A DISCRETE-ELEMENT ANALYSIS FOR ANISOTROPIC SKEW PLATES AND GRIDS

By

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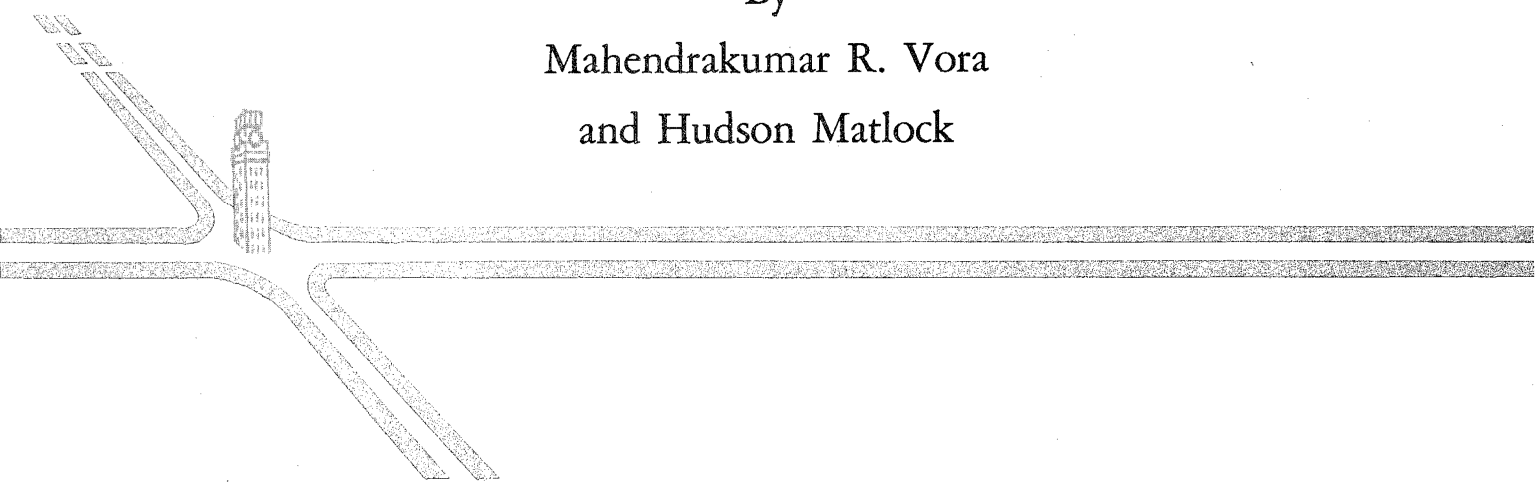

SUMMARY REPORT 56-18 (S)

SUMMARY OF
RESEARCH REPORT 56-18

PROJECT 3-5-63-56

COOPERATIVE HIGHWAY RESEARCH PROGRAM
WITH TEXAS HIGHWAY DEPARTMENT
AND
U. S. DEPARTMENT OF TRANSPORTATION
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SUMMARY REPORT 56-18 (S)

Foreword

Research Report 56-18 describes a discrete-element method of analysis for anisotropic skew-plate and grid-beam systems. Relationships are developed which simplify the computation of anisotropic slab stiffnesses. A computer program is included which is capable of analyzing anisotropic skew plates or slabs with grid-beams. This report is the eighteenth in a series of reports that describes work in Research Project 3-5-63-56, "Development of Methods for Computer Simulation of Beam-Columns and Grid-Beam and Slab Systems."

Introduction

Analysis of skew slabs or plates with skew ribs is always difficult, since there are no closed-form mathematical solutions available for even the simplest cases. The practicing engineer would use some approximate procedure for analysis, such as considering a strip of slab in the span direction, and analyze it as a beam. This kind of approximation may be reasonable for a rectangular slab but may be inappropriate in the case of a skew slab; generally, because there are large twisting effects, the largest principal moments are not in the span direction.

The objective of this report is to develop relationships for elastic compliances such that the computation of anisotropic plate stiffness is simplified and to develop a discrete-element method of analysis for anisotropic skew-plate and grid-beam systems in which the grid-beams may run in any three directions.

Discrete-Element Model

A mechanical model consisting of a tridirectional system of rigid bars and elastic joints is used to simulate anisotropic skew plates plus slab-and-grid systems in which the grid-beams may run in any three directions (Fig 1 and 2). The model allows for the free linear elastic variation of stiffnesses and support characteristics. Loads are applied at each joint to represent any degree of concentrated or uniform loading.

Using concepts of a continuum composed of interconnected fibers, stress-strain relationships for the anisotropic slab model are derived. Each grid-beam

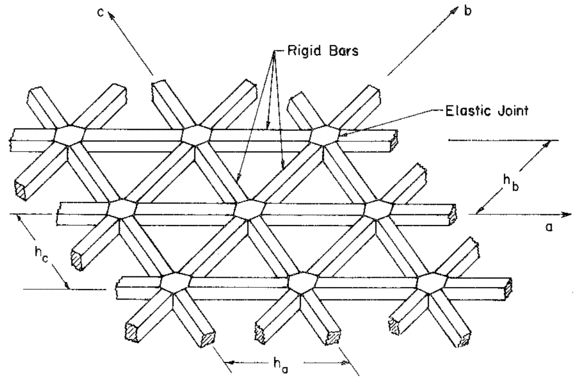

Fig 1. Discrete-element model for anisotropic skew plate showing all components.

may thus be considered the same as a previously developed discrete-element beam-column (Refs 3 and 4).

Anisotropic Relations

For plane stress problems involving generally anisotropic materials, the stress-strain relationships require computation of six elastic compliances or six elastic stiffnesses. Relationships are developed in which the six compliances and six stiffnesses are related to three moduli of elasticity with respect to any three directions and three Poisson's ratios related to these directions. Since the stiffnesses of an anisotropic plate or its discrete-element model are functions of these six elastic stiffnesses, the above simplification is helpful in understanding and in computing their values.

Method of Analysis

An equilibrium equation is applied at each joint of the discrete-element model and resulting equations

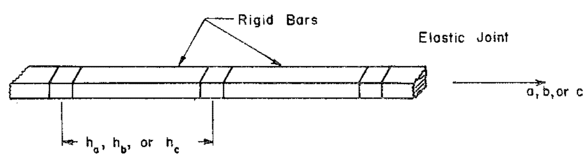

Fig 2. Discrete-element model for a typical grid-beam showing all components.

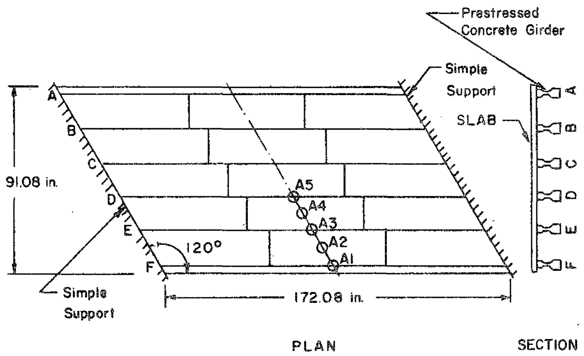

Fig 3. Model bridge investigated by Barboza (Ref 1).

are arranged and partitioned. A recursion-inversion solution procedure (Ref 2) is used to solve these equations.

Computer Program

A computer program, SLAB 44, is written to apply the discrete-element formulation of an anisotropic skew-plate and grid-beam system (Fig 1) in which the grid-beams may run in any three directions.

The computer program SLAB 44 is written in FORTRAN for the CDC 6600 computer. The program is easily made compatible with IBM 360, UNIVAC 1108, or other similar systems.

The input value of slab stiffness may be related to either orthogonal directions or three specific directions. The program output consists of deflections, bending moments and twisting moments (or bending moments in three directions), largest principal moments together with their directions, and support reactions (or a statics check at each joint of the discrete-element model).

Example Problems

Several series of example problems are included in the report to verify the discrete-element formulation. The results are compared with the solutions from other approximate methods, such as series, finite-element, conformal mapping, finite difference, and electrical analog, and with experimental results.

One of the problem series presented in the report is used to verify the program by comparison to the results of an actual experimental study. Figure 3 shows a simplified model of a Texas Highway Department standard simply-supported bridge with a skew angle of 30 degrees. Barboza (Ref 1) experimentally investigated the behavior of this prestressed concrete bridge under various loading conditions. Program SLAB 44 was used to analyze this model

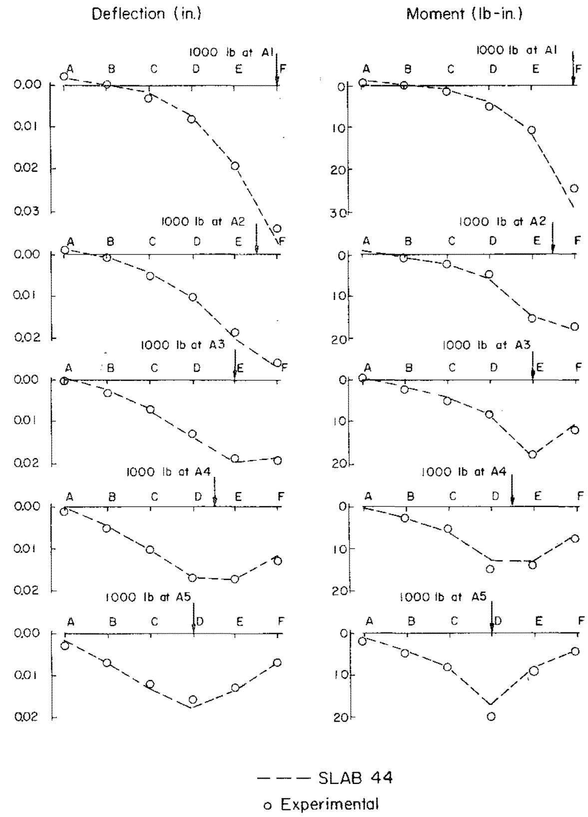

Fig 4. Comparison of SLAB 44 and experimental results (Ref 1).

bridge for five different test positions of concentrated load. The results of deflections and bending moments at the mid-spans of the girders are compared with the experimental results. Figure 4 shows these comparisons and it is evident that there is very close correlation between experimental results (Ref 1) and those from SLAB 44.

This problem series also effectively demonstrates the modeling of composite action, which was included as part of the supporting beam stiffnesses. It also shows that the diaphragms can be modeled very simply even though they run in neither the span direction nor the skew direction.

Conclusions

A discrete-element procedure of analysis of an anisotropic skew-plate and grid-beam system is presented. It has been observed from the literature studied that most of the work done on skew plates is for either isotropic or orthotropic properties and for simple load and support conditions. The method presented in this report is not limited by these considerations. The stiffnesses, loads, and supports can be freely varied from point to point and in any direction. Concentrated and distributed loads, foundation support springs, and external couples in three directions can be easily handled. The relations developed simplify the computation of anisotropic stiffnesses in terms of six conventional constants which could be experimentally determined by three uniaxial test specimens.

The full text of Research Report 56-18 can be obtained from R. L. Lewis, Chairman, Research and Development Committee, Texas Highway Department, File D-8 Research, 11th and Brazos Streets, Austin, Texas 78701 (512/475-2971).

References

1. Barboza, N. J., "Load Distribution in a Skewed Prestressed Concrete Bridge," Master's Thesis, The University of Texas at Austin, August 1970.
2. Endres, F. E., and Hudson Matlock, "An Algebraic Solution Process Formulated in Anticipation of Banded Linear Equations," Research Report No. 56-19, Center for Highway Research, The University of Texas at Austin, June 1970.
3. Matlock, Hudson, and T. A. Haliburton, "A Finite-Element Method of Solution for Linearly Elastic Beam-Columns," Research Report No. 56-1, Center for Highway Research, The University of Texas, Austin, September 1966.
4. Matlock, Hudson, and T. P. Taylor, "A Computer Program to Analyze Beam-Columns Under Movable Loads," Research Report No. 56-4, Center for Highway Research, The University of Texas at Austin, June 1968.

KEY WORDS: anisotropic elasticity, skew plate, skew grid, slab-grid system, skew bridge, discrete-element analysis, computers, bridges, plates.