

TEXAS
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TEXAS
HIGHWAY
DEPARTMENT

COOPERATIVE
RESEARCH

RE-EVALUATION OF TRUCK CLIMBING
CHARACTERISTICS FOR USE IN
GEOMETRIC DESIGN

in cooperation with the
Department of Transportation
Federal Highway Administration
Bureau of Public Roads

RESEARCH REPORT 134-2
STUDY 2-8-68-134
HIGHWAY DESIGN CRITERIA

RE-EVALUATION OF TRUCK CLIMBING CHARACTERISTICS
FOR USE IN GEOMETRIC DESIGN

by

John C. Glennon

and

Charles A. Joyner, Jr.

Research Report Number 134-2

Highway Design Criteria

Research Study No. 2-8-68-134

Sponsored by

The Texas Highway Department
In Cooperation with the
U. S. Department of Transportation
Bureau of Public Roads

August 1969

TEXAS TRANSPORTATION INSTITUTE
Texas A&M University
College Station, Texas

FOREWORD

This report describes one phase of Research Study No. 2-8-68-134 entitled "An Evaluation of the Basic Design Criteria as They Relate to Safe Operation on Modern High Speed Highways." Other reports published under this research study include: No. 134-1, The Passing Maneuver as it Relates to Passing Sight Distance Design Standards; No. 134-3, Evaluation of Stopping Sight Distance Design Criteria; and No. 134-4, State of the Art Related to Safety Criteria for Highway Curve Design. Separate reports and summary reports have been prepared for all phases of this research.

DISCLAIMER

The opinions, findings, and conclusions expressed or implied in this report are those of the research agency and not necessarily those of the Texas Highway Department or of the Bureau of Public Roads.

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ABSTRACT

An examination of the state of knowledge was conducted for the purpose of evaluating design criteria which relate truck operating characteristics on grades to the implementation of truck climbing lanes. The evaluation was specifically concerned with truck speed-distance characteristics on grades, truck weight-horsepower ratios related to climbing characteristics, and the speed reduction design criterion for initiating truck climbing lanes.

The evaluation was addressed to design criteria as presented in "A Policy on Geometric Design of Rural Highways, 1965," by the AASHO. The major findings were:

- The truck speed-distance curves presented in the AASHO Policy appear to be adequate for design. These curves were developed for a design vehicle with an approximate weight-horsepower ratio of 400:1, which represents a reasonable lower boundary for trucks presently on the highway.
- Based on a comparison of truck accident involvement rates, it was determined that the speed reduction criterion for initiating truck climbing lanes should be lowered from 15 mph to 10 mph.

SUMMARY AND FINDINGS

In the design of highway grades, consideration is given to the critical length of grade. The critical length of grade is that combination of grade percent and length which will cause a designated design vehicle to operate at some predetermined minimum speed. A lower speed is considered unacceptable for safety and operational efficiency. There are two alternatives which are considered when a designed grade is longer than critical; (1) adjust the grade line until it is no longer critical, or (2) add an auxiliary truck climbing lane in which slow-moving vehicles can operate adjacent to the main travel lane.

This study was conducted in response to an increasing concern by highway design engineers regarding the validity of geometric design criteria related to the safe operation of slow-moving vehicles on highway grades. The report presents a review of current AASHO (1) design criteria and an evaluation of these criteria, based on the existing state-of-the-art. The evaluation was specifically concerned with truck operating characteristics on grades, truck weight-horsepower ratios related to operating characteristics, and truck speed as it relates to operation characteristics and geometric design criteria.

Based on the evaluation of the state-of-the-art, which covered several truck gradability studies and prediction procedures, it was concluded that there was no substantiated justification for upgrading the truck gradability curves developed by Huff and Scrivner (2) as employed by the 1965 AASHO Policy (1). These curves were theoretically derived and validated by road tests of a heavily loaded truck with an approxi-

mate weight-horsepower ratio of 400:1. The trucking industry appears to have accepted this ratio as a performance control, although this does not account for overloading conditions which are occasionally practiced. From all indications of the trends in weight-horsepower ratios of trucks in operation, the 400:1 ratio appears to have continuing application as a design criterion.

The truck gradability curves developed by Huff and Scrivner utilize a 47-mph speed for trucks entering a grade from a level section. This represented the maximum sustained speed of the test truck on a level grade. This speed was the average of all trucks on Texas highways in 1953 and was considered as representative of a critical operating condition. Actually, a more representative critical speed would be that speed which is exceeded by, say, 85 percent of the trucks on the highway. The 1968 Texas Highway Department Speed Survey indicated that approximately 85 percent of the trucks exceeded 47 mph. It was concluded, therefore, that the 47-mph truck entering speed is applicable for current design considerations.

The AASHO Policy currently employs a 15-mph speed reduction criterion for determining critical lengths of grades. No objective basis could be found for this criterion. By applying some existing data, an objective basis for a speed-reduction criterion was established in the report.

From a study (3) conducted by the Bureau of Public Roads, a curve was developed which related accident involvement rate to deviation from the average speed of the traffic stream. This relationship showed

that the involvement rate increases logarithmically as this deviation increases. Employing this relationship and the 1968 Texas Highway Department Speed Survey data, accident involvement rates were computed for various speed reductions of 4-or-more-axle trucks. This relationship is plotted in Figure 16. This figure illustrates that the accident involvement rate related to a 15-mph speed reduction of the design vehicle is almost nine times that of a zero-mph reduction. It is also noted that the involvement rate increases very rapidly for increases in speed reduction beyond 10 mph. From this relationship, it was concluded that a 10-mph speed reduction criterion should be substituted for the present 15-mph criterion.

Highway engineers have been concerned that present design criteria are often responsible for truck climbing lanes that are too short for efficient operation. Operational problems are created because:

- 1) With the present 15-mph speed reduction criterion, it has been common practice to end a climbing lane when the design truck regains a speed equivalent to that speed for which the climbing lane was begun. This practice, for many profile conditions, allows the ending of the climbing lane shortly over the crest of the hill. This practice can create a lack of adequate operational sight distance to the end of the climbing lane, especially for slow-moving automobile drivers who choose to use the auxiliary lanes.
- 2) Truck drivers find it difficult to maintain desired

operation of their vehicles on short climbing lanes and therefore, by experience are often reluctant to use climbing lanes in areas where they know these auxiliary lanes tend to be short.

Although the report did not investigate the optimum length of truck climbing lanes, it was concluded that the substitution of a 10-mph speed reduction criterion in place of the current 15-mph criterion, would alleviate the operational problems discussed above.

Recommendations for Implementation

The following recommendations are proposed based on the findings of this report.

1. The truck gradability curves presented in the AASHO Policy should be retained as a design tool.
2. Consideration should be given to adopting a 10-mph speed reduction criterion for designing truck climbing lanes and critical lengths of grade.
3. As a general design principal, consideration should be given to extending the acceleration portion of the climbing lane on the steeper downgrades to allow trucks to obtain a re-entry speed closer to the average running speed on the highway. For the steeper downgrades, substantial reductions in the accident involvement potential are achieved with each small addition of lane length.
4. Consideration should be given to joining consecutive climbing

lanes which are separated by a short interval of highway. This would eliminate a hazardous weaving situation and would further encourage truck drivers to use the auxiliary lanes.

INTRODUCTION

Of all vehicles operating on our highways, the large transport trucks have the lowest engine power relative to their weight. Hence, these vehicles are generally the slowest on upgrades and require the longest distances to accelerate. Realistic design of highway grades and acceleration lanes should be based on the performance of these particular vehicles, inasmuch as all other vehicles can perform better.

The description of truck operating characteristics on grades used by the Texas Highway Department (1)* was developed from in-house studies (2). The design criteria for critical lengths of grade and truck climbing lanes are presented in the AASHO's "A Policy on Geometric Design of Rural Highways, 1965" (3). The AASHO Policy also presents the truck operating characteristics that were developed in the Texas study.

It was the purpose of this report to evaluate the validity of current design criteria for critical lengths of grade and truck climbing lanes. To perform this evaluation, an examination was conducted on the current state of knowledge concerning truck operating characteristics on grades, truck weight-horsepower ratios as they relate to truck operating characteristics, and truck speeds as they relate to safe truck operations on grades.

*Number denotes reference listed in the Bibliography.

STATE OF THE ART

This section attempts to present a comprehensive picture of the state of knowledge concerning design criteria which relate truck operating characteristics on grades to the implementation of critical lengths of grade and truck climbing lanes. The topics of discussion include: (1) truck operating characteristics on grades; (2) truck weight-horsepower ratios related to climbing characteristics; and (3) design criteria for critical lengths of grade and truck climbing lanes.

Truck Operating Characteristics on Grades

An extensive study (4) of truck performance was conducted in 1938-41 to determine the separate and combined effects of roadway grade, tractive effort, and gross vehicle weight. Data from this study were analyzed (5) to determine the effect of length of grade on the speed of trucks for a wide range in load, grade, and vehicle size. Speed-distance curves were developed using three weight classifications: light, medium, and heavy. These curves formed the basis for the 1954 AASHO Policy (6) design criteria for critical lengths of grade.

In 1949, Willey (7) documented the performance of trucks on grades. He developed speed profiles of truck performance on different mountainous grades in Arizona. The observed trucks were first grouped into gross vehicle weight classifications but, because of inconsistencies noted in the relationship between the groups, they were reclassified according to the following gross vehicle weights to brake

horsepower ratios:

Group A - Up to 199 lbs./BHP

Group B - 200 to 299 lbs./BHP

Group C - 300 to 399 lbs./BHP

Group D - over 400 lbs./BHP

Willey developed a gradability curve of heavily loaded trucks, (combination of Group C and Group D), which showed the probable average behavior to be expected from vehicles loaded to capacity, or nearly so, on various grades (See Figure 1).

Huff and Scrivner (2) used Willey's gradability curves in developing their simplified climbing-lane theory. This theory considered the forces acting upon a truck ascending a grade (See Figure 2) to develop the force equation:

$$\frac{W}{g} \frac{dv}{dt} = P - W \sin \theta \quad (1)$$

where

W = gross vehicle weight, in lbs.;

g = acceleration of gravity, 32.2 ft/sec²;

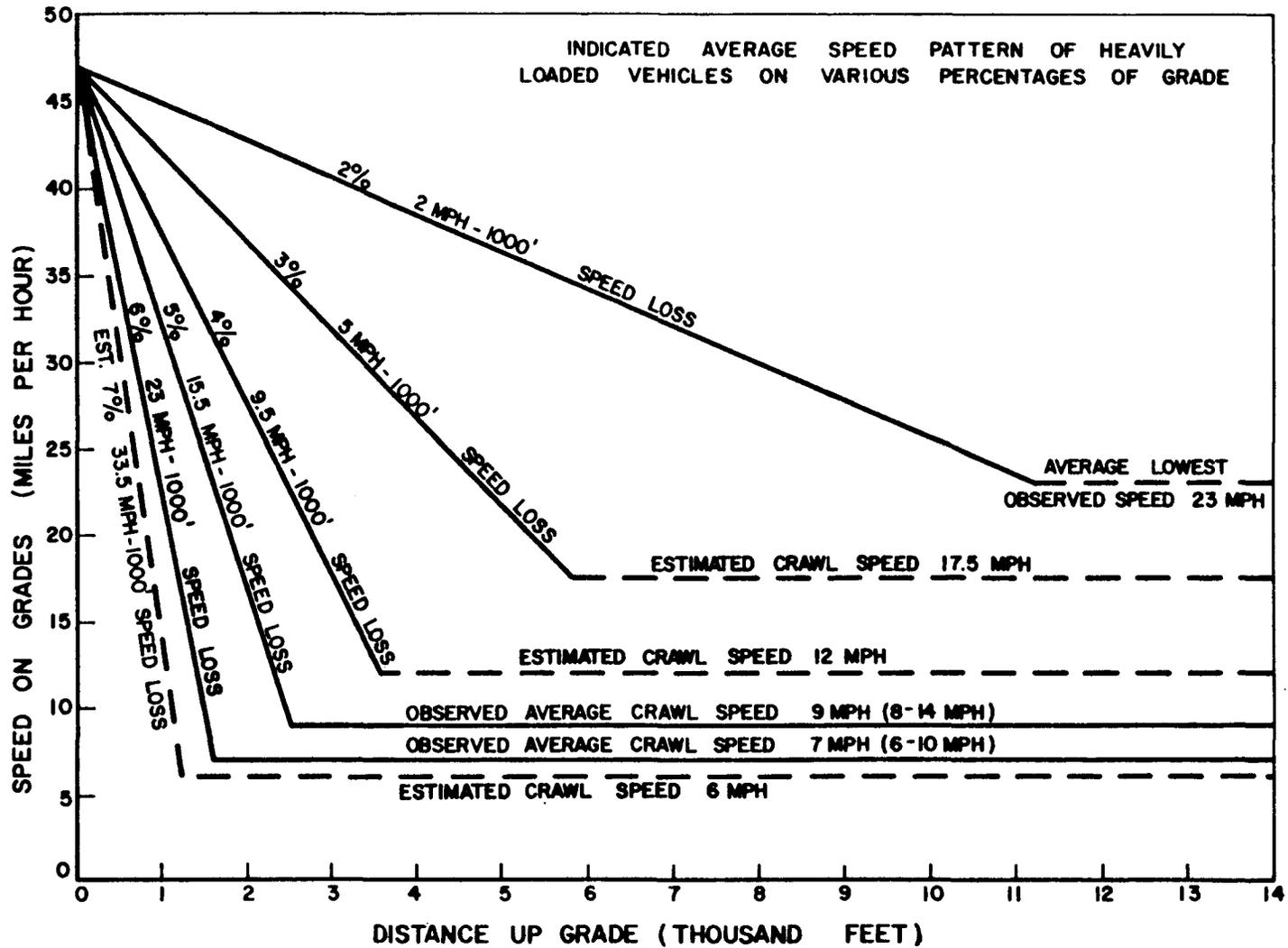
dv/dt = change in velocity with respect to time, in ft/sec²;

P = net driving force on the vehicle, in lbs., and

θ = the grade angle, in degrees

This equation holds when the driving force needed to impart angular acceleration to the rotating engine parts is neglected. Equation 1 may be written as:

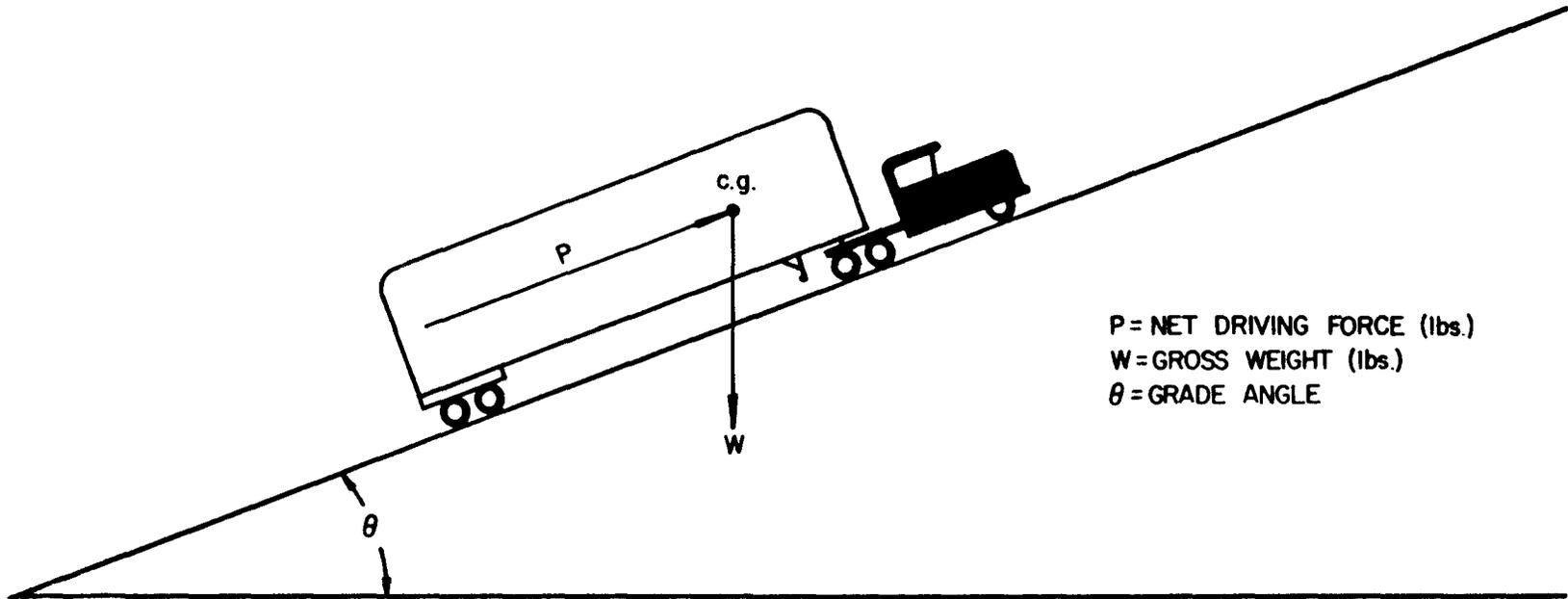
$$\frac{P}{W} = \frac{1}{g} \frac{dv}{dt} + \sin \theta \quad (2)$$



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Figure 1. Willey's Heavily Loaded Truck Gradability Curves on Different Grades.

5



P = NET DRIVING FORCE (lbs.)
 W = GROSS WEIGHT (lbs.)
 θ = GRADE ANGLE

MASS x ACCELERATION = FORCE

$$\frac{W}{g} \frac{dv}{dt} = P - W \sin \theta$$

Figure 2. Truck Ascending Diagram.

The net driving force acting on the vehicle, P , is the total traction exerted by the driving wheels against the road surface, less wind and road surface resistance.

Engine operation at partial throttle was not considered because it would mean that the driver's choice, rather than highway geometry, would determine the vehicle performance. Therefore, if the truck operates at the highest possible speed and within the manufacturer's recommendations, it is possible to approximate the total driving force as a function of the velocity only. If the following assumptions were made:

1. No inertial resistance to angular acceleration;
2. No wind exists, thereby considering air resistance as a function of the velocity; and
3. No change in pavement type or roughness, thereby considering surface resistance as a function of the velocity;

it was concluded, therefore, that although the net driving force must satisfy Equation 2, it may also be expressed as some function of velocity only.

If a truck operates at maximum sustained speed on any grade, the value of P/W may be calculated from Equation 2, which reduces to $P/W = \sin \theta$. This value of P/W will always exist at the respective speed, at least approximately, regardless of the value of the acceleration.

Figure 3 relates P/W to maximum sustained speeds, v , on various grades. The maximum sustained speeds were taken from the gradability curves in Figure 1. The points plotted in Figure 3 were connected by straight line segments to form a continuous graph. Each line segment

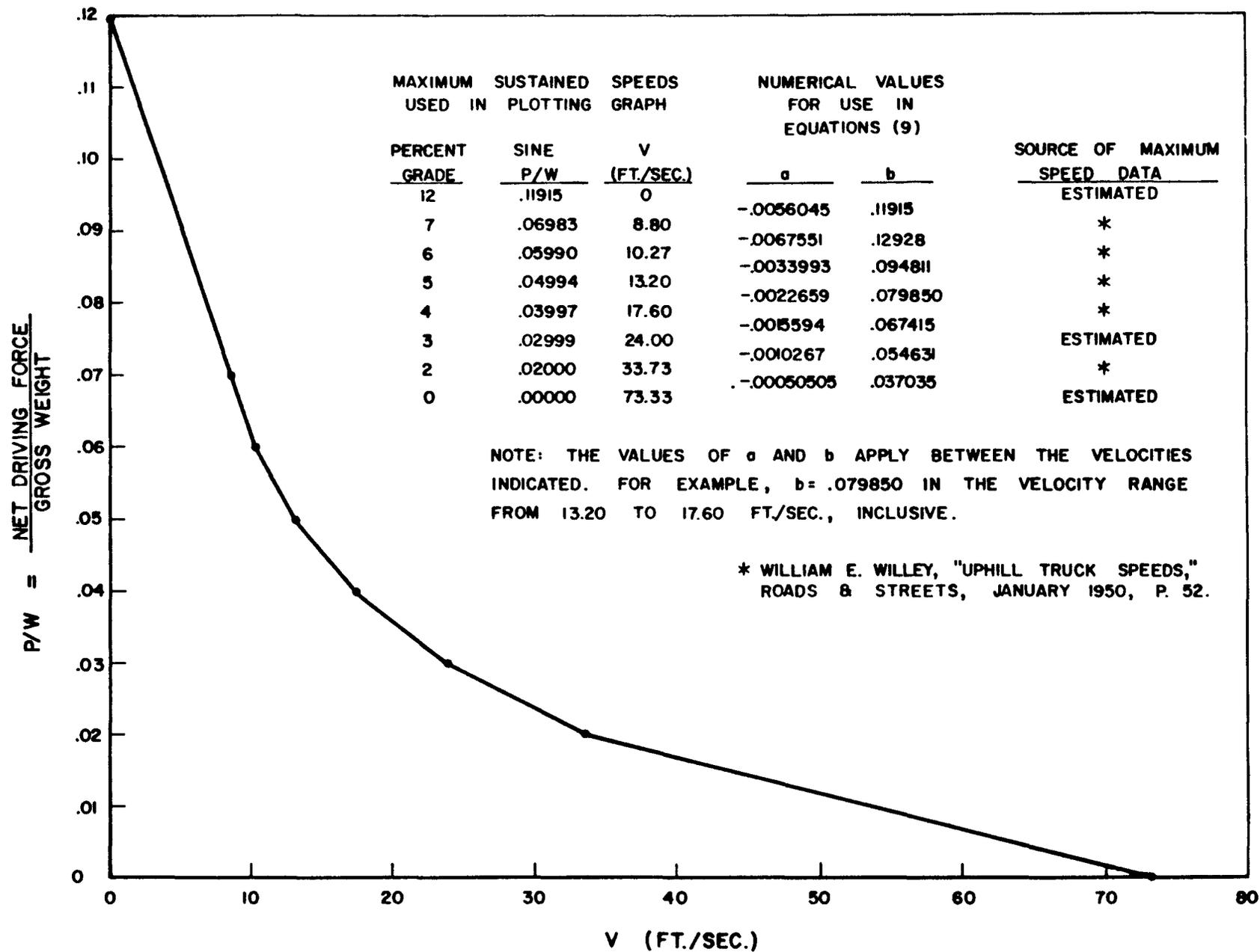


Figure 3. P/W Versus Maximum Sustained Speeds on Various Grades.

was represented by the general equation:

$$P/W = av + b \quad (3)$$

where v is the velocity at any point along a line segment, v_n to v_{n+1} , and a and b are constant along the same line segment. By substituting the P/W value of Equation 2 into Equation 3, a new general motion equation was derived:

$$\frac{dv}{dt} - gav + g(\sin \theta - b) = 0 \quad (4)$$

where v , a , and b are restricted as noted above.

The position of the truck along the grade may be represented at any instant by its coordinate, x , measured along the direction of the truck. If $\frac{dv}{dt}$ is the change in velocity, v , with respect to time, t , along any particular line segment and \bar{v} is the average speed along that line segment, then Equation 4 may be developed into an equation suitable for the construction of speed-distance and time-distance curves (See Appendix A):

$$x = \frac{v - v_o}{2g} + (\sin \theta - b) t \quad (5)$$

where:

$$t = \frac{1}{ag} \ln \left(\frac{av - \sin \theta + b}{av_o - \sin \theta + b} \right) \quad (5a)$$

To construct speed-distance curves using Equation 5, where the velocity change involves more than one line segment, the distance or time must be calculated over each interval and added, in order to obtain total distance or total time. Actually, by utilizing the same assumptions made by Huff and Scrivner in developing Equation 5 and 5a, a much simpler

singular speed-distance may be derived (See Appendix B):

$$X = \frac{1}{g} \frac{V_0^2 - V^2}{a(v_0 - v) - 2(\sin \theta - b)} \quad (6)$$

In December of 1953, Huff and Scrivner (2) conducted a road test of a heavy truck to determine whether the above theoretical equations applied to the actual performance on grades. The operating conditions and data for the truck tested are presented in Table 1. Eleven grades, ranging from 700 to 1,500 feet in length and from 0.16 to 7.62 percent in grade, were used in the tests.

Figure 4 was developed from the data obtained in the tests of the heavy truck. Each computed value of P/W was plotted against its corresponding velocity. The points represent any instant where the acceleration was not zero, and the circles represent any instant at which the truck was operating at maximum sustained speeds. Certain areas, where the points were scattered so as not to represent any consistency, were ignored and an average line was drawn through the remaining points. This line represented P/W as a function of velocity only.

The data presented in Table 1 were also used to compute the maximum sustained speeds using the SAE Truck Ability Prediction Procedure (8). The SAE computation sheet with the example of a 3-percent grade is shown in Table 2. This sheet was used in conjunction with the several graphs presented in the SAE publication to arrive at maximum sustained speeds. These speeds were plotted against the corresponding $\sin \theta$ and plotted on the same graph in Figure 4.

The average values of P/W versus velocity from the graph in Figure

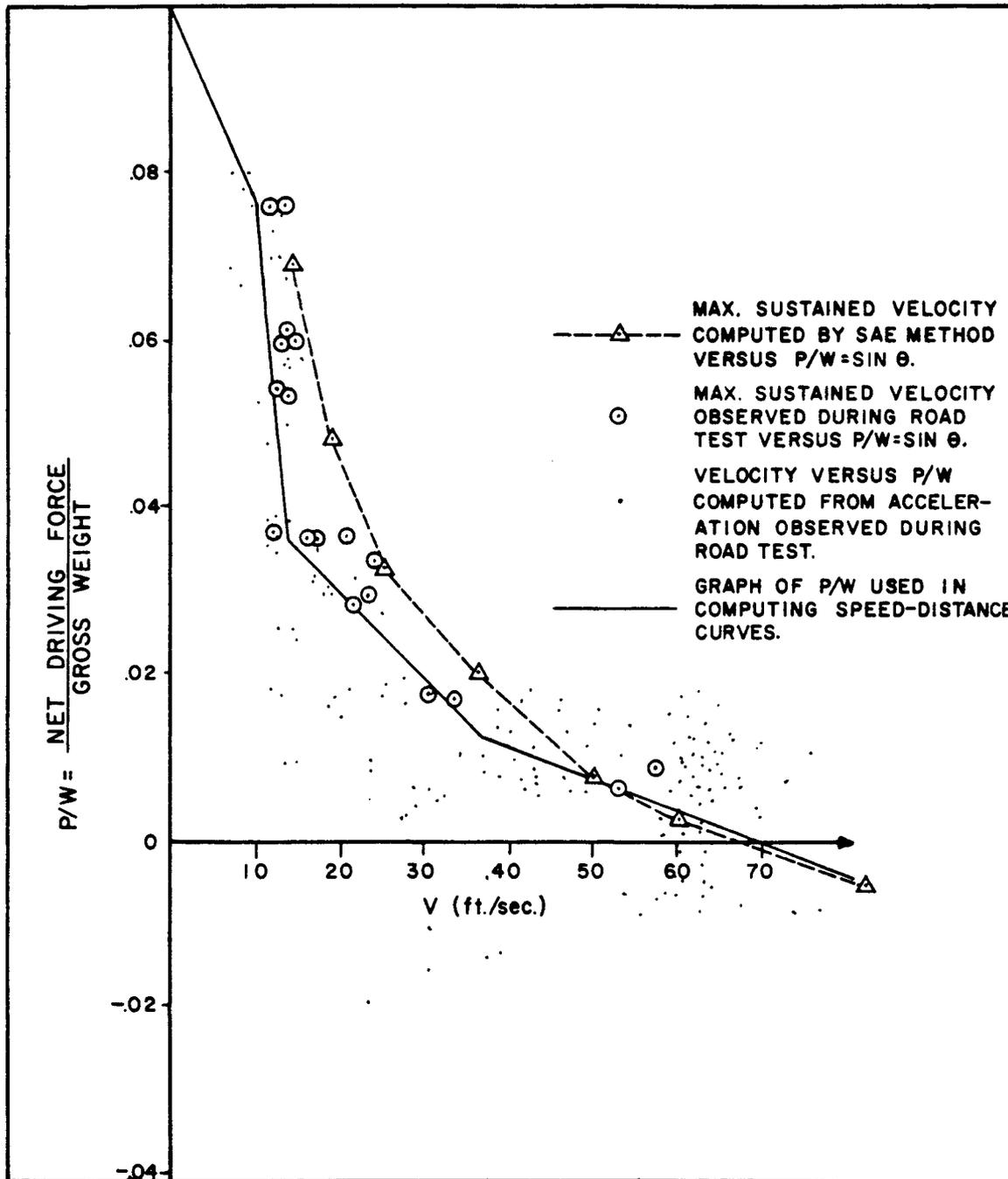


Figure 4. Graph of P/W Versus V for 1953 Road-Test Data.

TABLE 1
OPERATING CONDITIONS OF TEST TRUCK

Vehicle:	International R-195 Tractor with Hobbs tandem-axle, flat-bed trailer.
Dimensions: (a) Height (b) Width	7.75 feet 7.75 feet
Gross Vehicle Weight:	57,180 lbs.
Rated Gross Vehicle Weight:	50,000 lbs.
Gear Ratios: (a) Transmission (b) Auxiliary Transmission (c) Axle (d) Total Gear Reductions	6.98, 3.57, 1.89, 1.00, 0.825 None 6.50, 8.86 61.84, 45.37, 31.63, 23.21, 16.75, 12.28, 8.86, 6.50, 7.31, and 5.36
Tire Size:	10 x 20
Net Engine Hp at Sea Level:	146 hp at 2,600 RPM
Brake Horsepower	162 hp at 2,800 RPM
Altitude:	950 feet
Road Type and Condition:	bituminous, good
Net Weight-Horsepower Ratio:	391 lbs./hp
Weight to Rated Horsepower Ratio:	353 lbs./hp

TABLE 2

GRAPHICAL PROCEDURE SHEET FOR ESTIMATING MAXIMUM SPEED ATTAINABLE
IN STILL AIR USING THE SAE PREDICTION PROCEDURE (8)

Basic Information		
1.	Vehicle Identification <u>International R - 195</u>	
2.	Vehicle overall maximum dimensions: (a) Height <u>7.75</u> ft (b) Width <u>7.75</u> ft	
3.	Total gross weight in pounds <u>57180</u>	
4.	Manufacturers maximum gross vehicle weight rating for power unit in pounds <u>50,000</u>	
5.	Gear ratio (ideal, value 7) <u>16.44</u>	
6.	Tire size (driving wheels) <u>10.00 x 20.00</u>	
7.	Net engine power at sea level (a) <u>146</u> hp at (b) <u>2600</u> rpm	
8.	Altitude <u>950</u> ft	
9.	Road surface type and condition <u>bituminous, good</u>	
Steps		
	Procedure	Value
1.	Frontal area (Height, 3/4 ft) x Width <u>7 3/4</u> x <u>7 3/4</u>	40.25
2.	Net engine hp corrected for altitude (Altitude factor, Table 2) x Item 7 (a) <u>.962</u> x <u>146</u>	140.45
3.	<u>Net engine hp - Friction hp</u> Frontal area (Value 2) - (Factor, Table 7)/(Value 1) <u>140.45</u> - <u>27.0</u> / <u>40.25</u>	2.82
4.	<u>Total gross weight</u> Frontal area (Item 3)/(Value 1) = <u>57180</u> <u>40.25</u>	1420.62
5.	<u>Grade ability on Class 1</u> roads (good) (Specified net grade ability) + (Road factor, Table 9) <u>3%</u> + <u>.2</u>	3.2
6.	Speed (approximate) Select Figure No. 11 most closely approximating Value 5. Read across on Value 3 to Value 4, then down to speed.	18.5
7.	Gear ratio (ideal) (Item 7 (b))/(Value 6) x TF, Table 1 <u>2600</u> / <u>18.5</u> x <u>8.55</u> = <u>2600</u> <u>158.15</u>	16.44

4 were used to develop speed-distance curves for each of the eleven test grades and then compared against the actual gradability curves developed from the field test. If the curves for each grade coincided, then the computed curve was considered as representative of the measured test data, and if they did not coincide, then the opposite was assumed.

A comparison of the computed curves with the measured gradability curves showed a fair degree of consistency. There were, however, two major discrepancies:

1. There was some irregularity in the curves due to the motion of the truck, especially on some of the upgrade deceleration curves where maximum sustained speeds were reached.
2. The actual maximum sustained speeds were 1 to 3 mph greater than the maximum sustained speeds shown on the computed curves.

It was concluded that although the above discrepancies existed, the gradability curves in Figures 5 and 6, which were developed through the use of Equation 5 and Figure 4, represented the performance of the test truck on grades. Therefore, Equation 5, which assumes the net driving force as a function of velocity only, was considered satisfactorily accurate for use in the design of climbing lanes. The gradability curves shown in Figures 5 and 6 are those employed in the 1965 AASHO Policy.

Firey and Peterson (9) presented an equation which is almost identical to that of Huff and Scriver:

$$\frac{W}{g} \frac{dv}{dt} = F_T - F_R - W \sin \theta \quad (7)$$

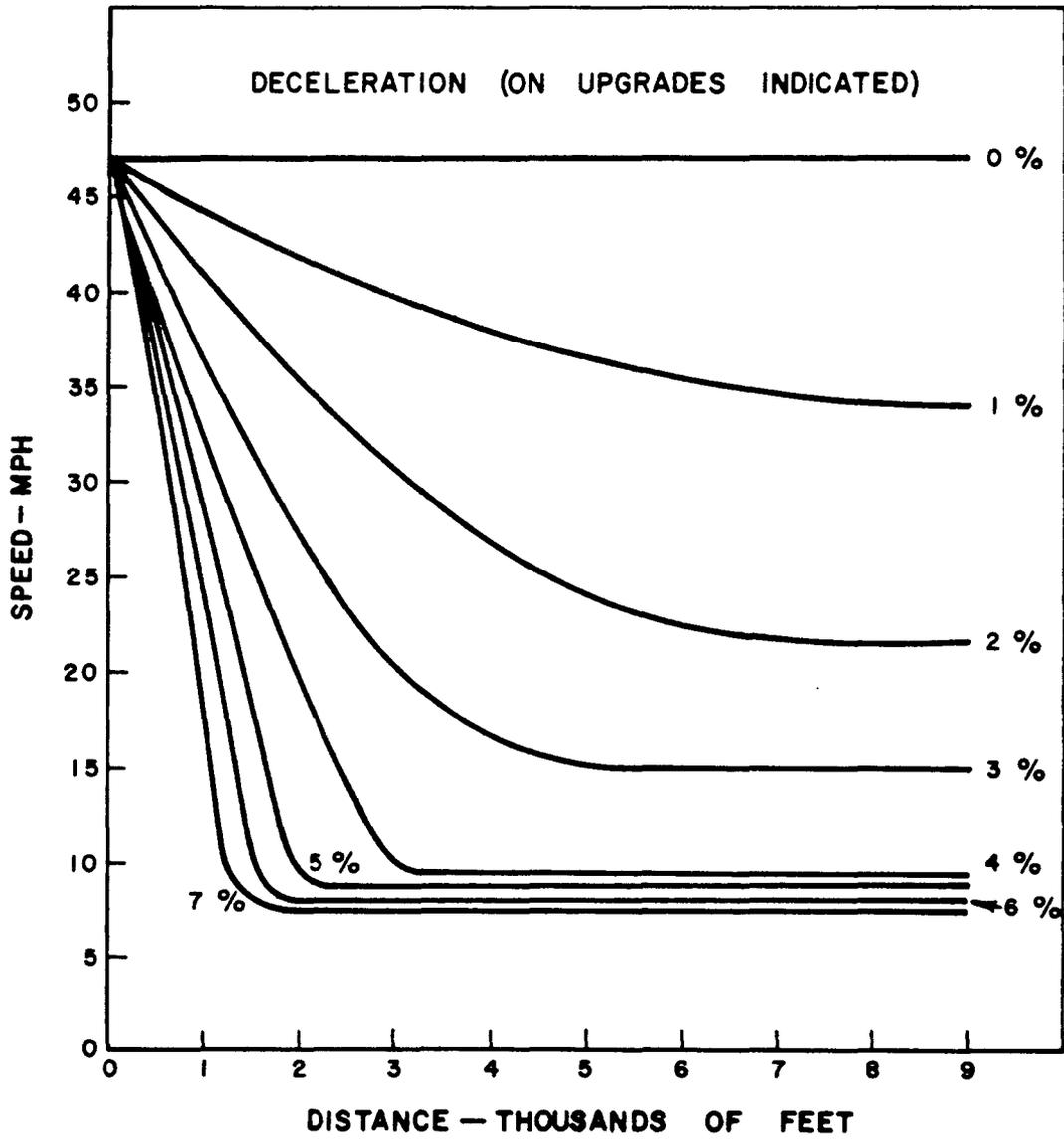


Figure 5. Speed-Distance Curves From Road Test Of A Typical Heavy Truck Operating On Various Grades AASBO Policy.

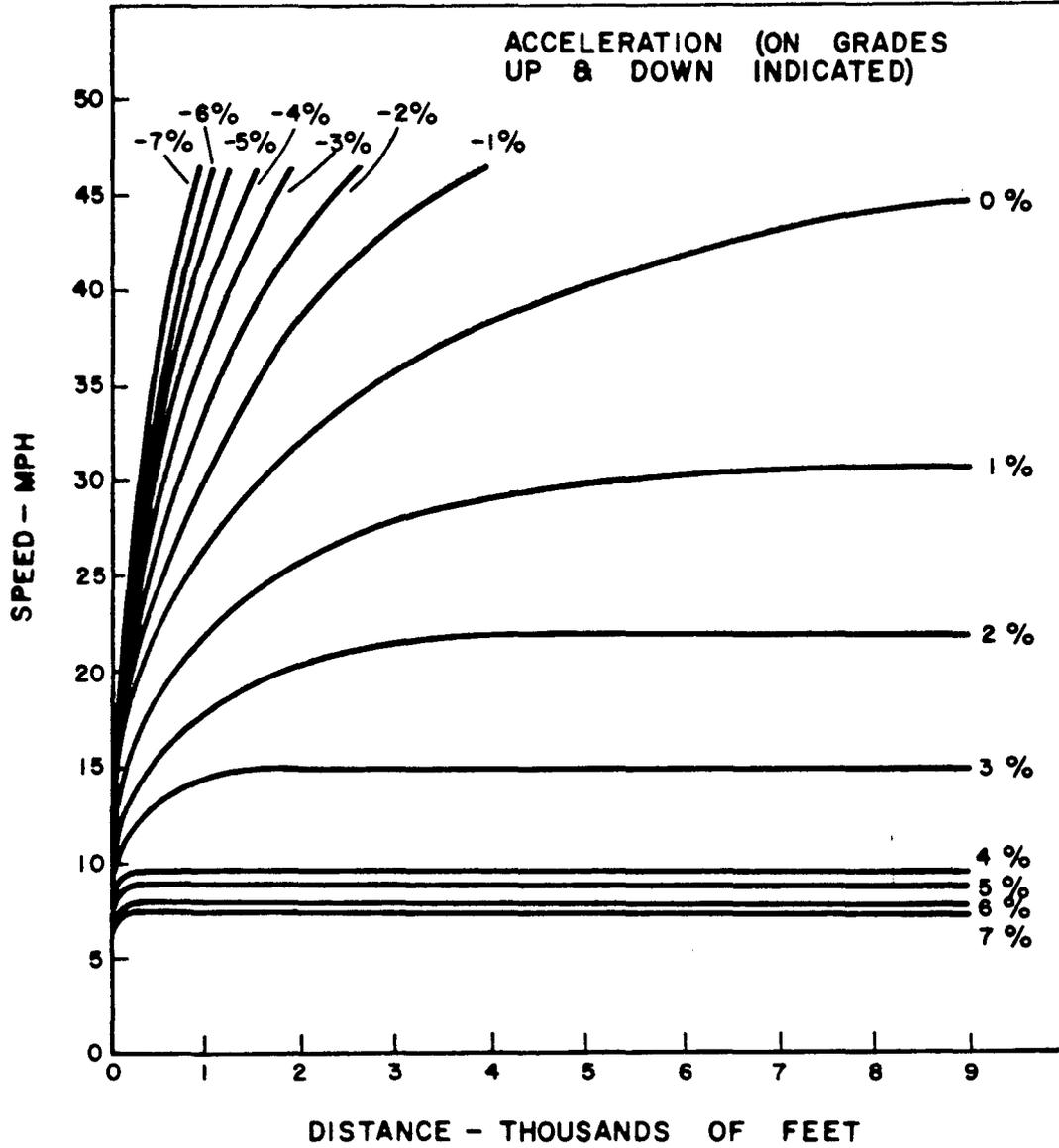


Figure 6. Speed-Distance Curves From Road Test Of A Typical Heavy Truck Operating On Various Grades - AASHO Policy.

where F_T is the truck engine thrust force and F_R is the truck rolling resistance force.

The engine thrust force, F_T , is zero when the clutch is disengaged and, assuming that the engine torque at wide-open throttle is constant over the operating speed range of the engine, F_T was calculated from the following equation:

$$F_T = \frac{E}{v_{\max}} (550) \quad (8)$$

where

E = engine rpm at wide-open throttle

v_{\max} = maximum truck speed attainable in a particular gear setting, ft/sec.

The truck rolling resistance force, F_R , was calculated from the following equation:

$$F_R = \frac{W}{148.5} + 195.0 \quad (9)$$

This is an empirical equation subject to the constraints of the coasting tests of several heavy trucks as described in another study (10). For significant upgrades, the exactness of F_R is Equation 9 is not very important because F_T is the dominant resisting force to vehicle motion.

The net force, F_0 , acting upon a truck was defined by the following equations:

at wide throttle;

$$F_0 = \frac{W}{g} \frac{dv}{dt} = \frac{E(550)}{v_{\max}} - \frac{W}{148.5} \quad (10)$$

with clutch disengaged, $F_T = 0$; therefore:

$$F_0 = \frac{-W}{148.5} - 195.0 - W \sin \theta \quad (11)$$

For computing speed-distance relationships on uniform grades, the following basic physics equations were used:

$$x = v_0 t + \frac{1}{2} a t^2 \quad (12)$$

$$v = v_0 + a t \quad (13)$$

Because the acceleration, a , in the above equations was considered equivalent to dv/dt , and because $dv/dt = F_0 g/W$, the following equations were derived for computing speed-distance relationships:

$$x = v_0 t + \frac{F_0 g t^2}{2W} \quad (14)$$

$$v = v_0 + \frac{F_0 g t}{W} \quad (15)$$

To calculate the velocity versus distance curves on uniform grades, the following steps were followed:

1. Values were assumed for W , W/H_p , θ , and initial v_0 .
2. These values were substituted into the vehicle motion equations, Equation 4 and 5.
3. On deceleration curves the first gear shift was assumed at $0.8 v_0$ and on acceleration curves it was assumed at $v_0/0.8$.
4. An average time of two seconds was determined (9) to shift the gears, and the vehicle was assumed to follow the vehicle motion equations for clutch disengagement during the gear shifting interval.
5. Steps 2 and 3 were repeated, using the vehicle motion equations for the clutch disengagement over the gear shifting interval.

6. For the second wide-open throttle periods, steps 2 and 3 were repeated, using the terminal speed from step 5, as v_0 in Equations 14 and 15.
7. The preceding steps were reiterated with values of v_0 until that value reached the established limitations: 10 mph on deceleration curves or 50 mph on acceleration curves.

The gradability curves developed by the use of the foregoing procedure are presented in Figure 7, Figure 8, and Figure 9, the deceleration and acceleration curves for trucks with W/H_p ratios equal to 400, 300, and 200, respectively.

The Effect of the Weight-Horsepower Ratio on Truck Operating Characteristics

In order to relate truck operations to design for highway grades, it is necessary to select a design vehicle which represents some lower boundary of operation. Willey (7) was the first to classify truck operating characteristics according to weight-horsepower ratios. Because the weight-horsepower ratios of trucks can be measured in field studies, this measure appears to be best suited as a parameter for determining a design vehicle.

In 1957, Saal (11) studied the relation between gross weights of motor trucks and their horsepower. This study indicated that the percentage of trucks in 1950 having a weight-horsepower ratio greater than 400 were as follows: 3-axle trucks, 10 percent; 2-axle truck-trailers with 1-axle semitrailers, 13 percent; 2-axle truck-trailers with 2-axle semitrailers, 41 percent; and all other combinations, 57 percent. He also stated that from 1955 to 1958 there had been an improvement in the performance ratio of at least ten percent for all

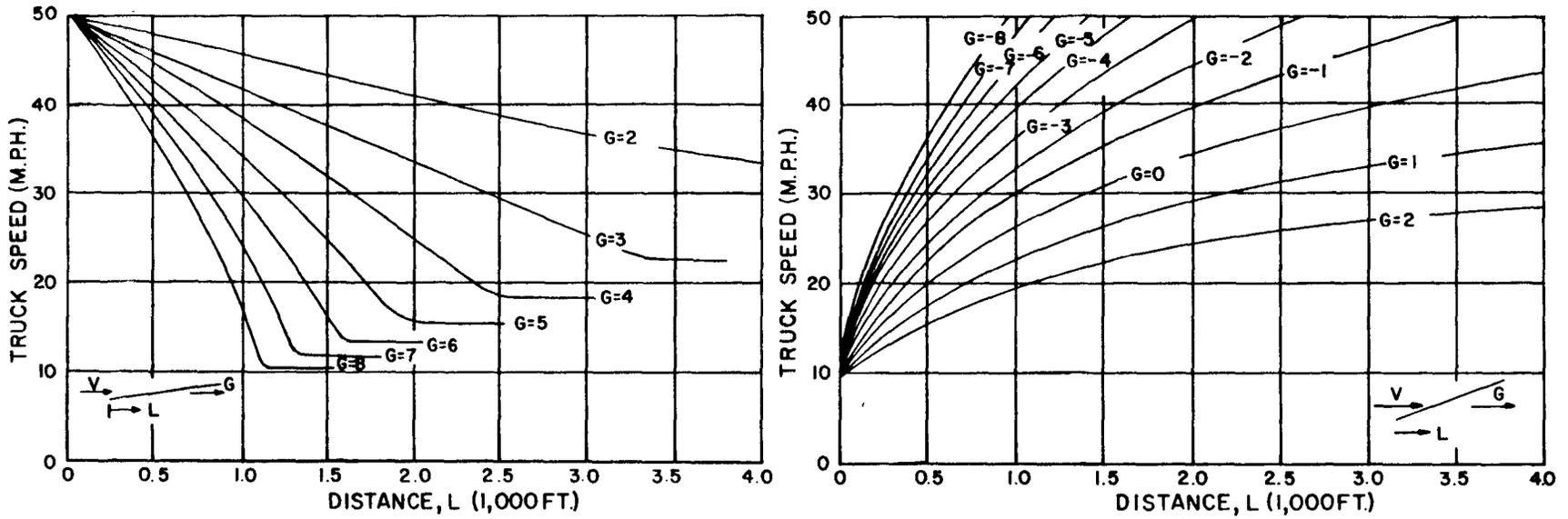


Figure 7. Deceleration and Acceleration Gradability Curves for Trucks with GVW/BHPW Ratio=400.

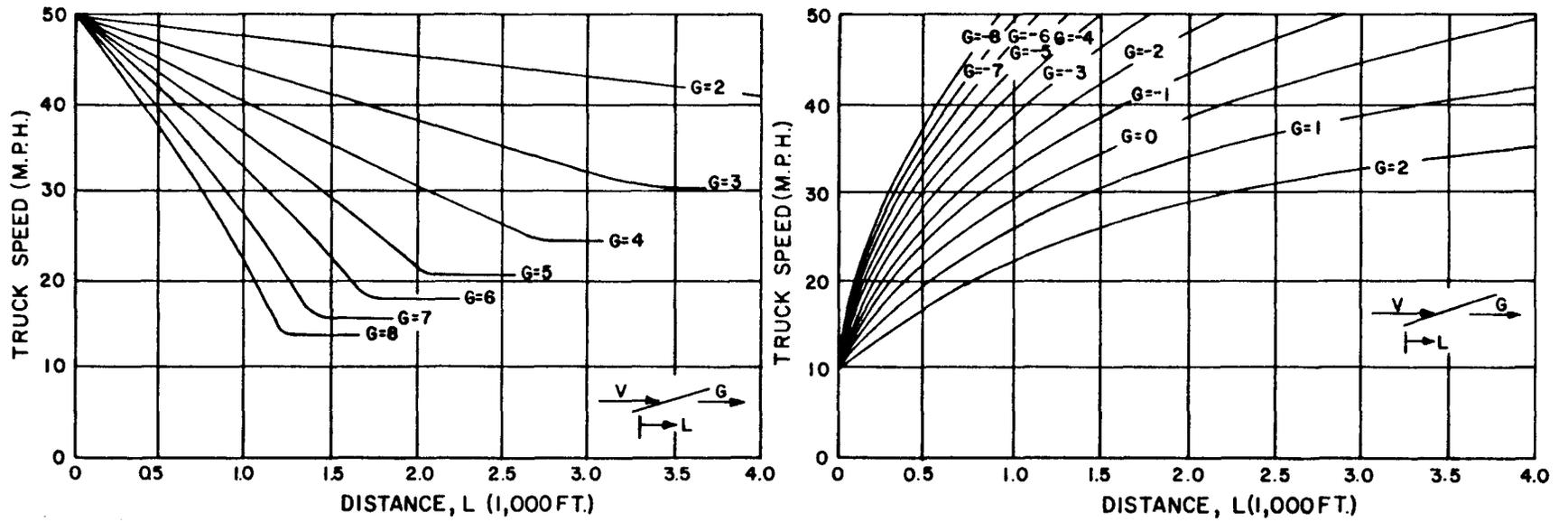


Figure 8. Deceleration and Acceleration Curves for Trucks with GVW/BHPW Ratio=300.

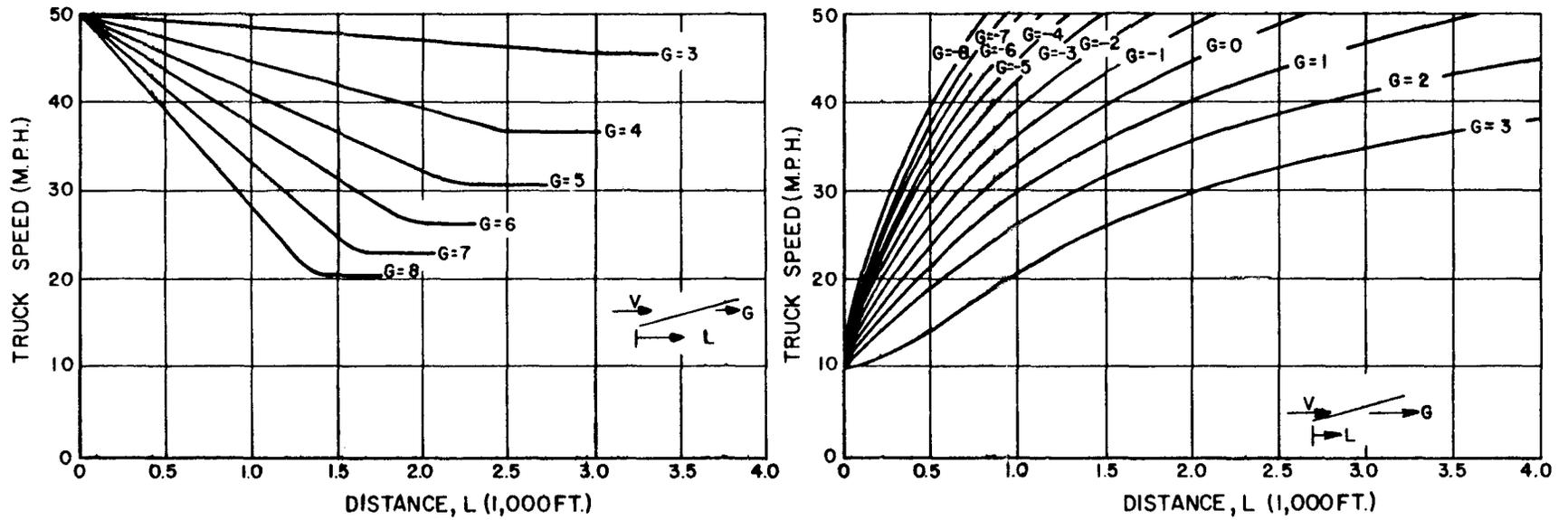


Figure 9. Deceleration and Acceleration Curves for Trucks with GVW/BHPW Ratio=200.

groups.

In 1963, Wright and Tignor (12) combined Saal's data and data from the 1963 brake study conducted by the Bureau of Public Roads. Table 3 shows the comparison of the average weight-horsepower ratios for all trucks involved in the 1949, 1950, 1955, and 1963 BPR brake studies. There was an overall reduction in the ratio of 12 percent from 1949 to 1955 and an overall reduction of 28 percent from 1955 to 1963.

Figure 10 shows cumulative frequency distributions of the weight-horsepower ratios from the 1963 study. It is important to note that only eight percent of all the loaded trucks did not meet the 400:1 ratio that had been accepted as a tolerable design performance ratio. Of all the trucks (loaded and unloaded) weighed in the 1963 study, only five percent could not meet a performance ratio of 400:1.

There has been a definite decreasing trend in the weight-horsepower ratio of the trucks operating over the highways. Figure 11 shows this trend for the 1949, 1955, and 1963 studies (12). Along with the trend to decrease the weight-horsepower ratio, there has also been a trend toward more heavy trucks on the highway (13). Figure 12 shows this trend from 1954 to 1967 and also predicts the trend will continue at least until 1980. The number of heavy trucks on the highways increased approximately 3.4 times from 1954 to 1967 and is predicted to increase 3 times from 1967 to 1980.

In 1968, more International Harvester trucks were registered across the United States in the heavy category, i.e., 26,000 pounds and

TABLE 3

COMPARISON OF AVERAGE WEIGHT-HORSEPOWER

FOR ALL TRUCKS BY TRUCK TYPES FOR 1949, 1950, 1955, 1963 (12)

Vehicle Type	Number of Trucks				Average Weight-Power Ratios				Percentage Reduction of Weight-Power Ratios		
	1949	1950	1955	1963	1949	1950	1955	1963	1949-55	1950-55	1955-63
2 - Single Tires	19	239	99	130	81	75	57	44	30	24	23
2 - Dual Tires	275	3,642	272	312	142	135	142	97	0	-5	32
3	38	263	67	42	227	244	231	145	-2	5	37
2-S1	228	3,900	117	108	291	294	264	149	9	10	44
2-S	87	1,991	145	217	369	357	301	227	18	16	25
3-S2	46	483	57	112	422	411	348	275	18	15	21
2-3, 3-2, and 2-S1-2	51	136	71	78	394	384	418	300	-6	-9	28
Other	38	72	34	27	428	421	374	290	13	11	22
Total Vehicles	782	10,726	862	1,026							
Weighted Vehicles					260	253	228	165	12	10	28

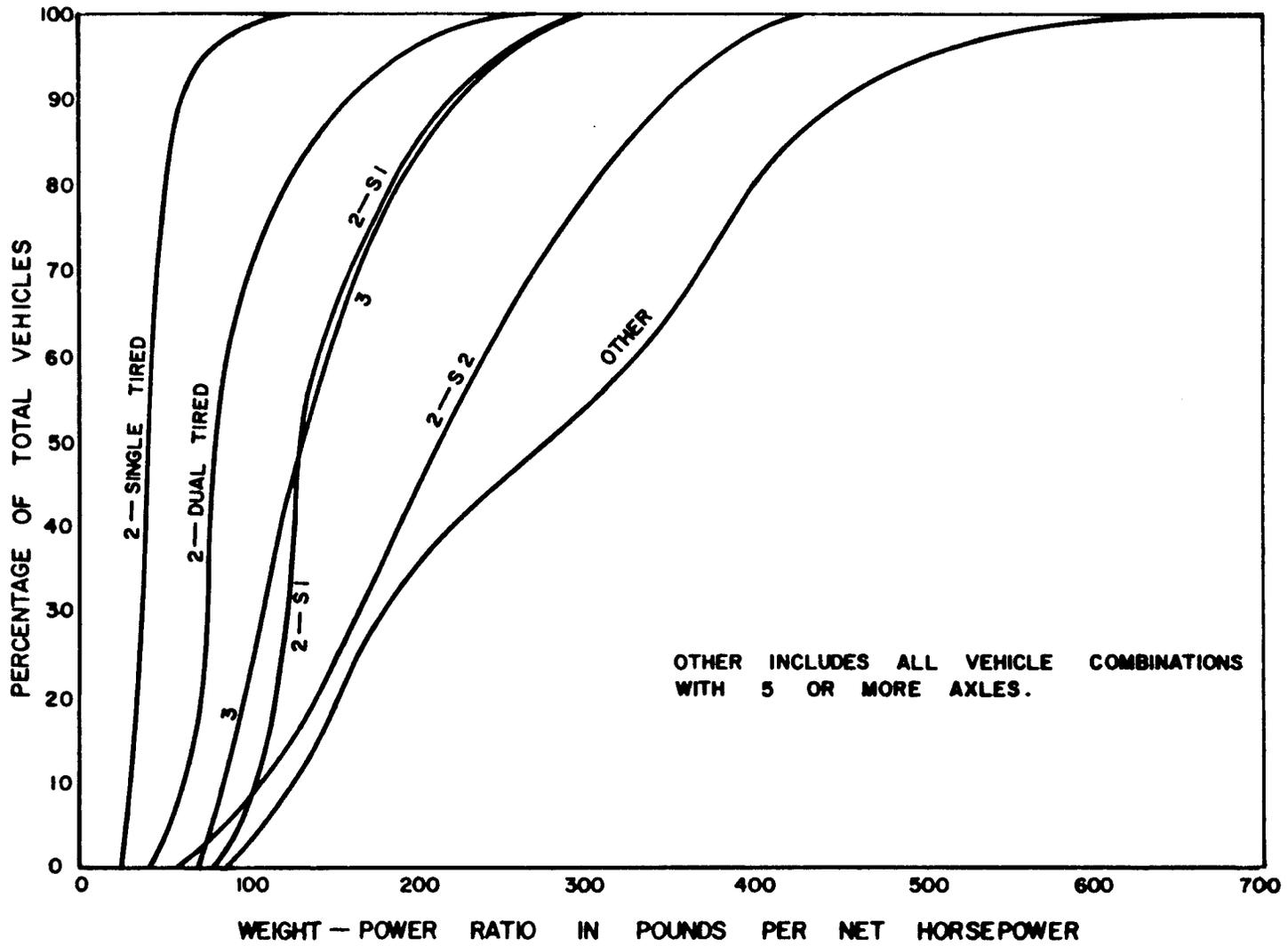


Figure 10. Cumulative Frequency Distributions Of Weight-Power Ratios Of All Commercial Vehicles Weighed In The 1963 Study (12).

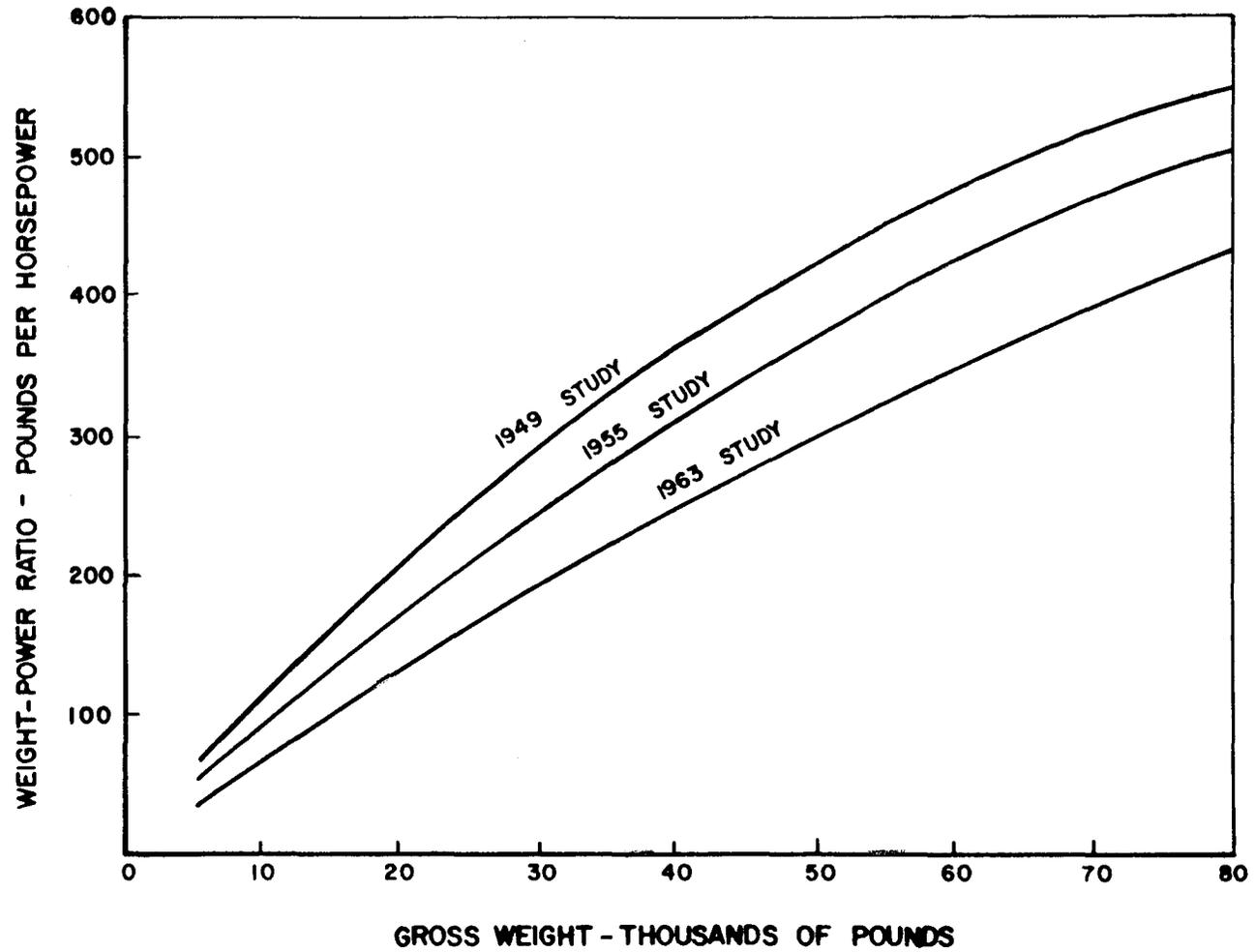
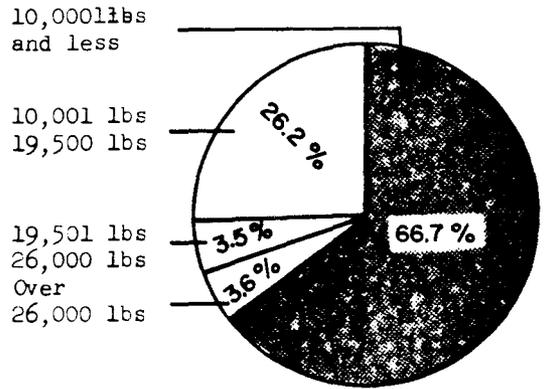
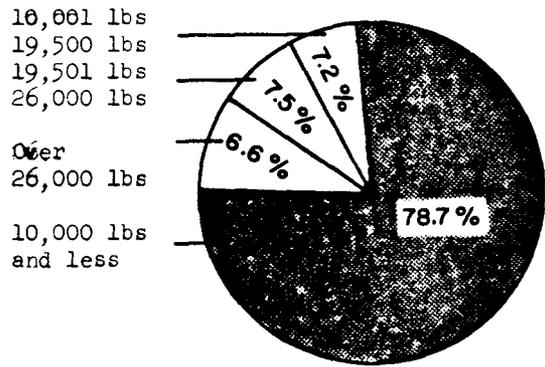


Figure 11. Trend In Weight-Power Ratio From 1949 to 1963
Based On Average Data For All Types Of Vehicles (12).



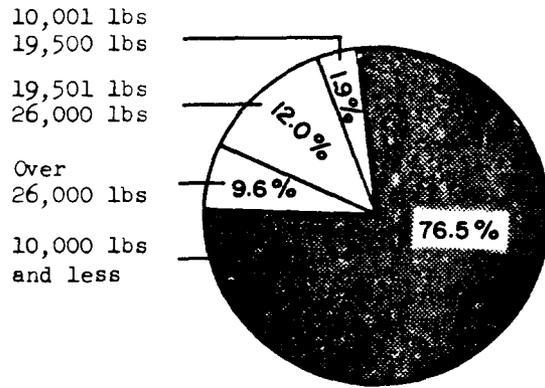
1954
GVW Class

10,000 lbs and less	552,848
10,001 lbs to 19,500 lbs	217,727
19,501 lbs to 26,000 lbs	29,091
Over 26,000 lbs	29,435
Total new truck registrations	829,101



1967
GVW Class

10,000 lbs and less	1,195,457
10,001 lbs to 19,500 lbs	109,854
19,501 lbs to 26,000 lbs	113,630
Over 26,000 lbs	99,511
Total new truck registrations	1,518,852



1980
GVW Class

10,000 lbs and less	2,407,513
10,001 lbs to 19,500 lbs	59,677
19,501 lbs to 26,000 lbs	379,370
Over 26,000 lbs	302,507
Total new truck registrations	3,149,067

Figure 12. Truck Registration Trend (13).

over. International Harvester offers five, 8-cylinder diesel engines to power its 65,000 pounds GVW trucks. The weight to net horsepower ratio of an IH truck powered by each of those engines would be 279:1, 298:1, 342:1, 392:1, or 414:1, depending upon which model was chosen. It should be noted that only 1 of the 5 engines offered would fall outside the accepted tolerable performance ratio of 400:1 (13).

The AASHO Policy on weight-horsepower ratios states that trucks with a weight-horsepower ratio of about 400 have acceptable operating characteristics from the standpoint of the highway user. It is stated that such a ratio will insure a maximum sustained speed of 15 mph on a three percent grade. There is also evidence that the industry is finding the 400 ratio a desirable goal and is voluntarily accepting it as a performance control, resulting in an improvement of the weight-horsepower ratios of trucks over the last several years. This improvement is illustrated by the trend curves shown in Figure 11. This means that trucks on the highways have greater power and improved climbing ability on grades.

The AASHO Policy calls attention to Wright and Tignor's study again to illustrate the fact that a weight-horsepower ratio of 400:1 is becoming the accepted standard. From Figure 10, it can be seen that in 1963 only thirty percent of the trucks with five axles or more and only eight percent of all trucks had a weight-horsepower ratio greater than 400.

Design Criteria Related to Truck Operations

The AASHO Policy indicates that the average truck speed is approximately 6 mph less than the average passenger car speed on a level highway section; increasing on downgrades of five percent or less, and decreasing on downgrades of seven percent or steeper. On upgrades, the maximum sustained speed that a truck can maintain is dependent upon the length and steepness of the grade and the weight-horsepower ratio of the truck. Factors affecting the average speed over the entire section are the trucks entering speed, wind resistance, and skill of the operator.

There are two factors which control truck speeds on grades: the steepness of the grade and the length of the grade. The AASHO Policy recommends for specific design speeds as presented in Table 4. The minimum grade is considered as that which will facilitate adequate drainage.

The "critical length of grade" is defined by the AASHO Policy as the maximum length of a designated upgrade upon which a loaded truck can operate without an unreasonable reduction in speed. If a truck is to reasonably operate on grades greater than "critical", then either the grade must be reduced or an additional climbing lane must be provided.

The AASHO Policy states that climbing lanes are necessary when the length of a specific grade causes truck speeds to reduce 15 mph or more, provided the volume of traffic and percentage of heavy trucks justify the added cost. Therefore, truck gradability, highway capacity, or both,

TABLE 4

AASHO POLICY'S COMPARATIVE GRADES AND DESIGN SPEEDS ON MAIN HIGHWAYS*

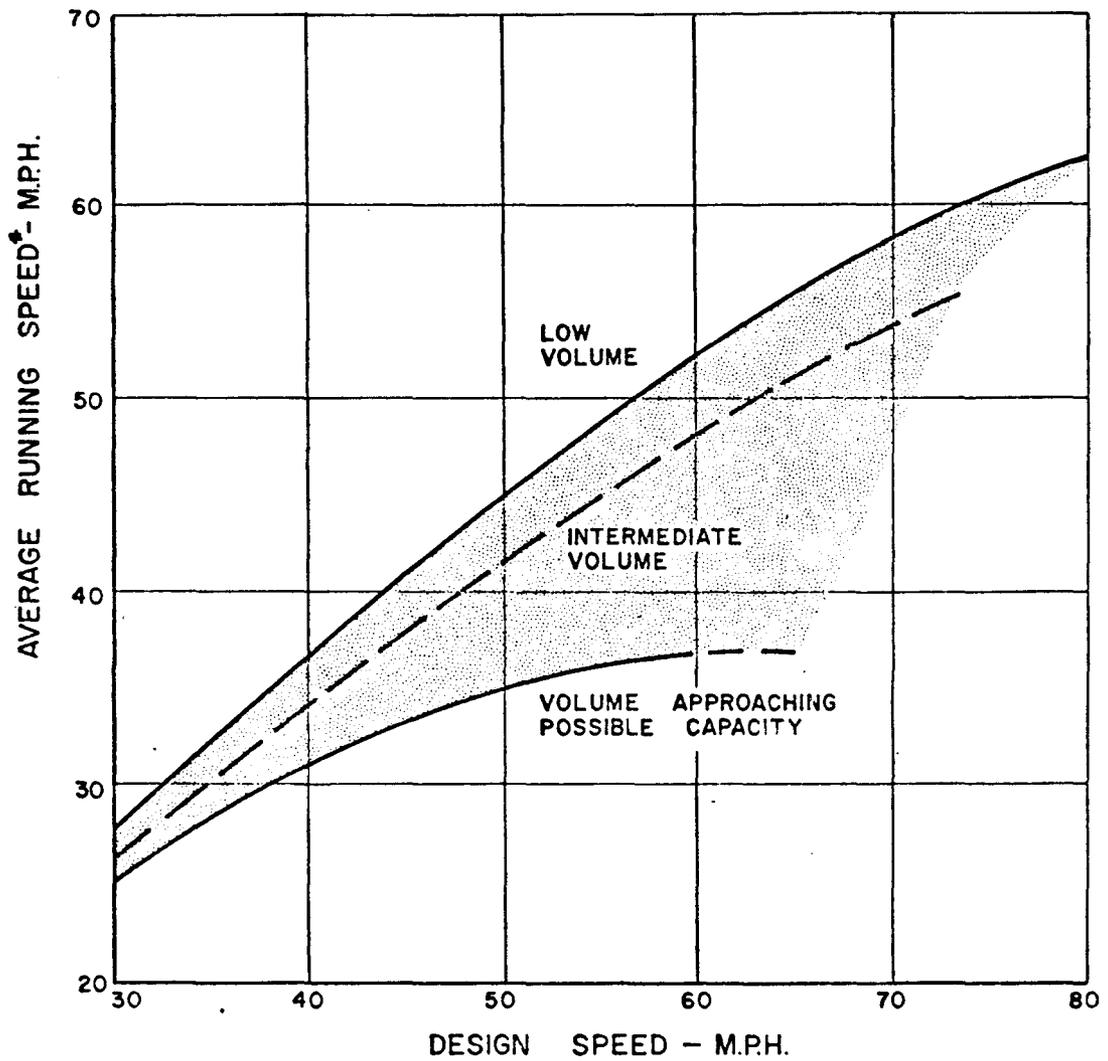
Type of Topography	Design Speed, mph							
	30	40	50	60	65	70	75	80
Flat	6	5	4	3	3	3	3	3
Rolling	7	6	5	4	4	4	4	4
Mountainous	9	8	7	6	6	5	-	-

*Highways of secondary nature may be 2 percent steeper.

can determine the "critical length of grade." If truck gradability governs, then the AASHO Policy considers that the following factors must be determined or assumed:

1. The size and power of the design truck along with the gradability data for this truck--The 400:1 weight-horsepower ratio is accepted as the national design vehicle; therefore, the gradability curves presented in Figures 5 and 6 are employed by the AASHO Policy.
2. Truck speed at entrance to critical length of grade--The average running speed as related to design speed can be used to approximate the average speed of vehicles beginning an uphill climb (See Figure 13). For downhill or uphill approaches, the entering speed should be adjusted accordingly.
3. The minimum tolerable speed at which a truck should operate on the grade--Although no specific data are available on the minimum tolerable speed of trucks, it seems logical that they would have a direct relationship with design speeds. Minimum speeds of 20 to 35 mph on highways with a design speed of 40 to 60 mph would be tolerable for a vehicle unable to pass on a two-lane highway, provided the no-passing interval is short. As the volume on a two-lane highway approaches capacity, the time interval will become more annoying. Multilane highways present more opportunity for and less difficulty in passing; therefore, they afford opportunities for lower tolerable truck speeds. In any case, highways should be designed to maintain a tolerable truck speed.

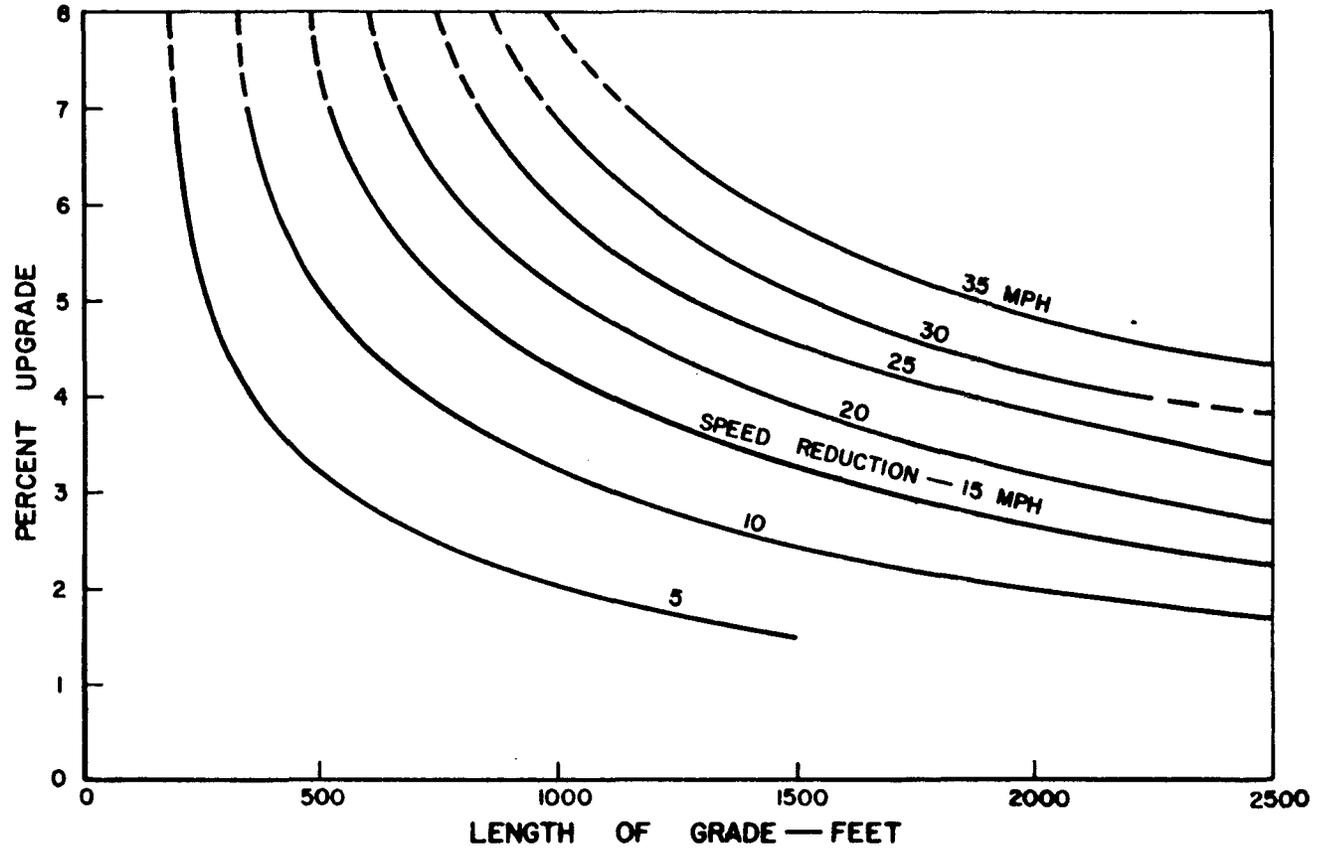
Although all states are not in agreement on what constitutes the critical length of grade, the most common determining factor is the 15-mph reduction in truck speed below the average truck running speed. Some states specify a minimum tolerable speed ranging from 20 to 35 mph instead of the 15-mph reduction. Figure 14 presents the critical length of grade for different speed reductions on specific grades (derived from Figure 5). The 15-mph curve in Figure 14 is suggested by AASHO as a general design guide for establishing critical lengths of



RUNNING SPEED IS THE SPEED (OF AN INDIVIDUAL VEHICLE) OVER A SPECIFIED SECTION OF HIGHWAY, BEING DIVIDED BY RUNNING TIME.

* AVERAGE RUNNING SPEED IS THE AVERAGE FOR ALL TRAFFIC OR COMPONENT OF TRAFFIC, BEING THE SUMMATION OF DISTANCES DIVIDED BY THE SUMMATION OF RUNNING TIMES. IT IS APPROXIMATELY EQUAL TO THE AVERAGE OF THE RUNNING SPEEDS OF ALL VEHICLES BEING CONSIDERED.

Figure 13 - Relationship Between Average Running Speed and Design Speed - AASHO Policy.



ASSUMED TYPICAL HEAVY TRUCK OF 400 POUNDS PER HORSEPOWER

Figure 14. Critical Lengths Of Grade For Design - AASHO Policy.

grades which are preceded by relatively level approaches. If there is an uphill approach to the grade, the critical length will be shorter and for downhill approaches the converse will be true.

Climbing lanes may be justified from the standpoint of highway capacity as well as truck gradability. The effect of trucks on highway capacity is primarily a function of the difference in average running speeds between trucks and passenger cars. Passenger car equivalents for trucks at various combinations of running speeds are given in Table 5. By selecting the appropriate values from Table 5 and from the gradability curves of Figures 5 and 6, the design capacity on any grade for a given percentage of trucks can be calculated.

The AASHO Policy also states that climbing lanes may be justified if the design hour volume (DHV) for a highway exceeds the design capacity of that highway by more than twenty percent. Table 6 shows the minimum design hour volumes for which climbing lanes should be considered. Before Table 6 is used, however, it would be advisable to make a detailed analysis of each specific situation because of the many variables involved.

The exact beginning of a climbing lane depends upon the entering speed of the truck on a grade. Again, Figure 5 may be used to determine when a truck's speed has decreased enough to be sufficient cause for the implementation of a climbing lane. The AASHO Policy recommends that the beginning of the climbing lane should be preceded by a tapered section at least 150 feet long.

TABLE 5

PASSENGER CAR EQUIVALENTS FOR TRUCKS AT VARIOUS
 AVERAGE TRUCK SPEEDS AS RELATED TO PASSENGER CARS FOR
 INDIVIDUAL GRADES ON TWO-LANE ROADS - AASHO POLICY (3)

Truck Speed, mph	Number of Passenger Cars to Which One Truck is Equivalent		
	For Average Passenger Car Running Speed of 45-50 mph	For Average Passenger Car Running Speed of 40-45 mph	For Average Passenger Car Running Speed of 35-40 mph
35	3.0	2.7	2.5
30	5.0	4.9	3.0
25	8.6	7.6	5.0
20	13.9	11.7	8.8
15	22.9	18.7	15.0
10	40.5	32.5	25.2
5	94.5	75.0	50.0

TABLE 6

THE AASHO POLICY'S MINIMUM TRAFFIC VOLUMES FOR CONSIDERATION
OF CLIMBING LANES ON GRADES ON TYPICAL TWO-LANE ROADS

Gradient, percent	Length of grade, miles	Minimum two-way DHV including trucks (not passenger equivalents) for consideration of climbing lane for various percentages of dual-tired trucks			
		3% Trucks	5% Trucks	10% Trucks	15% Trucks
4	1/3	4 lanes warranted for DHV over 750	4 lanes for DHV over 700	4 lanes over 600	4 lanes over 525
	1/2			550	450
	3/4		670	500	390
	1	750	640	470	370
	1 1/2	730	610	440	340
	2	710	590	420	340
5	1/3	4 lanes for DHV over 690	4 lanes over 640	4 lanes over 550	4 lanes over 480
	1/2			620	370
	3/4	650	540	380	300
	1	630	510	360	270
	1 1/2	600	490	340	260
	2	600	480	330	250
6	1/3	4 lanes over 625	4 lanes over 580	480	390
	1/2			570	250
	3/4	540	430	290	220
	1	530	420	280	210
	1 1/2	520	410	270	200
	2	510	410	270	200
7	1/3	470	410	310	240
	1/2	400	320	210	160
	3/4	380	300	200	150
	1	360	280	180	140
	1 1/2	350	270	170	130
	2	340	260	160	120

NOTE: Detailed analysis of each grade is recommended in lieu of tabular values.

It is desirable to end a climbing lane when the truck's speed has accelerated to a speed at least equal to the speed at which it entered the climbing lane. The AASHO Policy states that this may be impractical on many grades because of the long distance required to accelerate to such a speed; therefore, a practical point for ending the lane is where a truck can safely re-enter the normal flow of traffic. This would be at a point where the sight distance is sufficient to permit passing with safety. The AASHO Policy recommends that a taper of at least 200 feet should be provided to allow the truck to re-enter the flow of traffic.

A climbing lane should be at least 10 feet wide and preferably 12 feet wide. It should be easily distinguishable as an extra lane and signs should precede the lane to notify trucks that there is a climbing lane ahead (3).

EVALUATION OF THE DESIGN CRITERIA

The purpose of this section is to evaluate the design criteria for climbing lanes and critical lengths of grade. This will include an evaluation of the state-of-the-art as presented in the previous section to include:

1. Truck operating characteristics on grades.
2. The effect of weight-horsepower ratios on truck operating conditions.
3. Truck operating speeds.
4. The speed reduction criterion as it relates to safe operations.

Truck Operating Characteristics on Grades

Truck gradability procedures have been developed to predict the performance of trucks on grades in order to establish a design procedure that will enable all vehicles to operate safely on modern highways. Willey (7) documented the gradability characteristics of trucks, and classified the observed trucks according to their weight-horsepower ratios. Gradability curves were developed for the heavily loaded trucks on different grades; a heavily loaded truck being one with a weight-horsepower ratio greater than 300. Although Willey's observations may have been accurate at the time they were made, the report was not documented well enough to allow a verification of the number of heavily loaded trucks observed or what specific weight-horsepower ratio each heavily loaded truck had. Therefore, no direct comparison of Willey's

gradability curves could be made with those developed by any of the other truck ability prediction procedures.

Huff and Scrivner (2) developed a truck ability prediction procedure and compared this theoretical procedure with actual field tests of the performance of a heavily loaded truck with a weight-horsepower ratio of 391. From the field tests, it was concluded that the theoretical procedure compared fairly well with the actual truck performance on grades. Huff and Scrivner's procedure appears to describe the performance of trucks on grades, although their average curve of P/W versus v derived from the 1953 road test data ignored some of the plotted points. The truck gradability curves derived from this procedure have been adopted as part of the AASHO Policy.

Firey and Peterson (9) developed truck gradability curves for trucks with weight-horsepower ratio's of 200, 300, and 400. Figures 7 through 9 show the speed-distance curves for these three ratios.

From a design viewpoint, the controlling factor for climbing lane design criteria is the maximum sustained speed that a truck can maintain on a grade. The higher the sustained speed, the smaller length of climbing lane that is needed and the converse is also true. Table 8 lists a comparison of the maximum sustained speeds derived from the various truck gradability prediction procedures presented in this report. Also included are the maximum sustained speeds calculated using the SAE Procedure (9) for Huff and Scrivner's test truck.

It can be seen from Table 7 that there is considerable disparity among the various prediction methods. The Huff and Scrivner values are

TABLE 7
 GRADE VERSUS MAXIMUM SUSTAINED SPEED AS
 DETERMINED BY DIFFERENT GRADABILITY PROCEDURES

Grade	Willey	Huff and Scrivner	Firey and Peterson	SAE Procedure
%	MPH	MPH	MPH	MPH
1	NA	33.5	45.3	33.5
2	23.0	22.0	31.1	24.2
3	17.5	15.0	23.0	18.5
4	12.0	9.5	18.5	15.0
5	9.0	9.0	15.3	12.5
6	7.0	8.0	13.0	11.0
7	6.0	7.5	11.8	9.5

the lowest while the Firey and Peterson values are considerably higher than the others. However, the Huff and Scrivner values are the only values that were validated using a design vehicle, one which had a representative weight-horsepower ratio. Therefore, it appears that the Huff and Scrivner gradability curves adopted by the AASHO Policy are comparatively valid for design.

The Effect of Weight-Horsepower Ratios on Truck Operating Conditions

The weight-horsepower ratio of a truck determines how that truck will operate on grades. The higher the ratio, the more difficulty a truck will have ascending a grade and the maximum sustained speed attainable will be lower.

There is a definite trend toward a maximum tolerable ratio of 400:1. Figure 10 shows that only eight percent of all loaded trucks had a ratio greater than 400:1 in 1963. The AASHO Policy states that the 400:1 ratio has been accepted from the viewpoint of the highway user, and that the trucking industry has accepted the 400:1 ratio as a performance control. This can be shown by the fact that International Harvester offers only one out of five 8-cylinder engines for its heavy trucks which would result in a weight-horsepower ratio over 400:1. From all indications, it would seem reasonable to accept the 400:1 ratio as a design criterion until such time that legislation might be established to limit the weight-horsepower ratio to a level that will reduce the need for truck climbing lanes.

Truck Entering Speeds

Truck operating speeds along a highway, obviously, are determined

by the profile of that particular highway. Huff and Scrivner selected an entering speed on grades of 47 mph because it was the average speed of trucks on approximate level grades in Texas. Although this no longer represents the average speed, the Texas Highway Department's 1968 Statewide Speed Survey (14) indicates that a speed of 47 mph now represents the 15th percentile truck speed on Texas highways. Because the 15th percentile truck represents a reasonable lower boundary condition, the 47 mph entering speed is appropriate for design when considering entry to a grade from a level approach.

Because of the possibility of higher entry speeds on upgrades approached by a momentum grade, the AASHO Policy suggests use of the gradability curves by increasing the speed reduction criterion by an amount equal to the increase above 47 mph experienced on the down-grade. This procedure could be avoided by extrapolating the gradability curves to some higher entering speed, thereby allowing a range of entry speeds depending on profile conditions.

In using the gradability curves when considering adjacent grades of differing amounts, the highway engineer is always working with differential distances. Unless the curves are plotted to a large scale, these distances are difficult to determine accurately from the graph. This would suggest the development of gradability tables for use in design.

Speed Reduction Criterion

Truck speeds may be related to the average running speed of all traffic along a highway. In a study reported by Solomon for the Department

of Commerce (15), it was concluded that regardless of the average speed on the highway, the greater a vehicle's deviation from this average speed, the greater its chance of being involved in an accident. The accident involvement rates related to the deviation from the average speed are presented in Figure 15.

The speed distribution of vehicles traveling the Texas highways may be obtained from the Texas Highway Department (14). By utilizing this speed distribution and relating it to the accident involvement rates presented in Figure 15, the accident involvement rate may be obtained for 4-or-more-axle trucks operating on level grades. By assuming the reduction in the average speed of all vehicles on a grade to be 30 percent of the truck speed reduction on that same grade, the accident involvement rates for truck speed reductions of 5, 10, 15, and 20 mph may also be developed (See Appendix C).

The results of the analysis are presented numerically in Table 8 and graphically in Figure 16. It should be noted that most states base their climbing lane design on the criterion of 15-mph reduction of truck speed. From Table 8, the accident rate at a 15-mph reduction is 2,193 or almost nine times the involvement rate for a zero mph reduction and approximately 2.4 times the rate for a 10-mph reduction. The accident involvement rate increases, in absolute terms, 1,280 from the 10-mph to the 15-mph reduction. This is an increase of more than 5 times the increase from the zero to 5-mph reduction. This would indicate that a definite consideration should be given to the 10-mph reduction as a climbing lane design criterion, in place of the present

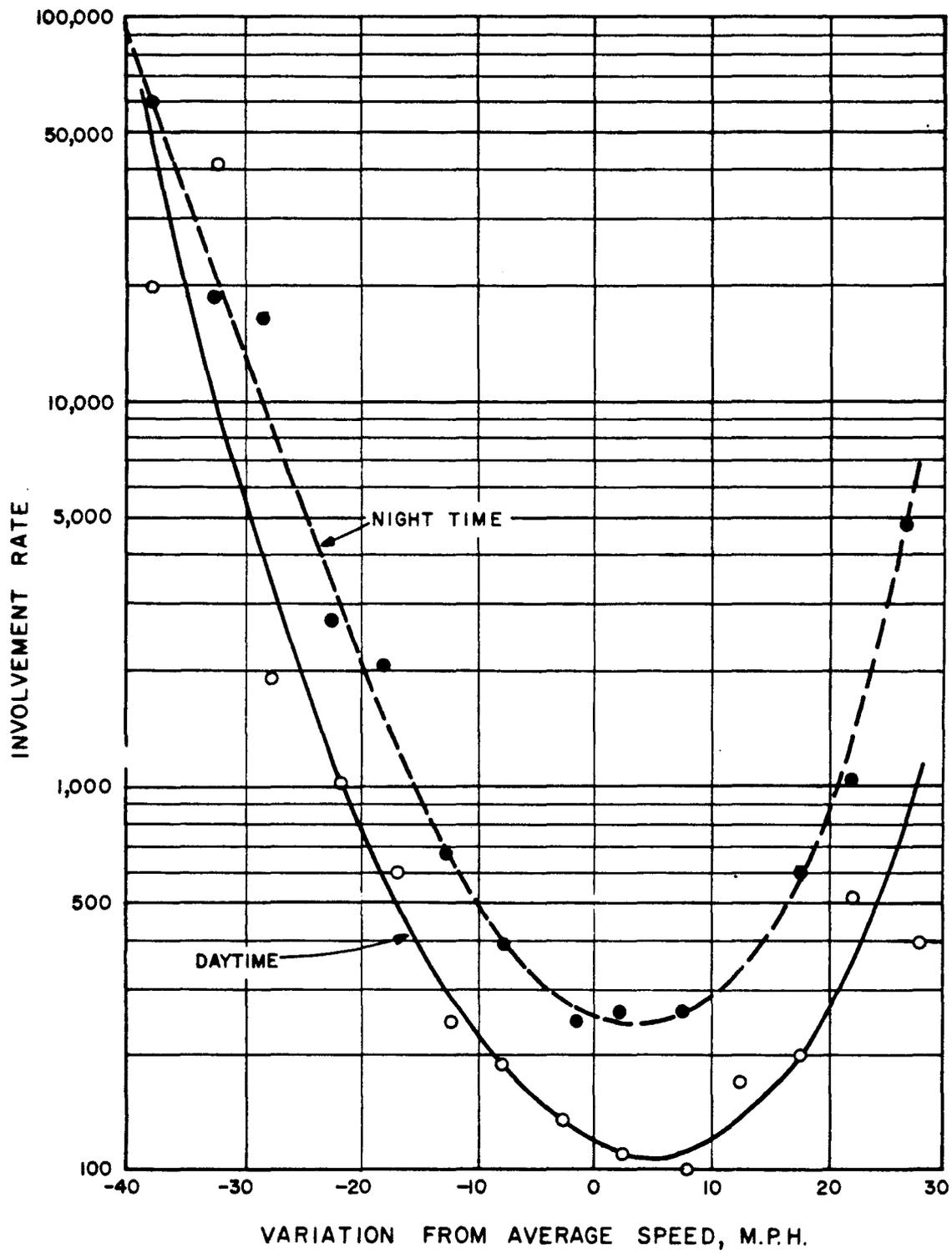


Figure 15. Involvement Rate By Variation From Average Speed On Study Section, Day And Night (15).

TABLE 8

Accident Involvement Rates On Grades Compared
To The Variation From The Average Speed Of All
Vehicles On A Highway

Speed Reduction	Accident Involvement Rate	Involvement Rate Ratio Related To 0 Speed Reduction
0	247	1.00
5	481	1.95
10	913	3.70
15	2193	8.90
20	3825	15.90

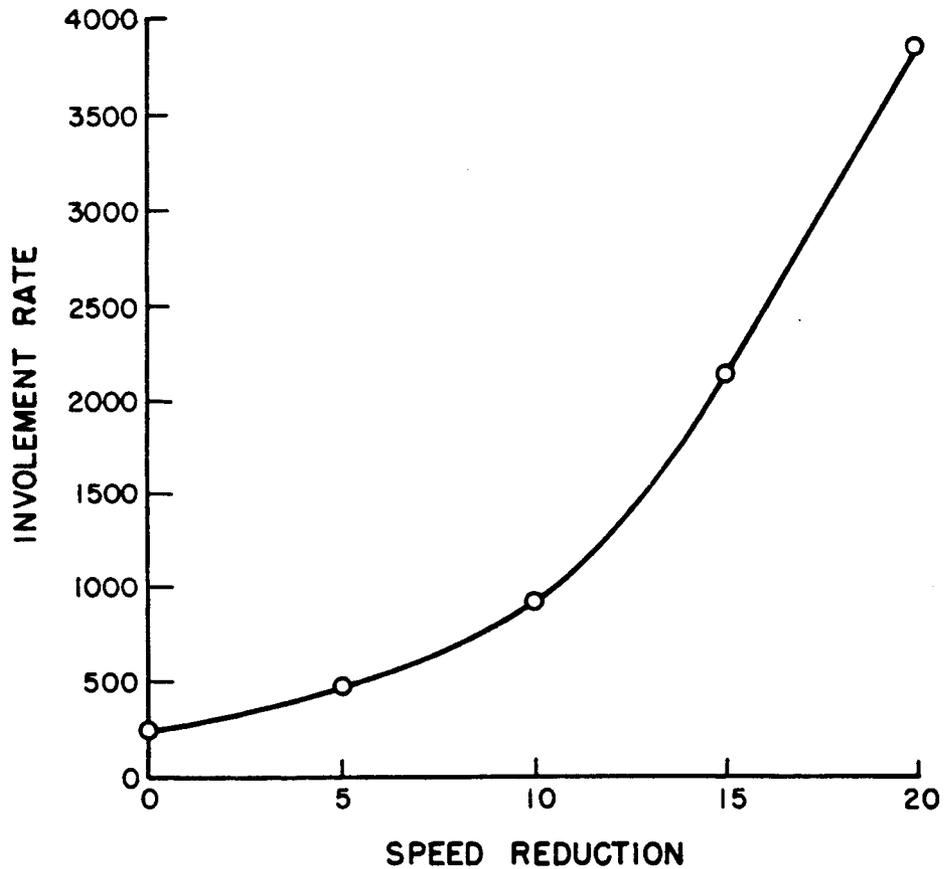


Figure 16 - Accident Involvement Rate Versus Speed Reduction From
the Average of All Vehicles on a Highway.

15-mph reduction.

For the steeper grades, consideration should be given to further reduction of the speed criterion. From Figure 14, it may be observed that a 5-mph decrease in the speed reduction criterion does not substantially increase the required climbing lane length for the steeper grades. This small increase in climbing lane length would be more than offset by the concomitant reduction of the accident involvement rate. These same considerations apply on the downstream end of the climbing lane where it is necessary to allow acceleration of the truck to a speed at which it can safely re-enter the normal traffic stream.

In terrain which dictates consecutive climbing lanes at short intervals, consideration should be given to joining the separate climbing lanes to form one continuous lane. This would eliminate the hazardous situation of re-entering the truck into the normal flow of traffic and then, in a short distance, removing the truck again.

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APPENDICES

Appendix A

The Derivation of Huff and Scrivner's Speed-Distance and Time-Distance Formulas

Through the summation of forces acting on a truck ascending any grade, a basic force equation may be developed:

$$\frac{W}{g} \frac{dv}{dt} = P - W \sin \theta \quad (1)$$

Dividing by W, Equation 1 becomes:

$$\frac{P}{W} = \frac{1}{g} \frac{dv}{dt} + \sin \theta \quad (2)$$

If it is stipulated that:

$$\frac{P}{W} = av + b \quad (3)$$

Then by substitution, an equation is formed which does not contain P/W:

$$\frac{dv}{dt} - gav + g(\sin \theta - b) = 0 \quad (4)$$

If $\frac{dv}{dt}$ is considered as the change in velocity with respect to time, $\frac{v_o - v}{t}$, and v is the average velocity, \bar{v} , then Equation 4 becomes:

$$\frac{v_o - v}{t} - ga\bar{v} + g(\sin \theta - b) = 0 \quad (5)$$

By multiplying by the time, t , and solving for $\bar{v}t$, Equation 5 may be written:

$$\bar{v}t = \frac{v_o - v}{ga} + \frac{gt(\sin \theta - b)}{ga}$$

Any distance, x , may be measured by the average velocity times time; therefore, Equation 6 becomes the following:

$$x = \frac{1}{a} \frac{v_0 - v}{g} + (\sin \theta - b)t \quad (7)$$

which is the first equation of Huff and Scrivner.

The second equation of Huff and Scrivner's may be derived by first solving for dt in Equation 4:

$$dt = \frac{dv}{g(av - \sin \theta + b)} \quad (8)$$

If we take the integral of both sides of Equation 8:

$$\int_{t_0}^t dt = \frac{1}{g} \int_{v_0}^v \frac{dv}{av - \sin \theta + b} \quad (9)$$

and consider $(\sin \theta + b)$ constant over any interval v_0 to v , then Equation 9 becomes:

$$t = \frac{1}{ag} \int_{v_0}^v \frac{adv}{av + (-\sin \theta + b)} \quad (10)$$

By integrating:

$$t = \frac{1}{ag} \ln \left[av + (-\sin \theta + b) \right] - \frac{1}{ag} \ln \left[av_0 + (-\sin \theta + b) \right] \quad (11)$$

or;

$$t = \frac{1}{ag} \ln \frac{av - \sin \theta + b}{av_0 - \sin \theta + b} \quad (12)$$

Appendix B

The Derivation of a Simplified Speed-Distance Formula

A simplified speed-distance formula may be derived by using the same assumptions made by Huff and Scrivner. If dv/dt is the change in velocity with respect to time and v is the average velocity, Equation 4, Appendix A, becomes:

$$\frac{v_o - v}{t} - ga\bar{v} + g(\sin \theta - b) = 0 \quad (1)$$

By dividing by the average velocity, \bar{v} , Equation 1 becomes:

$$\frac{v_o - v}{\bar{v}t} - \frac{ga + g(\sin \theta - b)}{\bar{v}} = 0 \quad (2)$$

Any distance, x , may be represented by an average speed times time, $\bar{v}t$; therefore, Equation 2 becomes:

$$\frac{v_o - v}{x} - \frac{ga + g(\sin \theta - b)}{\bar{v}} = 0 \quad (3)$$

Solving for x and substituting $\frac{v_o + v}{2}$ for \bar{v} , Equation 3 may be written:

$$x = \frac{1}{g} \frac{v_o^2 - v^2}{a(v_o + v) - 2(\sin \theta - b)} \quad (4)$$

Appendix C

An Analysis of 4-Axle Truck Accident Involvement Rates on Grades

This Appendix presents an analysis of accident involvement rates to ascertain whether the 15-mph design criterion is adequate for determining the critical length of grade. In a report for the Department of Commerce by Solomon (15), accident involvement rates were related to average running speeds of vehicles on a highway. It was concluded that, regardless of the average speed on a highway, the greater a vehicle's deviation from this average running speed of all traffic, the greater its chance of being involved in an accident. The involvement rates as they relate to the deviation from the average running speed of all traffic along the highway are shown in Figure 15 of this report.

Each year the Texas Highway Department's Planning Survey Division reports the speed distribution of all vehicles traveling on the highways in Texas. This survey is made by recording the actual speed of vehicles at 31 strategically located speed survey stations across the state. In 1968, the speeds of 48,253 vehicles were checked, 35,776 of which were passenger cars and 3,284 were 4-or-more-axle trucks.

The following assumptions were made to facilitate the analysis of accident involvement rates:

1. The statewide average speed determined by the Texas Highway Department was assumed to be the typical average speed of all vehicles operating on level grades along a highway.
2. The statewide speed distribution for 4-or-more-axle trucks determined by the Texas Highway Department was assumed to be the typical speed distribution for this type of truck operating on level grades along a highway.

3. The involvement rates were assumed to be those determined by the daytime graph of involvement rates versus deviation from the average speed (See Figure 15). The daytime graph was employed because it represented the lowest involvement rates and is considered to be conservative.
4. All 4-or-more-axle trucks were assumed to decelerate in the manner shown in Table C-1.
5. The average speed reduction of all vehicles on a grade was assumed to be thirty percent of the average truck speed reduction on that same grade.

The following procedure was used to determine the accident

involvement rates on grades:

1. The average speed of all vehicles on level grades and the speed distribution categories were obtained from the data reported by the Texas Highway Department.
2. The mid-point of each speed category was subtracted from the average speed of all vehicles to determine the difference from the average speed.
3. The deviation in speed from the average for each category was used to determine the involvement rate for that category from the daytime graph of involvement rates versus speed variation (See Figure 15).
4. This involvement rate for each category was multiplied by the percentage of 4-or-more-axle trucks within each speed category to obtain the weighted involvement rate.
5. All weighted rates were totaled and divided by 100.
6. The same procedure was followed, with one exception, to determine the involvement rates on grades which would cause a truck speed reduction of 5, 10, 15, and 20 mph. The average speed on the grade was established by subtracting 30 percent of the truck speed reduction from the average speed of all vehicles on level grades. All other steps, 2-5, were exactly the same. The calculated accident involvement rates are presented in Tables C-2 through C-6.

TABLE C-1

Assumed Speed Reduction of 4-Axle Trucks According To
Speed Categories For Various Speed Reductions of the Design Truck

Truck Speed Categories, mph	Speed Reduction of Design Truck, mph*				
	0	5	10	15	20
30-35	0	8	13	18	23
35-40	0	7	12	17	22
40-45	0	6	11	16	21
45-50	0*	5*	10*	15*	20*
50-55	0	4	8	12	16
55-60	0	3	6	9	12
60-65	0	2	4	6	8
65-70	0	1	2	3	4
70-75	0	0	0	0	0
Average Speed of All Traffic, mph	59.4	57.9**	56.4**	54.9**	53.4**

* Design truck operates within the 45-50 mph category.

** Assumed average speed of all traffic on grades is calculated by subtracting 30 percent of the design truck speed reduction from the average speed, 59.4 mph, of all vehicles on level grades.

TABLE C-2

Involvement Rate of 4-Axle Trucks on Level Grades

1*	2*	3*	4*	5*	6*	7*
Average Speed	Truck Speed Categories	Mid Point	Difference From Average	Percent Of Total 4-Axle Trucks	Involvement Rate	Product 5x6
59.4	30-34.9	32.5	- 26.9	0.9	2270	2493
	35-39.9	37.5	- 21.9	3.9	1080	4212
	40-44.9	42.5	- 16.9	6.1	480	2928
	45-49.9	47.5	- 11.9	18.3	270	4941
	50-54.9	52.5	- 6.9	19.8	180	3564
	55-59.9	57.5	- 1.9	37.4	135	5049
	60-64.9	62.5	+ 3.1	10.0	110	1100
	65-69.9	67.5	+ 8.1	3.4	118	401
	70-75.0	72.5	+ 13.1	0.2	148	30
				100.0		24,718
				Involvement Rate = $\frac{24,718}{100} = 247$		

1* 1968 average speed of all vehicles on highways in Texas; obtained from the Texas Highway Department's Planning Survey Division

2* Truck speed categories as established by the THD's Planning Survey Division

3* Midpoint of each truck speed category

4* Difference of the average truck speed from the average speed, 1 minus 3

5* Percentage of total 4-axle trucks in each speed category as determined by the THD's Planning Survey Division

6* Involvement rate taken from Figure 15

7* Product of the percentage of total 4-axle trucks and the involvement rate for the speed differential for each speed category, 5 times 6

TABLE C-3

Involvement Rate Of 4-Axle Trucks With An Assumed Speed Reduction On Grades Of 5 mph Below The Speed On Level Grades

1* Average Speed	2* Truck Speed Categories	3* Mid Point	4* Difference From Average	5* Percent Of Total 4-Axle Trucks	6* Involvement Rate	7* Product 5x6
57.9	22-27	24.5	- 33.4	0.9	13,000	11,700
	28-33	30.5	- 27.4	3.9	3100	12,090
	34-39	36.5	- 21.4	6.1	950	5795
	40-45	42.5	- 15.4	18.3	400	7320
	46-51	48.5	- 9.4	19.8	215	4257
	52-57	54.5	- 3.4	37.4	145	5423
	58-63	60.5	+ 2.6	10.0	110	1110
	64-69	66.5	+ 8.6	3.4	120	408
	70-75	72.5	+ 14.6	0.2	160	32
				100.0		48,135
Involvement Rate = $\frac{48,135}{100} = 481$						

1* Average speed of all vehicles on level grades less 30% of assumed reduction in truck speed on grades; $59.4 - (.3)(5) = 57.9$

2* Truck speed categories determined by subtracting the assumed truck speed reduction found in Table C-2 from the speed categories established by the THD's Planning Survey Division

3* Midpoint of each truck speed category

4* Difference of the average truck speed from the average speed, 1 minus 3

5* Percentage of total 4-axle trucks in each speed category as determined by the THD's Planning Survey Division

6* Involvement rate taken from Figure 15

7* Product of the percentage of total 4-axle trucks and the involvement rate for the speed differential for each speed category, 5 times 6

TABLE C-4

Involvement Rate of 4-Axle Trucks With An Assumed Speed Reduction On Grades Of 10 mph Below The Speed On Level Grades

1*	2*	3*	4*	5*	6*	7*
Average Speed	Truck Speed Categories	Mid Point	Difference From Average	Percent Of Total 4-Axle Trucks	Involvement Rate	Product 5x6
56.4	17-22	19.5	- 36.9	0.9	32,000	28,800
	23-28	25.5	- 30.9	3.9	6800	26,520
	29-34	31.5	- 24.9	6.1	1850	11,285
	35-40	37.5	- 18.9	18.3	640	11,712
	42-47	44.5	- 11.9	19.8	270	5346
	49-54	51.5	- 4.9	37.4	160	5984
	56-61	58.5	+ 2.1	10.0	115	1150
	63-68	65.5	+ 10.1	3.4	125	425
	70-75	72.5	+ 16.1	0.2	180	36
				100.0		91,258
Involvement Rate = $\frac{91,258}{100} = 91.3$						

1* Average speed of all vehicles on level grades less 30% of assumed reduction in truck speed on grades; $59.4 - (.3)(10) = 56.4$

2* Truck speed categories determined by subtracting the assumed truck speed reduction found in Table C-2 from the speed categories established by the THD's Planning Survey Division

3* The mid-point of each truck category

4* Difference of truck speed from the average speed, 1 minus 3

5* Percentage of total 4-axle trucks in each speed category as determined by the THD's Planning Survey Division

6* Involvement rate taken from Figure 15

7* Product of the percentage of total 4-axle trucks and the involvement rate for the speed differential for each speed category, 5 times 6

TABLE C-5

Involvement Rate Of 4-Axle Trucks With An Assumed Speed Reduction On Grades Of 15 mph Below The Speed On Level Grades

1* Average Speed	2* Truck Speed Categories	3* Mid Point	4* Difference From Average	5* Percent Of Total 4-Axle Trucks	6* Involvement Rate	7* Product 5x6
54.9	12-17	14.5	- 40.4	0.9	100,000	90,000
	18-23	20.5	- 34.4	3.9	17,000	66,300
	24-29	26.5	- 28.4	6.1	3700	22,570
	30-35	32.5	- 22.4	18.3	1180	21,594
	38-43	40.5	- 14.4	19.8	350	6,930
	46-51	48.5	- 6.4	37.4	175	6,545
	54-59	56.5	+ 1.6	10.0	118	1,180
	62-67	64.5	+ 9.6	3.4	123	4,182
	70-75	72.5	+ 17.6	0.2	200	40
				100.0		219,341
Involvement Rate = $\frac{219,341}{100} = 2193$						

1* Average speed of all vehicles on level grades less 30% of assumed reduction in truck speed on grades; $59.4 - (.3)(15) = 54.9$

2* Truck speed categories as determined by subtracting the assumed truck speed reduction found in table C-2 for the speed categories established by the TED's Planning Survey Division

3* The mid-point of each truck speed category

4* Difference of the average truck speed from the average speed, 1 minus 3

5* Percentage of total 4-axle trucks in each speed category as determined by the TED's Planning Survey Division

6* Involvement rate taken from Figure 15

7* Product of the percentage of total 4-axle trucks and the involvement rate for the speed differential for each speed category, 5 times 6

TABLE C-6

Involvement Rate Of 4-Axle Trucks With An Assumed Speed Reduction On Grades Of 20 mph Below The Speed On Level Grades

1* Average Speed	2* Truck Speed Categories	3* Mid Point	4* Difference From Average	5* Percent Of Total 4-Axle Trucks	6* Involvement Rate	7* Product 5x6
53.4	7-12	9.5	- 43.9	0.9	100,000	90,000
	13-18	15.5	- 37.9	3.9	46,000	179,400
	19-24	21.5	- 31.9	6.1	8,500	51,850
	25-30	27.5	- 25.9	18.3	2,350	43,005
	34-39	36.5	- 16.9	19.8	480	9,504
	43-48	45.5	- 7.9	37.4	190	7,106
	52-57	54.5	+ 1.1	10.1	120	1,200
	61-66	63.5	+ 10.1	3.4	128	435
	70-75	72.5	+ 19.1	0.2	230	46
					100.0	382,546
Involvement Rate = $\frac{382,546}{100} = 3825$						

1* Average speed of all vehicles on level grades less 30% of assumed reduction in truck speed on grades; $59.4 - (.3)(20) = 53.4$

2* Truck speed categories as determined by subtracting the assumed truck speed reduction found in Table C-2 from the speed categories established by the THD's Planning Survey Division

3* The mid-point of each truck speed category

4* Difference of the average truck speed from the average speed, 1 minus 3

5* Percentage of total 4-axle trucks in each speed category as determined by the THD's Planning Survey Division

6* Involvement rate taken from Figure 15

7* Product of the percentage of total 4-axle trucks and the involvement rate for the speed differential for each speed category, 5 times 6