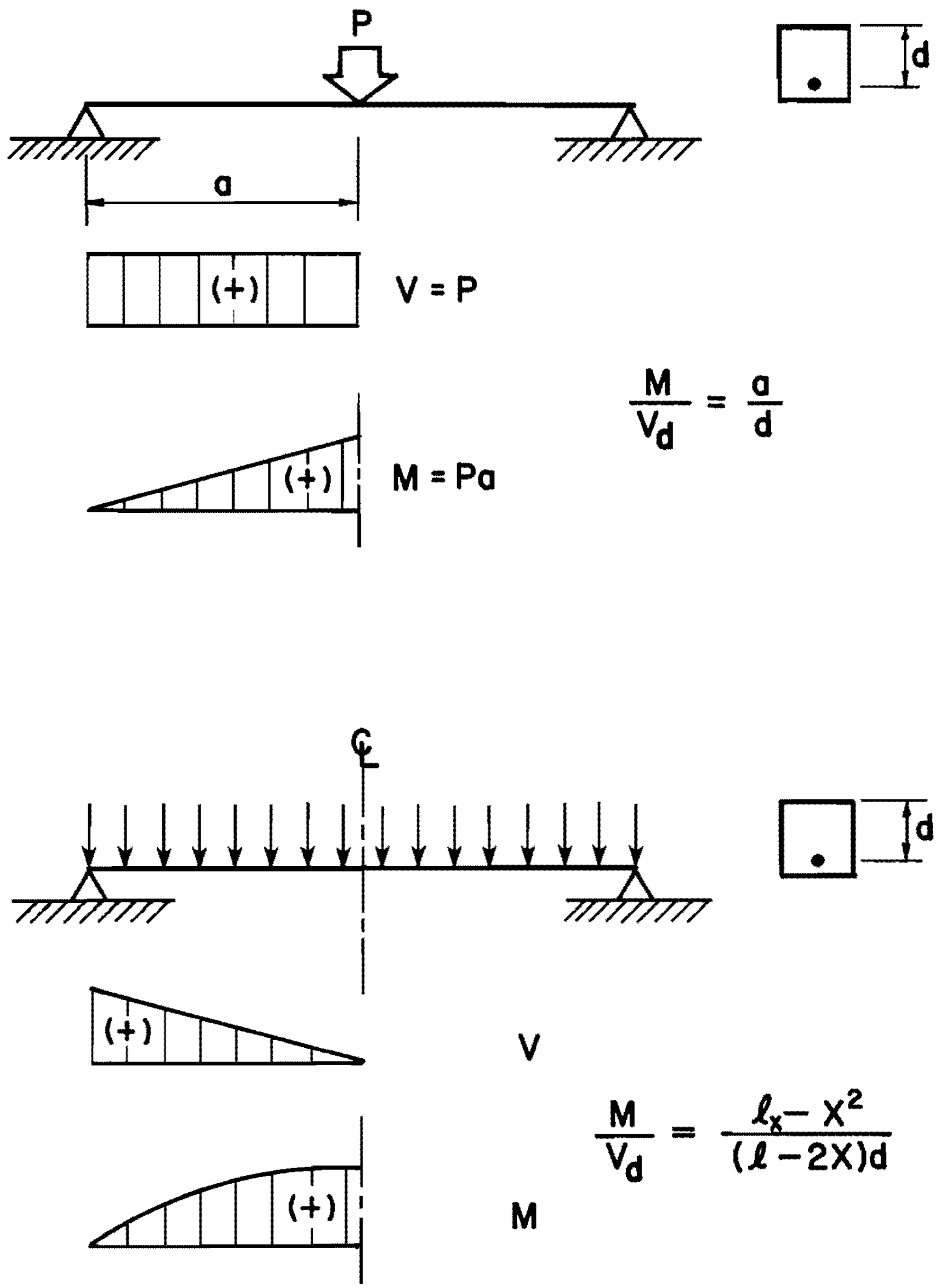


Fig. 2.6 Extension of the a/d ratio into an M/V_d ratio

ACI Committee 326 (26) in 1962, established the basis for current design procedures. The proposed procedure was based on the following assumptions:

1. For a beam with no web reinforcement, the shearing force which causes the first diagonal cracking can be taken as the shear capacity of the beam. For a beam which does contain web reinforcement, the concrete is assumed to carry a constant amount of shear, and web reinforcement need only to be designed for the shear force in excess of that carried by the concrete.

2. The amount of shear that can be carried by the concrete at ultimate is at least equal to the amount of shear that would cause diagonal cracking.

3. The amount of shear carried by the reinforcement (stirrups) is calculated using the truss analogy with a 45 degree inclination of the diagonal members.

It was also suggested that the refinement of using the internal lever arm, jd, in computing the average shear stress was not justified. It was recommended that average shear stresses should be calculated simply as:

$$v = V/bd \tag{2.8}$$

The ACI code of 1963 (21) in its ultimate strength design section (factored load) was based entirely on the 326 report for the design for shear in reinforced concrete beams. ACI 318-63 adopted the Eq. (2.7) to evaluate v_c and added a simplified and conservative alternative where the second term of such equation equals $0.1\sqrt{f'_c}$, so that v_c is taken as $2\sqrt{f'_c}$ (21). The area of steel required for vertical stirrups was evaluated on the basis of Ritter's equation:

$$A_v = [V_s s/(f_y d)] \tag{2.9}$$

where V_s was the difference between the total ultimate shear and the shear that could be carried by the concrete evaluated using Eq. (2.7). A conservative upper limit for the shear stresses of $10\sqrt{f_c'}$, was adopted based on test observations (22). In addition, a limit on the minimum amount of web reinforcement (where required) of $r = A_v/bs = 0.0015$ was established. Finally, based on tests of beams with stirrups of high yield strength, 60 ksi was set as the upper limit for the tensile strength of the steel used as web reinforcement.

The 1971 ACI Building Code (23) was the first ACI code to be based almost wholly on ultimate strength concepts. However, the design procedures were basically the same as those introduced in the 1963 code (21) in the ultimate strength design section. Only two changes were made; the minimum percentage of web reinforcement was now set as $r = 50/f_y$ (in psi), and a minimum amount of web reinforcement was always required when the shear stresses exceeded 1/2 of the shear that could be carried by the concrete alone.

The edition of the ACI Building Code published in 1977 (24) included only a minor change in its presentation format. Shear is now presented in terms of forces rather than stresses. This version reflects the same design concepts adopted in the 1971 and 1963 codes. In the proposed changes to the ACI Building Code for the 1983 edition the design procedures remain basically the same as in the 1977 Code; however, some changes were introduced. There is a redefinition of the web width "b_w," in the case of joists with a tapered web. It was proposed that if the web was in flexural tension, b_w should be taken as the average web width. This change was withdrawn in the standards action. The definition of maximum design shear as the shear existing at a distance "d" from the face of the support in the case where the support reaction in the direction of the applied shear introduces compression into the end regions of the member, was further restricted to cases where no abrupt change in shear, such as a heavy concentrated load, occurs between the face of the support and a section "d" away. This subject was previously addressed in the Commentary to the ACI 318-77 Building Code.

Previous to and including the 1973 AASHO Standard Specifications for Highway Bridges (10), the service load design method for concrete bridge beams subject to shear closely followed the ACI Building Code requirements.

The 1935 AASHO Specifications (1) allowed the following shear stresses (diagonal tension) in the concrete;

- For beams without web reinforcement 60 psi if the longitudinal bars were not anchored, and 90 psi if anchored.

- For beams with shear reinforcement and anchorage 160 psi.

Those values were based on concretes with f'_c = 3000 psi. For concretes having less strength, the unit stress should be proportionately reduced. The shearing unit stress was evaluated as $v = V/bjd$. The web reinforcement was designed using the formula

$$A_v = V's/(f_v jd) \tag{2.10}$$

where A_v = total area of web reinforcement in tension within a distance s; f_v = tensile unit stress of the web reinforcement assumed equal to 16,000 psi; jd = arm resisting internal flexure couple; s = spacing of web reinforcement bars measured at the neutral axis and in the direction of the longitudinal axis of the beam; V' = unfactored external shear at the section being considered after deducting that carried by the concrete.

The 1941 AASHO Standard Specifications for shear (2) were essentially the same as the 1935 edition except for a change in allowable shear stresses in beams with web reinforcement. It was adopted as 140 psi when longitudinal bars were not anchored, and to 180 psi if anchored.

In the 1944 (3) and 1949 (4) editions only one change was introduced. The allowable stresses were set as a function of the ultimate concrete compressive strength f'_c:

- For beams without web reinforcement $0.02f'_c$ if longitudinal bars not anchored, or $0.03f'_c$ if anchored.
- For beams with web reinforcement $0.046f'_c$ if longitudinal bars not anchored, $0.06f'_c$ if anchored.

However, the allowable stresses were limited to those values obtained from the previous relations using f'_c of 4500 psi.

The 1953 AASHO Specifications (5) continued to be based on working stress concepts. The evaluation of the amount of required web reinforcement remained the same as in previous editions and so did the allowable stresses for beams with no web reinforcement. However, for the case of beams with web reinforcement the distinction between anchored or not anchored was eliminated, and simply specified as $0.075f'_c$ for all cases. The allowable tensile stress was increased to 18000 psi for structural grade reinforcement.

The requirements for the 1957, 1961, and 1965 editions (6,7,8) remained essentially the same as the ones in the 1953 specifications. Only one change was introduced in the maximum allowed shear stresses in the case of beams without web reinforcement. The stresses were limited to 75 psi when longitudinal bars were not anchored, and 90 psi if anchored.

The 1969 version (9) was the last one completely based on the working stress design approach of unfactored loads and allowable stresses. In this edition the requirements remained the same, except for the allowable tensile stress of the reinforcement f_v, which was set as 20,000 psi for all grades of steel.

The 1973 AASHO Specifications (10) also allowed the concrete to resist an external shear stress of $0.03f'_c$ (90 psi maximum), with any excess shear stress to be resisted by the web reinforcement. The maximum allowable unit shear stress for beams with longitudinal and web reinforcement was limited to $0.75f'_c$. The equation for the area of vertical stirrups, $A_v = V's/f_v jd$, is exactly the same as that used in ACI 318-56 (12) where f_v is the tensile unit stress in the web reinforcement (see Fig. 2.7a).

In addition to the service load design method, the 1973 AASHO Specifications (10) also include a load factor design method. It requires that the shear stress capacity of the concrete, v_{uc}, shall not

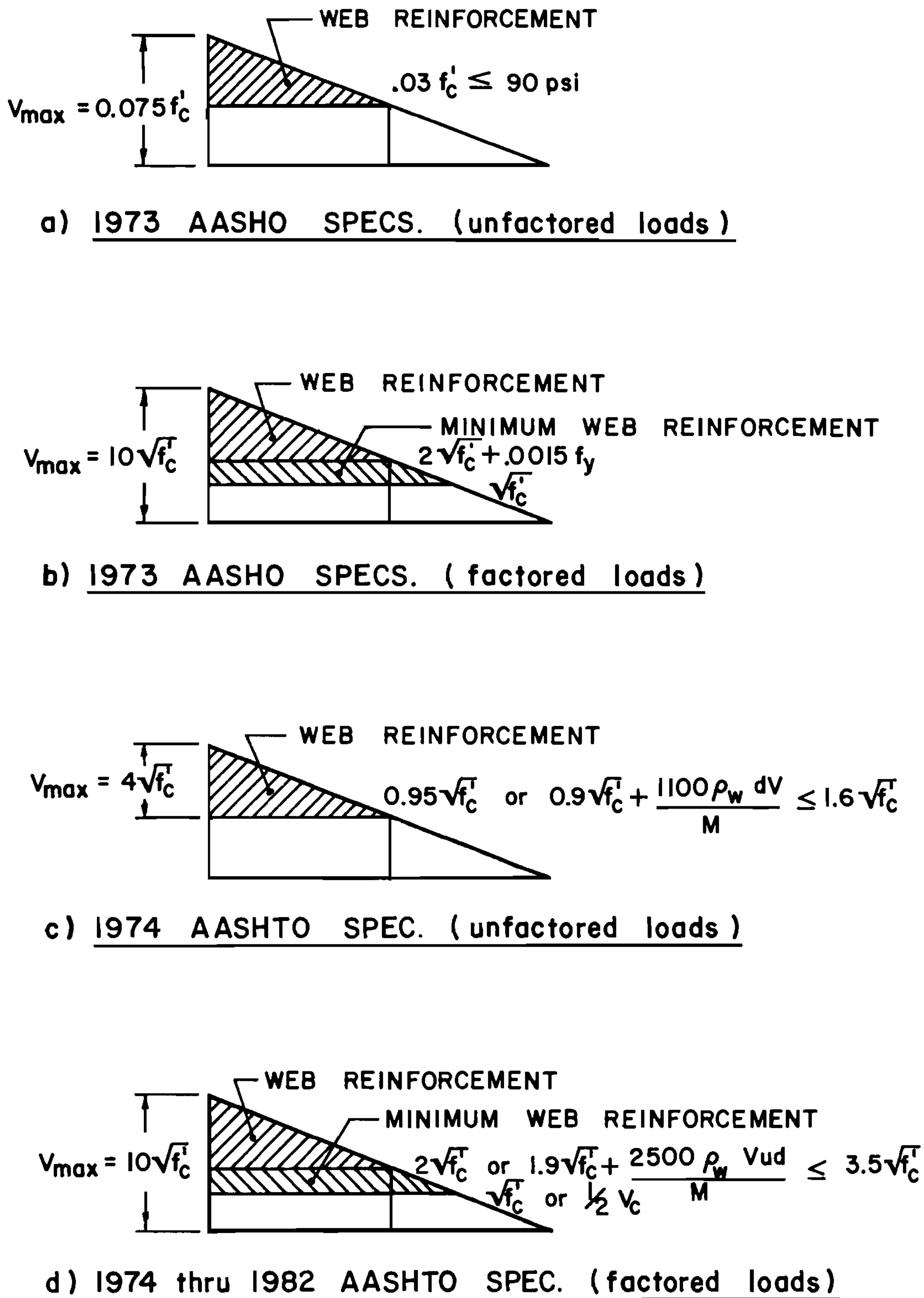


Fig. 2.7 Shear design in the AASHTO Specifications 1973-1982

exceed $2\sqrt{f'_c}$ and if the reinforcement ratio, ρ , is less than 1.2 percent, then $v_{uc} = (0.8 + 100\rho)\sqrt{f'_c}$. Like ACI, the amount of shear to be resisted by vertical web reinforcement was specified as $A_v d_y d/s$. Three requirements which were included in the 1973 AASHO Specifications also appeared in ACI 318-63 (21). The first one was the minimum amount of web reinforcement, $A_v = 0.0015 b_w s$, if v_u was larger than 1/2 v_c. Secondly, the yield point of the web reinforcement could not exceed 60 ksi. And lastly, the ultimate shear stress could not exceed $10\sqrt{f'_c}$ (see Fig. 2.7b).

The 1974 AASHTO Interim Specifications (11) varied significantly from the 1973 requirements. In the service load design method, the value of v_c was limited to $0.95\sqrt{f'_c}$ unless calculated by:

$$v_c = 0.9\sqrt{f'_c} \quad 1100\rho_w V_d/M \leq 1.6\sqrt{f'_c}$$

These specifications also required a minimum amount of web reinforcement when the design shear stress is greater than 1/2 v_c. This minimum area of web reinforcement is the same as that first found in the 1971 ACI Building Code (23):

$$A_v = 50\ b_w s/f_y$$

However, the expression for the amount of shear to be resisted by the web reinforcement, based on the truss analogy, is the same as that of the 1973 AASHO Specifications. Also included in the 1974 unfactored load design procedure was that the maximum shear stress to be resisted by the web reinforcement could not exceed $4\sqrt{f'_c}$ (see Fig.

2.7c). In the load factor design procedure the values of the shear stress to be resisted by the concrete are the same as those first appearing in the 1963 Building Code (21). The value of v_c was limited to $2\sqrt{f'_c}$ unless calculated by:

$$v_c = 1/9\sqrt{f'_c} + 2500b\rho_w V_u d/M_u \leq 3.5\sqrt{f'_c} \qquad (2.11)$$

The expression for the area of vertical web reinforcement is the same as that contained in the service load design procedure, but using yield strength of the reinforcement f_y instead of the allowable tensile stress f_v and with the same minimum area requirements. However, the maximum shear stress to be resisted by the web reinforcement was limited to $8\sqrt{f'_c}$ (see Fig. 2.7d). The values and expressions stated above also appeared in the 1977 AASHTO Standard Specifications for Highway Bridges (12) and remained unchanged in subsequent interim Specifications through 1982 (13,14,15,16,17). In the case of special problems, such as shear friction, the design approach followed is similar to the one presented in the ACI Building Code. However, in the case of deep beams no guidelines are given. In the 1982 AASHTO Interim Specifications the definition of maximum design shear near the supports as the shear at a distance "d" away from the face of the support in the case where the reaction introduces compression in the end regions of the member, is limited to cases when a major concentrated load is not imposed between that point and the face of the support. This problem which was addressed in the Commentary to the ACI Building Code of 1977 (24), will be treated in the ACI 318-83 Building Code.

In 1977 ACI Committee 426 (28) published suggested revisions to shear provisions for the ACI Building Code. These revisions have not yet been adopted by ACI Committee 318. The biggest change was in the calculation of V_c. It was completely rewritten based on the so-called "basic shear stress, v_b." This was done to unify the design for slender and deep beams, reinforced concrete beams with and without axial loads, and prestressed concrete beams (108). For nonprestressed members they recommended that $V_c = v_b b_w d$. For a/d ratios greater than 2 they recommended that the "basic shear stress," v_b be equal to $(0.8 + 120\ \rho_w)$ $\sqrt{f'_c}$ but not more than $2.3\sqrt{f'_c}$ nor less than $\lambda\sqrt{f'_c}$, where ρ = percentage of longitudinal steel reinforcement, and λ = 1.0 for normal weight concrete. This expression was originally proposed by Rajagopalan and Ferguson (141) with a coefficient of 100 instead of 120. It was felt that the shear span to depth ratio or M/V_d was a significant variable, but that for an a/d greater than 2 its effect was less pronounced. The expression for the amount of shear to be carried by the web reinforcement was the same basic equation of the truss analogy:

$$V_s = A_v f_y d/s \tag{2.12}$$

with an upper limit of $8b_w d\sqrt{f'_c}$, which first appeared in ACI 318-63 (21). However, a new limit was introduced in the Committee 426 recommendations. This limit was that the value of $(V_c + V_s)$ should not exceed

$$0.2b_w df'_c \tag{2.13}$$

The basis for this limit was that beams with vertical stirrups are subjected to inclined compressive stresses and, in addition, the diagonal compressive struts are subjected to a tranverse tensile stress introduced by bond from the stirrups crossing the cracks. These effects combined with nonuniformity of the distribution of compression stresses in the struts will cause crushing of the web at a stress considerably below f_c' (108). However, it was felt by Committee 426 that the previous limit of $10\sqrt{f_c'}$ was still conservative. Thus, a slight increase in the upper limit was allowed in the recommendations. This increase only makes a real difference in the higher f_c' range. Also, the 1970 CEB-FIP recommendations limit the shear in thin webs to 0.2 times the design compressive strength with vertical stirrups. However, none of the new recommendations were included in ACI 318-77 (24). The design recommendations proposed for the 1983 version of the ACI Building Code remained unchanged from the 1977 edition.

2.3 Shear in Prestressed Concrete Beams

Intensive research work has made the calculation of the flexural strength of prestressed concrete structures so rational that it is usually possible to closely predict the ultimate bending moment. Unfortunately, the knowledge of shear behavior is not of this high standard. In the USA, previous to about 1955, numerous prestressed concrete beams had been tested to determine their strength in flexure, but very few in shear. Between 1955 and 1961, a large number of specimens were actually tested to determine their strength in resisting shear, or combined moment and shear, with or without web reinforcement. The final result of these studies were the 1963 ACI Building Code design recommendations for shear in prestressed concrete members (21). Research in America continued in this area through the latter part of the 60's and early 70's. However, most of the work published in the American literature (37,39,44,47,73) focused on the refinement of the procedure proposed in the 1963 ACI Code. As a result of this situation, the design procedure for shear in prestressed concrete beams has remained virtually unchanged since 1963 in subsequent editions of both the ACI Building Code (24) and AASHTO Standard Specifications (12,13,14, 15,16,17).

In 1958 ACI-ASCE Committee 323 (25) published the first U.S. recommended practice for design of prestressed concrete. In this report the section pertaining to shear design was based on ultimate strength conditions (load factor method). The proposed procedure was based on the assumption that shear failure should not occur before the ultimate flexural strength of the member was attained.

The assumption of a shear resisting mechanism formed by the web concrete and the web reinforcement, used in reinforced concrete, was applied to prestressed concrete members. Hence, the total shear force that could be carried by the member at a given section, V_u, was evaluated as:

$$V_u = V_c + V_s$$

where V_c was the shear carried by the concrete prior to diagonal tension cracking, and was taken equal to 0.06 $f'_c b_w jd$, but not more than $180 b_w jd$.

The difference between the shear produced by the load required to develop the ultimate flexural capacity (V_u), and the shear required to produced inclined cracking (V_c), would have to be provided by the web reinforcement, V_s. From the examination of available test data from prestressed beams, it was concluded that the procedures used at that time for reinforced concrete beams were conservative for prestressed concrete. However, the 323 Committee made no reference to this available data as far as what was the level of prestress, levels of shear stress or percentage of web reinforcement in the specimens tested. However, based on this finding Committee 323 (25) recommended that a factor of 1/2 should be added to the formula used to evaluate the amount of web reinforcement in reinforced concrete members. As a result the following expression was proposed to compute the required amount of web reinforcement in prestressed concrete members:

$$A_v = \frac{1}{2} \frac{(V_u - V_c)}{f_{yv} jd} s \qquad (2.14)$$

where A_v = area of web reinforcement at spacing s, placed perpendicular to the longitudinal axis of the member, s = longitudinal spacing of web reinforcement, f_{yv} = yield strength of web reinforcement, and jd = internal lever arm. However, since the prestress force was not included as a variable, it was recommended that the factor of 1/2 be increased as the beam reached the condition of a conventionally reinforced concrete beam. No specific guidelines were given to differentiate between low and high levels of prestress. This increase was suggested to ensure that the equation yield conservative results (25). Based on limited experimental data Committee 323 (25) indicated that inclined tension cracks would not form and web reinforcement would not be required if the following condition was satisfied:

$$\frac{\rho f'_s}{f'_c} \leq \frac{0.3 f_{se}\, b_w}{f'_s b} \tag{2.15}$$

where b_w = thickness of the web, b = width of flange corresponding to that used in computing ρ, f'_s = ultimate strength of prestressing steel, f'_c = compressive strength of concrete at 28 days, f_{se} = effective steel prestress after losses, and $\rho = A_s/bd$, ratio of prestressing steel. However, they suggested that "because of the nature and limited knowledge of shear failures," some web reinforcement should be provided even though the above condition may be satisfied, and that this amount of minimum reinforcement be equal to 0.0025 $b_w s$.

The first AASHO design provisions for prestressed concrete appeared in the 1961 Standard Specifications for Highway Bridges (7). The formula for the web reinforcement required was the same as that suggested by ACI Committee 323 (25) in 1958.

However, the value of V_c was based on resisting a shearing stress of $0.03f'_c$, which was the allowable unit stress used for reinforced concrete but was utilized to evaluate shear under ultimate load conditions in prestressed concrete members. Also, as suggested by Committee 325, the minimum area of web reinforcement required was $0.0025b_w$, where b_w is the width of the web.

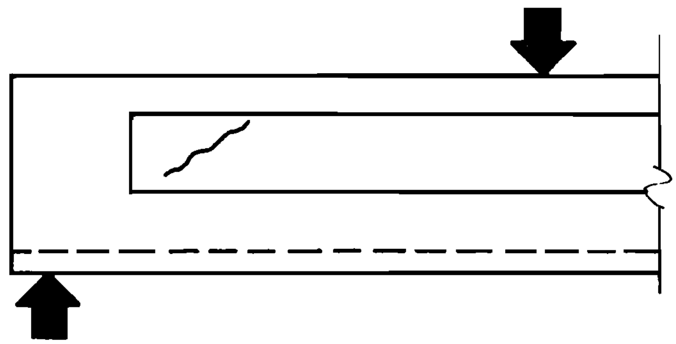
In ACI 318-63 (21), however, the value for the shear to be carried by the concrete V_c was changed from the original Committee 323 (25) recommendations. It was specified for two categories because of the two types of shear cracking that might occur.

Shear failures of beams are characterized by the occurrence of inclined cracks. Such inclined cracks in the web of a beam may develop either before a flexural crack occurs in their vicinity or as an extension of a previously developed flexural crack. The first type of inclined crack is often referred to as a "web-shear crack" (see Fig. 2.8a); the second type is identified as a "flexure-shear crack," and the flexural crack causing the inclined crack is referred to as the "initiating flexural crack" (see Fig. 2.8b). ACI 318-63 (21) referred to the shear required to produce these cracks as V_{ci} for the case of flexure-shear, and V_{cw} for web shear. Whichever is the smaller of these two values governs the design and is used as the concrete capacity V_c.

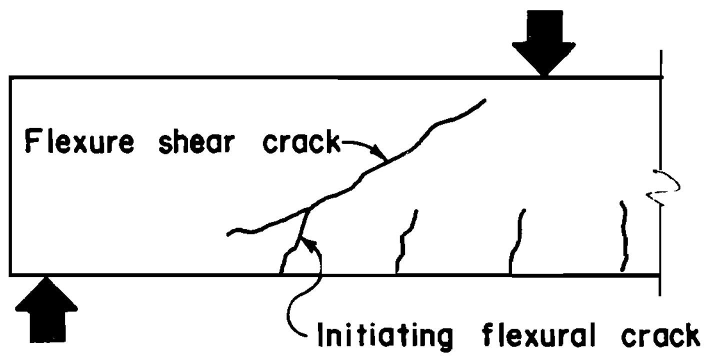
The flexure-shear capacity V_{ci} was determined as:

$$V_{ci} = 0.6 b_w d \sqrt{f'_c} + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d \tag{2.16}$$

where b_w = minimum width of web of a flanged member, d = distance from extreme compression fiber to centroid of the prestressing force, f'_c = compressive strength of concrete, M_{cr} = flexural cracking moment, M = moment due to factored externally applied loads at the section under consideration, V = shear due to factored externally applied loads at the section under consideration, and V_d = shear due to dead load at the section being investigated.



a. WEB SHEAR CRACK



b. FLEXURAL SHEAR CRACK

Fig. 2.8 Types of inclined cracks

The previous equation expressed the inclined cracking load V_{ci}, as the shear necessary to cause a flexure crack at a distance d/2 from the section under consideration, plus an increment of shear assumed to be necessary for this flexural crack to develop into an inclined crack and assumed to be a function of the dimensions of the cross section and the tensile strength of the concrete. It was postulated that in order to reduce the shear capacity of a beam, a diagonal crack should have a projection along the longitudinal axis of the beam equal to its effective depth "d" (see Fig.2.9). A flexural crack at a distance "d" away (in the direction of decreasing moment) may lead to a diagonal crack which could be critical for section B-B'. The principal tensile stresses along the path of the incipient diagonal crack will be increased by flexural cracking within the distance d. Since the maximum diagonal tensile stress occurs near the centroid of the beam, a flexural crack occurring at a distance d/2 from B-B' would mark the imminence of a flexure-shear crack. Therefore, consider section B-B' where, due to externally applied loads, the moment is M and the shear is V. The moment at section A-A necessary to produce a flexural crack will be M_{cr} with the corresponding shear V_{cr}. The change in moment between both cross sections is then given by the area of the shear diagram between the sections, $M-M_{cr} = V_d/2$, thus

$$V = \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} \tag{2.17}$$

The total shear due to both applied loads and dead loads when the critical flexural crack occurs is

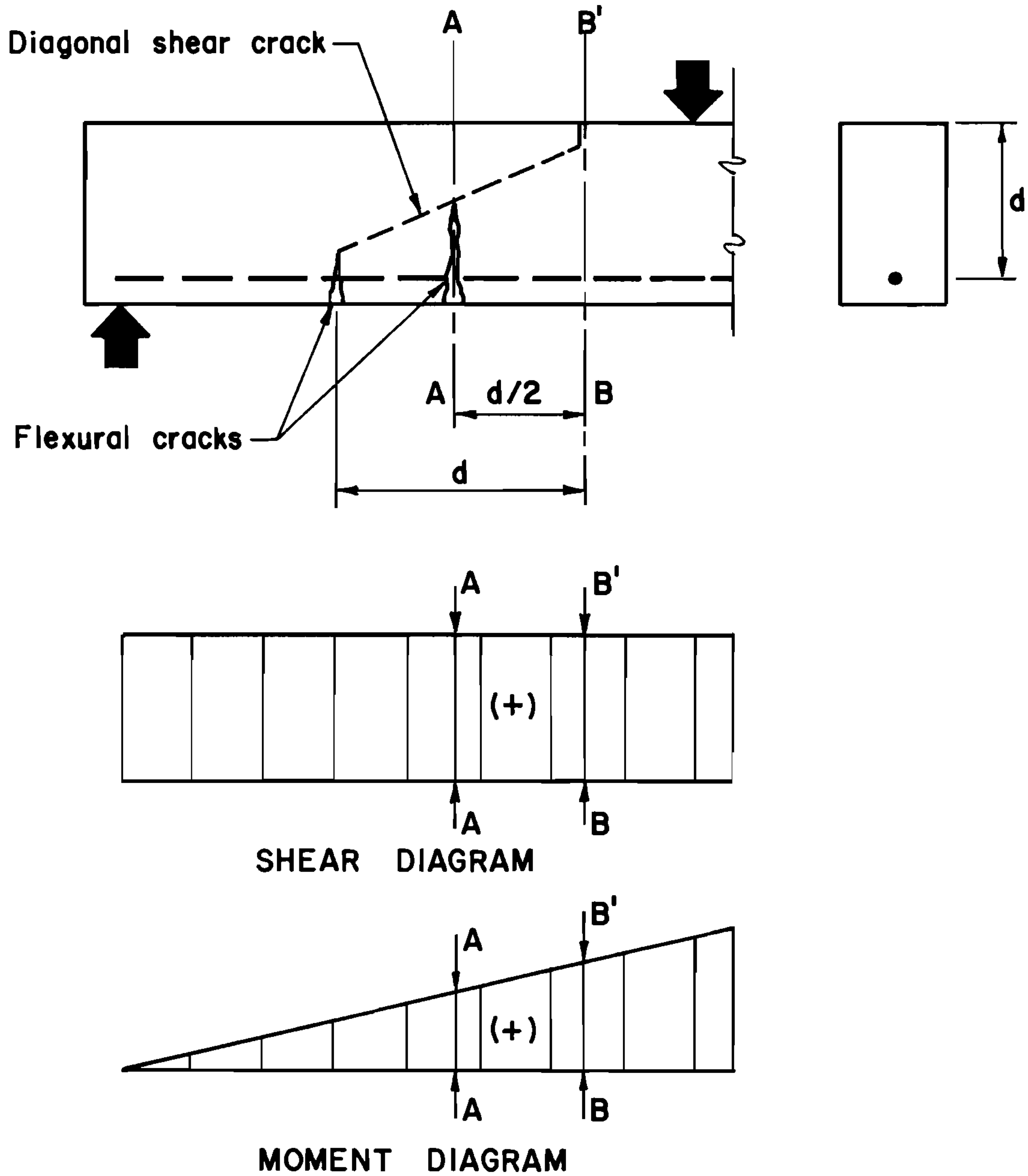


Fig. 2.9 Flexure-shear, V_{ci}

$$V = \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d \qquad (2.18)$$

The dead load was considered separately for two reasons:

1. Dead load is usually uniformly distributed whereas live loads can have any distribution.
2. The dead load effect is always computed for the prestressed section alone. The live load effect is computed for the composite section in composite construction.

The term M_{cr} was taken as the moment due to applied load when flexural cracking occurs at section A-A, and given by:

$$M_{cr} = I*[6\sqrt{f'_c} + f_{pe} + f_d]/y \qquad (2.19)$$

where $6\sqrt{f'_c}$ = modulus of rupture of the concrete, f_{pe} = compressive stress in the concrete due to prestress, f_d = stress due to dead load, I = moment of inertia of the section resisting external ultimate loads, and y = distance from centroidal axis of the section resisting the ultimate external loads to the extreme fiber in tension.

Lastly, the increment of shear necessary to turn the flexural crack into an inclined crack was taken equal to $0.6b_w d\sqrt{f'_c}$. From test data, a lower limit of V_{ci} was set at $1.7b_w d\sqrt{f'_c}$ because the only beams which failed below this limit had extremely low amounts of prestress.

The other shear mechanism, V_{cw}, is the shear in a nonflexurally cracked member at the time that diagonal cracking occurs in the beam web. The design for web shear cracking in prestressed concrete beams is based on the computation of the principal tensile stress in the web and the limitation of that stress to a certain specified value. The first part of this method, the computation of the principal tension based on the classical mechanics approach for combined stresses, is a theoretically correct procedure so long as the concrete has not cracked. The second part of this method, limiting the principal tension to a definite value, is not always an accurate approach, because there is evidence to show that the resistance of concrete to such principal tension is not a consistent value but varies with the magnitude of the axial compression stress. It seems, however, that when the axial compression is not too high (say less than about $0.5f'_c$) the resistance of concrete to principal tensile stresses is relatively consistent. Typical prestressed concrete beams have axial compression stress less than $0.5f'_c$. Hence, this computation of principal tensile stress can be regarded as a proper criterion for the stress conditions to determine when the concrete has cracked. The method of computing principal tensile stress in a prestressed concrete beam section is based on the elastic theory and on the classical approach for determining the state of stress at a point as explained in any treatise on mechanics of materials (154). Through Mohr's circle it can be shown that the value of the principal diagonal tension stress at the centroid of the web of a prestressed concrete beam prior to cracking is given by

$$f_t = [v_{cw} + (\frac{f_{pc}}{2})^2]^{0.5} - (\frac{f_{pc}}{2}) \qquad (2.20)$$

where f_t = principal diagonal tension stress, v_{cw} = shear stress, and f_{pc} = compressive stress due to prestress. This relation yields:

$$v_{cw} = f_t \left[1 + \left(\frac{f_{pc}}{f_t}\right)\right]^{0.5} \tag{2.21}$$

A value for f_t of $4\sqrt{f'_c}$ lower than the generally accepted value of $6\sqrt{f'_c}$ appeared to be substantiated by tests. But since Committee 318 wanted a nominal rather than maximal value, they choose to use $f_t = 3.5\sqrt{f'_c}$. For simplification they reduced the expression to the generally equivalent (see Fig.2.10) straight line function.

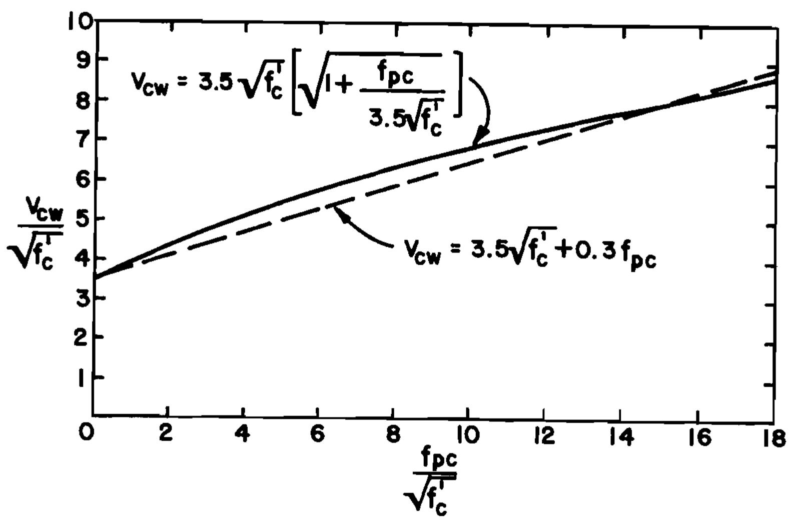


Fig. 2.10 Relationship between nominal stress at web-shear cracking and compressive stress at centroid

$$v_{cw} = 3.5\sqrt{f'_c} + 0.3f_{pc} \tag{2.22}$$

From equilibrium considerations the value of the counteracting vertical component of the prestressing force in inclined or draped strands, V_p, was added (23). Alternatively, the 1963 Code (21) stated that the value, V_{cw}, may be taken as the live load plus dead load shear which corresponds to a principal tensile stress of $4\sqrt{f'_c}$.

In the 1963 (21) ACI Building Code the factor of 1/2 in the equation used to compute the required amount of web reinforcement was eliminated, because it was considered that the beneficial effect of prestress on the shear capacity of the member was adequately accounted for in the proposed equations for V_{ci} and V_{cw}. Although the 1963 Code (21) had no provision for minimum web reinforcement for reinforced concrete members when V_u was less than V_c, there was a minimum reinforcement requirement for prestressed concrete. The minimum area required was:

$$A_{vmin} = \frac{A_s f'_s s}{80fy} \left[\frac{d}{b_w}\right]^{0.5} \tag{2.23}$$

where A_v = minimum area of shear reinforcement; f'_s = tensile strength of prestressing steel; A_s = area of prestressing steel; f_y = yield point of web steel; s = stirrup spacing, d = the greater of the distance from the extreme compression fiber to the longitudinal steel centroid or 80% of the overall beam depth, b_w = web width.

Combining $A_v f_y d/s$ and Eq. 2.23 yields:

$$(V_u - V_c)_{min} = \frac{A_s f'_s d}{80\sqrt{b_w d}} \tag{2.23a}$$

where $A_s f'_s d$ is a measure of the ultimate moment capacity of the member and thus of the required value of V_u. As the web thickness, b_w, increases, the danger of inclined cracking decreases, and hence the need for web reinforcement decreases. Thus, the minimum area is related to the flexural capacity and geometry of the member (21).

Because of uncertainty in shear design, it was considered that a minimum amount of web reinforcement was necessary to ensure that flexure would always control the type of failure in the member. Equation 2.23 has an empirical basis and came about as a result of research carried out at the University of Illinois in the late 1950's (76). It was developed in order to overcome the objections to the minimum steel equation, $A_v = 0.0025b_ws$ which appeared in the Committee 323 Tentative Recommendations (25). The objections to this formula resulted from the fact that the wider the web b_w the more steel required. This was contrary to the experience gathered from observed test values.

The design recommendations for shear in the 1965 (8) and 1969 (9) AASHO Standard Specifications were based on the ACI-ASCE Committee 323 report. In these editions the value of shear carried by the concrete V_c was different from the one not so clearly specified in the 1961 edition (7). The term V_c was set equal to $0.06f'_cb_wjd$ but could not be more than $180b_wjd$. These recommendations were a mixture of ultimate and service load conditions, and a capacity reduction factor for shear was not required.

There were only a few minor changes in the 1971 ACI Code (23). The value of V_{ci} was made more conservative by removal of the d/2 term. Rather than computing the flexural cracking load at a distance d/2 from the section under consideration, the flexural cracking load is computed at the section being investigated. Also in 1971 an addition was made to the Code (23) for computing the shear carried by the concrete v_c

$$v_c = 0.6\sqrt{f'_c} \quad 700V_c \tag{2.24}$$

The development of this equation is entirely empirical and was chosen as a lower bound of the shear strength of prestressed concrete beams. However, since this equation can be applied in lieu of the v_{ci} and v_{cw} equations an upper limit of $5\sqrt{f'_c}$ is imposed to act as a limit in the region where v_{cw} might control. However, the use of this simplified expression is limited to members having an effective prestress force equal to or greater than 40% of the tensile strength of the flexural reinforcement. The equation is just a simplified and generally conservative approximation of the equation for v_{ci}. The upper and lower bounds for v_c are derived from the expressions for v_{ci} and v_{cw} (23). When the minimum area of web reinforcement requirement,

$$A_v = 50b's/f_y \tag{2.25}$$

for most reinforced concrete members appeared in the 1971 ACI Code (23), it was made to apply to prestressed concrete members as well (23).

The 1973 AASHO Standard Specifications for Highway Bridges (10) stated that the requirements of ACI 318-71 (23) were acceptable but that the area of web reinforcement could not be less than $100b_ws/f_{sy}$. The reason being that Eq. 2.25 will require more minimum shear reinforcement for building-type prestressed members, and less for bridge type beams, than Eq. 2.23 of the 1963 ACI Building Code. Alternatively, AASHO had its own design requirements. The area of vertical web reinforcement required was the same as that of the 1961 specifications, but with the above mentioned minimum area. As in ACI 318-63, the yield point of the web reinforcement was limited to 60 ksi. The value of v_c was changed to $0.06f'_c$, but not greater than 180 psi. This is the same value recommended by ACI Committee 323 (25) in 1958.

The 1977 ACI Building Code (24) contains the same design specifications for shear as the 1971 ACI Code (23) except for the fact that the expressions were put in terms of forces rather than stresses (24).

The design provisions for shear in prestressed concrete of the 1977 AASHTO Standard Specifications for Highway Bridges (12) remained basically unchanged from the 1973 specifications (10). It allows web reinforcement to be designed in accordance with ACI 318-77 (24) but with the same required minimum area of reinforcement as in 1973. AASHTO's design for A_v and V_c also is the same as in 1973.

In 1977 ACI Committee 426 (28) suggested revisions to the shear previsions in prestressed concrete as well as in reinforced concrete. In an attempt to unify shear design provisions for reinforced and prestressed beams, they recommended that for members subject to axial compression, prestress, or both, the amount of shear, V_c, should be computed as the lesser of V_{ci} and V_{cw}. The amount of V_{cw} remained unchanged from that of ACI 318-71 (23), but the value of V_{ci} was changed. The proposed value of V_{ci} was

$$v_b b_w d + V_d + V_i M_o/M_{max} \tag{2.26}$$

where $$v_b = (0.8 + 120\rho_w)\sqrt{f'_c}$$

$$M_o = (I/y_t)(f_{pe} - f_d)$$

This empirical equation resulted from a comparative study carried out by Mattock (112) on prestressed, and reinforced concrete beams with and without axial load. Mattock (112) pointed out that axial loads affected the flexural cracking shear, but apparently did not affect the increment of shear between flexural and diagonal cracking. He suggested that the increment of shear stress varied both with percentage of flexural reinforcement ρ, and with the modular ratio $n = E_s/E_c$. He based this statement on the assumption that the intensity of a principal stress immediately above a flexure crack will depend upon the penetration of the flexure crack. The greater the penetration of the flexure crack, the greater the principal stress for a given applied shear. The flexure crack will penetrate almost to the neutral axis, the depth of which is a function of "$n\rho$". That is, the greater the value of "$n\rho$" the greater the depth of the neutral axis, and the less the penetration of the flexure crack. Hence, the greater the value of "$n\rho$" the less will be the principal stress for a given applied shear. Conversely, the greater the value of "$n\rho$" the greater must be the shear to cause the principal stress which will result in diagonal tension cracking. The value of V_c that appeared in ACI 318-71 (23) which was an approximation of V_{ci}, remained unchanged by Committee 426 (28). The recommended procedure for computing the amount of web reinforcement was the same as that for reinforced concrete. None of these new recommendations were adopted for ACI 318-77 (24). In the proposed changes to the ACI Building Code for the 1983 edition, although the design requirements for prestressed members remained basically the same a few changes were introduced. In the case of pretensioned members the value of V_{cw} was limited to that calculated using the reduced prestress force. This is in the case of pretensioned members where bonding of some tendons does not extend to the end of the member. Also, the prevision that waives the use of minimum web reinforcement was made more explicit in relation to the conditions required to meet the intent of this section.

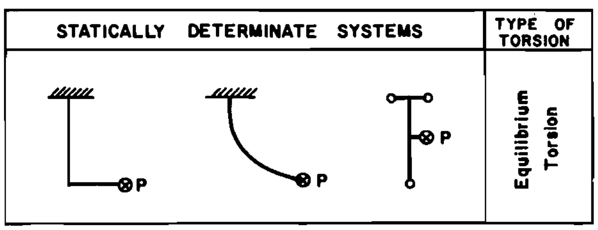
The 1980 AASHTO Interim Specifications (15) varied significantly from the 1977 requirements. The method for shear design of prestressed concrete beams completely followed the 1977 ACI-318 (24) recommendations, and the old equations $A_v = (V_u - V_c)s/2f_{sy}jd$ and $V_c = 0.06f_c'b_wjd$ were eliminated from the AASHTO Specifications. The minimum amount of steel required was changed to $A_v = 50b_ws/f_{sy}$ which is identical to the one specified in the 1977 ACI Building Code (24) for reinforced concrete beams. The maximum limit of shear strength that can be provided by the web reinforcement was limited to $8\sqrt{f_c'}b_wd$.

In summary, the Interim 1980 AASHTO Specifications (15) basically provide that design of prestressed concrete for shear is the same as current formulas utilized in the ACI 1977 Building Code (24). Normally the ACI Building Code shear provisions are used with static loads while in bridge analysis moving loads have to be considered. Proper input of moving live load shears and moments into the ACI formulas results in long and tedious calculations. In particular, the solution of the M/V_d portion of the V_{ci} equation gets complicated since different loading conditions must be used to evaluate the maximum moment and the maximum shear at the same section.

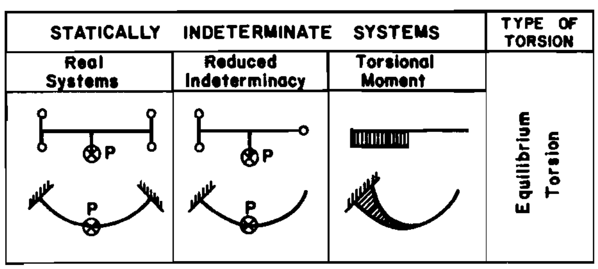
Where lane loadings govern, AASHTO specifies a uniform load of 640 lb/ft plus a concentrated load placed at the point of maximum effect. The concentrated load is 18 kips when computing design moments and 26 kips when design shears are evaluated. However, it is not clear whether the 18 kips or the 26 kips load should be used in the evaluation of M_{max}/V_i ratio. It gets especially complicated in the case of continuous beams where the load that would produce maximum shear at a given section does not produce maximum moment and vice versa. In addition, proper consideration of moving live load is extremely time consuming since calculations must be made for several points (suggested tenth points of L/2). Since design specifications for prestressed concrete first appeared in the ACI Building Code and AASHTO Specifications, there has been little change in the basic assumptions for shear design.

2.4 Torsion in Reinforced Concrete Beams

In the past, torsion effects have often has been ignored in the design of concrete structures. Designers felt that the traditional large safety factors for axial and flexural loadings provided sufficient margin to account for such neglected secondary problems of design as torsion. With the introduction of factored load design and continued downward revisions of safety factors these margins have been reduced to a point where these secondary effects should no longer be ignored. Today's engineers and architects often produce structural forms such as spiral staircases, cantilevers with eccentric loadings spandrel beams, and curved elevated roadways, in which torsion can be a primary consideration. Torsion may arise as a result of primary (equilibrium torsion) or secondary (compatability torsion) actions. The case of primary torsion occurs when the external load has no alternate path except to be resisted by torsional resistance. In such situations the torsional resistance required can be uniquely determined from static equilibrium. This case may also be referred to as equilibrium torsion. It is primarily a strength problem because the structure, or its component, will collapse if the torsional resistance cannot be supplied. Simple beams receiving eccentric line loadings along their span, and eccentrically loaded box girders, are examples of primary or equilibrium torsion (see Fig. 2.11a). In statically determinate structures only



(a)



(b)

Fig. 2.11 Equilibrium torsion

equilibrium torsion exists, while in indeterminate structures both types are possible. A given load P produces equilibrium torsion in an indeterminate structure if the torsion cannot be eliminated by releasing redundant restraints. Figure 2.11b gives an example of this type of torque. In this system both ends start torsionally restrained, as the load P is applied local cracking occurs at the right hand support and the torsional restraint is released. At this point the structure becomes a statically determinate system. The left support has to remain torsionally restrained to avoid the formation of a collapse mechanism, thus the torsional moment becomes zero to the right of the load "P", but remains P.e to the left of the load.

In statically indeterminate structures, torsion can also arise as a secondary action from the requirements of continuity. This case is referred to as compatability torsion. Torsional moments may be developed by resistance to rotation and may be relieved when local cracking occurs. Disregard for the effects of such restraint in design may lead to excessive crack widths but need not result in collapse if the cracked structure has alternate load paths which can resist the loading from an equilibrium standpoint more serious consequences. Designers often intuitively neglect such secondary torsional effects. The spandrel beams of frames which support slabs or secondary beams, are typical of this situation (see Fig. 2.12).

Consider the frame shown in Fig. 2.12. The deflection of the floor beam of this structure will force the flexurally unrestrained spandrel beam to twist causing torsion in the spandrel. In this case as

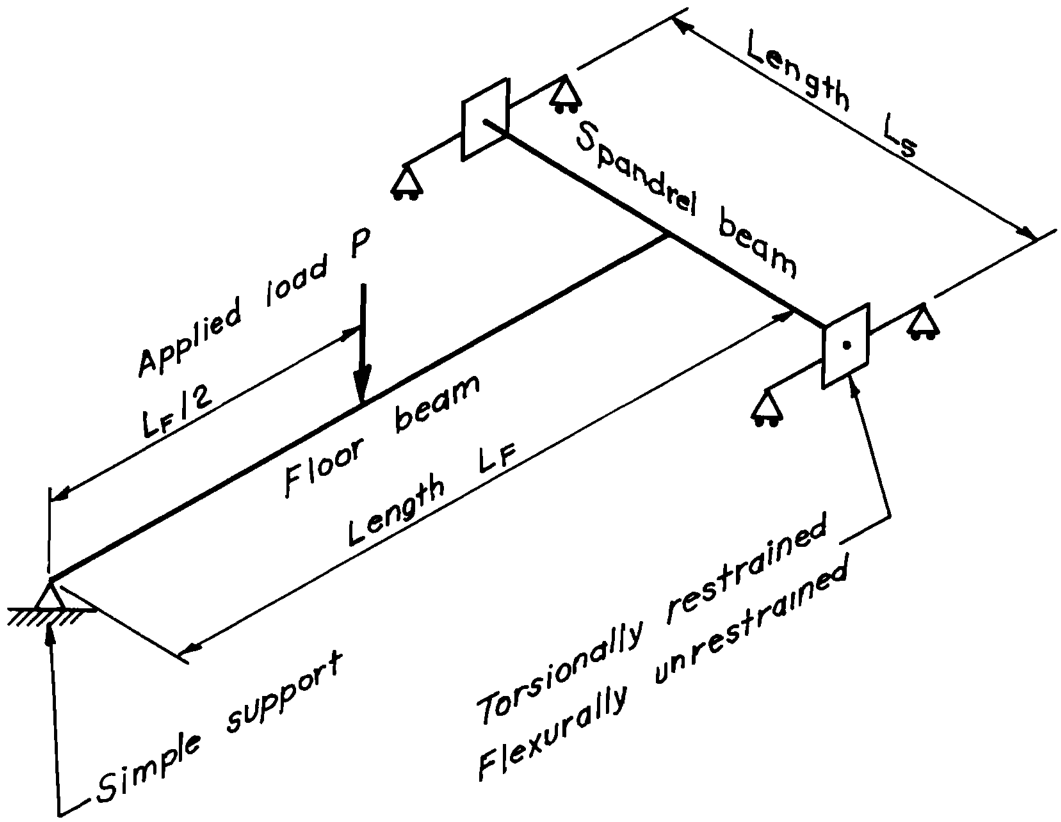
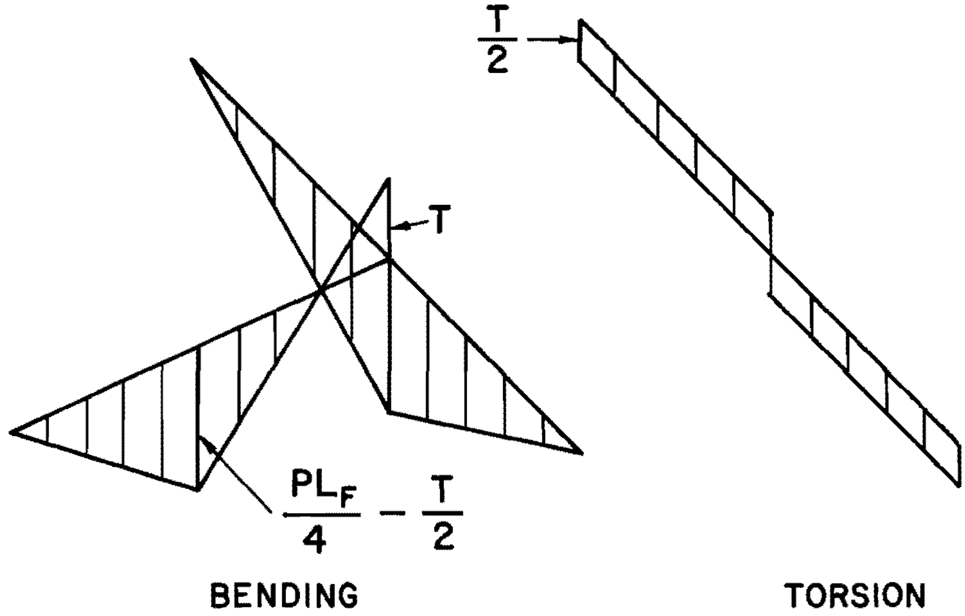
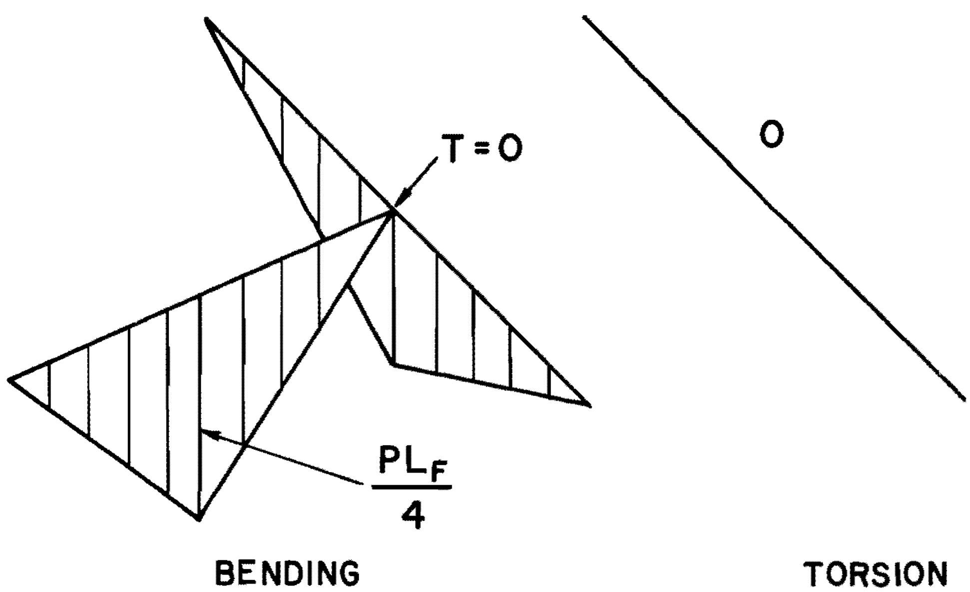


Fig. 2.12 Case of compatibility torsion in floor beam-spandrel beam structure

shown in Fig. 2.13a, the maximum torsion in each half of the spandrel will be additive and will provide a flexural restraining moment with magnitude equal to the torsional moment T, of the floor beam. The magnitude of T depends on the torsional stiffness of the spandrel. If the spandrel is infinitely stiff torsionally the floor beam would in effect be a propped cantilever and the restraining moment would be $3PL_F/16$ while the moment under the load would be $5PL_F/32$. As the



(a) Spandrel beam with torsional stiffness



(b) Spandrel beam with torsional stiffness
after torsional cracking

Fig. 2.13 Distribution of moment

spandrel becomes less stiff the torque in the spandrel and hence the restraining moment on the floor beam drops, causing the moment under the load to increase. When the spandrel beam has zero torsional stiffness, the T becomes zero and the floor beam becomes in effect simply supported with a moment of $PL_F/4$ under the load (see Fig. 2.13b).

The American Concrete Institute Building Code Committee 318 first provided torsion design guidance (in an indirect way) in the 1963 Building Code (ACI 318-63) (21). Section 921 (a) of ACI 318-63 required closed stirrups and at least one longitudinal bar in each corner of an edge beam or spandrel beam. This attention inspired ACI Committee 438 (torsion) to undertake (34) a broadly ranging program to:

1. Determine whether torsion was really a significant problem in concrete structures.
2. Survey other building codes for torsion design provisions.
3. Encourage research in the subject.
4. Write tentative design recommendations.
5. Educate Institute members.

Among the results of this program was a paper by Fisher and Zia which reviewed the building codes of 20 nations for torsional design requirements (68). At that time (1964), about one-third of the codes had reasonably thorough specifications, one-third had permissible stress specifications only, and the remaining made no explicit mention of torsion. The 1969 AASHO Standard Specifications were classed with the latter group.

The culmination of these efforts by Committee 438 was the publication of its tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion (34). These recommendations formed the basis for the design provisions in the 1971 ACI Building Code.

ACI 318-71 (23) was the first edition of the ACI Building Code to contain explicit provisions for the design of reinforced concrete members to resist torsion. ACI 318-71 followed the same approach found in both semirational and empirical studies of investigators such as Young, Sagar and Huges; Rausch; Turner and Davis; Anderson; Marshall and Tembe; Cowan; and Ernst (35). ACI 318-71 (23) expressed the equation for the torsional strength of a reinforced concrete beam as:

$$T_u = T_c + T_s \qquad (2.27)$$

where T_c = torsional strength of an equivalent plain concrete beam; and T_s = torsional strength contributed by steel. The principal difference among the various theories is the manner in which the contribution of the steel is determined. A common requisite of these theories was that torsional resistance required equal volumes of longitudinal and transverse steel.

The ACI 318-71 Building Code basic philosophy for torsion design assumed that the behavior of reinforced concrete members in torsion and in shear is similar, and suggested that the resistance of a member both to shear and to torsion is made up of two parts: one part is contributed by the web reinforcement while the other part is contributed by the concrete compression zone acting in either transverse shear or torsional shear. Based on past experience, gathered in both practice and in the laboratory, which had indicated that small torsional moments can be carried by reinforced concrete members without any significant reduction in their flexural and shear strength, ACI 318-71 (23) required members to be designed for torsion only when the torsion acting produced a nominal stress v_{tu} greater than $1.5\sqrt{f'_c}$. When the nominal torsional stress v_{tu} exceeded $1.5\sqrt{f'_c}$, ACI 318-71 required that in all cases the member be designed to carry the applied torsion, as well as flexure and shear.

The nominal torsional stress v_{tu} is evaluated on the basis that the distribution of torsional stresses in a concrete member before cracking, is intermediate between elastic and fully plastic, and can be expressed as:

$$v_{tu} = \frac{3\ T_u}{\Sigma x^2 y} \tag{2.28}$$

where, T_u is the factored design torque, "x" and "y" are the shorter and longer sides respectively of the component rectangles which make up the cross section and $x < y$.

The value of 3 is a minimum for elastic theory and a maximum for plastic theory. This equation was basically developed for rectangular sections. The ultimate torsional resistance for compound sections, such as T or I shapes, can be approximated by the summation of the contribution of the constituent rectangles. Box sections are the most efficient sections for resistance to torsion since they concentrate the entire cross-sectional area in the most highly stressed region, and at the greatest distance from the center of the section. Even when cracking takes place, the box remains the best section, because the material is at the greatest possible distance from the center. The torsional strength of a box section composed of a number of thin rectangles is far greater than the sum of the strengths of its component parts, and Eq. 2.28 is, therefore, in error. ACI 318-71 recognized this fact by allowing the treatment of box section as if it was a solid section, with certain restrictions depending on the wall thickness of the box.

ACI 318-71 (23) proposed that when torsion acts alone, the shear stress due to torsion, v_c, carried by an unreinforced beam could not exceed $2.4\sqrt{f'_c}$. This value was based on the contribution of the concrete to the ultimate torsional strength of a beam with web reinforcement. Hsu (82), in 1968, based on the analysis of test results of 53 reinforced concrete beams with web reinforcement which were subjected to pure torsion, proposed that the concrete contribution was equal to $2.4\sqrt{f'_c}\,K$, where K was a factor depending upon the cross section shape and relative dimensions, K was defined $[(x^2y)/x^{0.5}]$. In this relation, $1/x^{0.5}$ represented the scale factor effect in the torsional strength of the concrete for the range of test data considered, 6 in. $\leq x \leq$ 10 in. Since the nominal torsion stress at diagonal tension cracking of a beam subjected to pure torsion is about $6\sqrt{f'_c}$, the torsional stress of $2.4\sqrt{f'_c}$ proposed by ACI 318-71 represented about 40% of the cracking. Consequently, it was considered to be a lower bound value of the strength of beams without web reinforcement. However, ACI Committee 438 (34) considered that such conservatism was justified. Experiments (130) had shown that the torsional strength of a beam without web reinforcement might be reduced by up to 1/2 due to the simultaneous application of a bending moment. Therefore, by specifying a torsional shear stress which corresponded to 40% of the cracking torque in pure torsion, the effect of bending moment on the torsional strength of beams without web reinforcement could be neglected.

A fully rational theory for the interaction of shear and torsion in the presence of bending had not yet been developed. For this reason reliance was placed on empirical information derived from tests. By providing more than adequate flexural reinforcement, it is possible to experimentally study the failure criteria for combined shear and torsion. It is usual in such tests to keep the torsion to shear ratio constant while the load is being increased to failure. However, in practice one action may occur first, imposing its own crack pattern before the other action becomes significant. Thus, in 1971 it was felt advisable to be conservative in the interpretation of test results (135).

Such was the basis for the circular interaction relationship for shear and torsion proposed in the ACI 318-71 Code provisions (23). For convenience, the magnitude of the interaction of shear and torsional forces carried by a cracked section at ultimate load was presented in terms of nominal stress as:

$$\left(\frac{v_{tu}}{2.4\sqrt{f'_c}}\right)^2 + \left(\frac{v_u}{2\sqrt{f'_c}}\right)^2 = 1 \tag{2.29}$$

where v_{tu} = induced nominal torsional stress carried by the concrete at ultimate, given by Eq. 2.28, and v_u = induced nominal shear stress carried by the concrete at ultimate, given by Eq. 2.30:

$$v_u = V_u/b_w d \tag{2.30}$$

The 2.4 $\sqrt{f'_c}$ and 2.0 $\sqrt{f'_c}$ terms are the proposed values for the nominal ultimate torsional shear strength of the concrete after cracking without the presence of shear, and the nominal ultimate shear strength of the concrete without the presence of torsion, respectively. This interaction equation controls the design of beams with only nominal web reinforcement. The given stresses are assumed to be carried across a cracked section by mechanisms not involving the web reinforcement. Additional torsional and shear strength was to be derived from appropriate web reinforcement.

When the torsion and shear stresses due to the design (factored) load are greater than the torsion and shear stresses which can be carried by the concrete, then closed stirrups were to be provided (see Fig. 2.14). A space truss, consisting of stirrup tension members, diagonal concrete compression struts, and tension chord members provided by the longitudinal reinforcement, was the model on which the design of web reinforcement was based in the ACI 318-71 Building Code (23) (see Fig. 2.15). In this model the diagonal compression elements were

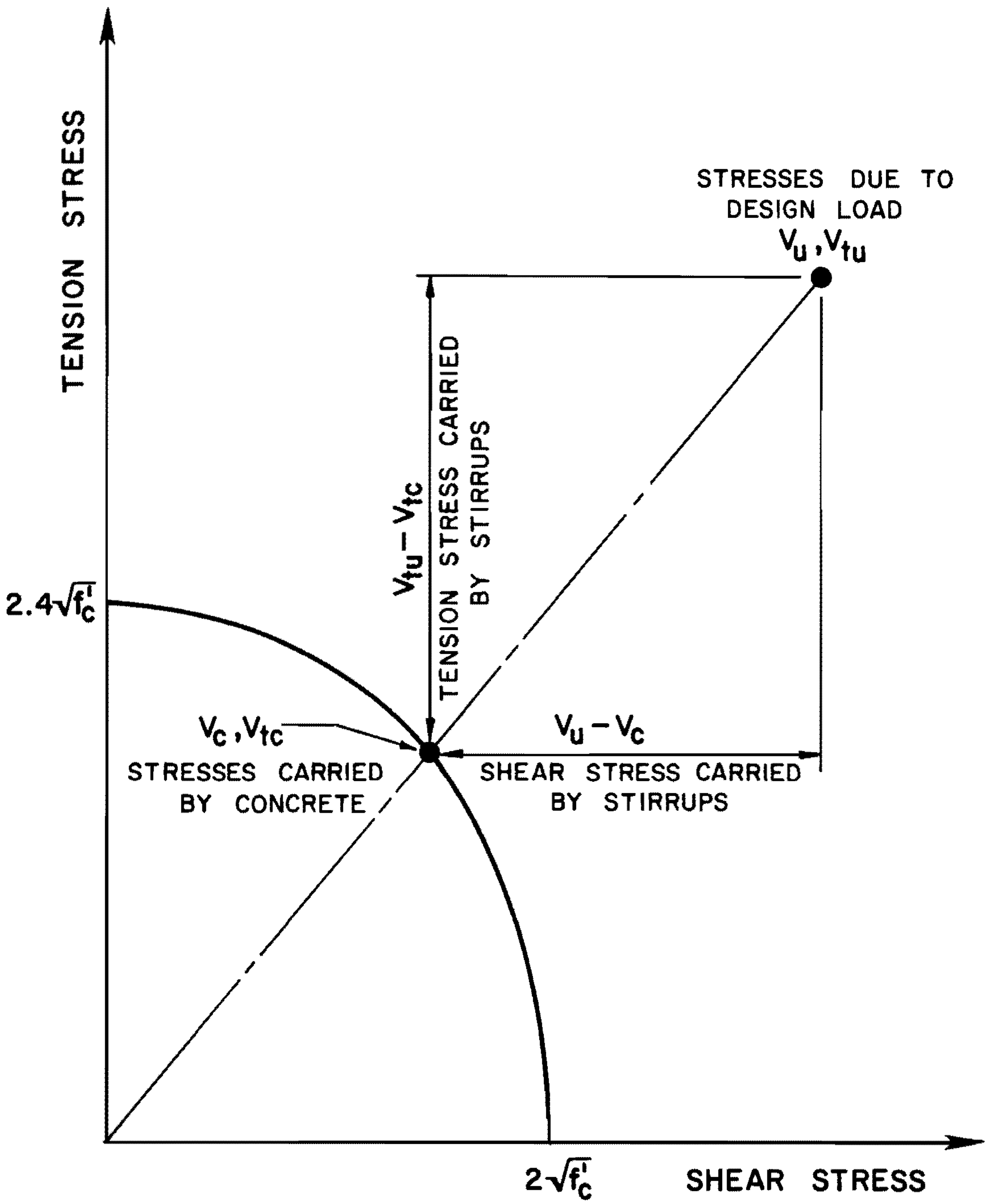


Fig. 2.14 Shear and torsion carried by web reinforcement

assumed to have an inclination of 45 degrees with respect to the longitudinal axis of the member. This assumption was based on the fact that initial cracking due to principal diagonal tension stresses caused by torsion generally takes place at 45 degrees. The stirrups were to be designed for the difference between the ultimate applied torsional stress and the torsional stress that could be carried by the concrete part of the ultimate mechanism. The amount was given by:

$$A_t = \frac{(T_u - T_c)s}{a_t x_1 y_1 f_y} \tag{2.31}$$

where

A_t = cross-sectional area of one stirrup leg

T_u = design torque

T_c = torque carried by concrete

s = stirrup spacing

a_t = 0.66 + 0.33(y_1/x_1) but $\leq$ 1.50

x_1 and y_1 = lengths of short and long sides of the closed stirrups respectively

f_y = yield strength of stirrups

The form of the equation can be obtained from a consideration of the mechanics of behavior of the analogous truss, but the coefficient a_t is empirical (82). It may be regarded as a reinforcement efficiency factor whose value varies between 0.99 and 1.50.

Note that torsion causes shear stresses on all faces of a beam and hence can cause diagonal tension cracking on every face of the beam.

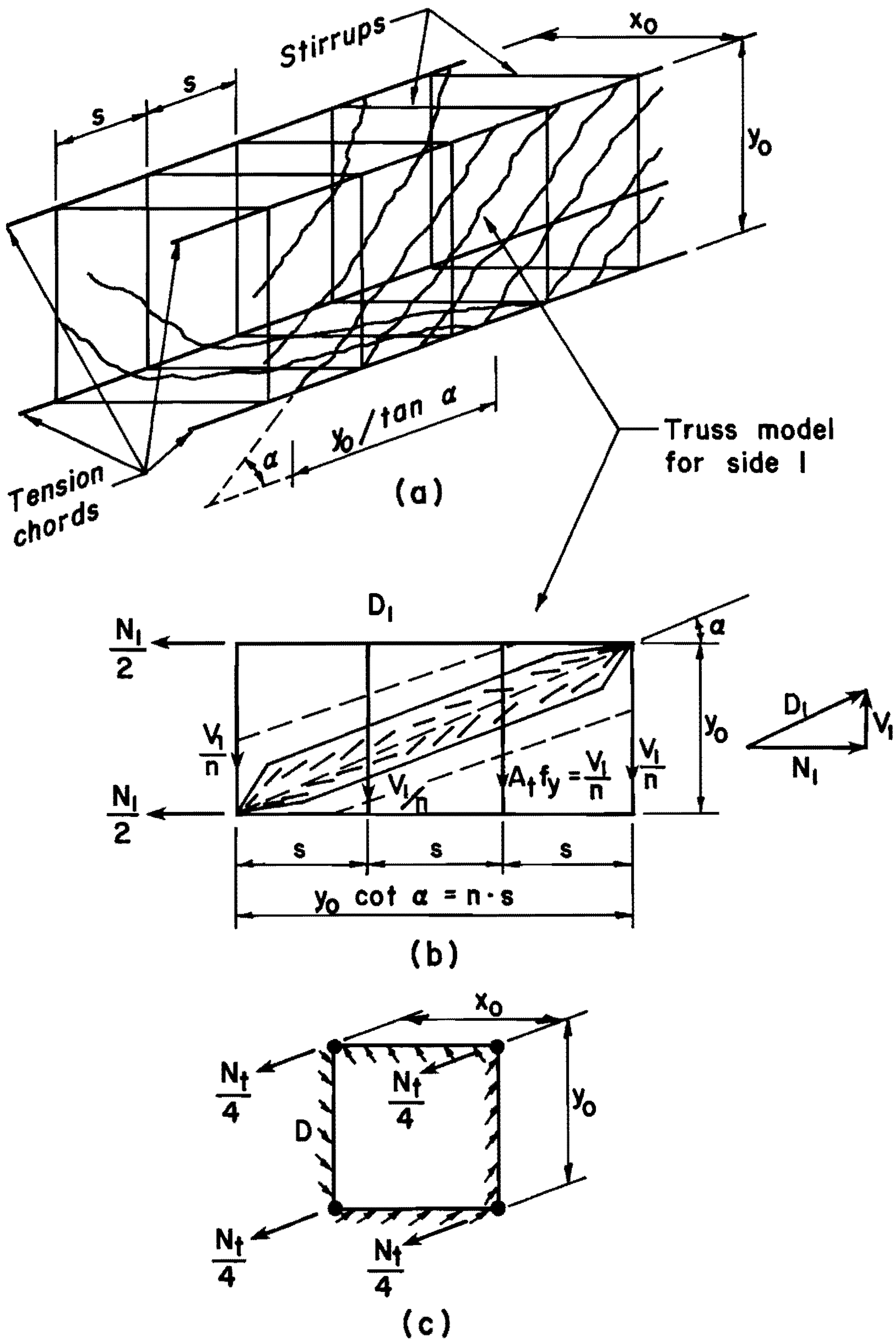


Fig. 2.15 Space truss model

If the torsional moment is sufficiently high, it will even cause diagonal tension cracking to occur on the flexural compression face. It was therefore required in ACI 318-71 (23) that web reinforcement be provided on every face. That is, closed stirrups must be used for torsion.

In the space truss resisting torsion longitudinal reinforcement must be specifically provided for torsion. The diagonal compression forces D from the diagonal compression struts resolve themselves at the joints of the truss into vertical and horizontal components V and N, respectively (see. Fig. 2.15b). The total horizontal force, which is the sum of the horizontal components of the diagonal compression forces, must then be balanced by an equal and opposite tension force requiring a total horizontal steel area, represented in the model of Fig. 2.15c by four longitudinal corner bars, of

$$A_l = \frac{2A_t f_y (x_0+y_0)}{f_{yl}\ s\ \tan^2\alpha} \qquad (2.32)$$

where A_l = total area of longitudinal steel provided, A_t = area of leg stirrup crossing the crack, f_y = yield strength of transverse reinforcement, f_{yl} = yield strength of longitudinal bars, α = inclination of the diagonal compression strut, s = stirrup spacing, x_0 = shorter dimension measured center-to-center of the longitudinal bars, and y_0 = longer dimension measured center-to-center of the longitudinal bars. The derivation of Eq. 2.32 is entirely based on equilibrium considerations of the space truss model shown in Fig. 2.15. In this case the summation of horizontal forces is set equal to zero.

The inclination of the diagonal compression strut can then be determined from the volumetric ratio of the longitudinal to transverse tension reinforcement of the space truss m_t.

$$m_t = \frac{\text{VOL } l}{\text{VOL } t} = \frac{A_l \; s}{2(x_0+y_0)A_t} = \frac{f_y}{f_{ly} \; \tan^2\alpha} \tag{2.33}$$

so that

$$\tan^2\alpha = \frac{f_y}{f_{ly} \; m_t} \tag{2.34}$$

hence:

$$A_l = \frac{2(x_0+y_0) \; A_t m_t}{s} \tag{2.35}$$

Based on the previous considerations, the total amount of longitudinal torsion reinforcement required by ACI 318-71 (23) is given by the equation:

$$A_l = 2A_t \frac{x_1+y_1}{s} \tag{2.36}$$

where A_1 = area of longitudinal torsional reinforcement, x_1 = shorter center-to-center dimension of a closed rectangular stirrup, y_1 = longer center-to-center dimension of a closed rectangular stirrup, A_t = area of one leg of a closed stirrup resisting torsion within a distance s, and s = stirrup spacing along the longitudinal axis of the member.

Since the inclination of the diagonal compression strut is set equal to 45 degrees, and the yield strengths of both reinforcements are assumed to be the same, this equation yields a volume of longitudinal reinforcement equal to the volume of the torsional web reinforcement. The validity of the equation for the cross section of the torsion stirrups is dependent upon the provision of this amount of longitudinal torsion reinforcement.

ACI 318-71 (23) specified in the case of torsion combined with shear, that the area of web reinforcement required for shear be computed by the equation:

$$A_v = (v_u - v_c)b_w s/f_y \tag{2.37}$$

where

A_v = cross-sectional area of both legs of the stirrup

v_u = shear stress = $V_u/b_w d$

s = stirrup spacing

d = effective depth of member

f_y = strength of stirrups

v_c = stress carried by the concrete compression zone

Then the total amount of stirrups required in a beam is the sum of the amounts required for shear and for torsion.

In addition, to ensure that the strength of a cracked beam would be at least a little greater than the load producing cracking (and so prevent a brittle failure at cracking), ACI 318-71 (23) required a minimum amount of shear and torsion reinforcement. In the case of pure shear a minimum web area equal to $50b_w s/f_y$ was required. When dealing with pure torsion however, it had been observed that the contribution of the concrete to the strength after cracking is very much less than the cracking torque. Because of this more web reinforcement was necessary to avoid brittle failure in the case of pure torsion than in the case of shear if equal volumes of web and longitudinal reinforcement are provided. However, for the very small quantities of reinforcement corresponding to minimum reinforcement, the ACI 318-71 Code (23) assumed that the contribution of the torsion reinforcement to the ultimate strength was proportional to the total volume of longitudinal and web reinforcement, and was independent of the ratio of web reinforcement to longitudinal reinforcement. Hence it was able to specify the same minimum web reinforcement for any combination of torsion and shear, that is;

$$(A_v + 2A_t)\text{min.} = 50 b_w s / f_y \tag{2.38}$$

This required the use of more than an equal volume of longitudinal reinforcement for torsion. Hence an area of longitudinal reinforcement

$$A_{1min} = \left[\frac{400xs}{f_y}\left(\frac{v_{tu}}{v_{tu}+v_u}\right) - 2A_t\right]\left(\frac{x_1+y_1}{s}\right) \tag{2.39}$$

where x = shortest dimension of the component rectangle of the cross section which contains the torsion reinforcement required.

This equation was supposed to provide enough longitudinal steel to ensure a ductile failure. In addition, it reduces the amount of minimum longitudinal torsion reinforcement as the ratio of torsion to shear decreases. This equation for A_1 will govern rather than Eq. 2.37 if A_t is less than

$$\frac{100xs}{f_y}\left(\frac{v_{tu}}{v_{tu}+v_u}\right) \tag{2.40}$$

ACI 318-71 (23) set an upper limit for the torsional stresses in order to ensure yielding of the reinforcement. This was done by the following equation:

$$v_{tu\ max} = \frac{12\sqrt{f'_c}}{\left[1+\left(\frac{1.2\ v_u}{v_{tu}}\right)^2\right]^{0.5}} \tag{2.41}$$

Since this equation represents an elliptical interaction relationship between maximum shear and torsion stresses, it also indirectly limited the value of the shear stress.

For the case of interaction between torsion and flexure, ACI 318-71 (23) implied that the torsion and the flexural reinforcement be designed independently and both amounts of reinforcement be provided in the beam. The reasoning behind this method was that beams designed in this way exhibit little or no interaction between torsion and flexure. Consequently it did not require any special consideration of interaction effects between bending and torsion when designing a reinforced concrete beam.

Consideration of torsional effects in the AASHTO Standard Specifications appeared for the first time in 1973 (10) in the Load Factor Design section. Even there, it merely specified whenever applicable effects of torsion shall be added to the nominal design shear stress. It did not specify how to evaluate such torsional stress. It also required that web reinforcement be provided whenever the value of the ultimate design shear stress, torsional effects included, exceeded $2\sqrt{f'_c}$. However, the reinforcement was designed using the same equation as in the case of pure shear. In summary, the torsional stresses were considered to increase the total ultimate design shear stress but the design method did not have to be any different than in the case of pure shear.

In the ACI 318-77 Building Code (24) some torsion design changes were introduced. A difference was established between the design for torsional moments that would produce equilibrium torsion and those that would cause compatibility torsion, by requiring that in statically indeterminate structures where reduction of torsional moments in a member could occur due to redistribution of internal forces, the maximum nominal torsional moment need not be taken greater than $4\sqrt{f'_c}\,\Sigma x^2y/3$. This value came from the clasical solution of St. Venant applied to the common rectangular plain concrete section. Accordingly, the maximum torsional shearing stress v_t is generated at the middles of the long side and can be evaluated as

$$v_t = [kT]/[x^2y] \tag{2.42}$$

where

T = torsional moment at the section

y,x = overall dimensions of the rectangular section, $x < y$

k = stress factor function of y/x.

In the case of compound sections it is customary to assume that a suitable subdivision of the section into its constituent rectangles is an acceptable approximation for design purposes, hence:

$$v_t = [kT]/[\Sigma x^2 y] \tag{2.43}$$

The stress factor k is obtained by applying the plastic solution of Nadais "sand heap analogy" (154) to the case of rectangular sections. According to this analogy the volume of sand placed over the given cross section is proportional to the plastic torque sustained by this section. Using this analogy:

$$k = \frac{2}{1 - \frac{x}{3y}} \tag{2.44}$$

It is evident that $k = 3$ when $x/y = 1$ and $k = 2$, when $x/y = 0$. Concrete is not ductile enough in tension to allow a perfect distribution of shear stresses. In reality the ultimate torsional strength will be between the fully elastic and fully plastic values. Based on this assumption ACI 318-71 (23) suggested the use of the value of $k = 3$. The value of the torsional shear stress v_t when computed using observed test values is between $4\sqrt{f'_c}$ and $7\sqrt{f'_c}$. ACI 318-71 adopted the value of $4\sqrt{f'_c}$ to provide a lower bound solution. x and y are the smaller and larger overall dimensions of the component rectangles of the cross section. The general format of the design process was changed from one of stresses to one of forces. An upper limit was set for the maximum torsional strength that could be provided by the use of web reinforcement T_s. It was not to be greater than four times the strength provided by the concrete T_c, so that the maximum torsional capacity of a member subject to equilibrium torsion was not to exceed fives times the torsional capacity provided by the concrete alone.

The 1977 AASHTO Standard Specifications (12), rather than specifying any specific method of design when torsion is present in reinforced concrete beams, simply suggests in both service load and load factor design sections, that the design criteria for torsion, or combined torsion and shear given in "Building Code Requirements for Reinforced Concrete ACI 318-77 (24)" may be used. Subsequent AASHTO Interim Specifications (13,14,15,16,17), 1982 included, have not changed the approach taken in 1977.

2.5 Torsion in Prestressed Concrete Beams

Currently, neither the ACI Building Code (24), nor the AASHTO Standard Specifications (17), provide any guidance concerning the design of prestressed concrete members for torsion. In order to meet this lack and because of the need for a better understanding of the behavior of prestressed concrete members, extensive research is being conducted to gain more knowledge of the effects of combined loadings on prestressed concrete members.

Past research in the area of torsion in prestressed concrete has followed the same path of development as research in shear in reinforced and prestressed concrete beams as well as torsion in reinforced concrete. Initially the behavior of beams with no transverse web reinforcement was studied. Later, the effect of web reinforcement on the ultimate torsional strength was studied.

Up until 1971, no design criteria for torsion in prestressed concrete beams had been presented in the American literature. In 1974, Zia and McGee (174) proposed a design procedure based on the analysis of 394 test results available in the literature. The empirical procedure for predicting the torsional strength of prestressed concrete beams suggested by Zia and McGee (174) was a development of the procedures incorporated in the 1971 ACI Building Code (23) for the torsional strength of reinforced concrete beams. The method presented by Zia and McGee had severe shortcomings. It was an empirical method, and as such it could be applied safely only in the range of the test data available to the authors. The procedure had been derived for rectangular members and it was quite conservative for flanged sections and box beams. The authors themselves stated (174):

> When more complete research data becomes available, especially for prestressed flanged and box members, the procedure can be further refined and simplified.

As a result of studies carried out in the late 60's European investigators (95,96) have proposed methods based on theory of plasticity in which a space truss with variable angle of inclination of the diagonal compression elements constitutes the lower (static) bound solution. These methods assume that diagonal compression fields form on the faces of the member when subjected to torsional stresses. During the 70's, in Canada, Mitchell and Collins (119,120) extended these concepts to relate truss statics with kinematics using appropriate stress-strain relationships.

The application of limit analysis and the space truss models leads to some very interesting conclusions, which shed light on some very old myths regarding the behavior of this type of member. First, regarding the effects of the longitudinal prestressing on the torsional behavior and strength of prestressed concrete beams, it shows that increasing the amount of longitudinal prestressed steel in a beam will increase the cracking torque more than it increases the ultimate torque. Furthermore, if the longitudinal steel yields at ultimate it makes no difference on the ultimate torsional capacity whether the longitudinal steel was prestressed or not. In this case the capacity depends on the total yield force of all the longitudinal steel in the section irrespective of the type of steel or of its location in the section, provided that for pure torsion the steel is symmetrically distributed around the perimeter of the cross section. However, if the longitudinal steel does not yield at ultimate then the torsion capacity will be a function of the magnitude of the tensile force in the longitudinal steel at ultimate, which will in turn depend on the level of prestress in the steel.

The truss model also shows that longitudinally prestressed thick-walled hollow members have the same torsional response as longitudinally prestressed solid members with otherwise indentical properties.

Finally, in relation to the volumetric ratio of reinforcement, the introduction of realistic stress-strain diagrams showed that there is a balanced amount of longitudinal reinforcement below which the longitudinal steel will yield at failure irrespective of the ratio of longitudinal steel to hoop steel. Furthermore, there is a balanced amount of hoop steel below which the hoops will yield at failure irrespective of the ratio of longitudinal steel to hoop steel. Mitchell and Collins' procedure is an ultimate load method which applied directly to the case of equilibrium torsion. In statically indeterminate structures, where reduction of the torsional moment in a member can occur (compatibility torsion), Mitchell and Collins suggest that the value of the maximum design factored torsional moment need not be greater than 67% of the torsional cracking moment of the section, provided that the corresponding adjustments to the moments and shears in adjoining members are made.

Both the European and Canadian approaches claim a unified design for shear and torsion in prestressed and reinforced concrete beams. Collins and Mitchell (56) in Canada have applied their compression field method to beams subjected to combined loading. So far the theory has not been fully extended to cover this case. They recommend the use of the space truss with variable angle of inclination of the diagonals to handle the analysis of such members and propose a conservative simplified design procedure, which basically requires the superposition of the effects of shear and torsion (56).

The Europeans, led by Thürlimann (166), have proposed a physical model based on the theory of plasticity. This model consists of a space truss with variable angle of inclination of the diagonal compression members. This procedure tests compatibility torsion as a restraint, and is basically used to analyze problems of equilibrium torsion in which the torsional stresses are resisted by a constant shear flow around the perimeter of the section. Nonconstant shear flows and warping torsion of open cross sections have also been treated. In this model the approach is to superimpose the effects of torsion and shear. The required reinforcement is generally determined separately for bending, torsion and shear although later proposals introduce general combined analyses. In the flexural tension zone all longitudinal reinforcement requirements must be added. In the compression zone, however, the required reinforcement can be reduced by considering the compression force due to the bending moment, since this compression offsets some of the longitudinal tensile component due to the torsional stresses. The stirrup reinforcement required for shear and torsion is added. This method recognizes the existence of different levels of stress or ranges in the member. The ranges are determined by consideration of the magnitude of the shear stresses existing due to shear and/or torsion. These ranges consist of uncracked, transition, and full truss ranges. In the uncracked and transition ranges, some tensile component of the concrete contributes significantly to the load carrying capacity of the member. The third range is the full truss range. It is defined by the magnitude of shear stress above which the total load carrying capacity of the member is based solely on the contribution of the compression diagonals or struts, the concrete longitudinal compression flange, and the transverse and longitudinal reinforcement tensile contribution (166). No tensile contribution from concrete is considered. Both the Compression Field Theory (56) and the Space Truss Method (166) impose an upper limit on the magnitude of the shear stress, in order to limit the cause of failure in the member to yielding of the reinforcement, rather than crushing of the diagonal compression elements or struts.

2.6 Summary

As was noted by Hognestad (78) over thirty years ago, reinforced and prestressed concretestructures were built in considerable numbers before rational design procedures were developed.

The knowledge of shear and torsion behavior in concrete members is unfortunately not as detailed or rational as the understanding of flexural behavior. This results in much more unsatisfactory shear and torsion design procedures. The present codes have confusing and overlapping empirical expressions to predict with apparent great refinement the effect of parameters such as M/V_d, ρ_w, and f_c' on shear, in addition to a_t in the case of torsion. Unfortunately, the largely empirical and complex equations fail to reflect the main emphasis that should be given to appreciating the action of the overall combined concrete and steel system for carrying shear and/or torsion. And, at present, there is a real need of a conceptual model for the designer upon which code or specification provisions can be based.

It may be noted that in the history of both reinforced and prestressed concrete basic concepts were at times understood correctly, even though incompletely, by early pioneers. Such correct concepts were occasionally not generally understood by other engineers of the time and hence, not accepted. Widespread practice, therefore, followed other trends for a number of years until a rediscovery was made, thus returning attention to the early findings (78).

Such is the case with the space truss model proposed by more recent European investigators such as Thürlimann et al. (95,96,165,166), which is a refinement of Ritter's and Morsch's pioneering truss models. This approach may greatly simplify the design for shear and torsion in reinforced and prestressed concrete members. Use of a conceptual model should reduce the confusion of the designer in comparison to current utilization of the highly empirical present AASHTO-ACI Specifications. In addition, use of such a model would provide the designer with guidance in those areas of torsion and combined shear and torsion where there is a complete absence of design recommendations.

The Compression Field Theory and the Space Truss with variable angle of inclination of the compression diagonals would yield the same ultimate load for a given underreinforced member since both procedures are based on the same conceptual model of an space truss with variable angle of inclination of the compression diagonals. The main difference between design proposals based on these two procedures is in the limitation of the angle of inclination, α. In the Space Truss Model a semi-empirical approach is followed to set the upper and lower limits of this angle. In the compression field theory the strain conditions existing in the diagonal strut are used to select the limits for the angle of inclination. Both approaches lead to the same general results. However, the beauty of the conceptual model of the space truss, is lost in the highly theoretical approach followed by the Compression Field Theory. In this study the Space Truss with variable angle of inclination of the diagonal compression struts is explored in detail, compared to test data, and proposed as a conceptual model for the treatment of prestressed and reinforced concrete beams subjected to torsion and/or shear. With the aid of this model, greatly simplified design specifications in the areas of shear and torsion are possible.

In the following chapter a discussion of the Space Truss model and its application in the areas of torsion and/or shear and bending is presented. A subsequent report (248-3) presents comparisons between this theory and test results. In the final report (248-4F) of this series, detailed proposals for codifying this theory are presented and compared with current procedures through use of design examples.

CHAPTER 3

THE SPACE TRUSS WITH VARIABLE INCLINATION DIAGONALS AS A DESIGN MODEL

3.1 Introduction

Improvements in materials technology, resulting in use of higher strength steels and concretes, as well as adoption of the generally less conservative ultimate strength design procedure, have resulted in more slender members than in the past. These changes have eroded the hidden factor of safety which helped guard against shear and torsional failures.

Because of the more abrupt nature of shear and torsion failures, and the difficulty of formulating reliable mathematical models for the behavior of beams in shear and torsion, research has tended to concentrate on predicting the collapse load of such members, usually on an empirical basis. In addition, because of the complexity in determining what role the concrete component plays in carrying shear in a structural member, the research in the U.S. has been very largely directed to studying beams with no or with very light web reinforcement. Unfortunately, from a scientific standpoint an empirical approach is only correct if the separation and control of the main variables in the test program are assured, and if sufficient tests are conducted to allow a statistical treatment of the results. In testing structural components or entire structures of reinforced or prestressed concrete these conditions are almost impossible to fulfill because of the time and financial constraints. Unfortunately, diverted by the large amount of test studies required to substantiate the empirical approaches, more basic studies of the behavior and modeling of the overall system carrying shear and torsional forces have been neglected. If this understanding were available, design rules would be much simpler and would not involve as much test verification.

The unsatisfactory nature of the shear and torsion "theories" currently used in North American design practice, which consist of a collection of complex, restricted, empirical equations, is a result of following such an empirical approach without a rational model that would provide an understandable central philosophy.

For this reason an approach based on a simplified model, considering the major variables is preferable. During the past two decades, research aimed at developing failure models to better understand the shear and torsional behavior of reinforced and prestressed concrete beams with web reinforcement has been conducted all over the world (57,58,60,93,95,164,165,166).

One of the most interesting failure models is the updated form of the Ritter and Morsch truss models. The model, a space truss with variable angle of inclination of the diagonals, permits a unified treatment of shear and torsion in both reinforced and prestressed concrete beams containing web reinforcement.

The theory of plasticity provides the mathematical foundation for this failure model by means of the upper and lower bound theorems of limit analysis (124). Substantiation of the space truss model has been provided by showing that the load for which a stable, statically admissible state of stresses exists in the truss, (a lower bound value), is equal to the load for which there exists an unstable, kinematically admissible state of motion (an upper bound value). This load is then the failure load (72,93,124,163). This substantiation is extremely important and will be briefly reviewed in this chapter. The excellent agreement with a wide body of test results is summarized in Report 248-3 (178). However, once the variable angle truss model is accepted the design procedures are quite simple. The plasticity proof is for introduction only and not for application.

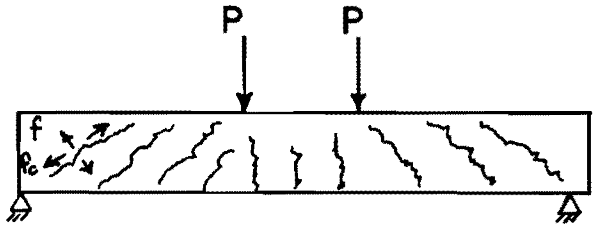
It may initially seem that a design based on a truss model with a variable inclination of the compression members would be quite complicated. However, the opposite is the case. Using relatively simple guidelines and limits, the designer is able to choose a truss model which is suitable to carry the applied loads, to determine the internal forces using the chosen truss model, to replace the truss members by compression struts with finite widths or by tension bars and to check whether these internal forces may be carried safely. Furthermore, such a truss analysis would pinpoint the locations where special attention for detailing of the reinforcement is required.

3.2 The Space Truss Model

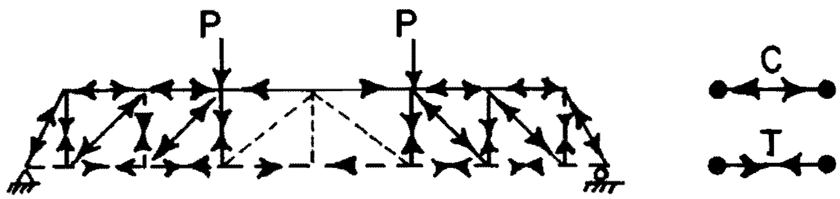
The truss model consists of the longitudinal tension reinforcement as a tension chord, the longitudinal compression reinforcement and the flexural concrete compression block acting as the compression chord, the vertical stirrups acting as ties and the concrete diagonals acting as inclined struts forming a continuous compression field. Unlike Ritter's original truss model (later extended by Morsch and by Rausch to beams subjected to torsion), where the diagonals are at 45 degrees, the space truss model uses a variable angle of inclination of the concrete struts α. This inclination of the struts is the inclination at ultimate and not at first inclined cracking.

The truss model representation for reinforced and prestressed concrete one-way members, subjected to shear and/or torsion, can be easily understood by observing the typical failure crack patterns of those members. The crack patterns at failure indicate the orientation of the principal stresses in the member.

A typical failure pattern for a beam subjected to moment and shear is shown by the crack pattern in Fig. 3.1a. The truss model is shown in Fig. 3.1b. The upper compression chord is formed by the concrete in flexural compression acting along with any compression steel. The lower tension chord is formed by the reinforcement acting across the flexural tension cracks. The principal diagonal tension stresses act perpendicular to the diagonal crack directions in the shear span. The principal inclined compression stresses would exist in the member at an orientation 90 degrees from the principal tensile stresses and, hence, would act parallel to the inclined crack direction. The concrete between these inclined cracks may then be considered to act as inclined compression struts. Finally, the truss tension verticals are



(a) Beam crack pattern



(b) Truss analogy

Fig. 3.1 Truss analogy in the case of bending and shear provided by the web reinforcement extending from the lower chord to the upper chord.

The model can be applied widely and used with torsion in circular or noncircular solid sections, as well as hollow closed sections (box beams). The same basic principles followed in forming the truss model representation in the case of shear in reinforced and prestressed concrete members can be applied in the case of torsion. By observing the typical crack patterns at failure of members subjected to torsion, the truss model representation for such members becomes obvious. Fig. 3.2(a) shows a beam subjected to torsion. The resulting shear flow on the section is shown in Fig. 3.2(b). In Fig. 3.2(c), typical crack patterns on each face at failure of a concrete member subjected to torsion are shown. The principal diagonal tension stresses act perpendicular to the diagonal crack directions. The principal compression stresses would exist in the member at an orientation 90 degrees from the principal tensile stresses and hence, would act parallel to the inclined crack direction. The truss model is shown in Fig. 3.2(d) in an exploded fashion for clarity. The concrete between the inclined cracks is then considered as acting as inclined compression struts in the analogous truss. The truss tension verticals are then provided by means of web reinforcement extending from the lower chord to the upper chord. The model is valid in the complete range of interaction between general bending, axial force and torsion. However, limits must be set in some fashion to preclude initial compression failures. Since concrete failure is excluded by these limits, then the

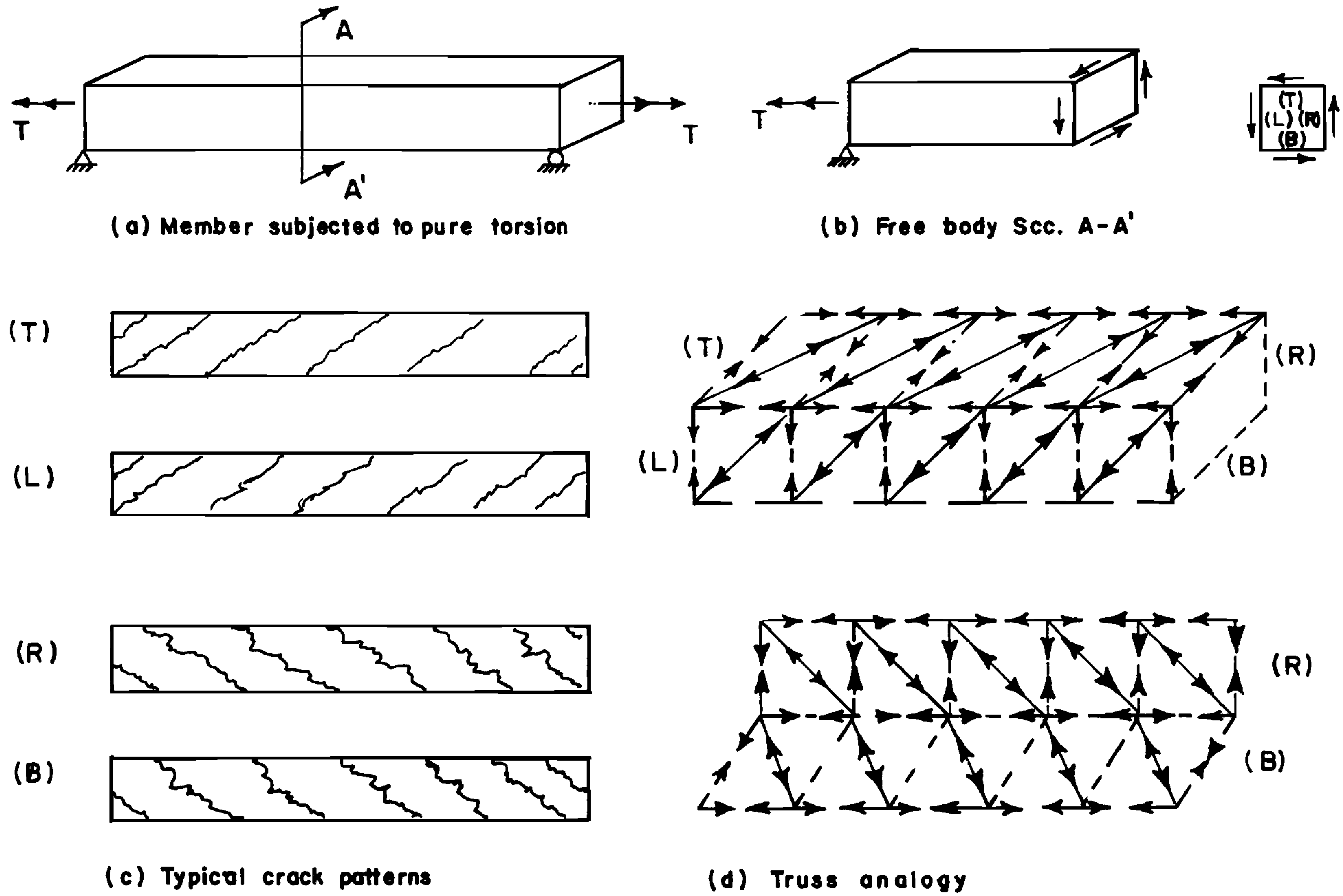


Fig. 3.2 Truss analogy in the case of pure torsion

assumption of an underreinforced member can be made. Thus, the failure load is determined by the yield forces of the longitudinal stringers and the stirrups. It is then possible to investigate the failure model by means of the upper and lower bound theorems of Theory of Plasticity. For this purpose an idealized elastic-plastic stress-strain curve for the reinforcing steel is assumed and equilibrium is formulated for the undeformed system (first order theory) (72,93,124).

In the development of this model, six important assumptions are made:

1. Prior to failure, yielding of the longitudinal reinforcement is required. This limits consideration to underreinforced members.

2. Diagonal crushing of the concrete does not occur prior to yielding of the transverse reinforcement. This requires an upper limit for the concrete stresses as well as limits on the angle of inclination of the diagonal compression struts.

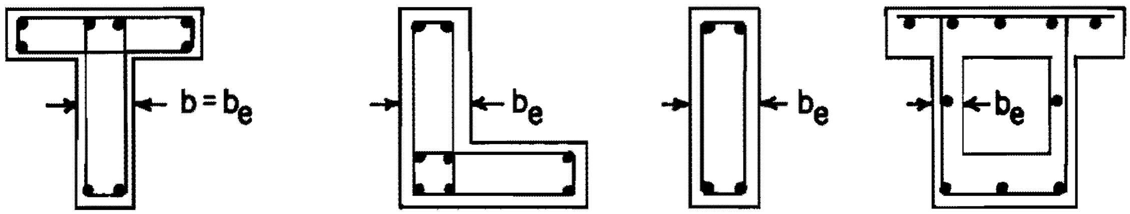
3. The concrete stress is compressive. This means the tensile strength of the concrete is neglected.

4. Only uniaxial forces are present in the reinforcement. (Thus dowel action is neglected.)

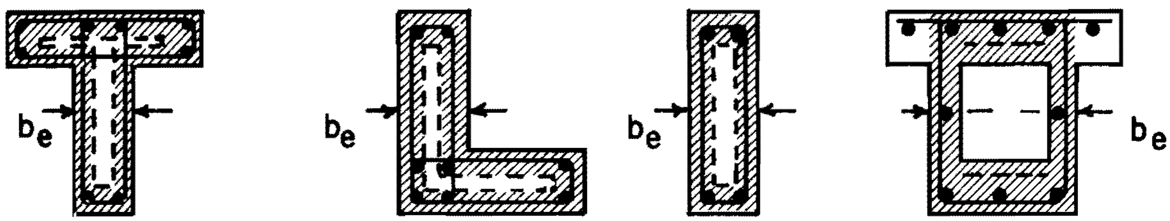
5. At ultimate load, after all elastic and inelastic deformations and the redistribution of internal forces have taken place, there is uniaxial yielding of the steel reinforcement and the opening of the failure cracks in the concrete is normal to the crack direction.

6. The steel reinforcement must be properly detailed so as to prevent premature local crushing and bond failures. (182)

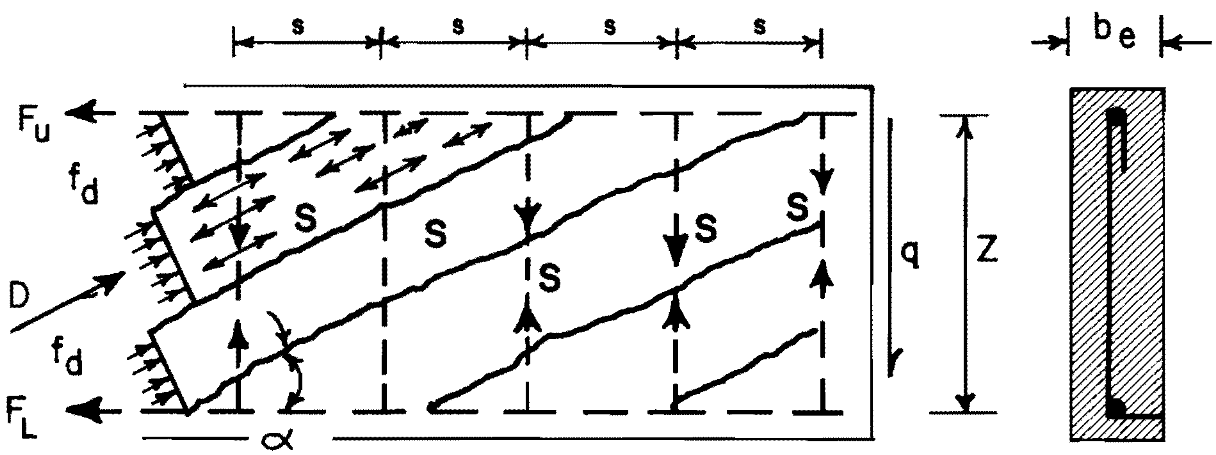
One of the basic concepts introduced by this failure model is the shear field element. A reinforced concrete beam of any particular shape (rectangular, L-beam, T-beam, or box section), can be subdivided into a number of shear fields (see Fig. 3.3). In these shear fields the



(a) In the case of shear



(b) In the case of torsion



(c) Forces in the shear field

Fig. 3.3 Shear field analogy applied to T, L, rectangular, and box beams in the case of shear and torsion longitudinal reinforcement or flexural compressive blocks are considered to be concentrated into chords at the corners, the stirrups or transverse reinforcement act as ties and the concrete between the inclined cracks provides the compression diagonals. A shear force or a torsional moment is assumed to produce a resultant constant shear flow in the shear fields. The state of stress and strain in the elements due to shear is hence similar to the one due to torsion. This similarity holds in the uncracked state, in the cracked state and at ultimate load. A unified approach to both actions is not only desirable but necessary to consider the combined loading case of bending, shear and torsion. The strength of the shear field element is determined using the equilibrium solution (lower bound) of the space truss model. The correctness has been confirmed by the agreement of the upper bound solution.

The Space Truss Model with variable angle of inclination of the compression diagonals departs from the traditional truss model with constant 45 degree angle diagonals proposed by Ritter (150) and Morsch (122). It is a descendant of the more general models suggested by Morsch. It is a more realistic truss model. The angle is such that in the field where failure occurs, both the longitudinal and stirrup reinforcement will reach their yield stresses. In this case a sufficient shear transfer by aggregate interlock across the initial inclined cracks is assumed so that the concrete diagonals can reach their final inclination under ultimate loads. Due to the fact that such shear transfer across a crack decreases with increasing crack widths, additional considerations become necessary. Hence, limits on the inclination of the concrete diagonals must be introduced.

In the study of the proposed failure model it becomes apparent that yielding of the longitudinal reinforcement and/or the stirrups must occur prior to failure of the concrete. Concrete failure before yielding of the reinforcement can be caused by:

a. crushing of the concrete compression diagonals

b. excessive shearing strains due to a large deviation of the angle of inclination of the compression struts from 45 degrees (128,180)

c. crushing of the concrete flexural compression zone

These types of failure must be avoided if the proposed failure model is to accurately predict the ultimate load.

In the application of this failure model, proper detailing of the cross section is of utmost importance. The model requires yielding of the reinforcement as well as an elimination of any type of local crushing or bond failures. Therefore, in addition to finding the correct internal forces, the designer must draw the necessary conclusions for the detailing of the reinforcement. The space truss model illustrates the manner in which reinforced concrete beams resist torsion and shear stresses, and enables the designer to visualize the functions of the concrete, the longitudinal steel, and the stirrups. Hence, the model also aids the designer in correctly detailing the member.

The model applies directly to both reinforced and prestressed concrete members. This is due to the fact that since only underreinforced sections are considered and, since initial shear failures are undesirable at ultimate load, the prestressed as well as the nonprestressed longitudinal reinforcement will yield. Prestressing of the longitudinal reinforcement basically influences behavior at service load levels. However, after the initial level of compression on the cross section induced by the prestress force is overcome, the strains in all reinforcement increase simultaneously. The crack patterns in the concrete at failure will be the same as for an ordinary reinforced beam and the yield force will be equal to the sum of the yield forces of both prestressed and nonprestressed reinforcement (48). Therefore, reinforced, prestressed and partially prestressed concrete members will exhibit the same fundamental behavior at ultimate load.

Consider a reinforced concrete member with area of longitudinal reinforcement, A_s, with yield stress, f_y, and consider a prestressed concrete member with area of longitudinal prestressed reinforcement, A_{ps}, and with yield stress, f_{yps}. The area of prestressed reinforcement can be expressed as an equivalent area of nonprestressed reinforcement A_q:

$$A_q = A_{ps}\, f_{yps}/f_y \qquad (3.1)$$

Thus, the yield force of the longitudinal chord of a reinforced concrete member can be expressed as:

$$F_y = A_s f_y \qquad (3.2)$$

That of a prestressed concrete member can be expressed as

$$F_y = A_{ps} f_{yps} = A_q f_y \qquad (3.3)$$

That of a partially prestressed concrete member can be expressed as

$$F_y = A_s f_y + A_{yps} = (A_s + A_q) f_y \tag{3.4}$$

Once the area of prestressed reinforcement is converted to an equivalent area, the truss model can be applied as for an ordinary reinforced concrete member with similar yield force of nonprestressed reinforcement.

Further studies by Mitchell and Collins (57,120) have corroborated these assumptions in the cases of beams subjected to torsion and/or shear and bending.

3.3 Inclination of the Diagonal Compression Elements of the Space Truss

Unlike Ritter's original truss model where the diagonals are assumed as always at 45 degrees. Thürlimann's model uses a variable angle of inclination of the concrete struts α. Note that α is the angle of inclination at ultimate and not at first inclined cracking. Sufficient shear transfer across the initial cracks is assumed so that the concrete diagonals can reach their final inclination under ultimate loads.

A redistribution of the internal forces between the cracking load level and the ultimate load level, corresponding to changing of the angle α, is only possible if a sufficient shear transfer by aggregate interlock across the previously formed crack occurs. If the cracks start to open widely, the shear transfer will deteriorate rapidly and no further redistribution becomes possible.

The effects of the inclination angle α on the mechanism of shear transfer across the cracks between the diagonal compression struts can be discussed using kinematic considerations.

Working under the assumptions that only underreinforced sections are considered, that the concrete is assumed rigid, and that all deformations are caused by elongation of the reinforcement, various investigators (162,163,165,72,93) as well as the Swiss Code and the CEB Model Code have introduced limitations on the value of the inclination angle of the diagonal compression strut at ultimate. Kinematic considerations regarding the relationships between crack width and the strains in the stringers and stirrups, as well as the rapid deterioration of the aggregate interlock in the concrete with increasing crack width require such limits.

In this study the kinematic relations are discussed only to the extent necessary to explain the limitations on the inclination of the concrete compression diagonals in the space truss model. More detailed treatments can be found in Refs. 93, 124, 163,and 164.

The direction of the crack opening of a section is first assumed to be normal to the crack direction. At ultimate, it is assumed that the angle of inclination of the principal compressive stress would coincide with the angle of inclination of the principal compressive strain (124).

Consider a shear field element subjected to shearing stresses: Because of the assumptions that only underreinforced sections are being considered and the additional assumption that the concrete compression diagonal is considered to be rigid, the energy is then dissipated entirely by yielding of the reinforcement. Consequently, the principal compressive strain ϵ_{ds} is assumed equal to zero. Following the assumption that the opening of the failure cracks in the concrete is normal to the crack direction the state of motion for a shear field element subjected to shearing stresses as shown in Fig. 3.4 is obtained.

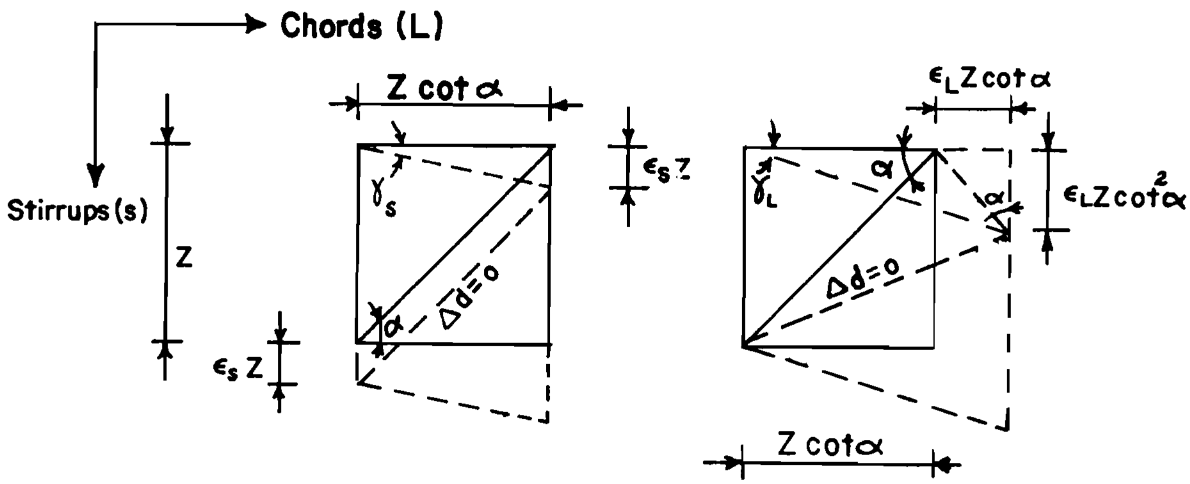


Fig. 3.4 Displacement diagram for a shear field element

The shearing strain due to stirrup strain for small angles is given by

$$\gamma_s = \frac{\varepsilon_s Z}{z \cot\alpha} = \varepsilon_s \tan\alpha \tag{3.5}$$

Similarly, the shearing strain in the shear field element due to the longitudinal strain can be found as:

$$\gamma_1 = \frac{\varepsilon_1 \cot\alpha}{1 + \varepsilon_1} \simeq \varepsilon_1 \cot\alpha \tag{3.6}$$

Thus, the total shearing strain is related to the elongation of the reinforcement as follows:

$$\gamma = \gamma_s + \gamma_l = \epsilon_s \tan\alpha + \epsilon_l \cot\alpha \tag{3.7}$$

The mean crack strain ϵ_r may be used as a convenient parameter, being defined as the mean crack width w, divided by the mean crack spacing distance d_{cr}. From the existing state of strain in the diagonal strut shown in Fig. 3.5, the relationship between the elongations ϵ_l and ϵ_s can be formulated as:

$$\epsilon_s = \epsilon_l \cot^2\alpha \tag{3.8}$$

The displacement due to the mean crack strain ϵ_r is related to the elongations of the reinforcement as follows:

$$\epsilon_s = \epsilon_r \cos^2\alpha \tag{3.9}$$

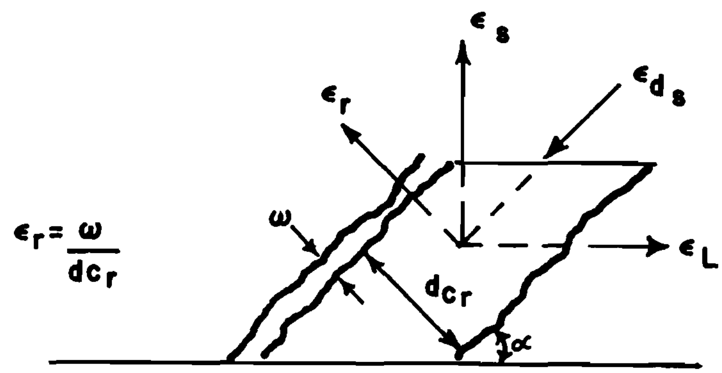
and

$$\epsilon_l = \epsilon_r \sin^2\alpha \tag{3.10}$$

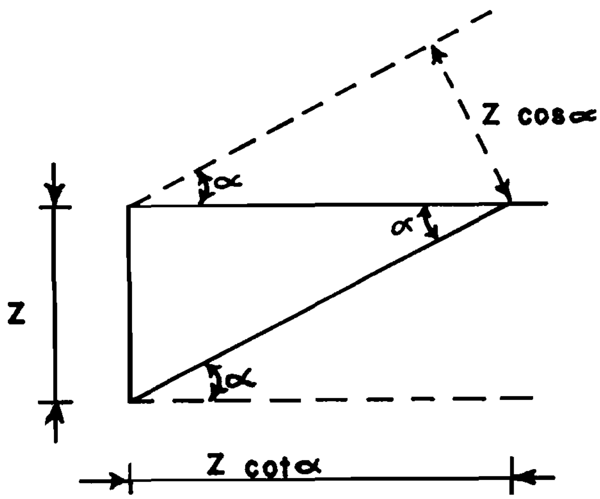
Addition of the two previous equations yields the relation:

$$\epsilon_s + \epsilon_l = \epsilon_r \tag{3.11}$$

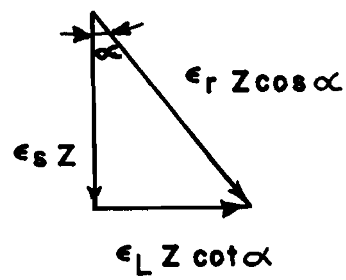
The same relations can be derived through the Mohr circle for the strains of the shear field element shown in Fig. 3.4. Assuming ϵ_{ds} equal to zero, a Mohr diagram at collapse can be drawn for the element as shown in Fig. 3.6 .



(a)



(b)



(c)

Fig. 3.5 State of strain in the diagonal strut

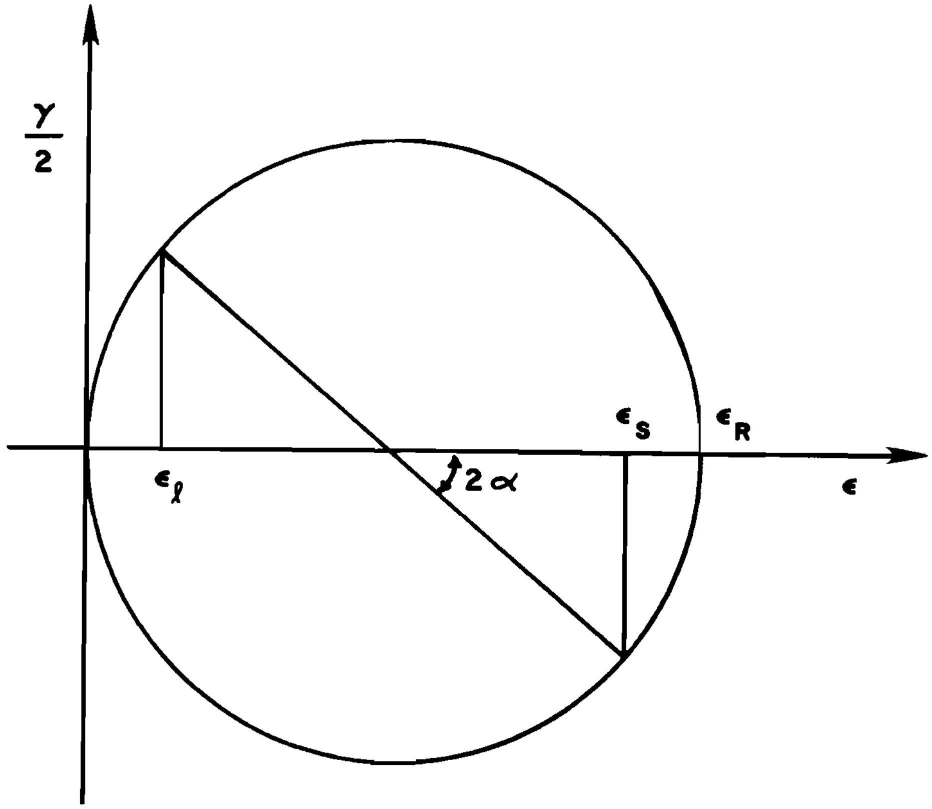


Fig. 3.6 Mohr's diagram for element of Fig. 3.4

From Eqs. 3.8 and 3.11, it follows that at yielding of the transverse reinforcement ($\epsilon_s = \epsilon_y$):

$$\epsilon_r = \epsilon_y(1 + \tan^2\alpha) \tag{3.12}$$

And at yielding of the longitudinal reinforcement ($\epsilon_1 = \epsilon_y$):

$$\epsilon_r = \epsilon_y(1 + \cot^2\alpha) \tag{3.13}$$

Equations 3.12 and 3.13 can be rearranged and plotted on the same graph as shown in Fig. 3.7. From this figure, it can be seen that if the compression struts are inclined at 45 degrees, the mean crack

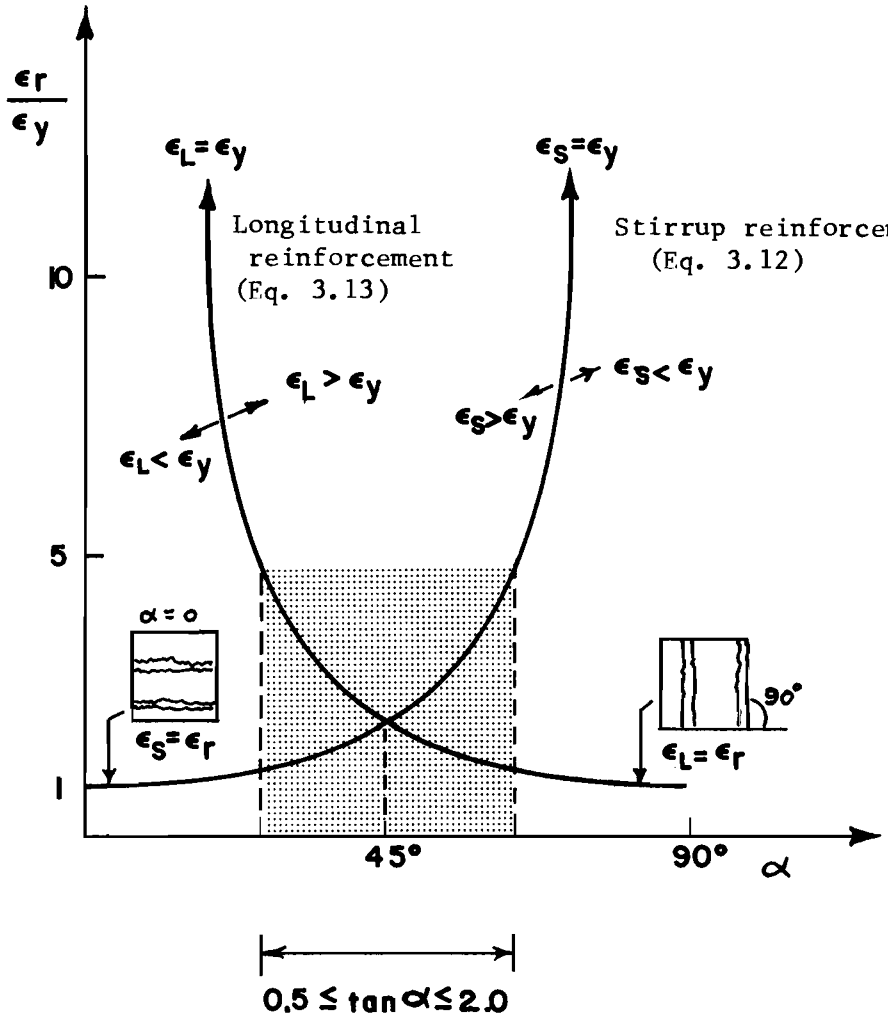


Fig. 3.7 Mean crack strain vs. yield strain in reinforcement (from Ref. 165)

strain and hence the mean crack width are at the minimum value for yielding of both the longitudinal and transverse reinforcement. In Fig. 3.7 it is also shown that if the angle of inclination is greater than 45 degrees, yielding of the stirrups demands larger mean crack strains. Conversely, for angles less than 45 degrees, yielding of the longitudinal reinforcement requires increasingly larger crack openings. The orthogonal crack opening as well as the inclination of the compression field are subjected to certain limits. The inclination α of the diagonal compression strut is the inclination at ultimate and not first inclined cracking. A redistribution of the internal forces from the cracking to ultimate load, due to changing of the angle α, is only possible if a sufficient shear transfer by aggregate interlock in the previously formed cracks occurs. If the cracks start to open at an accelerated rate the shear transfer deteriorates rapidly and no further redistribution becomes possible. Based on test observations, Thürlimann (165) proposes the following limits:

$$0.50 \leq \tan\alpha \leq 2.00$$

$$26^{o} \leq \alpha \leq 63^{o}$$

Where α is the angle of inclination of the diagonal compression strut at ultimate.

For the case of development of shear capacity after yielding of the flexural reinforcement so as to provide suitable ductility, both the longitudinal and transverse reinforcement of the shear field element have to yield in order to form a collapse mechanism. Equation 3.8, which expresses the orthogonal crack opening is only valid within the previously stated limits. Figure 3.7 shows that outside the limits $\tan\alpha = 0.5$ or 2.0 failure will require only yielding of the transverse or the longitudinal reinforcement together with large mean crack strains ϵ_r. The Swiss Code (156) taking into account service load considerations, and that Thürlimann's model is an ultimate load model, proposes more constrained limits for the variation of the angle of inclination of these diagonal elements by suggesting the values:

$$0.60 \leq \tan\alpha \leq 1.67$$

$$31^{o} < \alpha < 59^{o}$$

These limits were apparently later adopted by the CEB Model Code and the 1982 FIP Recommendations on Practical Design of Reinforced and Prestressed Concrete Structures (67).

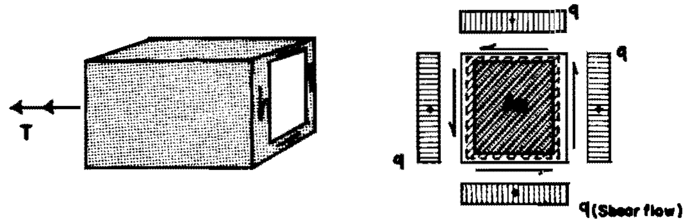
Basically these empirical limits must be introduced to compensate for the fact that procedures based on plastic analysis, such as the one presented by Thürlimann, cannot distinguish between underreinforcement and overreinforcement, i.e. yielding of the reinforcement prior to diagonal crushing, because they do not predict total deformations. Furthermore, the CEB, FIP, and Swiss Codes (67,156) lower limit of $\tan\alpha \geq 0.60$, which is intended to ensure adequate inclined crack width control at service load levels, made it necessary to introduce a transition region between uncracked and fully cracked behavior in order to avoid requiring more transverse reinforcement for low shear stresses than required by previous editions of such codes. So far only underreinforced sections have been considered. In such members the stirrups and the longitudinal reinforcement yield prior to failure of the concrete. Concrete failure can be caused by crushing of the bending compression zone or of the concrete compression diagonals. These kinds of failure should be avoided and therefore the concrete stresses must be checked.

3.4 The Space Truss Model for Torsion

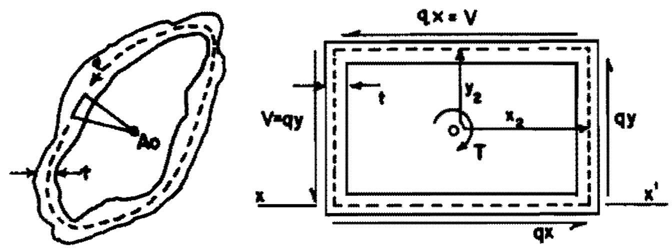
Although the space truss model intrinsically applies to thin-walled closed cross sections subjected to torsion and has been extended to open and thick-walled sections (180,181,182), the applicability of the same relations to solid cross sections has been well confirmed by test results (95,96,182).

The basic space truss model gives the torsional resistance of a thin-walled tube. If such a section is subjected to a torsional moment T, a constant shear flow "q" results around the circumference (154). (See Fig. 3.8a.)

The concept of the shear field element becomes of great importance, and it can be better illustrated by studying the box section. In this case, each of the side walls of the box becomes a shear field element. The basic components of such a side wall, now referred to as a shear field element, consist of upper and lower longitudinal tension chords stirrups as vertical ties, and a continuous compression field made up of the concrete compression diagonals inclined at an angle α. Similar to a thin walled tube, if a torsional moment is applied to the space truss, a constant shear flow around the



(a) Shear flow

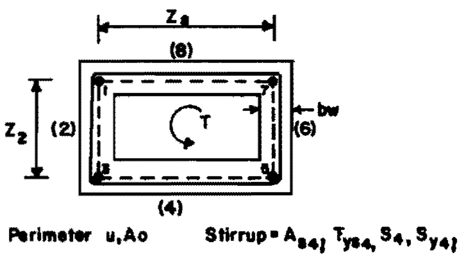


$$+\ \Sigma M_o = qx \cdot \frac{y}{2} + qy \cdot \frac{x}{2} + qx \cdot \frac{y}{2} + qy \cdot \frac{x}{2} - T$$

$$= 2\ qxy - T = 2q\ A_o - T$$

$$q = \frac{T}{2A_o}$$

(b) Shear flow formula



Chord: A_1, F_{y1}, F_1, F_{y1}

(c) Box section notation

Fig. 3.8 Shear flow "q" due to torsion in a thin walled closed section circumference results. From the principles of mechanics of materials (154), the shear flow in a thin walled tube is a function of the applied torsional moment and the enclosed area, A_o (see Fig. 3.8b). A_o is defined as the gross area enclosed by the perimeter connecting the longitudinal chords in the corners of the section (see Fig. 3.8c).

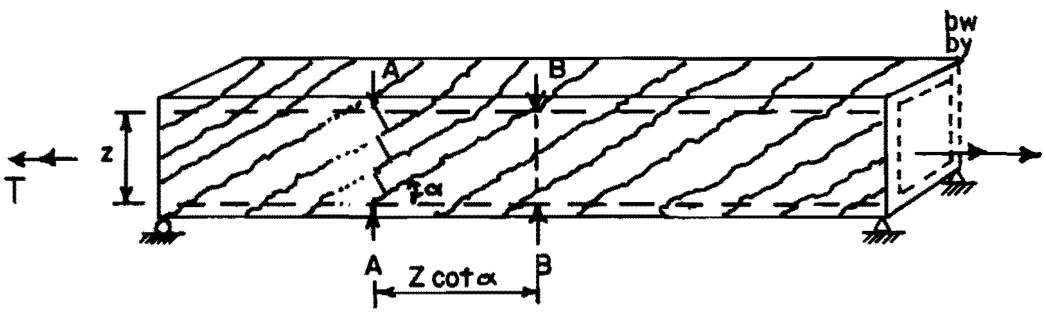
$$q = T/2A_o \tag{3.14}$$

The force in the compression diagonals can be obtained by cutting a free body which encompasses a single inclined crack as shown in Fig. 3.9(a). Note that the freebody along Sec. A-A is drawn normal to each compression diagonal which is cut. The truss forces produced by the shear flow "q" in a side wall of depth z may be found from the freebody shown in Fig. 3.9(b). Note that the effective area on which the inclined compressive forces act is always less than $(b_w)(z*\cos\alpha)$ which is always less than $(b_w)(z)$. This is because of the inclined section geometry. The assumption made at failure that the cracks will be wide enough to minimize aggregate interlock shear transfer across the cracks is the reason that no interface shear stresses are shown on the freebody along the crack in the lower portion. The compression resultant D of the compression field stress, f_d, is inclined at an angle α. From equilibrium the following relations are obtained (166):

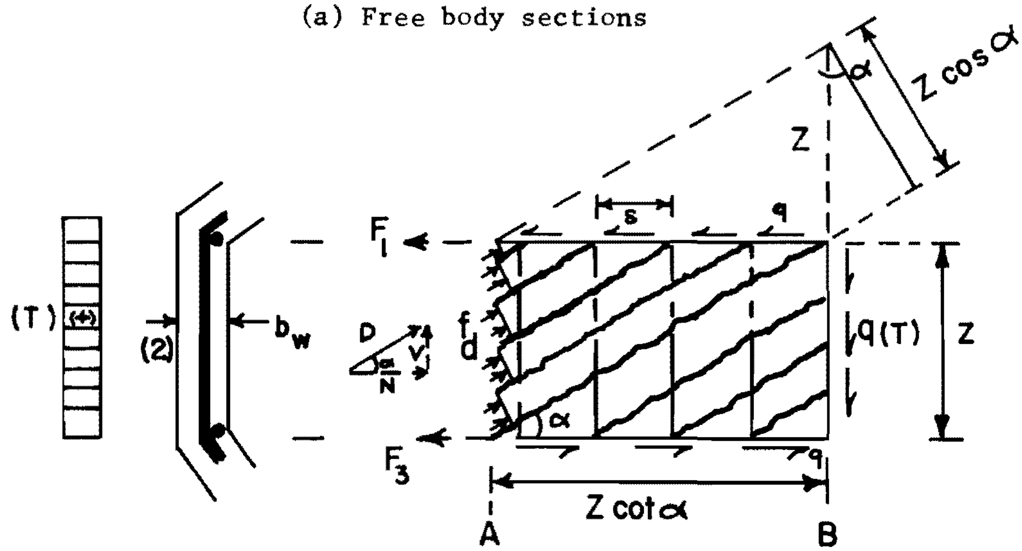
Diagonal force D: since $V = q*z$ and from vertical equilibrium $V = D*\sin\alpha$, then

$$D = q*z/\sin\alpha \tag{3.15}$$

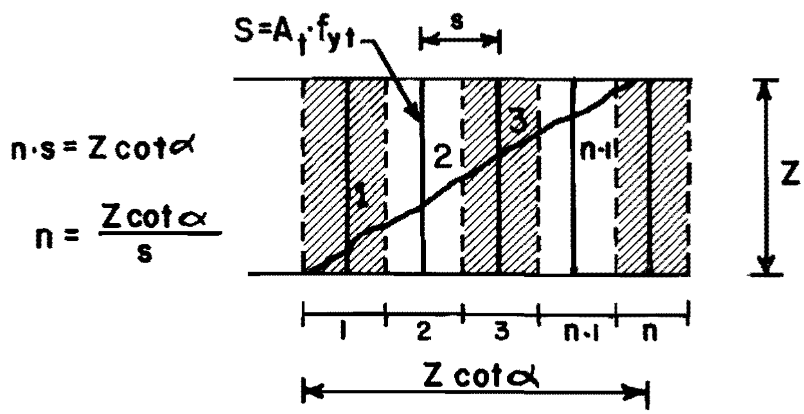
Since $D = f_d*b_w*z*\cos\alpha$ and, as shown, $D = q*z/\sin\alpha$ then the concrete compression stress is:



(a) Free body sections



(b) Side wall free body



(c) Stirrup zones

Fig. 3.9 Forces in the shear field element due to torsion

$$f_d = q/(b_w * \sin\alpha * \cos\alpha) \qquad (3.16)$$

The upper and lower chords forces are assumed equal chord forces: $[F_1 = F_3 = F]$, then from horizontal equilibrium $F_1 + F_3 = 2F = N$

$$F = q * z * \cot\alpha/2 \qquad (3.17)$$

However, $N = V*\cot\alpha = D*\sin\alpha* \cot\alpha = q*z*\cot\alpha$ or the total tension resultant for one side

$$2 * F = q * z * \cot\alpha \qquad (3.18)$$

As shown in Fig. 3.9c, a potential diagonal failure crack is crossed by "n" number of stirrup legs, where "n" is given by the relation:

$$n = z * \cot\alpha/s \qquad (3.19)$$

Where z is the straight portion of the vertical stirrup leg that can effectively cross the diagonal crack, and "s" is the stirrup spacing. Therefore, the total tension force developed across this crack at ultimate is:

$$V = q * z = n*A_t*f_{yt} = n * S \qquad (3.20)$$

Where A_t is the area of one leg of closed stirrup, f_{yt} is the yield stress in the stirrups, and S is the stirrup force. Consequently, since from Eq. 3.19 $n*s = z* \cot\alpha$, so $z/n = s/\cot\alpha = s \tan\alpha$, thus:

$$S = q * z/n = q*s*\tan\alpha \qquad (3.21)$$

If an entire cross section is subjected to a torsional moment T, the diagonal compression forces from the diagonal compression struts resolve themselves at the joints of the truss into vertical and horizontal components V and N respectively (see Fig. 3.9b). The total horizontal force which is the sum of the horizontal components of the diagonal compression forces, (this compression force) must be balanced by an equal and opposite tension force R provided by the chords of the truss. The stirrup reinforcement (vertical tension ties) is required to equilibrate the vertical component V of the diagonal compression force D (see Fig. 3.10).

The resultant compression force can be found using the static relations of the space truss. From Fig. 3.10:

$$N = \Sigma D \cos\alpha \tag{3.22}$$

or

$$N = V/\tan\alpha = (V_2/\tan\alpha_2) + (V_4/\tan\alpha_4) + (V_6/\tan\alpha_6) + (V_8/\tan\alpha_8) \tag{3.23}$$

If now a constant stirrup spacing "s" is assumed, the inclination of the compression diagonals will remain constant around the perimeter. Hence, for the case shown in Fig. 3.8, Eq. 3.23 reduces to

$$N_T = 2\,(V_2 + V_4)/\tan\alpha \tag{3.24}$$

Combining Eqs. 3.20 and 3.24 results in

$$N_T = q^*[2(z_2 + z_4)]/\tan\alpha \tag{3.25}$$

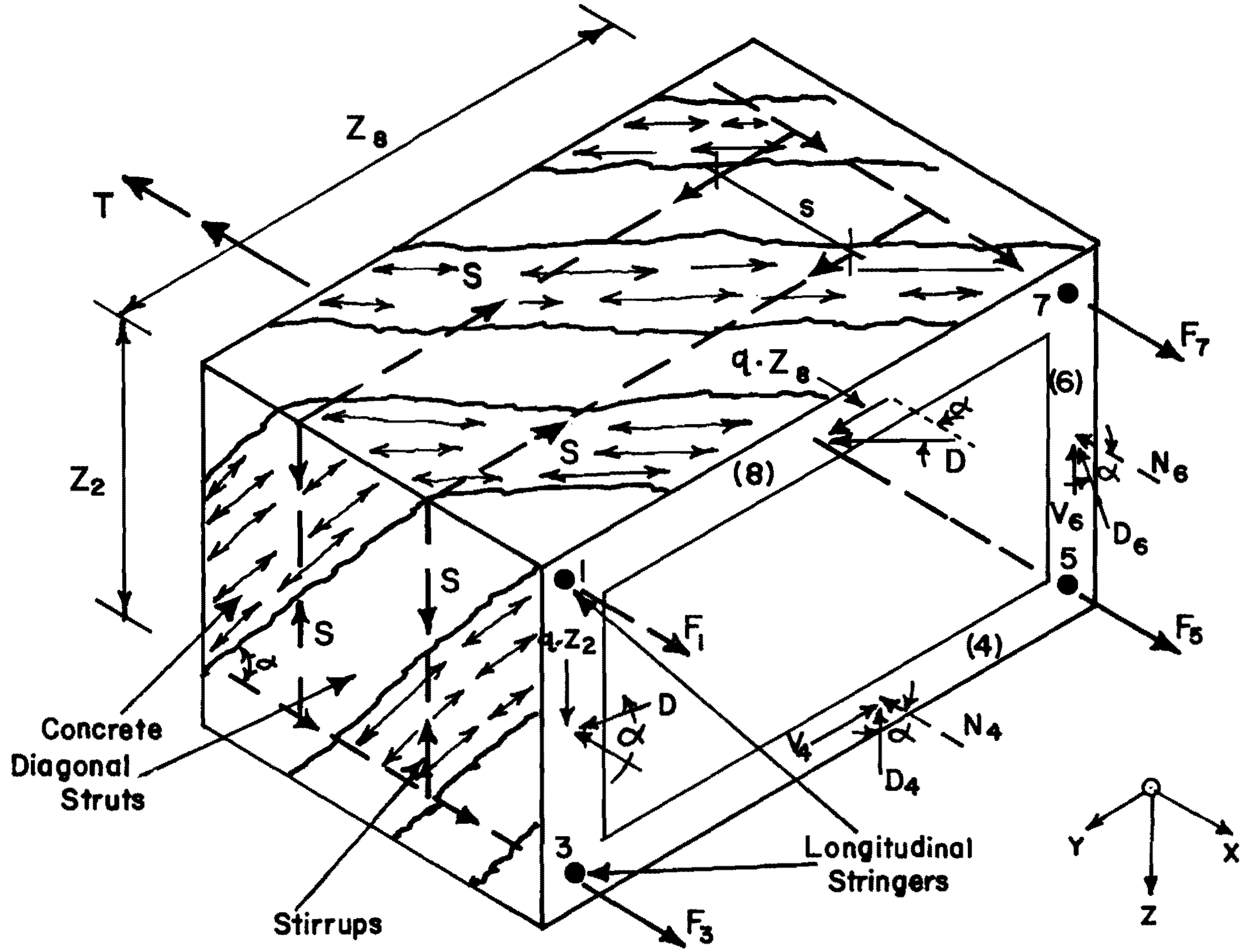


Fig. 3.10 Resultant forces in the Space Truss due to an applied torsional moment

where

$$u = z_2 + z_4 + z_6 + z_8 = 2(z_2 + z_4)$$

"u" being the perimeter connecting the longitudinal chords of the cross section. The resultant force R, acts at the centroid of the perimeter (not the centroid of cross section nor the centroid of enclosed area A_o), and is in equilibrium with the resultant of the axial components N of all compression diagonals. Hence

$$R = \Sigma N = q*u/\tan\alpha \qquad (3.26)$$

and together with Eq.3.14 yields the relation

$$R = T*u/(2*A_o* \tan\alpha) \qquad (3.27)$$

The stirrup forces S can be found from Eqs. 3.14 and 3.21 as

$$S = T*s*\tan\alpha/(2*A_o) \qquad (3.28)$$

Finally, the concrete compression stresses are given by Eqs. 3.14 and 3.16 in the form of:

$$f_d = \frac{T}{2A_0\ b_w}\left[\frac{1}{\sin\alpha\ \cos\alpha}\right] \qquad (3.29)$$

The state of stress described by Eqs. 3.27, 3.28 and 3.29 is in static equilibrium. Assuming that yielding of the steel will take place prior to crushing of the concrete, R and S are then limited by the yield forces R_y of all chords and S_y of the stirrups, respectively. Hence, the ultimate torsional resistance T_u is reached if both the chords and stirrups yield. Equations 3.27 and 3.28 give:

$$T_u = 2*A_o*R_y*\tan\alpha/u \qquad (3.30)$$

$$T_u = 2*A_o*S_y*\cot\alpha/s \quad (3.31)$$

Eliminating T_u or $\tan\alpha$ the final expressions are obtained:

$$\tan\alpha = [S_y*u/(R_y*s)]^{0.5} \quad (3.32)$$

$$T_u = 2*A_c*[(R_y*S_y)/(u*s)]^{0.5} \quad (3.33)$$

Equation 3.33 can be generalized for cross sections in which the chords are irregularly placed around its perimeter (see Fig. 3.11). In such case the minimum value of the resultant R_y governs the resistance. Taking an arbitrary axis B-B through a side, the resultant R_y at the centroid C_u of the perimeter can be determined as follows:

$$R_y = \Sigma[F_{yi} * z_i]/z_u \quad (3.34)$$

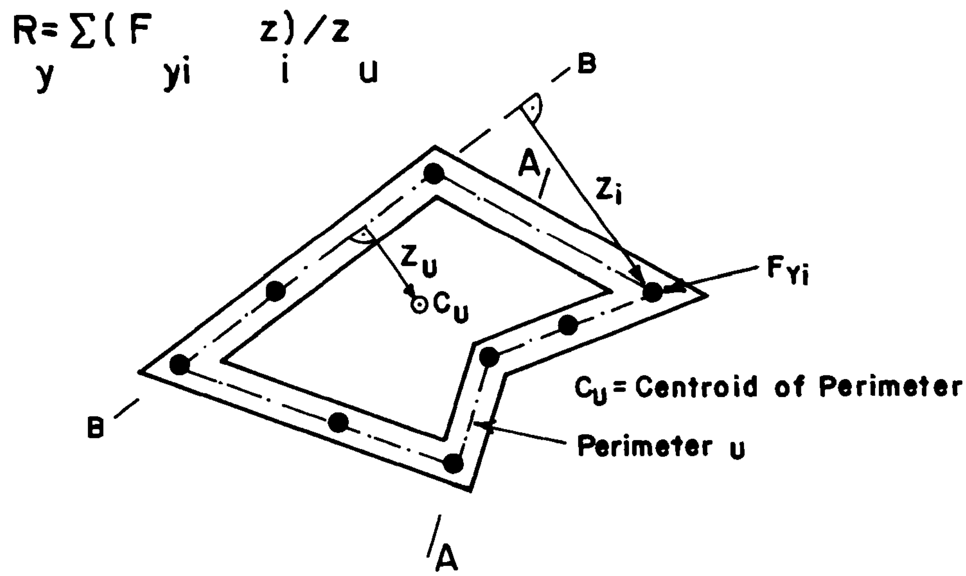


Fig. 3.11 General cross section

The axis B-B has to be varied to find the minimum of R_y. An axis A-A intersecting the section as shown in Fig. 3.11 is not admissible because it would put the chord i and hence the concrete of that region into compression.

Only underreinforced sections have been considered. In these elements yielding of the transverse and longitudinal reinforcement takes place prior to crushing of the concrete. Thus the diagonal compressive stress in the inclined members of the truss must be limited. The concrete compression diagonals carry the diagonal forces required to satisfy equilibrium in the plane of the reinforcement cage. In the case of thin walled hollow sections the diagonal compression stress acts over the web width of the member b_w. In the case of beams with solid cross section it has been shown (82,95,120,182) that the core offers no substantial contribution to the torsional strength. It is then reasonable to assume an effective outer shell for solid cross sections in computing the diagonal compression stresses. The effective thickness of this outer shell has been proposed (93) to be the smaller of the two values $b/6$ or $b_o/5$ where b and b_o are the diameters of the largest inscribed circles in the cross section and the area A_o, respectively (see Fig. 3.12). More comprehensive compatibility and plasticity approaches can predict the effective shell thickness.

The limits for the angle of inclination of the diagonals are the same as mentioned in Section 3.3 and they hold for any side of the truss. If they are reached no further redistribution of the internal forces is possible and the torsional strength provided by a constant shear flow "q" is reached.

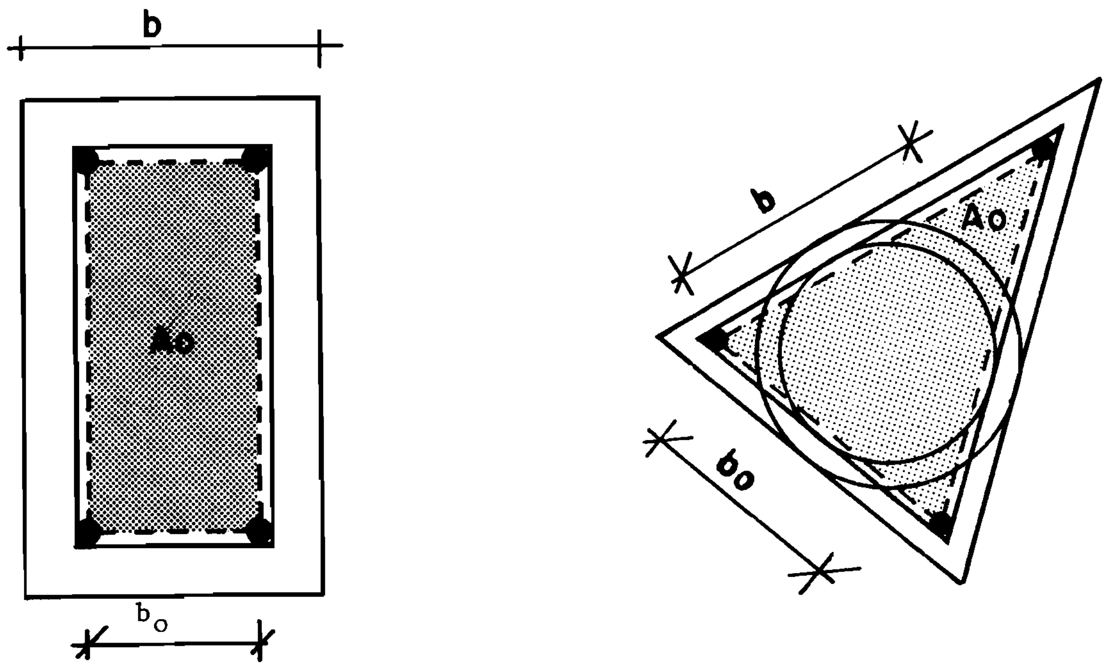


Fig. 3.12 Effective wall thickness for solid cross sections

3.5 Combined Actions and the Space Truss Model

The space truss model allows a general treatment of combined loading cases such as shear and bending, shear and torsion (48,93,96,124,164-166,180,182) as well as shear, torsion and bending. In most applications involving torsion, it will be combined with bending and/or shear. While most practical usage in design is for the case of combined bending and shear, the simpler case of torsion-bending will be first treated herein.

3.5.1 Torsion and Bending. All interaction equations for combined torsion and bending, may be derived from either the lower bound or upper bound solutions of the space truss. For consistency, the lower bound solution will be used herein. The load obtained from these equations will produce a stable, statically admissible state of stresses, and will be smaller than or equal to the collapse load.

Consider a beam with rectangular cross section and constant stirrup reinforcement. The forces acting on a cross section normal to the beam axis are shown in Fig. 3.13a. Consider the section to be symmetrical about the z-axis. The stirrup reinforcement is taken to be constant on all sides. In the corner detail of Fig. 3.13b when the forces in the x-direction are resolved it can be seen that the shear flow "q" must be constant around the whole perimeter. Equilibrium of forces in either the y or z direction gives:

$$S = q\ (s * \tan\alpha) \qquad (3.35)$$

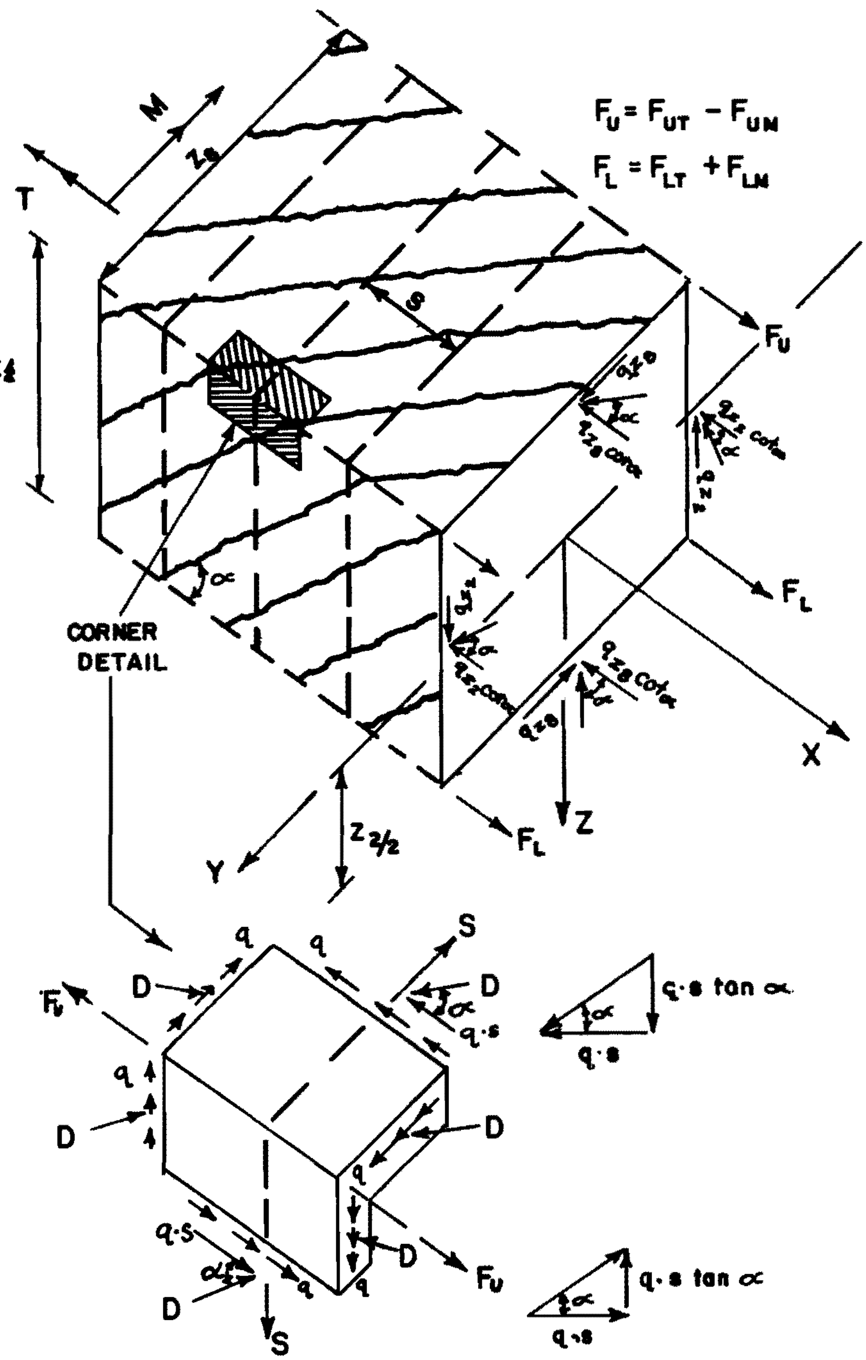
Consider a failure due to yielding of the stirrups. The yield force of one stirrup is assumed S_y. Because the stirrup reinforcement is constant, solving Eq. 3.35 for $\tan\alpha$, the angle α must be the same for all sides:

$$\tan\alpha = S_y/q * s \qquad (3.36)$$

The remaining equilibrium conditions are

$$\Sigma F_x = 0 = 2(F_u + F_1) - 2[q/\tan\alpha](z_2 + z_8) \qquad (3.37)$$

$$\Sigma M_y = 0 + \curvearrowright = 2(F_1 - F_u)\ z_2/2 - M \qquad (3.38)$$



(a) Cross section

(b) Corner detail

Fig. 3.13 Truss forces in beam with rectangular cross section

$$\Sigma M_x = T = [q * z_8]z_2 + [q*z_2]z_8 = 2A_o*q \tag{3.39}$$

The last equation (3.39) is usually given in the form $q = T/2A_o$. This relation represents the shear flow produced by a torsional moment acting on a thin-walled hollow section. The area A_o is defined as the area enclosed by the perimeter of the lines connecting the longitudinal reinforcements in the corners.

Applying to this same cross section a bending moment (which in fact will be a moment about the y-axis (see Fig. 3.13a)), the interaction between bending and torsion can be derived from the equilibrium conditions in the truss model. Considering a positive moment about the y-axis (compression at the top, tension at the bottom), taking summation of moments about the y-axis.

$$\Sigma M_y = 0 \circlearrowleft + = [-2F_u z_2/2] + [2F_1 z_2/2] - M \tag{3.40}$$

then

$$- F_u + F_1 = + M/z_2 \tag{3.41}$$

Directly from Fig. 3.13a, summation of horizontal forces gives

$$F_x = 0 \rightarrow + = 2F_u + 2F_1 - 2z_2 q \cot \alpha - 2qz_8 \cot \alpha \tag{3.42}$$

so that

$$2[F_u + F_1] = q*u/\tan \alpha \tag{3.43}$$

where "u" is the perimeter enclosing the centroids of the longitudinal chords of the space truss. The equilibrium condition dealing with summation of moments about the x-axis remains unaltered.

$$T = 2A_o q \tag{3.44}$$

Solving Eq. 3.44 for "q" and susbstituting in 3.43 yields

$$F_u + F_1 = (T*u)/(4A_o*\tan\alpha) \tag{3.45}$$

Combining Eq. 3.41, which represents the longitudinal force due to bending (F_{um},F_{1m}) and Eq. 3.45, which is the longitudinal force due to torsion (F_{ut},F_{1t}), and solving for F_1 yields

$$2F_1 = M/z_2 + T*u/(4A_o \tan\alpha) \tag{3.46}$$

Then, in the case of pure bending (T = 0), F_1 is given by

$$2F_1 = M/z_2 \tag{3.47}$$

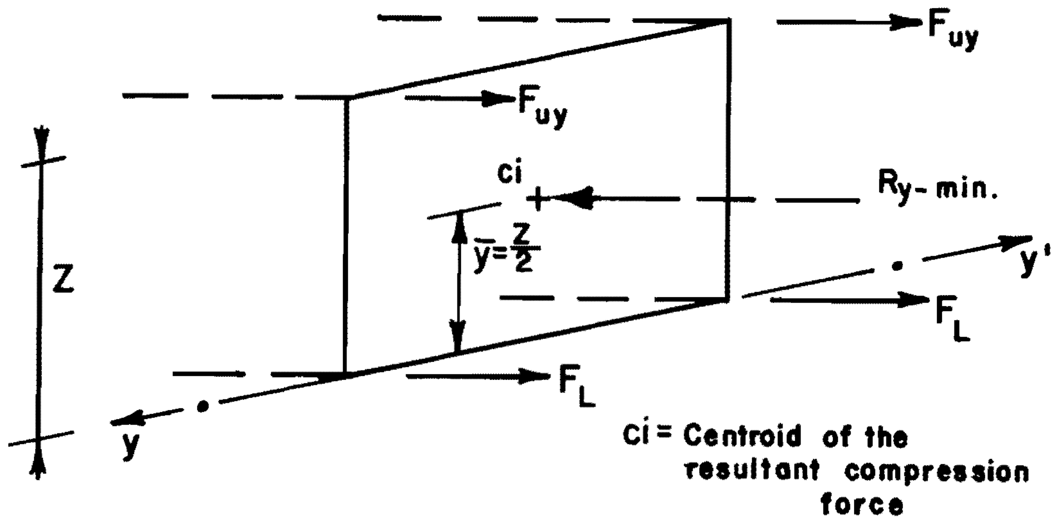
Since for failure due to a positive bending moment the bottom chords must yield, the ultimate bending moment for pure bending of the space truss model is

$$M_{uo} = 2F_{y1} z_2 \tag{3.48}$$

Consider the case of a beam with positive moment bending type reinforcement where $F_{y1} > F_{yu}$ because of the concrete compression block contribution in the case of bending. The ultimate torque in the case of pure torsion was calculated in Section 3.4, Eq. 3.33:

$$T_{uo} = 2A_o[R_y*S_y/(u*s)]^{0.5} \tag{3.49}$$

Where R_y was calculated as the minimum value of the resultant of the longitudinal forces in the chords. In the case of a beam symmetrical about the z-axis with bending type reinforcement (i.e. $F_{yl} > F_{yu}$), this minimum resultant R_{y-min} is obtained by passing an axis through the bottom reinforcement which in this case has the largest yielding force ($A_1 f_{yl}$). Summing moments about the axis y-y' in Fig. 3.14 yields:



$$+\curvearrowleft \Sigma M_{y-y'} = 0 = R_{y-min} \cdot \frac{Z}{2} - 2 F_{uy} \cdot Z$$

Fig. 3.14 Minimum resultant of the longitudinal forces in the chords

$$R_{y-min} = 4F_{uy} \qquad (3.50)$$

Therefore, somewhere in the range of combined torsion and bending there must be a change from tension yielding of the bottom to tension yielding of the top reinforcement. Hence, these two cases will be considered separately. The superposition of the chord forces F(T) and F(M) due to torsion and bending is shown in Fig. 3.15.

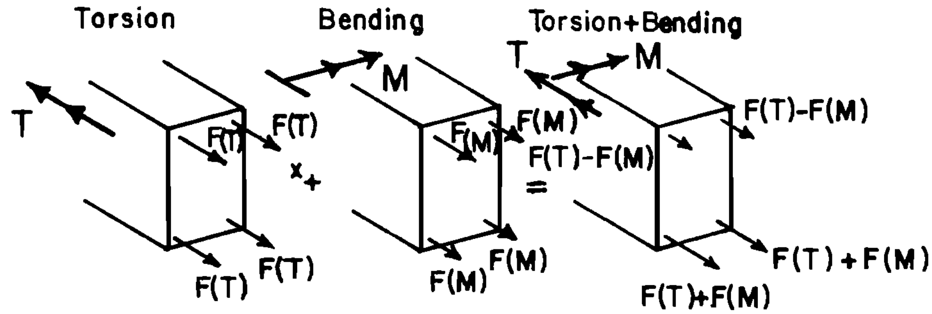


Fig. 3.15 Superposition of torsion and bending

Examine first the case when the combination is such that yielding of the lower chords (i.e. $F_1 = F_{y1}$), and the stirrups (i.e. $S = S_y$) will take place at failure. The stirrups contribute to the torsional resistance but not to the flexural resistance. The longitudinal forces in the bottom chords, are obtained from the equilibrium conditions of the space truss. In the case of yielding of the bottom reinforcement when a positive bending moment and a torsional moment are simultaneously applied, the longitudinal forces due to bending (Eq. 3.41, $\Sigma M_y = 0$) and torsion (Eq. 3.45, $\Sigma F_x = 0$) are both tension forces (see Fig.3.15). Hence, adding Eqs. 3.41 and 3.45 yields

$$2F_{y1} = + M/z_2 + [(T*u)/4A_o*\tan\alpha] \tag{3.46a}$$

The relationship for $\tan\alpha$ can be found by combining Eqs. 3.36 and 3.44

$$\tan\alpha = S_y * 2A_0/(T*s) \tag{3.51}$$

Substituting Eq. 3.51 in Eq. 3.46a, and dividing by $2F_{yl}$, yields

$$1 = \frac{M}{2F_{yl}\ z_2} + \frac{T^2 * u * s}{16\ F_{yl}\ A_0^2\ s_y} \tag{3.52}$$

Introducing the ultimate moments for pure bending and pure torsion from Eqs. 3.48 and 3.49 together with the value of the minimum resultant tension force R_{y-min} obtained from Eq. 3.50, the interaction equation that represents the behavior of beams with bending type reinforcement [$A_{lower\ chords} > A_{upper\ chords}$] in the case of yielding of the lower chords is obtained:

$$1 = \left(\frac{T_u}{T_{uo}}\right)^2 \frac{F_{yu}}{F_{yl}} + \frac{M_u}{M_{uo}} \tag{3.53}$$

Now consider the case of yielding of the top reinforcement (i.e. $F_u = F_{yu}$). In this case the tensile force produced by the torsional moment counteracts the compression force induced by the flexural moment (see Fig. 3.15). Again, using the equilibrium conditions of the space truss, Eq. 3.41 and Eq. 3.45 are combined and solved for $F_u = F_{uy}$ at yield,

$$2F_{uy} = -\frac{M}{z_2} + \frac{T * u}{4A_0\ \tan\alpha} \tag{3.54}$$

Dividing by $2F_{uy}$ and introducing the value for $\tan\alpha$ from Eq. 3.51 yields

$$1 = -\frac{M}{2\ F_{yu}\ z_2} + \frac{T^2 * u * s}{16\ A_0^2\ F_{yu}\ s_y} \tag{3.55}$$

Substituting the reference values for pure bending and pure torsion from Eqs. 3.48 and 3.49, the interaction equation between bending and torsion for the case of tensile yielding of the top reinforcement is derived:

$$1 = \left(\frac{T_u}{T_{uo}}\right)^2 - \frac{M_u}{M_{uo}}\frac{F_{yl}}{F_{yu}} \tag{3.56}$$

On the basis of Eqs. 3.53 and 3.56, the behavior of beams subjected to combined torsion and bending, can be studied. Define the ratio of the yield force of the top chord (F_{yu}) to the bottom (F_{yl}) chord longitudinal steel as "r". It becomes apparent from Eqs. 3.53 and 3.56 that for every value of "r" there exists a corresponding interaction diagram. The interaction curves for a rectangular beam, first assuming it has bending type reinforcement, r = 1/3, and later considering it symmetrically reinforced, r = 1, are shown in Fig. 3.16. It is seen that for a section conventionally reinforced for bending $F_{yl} > F_{yu}$, (r = 1/3) the torsional strength is actually increased by the simultaneous application of a bending moment since the compression stresses on the top due to bending offset the tensile stress on the top set up by torsion. On the other hand, the application of torque always decreases the flexural capacity. The maximum torsional strength is given by the intersection A of the two curves representing Eq. 3.53, tensile yielding of the bottom reinforcement, and 3.56, tensile yielding of the top reinforcement. For case A, both lower and upper chords yield in tension. For a symmetrically reinforced section ($F_{yl} = F_{yu}$) and r = 1, the maximum torque occurs when the bending moment M = 0. In this

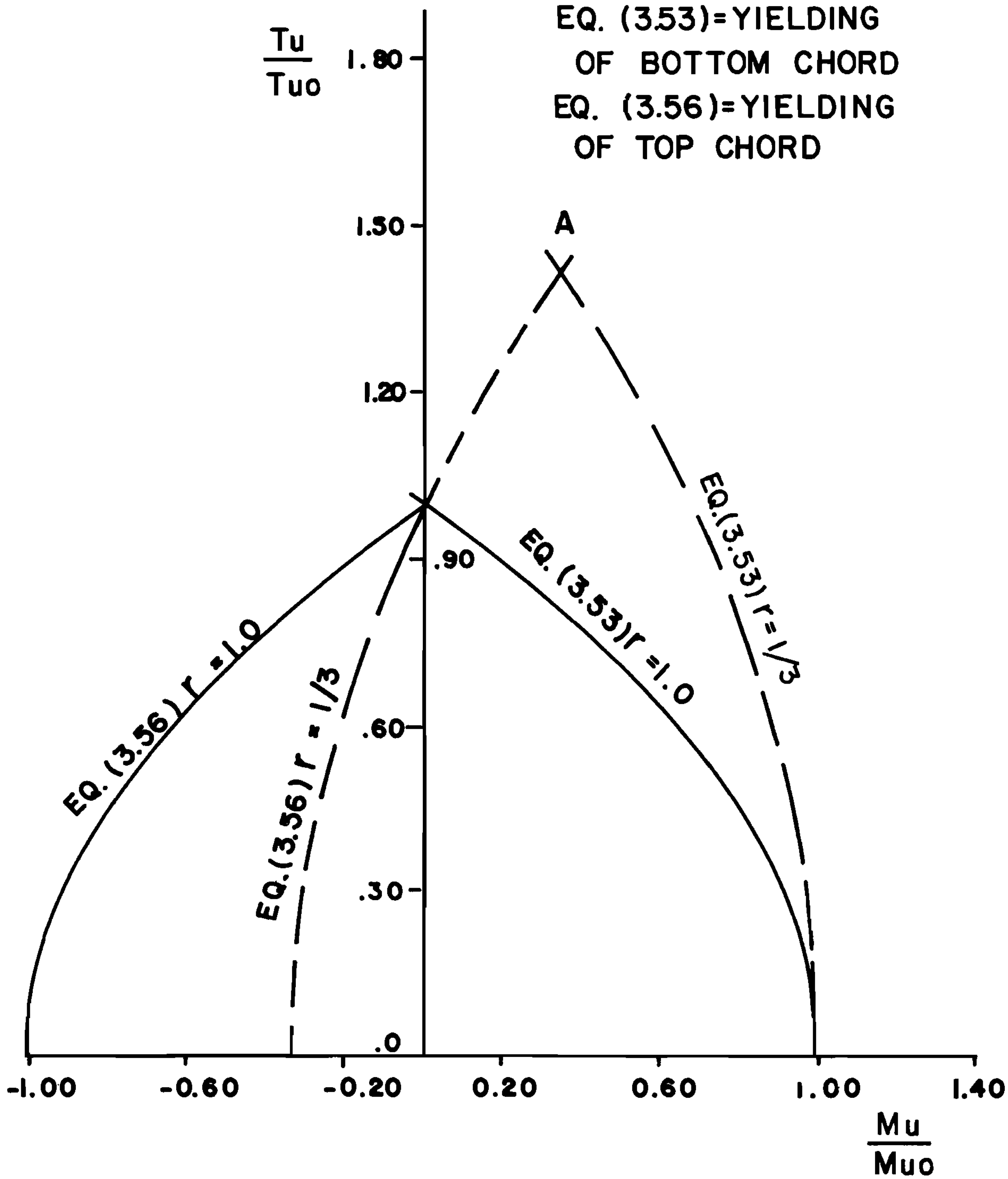


Fig. 3.16 Interaction torsion bending

case both the top and bottom longitudinal reinforcement and the stirrups yield in torsion. When $r = 1$ the addition of moment does not increase the torsional strength. Again, the addition of torque always decreases the moment capacity.

The form of these curves for the interaction between torsion and bending has been confirmed by tests (93,94).

3.5.2 Bending - Shear. The application of the Space Truss model to reinforced as well as prestressed concrete beams, has been studied by Grob and Thurlimann (72,163,165), by Collins and Mitchell (56), and important contributions have also been made by Muller (180.181). More recently in studies carried out at The University of Texas, Schaeffer(153) and Castrodale (50) have shown that there is good agreement between the truss model and observed test results in both reinforced and prestressed concrete beams subjected to different loading combinations of bending and shear.

Similar to the case of pure torsion, interaction equations for the case of combined bending and shear can be derived from the equilibrium solution of the space truss. The web of a concrete section can be idealized as the shear field element shown in Fig. 3.17, where the forces are acting in the plane of the web. From an equilibrium analysis the following relations can be developed: from Fig. 3.17 $\Sigma V = 0$ yields the diagonal force which is resisted by the concrete compression struts.

$$V = D \sin\alpha = q * z \tag{3.57}$$

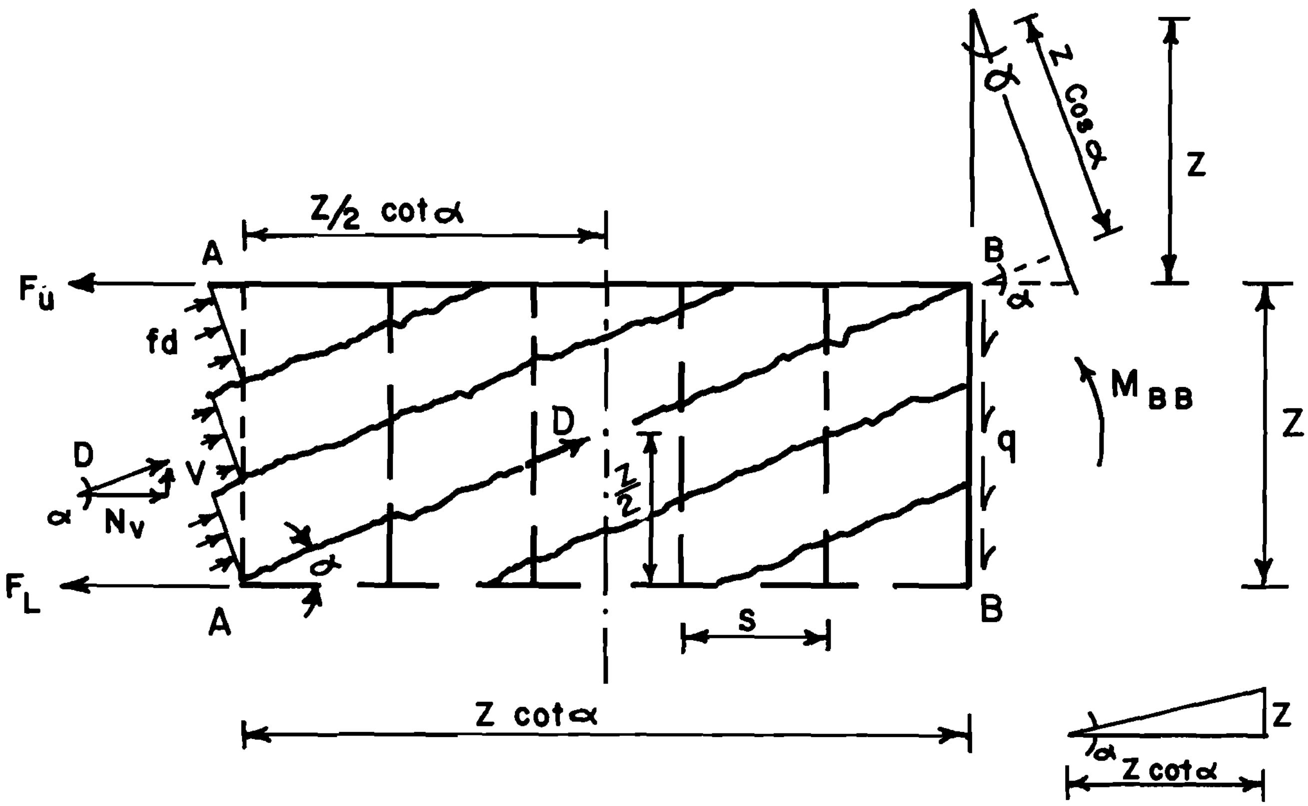


Fig. 3.17 Forces in the beam web-shear field element

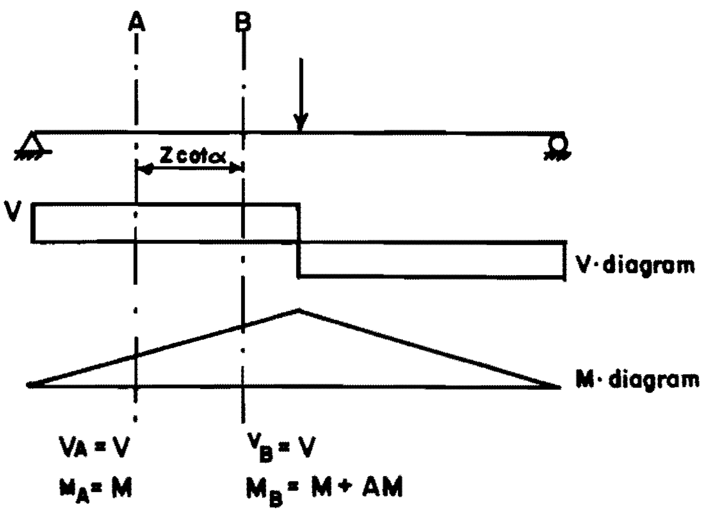
Since $D = f_d * b_w * z * \cos\alpha = V/\sin\alpha$, the compression stress in the diagonal concrete strut is given by:

$$f_d = \frac{D}{b_w\ z\ \cos\alpha} = \frac{V}{b_w\ z\ \sin\alpha\ \cos\alpha} \tag{3.58}$$

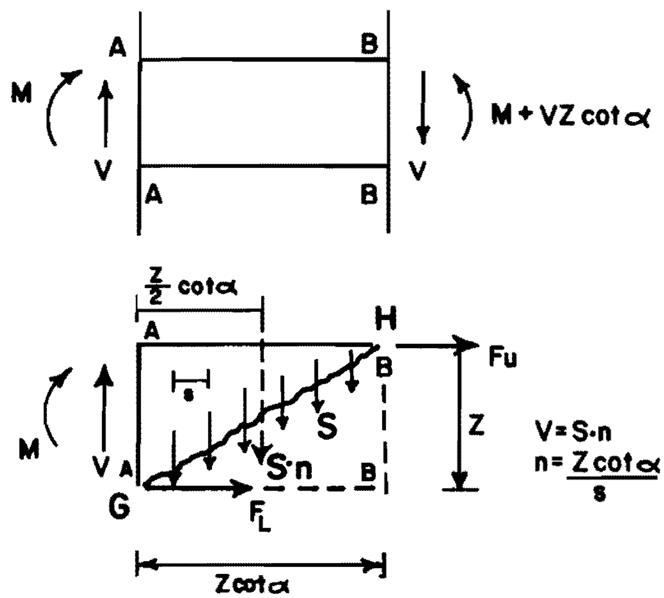
Examination of a free body bounded by an inclined crack and consideration of the stirrup forces S at a stirrup spacing "s" is necessary to determine the chord forces. As shown in Fig. 3.18b, $M_G = 0$ + yields the force in the upper chord

$$F_u = -\frac{M}{z} - \frac{V}{2}\cot\alpha \tag{3.59}$$

(Note that in Fig. 3.18b F_u was shown as a positive or tension force at Section BB). The force in the lower chord can be found from Fig.3.18b



(a) Actions



(b) Equilibrium of free body

Fig. 3.18 Forces in the Truss Model

using $M \curvearrowleft +$ about H

$$F_1 = \frac{M}{z} + \frac{V}{2}\cot\alpha \tag{3.60}$$

(Note that both of these chord forces differ from the commonly assumed value of M/z). The force in any stirrup S is obtained by dividing the total shear force across a given crack by the number of stirrups (n) of spacing s crossing the crack (see Fig. 3.18b). Hence, from summation of vertical forces:

$$S = V/n = V*s/\ (z(\cot\alpha\ )) \tag{3.61}$$

Using these equations derived from the equilibrium conditions in the truss model, the interaction between bending and shear in reinforced concrete sections can be studied.

Consider the case where yielding of the bottom reinforcement as well as the stirrups takes place. From Eq. 3.60 the yield force in the lower (tension) chord is given by:

$$F_{yl} = \frac{M_u}{z} + \frac{V_u}{2}\cot\alpha \tag{3.62}$$

From Eq. 3.61 the yield force in the stirrups is:

$$S_y = \frac{V_u * s * \tan\alpha}{z} \tag{3.63}$$

[where M_u and V_u are the ultimate values of moment and shear respectively]. Equation 3.63 can be solved for $\tan\alpha$:

$$\tan\alpha = \frac{S_y\ z}{V_u\ .\ s} \tag{3.64}$$

Combining Eqs. 3.62 and 3.64 yields:

$$F_{y1} = \frac{M_u}{z} + \frac{V_u^2 \, s}{2 \, S_y \, z} \tag{3.65}$$

Equation 3.65 represents the combined effects of bending and shear on the longitudinal tension reinforcement. In the case of pure bending ($V_u = 0$), Eq. 3.65 yields the reference value, M_{uo}:

$$M_{uo} = F_{y1} \; (z) \tag{3.66}$$

Using Eq. 3.65 and setting $M_u = 0$ for the case of pure shear, yields the maximum value of the shear force V_{uo} that can be obtained from a given combination of longitudinal and transverse reinforcement:

$$V_{uo} = [2F_{y1} \, S_y \, z/s]^{0.5} \tag{3.67}$$

Combining Eqs. 3.65, 3.66 and 3.67, the following expression for the interaction between shear and bending is obtained:

$$M_u/M_{uo} + [V_u/V_{uo}]^2 = 1 \tag{3.68}$$

On the basis of this equation, the interaction curve shown in Fig. 3.19 is obtained. However, the interaction equation is not valid unless $F_1 = F_{y1}$ and $S = S_y$, so it does not hold true for all inclinations of the angle as was discussed in Section 3.3. Typical limits for are shown on the figure.

Consider now the case where yielding of the top (compression) chord together with the stirrup reinforcement takes place. The yield force in the upper (compression) chord can be obtained directly from

Fig. 3.17 by taking $\Sigma F_H = 0 = F_1 + F_u - N_v$, where $N_v = V \cot\alpha$ and F_1 is found from Eq. 3.60. Hence, the yield force in the upper chord is:

$$F_{yu} = -\frac{M_u}{z} + \frac{V_u}{2}\cot\alpha \qquad (3.69)$$

(where M_u and V_u are the ultimate values of moment and shear respectively). The yield force in the stirrups is again given by Eq. 3.63. Solving for $\tan\alpha$ in Eq. 3.63, and combining with Eq. 3.69 yields:

$$F_{yu} = -\frac{M_u}{z} + \frac{V_u^2 * s}{2\,S_y\,z} \qquad (3.70)$$

Equation 3.70 represents the combined effects of bending and shear on the longitudinal top (compression) chord. In the case of pure bending ($V_u = 0$), Eq. 3.70 yields the reference value M_{uo}:

$$M_{uo} = -F_{yu} * z \qquad (3.71)$$

Setting $M_u = 0$ for the case of pure shear, Eq. 3.70 yields the maximum value of the shear force V_{uo}:

$$V_{uo} = [2F_{yu}S_y\ z/s]^{0.5} \qquad (3.72)$$

Combining Eqs. 3.70, 3.71, and 3.72 results in the same expression presented in Eq. (3.68). The conditions $F_u = F_{yu}$ and $S = S_y$ are required in order for this interaction equation to be valid.

The bending-shear interaction diagram, based on Eq. 3.68, can be divided into three types of failures mechanisms as shown in Fig. 3.19.

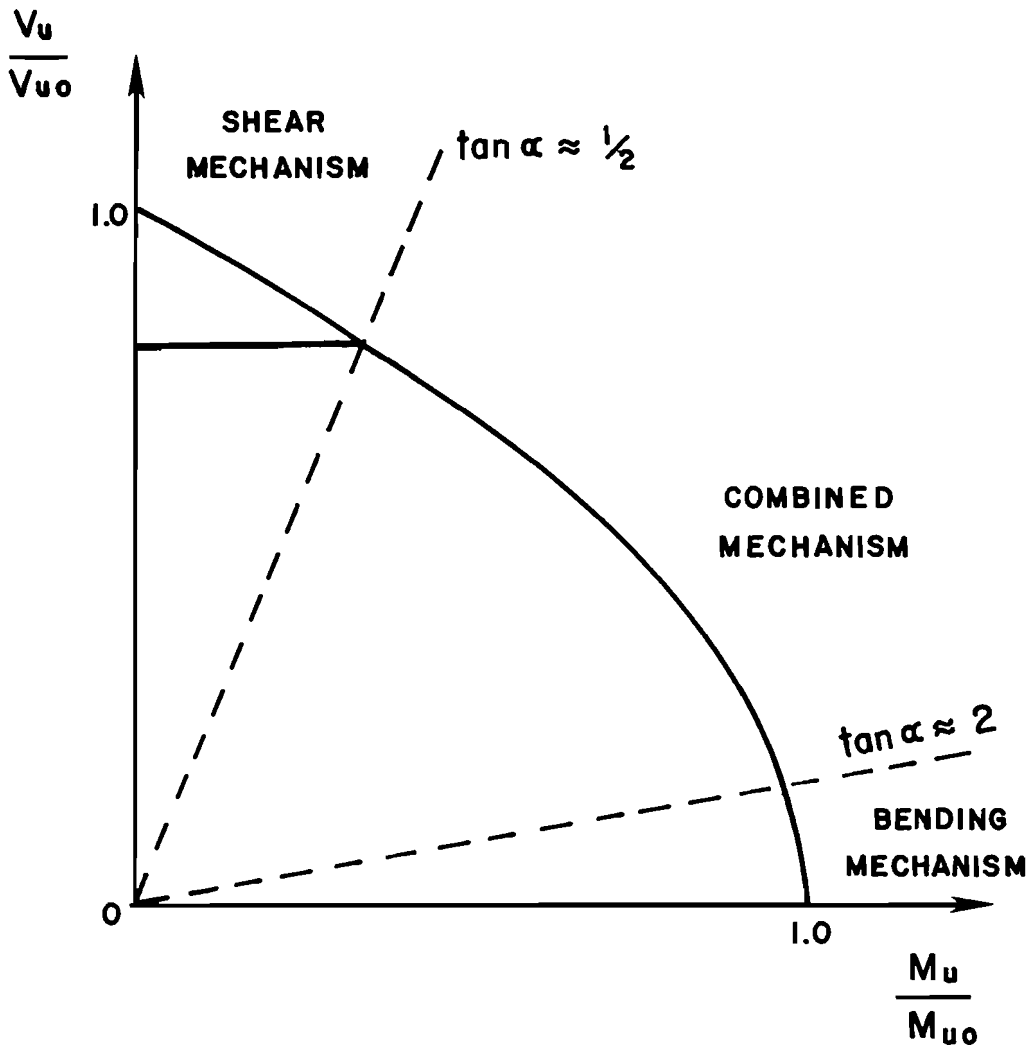


Fig. 3.19 Interaction diagram between bending and shear [from Ref. 72]

The type of failure mechanism is governed by the inclination of the angle α. As was previously suggested, the validity of Eq. 3.68 is limited by the angle of inclination at ultimate of the diagonal strut. In Section 3.3 the following limits are suggested for the angle α:

$$0.5 \leq \tan\alpha \leq 2.0$$

$$26^{\circ} \leq \alpha \leq 63^{\circ}$$

The limiting values of $\tan\alpha$ are represented as dashed lines in Fig. 3.19. The limits should be seen not as fixed values, but only as representing transitions between the different mechanisms. Thürlimann suggests that a combined mechanism where both the longitudinal and transverse reinforcement yield, occurs for values of $\tan\alpha$ between 0.5 and 2.0. A bending mechanism, yielding of the longitudinal reinforcement without yielding of the transverse reinforcement, is obtained for values of $\tan\alpha$ greater than 2.0. Lastly, for values of $\tan\alpha$ less than 0.5, a shear mechanism will occur, which is when the transverse reinforcement yields without yielding of the longitudinal steel.

3.5.3 Torsion - Bending - Shear. Consider the case where torsion, bending and shear interact on the section of Fig. 3.20a. The presence of shear will produce an additional shear flow, $q = V/(2z)$ acting on each of the side webs of the box section, which must be superimposed on the shear flow due to torsion $q = T/2A_o$, as shown in Fig. 3.20b. With this superposition, the resultant shear flows for the different sides are shown on Fig. 3.20c.

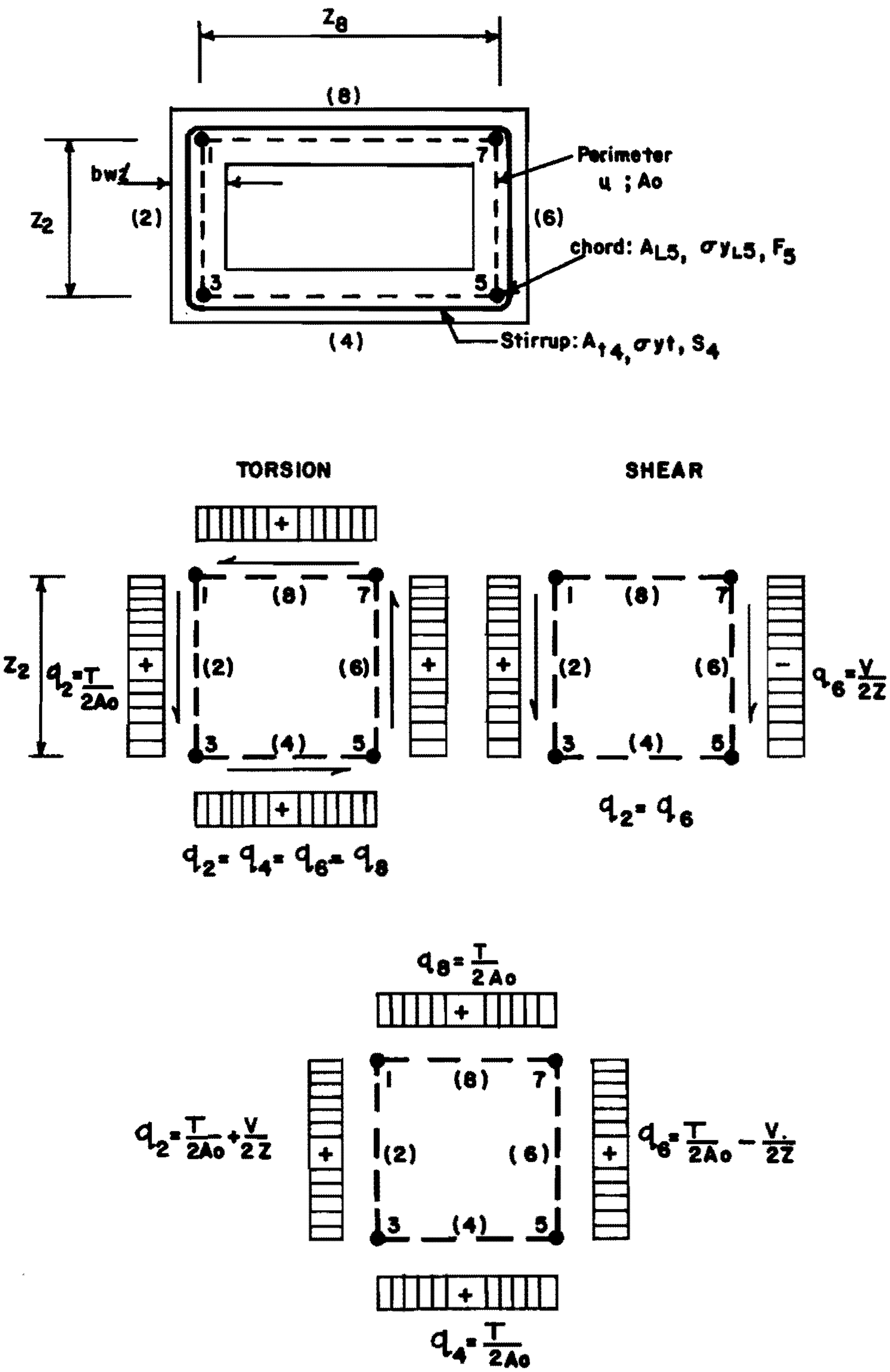


Fig. 3.20 Shear flows due to torsion and shear

The resultant static system is shown in Fig 3.21. Again the basic assumption in the space truss model is that all the sides may have different inclinations for the diagonal compression field but that tension yielding of two chords and the stirrups will occur at failure. Assuming that failure will produce yielding of the bottom chords ($F_3 = F_5 = F_{y1}$) as well as the stirrups ($S = S_y$), the minimum axial resultant of the longitudinal chords can be found by taking moments about the axis 1-7 in Fig. 3.21c. Summing moments about axis 1-7 results in the following relation:

$$\Sigma M_{1-7} = 0 \quad +\circlearrowright$$

$$0= -M_u+F_3z_2+F_5z_2-q_2 \frac{z_2^2}{2} \cot\alpha_2-q_4z_4z_2\cot\alpha_4-q_6z_6 \frac{z_2}{2} \cot\alpha_6 \qquad (3.73)$$

then

$$M_u = 2F_{y1}z_2 - q_2z_2^2 \cot\alpha_2/2 - q_4z_4z_2*\cot\alpha_4 - q_6z_2z_6\cot\alpha_6/2 \qquad (3.74)$$

From vertical equilibrium in the side walls of the truss the relationship between the stirrup forces and the inclination of the compression field can be obtained (see Fig. 3.22).

$$S_y = q*s/\cot\alpha \quad \text{or} \quad \cot\alpha = q*s/S_y \qquad (3.75)$$

Replacing the resultant shear flows from Fig. 3.21a and solving for $\cot\alpha$ yields:

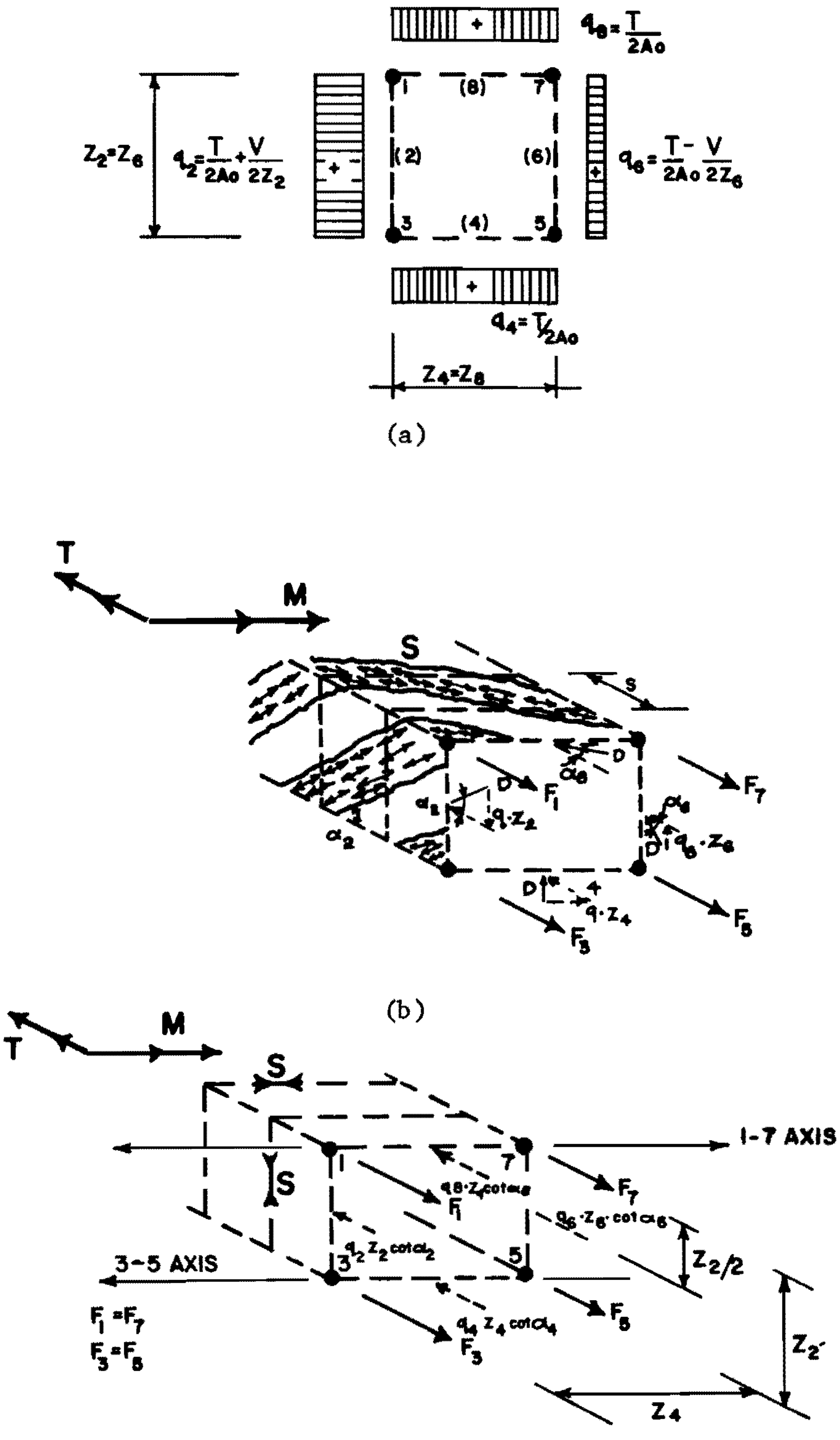


Fig. 3.21 Static system under torsion-bending-shear

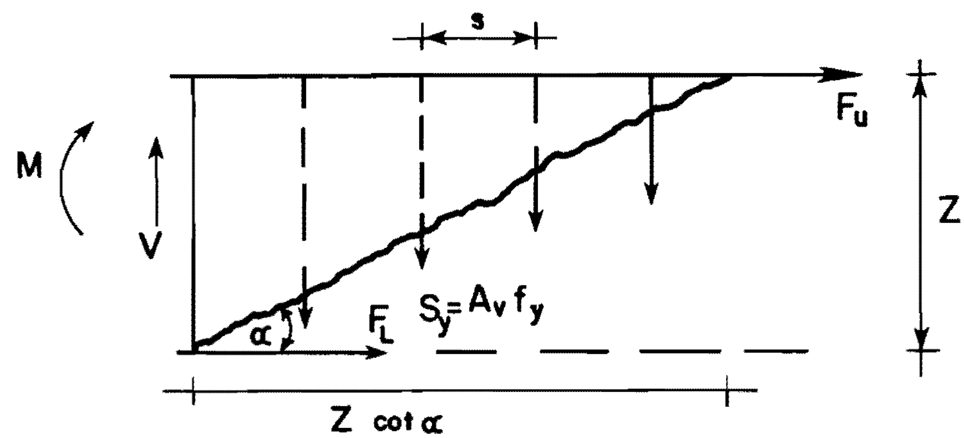
$$\cot\alpha_2 = s[T/(2A_o) + V/(2z_2)]/S_y \tag{3.76}$$

$$\cot\alpha_4 = \cot\alpha_8 = s[T/2A_o]/S_y \tag{3.77}$$

$$\cot\alpha_6 = s[(T/2A_o) - V/2z_2)]/S_y \tag{3.78}$$

The previous Eqs. 3.74 and 3.75 derived from equilibrium conditions ($\Sigma M = 0$ and $\Sigma F_V = 0$)in the truss model allow the study of the interaction between bending, torsion and shear in reinforced and prestressed concrete members.

Consider the case of a section with positive moment bending type reinforcement ($F_{yl} > F_{yu}$), then the following reference values are obtained:



n = number of stirrups = $\dfrac{z \cot\alpha}{s}$

$\Sigma F_v = 0 = V - S_y n = V - S_y z \dfrac{\cot\alpha}{s}$

Since $\dfrac{V}{z} = q$ = shear flow

then S_y = stirrup force = $q \cdot \dfrac{s}{\cot\alpha}$

Fig. 3.22 Relationship between stirrup forces and the inclination of the compression field

1. Since it is assumed that the bottom chords yield, the value of M_{uo} (T = 0, V = 0) will be given by Eq. 3.66.

2. The maximum value of the shear force V_{uo} (M = 0, T = 0) is given by Eq. 3.72 for the case of $F_{yl} > F_{yu}$. In the case in consideration, a box section with two side webs, each web has that value so the resisting shear becomes twice that value.

3. For the case of pure torsion (M = 0, V = 0) and $F_{yl} > F_{yu}$, Eqs. 3.49 and 3.50 yield the value:

$$T_{uo} = 2A_o[(4F_{yu}*S_y)/(u*s)]^{0.5} \tag{3.79}$$

The above expressions together with the values of $\cot\alpha$ from Eqs. 3.76, 3.77 and 3.78, and Eqs. 3.74 and 3.75 yield the interaction equation for bending, torsion and shear when yielding of the bottom reinforcement occurs in the case of beams with positive bending moment type reinforcement.

$$F_3=F_s=F_{yl} \qquad \frac{F_{yu}}{F_{yl}}\left[\left(\frac{T_u}{T_{uo}}\right)^2 + \left(\frac{V_u}{V_{uo}}\right)^2\right] + \frac{M_u}{M_{uo}} = 1 \tag{3.80}$$

Similarly an interaction equation can be derived for the case of yielding of the top reinforcement in the case of a beam with positive bending moment type reinforcement (tension at the bottom). In this case by taking moments about the 3-5 axis in Fig. 3.21c.

$$\Sigma M_{3-5} = 0 \; +\!\circlearrowright$$

$$0 = -F_1z_2-F_7z_2+q_2\frac{z_2^2}{2}\cot\alpha_2+q_6\frac{z_6^2}{2}\cot\alpha_6+q_8z_4z_2\cot\alpha_8-M_u \tag{3.81}$$

Since $F_1 = F_7 = F_{yu}$ and $z_2 = z_6$, then:

$$M_u = -2F_{yu}z_2+q_2\frac{z_2^2}{2}\cot\alpha_2+q_6\frac{z_2^2}{2}\cot\alpha_6+q_8z_4z_2\cot\alpha_8 \tag{3,82}$$

Since the same section is being analyzed ($F_{yl} > F_{yu}$), the previous reference values from Eqs. 3.66, 3.72 and 3.79 are still valid.

Together with the respective values of $\cot\alpha$ from Eqs. 3.76, 3.77, and 3.78, they yield the following interaction equation:

$$F_1 = F_7 = F_{yu} \qquad \left(\frac{T_u}{T_{uo}}\right)^2 + \left(\frac{V_u}{V_{uo}}\right)^2 - \frac{F_{yl}}{F_{yu}}\frac{M_u}{M_{uo}} = 1 \tag{3.83}$$

So far only the case of a section with a positive moment bending type reinforcement ($F_{yl} > F_{yu}$) subjected to torsion, shear and positive bending moment (tension in the lower chords) has been considered. However, the truss model is equally valid for the case of negative bending moment type reinforcement ($F_{yu} > F_{yl}$).

In the case of sections with positive moment type reinforcement, the reinforcement at the top is usually lighter than the one at the bottom and failure due to yielding of the upper chords takes place when the applied moment M is relatively very small with respect to the applied torsional moment, or negative. In such a case the upper chords yield in tension because the compression force at the top of the section is small or zero but the tensile stresses due to torsion when added onto the compression force will be large enough to produce yielding. From study of these interaction equations very interesting conclusions can be drawn. The relation between shear and torsion from those equations is a circular form as has been generally assumed. However, the relation between shear and torsion is not independent of the level of applied flexural moment or of the ratios of the top to bottom longitudinal reinforcement, as seems to be indicated by the proposed unit value for the radius of the circular interaction curve proposed in the ACI Building Code and AASHTO Specifications (17,24). The space truss model permits an evaluation of such effects. Rearranging Eq. 3.80 which represents the interaction between torsion, shear and bending when failure produces yielding of the positive moment tension reinforcement (bottom chord) yields:

$$\left(\frac{T_u}{T_{uo}}\right)^2 + \left(\frac{V_u}{V_{uo}}\right)^2 = \left[1 - \frac{M_u}{M_{uo}}\right] \frac{F_{yl}}{F_{yu}} \tag{3.84}$$

and Eq. 3.83 for yielding in tension of the positive moment compression reinforcement (top chord) becomes:

$$\left(\frac{T_u}{T_{uo}}\right)^2 + \left(\frac{V_u}{V_{uo}}\right)^2 = 1 + \frac{M_u}{M_{uo}} \frac{F_{yl}}{F_{yu}} \tag{3.85}$$

Fig. 3.23 shows plots of Eqs. 3.84 and 3.85 for several values of M_u/M_{uo} for the case of positive bending moment type reinforcement (F_{yl}/F_{yu} = 3). When the applied positive bending moment equals the ultimate capacity in pure bending of the cross section [M_u/M_{uo} = 1.0] failure is obviously controlled by yielding of the bottom reinforcement (Eq. 3.84) which plots as a point at the origin of the axis T_u/T_{uo} and V_u/V_{uo} in Fig. 3.23, indicating that no interaction between shear and torsion is possible. As the level of applied positive bending moment is reduced (M_u/M_{uo} = 0.66) the interaction between shear and torsion becomes feasible and failure remains controlled by yielding of the bottom (flexural tension) reinforcement as shown in Fig. 3.23 by the failure surface given by Eq. 3.84. For low values of applied positive bending moment (M_u/M_{uo} = 0.25) the interaction between shear and torsion increases and failure is then controlled by yielding of the top (flexural compression) reinforcement as shown in Fig. 3.23 by the

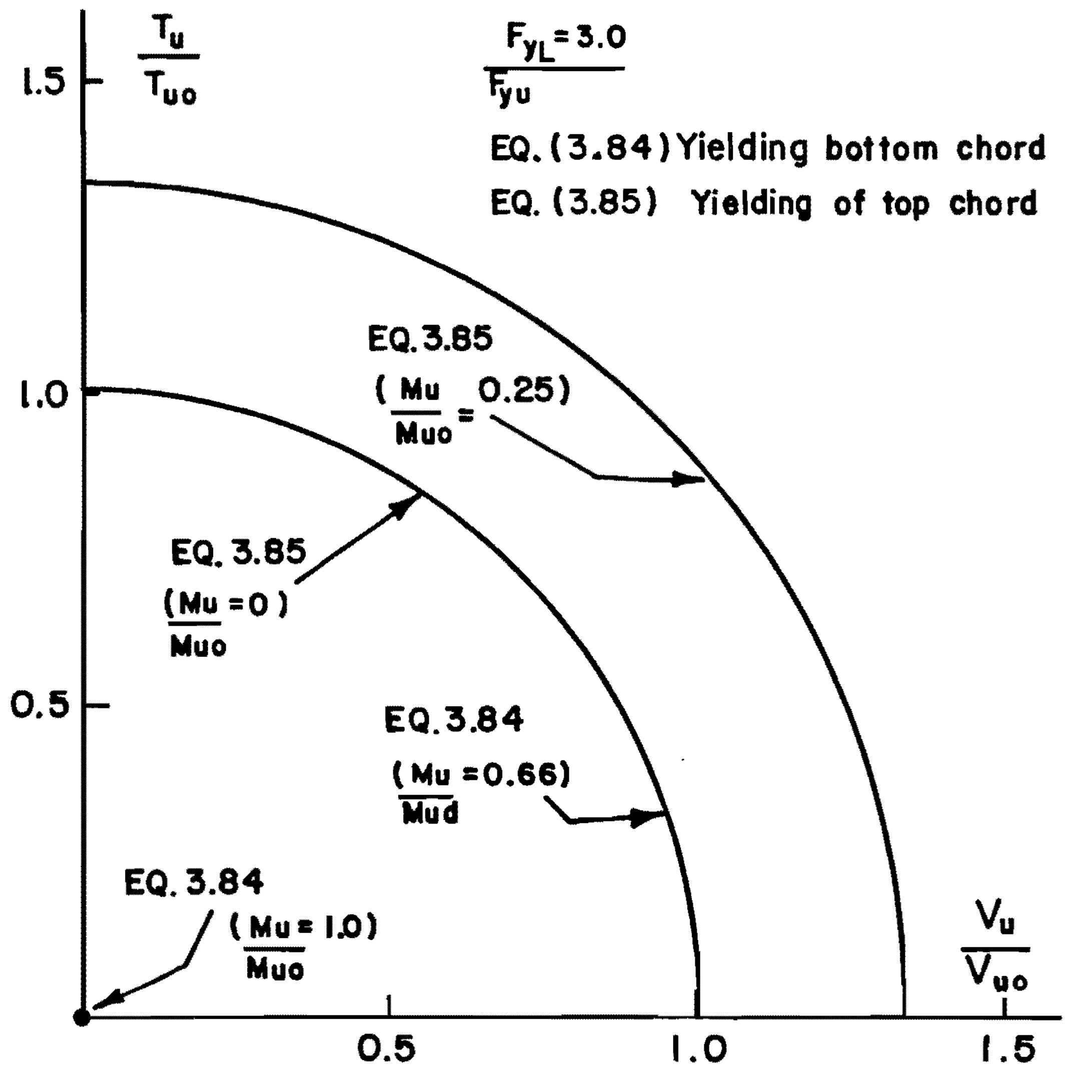


Fig. 3.23 Effect of the applied bending moment on the torsion-shear interaction of beams

circular failure surface given by Eq. 3.85. Note that when the applied moment is zero failure is controlled by yielding of the upper chord (flexural compression) in the case of members with bending type reinforcement ($F_{yu} > F_{yl}$), and the radius of the circular interaction is in fact equal to 1, as shown in Fig. 3.23 by the circular failure surface given by Eq. 3.85. Fig. 3.23 thus illustrates the effects of the applied bending moment in the interaction between torsion and shear in concrete members.

3.6 Design Approaches

The previous sections in this chapter outlined the basis for the truss model with variable inclination of the compression diagonal elements and used the model to develop interaction equations for flexure, shear and torsion. Such background is necessary for development but is not used in detail in application.

Although application of design procedures based on the truss model with variable inclination of the compression diagonal elements may appear to be rather complex, in reality the opposite is the case (50,72,93,153,165,166). Furthermore, the truss model is applicable to any type of cross section. It is suitable for: (a) the design of sections subjected to shear and bending; (b) shear and torsion; or (c) shear, torsion and bending. Finally, it allows a unified design of either prestressed or normally reinforced concrete sections containing web reinforcement. For clarity, the design approaches for the cases of bending-shear and torsion-bending-shear, are treated separately in the following subsections. Specific problems, limits in application, and a detailed comparison with experimental results are given in Report 248-3. Detailed design recommendations and example applications are given in Report 248-4F.

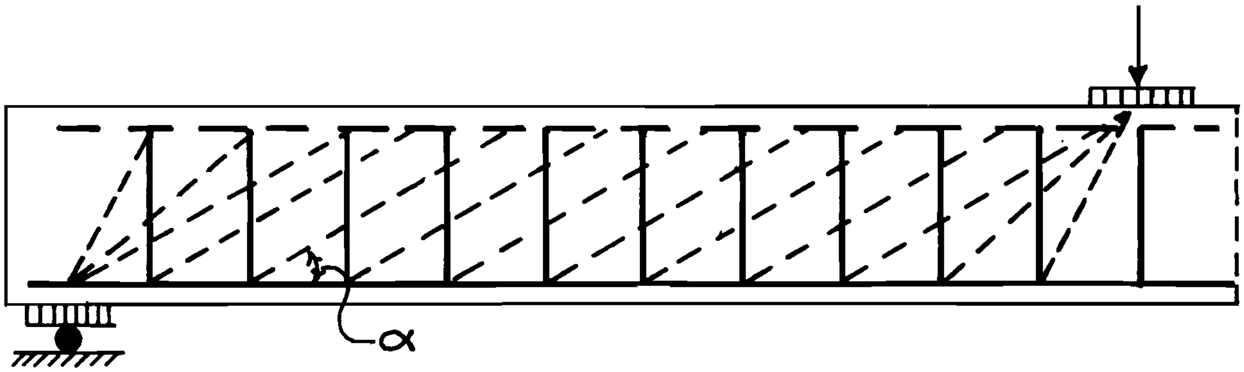
3.6.1 Bending and Shear. The design procedure based on the truss model is easy to conceptualize and use. Basically the procedure consists of 5 steps:

1. Select an appropriate truss system for the load pattern and structural constraints.
2. Assume a compression diagonal inclination that is within the limits established in Section 3.3 ($0.5 \leq \tan\alpha \leq 2.0$).
3. Compute the area of transverse reinforcement required as tension ties and select its spacing from equilibrium and spacing limits.
4. Determine the area of longitudinal reinforcement required for the combined actions.
5. Check the web concrete stresses f_d, in the diagonal compression elements of the truss to guard against web crushing.

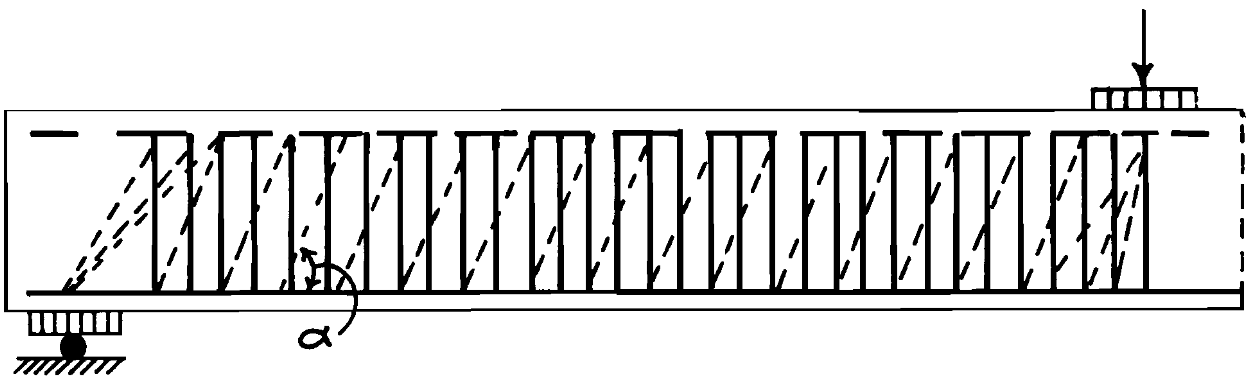
In contrast with the present ACI and AASHTO design procedure, the truss model approach allows for a variable inclination of the diagonal strut. In addition, the use of truss model panels imply design for a constant shear in a finite zone equal to the horizontal projection of the inclined crack ($z*\cot\alpha$). This is different than current ACI or AASHTO design procedures which imply use at a given location along the span of the member and hence require continual stirrup changes as the shear changes. This becomes of significance when designing for uniformly loaded beams where the truss model procedures are quite simple.

In a normal design situation, the applied moment and shear would be known in addition to the material properties, f_{yl}, f_{ys}, and f_c'. Also selected by general proportioning considerations would be the overall depth of the section and possibly the width. The value of z should be taken as the distance between the tension and compression resultants caused by the applied moment and can often be approximated by the distance between the longitudinal compression and tension reinforcement enclosed by the stirrups The most important step in the design procedure is the selection of an adequate truss system for the given load pattern and structural constraints. In the design procedure using the truss model the designer has the freedom to choose the angle of inclination of the diagonal compression elements within the limits presented in Sec. 3.3. The freedom in the selection of the angle of inclination α can produce, for the same loading conditions, a number of different truss possibilities (see Fig. 3.24).

As shown in Fig. 3.24, the freedom in choice of inclination angle α between the stated limits allows a variety of truss forms with highly different reinforcement patterns. Choice of low α angles requires lighter stirrups while choice of higher α angles results in heavier stirrups. Detailing requirements impose practical limits on stirrup spacing. Even within a given member it is usually necessary to vary the angle of inclination of the compression struts near reactions and in the vecinity of concentrated loads as shown in Fig. 3.25.



(a) α - small ($\tan\alpha = 0.2$)



(b) α - large ($\tan\alpha = 2.0$)

Fig. 3.24 Different truss models

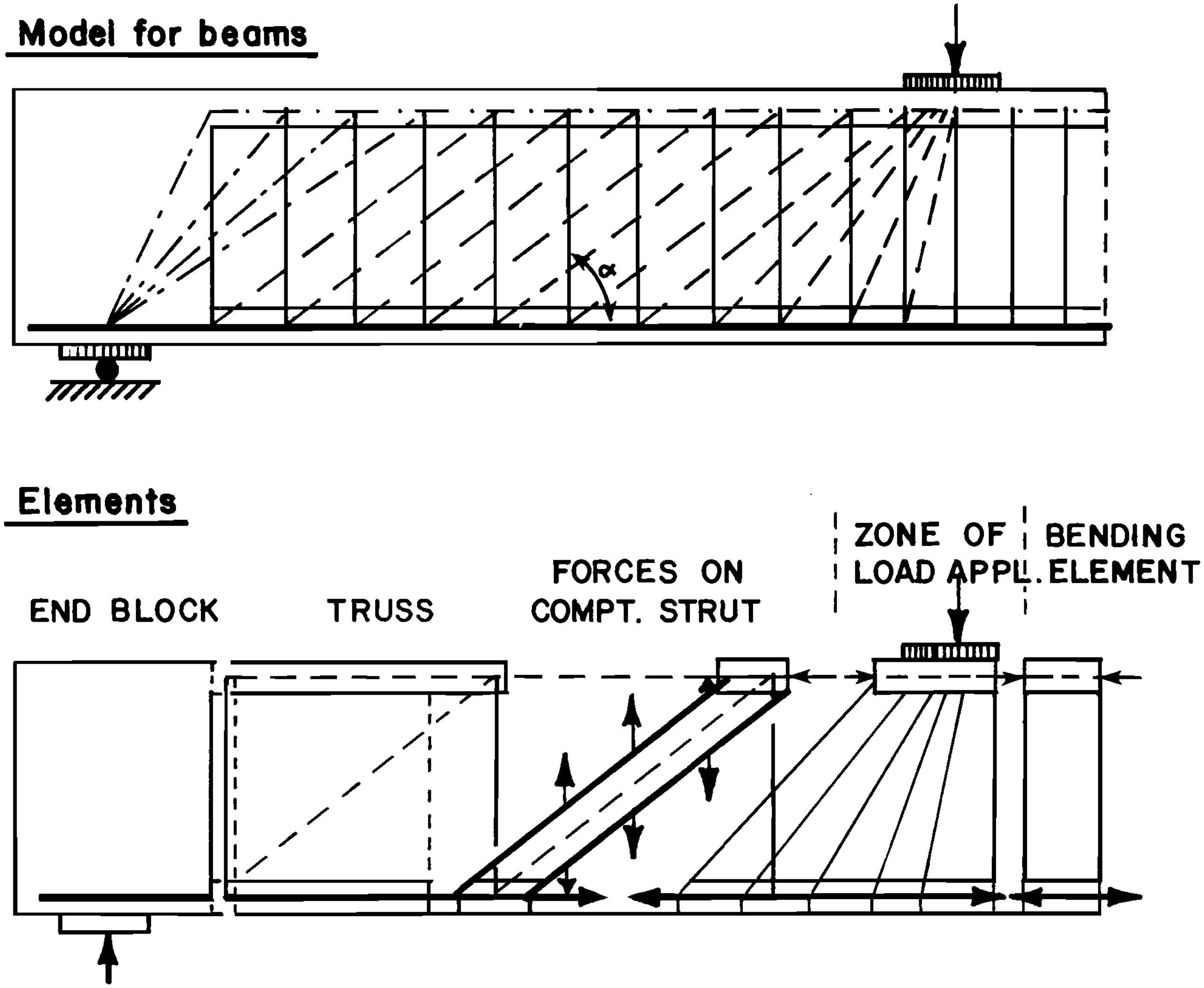


Fig. 3.25 Truss model and its elements for the case of bending and shear

The different angles of inclination of the diagonal compression strut lead to different combinations of transverse and longitudinal reinforcement for the same ultimate load.

Using the simple free body diagram A-A'-B of the beam shown on Fig. 3.26a, and based on equilibrium considerations, the effects of choosing different angles of inclination of the diagonal compression strut can be studied. Summation of moments about B in Fig. 3.26b yields:

$$A_1 = A_1(M) + A_1(V) = M/zf_y + V \cot\alpha/(2f_y) \qquad (3.86)$$

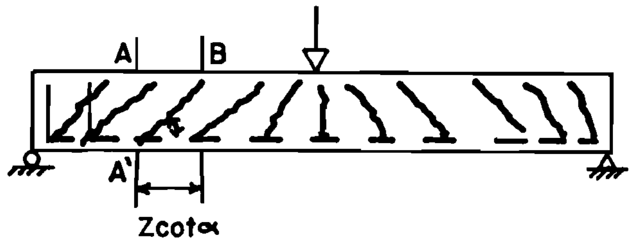
where $A_1(M)$ = area of longitudinal steel in tension chord due to flexure and $A_1(V)$ = area of longitudinal steel in tension chord due to the presence of shear.

Summation of vertical forces on the free body shown in Fig. 3.26b yields the relationship shown in Fig. 3.26c between the angle of inclination of the diagonal strut and the transverse reinforcement:

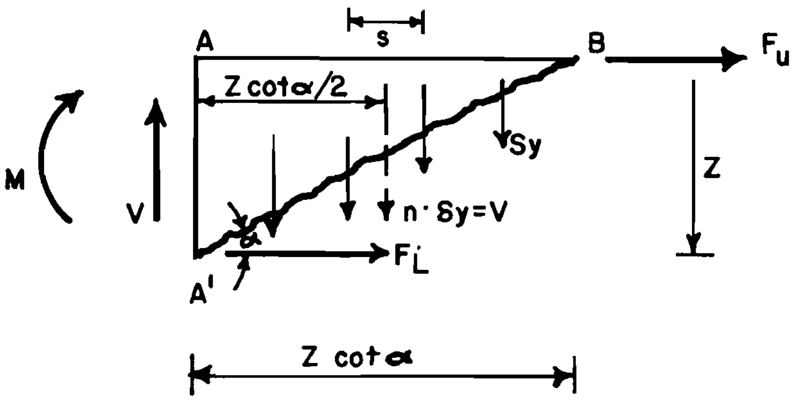
$$A_v/s = V/(f_y{*}z{*}\cot\alpha) \qquad (3.87)$$

where A_v/s = area of web reinforcement perpendicular to the longitudinal axis of the beam, per unit length of stirrup spacing "s".

From the study of the two previous equations it becomes apparent that for decreasing values of the stirrup requirements decrease while the amount of longitudinal reinforcement increases and vice versa. The designer must analyze the internal forces in the member using the chosen truss model. The steps of a routine design once the truss model has been selected are very straight forward.



(a) Beam subjected to bending and shear



$$+\Sigma M_B = 0 = M + V\,z\,\cot\alpha - \frac{V\,z\,\cot\alpha}{2} - F_L\,z$$

$$F_L = A_\ell f_y = \frac{M}{z} + \frac{V}{2}\cot\alpha$$

$$A_\ell = \frac{M}{z f_y} + \frac{V}{2 f_y}\cot\alpha$$

(b) Force in the longitudinal tension cord FL

$$\Sigma F_v = 0 = n\,S_y - V;\; n = \frac{z\,\cot\alpha}{s} \quad \text{(constant stirrup spacing)}$$

Thus: $S_y = A_v f_y = V\,s/z\,\cot\alpha$

and $A_v = \dfrac{V_s}{z\,f_y\,\cot\alpha}$

(c) Summation of vertical forces

Fig. 3.26 Effects of the variation of the angle of inclination of the diagonal strut in the design process

First the longitudinal reinforcement must be designed in two parts. The longitudinal chord must be proportioned to resist the tensile force of the couple caused by the applied moment, "M." The area of longitudinal reinforcement to resist the moment is

$$A_l(M) = M_u/zf_{yl} \tag{3.88}$$

The chord must also be able to resist the additional longitudinal force due to the horizontal component of the diagonal compression force in the concrete strut produced by the applied shear force. This is evident from Fig. 3.26b. The area of longitudinal reinforcement required due to the presence of shear "V_u," in addition to the one required for bending is

$$A_l(V) = V_u \cot\alpha/2f_{yl} \tag{3.89}$$

In the use of the truss model it can be observed that the resultant diagonal compression force due to the presence of shear creates vertical and horizontal compression forces in the concrete which must be balanced by vertical and horizontal tension forces. Therefore, the presence of a shear force induces not only vertical tension forces which must be resisted by the stirrup reinforcement, but longitudinal tension forces as well. In the current ACI and AASHTO provisions (24,17) the need for longitudinal reinforcement due to shear is only implicitly recognized in the provisions dealing with development of flexural reinforcement.

As a result the total area of longitudinal tension steel required in any panel of the tension chord of the truss becomes

$$A_1 = A_1\ (M) + A_1\ (V) \tag{3.90}$$

Secondly, the area of transverse reinforcement is computed from the free body A-A'-B shown in Fig. 3.26b. Summation of vertical forces (see Fig. 3.26c) yields $A_v/s = V_u \tan\alpha/(f_y{*}z)$ (stirrup spacing limits, s max, will be introduced under detailing provisions).

Finally, the compression stresses in the diagonal strut must be checked to avoid premature web crushing failures using Eq. 3.58:

$$f_d = \frac{V_u}{b_w z}\left[\frac{1}{\frac{1}{\sin\alpha\ \cos\alpha}}\right] \le f_c \tag{3.91}$$

Where f_c is a prescribed maximum allowable compression stress. This allowable stress is a function of the design concrete compressive strength and of the state of strains in the web section. Further discussion on this subject and specific values are presented in subsequent reports in this series.

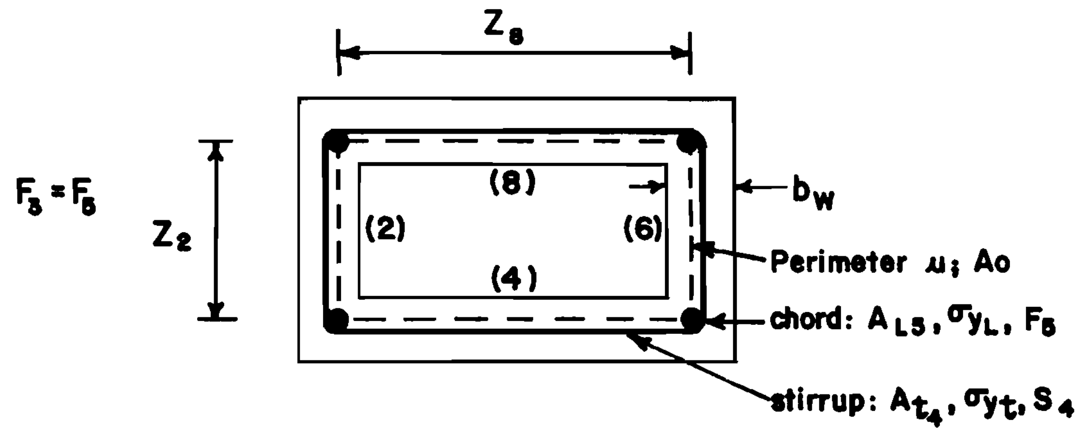
So far in this discussion only full truss action requiring considerable diagonal cracking in the web has been considered. If, however, the nominal shear stress v = V/bz in the web is small at ultimate load, no or very limited diagonal cracking will take place. In this transition range between an uncracked condition and the full truss action, there is the transmission of a diminishing amount of shear forces by such mechanisms as aggregate interlock. Hence, the concrete in the web will provide an additional continuously diminishing shear resistance, which becomes equal to zero as soon as the full truss action is obtained. Since many lightly loaded flexural members are used in practice it is important to also consider the role of concrete tensile forces in this transition range and the possible reduction in stirrup reinforcement when such forces are considered. This will be discussed in a later report in this series.

3.6.2 Torsion, Bending and Shear. The design procedure for combined torsion, bending and shear remains basically the same as in the case for bending and shear:

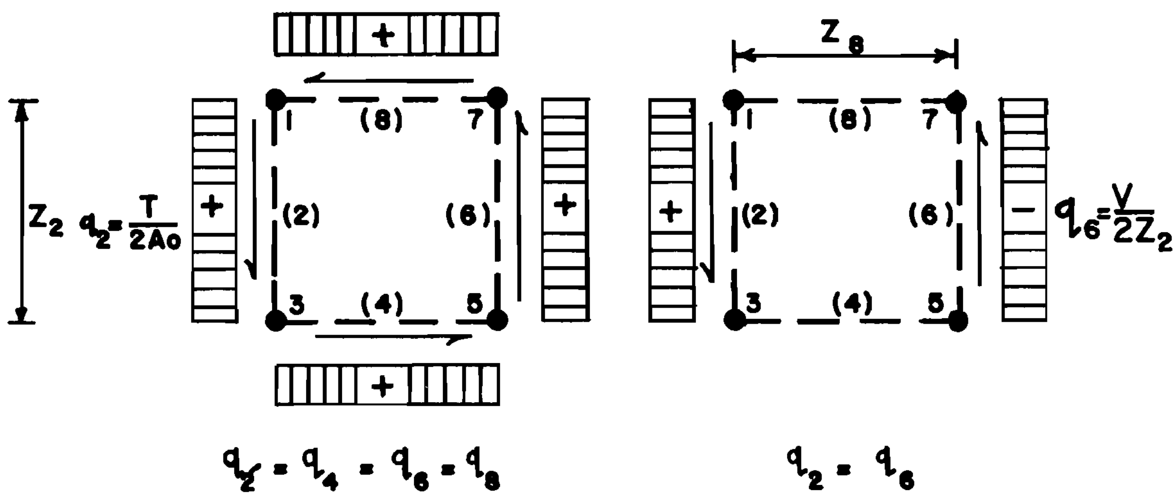
1. Select an appropriate truss system for the load pattern and structural constraints.
2. Assume a compression diagonal inclination that is within the limits established in Section 3.3. ($0.5 \leq \tan\alpha \leq 2.0$)
3. Compute the area of transverse reinforcement required as tension ties and select its spacing from equilibrium and spacing limits.
4. Determine the area of longitudinal reinforcement required for the combined actions.
5. Check the web concrete stresses, f_d, in the diagonal compression elements of the truss to guard against web crushing.

In general the values of the torsional moment "T", as well as the bending moment "M", and applied shear "V" would be known. Also given would be the material properties f_{yl}, f_{ys}, and f'_c. In most cases the overall cross section dimensions would be known from preliminary proportioning.

Consider the design of a box section (see Fig. 3.27a) subjected to shear, torsion and bending. The procedure that must be followed is essentially the same used in the case of combined bending and shear. The first step would be to select an adequate truss system for the given load pattern and structural constraints.



(a) Box section

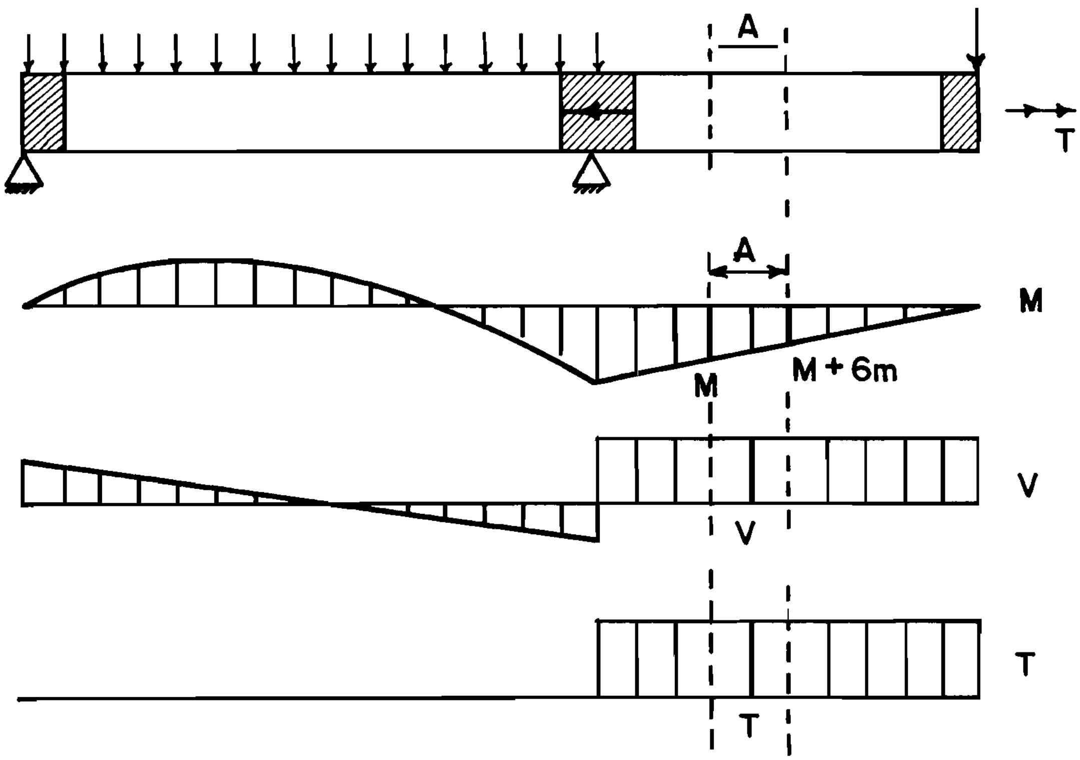


(b) Shear flows due to shear and torsion

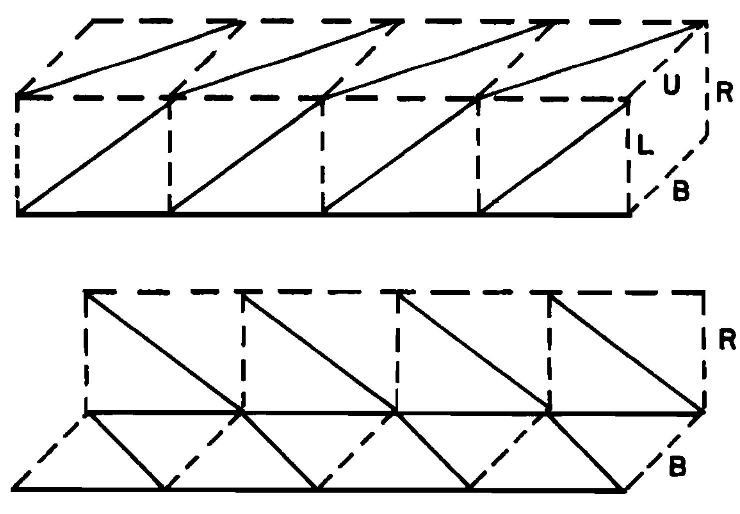
Fig. 3.27 Box section for the case of combined shear, torsion, and bending

Consider the case of the box section of Fig. 3.27a loaded in the manner shown in Fig. 3.28a. The beam will be subjected in the zone "A" to a combination of negative bending moment M (tension at the top), shear force V, and torsional moment T. The corresponding truss analogy is shown in Fig. 3.28b. If the box section is subdivided in its corresponding shear field elements or webs, namely U, L, R and B (see Fig. 3.29a); then the truss models for each of these shear field elements can be selected taking into account the actions taking place on each of the particular shear field elements or webs of the box section (see Fig. 3.29b). Next the angle of inclination of the diagonal compression elements in the truss model must be selected in accordance with the limits prescribed in Sec. 3.3. Once the truss model has been selected and the internal forces computed for the chosen model, the design procedure becomes simple and straight forward. As in the case of combined bending and shear the longitudinal tension reinforcement must be designed in two parts. Consider the side shear field element (2) of the box section shown in Fig. 3.27a. Summing moments about B in Fig. 3.30b yields the value of the longitudinal tension force in the bottom chord due to a bending moment and a shear flow "q".

From this relation it becomes apparent that the longitudinal tension force is a function of both the applied moment (M/z), and the horizontal component of the diagonal compression strut ($q*z_i*\cot\alpha$) which has to be balanced by a longitudinal tension force.

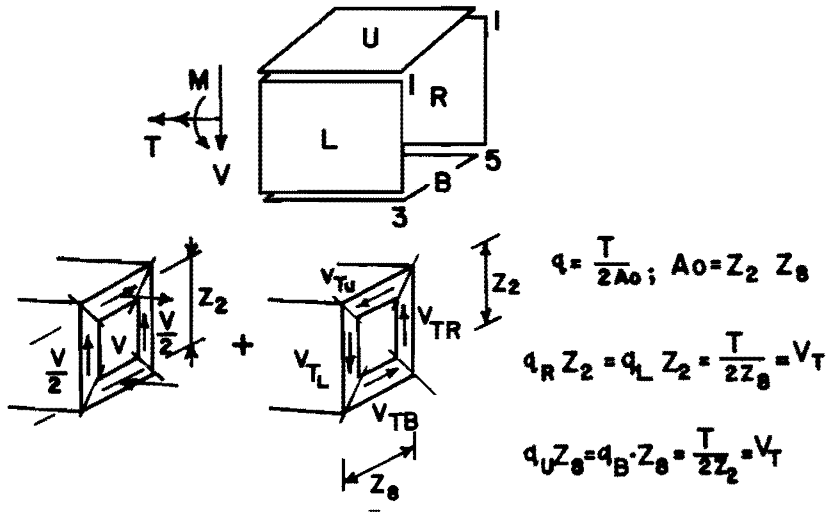


(a) Loading combination

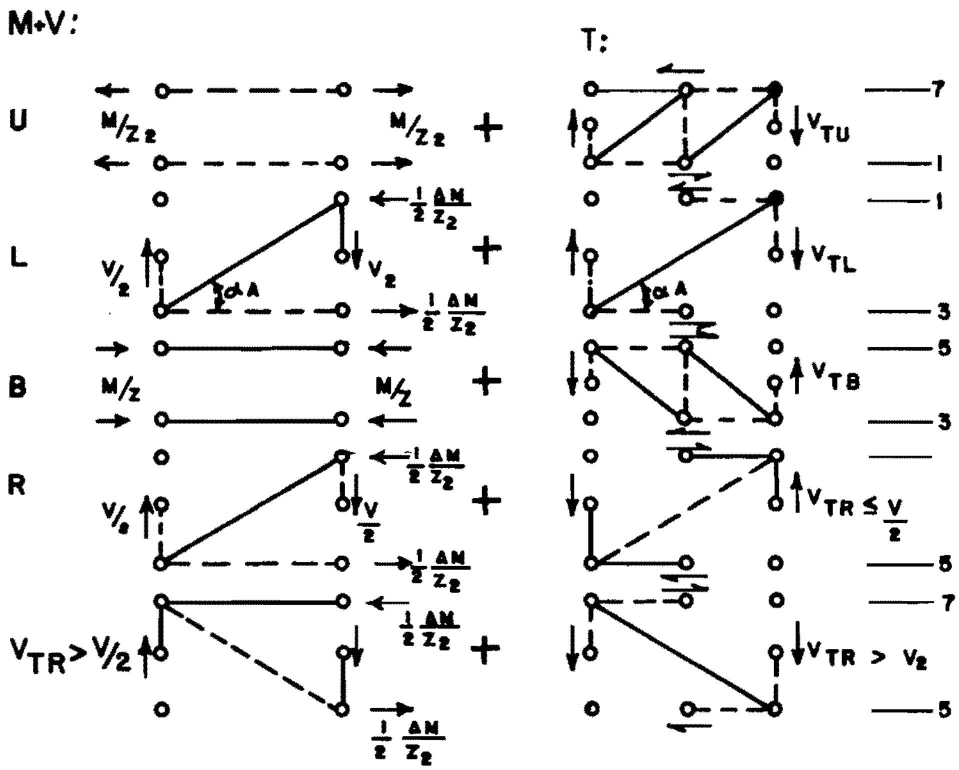


(b) Truss analogy for zone A

Fig. 3.28 Beam subjected to bending, shear, and torsion

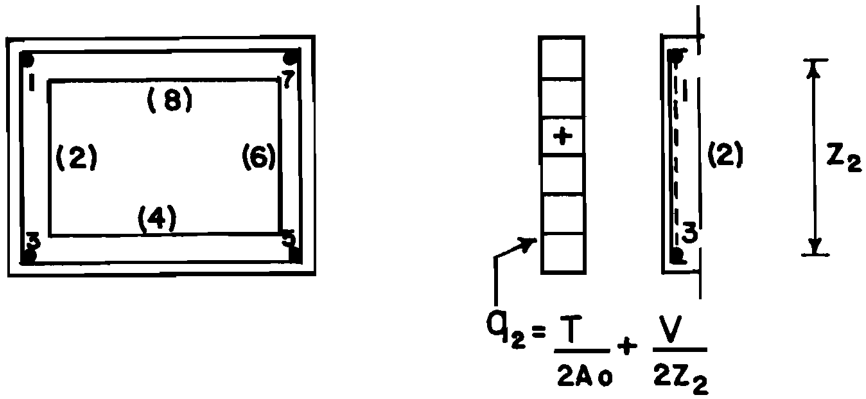


(a) Forces acting on the cross section

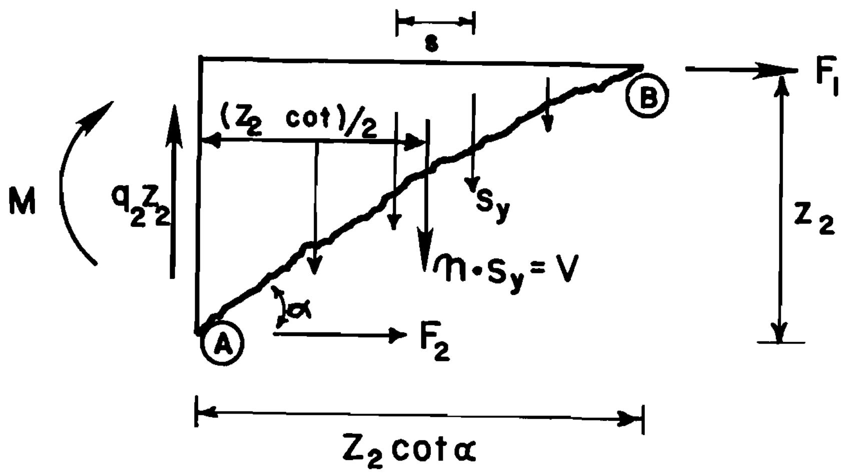


(b) Truss models

Fig. 3.29 Determination of the truss model for the box section subjected to bending, shear, and torsion



(a) Side field element (2)



$$\Sigma M_b = \curvearrowleft + = M + q_2 \circ z_2 \cdot z_2 \cot\alpha - \frac{V_2}{2} z_2 \cot\alpha - F_3 z_2$$

From $\Sigma V = 0$ $\quad V_2 = q_2 \cdot z_2$ $\quad$ where by definition $V_2 = n \cdot S_y$

hence: $F_3 = \dfrac{M}{z_2} + q_2 \cdot \dfrac{z_2}{2} \cot\alpha$

(b) Force on the longitudinal tension chord F_3

Fig. 3.30 Force in the longitudinal tension chord, $F_3 = F_5$

The area of longitudinal steel required for flexure is determined by dividing the tension force due to flexure (M/z) by the yield stress of the longitudinal reinforcement (f_{yl}):

$$A_1(M) = M/(z\, f_{yl}) \tag{3.92}$$

$A_1(M)$ is the total area required for flexure; since for the box beam shown in Fig. 3.27a the areas of chord 3 = chord 5, and chord 1 = chord 7, then the area of longitudinal tension steel due to flexure per individual flexure tension chord (3 or 5) becomes:

$$A_{13}\ (M) = A_{15}\ (M) = M/2z_2\ f_{yl} \tag{3.93}$$

Next the shear flows due to shear, q(V), and torsion, q(T), acting on all the sides, must be evaluated:

$$q(V) = V/n*z \tag{3.94}$$

Where, V = total shear force acting on the section, and n = number of shear field elements (webs) resisting this shear force, and "z" is the effective depth. The torsional shear flow is

$$q(T) = T/2A_o \tag{3.95}$$

Where, T = torsional moment acting on the section, and A_o = area of the perimeter enclosing the centroids of the longitudinal chords. The additional longitudinal reinforcement required for each shear field element is then determined from the horizontal components of those vertical shear flows, $N = \Sigma q_i * z_i * \cot\alpha_i$ (see Fig. 3.32). This total horizontal force (N), is then equally distributed between both longitudinal chords in the shear field element or may be uniformly distributed over each shear field element and resisted by distributed load. Consider the box section shown in Fig. 3.27a subdivided in its constituent side webs as shown in Fig. 3.31.

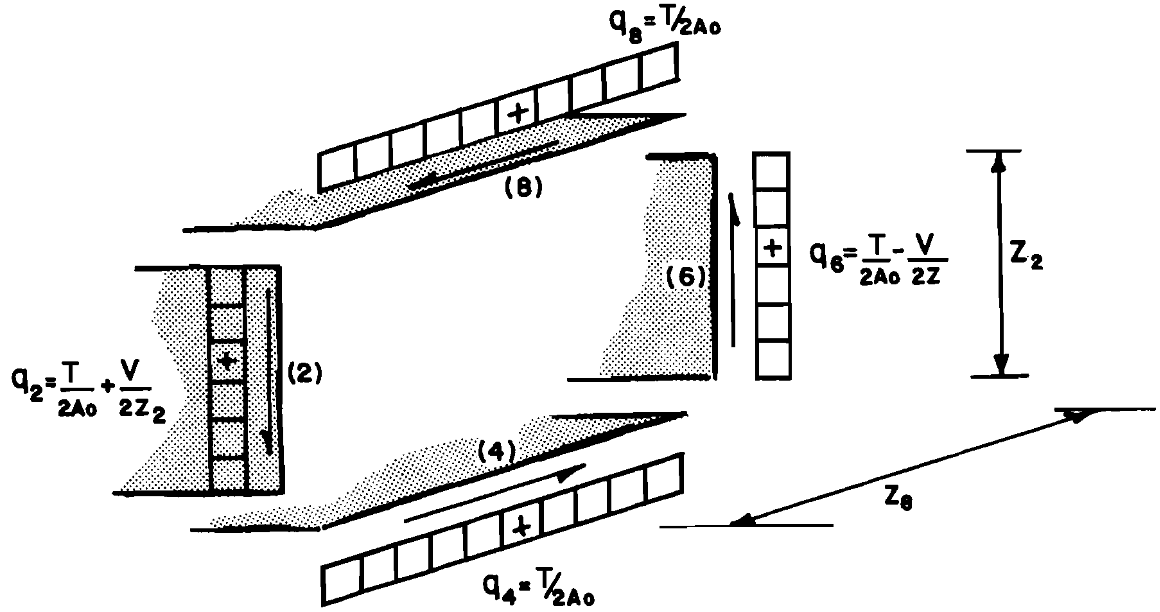


Fig. 3.31 Constituent side webs of the box section

Analyzing each of the side webs separately, the horizontal component of the diagonal compression strut induced by the presence of a vertical shear flow due to shear and torsion, can be found from geometric considerations of the diagonal compression strut as shown in Fig. 3.32.

For the case of the box section shown in Fig. 3.27a the longitudinal chords 1 and 3 in the side shear field element are assumed equal; hence, the horizontal component of the diagonal compression strut

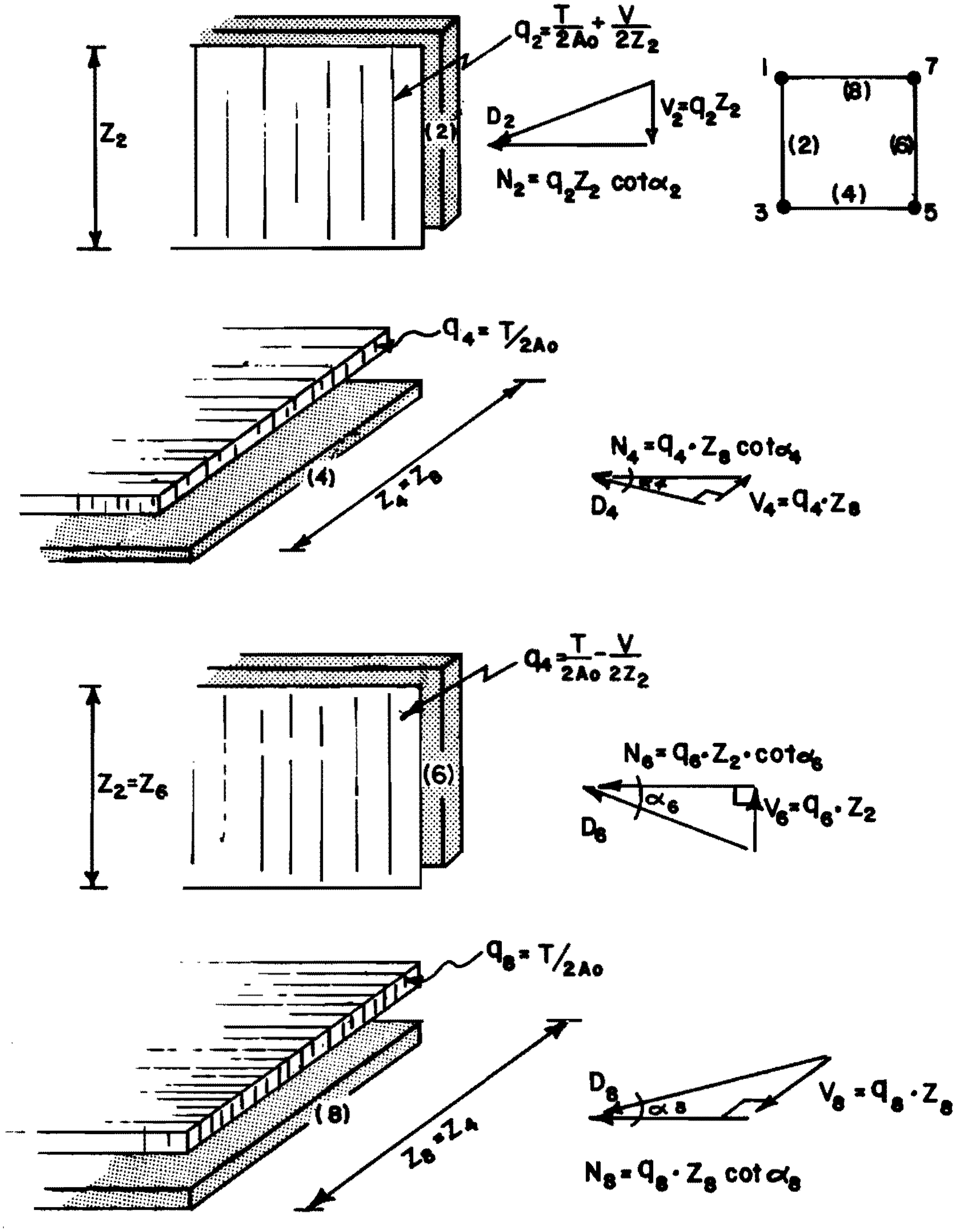


Fig. 3.32 Horizontal components of the diagonal compression strut in the different side webs of the box section

due to the presence of the vertical shear flow "q_2" is balanced by the chords 1 and 3, each contributing an equal amount, thus the area of longitudinal steel required becomes

$$A_{11}(q_2) = A_{13}(q_2) = \frac{1}{2f_{y1}} (q_2 z_2 \cot\alpha_2) \tag{3.96}$$

Consider now the top side shear field element 8. In this case, the longitudinal chords 1 and 7 are assumed equal again; hence, the horizontal component of the diagonal compression strut produced by the shear flow "q_8" is balanced in equal amounts by the longitudinal chords 1 and 7. Thus, the area of longitudinal steel required for the chords in the side field element 8 becomes:

$$A_{11}(q_8) = A_{17}(q_8) = \frac{1}{2f_{y1}} (q_8 z_8 \cot\alpha_8) \tag{3.97}$$

Therefore, the total area of longitudinal steel required for chord 1 due to shear and torsion is: $A_{11}(T,V) = A_{11}(q_2) + A_{11}(q_8)$, thus

$$A_{11} = (T,V) = \frac{1}{2f_{y1}} [q_2 z_2 \cot\alpha_2 + q_8 z_8 \cot\alpha_8] \tag{3.98}$$

Substituting the values of q_2 and of q_8 shown in Fig. 3.31 yields:

$$A_{11}(T,V) = \frac{1}{2f_{y1}} [(\frac{T}{2A_0} + \frac{V}{2z_2}) z_2 \cot\alpha_2 + \frac{T}{2A_0} z_8 \cot\alpha_8] \tag{3.99}$$

If a constant angle of inclination of the diagonal compression strut on all the sides of the box section is assumed in the design process, then $\cot\alpha_2 = \cot\alpha_8$, and Eq. 3.99 becomes

$$A_{11}(T,V) = \frac{1}{2f_{y1}} \left[\frac{T*u}{4A_0} + \frac{V}{2}\right] \cot\alpha_2 \qquad (3.100)$$

A similar procedure can be followed to compute the areas of longitudinal steel required due to shear and torsion in the longitudinal chords 3, 5 and 7. The longitudinal reinforcement required for bending on the tension side must always be added to the shear and torsion components. On the flexural compression side, the tension force due to shear and torsion is counteracted by the compression force due to bending. Hence, the tensile reinforcement can be proportionally reduced (see Fig. 3.15).

$$A_1 = A_1(T,V) - M/(2zf_{y1}) \qquad (3.101)$$

The number 2 in the denominator of the second term comes from the fact that the 2 top chords, 1 and 7, are assumed to be equal. Hence, the compression force is equally distributed between them. The summation sign in this equation emphasizes the fact that even though the computations can be done for each shear field element (sides), one must always keep in mind the overall system and must add all effects for the overall system. Hence, for example, it indicates that the longitudinal components of the shear flows on both adjacent sides 2 and 8 in Fig. 3.32 should be added when computing the total area of chord 1.

Next, the stirrup reinforcement has to be evaluated as from Fig. 3.26c where $S = S_y = A_v f_{yv}$ and $V = q(T,V)*z$, hence:

$$A_v = S_y/f_{yv} = q(T,V)*s/(\cot\alpha f_{ys}) \qquad (3.102)$$

Summing the shear flows due to shear and torsion for each shear field element, the required stirrup reinforcement for each element is obtained.

Finally, the concrete compression stresses must be checked in order to ensure that no premature failure, due to crushing of the web prior to yielding of the reinforcement, would take place. The concrete compression stresses are obtained from Fig. 3.33.

$$f_d = q(T,V)/[b_w \sin\alpha \cos\alpha] \tag{3.103}$$

This value must not exceed a specified limit, f_c, which will be discussed in more detail in subsequent reports in this series.

3.7 Summary

A failure model has been introduced which permits the analysis and design of reinforced and prestressed concrete beams subjected to shear and torsion using basic equilibrium relationships.

The model is completely applicable only for the "full truss action" range. Beams in this range are those subjected to high nominal shear stresses, where other mechanisms of shear transfer which act in the transition range have diminished. In the case of prestressed concrete members, this also implies that the initial compression has been overcome and that the behavior is then similar to that of a reinforced concrete beam with the same longitudinal reinforcement yield force. The strain and crack patterns of two such beams at ultimate will be similar (165,166).

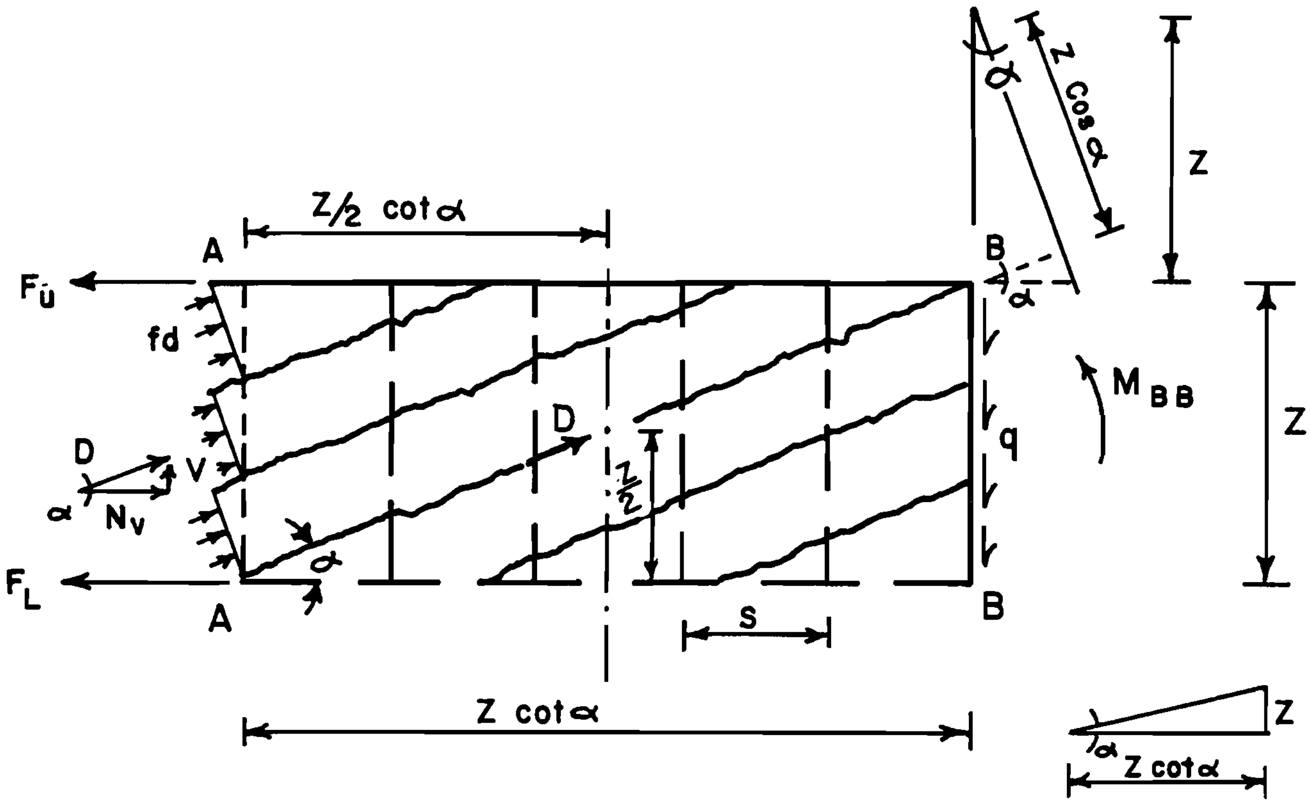


Fig. 3.33 Compression stress in the diagonal strut due to shear and torsion

In this discussion, so far, only ultimate load behavior has been considered. If the model is to be used as a basis for design procedures, service load considerations should be included in addition to limits for the amounts and spacing of web reinforcement. These limits will be discussed in subsequent reports in this series.

The variable angle truss model provides the designer with a conceptual model to analyze the behavior of members subjected to combined actions. The designer can visualize the effects that such actions will have on the different components of the member. A more complete understanding of this behavior should lead to a simpler and more effective design process.

In Report 248-3, the effects of special conditions on the space truss model will be studied. Such conditions include:

- Different loading conditions
- Strength of the compression diagonal strut
- Noncirculatory torsion
- Strand draping
- Detailing of reinforcement
- Uncracked, transition and full truss states

C H A P T E R 4

CONCLUSIONS

A historical review of the development of American design recommendations for shear and torsion in reinforced and prestressed concrete showed that the AASHTO Specifications and the ACI Building Code tend to use very similar approaches to the problem. In many cases they are identical in formulation.

Many very successful structures have been built under current regulations and the reported unhappiness of designers with current approaches stems from the difficulty of understanding and applying the regulations to unusual and unfamiliar cases.

The general design philosophy for shear and torsion members does not have an apparent rational basis similar to that used for flexural members or with combined axial load and flexure. This has resulted in highly empirical and somewhat confusing shear and torsion design procedures. The present codes have confusing and overlapping empirical expressions to predict with apparent great refinement the effect of parameters such as M/V_d, ρ_w, and f_c' on shear, in addition to a_t, in the case of torsion. The provision of the detailed equations is barely acceptable for dimensioning of conventional beams but masks the understanding of the overall combined concrete and steel system required to carry combinations of moment and shear, with or without torsion.

The review of current design procedures indicated a real need of a conceptual model upon which code or specification provisions can be based. Examination of the Space Truss Model with Variable Inclination of Diagonals indicated that this procedure has great promise as the rational design approach required to be a useful and understandable design model for combined effects of moment, shear and torsion. The general background of the procedure was outlined in Chapter 3 and the basic equilibrium equations relating truss member forces to applied load effects were developed. Although not needed in the design process, interaction equations illustrating the relations between moment, shear, and torsion were developed. Such relationships help clarify the ability of members to carry various loading combinations.

The treatment of the space truss model in this report only deals with general requirements and equilibrium relationships. Detailed examination of a number of special considereations required before the method can be applied in a design framework will be given in the following report in this series. Similarly, a thorough comparison of the space truss model with existing test results will be given in that report.

Specific AASHTO Specification type language to implement the space truss model as the basic design approach for shear and torsion will be given in the final report in this series. Several design examples will be included to illustrate application of the procedures.

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