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PROCEDURES FOR THE STATIC AND REPEATED-LOAD
INDIRECT TENSILE TEST

by

Thomas W. Kennedy
James N. Anagnos

Research Report Number 183-14

Tensile Characterization of Highway Pavement Materials
Research Project 3-9-72-183

conducted for

Texas
State Department of Highways and Public Transportation

in cooperation with the
U. S. Department of Transportation
Federal Highway Administration

by the

Center for Transportation Research
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The University of Texas at Austin

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The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

PREFACE

This report is the fourteenth in a series of reports for Project 3-9-72-183, "Tensile Characterization of Highway Pavement Materials," and is concerned with the indirect tensile test. The use of the static and repeated-load indirect tensile tests is discussed and tentative test procedures and equipment are recommended.

Special appreciation is extended to Pat Hardeman, Harold H. Dalrymple, Victor N. Toth, and Eugene Betts for their assistance; to Avery Smith, Gerald B. Peck, James L. Brown, Robert E. Long and Frank E. Herbert of the Texas State Department of Highways and Public Transportation, who provided technical liaison; to A. W. Eatman, Larry G. Walker, and Billy R. Neeley, who served as the Materials and Tests Division (D-9) Engineers during the study and who provided the support of the Materials and Tests Division. Appreciation is also extended to the staff of the Center for Transportation Research, whose assistance has been essential to the conduct of the study.

LIST OF REPORTS

Report No. 183-1, "Tensile and Elastic Characteristics of Pavement Materials," by Bryant P. Marshall and Thomas W. Kennedy, summarizes the results of a study on the magnitude of the tensile and elastic properties of highway pavement materials and the variations associated with these properties which might be expected in an actual roadway.

Report No. 183-2, "Fatigue and Repeated-Load Elastic Characteristics of Inservice Asphalt-Treated Materials," by Domingo Navarro and Thomas W. Kennedy, summarizes the results of a study on the fatigue response of highway pavement materials and the variation in fatigue life that might be expected in an actual roadway.

Report No. 183-3, "Cumulative Damage of Asphalt Materials Under Repeated-Load Indirect Tension," by Calvin E. Cowher and Thomas W. Kennedy, summarizes the results of a study on the applicability of a linear damage rule, Miner's Hypothesis, to fatigue data obtained utilizing the repeated-load indirect tensile test.

Report No. 183-4, "Comparison of Fatigue Test Methods for Asphalt Materials," by Byron W. Porter and Thomas W. Kennedy, summarizes the results of a study comparing fatigue results of the repeated-load indirect tensile test with the results from other commonly used tests and a study comparing creep and fatigue deformations.

Report No. 183-5, "Fatigue and Resilient Characteristics of Asphalt Mixtures by Repeated-Load Indirect Tensile Test," by Adedare S. Adedimila and Thomas W. Kennedy, summarizes the results of a study on the fatigue behavior and the effects of repeated tensile stresses on the resilient characteristics of asphalt mixtures utilizing the repeated-load indirect tensile test.

Report No. 183-6, "Evaluation of the Resilient Elastic Characteristics of Asphalt Mixtures Using the Indirect Tensile Test," by Guillermo Gonzalez, Thomas W. Kennedy, and James N. Anagnos, summarizes the results of a study to evaluate possible test methods for obtaining elastic properties of pavement materials, to recommend a test method and preliminary procedure, and to evaluate properties in terms of mixture design.

Report No. 183-7, "Permanent Deformation Characteristics of Asphalt Mixtures by Repeated-Load Indirect Tensile Test," by Joaquin Vallejo, Thomas W. Kennedy, and Ralph Haas, summarizes the results of a preliminary study which compared and evaluated permanent strain characteristics of asphalt mixtures using the repeated-load indirect tensile test.

Report No. 183-8, "Resilient and Fatigue Characteristics of Asphalt Mixtures Processed by the Dryer-Drum Mixer," by Manuel Rodriguez and Thomas W. Kennedy, summarizes the results of a study to evaluate the engineering properties of asphalt mixtures produced using a dryer-drum plant.

Report No. 183-9, "Fatigue and Repeated-Load Elastic Characteristics of Inservice Portland Cement Concrete," by John A. Crumley and Thomas W. Kennedy, summarizes the results of an investigation of the resilient elastic and fatigue behavior of inservice concrete from pavements in Texas.

Report No. 183-10, "Development of a Mixture Design Procedure for Recycled Asphalt Mixtures," by Ignacio Perez, Thomas W. Kennedy, and Adedare S. Adedimila, summarizes the results of a study to evaluate the fatigue and elastic characteristics of recycled asphalt materials and to develop a preliminary mixture design procedure.

Report No. 183-11, "An Evaluation of the Texas Blackbase Mix Design Procedure Using the Indirect Tensile Test," by David B. Peters and Thomas W. Kennedy, summarizes the results of a study evaluating the elastic and repeated-load properties of blackbase mixes determined from current blackbase design procedures using the indirect tensile test.

Report No. 183-12, "The Effects of Soil Binder and Moisture on Blackbase Mixtures," by Wei-Chou V. Ping and Thomas W. Kennedy, summarizes the results of a study to evaluate the effect of soil binder content on the engineering properties of blackbase paving mixtures.

Report No. 183-13, "Evaluation of the Effect of Moisture Conditioning on Blackbase Mixtures," by James N. Anagnos, Thomas W. Kennedy, and Freddy L. Roberts, summarizes the results of a study to evaluate the effects of moisture content on the engineering properties of blackbase paving mixtures.

Report No. 183-14, "Procedures for the Static and Repeated-Load Indirect Tensile Test," by Thomas W. Kennedy and James N. Anagnos, summarizes indirect tensile testing and recommends testing procedures and equipment for determining tensile strength, resilient properties, fatigue characteristics, and permanent deformation characteristics.

ABSTRACT

This report establishes the test procedure and equipment required to conduct the indirect tensile test and the repeated-load indirect tensile test. The equations and their use to determine the various elastic properties are presented. An ASTM test procedure to conduct the repeated-load indirect tensile test was developed from this effort.

KEY WORDS: asphalt mixtures, indirect tensile test, pavement materials, elastic properties, permanent deformation, fatigue, resilient modulus, tensile strength, modulus of elasticity, Poisson's ratio, tensile strains, test procedures

SUMMARY

The indirect tensile test is a practical and effective test for determining the elastic tensile properties and distress related properties of asphalt mixtures. Simple mathematical equations have been developed to calculate the elastic properties for nominal 4- or 6-inch diameter specimens. Equations with coefficients are also presented whereby the coefficients change as related to exact specimen diameter.

The basic equipment such as loading head, curved face loading strips, load and deformation measurement devices, and recording equipment required to conduct the static indirect tensile test or the repeated-load tensile test are described. Also described are the test procedures concerned with test temperature, loading rate, shape of the loading curve, and load frequency and duration as related to the static or repeated-load test.

IMPLEMENTATION STATEMENT

The indirect tensile test can be used to determine the engineering properties of asphalt mixtures and to design mixtures on the basis of the engineering properties. As a result, the test has gained wide acceptance and has led to an ASTM standard for determining resilient modulus of asphalt mixtures.

Properties which can be estimated are tensile strength, static modulus of elasticity and Poisson's ratio, resilient modulus of elasticity and Poisson's ratio, fatigue properties, and permanent deformation properties. These tensile properties obtained from this test are expressed in terms of standard engineering units and thus are more meaningful than empirical numbers in theoretical design procedures which require the use of elastic constants. Steps should be taken to routinely use the test in Texas for mixture designs and to develop test values for construction control.

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DEFINITION OF SYMBOLS

DR	=	deformation ratio
E	=	modulus of elasticity
E_{RI}	=	instantaneous resilient modulus of elasticity
E_{RT}	=	total resilient modulus of elasticity
H_D	=	horizontal deformation
H_I	=	instantaneous resilient horizontal deformation
H_T	=	total resilient horizontal deformation
I	=	intercept with permanent strain axis (arithmetic)
K_1	=	antilog of the intercept of the logarithmic relationship between fatigue life and tensile strain
K_2	=	antilog of the intercept of the logarithmic relationship between fatigue life and tensile stress
K_2'	=	antilog of the intercept of the logarithmic relationship between fatigue life and stress difference
N	=	number of load applications
N_f	=	fatigue life
n_1	=	slope of the logarithmic relationship between fatigue life and initial strain
n_2	=	slope of the logarithmic relationship between fatigue life and tensile stress
P	=	repeated-load
P_{FAIL}	=	static failure load
S	=	slope of the linear portion of the logarithmic relationship
S_H	=	horizontal tangent modulus
S_T	=	tensile strength
t	=	thickness or height of specimen

V_D	= vertical deformation
V_I	= instantaneous resilient vertical deformation
V_T	= total resilient vertical deformation
$\Delta\sigma$	= stress difference
ϵ_c	= compressive strain
ϵ_i	= initial tensile strain or repeated strain
ϵ_r	= resilient strain
ϵ_T	= tensile strain
ϵ_α	= accumulated permanent strain
ν	= Poisson's ratio
ν_{RI}	= instantaneous resilient Poisson's ratio
ν_{RT}	= total resilient Poisson's ratio
σ_T	= applied tensile stress

CHAPTER 1. INTRODUCTION

The ability to characterize pavement materials in terms of fundamental properties is becoming increasingly important, partially due to the fact that many agencies are beginning to use mechanistic pavement design methods based on elastic or viscoelastic theory. Empirical tests required for previous design procedures do not provide fundamental engineering properties required by these newer design procedures and generally cannot be used to evaluate new materials, such as recycled asphalt mixtures, sulphur-asphalt mixtures, or marginal materials that have no performance history. In addition, it is highly desirable to be able to evaluate the material properties that are related to three very important pavement distress modes:

- (a) thermal or shrinkage cracking,
- (b) fatigue cracking, and
- (c) permanent deformation, or rutting.

One of the important inputs to these mechanistic evaluations is the response of the various materials when subjected to tensile stresses or strains, especially repeated tensile stresses or strains. For each material the following basic materials properties are required as inputs for an elastic layer analysis of a flexible pavement:

- (a) modulus of elasticity and Poisson's ratio, including variations with temperature and rate of loading,
- (b) tensile strength, which is primarily required for thermal or shrinkage cracking analysis, and
- (c) repeated-load characteristics of the materials, which include the fatigue and permanent deformation characteristics.

In addition, a viscoelastic analysis may include other properties such as creep compliance.

Most structural design methods and the various elastic or viscoelastic programs have been developed semi-independently resulting in the use of a wide variety of different field and laboratory tests. Because field testing is usually time consuming and not always practical, laboratory

methods have received considerable emphasis. Many of the more commonly used laboratory tests are empirical and used primarily for one material, making it difficult to compare materials, evaluate new materials, or provide input into elastic or viscoelastic design and analysis procedures except through the use of correlations.

Thus, there has been a need for simple, effective laboratory tests for characterizing materials in terms of the required fundamental properties. As a result, the static and repeated-load indirect tensile tests were developed to evaluate the engineering properties of pavement materials.

The indirect tensile test has been described under a series of names including: Brazilian Split Test, Split Test, Splitting Tensile Test, Diametral Test, Resilient Modulus Test, Schmidt Test, as well as the (Static) Indirect Tensile Test and Repeated-Load Indirect Tensile Test. The test can be performed in a repeated-load configuration or as a static, single load to failure mode. Regardless of the name, it should be emphasized that the test basically provides the same information on measurements of either strengths, elastic properties, or viscoelastic properties. The equipment and setup prescribed for use in various supporting documents will vary somewhat but the results are the same, in terms of strength, elastic or viscoelastic properties, and properties related to distress. This report describes the indirect tensile tests, test procedures, and methods of analyses. In addition, equipment which has been used successfully is described and discussed.

CHAPTER 2. THE INDIRECT TENSILE TEST

This chapter summarizes the findings and developments of the indirect tensile test which occurred as part of Research Project 98 and Research Project 183, the latter of which is summarized in Research Reports 183-3, 183-4, 183-6, and 183-7 (Refs 1, 2, 3, and 4).

HISTORY

The indirect tensile test was developed simultaneously but reported independently by Carneiro and Barcellos (Ref 5) in Brazil and Akazawa (Ref 6) in Japan in 1953. Testing involves loading a cylindrical specimen with compressive loads distributed along two opposite generators. This condition results in a relatively uniform tensile stress perpendicular to and along the diametral plane containing the applied load. Failure usually occurs by splitting along this loaded plane.

Prior to 1965, use of this test had generally been on concrete or mortar specimens, primarily to measure the tensile strength. However, in 1965 Thompson (Ref 7) reported that the test was satisfactory for the evaluation of the tensile characteristics of lime-soil mixtures while Messina (Ref 8) and Breen and Stephens (Refs 9 and 10) reported use of the test for the study of asphaltic concrete in 1966. Also, Livneh and Shklarsky (Ref 11) reported the use of the test to evaluate anisotropic cohesion of asphaltic concrete in 1962.

Because of the many practical advantages of the test, adaptation of this test to other pavement materials was begun in 1965 at The University of Texas at Austin as part of two research projects. Reports from the initial project were concerned with the adaptation of the theory and test procedure, whereas the second project involved the evaluation of both the static and repeated-load, elastic and permanent deformation characteristics of pavement materials. Since that time the test has been used successfully to evaluate engineering properties of sulphur-asphalt, sulphlex, and recycled asphalt mixtures (Refs 12, 13, and 14); modified asphalt-aggregate mixtures have also been evaluated and the test has been used to evaluate

the moisture susceptibility of asphalt-aggregate mixtures with and without antistripping additives.

From a literature review related to the indirect tensile test a number of advantages were attributed to the test, as follows (Refs 15 and 16):

- (1) it is relatively simple,
- (2) the types of specimens and equipment are the same as those used for compression testing,
- (3) failure is not seriously affected by surface conditions,
- (4) failure is initiated in a region of relatively uniform tensile stress,
- (5) the coefficient of variation of the test results is low, and
- (6) Mohr's theory is a satisfactory means of expressing failure conditions for brittle crystalline materials such as concrete.

GENERAL THEORY AND TEST DESCRIPTION

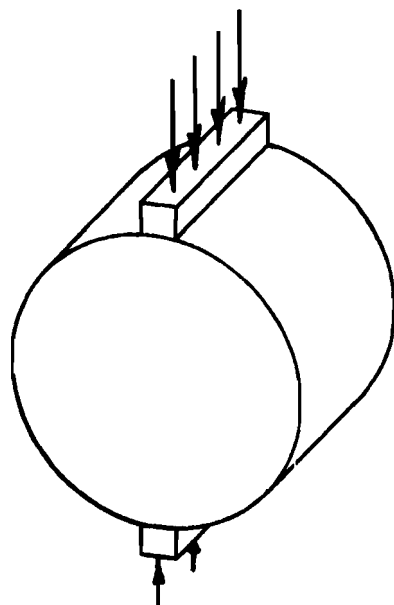
The indirect tensile test involves loading a cylindrical specimen with a single or repeated compressive load which acts parallel to and along the vertical diametral plane of the specimen (Fig 1a). To distribute the load and maintain a constant loading area the compressive load is applied through a stainless steel loading strip which is curved at the interface with the specimen and has a radius equal to that of the specimen.

This loading configuration develops a relatively uniform tensile stress perpendicular to the direction of the applied load and along the vertical diametral plane (Fig 1b). Ultimately, the specimen fails by splitting along the vertical diameter due to a single applied load or repeated-load applications (fatigue).

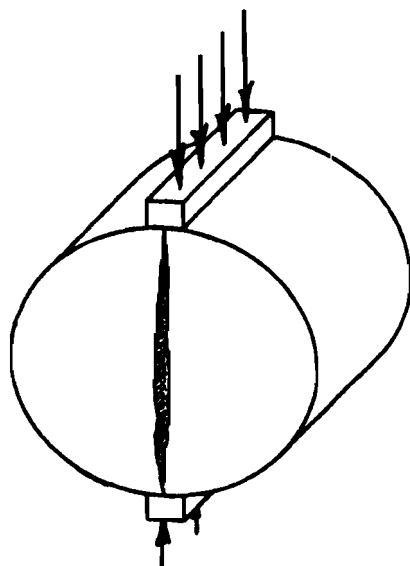
Theory

Development of stresses within a cylindrical specimen subjected to a line load was reported by Kennedy and Hudson (Refs 15 and 16). The significant stress distributions along the horizontal and vertical axes are shown in Figure 2.

Under conditions of a line load, the specimen fails near the load points due to compressive stresses and not in the center portion of the



(a) Compressive load being applied.



(b) Specimen failing in tension.

Fig 1. Indirect tensile test loading and failure.

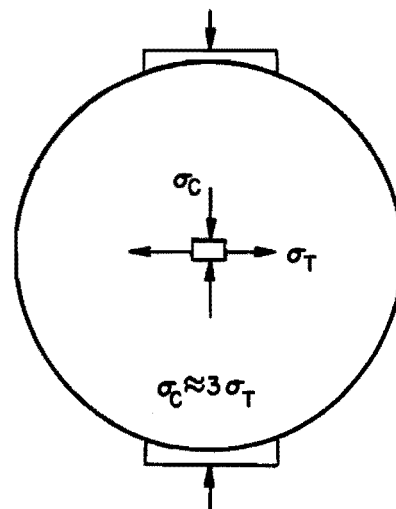
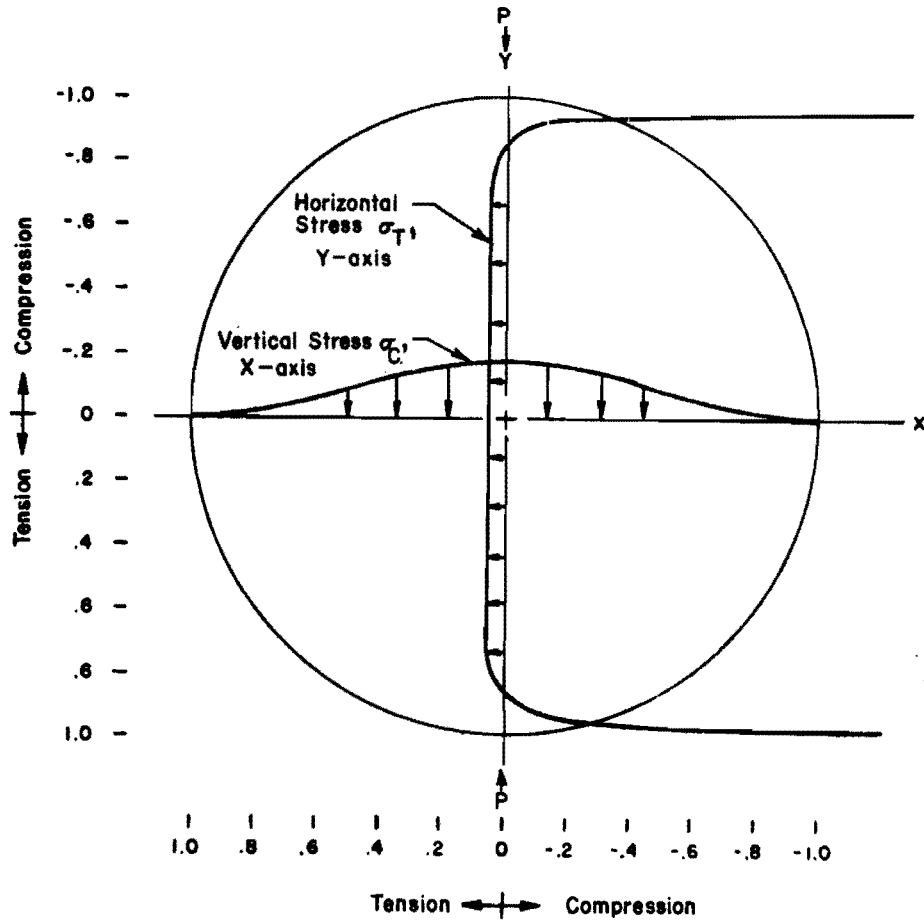


Fig 2. Relative stress distributions and center element showing biaxial state of stress for the indirect tensile test.

specimens due to tensile stresses. However, these compressive stresses are greatly reduced by distributing the load through a loading strip, which not only reduces the vertical compressive stresses but also changes the horizontal stresses along the vertical diameter from tension to compression near the points of load application. In addition, as previously noted, a biaxial state of stress is developed within the specimen. At the center of the specimen the vertical compressive stress is approximately three times the horizontal tensile stress. Curved loading strips 0.50 in. (13 mm) wide for 4-in.-diameter specimens or 0.75 in. (19 mm) wide for 6-in.-diameter specimens are recommended for use since the stress distributions are not altered significantly. Calculations of modulus of elasticity and Poisson's ratio are facilitated by maintaining a constant loading width rather than a constantly changing loading width, which occurs with a flat strip.

Equations were developed that permit the computation of the tensile strength, tensile strain, modulus of elasticity, and Poisson's ratio (Refs 17 and 18). These equations require that the integration be carried out using a computer program; however, for a given diameter and width of loading strip the equations can be simplified and used without the aid of a computer. These equations for 4-inch and 6-inch diameter specimens and a description of the input parameters are contained in Chapter 3 and in Appendix A.

General Test Procedures

In the static test a cylindrical specimen is loaded generally at a rate of 2 in. (50 mm) per minute. Slower rates can be used, especially for colder temperatures, since the material behaves more elastically and since loads or deformation associated with thermal cracking develop slowly, and for more brittle materials such as portland cement concrete. The testing temperature normally has been at room temperature, approximately 75°F (24°C), to eliminate the need for special heating or cooling facilities; however, other temperatures can be used. To completely characterize a material such as asphalt concrete at least three temperatures of 41, 77 (room temperature), and 104°F (5, 25, and 40°C) should be used to obtain the effects of temperature. The total horizontal (tensile) deformations

and vertical (compressive) deformations should be measured continuously during loading.

In the dynamic or repeated-load indirect tensile test method the same basic equations are used but it is not necessary to characterize the entire load-deformation relationship. Resilient modulus of elasticity can be obtained by measuring the recoverable vertical and horizontal deformations and assuming a linear relationship between load and deformation. In addition, this method can also provide an estimate of permanent deformation which occurs under repeated loads. Generally, the repeated stress is applied in the form of a haversine and a small preload is used in order to maintain constant contact between the loading strip and specimen. Typical load-time pulse and deformation-time relationships are shown in Figures 3 and 4. It is recommended that a shorter load duration be used if adequate recording and loading equipment is available. Other load-time pulses, e.g., square wave or trapezoidal wave forms, can also be used.

PROPERTIES RELATED TO DISTRESS

In addition to the basic elastic and viscoelastic inputs, properties related to the basic distress modes of thermal and shrinkage cracking, fatigue cracking, and permanent deformation are required and can be obtained using the static and repeated-load indirect tensile tests.

Thermal or Shrinkage Cracking

Tensile strengths required by the thermal or shrinkage cracking subsystem can be obtained using the direct tension or the static indirect tensile test. The direct tension test, however, is extremely difficult and time-consuming to conduct while the indirect tensile test is simple and can be conducted at a rate of 25 tests per hour. Values for asphalt concrete generally have varied from 50 to 600 psi depending on the temperature. At 77°F, values generally have been in the range of 100 to 200 psi. These strengths are typical and realistic for asphalt concrete. Realistic values have also been obtained for portland cement concrete and other materials.

Because of the ease of conducting the static test, the test can be used for quality control and has definite application for the evaluation of

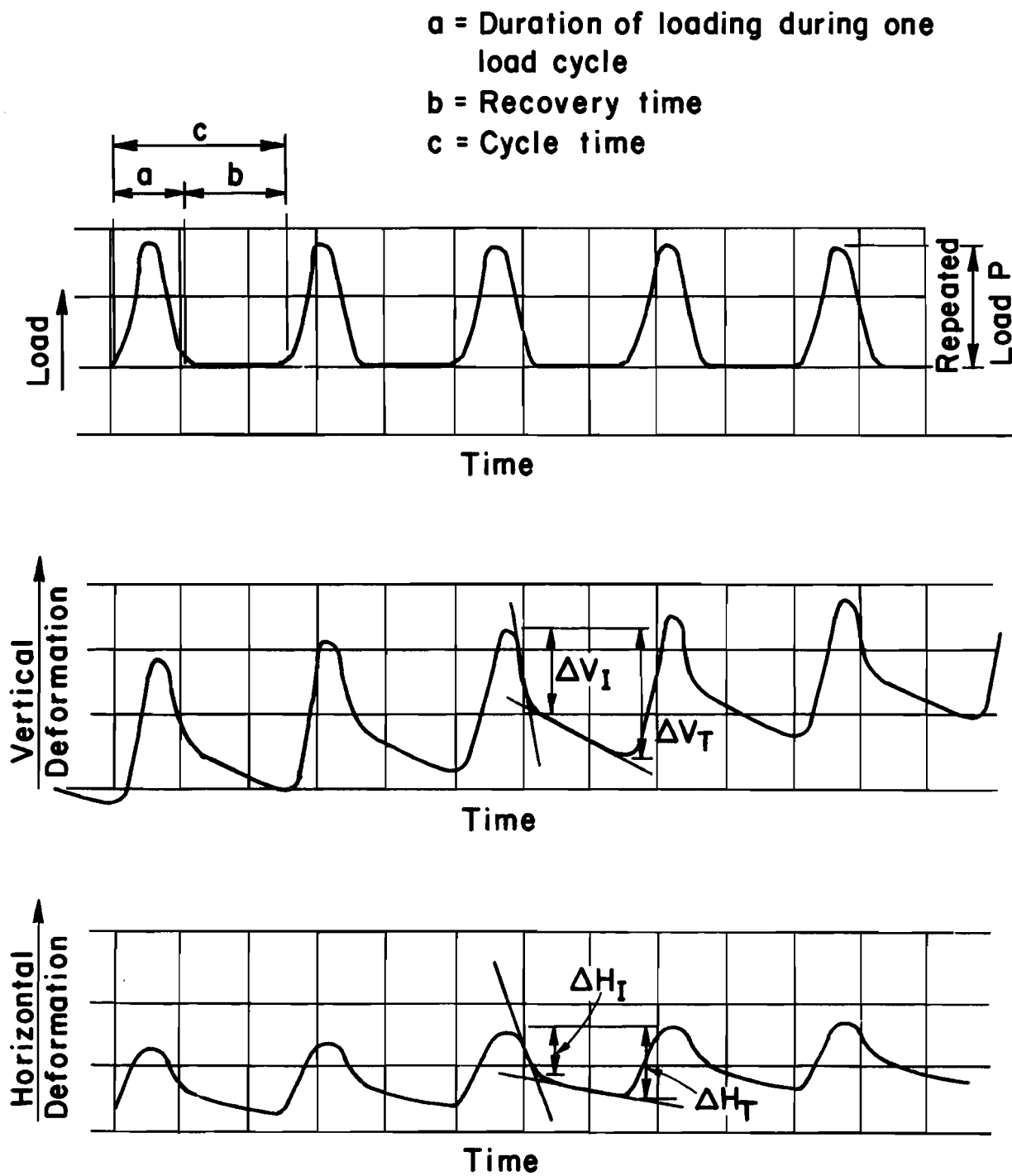


Fig 3. Load pulse and associated deformation relationships for the repeated-load indirect tensile test.

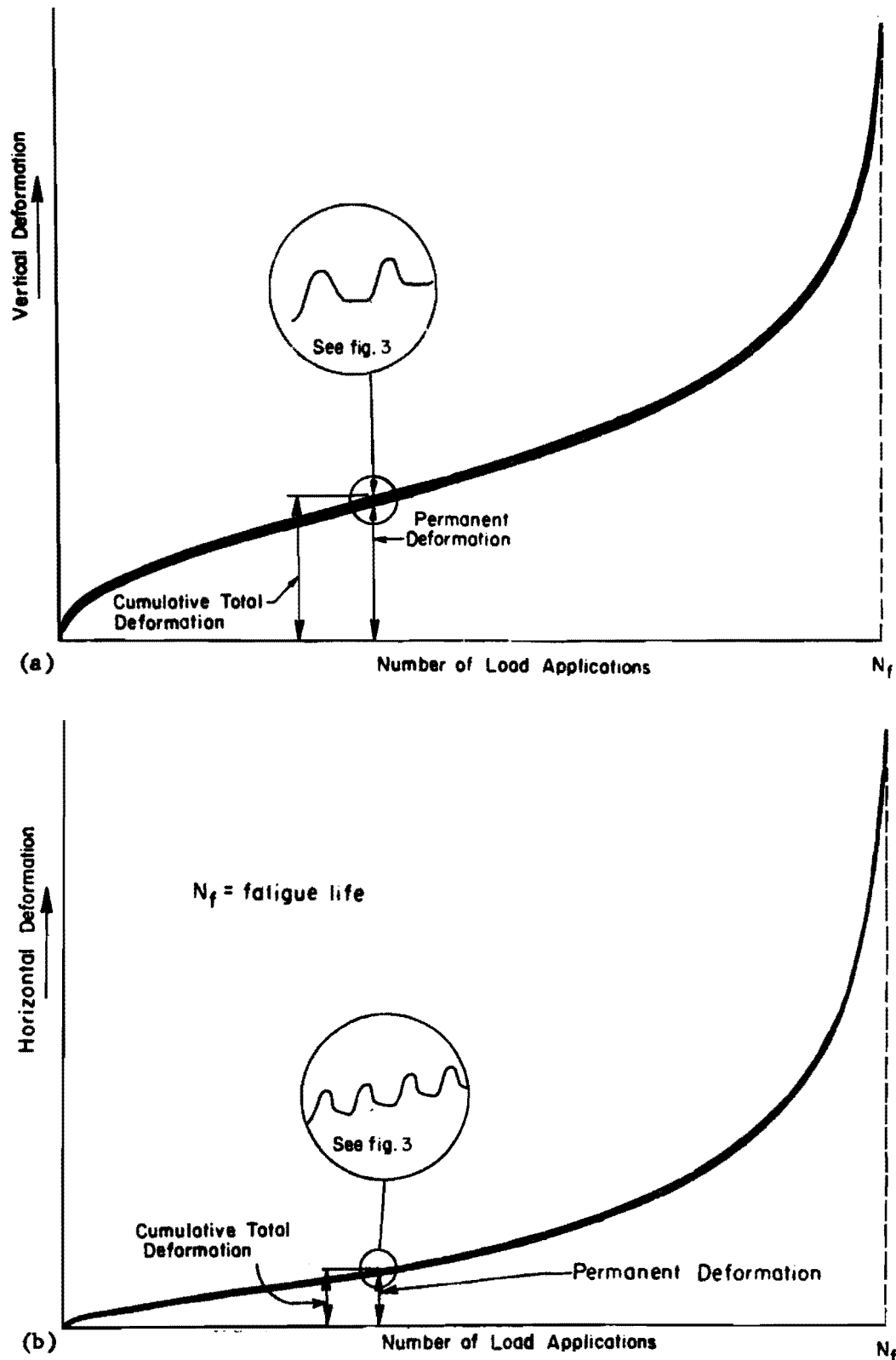


Fig 4. Relationships between number of load applications and vertical and horizontal deformation for the repeated-load indirect tensile test.

pavement materials in areas which do not have easy access to testing laboratories. It is also possible that tensile strength or the static modulus of elasticity can be related to the behavior under repeated loads, or that mixture designs can be based on static tests.

Fatigue Cracking

Various types of tests have been used to study the fatigue behavior of asphalt mixtures and other pavement materials. Those tests which have been used significantly for asphalt materials are the flexure test, rotating cantilever test, axial load test, and repeated-load indirect tensile test.

In addition, two basic types of loading are used in laboratory tests, controlled-strain or controlled-stress. Controlled-strain tests involve the application of repeated-loads which produce a constant repeated deformation or strain. In the controlled-stress tests a constant stress or load is repeated. Materials in thick flexible pavements are best tested using controlled-stress. The controlled-strain test is more applicable to thin flexible pavements.

In all of the above tests, a linear relationship is assumed to exist between the logarithm of the applied tensile stress and the logarithm of fatigue life, which can be expressed in the form

$$N_f = K_2 \left(\frac{1}{\sigma_T} \right)^{n_2} \quad (2.1)$$

where N_f = fatigue life,
 σ_T = applied tensile stress,
 n_2 = slope of the logarithmic relationship between fatigue life and tensile stress, and
 K_2 = antilog of the intercept of the logarithmic relationship between fatigue life and tensile stress.

It was found (Refs 19 and 20) that values of n_2 obtained using the indirect tensile test compared favorably with those reported by other investigators using other test methods (Refs 21, 22, and 23); however, the values of K_2 were significantly smaller, resulting in much lower fatigue lives. Thus, the results obtained from other test methods were analyzed

and compared with the characteristics of these tests and it was concluded that the results obtained from the repeated-load indirect tensile test were compatible if the applied stress was expressed in terms of stress difference, or deviator stress, to account for the biaxial state of stress which exists in the indirect tensile test (Fig 2). Figure 5 illustrates the relationships between fatigue life and stress difference for various tests. The dashed line illustrates the relationship between fatigue life and stress using the repeated-load indirect tensile test. For the indirect tensile test, stress difference is approximately equal to $4\sigma_t$ while stress difference for the uniaxial tests is equal to the applied stress. As seen in Figure 5, the differences in the results were greatly reduced when the stress difference is taken into account.

Expressing fatigue life in terms of stress difference merely shifts the position of the stress-fatigue life relationship and does not change the slope. Therefore, the K_2 values are significantly increased but values of n_2 are not affected and the relationship can be expressed in the form

$$N_f = K_2' \left(\frac{1}{\Delta\sigma} \right)^{n_2} \quad (2.2)$$

where $\Delta\sigma$ = stress difference, and

K_2' = the antilog of the intercept value of the logarithmic relationship between fatigue life and stress difference.

Values of K_2' , which are based on stress difference, were found to be comparable to values obtained for similar mixtures using other test methods.

In addition, fatigue life is significantly increased if the duration of the applied stress is reduced. In the above tests, the duration was 0.4 seconds, which ideally should be reduced to about 0.1 seconds. Such a change will improve the fatigue life predictive capabilities since laboratory fatigue tests underestimate the actual fatigue life of inservice pavements.

The relationship between initial strain and fatigue life can also be expressed as

$$N_f = K_1 \left(\frac{1}{\epsilon_i} \right)^{n_1} \quad (2.3)$$

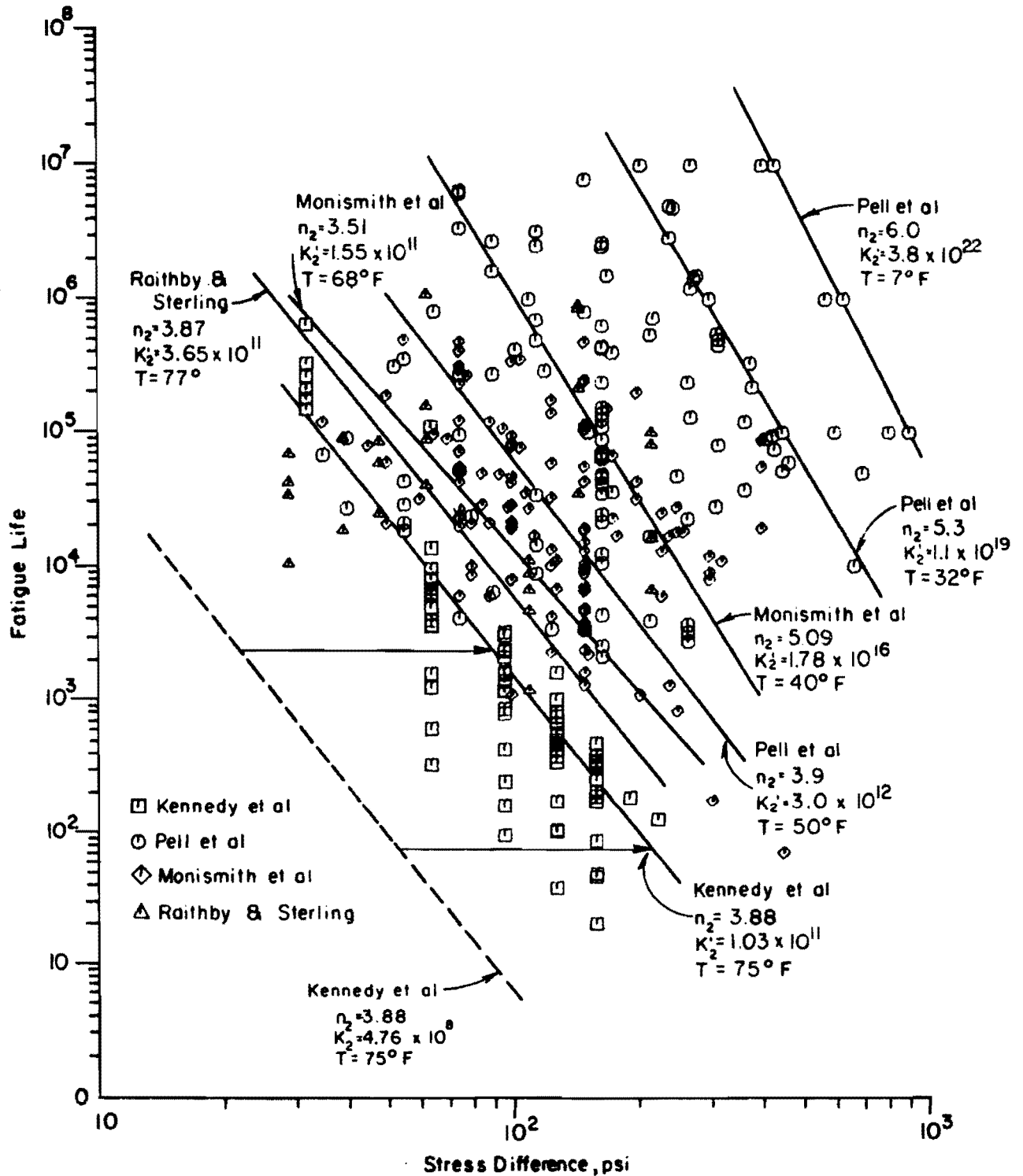


Fig 5. Typical stress difference-fatigue life relationships for various test methods.

where ϵ_i = initial tensile strain or repeated strain,
 n_1 = slope of the logarithmic relationship between fatigue life
and initial strain, and
 K_1 = antilog of the intercept of the logarithmic relationship
between fatigue life and tensile strain.

Values of K_1 compared favorably with previously reported values for similar mixtures with the same asphalt contents and tested at the same temperature.

Thus, it has been demonstrated in a number of studies that the fatigue characteristics obtained using the repeated-load indirect tensile test are comparable to the results obtained from other tests if stress is expressed in terms of stress difference or strain. This is significant since the repeated-load indirect tensile test is easier and more rapid to conduct than other commonly used fatigue tests and uses cylindrical specimens and cores.

Permanent Deformation

Three basic repeated-load tests have been used to obtain permanent strain information for asphalt materials. These tests are the:

- (a) triaxial compression test,
- (b) triaxial test in which the axial stress is tension, and
- (c) repeated-load indirect tensile test.

On the basis of a comparison of values obtained for the Brampton Road Test (Ref 24) with values obtained using the repeated-load indirect tensile test, it was concluded that the repeated-load indirect tensile test and the triaxial test in which the axial stress is tension provides reasonable estimates of permanent strain (Ref 4).

In addition to normal permanent strain characteristics, the permanent strain properties used by VESYS can be determined using the repeated-load indirect tensile test and the triaxial test. Two basic parameters, GNU and ALPHA, are used to describe the permanent deformation characteristics of asphalt mixtures and to predict rutting.

The theory (Ref 25) assumes that the logarithmic relationship between the number of repeated loads and permanent strain is essentially linear

over a range of load applications (Fig 6) and can be described by the equation

$$\epsilon_{\alpha} = IN^S \quad (2.4)$$

where ϵ_{α} = accumulated permanent strain,
 I = intercept with permanent strain axis (arithmetic strain value, not log value) (Fig 6),
 N = number of load applications, and
 S = slope of the linear portion of the logarithmic relationship.

GNU is defined as
$$\mu = \frac{IS}{\epsilon_r} \quad (2.5)$$

and ALPHA is defined as
$$\alpha = 1 - S \quad (2.6)$$

where ϵ_r = resilient strain, which is considered to become constant after a few load applications (Fig 7).

An evaluation of the three tests listed above (Ref 4) indicated that the permanent strain relationships for the latter two tests, which involve tensile stresses, are similar but different from those for the triaxial compression tests. Typical compressive and tensile test relationships are shown in Figure 8.

For compressive tests, the semilogarithmic relationship has a linear portion; however, the logarithmic relationship is nonlinear. For the tensile tests, the arithmetic relationship has a significant linear portion, but, as with the compressive stress relationship, the logarithmic relationship is nonlinear. This behavior is characteristic of the relationships obtained from both the repeated-load indirect tensile test and the triaxial test in which the axial stress is tensile (Refs 24, 26, and 27).

Because of the differences in the permanent strain relationships for the various tests and the fact that all three differ from the assumed relationship, the concept of GNU and ALPHA should be re-evaluated in order to improve the ability to characterize the permanent strain relationships of asphalt mixtures for use in VESYS. Nevertheless, the indirect tensile

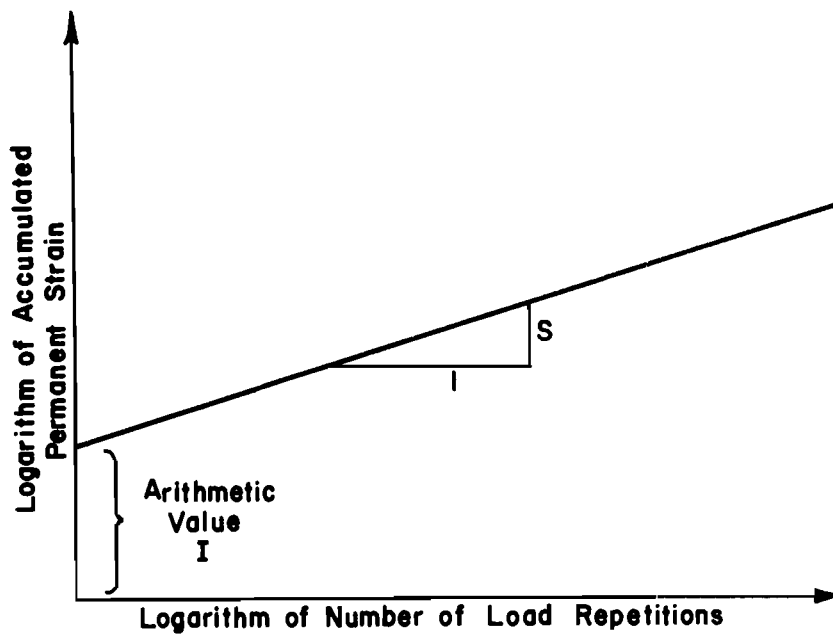


Fig 6 . Assumed logarithmic relationship between permanent strain and number of load repetitions (Ref 25).

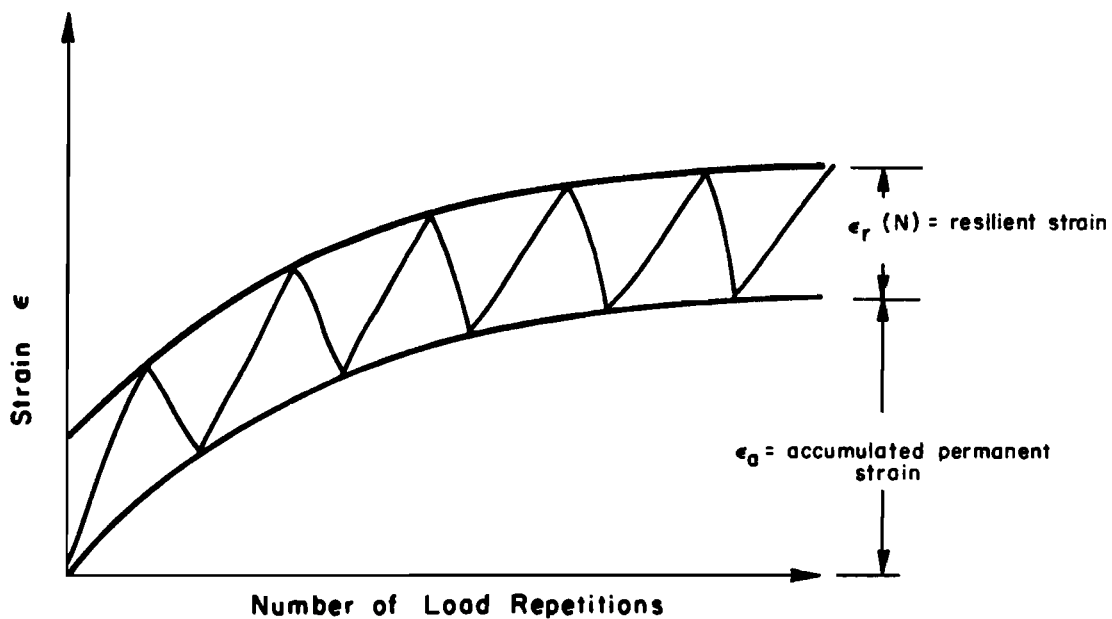


Fig 7 . Typical relationship between strain and number of load repetitions (Ref 25).

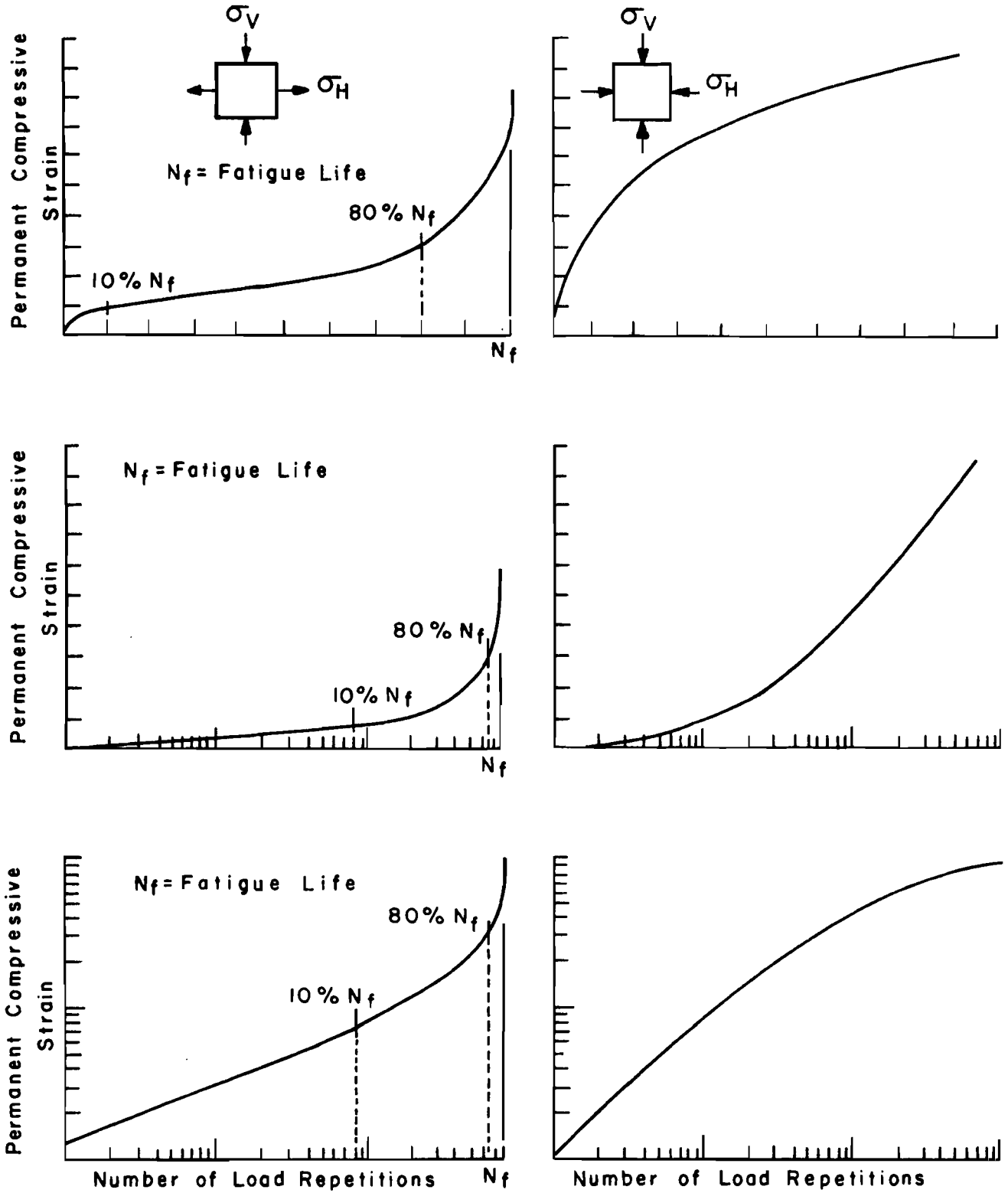


Fig 8. Typical arithmetic and logarithmic permanent strain relationships for relationships for tensile and compressive tests (Ref 4).

test can be used to obtain acceptable values for GNU and ALPHA by characterizing the initial portion of the logarithmic relationship for permanent strain.

Summary

Table 1 contains a subjective comparison of various test methods currently used to obtain fundamental materials' characteristics inputs. The various tests, as commonly conducted, are evaluated and summarized in terms of their ability to provide elastic and viscoelastic properties plus information related to the distress modes in terms of the previously discussed criteria.

An examination of the comparisons suggests that the indirect tensile test has certain advantages of economy and simplicity for bound, or cohesive, materials. In the case of unbound materials, the triaxial test remains the only variable method for laboratory evaluation, although a triaxial form of indirect tensile test is currently being developed.

Prior to 1965, the indirect tensile test was used primarily to measure the tensile strength of concrete. However, since 1965 the test has been used to evaluate many types of paving mixtures.

In addition, during the past few years, the indirect tensile test has been used nationwide to evaluate the engineering properties of asphalt mixtures, and an ASTM standard (Ref 28) for the determination of the resilient modulus of elasticity and Poisson's ratio is available.

TABLE 1. COMPARISON OF COMMON TEST METHODS (REF 29)

	Basic Test	Variations of Basic Test	Fundamental Properties Usually Determined By Test	Relationships Test Commonly Used For	Structural Subsystem Applicability			Criteria		Remarks
					Fatigue	Permanent Deformation	Low-Temperature Cracking	Ease of Testing and Economy	Reproducibility	
Tests for Elastic Properties	Indirect Tensile Test	Static	Stiffness Modulus, S	Fatigue Permanent Deformation Strain vs Temperature	Yes	Yes	Yes	Excellent	Good	Easy acquisition of specimens (i.e., from Marshall test or field cores)
		Dynamic Repeated	Resilient Modulus, E_R							
	Triaxial	Complex Modulus Test ¹	Complex Modulus, E		No	Yes	No	Good	Good	Output of test used for layer analyses rather than for fatigue, permanent deformation, or cracking relationships
		Resilient Modulus Test ²	Resilient Modulus, M_R							
	Beam Bending		Stiffness Modulus, S	Fatigue	Yes	No	No	Fair	Fair	
	Direct Tension	Triaxial ³		Permanent Deformation	Yes	Yes	No	Poor	Good	Specimen preparation usually requires sawing
Beam Test		Stiffness Modulus, S	Strain vs Temperature	Yes	No	Yes	Good	Poor		
Tests for Viscoelastic or Time-Dependent Properties	Triaxial	Static Creep ⁵	Creep Compliance	Permanent Deformation	No	Yes	No	Good	Good	
		Dynamic Repeated	GNU and ALPHA ⁶					Fair	Fair	
	Indirect Tensile Test ⁷	Static Creep ⁵	Creep Compliance	Permanent Deformation	Yes	Yes	No	Excellent	Good	Limited experience in applying this test to viscoelastic materials
		Dynamic Repeated	GNU and ALPHA ⁶							

CHAPTER 3. RECOMMENDED TEST PROCEDURES

The following provides detailed test procedures for conducting the Static and Repeated-Load Indirect Tensile Tests for estimating elastic and tensile properties of paving mixtures with emphasis on the properties of asphalt mixtures. In addition, the necessary equations for calculating these properties and the method of obtaining other distress-related properties are provided.

CALCULATIONS AND EQUATIONS

Properties which can be estimated from each test are

Static Indirect Tensile Test

Tensile strength,
Static modulus of elasticity, and
Static Poisson's ratio.

Repeated-Load Indirect Tensile Test

Resilient modulus of elasticity,
Resilient Poisson's ratio,
Fatigue characteristics, and
Permanent deformation characteristics.

Tensile Strength and Elastic Properties

Equations for calculating the tensile strength, static and resilient moduli of elasticity and Poisson's ratios, and strains for nominal 4-inch and 6-inch-diameter specimens are contained in Table 2. More generalized equations for other diameters and for strains with inputs in the English system (psi) and Metric or SI Systems (MPa) are contained in Appendix A. The equations and inputs in this appendix can be used to develop simplified equations similar to those in Table 2 for specimens with diameters which are different than 4 or 6 inches.

TABLE 2. SIMPLIFIED EQUATIONS FOR CALCULATING TENSILE PROPERTIES

Properties		4-inch Diameter*	6-inch Diameter**
Static	Tensile strength S_T , psi	$\frac{0.156 P_{FAIL}}{t}$	$\frac{0.104 P_{FAIL}}{t}$
	Poisson's ratio ν	$\frac{3.59}{DR} - 0.27$	
	Modulus of elasticity E , psi	$\frac{S_H}{t} (0.27 + \nu)$	
Repeated-Load	Instantaneous resilient Poisson's ratio ν_{RI}	$3.59 \left(\frac{H_I}{V_I} \right) - 0.27$	
	Instantaneous resilient modulus of elasticity E_{RI} , psi	$\frac{P}{H_I t} (0.27 + \nu_{RI})$	

P_{FAIL} = failure load (Fig 13), pounds

P = repeated-load (Fig 3), pounds

t = height or thickness of specimen, inches

DR = deformation ratio $\frac{\Delta V_D}{\Delta H_D}$ (the slope of the relationship between vertical deformation V_D and the corresponding horizontal deformation H_D for the approximately linear portion of the relationship, Fig 13. The linear relationship can be established by eye or by regression using the method of least squares. The first 50 to 75 percent of the curve should be used.)

S_H = horizontal tangent modulus $\frac{\Delta P}{\Delta H_D}$ (the slope of the relationship between load P and horizontal deformation H_D)

H_I, V_I = instantaneous resilient horizontal and vertical deformations (Fig 3), respectively, inches

*0.5-inch-wide loading strip

**0.75-inch-wide loading strip

Fatigue Characteristics

Fatigue life. Fatigue life (N_f) is the number of repeated load applications required to completely fracture or fail the specimen and is stress- or strain-dependent. Tests can be conducted using a controlled stress (load) or controlled strain (deformation) mode of testing. In a controlled stress test, which is the most common, the repeated applied load or stress is kept constant until failure occurs. If the deformation or strain level is kept constant, the test is called a controlled strain test. Controlled stress loadings are more applicable to the evaluation of mixtures in thick pavement sections, while controlled strain tests are more applicable to thin pavement layers.

Fatigue Constants. Since fatigue life is stress- or strain-dependent, the fatigue characteristics are generally expressed in terms of the relationship between the logarithm of fatigue life and either the logarithm of stress or stress difference (Eqs 2.1 or 2.2) or the logarithm of strain (Eq 2.3). Since these relationships are generally considered to be linear, the fatigue life over a range of stresses or strains can be defined in terms of the slope and intercept constants of these linear relationships (Fig 5), i.e., n_2 , K_2 or K_2' and n_1 , K_1 as defined below for Equations 2.1, 2.2 and 2.3:

$$N_f = K_2 \left(\frac{1}{\sigma_\tau} \right)^{n_2} \quad (2.1)$$

where N_f = fatigue life,
 σ_τ = applied tensile stress,
 n_2 = slope of the logarithmic relationship between fatigue life and tensile stress, and
 K_2 = antilog of the intercept of the logarithmic relationship between fatigue life and tensile stress.

$$N_f = K_2' \left(\frac{1}{\Delta\sigma} \right)^{n_2} \quad (2.2)$$

where $\Delta\sigma$ = stress difference, and
 K_2' = the antilog of the intercept value of the logarithmic relationship between fatigue life and stress difference.

$$N_f = K_1 \left(\frac{1}{\epsilon_i} \right)^{n_1} \quad (2.3)$$

where ϵ_i = initial tensile strain,
 n_1 = slope of the logarithmic relationship between fatigue life and initial strain, and
 K_1 = antilog of the intercept of the logarithmic relationship between fatigue life and tensile strain.

Permanent Deformation Characteristics

Permanent strain. Permanent strain for any given number of load applications can be calculated using the equations for tensile strain or compressive strain in Tables A1 or A3, Appendix A.

GNU, ALPHA. Values of GNU and ALPHA for use in the VESYS Pavement Program can also be estimated although additional work is still required on the basic concept of GNU and ALPHA and the methods of estimating. Discussion of these two parameters is contained in Chapter 2 and they are defined in Equations 2.4, 2.5, and 2.6.

STATIC INDIRECT TENSILE TEST

Equipment

Loading machine. The loading machine should be capable of applying a compressive load at a uniform deformation rate of 2 in. (50 mm) per minute. Any good mechanical screw jack system, electro-hydraulic system, or hydraulic system having sufficient capacity may be used.

Loading head. The loading head must have guided upper and lower steel loading strips with concave surfaces having a radius of curvature equal to the nominal radius of the test specimen. Specimens will normally be either a nominal 4 in. (102 mm) or 6 in. (152 mm) diameter. The loading strips should be either 0.50 in. (13 mm) or 0.75 in. (19 mm) wide for 4 in. or 6 in. diameter specimens, respectively. Loading strip edges should be rounded to remove the sharp edge in order not to cut the sample during testing.

It is important to maintain parallel and vertically aligned load strips during testing. This can be achieved by a variety of methods. In addition, the preload on the specimens should be as small as possible to reduce creep deformations prior to actual testing. From the standpoint of effectiveness and cost, a Marshall test head may be easily modified to accommodate a 4 in. diameter specimen (Figs 9 and 10). Also, commercially available die sets may be altered to fit a 4-in. or 6 in. diameter specimen (Fig 11). It is recommended that a modified Marshall test head be used in order to minimize the dead load on the specimen, which is especially important at the higher test temperatures.

Measurement Systems. The capability of measuring load, and horizontal and vertical deformations, are required to determine tensile strength, Poisson's ratio, and modulus of elasticity. If tensile strength is the only parameter desired, then the applied load is the only measurement required.

1. Load - the applied compressive load should be measured by either an electronic load cell, pressure transducer, or hydraulic gauges with sufficient capacity to indicate the failure load. However, it is imperative that the system have sufficient resolution and accuracy to obtain meaningful load measurements.
2. Deformations - vertical deformations can be measured using linear variable differential transducers (LVDT's) or other suitable devices. If an assumed value for Poisson's ratio is used, the vertical deformation need not be measured. Horizontal deformations can be measured using a "C" type strain gage device in contact about the mid-point of the specimen. Such a device is illustrated in Figure 12. Details concerned with fabricating such a device may be obtained in Research Report 98-10 (Ref 14).
3. Recorders - two X-4 plotters are required to continuously record load and vertical and horizontal deformations. Care should be exercised in selecting recorders with adequate response times.

Test Procedures

1. Record thickness and diameter of test specimens. Thickness ideally should be equal to at least one-half of the diameter;



Fig 9. Modified Marshall test head.



Fig 10. Modified Marshall test head with specimen.

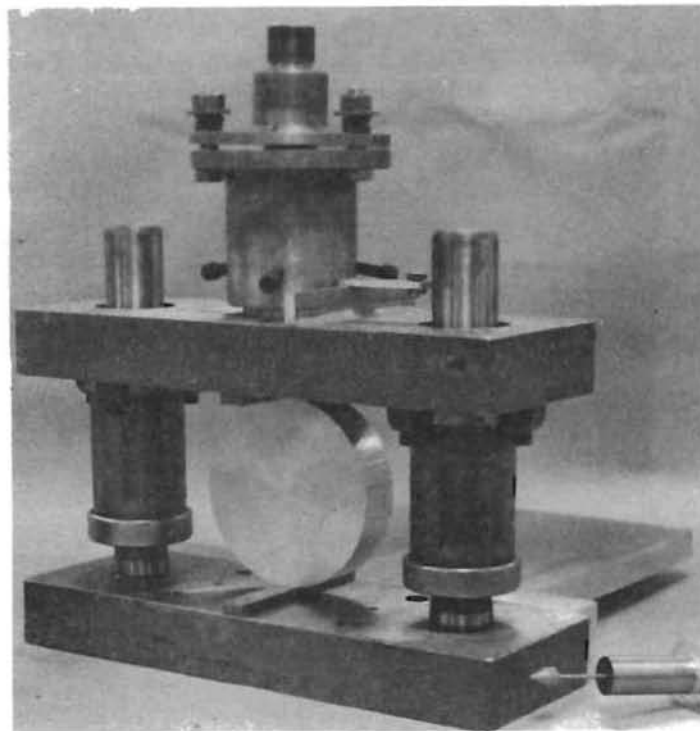


Fig 11. All steel precision die set with loading strips and specimen in place.

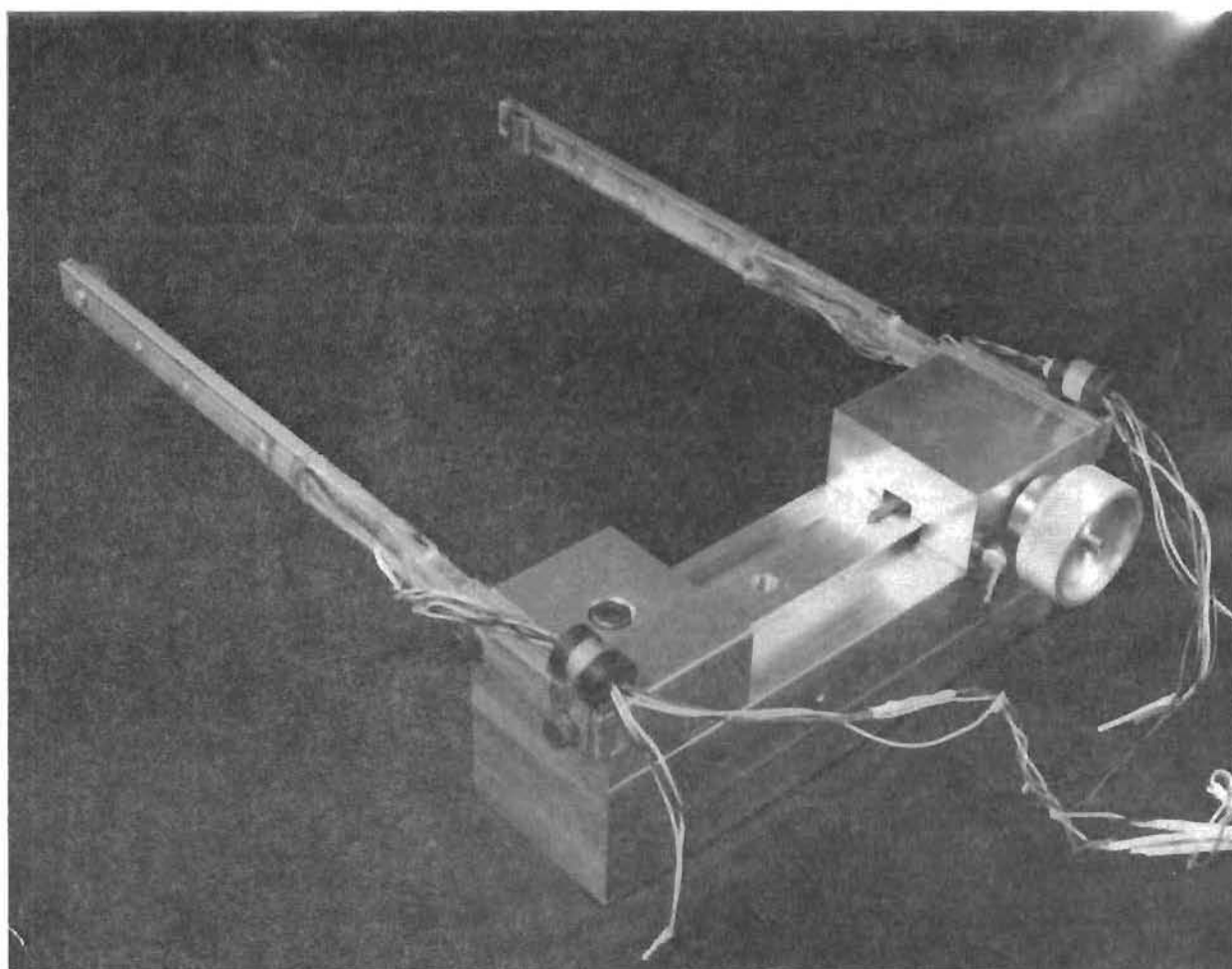


Fig 12. Horizontal deformation device.

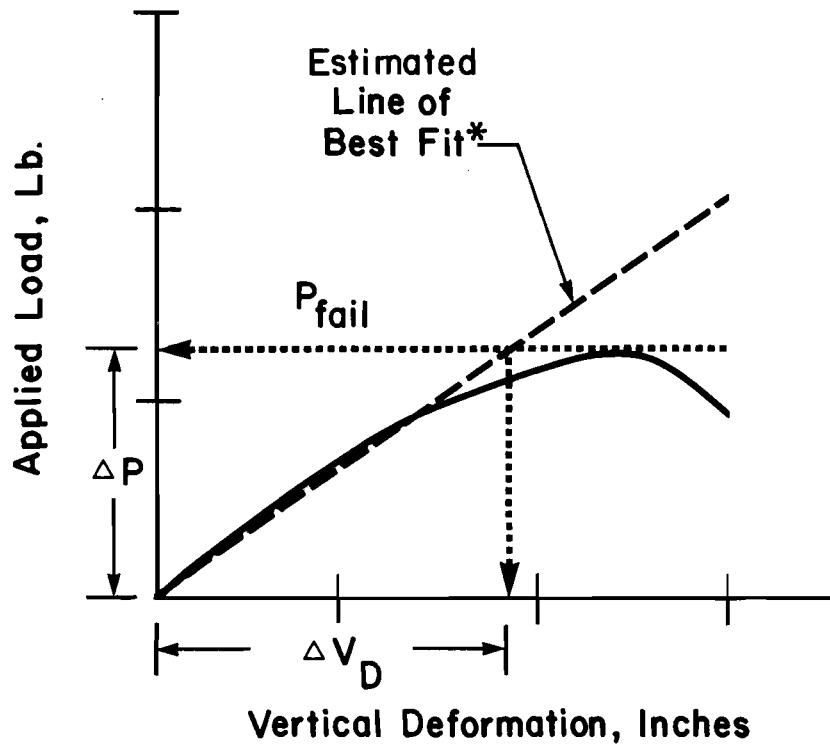
however, for cores tests can be conducted on 1.5 and 2 inch thick specimens with 4-inch and 6-inch diameters, respectively. A minimum of three test specimens should be used for any given condition.

2. Test specimens should be placed in a controlled temperature environment for a minimum of three hours prior to testing. Suggested test temperatures are 41, 77, and $104 \pm 2^\circ\text{F}$ (5, 25, and 40°C).
3. Center specimen in loading head containing loading strips ensuring that the specimen is properly aligned.
4. Apply a minimum preload to ensure the load strips are seated and to prevent impact load. The preload should be as small as possible and definitely should not exceed 20 pounds.
5. Position horizontal deformation measuring device.
6. Adjust and balance all measuring devices as required.
7. Apply a compressive load at the rate of 2 in. per minute head travel.
8. Continuously record load and horizontal and vertical deformations with a continuous recording device. Typical load-deformation curves are illustrated in Figure 13.
9. To determine tensile properties use equations in Table 2, as discussed under Calculations and Equations, page 21.

REPEATED-LOAD INDIRECT TENSILE TEST

Equipment

Loading Machine. This equipment should be capable of applying a compressive load over a range of frequencies, load durations, and load levels. An electro-hydraulic machine with a function generator has been used for this test. Other commercially available or laboratory constructed loading machines such as those using pneumatic repeated-loading can also be used. However, it must be recognized that some of this equipment may be limited in its capability to handle large loads at the colder test temperatures.



*Determined by eye or method of least squares

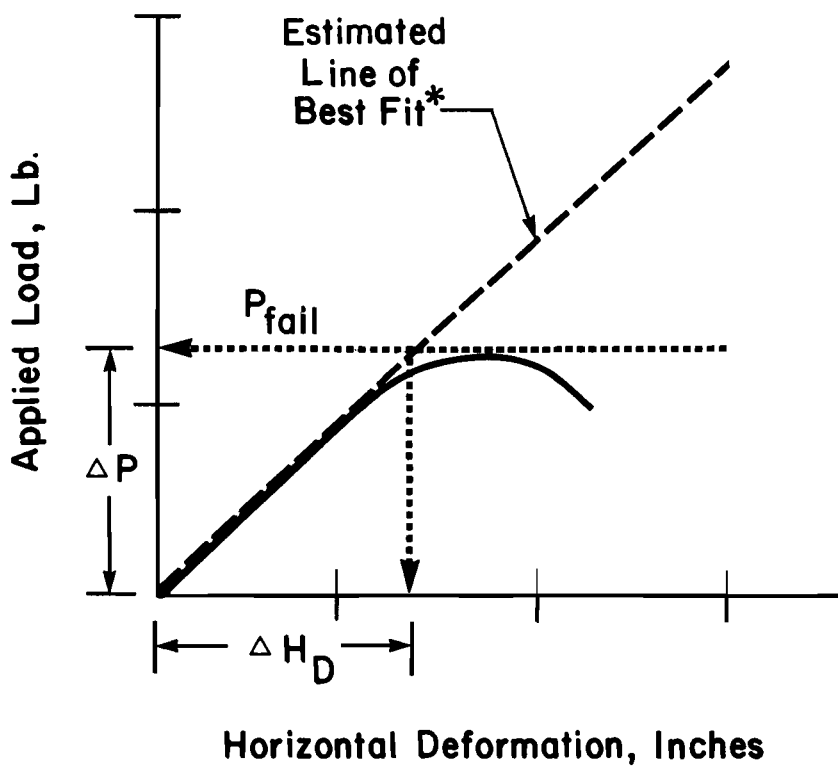


Fig 13. Load-horizontal deformation curve.

Loading Head. Same as used in static tests except that heavy loading heads such as commercially available die sets should be avoided if possible.

Measurement System. The capability of measuring load, and horizontal and vertical deformations, is required to measure resilient Poisson's ratio and resilient modulus of elasticity. If an assumed value is used for Poisson's ratio, vertical deformation measurements may be omitted.

1. Load - the applied compressive load should be set using a load cell, air or hydraulic pressure gauge, or a pressure transducer. The ability to monitor the minimum and maximum stress is extremely important.
2. Deformations - the vertical and horizontal deformations can be measured using linear variable differential transducers (LVDT's) or other suitable devices. A sensitivity of 0.00001 inch (0.00025 mm) is required for the horizontal deformation. A positive contact by spring loading or gluing attachments to the specimen should be provided to ensure direct contact between the specimen and the measuring device.

Transducers such as Trans-Tex Model 350-000 LVDT's and Statham UC-3 have been found to be satisfactory for this purpose. The gauges should be wired to preclude the effects of eccentric loading so as to give the algebraic sum of the movement of each side of the specimen. Alternatively, each gauge can be read independently and the results summed for use in the calculations. Figure 14 illustrates a typical horizontal deformation measurement system.

3. Recorders - Any recorder may be used that has a response frequency of at least 1.0 Hz.

Test Procedures

1. Record thickness and diameter of test specimens. The specimen thickness should be equal to at least one-half of the diameter. A minimum of three test specimens should be used for any given condition.

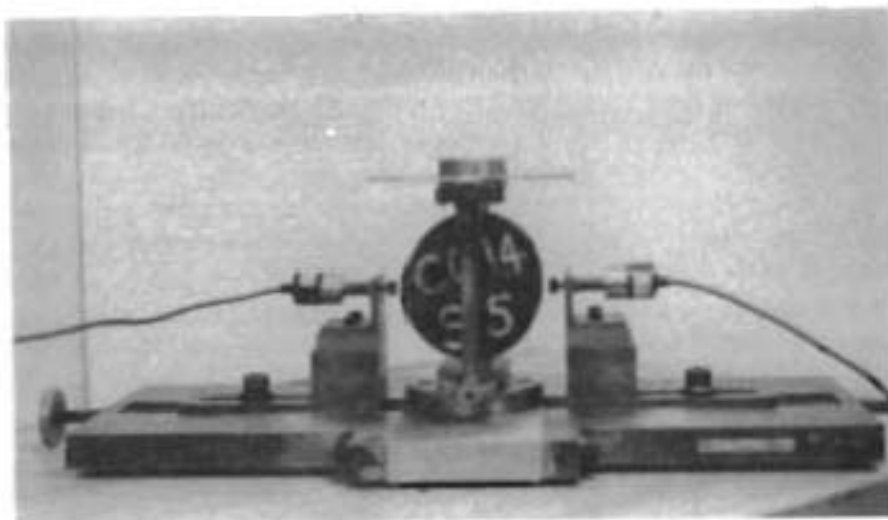


Fig 14. Horizontal measuring device for repeated-load indirect tensile test.

2. Test specimens should be placed in a controlled temperature environment for a minimum of three hours prior to testing. Suggested test temperatures are 41, 77, and $104 \pm 2^\circ\text{F}$ (5, 25, and 40°C).
3. Center specimen in loading head containing loading strips taking care to ensure that the specimen is properly aligned.
4. Apply a minimal preload to ensure the specimen is seated and to prevent impact loading. This preload should be as small as possible.
5. Bring horizontal deformation transducers into contact with specimen.
6. Adjust and balance all measuring devices as required.
7. Apply a repeated haversine or other suitable waveform load at a preselected frequency of 0.33, 0.50, or 1.0 Hz for a minimum period sufficient to obtain uniform deformation readings. Generally, a minimum of 50 to 200 load repetitions is sufficient. Recommended load range is between 10 to 50 percent of the tensile strength as determined in the static indirect tensile test procedures. In lieu of tensile strength data, load ranges from 25 to 200 lb per inch of specimen thickness can be used.

It should be recognized that load duration is a very important variable and will greatly affect values of modulus of elasticity and fatigue life. It is recommended that the frequency be at 1 Hz and that a load duration of 0.4 sec be used.

8. Monitor the horizontal and, if measured, the vertical deformations during the test. A typical load pulse-deformation trace is shown in Figure 3.
9. Measure the average instantaneous resilient horizontal and vertical deformations over at least three consecutive load cycles (Fig 3) after the repeated resilient deformations have stabilized. Again, vertical deformation measurements may be omitted if a value for Poisson's ratio is to be assumed.
10. Determine the resilient properties using the equations in Table 2 as discussed under Calculations and Equations, page 21.

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APPENDIX A

EQUATIONS AND COEFFICIENTS
FOR VARIABLE SPECIMEN DIAMETERS

TABLE A.1. EQUATIONS FOR CALCULATING TENSILE PROPERTIES
IN THE ENGLISH SYSTEM

Static Properties	
(1) Tensile strength S_T , psi	$= \frac{P_{Fail}}{t} \cdot K_1$
(2) Poisson's ratio ν	$= \frac{DR \cdot K_2 + K_6}{DR \cdot K_3}$
(3) Modulus of elasticity E , psi	$= \frac{S_H}{t} (0.27 + \nu)$
(4) Tensile strain ϵ_T	$= H_D \left[\frac{K_4 - \nu \cdot K_5}{K_2 - \nu \cdot K_3} \right]$
(5) Compressive strain ϵ_C	$= V_D \left[\frac{K_5 - \nu \cdot K_4}{K_6 - \nu \cdot K_7} \right]$
Repeated-Load Properties	
(6) Instantaneous resilient Poisson's ratio ν_{RI}	$= \frac{\frac{V_I}{H_I} K_2 + K_6}{\frac{V_I}{H_I} K_3}$
(7) Instantaneous resilient modulus of elasticity E_{RI} , psi	$= \frac{P}{H_I t} (0.27 + \nu_{RI})$
(8) Total resilient Poisson's ratio ν_{RT}	$= \frac{\frac{V_T}{H_T} K_2 + K_6}{\frac{V_T}{H_T} K_3}$
(9) Total resilient modulus of elasticity E_{RT} , psi	$= \frac{P}{H_T t} (0.27 + \nu_{RT})$

(continued)

TABLE A.1. (Continued)

P_{Fail}	=	failure load (Fig 13), pounds
P	=	repeated-load (Fig 3), pounds
t	=	height of specimen, inches
H_D, V_D	=	total horizontal and vertical deformations, respectively, inches
DR	=	deformation ratio $\frac{\Delta V_D}{\Delta H_D}$ (the slope of the relationship between vertical deformation V_D and the corresponding horizontal deformation H_D for the approximately linear portion of the relationship, Fig 13)
S_H	=	horizontal tangent modulus $\frac{\Delta P}{\Delta H_D}$ (the slope of the relationship between load P and horizontal deformation H_D , Fig 13)
H_I, V_I	=	instantaneous resilient horizontal and vertical deformations (Fig 3), respectively, inches
H_T, V_T	=	total resilient horizontal and vertical deformations (Fig 3), respectively, inches
$K_1, K_2, K_3, K_4, K_5, K_6, K_7$	=	constants (see Tables A.2 and A.3)

TABLE A.2. CONSTANTS FOR EQUATIONS* FOR
INDIRECT TENSILE PROPERTIES
FOR 0.5-INCH LOADING STRIP

Diameter, inches	K ₁	K ₂	K ₃	K ₄	K ₅	K ₆	K ₇
3.5	.178	.077	-.286	.051	-.155	-.979	-.020
3.6	.172	.075	-.278	.048	-.147	-.961	-.019
3.7	.168	.073	-.270	.046	-.139	-.944	-.018
3.8	.164	.071	-.263	.043	-.132	-.928	-.017
3.9	.160	.069	-.256	.041	-.125	-.912	-.016
4.0	.156	.068	-.250	.039	-.119	-.897	-.016
4.1	.153	.066	-.244	.037	-.113	-.882	-.015
4.2	.149	.064	-.238	.036	-.108	-.868	-.014
4.3	.146	.063	-.233	.034	-.103	-.855	-.014
4.4	.142	.062	-.227	.032	-.098	-.842	-.013
4.5	.139	.060	-.222	.031	-.094	-.829	-.012
4.6	.136	.059	-.217	.023	-.090	-.817	-.012
4.7	.134	.058	-.213	.028	-.086	-.805	-.011
4.8	.131	.056	-.208	.027	-.083	-.794	-.011
4.9	.128	.055	-.204	.026	-.079	-.783	-.010
5.0	.126	.054	-.200	.025	-.076	-.772	-.010
5.1	.123	.053	-.196	.024	-.073	-.762	-.010
5.2	.121	.052	-.192	.023	-.070	-.752	-.009
5.3	.119	.051	-.189	.022	-.068	-.742	-.009
5.4	.117	.050	-.185	.022	-.065	-.733	-.009
5.5	.115	.049	-.182	.021	-.063	-.723	-.008
5.6	.113	.048	-.179	.020	-.061	-.714	-.008
5.7	.111	.048	-.175	.019	-.057	-.697	-.007
5.8	.109	.047	-.172	.019	-.057	-.697	-.007
5.9	.107	.046	-.170	.018	-.055	-.689	-.007
6.0	.105	.045	-.166	.018	-.053	-.681	-.007
6.1	.104	.045	-.164	.017	-.051	-.673	-.007
6.2	.102	.044	-.161	.016	-.050	-.666	-.006
6.3	.100	.043	-.159	.016	-.048	-.658	-.006
6.4	.099	.042	-.156	.015	-.047	-.651	-.006
6.5	.097	.042	-.154	.015	-.045	-.644	-.006

*Equations given in Table A.1, page 37

TABLE A.3. CONSTANTS FOR EQUATIONS* FOR
INDIRECT TENSILE PROPERTIES
FOR 0.75-INCH LOADING STRIP

Diameter, inches	K_1	K_2	K_3	K_4	K_5	K_6	K_7
3.5	.172	.075	-.286	.492	-.154	-.839	-.031
3.6	.168	.073	-.278	.466	-.146	-.825	-.029
3.7	.164	.071	-.270	.442	-.138	-.812	-.028
3.8	.160	.070	-.263	.421	-.131	-.799	-.026
3.9	.156	.068	-.256	.400	-.124	-.786	-.025
4.0	.153	.066	-.250	.381	-.118	-.774	-.024
4.1	.149	.065	-.244	.364	-.112	-.763	-.022
4.2	.146	.063	-.238	.347	-.107	-.751	-.021
4.3	.143	.062	-.233	.332	-.102	-.740	-.020
4.4	.140	.061	-.227	.318	-.098	-.730	-.019
4.5	.137	.059	-.222	.304	-.094	-.720	-.019
4.6	.134	.058	-.217	.292	-.090	-.710	-.018
4.7	.131	.057	-.213	.280	-.086	-.700	-.017
4.8	.129	.056	-.208	.268	-.082	-.691	-.016
4.9	.126	.055	-.204	.258	-.079	-.682	-.016
5.0	.124	.054	-.200	.248	-.076	-.673	-.015
5.1	.122	.053	-.196	.238	-.073	-.665	-.014
5.2	.119	.052	-.192	.230	-.070	-.656	-.014
5.3	.117	.051	-.189	.221	-.068	-.648	-.013
5.4	.115	.050	-.185	.213	-.065	-.641	-.013
5.5	.113	.049	-.182	.206	-.063	-.633	-.012
5.6	.111	.048	-.179	.199	-.061	-.626	-.012
5.7	.109	.047	-.175	.192	-.058	-.618	-.012
5.8	.108	.046	-.172	.186	-.057	-.612	-.011
5.9	.106	.046	-.170	.179	-.055	-.605	-.011
6.0	.104	.045	-.167	.174	-.053	-.598	-.010
6.1	.102	.044	-.164	.168	-.051	-.592	-.010
6.2	.101	.044	-.161	.163	-.049	-.585	-.010
6.3	.099	.043	-.159	.158	-.048	-.579	-.009
6.4	.098	.042	-.156	.153	-.046	-.573	-.009
6.5	.096	.042	-.154	.148	-.045	-.567	-.009

*Equation given in Table A.1, page 37

TABLE A.4. EQUATIONS FOR CALCULATING TENSILE PROPERTIES
IN THE METRIC OR SI SYSTEM

Static Properties	
(1) Tensile strength S_T , MPa	$= \frac{P_{Fail}}{t} \cdot D_1$
(2) Poisson's ratio ν	$= \frac{DR \cdot D_2 + D_6}{DR \cdot D_3}$
(3) Modulus of elasticity E , MPa	$= \frac{S_H}{t} (0.27 + \nu)$
(4) Tensile strain ϵ_T	$= H_D \left[\frac{D_4 - \nu \cdot D_5}{D_2 - \nu \cdot D_3} \right]$
(5) Compressive strain ϵ_C	$= V_D \left[\frac{D_5 - \nu \cdot D_4}{D_6 - \nu \cdot D_7} \right]$
Repeated-Load Properties	
(6) Instantaneous resilient Poisson's ratio ν_{RI}	$= \frac{\frac{V_I}{H_I} D_2 + D_6}{\frac{V_I}{H_I} D_3}$
(7) Instantaneous resilient modulus of elasticity E_{RI} , MPa	$= \frac{P}{H_I t} (0.27 + \nu_{RI})$
(8) Total resilient Poisson's ratio ν_{RT}	$= \frac{\frac{V_T}{H_T} K_2 + K_6}{\frac{V_T}{H_T} K_3}$
(9) Total resilient modulus of elasticity E_{RT} , MPa	$= \frac{P}{H_T t} (0.27 + \nu_{RT})$

(continued)

TABLE A.4. (Continued)

P_{Fail}	=	failure load (Fig 13), Newtons
P	=	repeated-load (Fig 3), Newtons
t	=	height of specimen, mm
H_D, V_D	=	horizontal and vertical deformations, respectively, mm
DR	=	deformation ratio $\frac{\Delta V_D}{\Delta H_D}$ (the slope of the relationship between vertical deformation V_D and the corresponding horizontal deformation H_D for the approximately linear portion of the relationship, Fig 13)
S_H	=	horizontal tangent modulus $\frac{\Delta P}{\Delta H_D}$ (the slope of the relationship between load P and horizontal deformation H_D , Fig 13)
H_I, V_I	=	instantaneous resilient horizontal and vertical deformations (Fig 3), respectively, mm
H_T, V_T	=	total resilient horizontal and vertical deformations (Fig 3), respectively, mm
$D_1, D_2, D_3, D_4, D_5, D_6, D_7$	=	constants (see Tables A.5 and A.6)

TABLE A.5. CONSTANTS FOR EQUATIONS* FOR
INDIRECT TENSILE PROPERTIES
FOR 0.5-INCH LOADING STRIP

Diameter, mm	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
90	.0069	.0030	-.0111	.000077	-.00023	-.038	-.00079
92	.0068	.0030	-.0109	.000074	-.00022	-.038	-.00075
94	.0066	.0029	-.0106	.000071	-.00022	-.037	-.00072
96	.0065	.0028	-.0104	.000068	-.00021	-.037	-.00069
98	.0064	.0028	-.0102	.000065	-.00020	-.036	-.00066
100	.0062	.0027	-.0100	.000062	-.00019	-.036	-.00064
102	.0061	.0026	-.0098	.000060	-.00018	-.035	-.00061
104	.0060	.0026	-.0096	.000058	-.00018	-.035	-.00059
106	.0059	.0025	-.0094	.000056	-.00017	-.034	-.00057
108	.0058	.0025	-.0093	.000054	-.00016	-.034	-.00055
110	.0057	.0025	-.0091	.000052	-.00016	-.034	-.00053
112	.0056	.0024	-.0089	.000050	-.00015	-.033	-.00051
114	.0055	.0024	-.0088	.000048	-.00015	-.033	-.00049
116	.0054	.0023	-.0086	.000047	-.00014	-.032	-.00047
118	.0053	.0023	-.0085	.000045	-.00014	-.032	-.00046
120	.0052	.0023	-.0083	.000044	-.00013	-.032	-.00044
122	.0052	.0022	-.0082	.000042	-.00013	-.031	-.00043
124	.0051	.0022	-.0081	.000041	-.00012	-.031	-.00041
126	.0050	.0022	-.0079	.000040	-.00012	-.031	-.00040
128	.0049	.0021	-.0078	.000038	-.00012	-.030	-.00039
130	.0048	.0021	-.0077	.000037	-.00011	-.030	-.00038
132	.0048	.0021	-.0076	.000036	-.00011	-.030	-.00037
134	.0047	.0020	-.0075	.000035	-.00011	-.029	-.00035
136	.0046	.0020	-.0074	.000034	-.00010	-.029	-.00034
138	.0046	.0020	-.0072	.000033	-.00010	-.029	-.00033
140	.0045	.0019	-.0071	.000032	-.00010	-.028	-.00032
142	.0044	.0019	-.0070	.000031	-.00009	-.028	-.00032
144	.0044	.0019	-.0069	.000030	-.00009	-.028	-.00031
146	.0043	.0019	-.0068	.000030	-.00009	-.028	-.00030
148	.0043	.0018	-.0068	.000029	-.00009	-.027	-.00029
150	.0042	.0018	-.0067	.000028	-.00008	-.027	-.00028
152	.0042	.0018	-.0066	.000027	-.00008	-.027	-.00028
154	.0041	.0018	-.0065	.000027	-.00008	-.027	-.00027
156	.0040	.0017	-.0064	.000026	-.00008	-.026	-.00026
158	.0040	.0017	-.0063	.000025	-.00008	-.026	-.00025
160	.0039	.0017	-.0063	.000025	-.00007	-.026	-.00025

*Equations given in Table A.4, page 40

TABLE A.6. CONSTANTS FOR EQUATIONS* FOR
INDIRECT TENSILE PROPERTIES
FOR 19.05-MM LOADING STRIP

Diameter, mm	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
90	.0067	.0029	-.0111	.000074	-.00023	-.033	-.00119
92	.0066	.0029	-.0109	.000071	-.00022	-.032	-.00114
94	.0064	.0028	-.0106	.000068	-.00021	-.032	-.00109
96	.0063	.0027	-.0104	.000066	-.00020	-.032	-.00104
98	.0062	.0027	-.0102	.000063	-.00020	-.031	-.00100
100	.0061	.0026	-.0100	.000061	-.00019	-.031	-.00096
102	.0060	.0026	-.0098	.000059	-.00018	-.030	-.00092
104	.0059	.0025	-.0096	.000056	-.00017	-.030	-.00089
106	.0058	.0025	-.0094	.000054	-.00017	-.030	-.00085
108	.0057	.0025	-.0092	.000052	-.00016	-.029	-.00082
110	.0056	.0024	-.0091	.000051	-.00016	-.029	-.00079
112	.0055	.0024	-.0089	.000049	-.00015	-.029	-.00076
114	.0054	.0023	-.0088	.000047	-.00014	-.028	-.00074
116	.0053	.0023	-.0086	.000046	-.00014	-.028	-.00071
118	.0052	.0023	-.0085	.000044	-.00014	-.028	-.00069
120	.0051	.0022	-.0083	.000043	-.00013	-.027	-.00066
122	.0051	.0022	-.0082	.000042	-.00013	-.027	-.00064
124	.0050	.0022	-.0081	.000040	-.00012	-.027	-.00062
126	.0049	.0021	-.0079	.000039	-.00012	-.027	-.00060
128	.0048	.0021	-.0078	.000038	-.00012	-.026	-.00058
130	.0048	.0021	-.0077	.000037	-.00011	-.026	-.00057
132	.0047	.0020	-.0076	.000036	-.00011	-.026	-.00055
134	.0046	.0020	-.0075	.000035	-.00010	-.026	-.00053
136	.0046	.0020	-.0074	.000034	-.00010	-.025	-.00052
138	.0045	.0019	-.0072	.000033	-.00010	-.025	-.00050
140	.0044	.0019	-.0071	.000032	-.00010	-.025	-.00049
142	.0044	.0019	-.0070	.000031	-.00009	-.025	-.00047
144	.0043	.0019	-.0069	.000030	-.00009	-.024	-.00046
146	.0043	.0018	-.0068	.000029	-.00009	-.024	-.00045
148	.0042	.0018	-.0068	.000028	-.00009	-.024	-.00044
150	.0042	.0018	-.0067	.000028	-.00008	-.024	-.00042
152	.0041	.0018	-.0066	.000027	-.00008	-.024	-.00041
154	.0041	.0017	-.0065	.000026	-.00008	-.023	-.00040
156	.0040	.0017	-.0064	.000026	-.00008	-.023	-.00039
158	.0040	.0017	-.0063	.000025	-.00008	-.023	-.00038
160	.0039	.0017	-.0062	.000024	-.00007	-.023	-.00037
162	.0039	.0017	-.0062	.000024	-.00007	-.023	-.00036

*Equations given in Table A.4, page 40