

Southwest Region University Transportation Center

**Energy Conservation Through Enhanced
Traffic Signal Responsiveness**

SWUTC/95/60063-1



**Center for Transportation Research
University of Texas at Austin
3208 Red River, Suite 200
Austin, Texas 78705-2650**

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16. Abstract <p>Traditional traffic system management objectives are based on operational efficiency, including capacity, delay reduction, and safety. Generally, criteria for evaluating the effectiveness of signalized intersections are: minimization of total or stopped delay and numbers of stops, minimizing fuel consumption, cost-efficiency, and trade-offs of these factors.</p> <p>Fuel consumption is an important traffic control criterion. A new fuel consumption model called the Analytical Fuel Consumption Model is proposed in this research based on queuing model concepts and different vehicle operational states. The model, aiming to include the impact of traffic characteristics, fuel consumption rates, and control variables, includes different vehicle operational states describing operations on three intersection elements: inbound approach, intersection itself, and outbound approach. For each element, vehicle operational states are described in three signal cycle stages.</p> <p>Numerical experiments are conducted to calibrate fuel consumption rates of the new model for different traffic volumes and cycle lengths. Results show consistency with those of the TEXAS simulation model.</p> <p>Results for both fuel consumption and delay minimization show that short cycle time lengths are preferred in low volume cases, and likewise, long cycle lengths are preferred in high volume cases.</p>					
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**ENERGY CONSERVATION THROUGH ENHANCED
TRAFFIC SIGNAL RESPONSIVENESS**

by

Tsai-Yun Liao
Randy Machemehl

Research Report SWUTC 95/60063

Southwest Region University Transportation Center
Center for Transportation Research
The University of Texas
Austin, Texas 78712

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EXECUTIVE SUMMARY

Within urban street networks, intersections create most vehicular stops, queues, and delays and limit maximum possible flows. As such, intersections typically are fuel consumption "hot spots". Conventional methods of designing intersection traffic control minimize vehicular delay or maximize flow but rarely consider effects upon vehicular fuel consumption.

This study constitutes a first major installment in development of a fuel consumption based, intersection traffic control optimization technique. Specifically, a limited version of an at-grade intersection signal timing optimization procedure has been developed. This procedure will estimate basic signal timing parameters which minimize vehicular fuel consumption within an intersection influence area. Models describing vehicular fuel consumption for inbound and outbound intersection approaches as well as the intersection itself are provided. Calibration and testing of the models has utilized NETSIM and TEXAS microsimulation models.

Comparisons of the fuel consumption based optimization with conventional delay minimization techniques indicate significant differences. Work currently underway, beyond the scope of the original study, will attempt to generalize the developed modeling procedures and remove several limitations.

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ABSTRACT

Traditional traffic system management objectives are based on operational efficiency, including capacity, delay reduction, and safety. Generally, criteria for evaluating the effectiveness of signalized intersections are: (1) minimization of total or stopped delay, (2) reduction of numbers of stops (3) minimizing a combination of delay and numbers of stops, (4) minimizing fuel consumption, (5) cost-efficiency, and (6) trade-offs of these factors.

Fuel consumption is an important traffic control criterion. A new fuel consumption model called the Analytical Fuel Consumption Model is proposed in this research based on queueing model concepts and different vehicle operational states. The model, aiming to include the impact of traffic characteristics, fuel consumption rates, and control variables, includes different vehicle operational states describing operations on three intersection elements: inbound approach, intersection itself, and outbound approach. For each element, vehicle operational states are described in three signal cycle stages, namely, effective red time, time from green onset to time t_0 , during which vehicles pass the stop line at saturation flow rate, and time from t_0 to the effective green end.

Numerical experiments are conducted to calibrate fuel consumption rates of the new model for different traffic volumes and cycle lengths. Results show consistency with those of the TEXAS simulation model. Fuel consumption increases in the effective red time on the inbound approach, increases dramatically in t_0 while vehicles are accelerating from a stopped condition, decreases at the end of t_0 , and remains stable when vehicles travel on the outbound approach.

For fuel consumption minimization, optimal cycle lengths for the low, medium, and high traffic volume cases are 50, 80, and 100 seconds from the Analytical Fuel Consumption Model compared to 40, 60, and 120 seconds for delay minimization. However, results for both fuel consumption and delay minimization show that short cycle lengths are preferred in low volume cases and likewise, long cycle lengths are preferred in high volume cases.

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CHAPTER 1 INTRODUCTION

MOTIVATION

Fuel consumed by ground transport vehicles represents more than 75% of all transportation energy use. The problem of fuel consumption by automobiles and trucks in urban networks has received increasing attention recently because of both energy conservation and environmental issues.

Traditional traffic system management objectives are based upon operational efficiency, including capacity, delay reduction, and safety. Generally, criteria for evaluating the effectiveness of signalized intersections are: (1) minimization of total or stopped delay, (2) reduction of numbers of stops (3) minimizing a combination of delay and numbers of stops, (4) minimizing fuel consumption, (5) cost-efficiency, and (6) trade-offs of these factors.

Fuel consumption is an important traffic control criterion. In recent years, more than 150 million vehicles consume about 75 billion gallons of gasoline per year in the United States. A number of studies have tackled the problem of vehicle fuel consumption in urban traffic systems and produced approaches to evaluate fuel economy and predict fuel consumption based on different vehicle types, vehicle engines, roadway geometric conditions, and traffic situations.

This research develops a comprehensive model to estimate fuel consumption at signalized intersections. Most fuel consumption models consider overall travel conditions, however, the model in this research specifically considers vehicle fuel consumption at a signalized intersection where the intersection causes vehicles to slow, stop, and accelerate consuming excess fuel.

This model, called the Analytical Fuel Consumption Model, includes the intersection and street sections up to 600 ft from the intersection. These elements are called inbound approaches (600 ft prior to the intersection), outbound approaches (600 ft after the

intersection) and the connection of inbound and outbound approaches is called the intersection itself.

Fuel consumption for vehicles within these intersection elements is estimated using different sub-models in the Analytical Fuel Consumption Model and based on vehicle volume, cycle time, effective red time, effective green time, vehicle speed, and vehicle travel time. In order to calibrate these models, fuel consumption during a full signal cycle has been separated into three stages: fuel consumption during the effective red time, fuel consumption during queue departure after the red signal changes to green (called t_0), and fuel consumption during the effective green time minus the t_0 time.

OBJECTIVES

The aims of the study are to develop a model to estimate fuel consumption at an isolated intersection, and to analyze the relationship between signal cycle length and fuel consumption. The specific objectives are as follows:

1. To analyze the main factors associated with fuel consumption and develop a suitable model to estimate fuel consumption at an isolated intersection.
2. To compare the fuel consumption model results with those of the TEXAS simulation model and verify the effects of fuel consumption at different intersection elements namely the inbound approach, the intersection itself, and the outbound approach.
3. To test the validity and reliability of this fuel consumption estimation method using a set of experimental design data and compare the results with other models.
4. To draw conclusions about the fuel consumption development and propose recommendations for future research.

STUDY OVERVIEW

Traditional criteria for evaluating the effectiveness of signalized intersections are: (1) minimization of total or stopped delay, (2) reduction of numbers of stops (3)

minimizing a combination of delay and numbers of stops, (4) minimizing fuel consumption, (5) cost-efficiency, and (6) trade-offs of these factors.

Fuel consumption is an important traffic control criterion. A new fuel consumption model called the Analytical Fuel Consumption Model is proposed in this research based on queueing model concepts and different vehicle operational states. The model, aiming to include the impact of traffic characteristics, fuel consumption rates, and control variables, includes different vehicle operational states describing operations on three intersection elements: inbound approach, intersection itself, and outbound approach. For each element, vehicle operational states are described in three signal cycle stages, namely, effective red time, time from green onset to time t_0 , during which vehicles accelerate from a stopped condition, and time from t_0 to the effective green.

An experimental design is setup to calibrate model fuel consumption parameters and analyze the new model. Thus, fuel consumption characteristics and the relationship between fuel consumption minimization and delay minimization can be investigated.

In this report, the motivation and objectives are described in Chapter 1. Chapter 2 reviews different fuel consumption models based on a model hierarchy proposed by Akcelik (1). His model hierarchy includes macro-level, speed-type, delay-type, and pure fuel consumption models. The Analytical Fuel Consumption Model, related to the delay-type fuel consumption models, is developed in Chapter 3. It is followed, in Chapter 4, by an experimental design description which is intended to calibrate the model and analyze fuel consumption based on traffic volume and cycle length. Conclusions and future research suggestions are discussed in Chapter 5.

CHAPTER 2 LITERATURE REVIEW

INTRODUCTION

This chapter reviews approaches that have been applied to develop fuel consumption models describing urban network fuel economy and consumption. Section 2.1 describes a model hierarchy proposed by Akcelik (1). Section 2.2 presents macro-level fuel consumption models based on aggregate data. Fuel consumption models based on velocity change are described in Section 2.3 and Section 2.4 presents models based on measures of effectiveness, such as delay and stops. Models developed according to vehicle types and roadway conditions are discussed in Section 2.5.

Generally, fuel consumption varies with vehicle types, roadway geometric conditions, traffic control measures, and traffic demand. Fuel consumption models must describe how fuel is consumed under a variety of roadway design and traffic control changes. The fuel economy problem has motivated researchers to develop comprehensive models in order to understand the relationship between fuel consumption and traffic control measures.

Generally, four different fuel consumption model approaches have been applied. The first approach uses aggregate data to derive a relationship between fuel consumption and measured network-wide parameters, such as average travel time and average travel distance. The second approach considers fuel consumption as a function of speed and other parameters that aim to capture speed change effects through kinetic energy or inertial power. The third approach derives fuel consumption models based on other commonly used measures of effectiveness, such as delay and stops. The last approach considers the impact of vehicle design and roadway geometric conditions.

A classification proposed by Akcelik (1) divides fuel consumption models into four

levels. The proposed hierarchy of vehicle fuel consumption models, as shown in Figure 2.1, classifies different levels of fuel consumption models and illustrates their interrelationships among different components. These four levels of consumption models are briefly described hereafter.

Level 0: Basic Models

This level considers fuel consumption of individual vehicles as effected by vehicle components, such as engines, transmissions, and other vehicle characteristics. This level of fuel consumption models aims to provide a vehicle design aid.

Level 1: Micro Models

This model level has the form of an instantaneous fuel consumption function as defined by speed and acceleration/deceleration. Several simulation models, such as NETSIM and the TEXAS model, have the ability to predict the speed-time profiles and utilize this information to obtain fuel consumption estimates. This approach provides detailed insights to estimate fuel consumption in response to traffic conditions in terms of speed and speed change.

Level 2: Micro/Macro Models

These models consider aggregate and simplified information that are obtained from Level 1. They provide a simpler form to estimate fuel consumption, but are capable of responding to small traffic condition changes , such as signal timing. Therefore, these models are suitable for traffic and transport management purposes.

Level 3: Macro Models

Macro-level models, aiming to provide simple traffic system analyses, are derived by simple regression models that use as input data total travel time and distance.

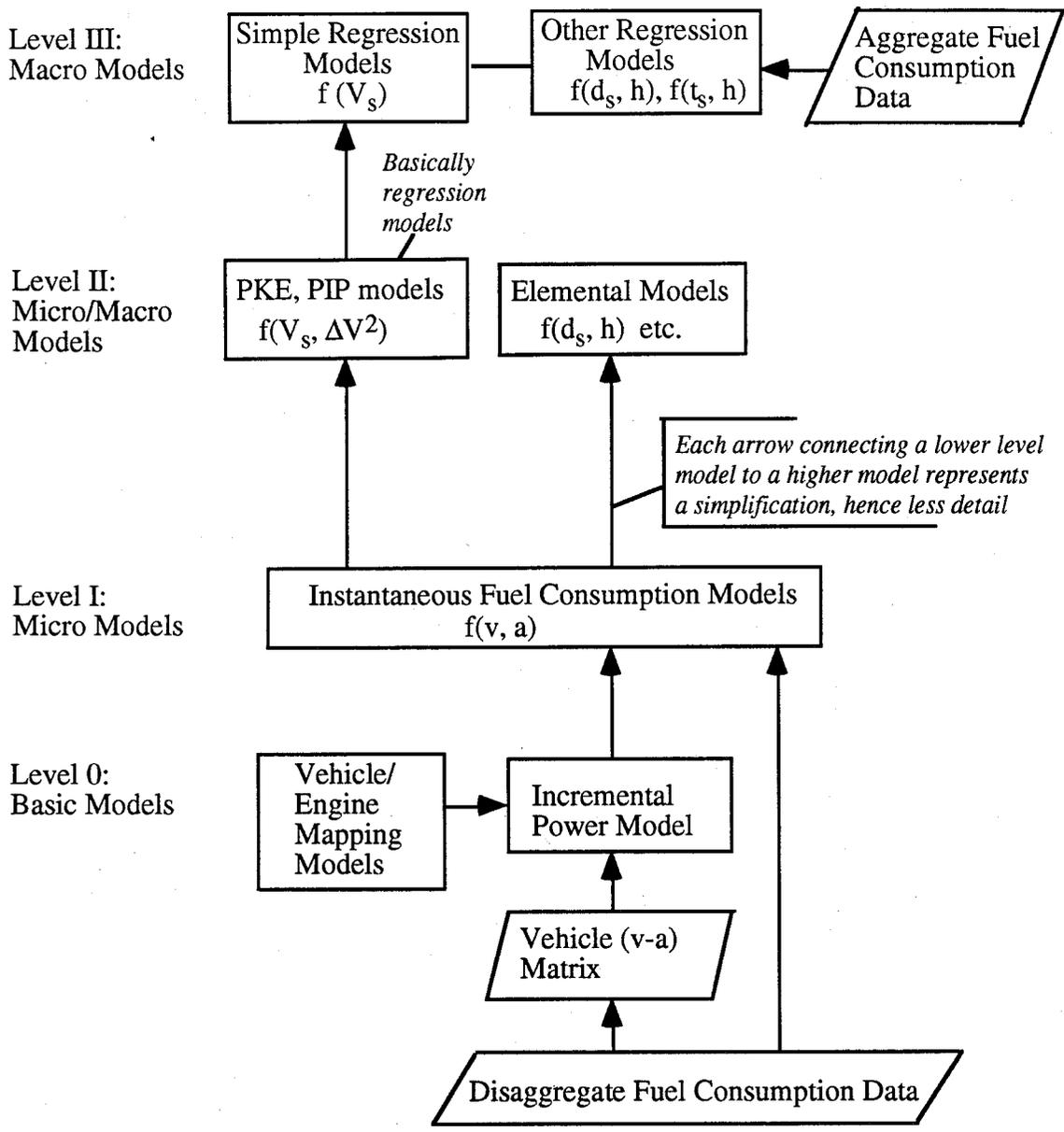


Figure 2.1 Hierarchy of Vehicle Fuel Consumption Models

Source: Akcelik et al. (1983)

MACRO-LEVEL FUEL CONSUMPTION MODELS

Macro-level fuel consumption models use regression analysis to derive a relationship between fuel consumption and network-wide variables, such as average travel time, average travel distance, and numbers of stops. Because these models do not consider speed change in the fuel consumption estimation, they are insensitive to small traffic condition changes.

Research conducted at General Motors Corporation was among the first to establish macro-level fuel consumption models. Evans, Herman, and Lam (11) investigated 17 variables describing the effects of fuel consumption, including average trip speed, largest instantaneous deceleration and acceleration, average trip time per unit distance, and number of complete vehicle stops, and found that fuel consumption estimation for urban trips F , can be estimated using average distance D , and average travel time T , i.e., $F = k_1 D + k_2 T$. Thus the fuel consumed per unit distance can be described as:

$$f = k_1 + k_2 t \quad [2.1]$$

where,

f : fuel consumption per unit distance,

k_1 : a parameter associated with fuel consumption per unit distance to overcome rolling resistance and is approximately proportional to vehicle mass,

k_2 : a parameter that is approximately proportional to the idle fuel flow rate, and

t : average trip time per unit distance.

Parameters k_1 (gallons per mile) and k_2 (gallons per hour) are coefficients related to vehicle characteristics. According to the model, fuel consumption can be estimated appropriately where vehicle speed is less than 35 mph. Chang and Herman (7) used two instrumented vehicles to estimate fuel consumption on two routes under different traffic conditions in Milwaukee. The results show that fuel consumption is independent of metropolitan areas and is approximately linearly related to average trip time. The impact of speed change on fuel consumption was described by Chang and Herman (6), and Evans et al. (10). The

results show that conservative driving behavior and proper traffic maneuvers, which usually have fewer speed change, can reduce fuel consumption.

The fuel consumption model was improved by considering the influence of vehicle stops in urban traffic systems by Herman and Ardekani in 1985 (14):

$$f = k_1 + k_2 t + k_3 \Delta N_s \quad [2.2]$$

where,

t: average trip time per unit distance,

ΔN_s : the difference between N_s and $N_s(t)$,

N_s : number of stops for a given datum point, and

$N_s(t)$: average number of stops associated with the trip time interval in which the datum point falls.

The results from regression analysis show that t and ΔN_s are independent; therefore, the model, including the additional variable ΔN_s , is more appropriate to estimate fuel consumption in urban traffic systems.

Results from several other studies (17, 18) are consistent with the models described earlier and have a similar fuel consumption model form. Pienaar (17) estimated the average fuel consumption rate in South Africa and found that minimum fuel consumption occurred at an average journey speed of about 64 km/hr. Pitt et al. (18) evaluated seven fuel consumption models during a Perth traffic pattern study utilizing data from an instrumented four-speed manual vehicle. Four of the models are macro-level and three are based on vehicle design and roadway geometric conditions. The results show that the performance of macro-level models are similar and independent of data used for calibration and/or testing.

SPEED-TYPE FUEL CONSUMPTION MODELS

Fuel consumption models based on average speed and speed change are categorized as speed-type fuel consumption models. The model forms depend on the derivation

assumptions. Examples include the PKE (Positive Kinetic Energy) and PIP (Positive Inertial Power) models (1).

The simplest form of speed-type fuel consumption model is proposed by Fwa and Ang (13). This can be described as:

$$f = k_1 + k_2 / V \quad [2.3]$$

where V is the average speed. The model is actually a macro-level fuel consumption model; however, this basic speed-type model can be combined with other variables that describe vehicle characteristics. Of course, this model is not sophisticated enough to capture traffic network speed changes.

Watson (21) derived a fuel consumption model as a function of speed and energy changes. The function can be described as:

$$f = k_1 + k_2 / V + k_3 V + k_4 \text{ PKE} \quad [2.4]$$

where k_1 to k_4 are coefficients, V is average speed, and PKE is the sum of positive acceleration kinetic energy changes. The PKE term aims to capture the dynamic effect of acceleration upon additional fuel consumption. One of the major shortcomings in this model is the difficulty to measure PKE, and thus a meaningful regression analysis is difficult. Everall (12) described the relationship between the variation of average fuel consumption and traffic speed in urban and rural roads as:

$$f = k_1 + k_2 / V + k_3 V^2 \quad [2.5]$$

Several micro traffic simulation models, such as NETSIM and the TEXAS model, also have the ability to estimate fuel consumption based upon speed and speed change of individual vehicles. For example, the TEXAS model, estimates fuel consumption using instantaneous vehicle speeds and acceleration.

DELAY-TYPE FUEL CONSUMPTION MODELS

This type of fuel consumption model aims to establish the relationship between fuel consumption and commonly used traffic measures of effectiveness, such as delay. Since

delay is a very popular measure of effectiveness in traffic analysis work, its use in a fuel consumption model is advantageous.

A fuel consumption model that was developed by stepwise multiple regression analysis is incorporated into the TRANSYT-7F model (19), so TRANSYT-7F can be applied to traffic signal optimization problems using a fuel consumption criterion. The model can be expressed as:

$$f = \sum_{i=1}^N [k_{i1} T + k_{i2} D + k_{i3} S] \quad [2.6]$$

where,

f: fuel consumption in gallons per hour,

T: total travel in vehicle-miles per hour,

D: total delay in vehicle-hours per hour,

S: total stops in stops per hour, and

k_{ij} : model coefficients which are functions of cruise speed on each link i:

$$k_j = A_{j1} + A_{j2} / V + A_{j3} V^2$$

Several studies (2, 8, 9) have focused on the study of traffic signal timing and fuel consumption. Cohen and Euler (8) used NETSIM to evaluate fuel consumption for different signal timing plans and found that the optimal cycle lengths for minimizing delay and for minimizing fuel consumption are the same. However, the result is different from the studies of Bauer (2) and Courage and Parapar (9) where the optimum cycle length for minimizing fuel consumption is much longer than the cycle length for minimizing isolated intersection delay. Bauer used an incremental fuel consumption model to analyze the change in fuel consumption due to signal cycle time. The form is expressed as:

$$\Delta E(c) = (E_{idle}) \sum_{j=1}^N d_j q_j + (E_{start}) \sum_{j=1}^N p_j q_j \quad [2.7]$$

where

$\Delta E(c)$: total incremental energy consumption resulting from one hour of intersection operation at a cycle time c,

E_{idle} : idling energy consumption of an average vehicle in the traffic mix using the intersection (gallons/hour),

E_{start} : energy consumption of an average vehicle in the mix using the intersection during a 0 to 30 mph acceleration maneuver (gallons),

N : number of approaches to the intersection,

d_j : delay in vehicle-hours for vehicles on the j th approach
(Webster's equation) (22),

p_j : average number of stops per vehicle for vehicles on the j th approach (Webster's equation) (22),

q_j : flow in vehicles/hour on the j th approach, and

c : cycle length used for signal timing.

Incremental fuel consumption based on different cycle lengths is related to idling energy consumption, acceleration energy consumption, vehicle flow rates, vehicle delay, and numbers of stops. The vehicle delay and numbers of vehicle stops are obtained from Webster's equation (22). Courage and Parapar's results are similar to Bauer's.

FUEL CONSUMPTION MODELS RELATED TO VEHICLE AND ROADWAY CONDITIONS

This section discusses the effects of vehicle design, roadway geometric design, and pavement type and condition on vehicle fuel consumption and describes relative fuel consumption models.

The engine of a moving vehicle must overcome resistance due to rolling, air, and gradients. It is obvious that pavement type affects fuel consumption through rolling resistance and roadway geometric design affects it through rolling resistance and gradient resistance. Vehicle design affects rolling, air, and gradient resistance.

A fuel consumption model based upon resistance to motion was derived by Bester in 1981 (3). The form of the model is:

$$f = P_1 + P_2 / V + P_3 V_2 + P_4 G \quad [2.8]$$

where,

V: speed,

G: gradient,

P₂: a constant that is related to idling fuel consumption, and

P₁, P₃, and P₄: constants derived from the rolling, air, and gradient resistance.

Bester used the model to investigate the effect of pavement type and condition on fuel consumption and found that pavement type has a minor effect on fuel consumption, yet pavement condition has a strong fuel consumption effect.

The ARFCOM (ARRB Road Fuel Consumption Model, 4) covers each level of the hierarchy shown in Figure 2.1 and includes three sub-models: an instantaneous model, a four mode elemental model, and a running speed model. The instantaneous model is a detailed engine-map based model that is related to engine power, engine drag and efficiency, and engine speed. The model form is expressed as:

$$f = \beta (P_{out} + P_{eng}) \quad [2.9]$$

or

α ,

whichever is greater

where,

f: the fuel consumption rate per unit time (ml/s),

α : the idle fuel consumption rate with accessories operating (ml/s),

β : the fuel-to-power efficiency factor (ml/s/kW),

P_{eng}: the power to overcome internal engine drag (KW), and

P_{out}: the total external engine power (KW) required to overcome rolling and air resistance, inertia and grade forces and provide power to run accessories.

The instantaneous model requires detailed individual vehicle design factors and is suitable for microscopic traffic models.

The four mode elemental models of ARFCOM include fuel consumption models describing idle, cruise, acceleration, and deceleration. Idle fuel consumption is a function of the idle fuel consumption rate and idling time. Cruise fuel consumed depends on the cruise speed and speed fluctuation impacts. Acceleration fuel consumption mainly depends on vehicle power components and deceleration fuel consumption is related to deceleration time and idle fuel consumption rate. The expressions of the four mode elemental models in ARFCOM are:

Idle fuel consumption model:

$$F_i = \alpha t_i \quad [2.10]$$

Cruise fuel consumption model:

$$F_c = \beta_b (1 + e_{hp} k_2 P_{out} / P_{max}) (P_{out} + P_{eng}) 3600 / V_c$$

or $3600 \alpha / V_c$, whichever is greater [2.11]

Acceleration fuel consumption model:

$$F_a = \beta_b (1 + e_{hp} k_2 P_{out} / P_{max}) (P_{out} + P_{eng}) t_a$$

or α , whichever is greater [2.12]

Deceleration fuel consumption model:

$$F_d = \beta_b (1 + e_{hp} k_2 P_{out} / P_{max}) (P_{out} + P_{eng}) t_d \quad [2.13]$$

where,

F_i : idle fuel consumption (ml),

F_c : cruise fuel consumption (ml),

F_a : acceleration fuel consumption (ml),

F_d : deceleration fuel consumption (ml),

α : idle fuel consumption rate with accessories operating (ml/s),

t_i : idle (stopped) time (s),

β_b : base engine fuel efficiency factor (ml/s/kW),

e_{hp} : proportionate decrease in engine fuel efficiency at maximum power,

P_{max} : maximum rated engine power (kW),

P_{out} : total output power of the engine (kW),

P_{eng} : power required to overcome engine drag (kW),

V_c : cruise speed (km/h),

t_a : acceleration time (s), and

t_d : deceleration time (s).

The ARFCOM running speed model is a macro level expression. It requires average running speed, idle time (stopped time), and travel distance. The function and characteristics of the running speed model are similar to the models described in section 2.2. The model is expressed as:

$$F_s = \alpha t_i + f_r x_s \quad [2.14]$$

where,

f_r : the fuel consumption per unit distance (ml/km) for a given average running speed, V_r , and sum of positive kinetic energy changes, denoted as E_{k+} ,

x_s : the section distance (km),

t_i : the idle (stopped) time (s), and

α : the idle fuel consumption rate (ml/s).

HDM-III (Highway Design and Maintenance Standards Model, 20) fuel consumption model was developed based on an experimental study in Brazil. It describes fuel consumed for an individual vehicle on any section of a specified geometric alignment. The fuel consumption is defined as:

$$F_L = 500 \alpha_1 \alpha_2 (UFC_u / V_u + UFC_d / V_d) \quad [2.15]$$

where,

F_L : average round trip fuel consumption (liters/1000 vehicle-km),

α_1 : relative energy-efficiency factor,

α_2 : fuel adjustment factor,

UFC_u : the predicted unit fuel consumption for the uphill segment (ml/s),

UFC_d : the predicted unit fuel consumption for the downhill segment (ml/s),

V_u : predicted steady-state speed for the uphill segment (m/s), and

V_d : predicted steady-state speed for the downhill segment (m/s).

A number of truck fuel consumption models have been developed although most are engine-map models. The fuel consumption model for heavy-duty trucks in the TEXAS model (15) was developed at the University of Texas at Austin. It uses engine speed and torque as predictor variables:

$$\begin{aligned} FF = & \alpha_1 + \alpha_2 \text{TRQ} + \alpha_3 (\text{RPM}) (\text{TRQ}) + \alpha_4 (\text{TRQ} + \text{RPM}) \\ & - \alpha_5 (\text{TRQ})^{1/2} \end{aligned} \quad [2.16]$$

where,

FF: fuel consumption (grams/second),

$\alpha_1, \alpha_2, \alpha_3, \alpha_4,$ and α_5 : constant coefficients,

RPM: engine speed in revolutions per minute, and

TRQ: engine torque in foot-pounds.

SUMMARY

Fuel consumption models have been developed for different purposes and have different prediction capabilities. It is important to select a suitable model to evaluate fuel consumption accurately.

In this chapter, four levels of fuel consumption models have been reviewed based on a model hierarchy proposed by Akcelik. These models, namely macro-level, speed-type, delay-type, and fuel consumption models related to vehicle and roadway conditions, were developed to estimate fuel consumption according to traffic situations and roadway conditions. Among these models, delay-type fuel consumption models are related to traffic signal timing design and thus are emphasized. In the next chapter, a new fuel consumption model is proposed based on queueing model concepts. Numerical experiments, results, and comparisons are discussed in Chapter 4.

CHAPTER 3 RESEARCH METHODOLOGY OF THE ANALYTICAL FUEL CONSUMPTION MODEL DEVELOPMENT

INTRODUCTION

This chapter presents the Analytical Fuel Consumption Model developed using queuing model concepts. The same techniques have been applied to develop delay equations, but are seldom used in fuel consumption models. Section 3.1 describes the fuel consumption model background. Section 3.2 explains the underlying queuing model concepts and defines a set of variables. The models for inbound approach, intersection, and outbound approach are presented in Section 3.3. Section 3.4 summarizes the fuel consumption model development.

Fuel consumption models must describe how fuel is consumed under existing traffic conditions and must have the ability to predict fuel consumption for a variety of roadway design and traffic control changes. The significance of factors affecting fuel consumption has been evaluated by many authors (1). Elements that have dominated model development include: (1) area and facility type, such as urban networks, rural areas, or freeways; (2) prediction ability for individual vehicles or an aggregate system; and (3) availability of suitable data from experimental tests.

Generally, fuel consumption models from previous studies include effects due to vehicle types, roadway geometric conditions, traffic control measures, and traffic conditions. Fuel consumption models for an urban signalized intersection must include effects primarily due to vehicle stops and vehicular stopped delay.

DEFINITION OF TERMS AND ASSUMPTIONS OF THE ANALYTICAL FUEL CONSUMPTION MODEL DEVELOPMENT

In order to study the impact of traffic control measures, such as green signal time and cycle length, a fuel consumption model must explicitly consider these control variables.

The Analytical Fuel Consumption Model is developed based on queueing models, and is similar to several popular delay models.

The total fuel consumption at a signalized intersection can be estimated from several variables, such as an average fuel consumption rate, traffic characteristics, and associated control measures. Average fuel consumption rates for different operating modes can express the impact of vehicle operations upon fuel consumption. For example, idling vehicles consume less fuel than accelerating vehicles. Calculation of average fuel consumption rates is discussed in Chapter 4. Traffic characteristics include the vehicle arrival pattern, average flow rate, and saturation flow rate. Pretimed signal control variables include cycle length, effective green time, and effective red time. Assumptions and notations are defined in this section.

To simplify the model analysis and calibration, the vehicle arrival pattern is assumed to be uniform with a constant rate, as the continuum model proposed by May (16). Another assumption is that only straight movements are considered, i.e. no left or right turns. Note here the basic model considers only undersaturated flow conditions, and no residual queue exists in any cycle. In an ideal undersaturated flow situation, any queue accumulated during the effective red time is cleared during the next available green and some vehicles go through the intersection without stopping or decelerating. However, these conditions will be relaxed in a more general model that is under development. Vehicle flow rates are expressed in passenger car units (pcu) and a truck is assumed equal to 1.5 to 2.0 pcu. There are no particularly conservative or aggressive drivers, i.e., all are assumed driving at desired speeds.

The model considers the intersection and street sections up to 600 ft from the intersection. These elements are called inbound approaches (600 ft prior to the intersection), outbound approaches (600 ft after the intersection) and the intersection itself. Specification of the lengths of studied sections is important because vehicles consume fuel continually.

Fuel consumption in a pretimed signal cycle is separated into three stages: the effective red time, the time from green onset to time t_0 , which is the time during which vehicles pass the stop line at saturation flow rates, and the time from t_0 to the end of the effective green. Vehicle operations for each stage are briefly described as follows:

1. The effective red time ($0 \leq t \leq r$)

On the inbound approach, vehicles decelerate and stop, and the number of queued vehicles increases according to the arrival flow rates. Vehicles in the intersection and outbound approach can travel at desired speeds until they leave the system.

2. Time from green onset to time t_0 ($r < t \leq r + t_0$)

On the inbound approach, vehicles move from the queue and accelerate to enter the intersection itself. In the intersection and outbound approach, vehicles accelerate, disperse on the road, and try to reach a desired speed.

3. Time from t_0 to the end of the effective green ($r + t_0 < t \leq r + g = c$)

In this stage, there is no stopped queue, so vehicles are assumed to travel at a constant velocity, i.e. no acceleration/deceleration.

The notations presented in the models are:

q : average flow rate on the approach (vehicle/sec),

s : saturation flow rate on the approach (vehicle/sec),

c : cycle time (sec), $c = r + g$,

g : effective green time (sec),

r : effective red time (sec),

y : g / s ,

t_0 : $y r / (1 - y)$ (After the green time starts, at time t_0 the arrivals equal the discharge, i.e., $q (r + t_0) = s t_0$. Therefore, $t_0 = y r / (1 - y)$), and

f_{ij} : fuel consumption for vehicles moving from status i to status j , where status means the speed change.

The calibration of the f_{ij} value described in the following models will be discussed

in Chapter 4. The following sections describe the Analytical Fuel Consumption Model for the inbound approach, the intersection itself, and the outbound approach.

The Analytical Fuel Consumption Model

In this section, the Analytical Fuel Consumption Model is discussed in three parts, inbound, intersection, and outbound, and each part is discussed in three different stages, as described in the previous section. Regarding the inbound approach, the model considers how vehicles move to an intersection, form a queue, and discharge. Model components for the intersection and outbound approach describe how vehicles consume fuel during the discharge stages, i.e. vehicles accelerate from the start of green to a desired speed or a speed limit. The inbound approach is a dominant factor in determining overall vehicle behavior and thus determines fuel consumption.

Inbound Approach Fuel Consumption Model

On the inbound approach, vehicles arriving during the effective red time must stop; therefore, two different operations are considered in the model, namely, deceleration and stopping. The number of queued idling vehicles increases during the effective red time and arriving vehicles have a shorter distance to stop due to the queue length increase. After the onset of green, vehicles in the queue are discharged at the saturation flow rate until t_0 . In this stage, vehicles are accelerating and moving from the queue to the intersection and outbound approach. In the last stage, where vehicles are still moving on the inbound approach, vehicles are assumed to enter the intersection without accelerating; therefore, the fuel consumption rate depends on the desired speed or speed limit. The models describing these operations are described in the following paragraphs.

(1) The effective red time ($0 \leq t \leq r$). During the effective red time, arriving vehicles decelerate to a stop before the stop line and fuel consumption varies from a moving to an idle status. Assuming vehicles enter the inbound approach at desired speed V_r (35

mph) and have speed V_0 when stopped. The average fuel consumption rate for speeds changing from V_r to V_0 is f_{r0} . The rate for idling vehicles is respectively f_0 . Fuel consumption F at any instant in time can be expressed as:

$$\begin{aligned}
 F &= (\text{idle vehicles}) f_0 + (\text{moving vehicles}) f_{r0} \\
 &= n_1 f_0 + n_2 f_{r0} \\
 &= q t f_0 + q T_1 f_{r0}
 \end{aligned}
 \tag{3.1}$$

where,

T_1 : estimated travel time on the inbound approach, time lag,

f_0 : fuel consumption rate for idle vehicles, and

f_{r0} : fuel consumption for vehicles moving from V_r (35 mph) to V_0 (0 mph).

Therefore, if r is the effective red duration, the total fuel consumption TF is:

$$\begin{aligned}
 TF &= \int_0^r [q t f_0 + q T_1 f_{r0}] dt \\
 &= \frac{1}{2} q r^2 f_0 + q T_1 r f_{r0}
 \end{aligned}
 \tag{3.2}$$

T_1 is defined as a time lag, which is determined by the length considered (600 ft) and the arrival flow rate (i.e. average queue length), and is used to estimate the number of arriving vehicles that are currently in the deceleration process.

(2) Time from green onset to time t_0 ($r < t \leq r + t_0$). In this stage, vehicles in the queue are discharged at the saturation flow rate, and vehicles are accelerating and moving from the queue through the intersection. Arriving vehicles are still delayed by the queue; therefore, they decelerate and join the moving queue. Vehicles in the queue are assumed to accelerate to speed V_2 and the fuel consumption rate for this acceleration is defined as f_{02} . The first arriving vehicle after the onset of green must fully stop, and the last vehicle arriving at time t_0 decelerates to speed V_2 . Arriving vehicles are assumed to have initial speed V_r and pass the stop line at speed V_3 , which is the average of V_r and V_2 . Therefore, fuel consumption F at any instant in time is:

$$F = (\text{number of vehicles in queue}) f_{02} + (\text{number of arriving vehicles}) f_r \tag{3.3}$$

where,

f_{02} : fuel consumption for queued vehicles accelerating from V_0 to V_2 , and
 f_{r3} : fuel consumption for moving vehicles changing speed from V_r to V_3 , where
 V_3 is the average of V_r and V_2 .

Total fuel consumption during this stage is:

$$\begin{aligned} TF &= \int_0^{t_0} \left[\frac{qr}{2} f_{02} + qt f_{r3} \right] dt \\ &= \frac{qr}{2} t_0 f_{02} + \frac{1}{2} q t_0^2 f_{r3} \end{aligned} \quad [3.4]$$

The term $\frac{qr}{2}$ defines the average number of vehicles accelerating from the queue in this stage, and the term qt represents the total number of arriving vehicles. In this expression, the total fuel consumption depends significantly upon t_0 which expresses the congestion level. Also, the average fuel consumption rates at these stages are usually high because vehicles are accelerating to reach speed V_2 .

(3) Time from t_0 to the end of the effective green ($r + t_0 < t \leq r + g = c$). Vehicle maneuvers will be back to normal, non-stopping conditions after t_0 . Although vehicle interaction will affect speed fluctuations, all vehicles are assumed moving at their desired speed V_r . At any instant in time, fuel consumption F is:

$$F = (\text{moving vehicle}) f_r \quad [3.5]$$

Total fuel consumption is:

$$\begin{aligned} TF &= \int_{t_0}^g q T_2 f_r dt \\ &= q T_2 f_r (g - t_0) \end{aligned} \quad [3.6]$$

where,

g : effective green time (sec),

T_2 : estimated travel time on the inbound approach from time t_0 to the end of the effective green, and

f_r : fuel consumption for vehicles moving at their desired speed V_r .

The term qT_2 defines the number of vehicles that are approaching the intersection.

Intersection Fuel Consumption Model

Vehicles accelerate to enter the intersection when the signal changes to green. During the initial green time, namely t_0 , vehicles will pass into the intersection at saturation flow rate s and initial speed V_2 and reach speed V_4 before entering the outbound approach. Since the intersection width is relatively small, the number of vehicles is assumed fixed. Therefore, the total fuel consumption TF in the intersection is defined as:

(1) **The effective red time ($0 \leq t \leq r$).** TF = 0, because no vehicle may enter the intersection during the effective red time.

(2) **Time from the onset of green to the time t_0 ($r < t \leq r + t_0$).**

Within the limited space comprising the intersection itself, the number of vehicles is fixed.

If the average travel time across the intersection is k , the total fuel consumption is:

$$\begin{aligned} TF &= \int_0^h \frac{sk}{2} f_{24} dt \\ &= \frac{sk}{2} f_{24} h \quad \text{if } 0 < t \leq h \end{aligned} \quad [3.7]$$

$$\begin{aligned} TF &= \int_h^{t_0} sk f_{24} dt \\ &= sk f_{24} (t_0 - h) \quad \text{if } h < t \leq t_0 \end{aligned} \quad [3.8]$$

where,

f_{24} : fuel consumption for vehicle changing speed from V_2 to V_4 ,

k : average travel time across the intersection, and

h : the time for the queued vehicles fill to the intersection.

In this expression, h is the time for vehicles in the queue to move into and fill the intersection. The magnitude of h is obviously determined by the intersection size.

(3) **Time from t_0 to the end of the effective green ($r + t_0 < t \leq r + g = c$).** In this stage, vehicles are not affected by signal operation, and are assumed to be traveling at desired speed V_r and have fuel consumption rate f_r . The total fuel consumption is:

$$TF = \int_{t_0}^g q k f_r dt$$

$$= q k f_r (g - t_0) \quad [3.9]$$

where,

g : effective green time (sec),

f_r : fuel consumption for vehicles moving at desired speed V_r , and

k : average travel time across the intersection.

Outbound Approach Fuel Consumption Model

The analysis in this section is more complicated because the number of vehicles in the outbound approach is varying. The number of vehicles on the outbound approach depends on the rate at which vehicles are entering and leaving the outbound approach. Vehicles enter the outbound approach during the green and exit the system after traversing the outbound approach. On the outbound approach, most fuel consumption occurs during the green time because vehicles are moving through the intersection and accelerating to their desired speed. Vehicles that have not exited the system during the cycle time will affect the total fuel consumption in the next cycle. Since the dominant factor is the number of vehicles moving on the outbound approach, fuel consumption is more related to travel time and distance on the outbound approach than to the inbound approach and the intersection. For this reason, the time lag τ is introduced as the time for a vehicle to travel from the start to the end of the outbound approach.

(1) **Time from green onset to time t_0 ($r < t \leq r + t_0$).** Vehicles enter the outbound approach during the green signal and leave the outbound approach after the time period " τ ". The number of vehicles on the outbound approach at any time will be the number of vehicles that have entered the outbound approach minus the number that exited. The total fuel consumption TF can be defined as:

$$TF = (\text{number of vehicles}) f_{4r} \quad [3.10]$$

Where f_{4r} is the fuel consumption rate for vehicles changing from V_4 , speed when leaving the intersection, to V_r which is the desired speed. Because no vehicles occupy the

outbound approach in the first few green time seconds, k , the TF in this stage $0 < t < k$ is

$$TF = 0 \quad \text{if } 0 < t < k$$

When vehicles are entering and leaving the outbound approach, the TF in the stage $k < t < t_0$ is

$$TF = \int_0^{t_0} (s t - \max\{0, (t - \tau) s\}) f_{4r} dt \quad \text{if } k < t < t_0 \quad [3.11]$$

where,

s : saturation flow rate on the approach (vehicle/sec),

τ : time lag on outbound approach travel time,

f_{4r} : fuel consumption for a vehicle changing speeds from V_4 to V_r , and

k : the time for the first vehicle to enter the outbound approach.

(2) Time from t_0 to the end of the effective green ($r + t_0 < t \leq r + g$

= c). After t_0 , some vehicles on the outbound approach have reached their desired speed V_r , and some are accelerating to reach that speed. If vehicles that are still accelerating have an average speed V_6 , the total fuel consumption TF is defined as:

$$TF = \int_{t_0}^g ((s t_0 - \min\{s t_0, \max\{0, (t - \tau) s\}\}) f_{6r} + (q (t - t_0) - \max\{0, (t - t_0 - \tau) q\}) f_r) dt \quad [3.12]$$

where,

g : effective green time (sec), and

f_{6r} : fuel consumption rate for vehicles accelerating from V_6 to V_r , where V_6 is defined as the average of V_4 and V_r .

(3) The effective red time ($0 \leq t \leq r$). After the end of the green time, some vehicles are still traveling on the outbound approach. If vehicle trajectories follow the same pattern as the green time and total fuel consumption is:

$$TF = \int_g^{g+r_1} ((s t_0 - \min\{s t_0, \max\{0, (t - \tau) s\}\}) f_{6r} + (q (g - t_0) - \max\{0, (t - t_0 - \tau) q\}) f_r) dt \quad [3.13]$$

where,

g : effective green time (sec),

r_1 : the elapsed time required for all vehicles to leave the system,

τ : time lag on outbound approach travel time,

f_{G_r} : fuel consumption for a vehicle changing from speed V_G to V_r , and

f_r : fuel consumption for vehicles moving at desired speed V_r .

SUMMARY

In this chapter, the Analytical Fuel Consumption Model is proposed based on queueing model concepts and vehicle operation stages. The model, aiming to include the impact of traffic characteristics, fuel consumption rates, and control variables, includes three different vehicle operating conditions describing operations on an inbound approach, the intersection itself, and an outbound approach. For each condition, three flow characteristic stages are discussed. The flexible design of this model can be extended to include the impact of residual queues, turning movements, and different arrival patterns. Experimental setups and fuel consumption rates are discussed in the next chapter illustrating the proposed fuel consumption model and the relationship between fuel consumption and cycle length.

CHAPTER 4 EXPERIMENTAL DESIGN AND COMPARISONS

INTRODUCTION

In this chapter, numerical experiments designed to develop insight into fuel consumption modeling are described. Section 4.1 outlines the experimental objectives. Section 4.2 discusses estimation of fuel consumption rates, which is critical to the Analytical Fuel Consumption Model. Experimental factors are discussed in Section 4.3 and numerical results and comparisons are discussed in Section 4.4. A brief summary of the experimental design and comparison is described in Section 4.5.

As discussed in Chapter 3, fuel consumption at a signalized intersection is estimated using average fuel consumption rates, traffic characteristics, and associated control measures. Due to the system complexity, numerical experiments are conducted to explore the Analytical Fuel Consumption Model effectiveness and accuracy. Two important objectives of these numerical experiments are to establish the credibility of the Analytical Fuel Consumption Model and explore utilization of the model to optimize signal timing.

To establish the model credibility, results from the Analytical Fuel Consumption Model are compared with TEXAS model results. The fuel consumption model in the TEXAS model was calibrated from field experiments. Since both the TEXAS and NETSIM models are capable of predicting total fuel consumption, NETSIM is used to calibrate the Analytical Fuel Consumption Model and TEXAS is used for independent comparative analysis. Under the same control measures and traffic characteristics, patterns of total fuel consumption are compared.

Since TEXAS and NETSIM are descriptive-type models, they do not have the ability to optimize signal attributes based on fuel consumption minimization. The Analytical Fuel Consumption Model is applied to study the relationship between control measures and total fuel consumption. As discussed in Chapter 2, results from delay-type

fuel consumption models show inconsistent conclusions. Through the Analytical Fuel Consumption Model, one should be able to examine the relationship more closely.

To explore utilization of the Analytical Fuel Consumption Model to optimize signal timing, the optimal cycle length for fuel consumption minimization is provided and compared with the results of Webster's delay equation and other fuel consumption models.

CALCULATION OF FUEL CONSUMPTION RATES

The intersection is divided into three physical elements: inbound, intersection, and outbound. A signal cycle is divided to three stages: effective red time, green time with saturation flow rate, and remaining green time. During each stage, vehicle trajectories for the three elements are different; therefore, fuel consumption rates may be different. The calculation of the fuel consumption rate is based on experimental NETSIM results, and these data are in Appendix A.

As described in equation (3.2), the total fuel consumption is given by

$$\frac{1}{2} q r^2 f_0 + q T_1 r f_{r0}$$

where f_0 is idle fuel consumption and is about 13×10^{-5} gallons/sec (0.33098 grams/sec). The fuel consumption rate for a vehicle traveling at constant velocity V_r , is f_r and if V_r is 35 mph, f_r is 34×10^{-5} gallons/sec (0.86564 grams/sec). The fuel consumption rate, f_{r0} , is the rate for a vehicle decelerating from desired speed V_r to a stopped or an idle state. The model assumes V_r is 35 mph (51.5 ft/second) and acceleration as a function of time is a constant value:

$$A(t) = -a, \text{ and}$$

$$V(t) = -at + V_r, \text{ so}$$

$$a = \frac{V_r}{t}$$

In the above equations, a is the deceleration rate and t is the time for the vehicle to decelerate to a stop. The maximum distance for a vehicle to decelerate is assumed to be the physical length of the inbound approach and all vehicles use constant deceleration, the

values of deceleration rate a and time t can be calculated by:

$$S = V_r t - \frac{1}{2} a t^2$$

$$0 = V_r - a t.$$

Thus t is 11.65 seconds and a is 4.4 ft/sec² if no stopped queue is present. If there is a queue at the intersection, then the deceleration distance will reflect this and the deceleration rate will be different.

Since the fuel consumption rates are different for different speeds, an approximation is used to estimate the fuel consumption rate.

$$f_{r0} = \int_0^t FF_v dt \cong FF_v \Delta t \quad [4.1]$$

where,

f_{r0} : fuel consumption rate for moving vehicles changing speed from V_r to V_0 , and

FF_v : fuel consumption rate at speed v and constant acceleration rate $a = 4.4$ ft/sec².

The fuel consumption rates FF_v are given in the following table and the average fuel consumption f_{r0} is calculated as 16.8077×10^{-5} gallons/sec (0.42792 grams/sec).

Time (sec)	0	1	2	3	4	5	6	7	8	9	10	11	12
Speed (ft/sec)	51.5	47.1	42.7	38.3	33.9	29.5	25.1	20.7	16.3	11.9	7.5	3.1	0
FF_v (10^{-5} gallons/sec)	17	16	17	18	18	18.5	18	18	18	17	17	14	13

The parameter f_{02} , used in equation 3.4, is defined as the fuel consumption rate for a vehicle accelerating from idle to speed V_2 . If a vehicle has an acceleration rate a for a short duration t , one can estimate a using the design acceleration from the AASHTO publication "A Policy on Geometric Design of Highways and Streets". From Figure II-16 (P.40, 1990) (22), the distance for a vehicle to accelerate from idle to speed 10 mph is about 20 feet. By the equations:

$$V_2 = V_0 + a t$$

$$a = \frac{V_2}{t}$$

$$S = \frac{1}{2} a t^2,$$

$a = 5.5 \text{ ft/sec}^2$ and $t = 2.72$ seconds. The procedure assumes vehicles accelerate at the constant acceleration rate 5.5 ft/sec^2 for a short 2 second duration. The fuel consumption rates FF_v are given in the following table and the average fuel consumption f_{02} is calculated as $(44.75 + 67) / 2 = 55.875$ gallons/second (1.42258 grams/second).

time (sec)	1	2
speed (ft/sec)	5.5	11
FF_v (10^{-5} gallons/sec)	44.75	67

The fuel consumption rate for a vehicle changing speed from V_r to V_3 is called f_{r3} in equation 3.4. On the inbound approach, moving vehicles have initial speed V_r and must reduce speed to V_3 to join the moving queue. If the deceleration distance is 600 feet, the average speed $V_3 = (V_1 + V_2) / 2 = 31.25 \text{ ft/second}$.

$$V_3 = V_r - a t$$

$$31.25 = 51.5 - a t$$

$$S = L = 600 = V_r t - \frac{1}{2} a t^2$$

$a = 1.4 \text{ ft/sec}^2$ and $t = 14.5$ seconds. One can calculate f_{r3} from the above procedure as $f_{r3} = 17.1714$ gallons/second (0.43718 grams/second).

All fuel consumption rates in the Analytical Fuel Consumption Model are summarized in Table 4.1.

**TABLE 4.1 FUEL CONSUMPTION RATES IN THE
ANALYTICAL FUEL CONSUMPTION MODEL**

Variable Name	Definition	Fuel Consumption Rate	
		gallon/sec (10 ⁻⁵)	gram/sec
f ₀	Idle	13	0.33098
f _{r0}	Change speed from V _r to V ₀	16.8077	0.42792
f _r	Traveling at a constant speed	34	0.86564
f ₀₂	Change speed from idle to V ₂	55.875	1.42258
f _{r3}	Change speed from V _r to V ₃	17.1714	0.43718
f ₂₄	Change speed from V ₂ to V ₄	112	2.85152
f _{4r}	Change speed from V ₄ to V _r	152.9	3.89283
f _{6r}	Change speed from V ₆ to V _r	178.25	4.53824

EXPERIMENTAL DESIGN

In all experiments, a two by two intersection (one inbound and one outbound lane on each leg) is used. The length for each inbound and outbound approach is 600 feet and the width of intersection is 40 feet. Two major factors considered in experimental design and numerical analysis are traffic volume and cycle length. Volume is a dominant factor and when volume is low, vehicles have little interaction with each other. Values of traffic volume, include 400, 600, and 750 vehicles per hour (vph) reflecting different traffic conditions. Cycle length is varied from 20 to 150 seconds with an interval of 10 seconds. Each cycle length has 50%-50% green splits and 3 second clearance intervals.

In these experiments, results from the Analytical Fuel Consumption Model and the TEXAS model are compared across different cycle lengths and street elements. The comparisons are based on both the variation of fuel consumption in each unit time (one second) and total fuel consumption in each cycle. Thus one can recognize the relationship

between volume and fuel consumption, and analyze the fuel consumption changes during one cycle.

Traffic volume and signal cycle length have important fuel consumption impacts. The optimal cycle length for minimizing fuel consumption is discussed in this research, and results are compared with those obtained from delay minimization considerations.

NUMERICAL RESULTS AND COMPARISONS

The variation of fuel consumption for flow rate 600 vph in a 60 second cycle is shown in Figure 4.1. In this figure, the elapsed time 0 to 30 seconds is the effective red time, the time 30 to 60 seconds is the effective green time, and t_0 is 18 seconds. During the effective red time, because of the increasing number of inbound approach vehicles, fuel consumption increases as the elapsed time increases as shown in Figure 4.1(a). When the signal changes to green, vehicles accelerate to reach a desired speed traveling on the outbound approach. The highest fuel consumption rate during a cycle occurs during acceleration. This means that fuel consumption per unit time reaches a maximum during time t_0 and decreases after t_0 . This situation can be observed clearly from the elapsed time 30 to 48 seconds in Figure 4.1(c), in which the fuel consumption increases dramatically due to the high acceleration rate. Note that in Figure 4.1(c), fuel consumption exists in the first few seconds of the effective red time because vehicles traveling on the outbound approach have not been discharged completely.

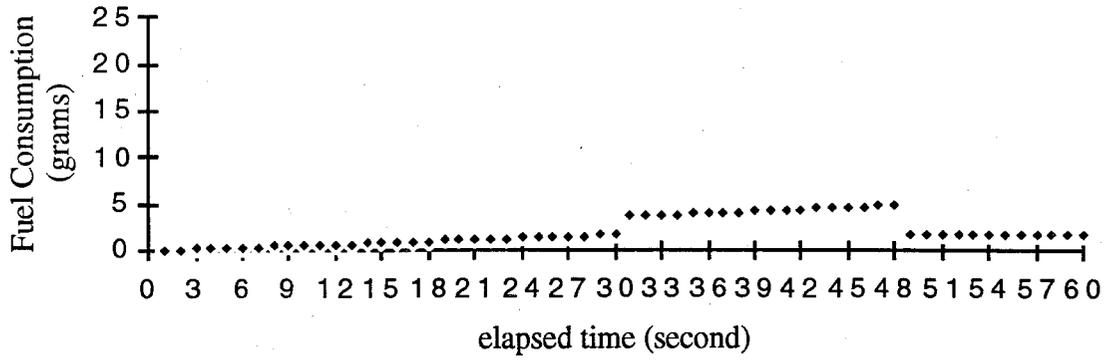
Variations of fuel consumption for volumes 400 and 750 vph are shown in Figure 4.2 and 4.3, respectively. Both variations exhibit a pattern similar to Figure 4.1. In the case of volume 750 vph, there is more fluctuation in fuel consumption due to increased vehicle interactions.

Figure 4.4 depicts fuel consumption variation obtained from the TEXAS model for a period from 690 seconds to 750 seconds. The pattern is similar and consistent with results in Figure 4.1. Fuel consumption changes from effective red time to effective green

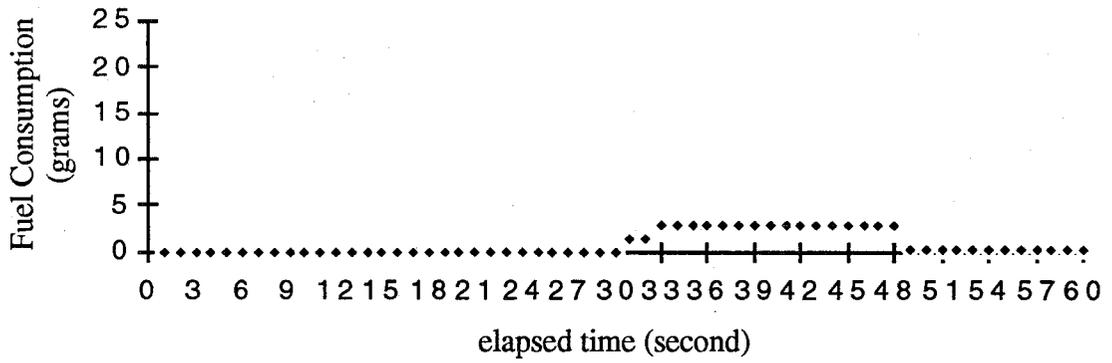
time match results of the Analytical Fuel Consumption Model. For instance, fuel consumption increases in the effective red time on the inbound approach, increases dramatically in t_0 as vehicles accelerate into the intersection and the outbound approach, decreases at the end of t_0 , and remains stable when vehicles travel on the outbound approach.

The new queueing theory based model is acceptable when compared with the TEXAS model. One can investigate the impact of different volume levels on fuel consumption and the variation of average fuel consumption for different cycle lengths, and thus derive an optimal cycle length for fuel consumption minimization.

(a) Inbound Approach



(b) Intersection



(c) Outbound Approach

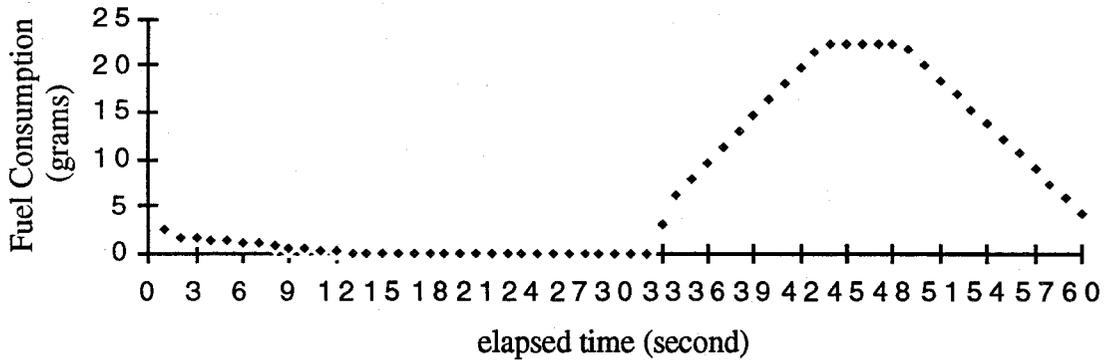
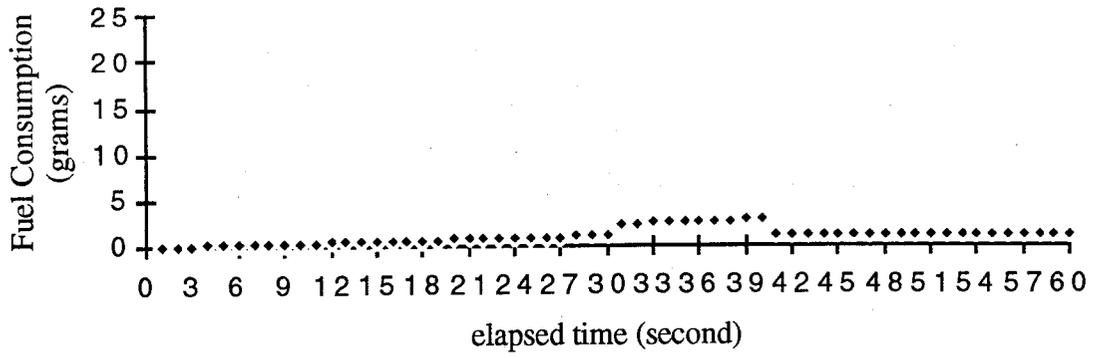
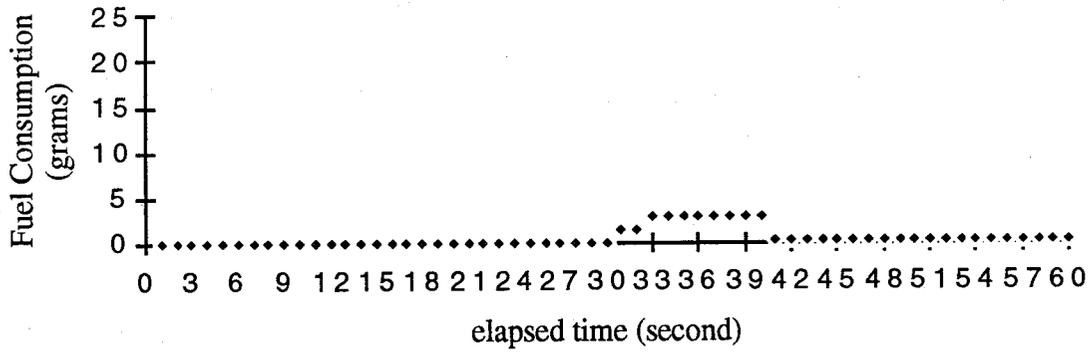


Figure 4.1. Fuel Consumption Versus Elapsed Time from the Analytical Fuel Consumption Model - 600 vph case

(a) Inbound Approach



(b) Intersection



(c) Outbound Approach

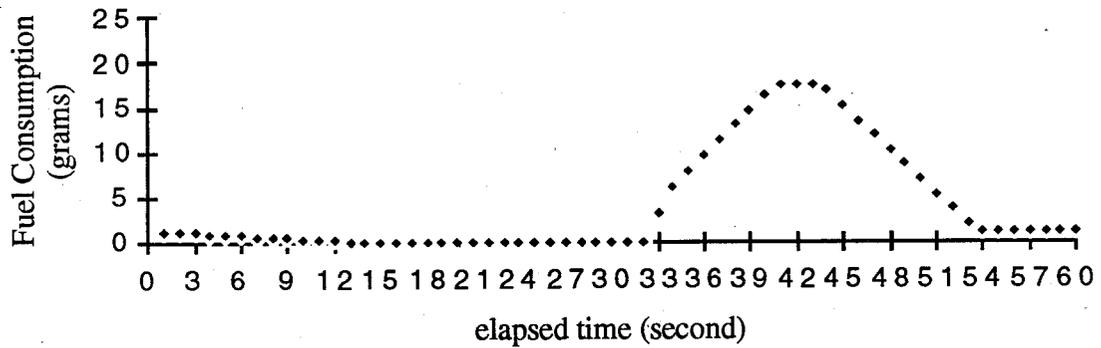
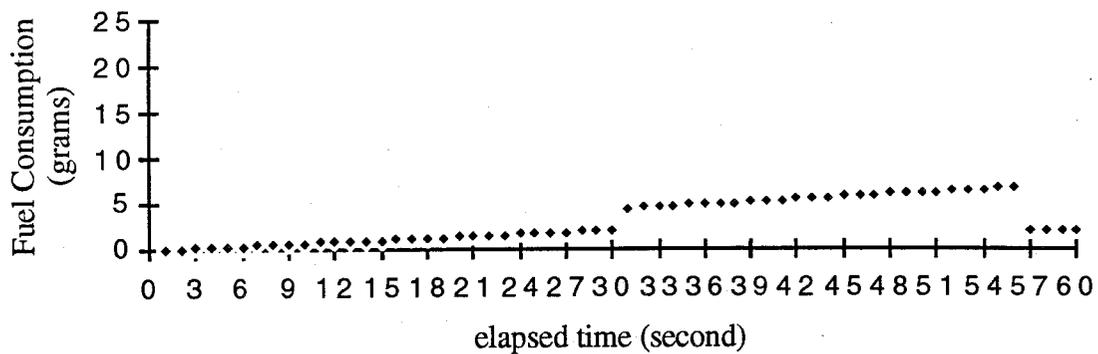
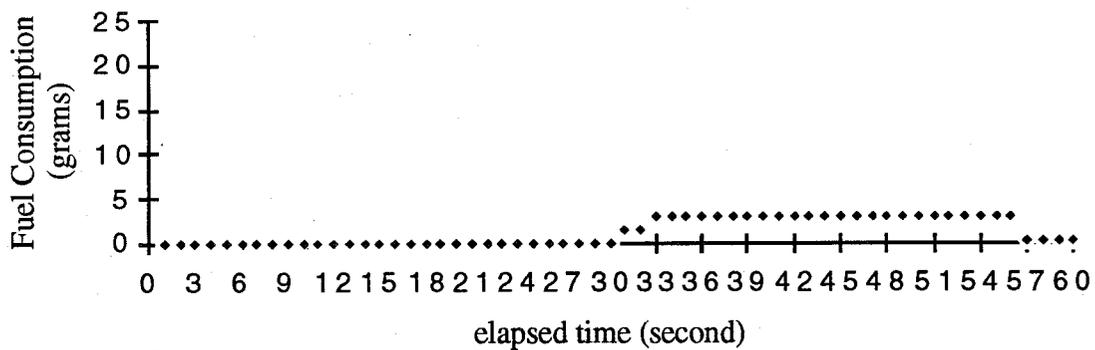


Figure 4.2. Fuel Consumption Versus Elapsed Time from the Analytical Fuel Consumption Model - 400 vph case

(a) Inbound Approach



(b) Intersection



(c) Outbound Approach

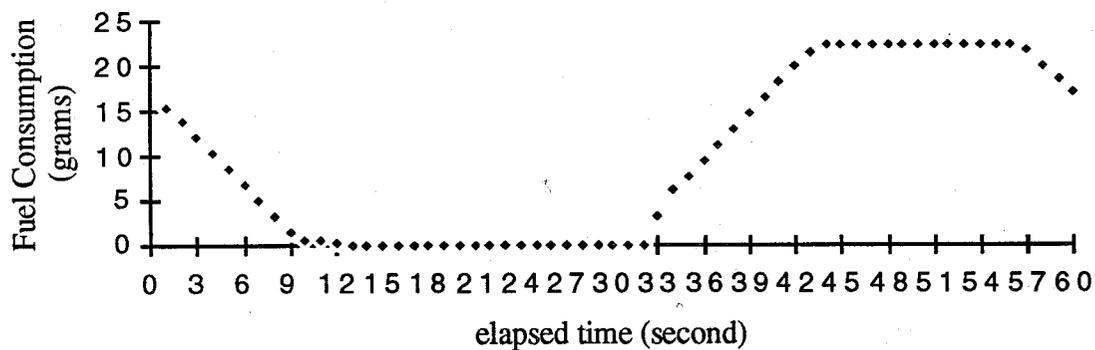
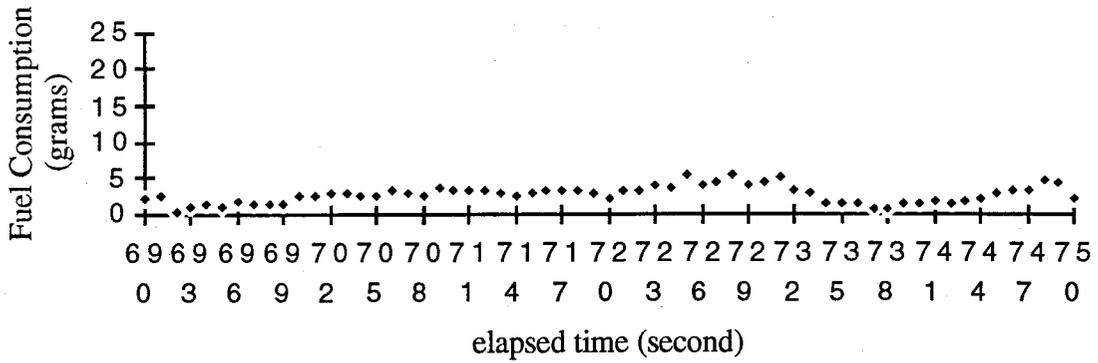
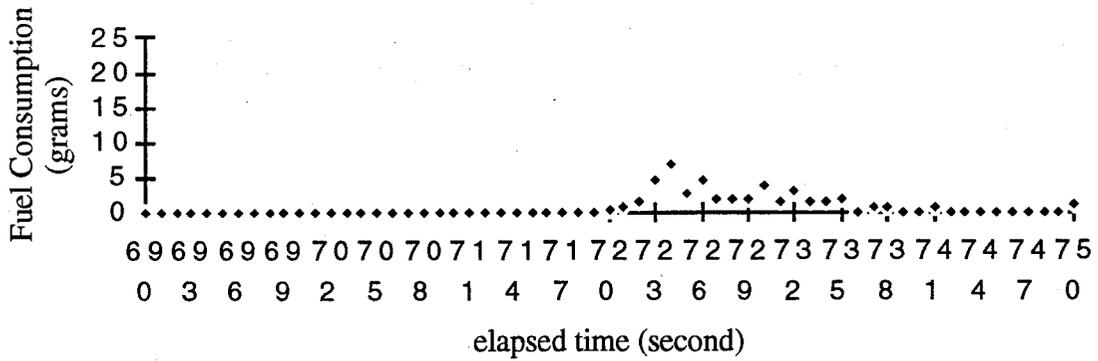


Figure 4.3. Fuel Consumption Versus Elapsed Time from the Analytical Fuel Consumption Model - 750 vph case

(a) Inbound Approach



(b) Intersection



(c) Outbound Approach

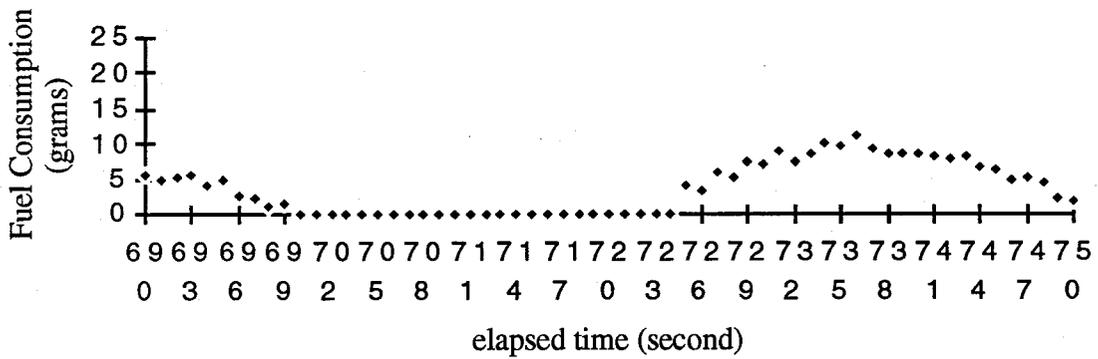


Figure 4.4. Fuel Consumption Versus Elapsed Time
from the TEXAS Model - 600 vph case

SIGNAL SETTING FOR FUEL CONSUMPTION MINIMIZATION

Since it is difficult to derive an optimal cycle length for minimizing fuel consumption by mathematical optimization techniques, numerical analysis is applied to find an approximate optimal result by varying cycle lengths from 20 to 150 seconds with a 10 second increment. Variations of fuel consumption with respect to cycle lengths for volumes 400, 600, and 750 vph are illustrated in Figures 4.5 and 4.6. The figures show results obtained from the Analytical Fuel Consumption Model and the TEXAS model, respectively. Patterns in both figures are similar; however, more fluctuation in Figure 4.6 is probably due to the nature of simulation. In both figures, the change of cycle length has significant impacts on fuel consumption in the high volume case, but not in the low volume 400 vph case. Although the 400 vph curve is rather flat, one can still find an optimal cycle length for fuel consumption minimization. In the 750 vph case, the long cycle length fuel consumption is less than that of the short cycle length.

Generally speaking, all curves shown in Figure 4.5 are convex, and an optimal cycle length can be expected for each case. Numerical results of the optimal cycle length based on fuel consumption minimization are listed in Table 4.2 which shows that the optimal cycle length is 100 seconds for the 750 vph case compared to 50 seconds for the 400 vph case and 80 seconds for the 600 vph case. However, results from the TEXAS model show a little different pattern, in which the optimal cycle lengths are 40, 80, and 70 seconds, respectively. Actually, longer cycle lengths are expected for high volume cases because of more acceleration and deceleration maneuvers. Generally, for fuel consumption minimization, all cycle lengths are longer for high than for low volume.

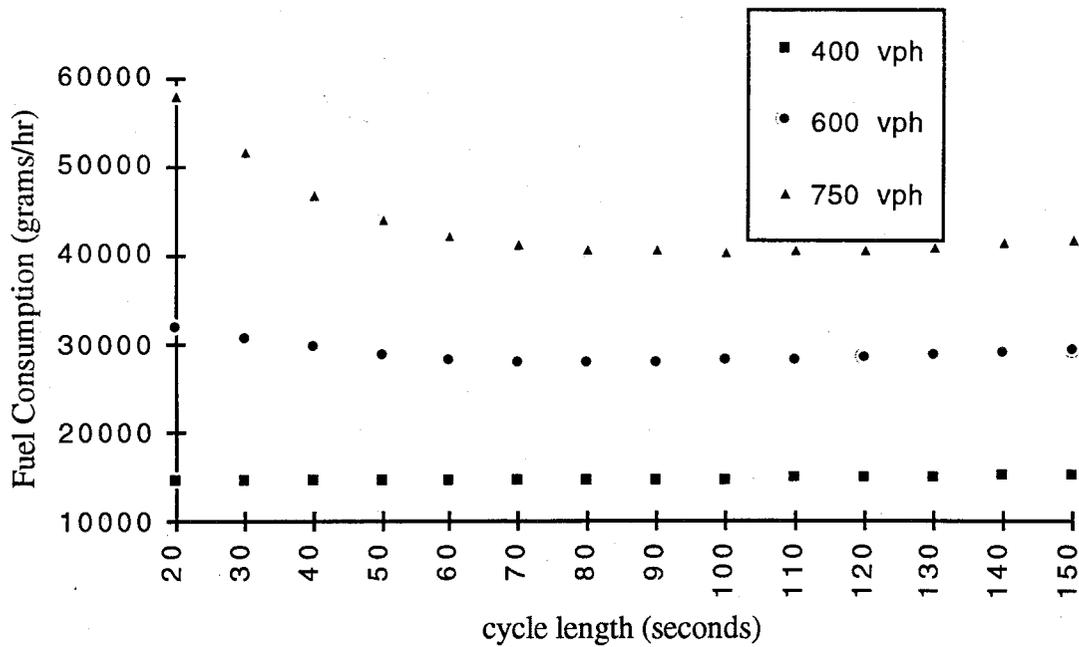


Figure 4.5. Optimal Cycle Length Versus Traffic Volume from the Analytical Fuel Consumption Model

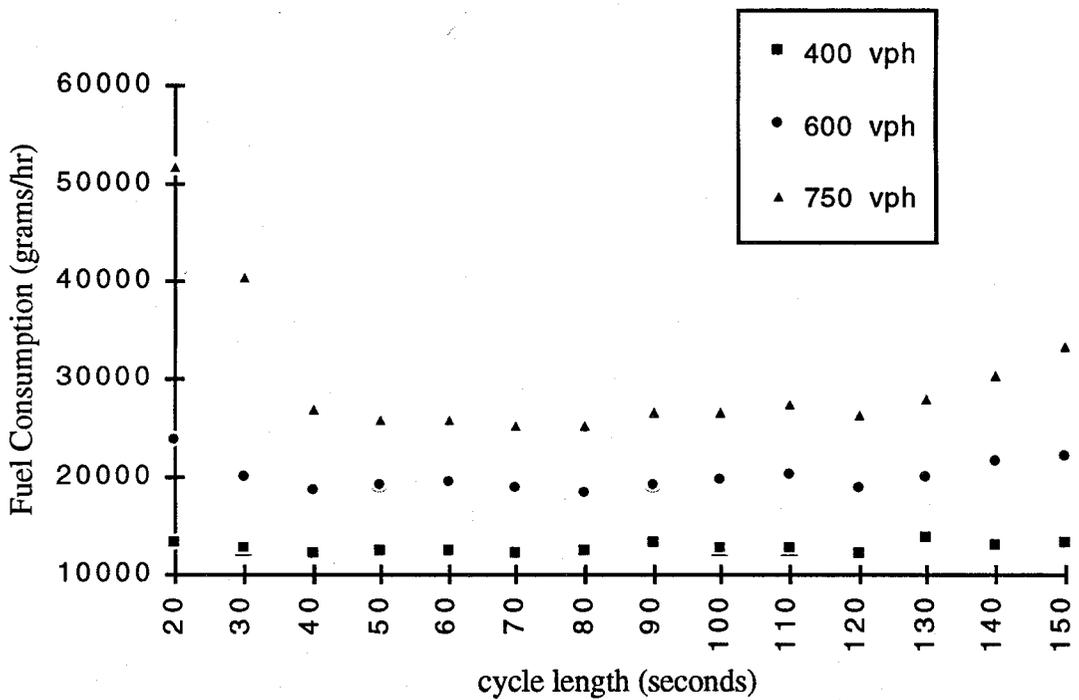


Figure 4.6. Optimal Cycle Length Versus Traffic Volume from the TEXAS Model

TABLE 4.2 OPTIMAL CYCLE LENGTH FOR DIFFERENT TRAFFIC VOLUMES

Phase I		Phase II		The Analytical Fuel Consumption Model	The TEXAS Model
Volume	$\frac{g}{c}$	Volume	$\frac{g}{c}$		
400 vph	0.5	400 vph	0.5	50 seconds	40 seconds
600 vph	0.5	600 vph	0.5	80 seconds	80 seconds
750 vph	0.5	750 vph	0.5	100 seconds	70 seconds

For an intersection with pretimed traffic signals, fuel consumption changes during the 24 hours of a day due to changing traffic demands. These changing demands are sometimes described as three or four generically different conditions. These are sometimes considered as low volume during late night, medium volume in off-peak hours, and high volume in peak hours. In order to minimize fuel consumption, the cycle length should be adjusted in the different time periods according to traffic volume changes. For instance, if flow rates in an intersection are 750 vph in the morning and afternoon peak hours, 400 vph at night, 600 vph for the rest of a day, the optimal cycle length should be 100 seconds in the peak hours, 50 seconds at night, and 80 seconds in the off-peak hours.

COMPARISONS

Signal Settings for Delay Minimization

Optimization of traffic signals, during all except peak hours traditionally, has been based on a delay minimization criterion. A number of authors have tackled the optimum signal setting problem based on different assumptions and conditions. Webster (21) derived optimal cycle time using his empirically developed delay equation and his equation has been used extensively in practice.

Webster's results, therefore, are used to compare signal settings for fuel

consumption minimization and delay minimization in this experimental analysis. For delay minimization, optimal cycle time is given by Webster's Equation:

$$C_0 = \frac{1.5L + 5}{1 - Y} \text{ seconds} \quad [4.2]$$

where,

Y: the sum for all signal phases of the highest ratios of flow to saturation flow,

L: $n l + R$,

n: the number of phases,

l: the average lost time per phase (excluding all-red times), and

R: all-red times.

Using the same experimental design described in Section 4.3, Figure 4.7 depicts for different volumes the relationship between cycle length and delay. One can find the optimal cycle lengths for three different volumes. The optimal cycle lengths for 400, 600, and 750 vph cases are 40, 60, 120 seconds, respectively. Results of cycle lengths under fuel consumption minimization and delay minimization are listed in Table 4.3.

TABLE 4.3 OPTIMAL CYCLE LENGTH FOR DELAY
AND FUEL CONSUMPTION MINIMIZATION

Phase I		Phase II		Delay Minimization	Fuel Consumption Minimization (The Analytical Fuel Consumption Model)
Volume	$\frac{g}{c}$	Volume	$\frac{g}{c}$		
400 vph	0.5	400 vph	0.5	40 seconds	50 seconds
600 vph	0.5	600 vph	0.5	60 seconds	80 seconds
750 vph	0.5	750 vph	0.5	120 seconds	100 seconds

The optimal cycle lengths based on delay minimization for the 400 and 600 vph cases are 40 and 60 seconds and are shorter than those of fuel consumption minimization.

For the high volume 750 vph case, the optimal cycle length for delay minimization is longer than for fuel consumption minimization. However, the results for both criteria show that short cycle lengths are preferred in low volume cases and likewise, long cycle lengths are preferred in high volume cases .

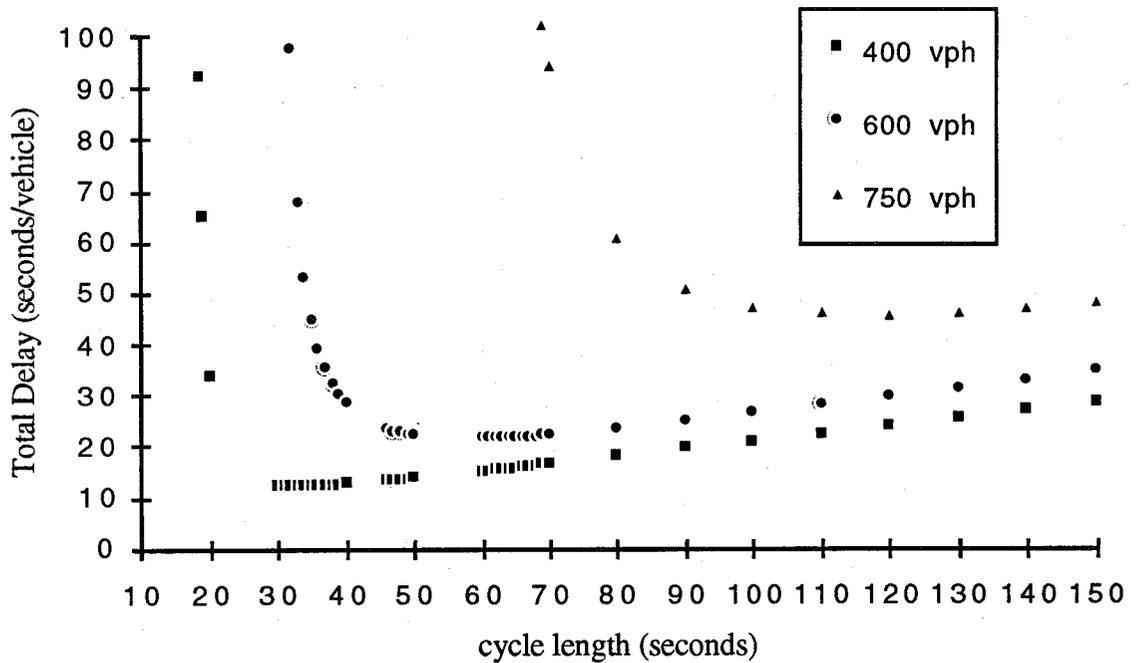


Figure 4.7 Optimal Cycle Length Versus Traffic Volume from Webster's Delay Model

Signal Settings for Fuel Consumption Minimization from Other Models

Several studies (2, 8, 9) have focused on traffic signal timing and fuel consumption. The optimal cycle lengths for fuel consumption minimization provided by these studies are different from each other, and they are different from the Analytical Fuel Consumption Model. Bauer (2) used an incremental fuel consumption model to analyze the variation of fuel consumption due to signal cycle length changes and found that the optimal cycle length for minimizing fuel consumption, as shown in Table 4.4, is much longer than

the cycle length for minimizing isolated intersection delay.

TABLE 4.4 OPTIMAL CYCLE LENGTH VERSUS
TRAFFIC VOLUME FROM BAUER'S MODEL

Σ volume	Delay Minimization	Fuel Consumption Minimization
800 vph	28 seconds	48 seconds
1200 vph	53 seconds	80 seconds
1400 vph	95 seconds	127 seconds

Courage and Parapar (9) used the similar approach and obtained the same results as Bauer's, i.e., the optimal cycle length for minimizing fuel consumption is much longer than the cycle length for minimizing isolated intersection delay. However, the results are different from the studies of Cohen and Euler (8) where optimal cycle lengths for minimizing delay and fuel consumption are very similar. Cohen and Euler (8) used NETSIM to evaluate fuel consumption for different signal timing plans and obtained results as shown in Table 4.5.

TABLE 4.5 OPTIMAL CYCLE LENGTH VERSUS TRAFFIC
VOLUME FROM COHEN AND EULER'S MODEL

Phase I ¹	Phase II ²	$\frac{g_1}{g_2}$	Delay Minimization	Fuel Consumption Minimization
Volume	Volume			
1600 vph	500 vph	1.525	60 seconds	60 seconds
1800 vph	400 vph	2.143	80 seconds	80 seconds
1000 vph	800 vph	0.608	100 seconds	80 seconds

1. East-west, 2 lanes per approach and one left turn bay on east approach.
2. North-south, one lane per approach.

Results from these models are based on different experimental designs and different modeling approaches, and this may partially account for different results. Results in this

research, however, are close to those of Cohen and Euler where the optimal cycle lengths for minimizing delay and fuel consumption are similar.

SUMMARY

An experimental design is described and analysis results are discussed in this chapter. Relative fuel consumption rates for the new model developed in Chapter 3 are used to calculate fuel consumption during different stages of a signal cycle. Optimal cycle lengths are provided for different traffic volumes based on fuel consumption and delay minimization.

For fuel consumption minimization, the optimal cycle lengths for the 400, 600, 750 vph cases are 50, 80, and 100 seconds from the Analytical Fuel Consumption Model compared to 40, 80, and 70 seconds from the TEXAS model. For delay minimization, the optimal cycle lengths are 40, 60, and 120 seconds, respectively.

Results from the Analytical Fuel Consumption Model and the TEXAS model are different because the TEXAS model includes random traffic stream characteristic. However, the results for both fuel consumption and delay minimization show that short cycle lengths are preferred in low volume cases and likewise, long cycle lengths are preferred in high volume cases.

A much more general version of the analytical Fuel Consumption Model is currently under development. This version will include provisions that will significantly extend it's usefulness.

CHAPTER 5 CONCLUSION

OVERALL CONCLUSIONS

This research develops a comprehensive model framework to estimate fuel consumption at a signalized intersection and analyzes the relationship between cycle length and fuel consumption. A new model (the Analytical Fuel Consumption Model) is developed as nine sub-models dealing respectively with three signal cycle stages and three street elements. The conclusions of the research are:

1. The Analytical Fuel Consumption Model is developed for three street elements including inbound and outbound approaches and the intersection. For each street element, a model for each of three signal cycle stages is described. The flexible design of this model can be extended to include impacts of residual queues, turning movements, and arrival patterns.
2. Results of the fuel consumption predictions from the Analytical Fuel Consumption Model are consistent with those of the TEXAS simulation model. The patterns of the variation of fuel consumption within a cycle from the Analytical Fuel Consumption Model and the TEXAS model are similar. Fuel consumption increases in the effective red time on the inbound approach, increases dramatically which vehicles accelerate into the intersection, decreases at the end of t_0 , and remains stable as vehicles travel on the outbound approach.
3. From the Analytical Fuel Consumption Model, the optimal cycle length for minimizing fuel consumption is 50 seconds for the 400 vph case, 80 seconds for the 600 vph case, and 100 seconds for the 750 vph case. The results are slightly different from the TEXAS model where the optimal cycle lengths for 400, 600, 750 vph cases are 40, 80, and 70 seconds, respectively.
4. The optimal cycle lengths based on delay minimization for volume 400 and 600 vph

cases are 40 and 60 seconds from Webster's delay equation and are shorter than those for fuel consumption minimization. For the high volume 750 vph case, the optimal cycle length for delay minimization is 120 seconds and is higher than that for fuel consumption minimization. However, results for both criteria show that short cycle lengths are preferred in low volume cases and likewise, long cycle lengths are preferred in high volume cases.

FURTHER RESEARCH

The Analytical Fuel Consumption Model, developed based on the assumptions that vehicle arrival patterns are uniform and constant and straight movement only, is not practical for real traffic situations and roadway conditions. The model should be extended in the following ways:

1. Incorporate Poisson and binomial arrival flow patterns and permit residual queues.
2. Include effects of a mixed vehicle fleet, turning movements, and channelization.

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APPENDIX A

FUEL CONSUMPTION RATE AT DIFFERENT SPEED AND ACCELERATION/DECELERATION (10⁻⁵ gallon/sec)

speed (ft/sec)	acceleration/deceleration (ft/sec ²)																		
	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
0	13	13	13	13	13	13	13	13	13	13	18	19	21	23	25	28	31	33	35
1	13	13	13	13	13	13	13	13	13	13	18	20	22	25	27	30	33	36	39
2	14	14	14	14	14	14	14	14	14	14	18	21	24	27	29	32	36	40	42
3	14	14	14	14	14	14	14	14	14	14	18	22	26	29	33	36	40	44	47
4	15	15	15	15	15	15	15	15	15	15	19	23	28	32	36	40	44	49	52
5	16	16	16	16	16	16	16	16	16	16	20	25	30	36	40	45	49	54	58
6	16	16	16	16	16	16	16	16	16	16	20	26	32	39	44	50	54	60	65
7	17	17	17	17	17	17	17	17	17	17	22	28	35	42	48	54	59	65	71
8	17	17	17	17	17	17	17	17	17	17	23	30	38	45	52	58	64	71	78
9	17	17	17	17	17	17	17	17	17	17	24	32	41	48	56	63	70	77	84
10	17	17	17	17	17	17	17	17	17	17	26	34	43	51	60	67	76	83	91
11	17	17	17	17	17	17	17	17	17	17	27	36	45	54	63	71	81	89	98
12	17	17	17	17	17	17	17	17	17	17	29	38	48	58	67	75	85	94	104
13	17	17	17	17	17	17	17	17	17	18	30	39	50	61	70	78	89	100	109
14	17	17	17	17	17	17	17	17	17	18	32	42	53	64	74	82	93	105	115
15	17	17	17	17	17	17	17	17	17	18	35	44	56	67	77	86	98	110	120
16	17	17	17	17	17	17	17	17	17	18	31	47	59	70	80	91	103	116	126
17	18	18	18	18	18	18	18	18	18	19	32	50	62	74	83	96	109	121	132
18	18	18	18	18	18	18	18	18	18	19	33	44	65	77	88	101	114	127	138
19	18	18	18	18	18	18	18	18	18	19	35	46	69	80	93	107	120	133	144
20	18	18	18	18	18	18	18	18	18	20	35	48	73	84	98	113	126	139	148
21	18	18	18	18	18	18	18	18	18	20	36	50	77	87	103	118	132	145	151
22	18	18	18	18	18	18	18	18	18	20	37	52	73	91	108	123	137	151	155
23	18	18	18	18	18	18	18	18	18	21	38	54	76	94	111	128	143	157	159
24	18	18	18	18	18	18	18	18	18	21	39	56	78	97	115	132	148	161	162

25	18	18	18	18	18	18	18	18	18	22	40	57	80	100	119	137	153	164	165
26	18	18	18	18	18	18	18	18	18	22	42	57	83	104	122	141	158	167	168
27	18	18	18	18	18	18	18	18	18	23	43	59	85	108	126	146	162	169	171
28	18	18	18	18	18	18	18	18	18	23	44	61	88	112	129	150	166	172	173
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48	17	17	17	17	17	17	17	17	17	36	61	100	129	181	238	249	251	251	251
49	17	17	17	17	17	17	17	17	17	28	63	102	132	185	243	252	253	253	253
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61	18	18	18	18	18	18	18	18	19	37	67	121	179	225	239	239	239	239	239
62	18	18	18	18	18	18	18	18	19	38	69	125	183	227	239	239	239	239	239
63	18	18	18	18	18	18	18	18	19	38	71	127	187	229	238	239	239	239	239
64	18	18	18	18	18	18	18	18	20	39	74	129	191	231	237	237	237	237	237
65	18	18	18	18	18	18	18	18	20	39	77	132	195	232	235	235	235	235	235
66	19	19	19	19	19	19	19	19	20	39	81	134	198	224	227	227	227	227	227
67	19	19	19	19	19	19	19	19	21	39	84	136	202	224	227	227	227	227	227
68	19	19	19	19	19	19	19	19	21	40	86	139	205	225	227	227	227	227	227
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70	20	20	20	20	20	20	20	20	23	43	89	144	212	226	228	228	228	228	228